This are my lecture notes

Title of the lecture, ST 2024

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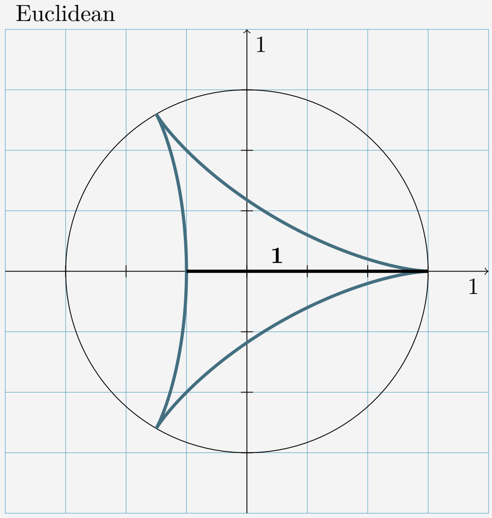
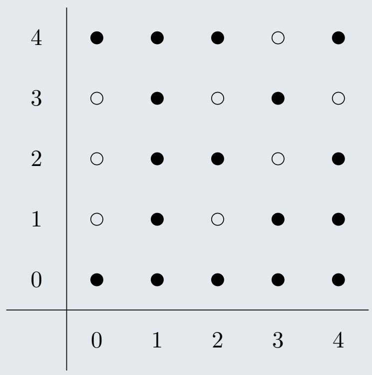
... Kakeya problem

Euclidean



Finite



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Contents

1 Preliminaries

All sorts of things are explained and introduced here. One can also put stuff in fancy boxes. A tasty example:

Definition 1: Group action

Let G be a group and X a set. A mapping $\alpha : G \times X \to X$ that satisfies

(A1) $\alpha(e, x) = x$,

(A2) $\alpha(g, \alpha(h, x)) = \alpha(gh, x)$

for all g, $h \in G$ and $x \in X$ is called a *group action* of G on X.

Remark 2. It is very common that one replaces α with a dot. Then the two above axioms read as $e \cdot x = x$ and $g \cdot (h \cdot x) = (gh) \cdot x$. $\diamond$

2 Digression on finite Kakeya sets

Example 3 (Kakeya set in $\mathbb{F}_3^2$)
We consider the finite field $\mathbb{F}_3 = \mathbb{Z}/3\mathbb{Z} = \{0+3\mathbb{Z},\ 1+3\mathbb{Z},\ 2+3\mathbb{Z}\}$. For the sake of readability, we will denote the representative for each residue class in the following discussion. Our goal is to find a non-trivial Kakeya set, denoted as $K \subseteq \mathbb{F}_3^2$, which contains a line in each direction. The directions we are considering are as follows: $[(1,0)] = \{(1,0),\ (2,0)\}$, $[(0,1)] = \{(0,1),\ (0,2)\}$, $[(1,1)] = \{(1,1),\ (2,2)\}$, and $[(1,-1)] = \{(1,2),\ (2,1)\}$ (note: $(1,-1) = (1,2)$). As an example, we select the following lines:

$$\begin{aligned} \mathcal{L}_{(1,0),\,(0,0)} &= \{(0,0) + t(1,0) : t \in \{0,\ 1,\ 2\}\} = \{(0,0),\ (1,0),\ (2,0)\}, \\ \mathcal{L}_{(0,1),\,(2,0)} &= \{(2,0),\ (2,1),\ (2,2)\}, \quad \mathcal{L}_{(1,1),\,(0,0)} = \{(0,0),\ (1,1),\ (2,2)\} \\ \mathcal{L}_{(1,-1),\,(1,2)} &= \{(1,2),\ (2,1),\ (0,0)\}, \end{aligned}$$

resulting in the Kakeya set $K \subseteq \mathbb{F}_3^2$:

$$\begin{aligned} K &\coloneqq \mathcal{L}_{(1,0),\,(0,0)} \cup \mathcal{L}_{(0,1),\,(2,0)} \cup \mathcal{L}_{(1,1),\,(0,0)} \cup \mathcal{L}_{(1,-1),\,(1,2)} \\ &= \{(0,0),\ (1,0),\ (1,1),\ (1,2),\ (2,0),\ (2,1),\ (2,2)\}. \end{aligned}$$

$\triangle$

3 Improvements?

Feel free to adapt/polish this template in any way you like. I am happy to discuss ideas and suggestions for general improvement of this template!

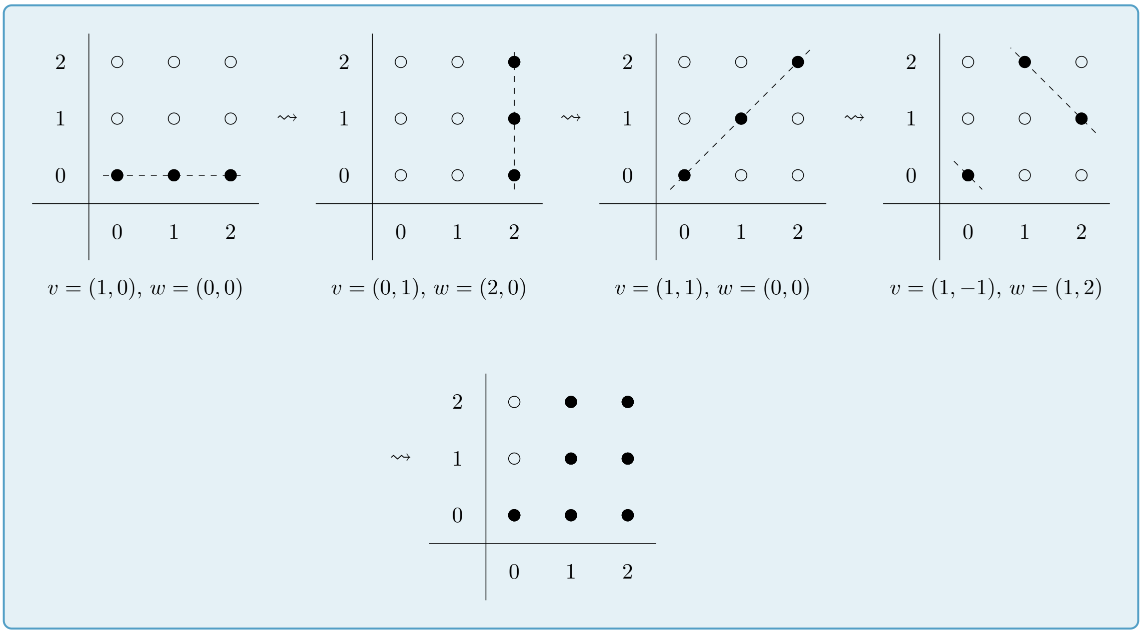

Figure 1: Kakeya set in $\mathbb{F}_3^2$ of the Example 3

References

[1] T.W. Hungerford. *Algebra*. Graduate Texts in Mathematics. Springer New York, 2003. ISBN: 9780387905181.