Loading Service - Actuarial Documentation

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Contents

1 What has changed?

1.1 June 2010

- Introduction of new RDS parameter allowing to split cost (as in Life and Health) into fixed and variable cost. According section added with explanation what to store into RunOff and Acquisition cost.
- Bug-Fix: Regulatory and Rating Agency capital cost had a copy-paste error in them

1.2 April, Mai 2010

The following changes were included:

- More input arguments (cash flows for premium, claims)
- Cash-flow based calculation
- Changed premium discounting
- Changed loss discounting
- Changed capital cost calculation
- Changed output object (cash flow streams for relevant quantities)
- Nat Cat price guidance separately defined for treaty and fac and thus to be fetched like that from RDS

1.3 August 2009

The following changes were included:

- Nat cat price guidance (see Jira 196, 220, 286, 305)
- CMR (see Jira 196)
- Loading service light (see Jira 115, not described in this document)

1.4 August 2008

The main changes in the loading service compared to the previous version are:

- The capital cost have to be deliverred in much more detail (6 components instead of 3)
- The capital cost, economic profit and economic return have to be calculated not only pre tax (as they are now) but also post tax
- New attributes such as the EVM capital, double taxation and taxes have to be calculated.
- EROC is used instead of ROE.

2 In which order has the calculation to be done?

In general the order of the calculation is the same as the order in this document thus the order of calculation is

- Nominal Expected loss
- Average Settlement time and Average Interest rate
- Discounted expected loss and loss discount factor
- Reserves
- Scenario Shortfall and Risk measure
- Capitals (breakdown)
- Capital Costs pre and post tax (breakdown)
- Economic return and profit (breakdown)
- Taxes and double taxation (breakdown)
- internal expenses (breakdown)
- All values of Accept level
- All values for the realized price level which depend on the realized premium

3 General Structure

The Loading Service reflects the general structure of a Swiss Re reinsurance contract, which consists of three hierarchical levels:

- The pricing (or *program*) level, where the general contract attributes are established.
- The acceptance level: Each pricing is a set of acceptances that are characterized by a price and a Swiss Re share.
- The breakdown (or *section*) level: Each acceptance further consists of a collection of N breakdowns, which we shall label by an index $i = 1, \dots, N$.

Remark: Although the Loading Service takes both the *structured* and *unstructured* nominal expected loss as input, it does not currently use the latter value.

4 Input Values

There are mandatory and optional input values.

4.1 Input Values at Pricing Level

4.1.1 Mandatory Input Values at Pricing Level

Description	Java Identifier	Java Type	Symbol
Market	market	*string*	None
Business Unit	businessUnit	*string*	None
Country	exposedCountry	*string*	None
Carrier	carrier	*string*	None
Currency	currency	*string*	None
Reference Date	referenceDataPer	*date*	None
Type of Business	typeOfBusiness	*string*	None
Type of Treaty	typeOfTreaty	*string*	None
Type of Agreement	typeOfAgreement	*string*	None

4.1.2 Optional Input Values at Pricing Level

Description	Java Identifier	Java Type	Symbol
Type of Interest Rates	typeOfInterestRates	*string*	None

4.2 Input Values at Acceptance Level

4.2.1 Mandatory Input Values at Acceptance Level

Description	Java Identifier	Java Type	Symbol
Limit	limit [0..1]	*double*	C
Deductible	deductible	*double*	D
Effective Share	effectiveShare	*double*	ζ
Deductions	deductionsPct	*double*	e_d
Commission	commissionPct	*double*	e_c
Realised Premium Nominal	premiumNomReal	*double*	$P_{real}^{\mathsf{INPUT}}$

Please note that the *Realised Premium Nominal Pattern* is of the format *double[51]*; this implies that a premium pattern is expected as input. The values of this pattern range from 0 to 1 and increas monotonously. Client applications are advised to stop filling this array of numbers once the final value of 1 has been reached and to pass NULL for subsequent years. The same is true for all subsequent items of the format *double[51]*.

4.2.2 Optional Input Values at Acceptance Level

Description	Java Identifier	Java Type	Symbol
Overall Shortfall	overallShortfall	*double*	$SFOverall^{INPUT}$
Internal Cost Multiplier	internalCostMultiplier	*double*	λ_A
Overall Shortfall Pattern	overallShortfallPat	*double[51]*	$SFOverall_y^{pat}$
Average Premium Payment Time	averagePaymentTime	*double*	$\tau^{\mathsf{Prem,\ INPUT}}$
Realised Premium Pattern	premiumNomRealPat	*double[51]*	$p_y^{\mathsf{Prem,\ INPUT}}$

4.2.3 Decommissioned Loading Service 1.0 Input Values

Description	Java Identifier	Java Type	Symbol
Realised Premium Discounted	premiumPvReal [0..1]	*double*	P_{disc}
Premium Discount Rate	premiumDiscRate	*double*	$d(1)_M$

4.3 Input Values at Breakdown Level

4.3.1 Mandatory Input Values at Breakdown Level

Description	Java Identifier	Java Type	Symbol
Line of Business	lineOfBusiness	*string*	None
(Nat Cat) Scenario	scenario	*string*	None
Structured Nominal Expected Loss	structuredNominalExpectedLoss	*double*	$L_{N,i}$

4.3.2 Optional Input Values at Breakdown Level

Description	Java Identifier	Java Type	Symbol
Industry Segment	industrySegment	*string*	None
Scenario (Nat Cat) Shortfall	scenarioShortfall	*double*	$SF_i^{\mathsf{Scen,INPUT}}$
Event Set Based Shortall	ESBShortfallPart	*double*	SF_i^{ESB}
Exposed Sublimit	exposedSublimit	*double*	SL_i
Number of risks	numberOfRisks	*double*	NR_i
Structured Nominal EL Pattern	structuredNominalExpectedLossPat	*double[51]*	$p_{L,i,y}^{\mathsf{INPUT}}$
Average Settlement Time	averageSettlementTime	*double*	τ_i^{INPUT}
Type of Exposure	typeOfExposure	*string*	None
Exposure	exposure	*double*	Exp_i
Exposure Pattern	exposurePat	*double[51]*	$p_{i,y}^{Exp}$
Event Set Based Shortfall Pattern	ESBShortfallPat	*double[51]*	$p_{i,y}^{SF,ESB,INPUT}$
Conditional Shortfall Pattern	conditionalShortfallPat	*double[51]*	$p_{i,y}^{Scenario,INPUT}$

4.3.3 Decommissioned Loading Service 1.0 Input Values

Description	Java Identifier	Java Type	Symbol
Structured Discounted Expected Loss	discountedStructuredLoss [0..1]	*double*	$L_{D,i}^{INPUT}$
Unstructured Nominal Expected Loss	unstructuredNominalExpectedLoss	*double*	$L_{N,i}^{un}$

5 Output Values

The output structure consists of two sets of values. The first are general values, whereas the second set is provided for each individual price level: *Underwriting Cost*, *Production Cost*, *Cycle Reference*, *Renewal Target* and *Realised*.

Note: Not all 'price level specific' values do actually depend on the price level from a mathematical point of view.

5.1 Output Values at Acceptance Level

General values:

Description	java identifier	java type	symbol
Proposed Overall Shortfall	proposedOverallShortfall	*double*	SF^{PROP}
Applied Overall Shortfall	appliedOverallShortfall	*double*	$SF^{Applied}$
Applied Overall Shortfall Nominal	appliedOverallShortfallNominal	*double*	$SF^{AppliedN}$
Applied Overall Shortfall Pattern	appliedOverallShortfallPat	*double[51]*	$SF_y^{Applied,pat}$
Nat Cat Price Guidance	NCPG	*double*	$NCPG$

Price Level specific values:

Description	java identifier	java type	symbol
Discounted Gross Premium	grossPremium	*double*	P_D
Discounted Net Premium	netPremium	*double*	P_D^{net}
Discounted Expected Loss	expectedLossDiscounted	*double*	L_D
Basic Taxes	basicTaxes	*double*	T
Underwriting Taxes	underwritingTaxes	*double*	T_{UW}
Underwriting Taxes Nominal	underwritingTaxesNominal	*double*	T_{UWN}
Underwriting Taxes Pattern	underwritingTaxesPat	*double[51]*	$T_{UW,y}^{pat}$
Replicating Portfolio Taxes	replicatingPortfolioTaxes	*double*	T_{RP}
Replicating Portfolio Taxes Nominal	replicatingPortfolioTaxesNominal	*double*	T_{RPN}
Replicating Portfolio Taxes Pattern	replicatingPortfolioTaxesPat	*double[51]*	$T_{RP,y}^{pat}$
External Expenses	externalExpenses	*double*	E
External Expenses Nominal	externalExpensesNominal	*double*	E_N
External Expenses Pattern	externalExpensesPat	*double[51]*	E_y^{pat}
Internal Expenses Run-off	internalExpensesRunoff	*double*	I_{runoff}
Internal Expenses Run-off Nominal	internalExpensesRunoffNominal	*double*	$I_{runoffN}$
Internal Expenses Run-off Pattern	internalExpensesRunoffPat	*double[51]*	$I_{runoff,y}^{pat}$
Internal Expenses Acquisition	internalExpensesAcquisition	*double*	I_{acq}
Internal Expenses Acquisition Nominal	internalExpensesAcquisitionNominal	*double*	I_{acq_N}
Internal Expenses Acquisition Pattern	internalExpensesAcquisitionPat	*double[51]*	$I_{acq,y}^{pat}$
Capital Cost, Scenario Part after tax	capitalCostScenarioPartPost	*double*	C^{Scen}
Capital Cost, Volume Part after tax	capitalCostVolumePartPost	*double*	C^{VOL}
Capital Cost, Overall Part after tax	capitalCostOverallPost	*double*	C^{OVR}
Economic Profit	economicProfit	*double*	EP
Economic Return on Capital	EROC	*double*	$EROC$
CMR	CMR	*double*	CMR
CMRoC	CMRoC	*double*	$CMRoC$

5.1.1 Decommissioned Loading Service 1.0 Output Values

Description	Java Identifier	Java Type	Symbol
Upfront Premium	upfrontPremium	*double*	$P_{upfront}$
Return on equity	returnOnEquity	*double*	ROE
Capital Cost, Scenario Part before tax	capitalCostScenarioPartPre	*double*	$\tilde{C}^{Scen}$
Capital Cost, Volume Part before tax	capitalCostVolumePartPre	*double*	$\tilde{C}^{VOL}$
Capital Cost, Overall Part before tax	capitalCostOverallPre	*double*	$\tilde{C}^{OVR}$

5.2 Output Values at Breakdown Level

General values:

Description	Java Identifier	Java Type	Symbol
Average Settlement Time	averageSettlementTime	*double*	τ_i
Proposed Scenario Shortfall	proposedShortfall	*double*	$SF_i^{Scenario,PROP}$
Applied Scenario Shortfall	appliedScenarioShortfall	*double*	$SF^{Scenario,Applied}$
Applied Scenario Shortfall Nominal	appliedScenarioShortfallNominal	*double*	$SF^{Scenario,AppliedN}$
Applied Scenario Shortfall Pattern	appliedScenarioShortfallPat	*double[51]*	$SF_y^{Scenario,Applied,pat}$
Average Interest Rate	interestRate	*double*	d_i
Loss Discount Factor	lossDiscountFactor	*double*	$f_{d,i}$
Nominal Reserves-Time, Nominal	NRT_N	*double*	$NRT_{N,i}$
Nominal Reserves-Time, Discounted	NRT_D	*double*	$NRT_{D,i}$
Nominal Reserves-Time Pattern	NRTPat	*double[51]*	$NRT_{i,y}^{pat}$
Discounted Reserves-Time, Nominal	DRT_N	*double*	$DRT_{N,i}$
Discounted Reserves-Time, Discounted	DRT_D	*double*	$DRT_{D,i}$
Discounted Rserves-Time Pattern	DRTPat	*double[51]*	$DRT_{i,y}^{pat}$
Nat Cat Price Guidance Premium	NCPG	*double*	$NCPG$

5.2.1 Decommissioned Loading Service 1.0 Output Values

Description	Java Identifier	Java Type	Symbol
Nominal Expected Loss	nominalExpectedLoss	*double*	$L_{N,i}$

Price Level specific values:

Description	Java Identifier	Java Type	Symbol
Internal Expenses Run-off	internalExpensesRunoff	*double*	$I_{run-off,i}$
Internal Expenses Acquisition	internalExpensesAcquisition	*double*	$I_{acq,i}$
Discounted Expected Loss	expectedLossDiscounted	*double*	$L_{D,i}$

EVM Capital

Description	Java Identifier	Java Type	Symbol
Regulatory Capital	CapitalRegulatory	*double*	$K_{D,i}^{Reg}$
Regulatory Capital Nominal	CapitalRegulatoryNominal	*double*	$K_{N,i}^{Reg}$
Regulatory Capital Pattern	CapitalRegulatoryPat	*double[51]*	$K_{i,y}^{Reg,pat}$
Rating Agency Capital Volume	CapitalRatingAgencyVolume	*double*	K_{i}^{RaVol}
Rating Agency Capital Volume Nominal	CapitalRatingAgencyVolumeNominal	*double*	$K_{N,i}^{RaVol}$
Rating Agency Capital Volume Pattern	CapitalRatingAgencyVolumePat	*double[51]*	$K_{i,y}^{RaVol,pat}$
Rating Agency Capital Scenario	CapitalRatingAgencyScenario	*double*	K_{i}^{RaScen}
Rating Agency Capital Scenario Nominal	CapitalRatingAgencyScenarioNominal	*double*	$K_{N,i}^{RaScen}$
Rating Agency Capital Scenario Pattern	CapitalRatingAgencyScenarioPat	*double[51]*	$K_{i,y}^{RaScen,pat}$
Risk Capital Overall	CapitalRiskOverall	*double*	$K_{i}^{RiskOvr}$
Risk Capital Overall Nominal	CapitalRiskOverallNominal	*double*	$K_{N,i}^{RiskOvr}$
Risk Capital Overall Pattern	CapitalRiskOverallPat	*double[51]*	$K_{i,y}^{RiskOvr,pat}$
Risk Capital Volume	CapitalRiskVolulme	*double*	$K_{i}^{RiskVol}$
Risk Capital Volume Nominal	CapitalRiskVolulmeNominal	*double*	$K_{N,i}^{RiskVol}$
Risk Capital Volume Pattern	CapitalRiskVolulmePat	*double[51]*	$K_{i,y}^{RiskVol,pat}$
Risk Capital Scenario	CapitalRiskScenario	*double*	$K_{i}^{RiskScen}$
Risk Capital Scenario Nominal	CapitalRiskScenarioNominal	*double*	$K_{N,i}^{RiskScen}$
Risk Capital Scenario Pattern	CapitalRiskScenarioPat	*double[51]*	$K_{i,y}^{RiskScen,pat}$
Total EVM Capital	EVMCapital	*double*	K_{i}^{EVM}
Total EVM Capital Nominal	EVMCapitalNominal	*double*	$K_{N,i}^{EVM}$
Total EVM Capital Pattern	EVMCapitalPat	*double[51]*	$K_{i,y}^{EVM,pat}$

Capital Cost

Description	Java Identifier	Java Type	Symbol
Regulatory Capital Cost after tax	CapitalCostRegulatoryPost	*double*	C_i^{Reg}
Rating Agency Capital Cost Volume after tax	CapitalCostRaVolumePost	*double*	C_i^{RaVol}
Rating Agency Capital Cost Scenario after tax	CapitalCostRaScenarioPost	*double*	C_i^{RaScen}
Risk Capital Cost Overall after tax	CapitalCostRiskOverallPost	*double*	$C_i^{RiskOvr}$
Risk Capital Cost Volume after tax	CapitalCostRiskVolumePost	*double*	$C_i^{RiskVol}$
Risk Capital Scenario after tax	CapitalCostRiskScenarioPost	*double*	$C_i^{RiskScen}$
Capital Cost Scenario Part after tax	capitalCostScenarioPartPost	*double*	C_i^{Scen}
Capital Cost Volume Part after tax	capitalCostVolumePartPost	*double*	C_i^{Vol}
Capital Cost Overall Part after tax	capitalCostOverallPartPost	*double*	C_i^{Ovr}
Capital Cost Total after tax	capitalCostTotalPost	*double*	C_i^{Tot}
Capital Cost Total after tax Nominal	capitalCostTotalPostNominal	*double*	$C_{N,i}^{Tot}$
Capital Cost Total after tax Pattern	capitalCostTotalPostPat	*double[51]*	$C_{i,y}^{Tot,pat}$
Hedging Cost	hedgingCost	*double*	HC_i
Hedging Cost Nominal	hedgingCostNominal	*double*	$HC_{N,i}$
Hedging Cost Pattern	hedgingCostPat	*double[51]*	$HC_{i,y}^{pat}$
Funding Cost	fundingCost	*double*	FC_i
Funding Cost Nominal	fundingCostNominal	*double*	$FC_{N,i}$
funding Cost Pattern	fundingCostPat	*double[51]*	$FC_{i,y}^{pat}$
Other Cost	otherCost	*double*	OC_i
Other Cost Nominal	otherCostNominal	*double*	$OC_{N,i}$

5.2.2 Decommissioned Loading Service 1.0 Output Values

Description	Java Identifier	Java Type	Symbol
Regulatory Capital Cost before tax	CapitalCostRegulatoryPre	*double*	$\tilde{C}_i^{Reg}$
Rating Agency Capital Cost Volume before tax	CapitalCostRaVolulmePre	*double*	$\tilde{C}_i^{RaVol}$
Rating Agency Capital Cost Scenario before tax	CapitalCostRaScenarioPre	*double*	$\tilde{C}_i^{RaScen}$
Risk Capital Cost Overall before tax	CapitalCostRiskOverallPre	*double*	$\tilde{C}_i^{RiskOvr}$
Risk Capital Cost Volume before tax	CapitalCostRiskVolumePre	*double*	$\tilde{C}_i^{RiskVol}$
Risk Capital Scenario before tax	CapitalCostRiskScenarioPre	*double*	$\tilde{C}_i^{RiskScen}$
Capital Cost Scenario Part before tax	capitalCostScenarioPart	*double*	$\tilde{C}_i^{Scen}$
Capital Cost Volume Part before tax	capitalCostVolumePart	*double*	$\tilde{C}_i^{Vol}$
Capital Cost Overall Part before tax	capitalCostOverallPart	*double*	$\tilde{C}_i^{Ovr}$
Capital Cost Total before tax	capitalCostTotal	*double*	$\tilde{C}_i^{Tot}$

Return, Profit and taxes

Description	Java Identifier	Java Type	Symbol
Economic Return after tax	EconomicReturnPost	*double*	R_i
Economic Profit after tax	EconomicReturnPost	*double*	EP_i
Double taxation	DoubleTaxation	*double*	Θ_i
Double taxation Nominal	DoubleTaxationNominal	*double*	$\Theta_{N,i}$
Double taxation Pattern	DoubleTaxationPat	*double[51]*	$\Theta_{i,y}^{pat}$
Replicating Portfolio Taxes	replicatingPortfolioTaxes	*double*	T_i^{RP}
Underwriting Taxes	underwritingTaxes	*double*	T_i^{UW}
Basic Taxes	BasicTaxes	*double*	T_i

5.2.3 Decommissioned Loading Service 1.0 Output Values

Description	Java Identifier	Java Type	Symbol
Economic Return before tax	EconomicReturnPre	*double*	$\tilde{R}_i$
Economic Profit before tax	EconomicProfitPre	*double*	$\tilde{EP}_i$

6 Calculations for the four Reference Price Levels

The following formulas are implemented for the four reference price levels: *Underwriting Cost*, *Production Cost*, *Cycle Reference*, *Renewal Target.*

6.1 Calculation prerequisites

All quantities relative to a section $i = 1, \ldots, N$ have i as subscript. N counts the number of acceptances.

Nominal Expected Loss:

Set

$$L_{N,i} := \begin{cases} L_{N,i}^{INPUT}, & L_{N,i}^{INPUT} \neq 0, \\ 10^{-8}, & L_{N,i}^{INPUT} = 0. \end{cases} \tag{1}$$

Interest Rate Curve

We assume that an interest rate curve for the specified currency for the contract has been fetched from RDS. It's symbol from now on is d_y, $0 \leq y \leq V_{\max}$ where $V_{\max}$ is the maximum size of a vector in the implementation we look at (usually 50).

Loss pattern

From now on, we will assume that all patterns are denoted with a development year index y and range from $0 \ldots V_{\max}$ with $V_{\max} = 50$ unless specified otherwise. The pattern for the expected loss on breakdown i will be called $p_{L,i,y}$. We either take it from the input or as a default from RDS. For that purpose, we fetch (with Country, LoB and Product) the default pattern from RDS and call it $p_{L,i,y}^{\mathsf{RDS}}$.

$$p_{L,i,y}^{\mathsf{RDS}} = \mathsf{getPatternFromRDS}(\mathsf{Country}, \mathsf{LoB}, \mathsf{Product}) \tag{2}$$

As well, we deduct – if $\tau_i^{\mathsf{INPUT}} \neq \texttt{null}$ – a Poisson pattern from the average settlement time given and call it as follows.

$$p_{L,i,y}^{\mathsf{Poi}} = e^{-\tau_i^{\mathsf{INPUT}}} \frac{\left(\tau_i^{\mathsf{INPUT}}\right)^y}{y!} \tag{3}$$

With these quantities, we select the pattern to be used for the expected loss on breakdown i with the following logic. This has to be read as follows: if an input pattern $p_{L,i,y}^{\mathsf{RDS}}$ is given, then take that one. Else, take a poisson-approximated pattern if an average settlement time has been given as an input argument. Finally, if no pattern information has been given at all, use the default pattern from RDS.

$$p_{L,i,y} = \begin{cases} p_{L,i,y}^{\mathsf{INPUT}} & \text{if } p_{L,i,y}^{\mathsf{INPUT}} \neq \texttt{null} \\ p_{L,i,y}^{\mathsf{Poi}} & \text{if } \tau_i^{\mathsf{INPUT}} \neq \texttt{null} \\ p_{L,i,y}^{\mathsf{RDS}} & \text{else} \end{cases} \tag{4}$$

Expected Loss Cash Flow

With that, we define the expected loss cash flow L for section i as

$$L_{i,y} = L_{N,i} \cdot p_{L,i,y}. \tag{5}$$

Risk Measures: Overall, conditional, ESB

There are three types of risk measures which are called overall risk measure, conditional shortfall, ESB (event set based) shortfall. This naming convention is used in the following way. The overall risk measure is one which depends on the result shortfall on an acceptance level (thus, includes also effects as reinstatement premium, ... which are not even modelled here). In order to come up with capital cost on a breakdown level, this risk measure is broken down onto this lower level with the help of nominal loss reserves. Then, there is the conditional shortfall, which is per definition the shortfall of one breakdown under the condition that all the others are kept constant. This is only in use for nat cat, and estimated by various means, in particular with multipliers on the expected loss. The third "type" of shortfall is the ESB shortfall, which is the result of a scenario-based calculation (Multisnap) and exists only for the main scenarios. The main difference between ESB and condition shortfall is that ESB is modelled in greater detail and the result is already the "diversified" contribution to the group shortfall.

Now, as there is not always a means of estimating the overall shortfall by the means of evaluating the tail of a distribution (and getting this in the loading service as an input argument), we do need simple estimations for an overall shortfall in order to allocate a fair amount of capital cost to individual transactions. We call this estimate "proposed overall risk measure" $SF^{\mathsf{Ovr,\ PROP}}$, and its estimation depends on the type of agreement. Let

$$L_N = \sum_i L_{N,i} \tag{6}$$

In the treaty case,

$$SF^{\mathsf{Ovr,\ PROP}} := 100 \cdot L_N. \tag{7}$$

In the facultative case or for large corporate risks,

$$SF^{\mathsf{Ovr,\ PROP}} := k_1 \cdot L_N + k_2 \cdot g(L_N, D, C) \tag{8}$$

with

$$g(L_N, D, C) := \sqrt{L_N\,(D+a)} \cdot \frac{\sqrt{\alpha - 1}}{\frac{\alpha}{2} - 1} \cdot \frac{1 - h^{\frac{\alpha}{2}-1}}{\sqrt{1 - h^{\alpha-1}}}. \tag{9}$$

and

$$h = \frac{D+a}{D+C+a}. \tag{10}$$

Recall that D denotes the deductible and C the cover of the acceptance. The Loading Service throws an exception if either $D < 0$ or $C < 0$. For an infinite cover ($C = \infty$), we set $h = 0$ in (9), which then reduces to

$$g(L_N, D, C) = \sqrt{L_N\,(D+a)} \cdot \frac{\sqrt{\alpha - 1}}{\frac{\alpha}{2} - 1}. \tag{11}$$

The constants α, a, k_1, k_2 are provided by RDS. Their current values are the following:

$$\begin{aligned} \alpha &= 2.1 \\ a &= 120'000 \text{ CHF} \\ k_1 &= 3.8 \\ k_2 &= 0.2 \end{aligned}$$

The overall shortfall measure SF^{Ovr} on an acceptance level is either given as input, or the proposition calculated above is taken.

$$SF^{\mathsf{Ovr}} = \begin{cases} SFOverall^{\text{INPUT}} & \text{if } SFOverall^{\text{INPUT}} \neq \text{null} \\ SF^{Ovr,PROP} & \text{else} \end{cases} \tag{12}$$

Breakdown of overall risk measure and patterns

The attributed risk measure per breakdown i is distributed with the nominal loss reserves

$$SF_i^{\mathsf{Ovr}} = SF^{\mathsf{Ovr}} \cdot \frac{\sum_y \mathrm{RN}_y\,[L_i]}{\sum_{i,y} \mathrm{RN}_y\,[L_i]}. \tag{13}$$

Pattern for the overall Risk measure

The patterns $p_{SF,y}$ for the overall shortfall on an acceptance level is either given as input or we take the nominal loss reserves pattern.

$$p_{SF,y} = \begin{cases} p_y^{SF,Ovr,\mathrm{INPUT}} & \text{if } p_y^{SF,Ovr,\mathrm{INPUT}} \neq \text{null} \\ \frac{\sum_i \mathrm{RN}_y[L_i]}{\sum_{i,y} \mathrm{RN}_y[L_i]} & \text{else} \end{cases} \tag{14}$$

Conditional (scenario) shortfall

The proposed nominal amount of the proposed conditional (scenario-specific) shortfall is

$$SF_i^{Scenario,PROP} := \min\left\{C, K_i \cdot L_{N,i}\right\}. \tag{15}$$

This leads to the conditional shortfall of

$$SF_i^{\mathrm{scenario}} = \begin{cases} SF_i^{Scenario,INPUT} & \text{if } SF_i^{Scenario,INPUT} \neq \text{null} \\ SF_i^{Scenario,PROP} & \text{else} \end{cases} \tag{16}$$

Fur further use, we define an "immediate pattern" as follows.

$$p_y^{\mathrm{imm}} = \begin{cases} 1 & y = 0 \\ 0 & \text{else} \end{cases} \tag{17}$$

The pattern for the conditional shortfall is

$$p_{i,y}^{SF,CS} = \begin{cases} p_{i,y}^{SF,CS,\mathrm{INPUT}} & \text{if } p_{i,y}^{SF,CS,\mathrm{INPUT}} \neq \text{null} \\ p_y^{\mathrm{imm}} & \text{else} \end{cases} \tag{18}$$

ESB shortfall

The ESB (Event Set based) shortfall $\mathrm{SF}_i^{\mathrm{ESB}}$ is given by

$$SF_i^{\mathrm{ESB}} = \begin{cases} SF_i^{\mathrm{ESB,\ INPUT}} & \text{if } SF_i^{\mathrm{ESB,\ INPUT}} \neq \text{null} \\ 0 & \text{else} \end{cases} \tag{19}$$

The corresponding pattern is

$$p_{i,y}^{SF,ESB} = \begin{cases} p_{i,y}^{SF,ESB,\mathrm{INPUT}} & \text{if } p_{i,y}^{SF,ESB,\mathrm{INPUT}} \neq \text{null} \\ p_y^{\mathrm{imm}} & \text{else} \end{cases} \tag{20}$$

6.1.1 Parameters to fetch from RDS

To calculate the EVM capital the following parameters have to be taken from RDS:

Description	Java Identifier	Java Type	Symbol
Uncertainty Parameter	UncertaintyParameter	*double*	s_C^{uncert}
Solvency premium intensity	SolvencyPremiumIntensity	*double*	s_P
Average discounted loss Ratio	DiscountedLossRatio	*double*	LR_D
Solvency Loss Intensity on Nom	SolvencyLossIntensityNom	*double*	$s_{R,N}$
Regulatory Reserve intensity	RegulatoryReserveIntensity	*double*	w_r
Solvency Loss Intensity on Disc	SolvencyLossIntensityDisc	*double*	$s_{R,D}$
Overall Shortfall intensity	RiskOvrIntensity	*double*	α_0
Scenario Shortfall intensity	ScenarioIntensity	*double*	α
ESB shortfall intensity	ESBIntensity	*double*	λ
Risk reserve intensity	RiskReserveIntensity	*double*	γ
RA Capital intensity applied to the premium	VolumePremiumIntensity	*double*	r_P
RA Capital intensity applied to the nom loss	VolumeLossIntensity	*double*	$r_{R,N}$
RA Capital intensity applied to the pv loss	VolumeLossIntensity	*double*	$r_{R,D}$
Rating Agency Adjustment Factor	RaAdjustmentFactor	*double*	SP^{adj}
Rating Agency Increase factor	RaIncreaseFactor	*double*	SP^{inc}
Rating Agency Capital Charge	RaCapitalCharge	*double*	C_{Ra}
Regulatory Capital Charge	RegCapitalCharge	*double*	C_{Reg}
Risk Capital Charge	RiskCapitalCharge	*double*	C_{Risk}
Tax rate	TaxRate	*double*	ϑ
Regulatory intensity on overall shortfall	RegOvrSFIntensity	*double*	s_{SF}
Regulatory intensity on conditional Cat shortfall	RegCCSFIntensity	*double*	$s_{SF,CS,Cat}$
Regulatory intensity on ESB Cat shortfall	RegESBSFIntensity	*double*	$s_{SF,\mathrm{ESB},\mathrm{Cat}}$
Regulatory intensity on Exposures	RegIntensitySumAtRisk	*double*	$i_{Reg,ToE}$
Rating Agency intensity on overall shortfall	RegOvrSFIntensity	*double*	r_{SF}
Rating Agency intensity on conditional Cat shortfall	RegCCSFIntensity	*double*	$r_{SF,CS,Cat}$
Rating Agency intensity on ESB Cat shortfall	RegESBSFIntensity	*double*	$r_{SF,\mathrm{ESB},\mathrm{Cat}}$
Rating Agency intensity on Exposures	RegIntensitySumAtRisk	*double*	$i_{RA,ToE}$
Part of internal expensed to be considered as fixed	CmrFixPct	*double*	w_{Fix}

The following parameters are needed to calculate the internal expenses.

Description	Java Type	Symbol	
Fixed Cost per Acceptace	FixedCostAccept	*double*	$I^{Fix}_{BU,Prod}$
Shift of Cost int. function	LossShift	*double*	EL_1
Average Loss Size	AverageLossSize	*double*	EL_0
Nonlinear Exponent for Intensity	NonLinFactor	*double*	ρ
Volume intensity: Business Unit Factor	VolumeIntBU	*double*	f^{Vol}_{BU}
Volume intensity: Line of Business Factor	VolumeIntLOB	*double*	g^{Vol}_{LoB}
Volume intensity: Product Factor	VolumeIntProduct	*double*	h^{Vol}_{Prod}
Volume intensity	VolumeInt	*double*	c^{Vol}
Overhead Cost Percentage, before tax	OverheadPctBT	*double*	$\tilde{c}^{OH,Capital}$
Overhead Cost Percentage, after tax	OverheadPctAT	*double*	$c^{OH,Capital}$

For an overview of the tables from which these numbers have to be taken see Appendix A.

6.2 Algorithmics of the actual calculation

6.2.1 EVM capital, first estimate before statutory capital correction

First, we calculate an approximation to EVM capital which will later be corrected by economic reserves we choose to hold based on economic (and not regulatory) reasons. Please note: in the volume components, there is a field called "ToE" which corresponds to "Type of Exposure". A table with the same name exists in RDS, and content-wise as of May 2010, we use it only to take into account an intensity on Sum at Risk. However, in the future, other intensities might follow.

Regulatory Capital Volume part, uncorrected for statutory capital

Note that the regulatory capital here is a vector with index y, $0 \leq y \leq N$. Note as well that we need to calculate the regulatory capital in two iterations. The first iteration will be $K^{Reg,1}_{i,y}$, this is not the final value. The final value will be called again $K^{Reg}_{i,y}$ as before.

$$
\begin{aligned}
K^{RegVol,1}_{i,y} \quad = \quad & s^{uncert}_{C,i}\left(\frac{s_{P,i}}{LR_{D,i}} \cdot L_{i,y} + s_{R,N} \cdot \mathrm{RN}_y\left[L_i\right] + s_{R,D} \cdot RD_y\left[L_i\right]\right. \\
& \left. + \sum_{ToE} i_{Reg,ToE} \cdot \rho^{\mathrm{ToE}}_{i,y}\right) + w_r\left(\mathrm{RN}_y\left[L_i\right] - RD_y\left[L_i\right]\right) \qquad (21)
\end{aligned}
$$

Regulatory Capital Nat Cat part

$$
K^{RegCat}_{i,y} \quad = \quad s^{uncert}_{C,i}\left(s_{SF,CS,Cat} \cdot SF^{\mathrm{Scenario}}_{i,y} + s_{SF,\mathrm{ESB},\mathrm{Cat}} \cdot SF^{ESB}_{i,y}\right) \qquad (22)
$$

Regulatory Capital Overall part

$$
K^{RegOvr}_{i,y} \quad = \quad s^{uncert}_{C,i} \cdot s_{SF} \cdot SF_{i,y} \qquad (23)
$$

Rating Agency Capital Volume part

$$K_{i,y}^{Ra,Vol} = s_{C,i}^{uncert}\left(\frac{r_{P,i}SP_i^{adj}}{LR_{D,i}} \cdot L_{i,y} + r_{R,N} \cdot SP_i^{adj} \cdot \mathrm{RN}_y\left[L_i\right] + r_{R,D} \cdot RD_y\left[L_i\right] + \sum_{ToE} i_{Reg,ToE} \cdot \rho_{i,y}^{\mathrm{ToE}}\right) \tag{24}$$

Rating Agency Overall part

$$K_{i,y}^{Ra,Ovr} = s_{C,i}^{uncert} \cdot r_{SF} \cdot SF_{\rho,i,y} \tag{25}$$

Rating Agency Capital Nat Cat part

$$K_{i,y}^{Ra,Scen} = s_{C,i}^{uncert}\left(r_{SF,CS,Cat} \cdot SP_i^{inc} \cdot SF_{i,y}^{Scenario} + \lambda_{cat,i}SP_i^{inc}\Delta R_y\right) \tag{26}$$

where $\Delta R_y = SF_{i,y}^{ESB} - L_{i,y}$.

Risk Capital Overall part

$$K_{i,y}^{Risk,Overall} = s_{C,i}^{uncert} \cdot \alpha_{0,i} \cdot SF_{i,y} \tag{27}$$

Risk Capital Nat Cat part

$$K_{i,y}^{Risk,scen} = s_{C,i}^{uncert}\left(\alpha_i \cdot SF_{i,y}^{Scenario} + \lambda_{cat,i} \cdot \Delta R_y\right) \tag{28}$$

Risk Capital Volume part

$$K_{i,y}^{Risk,Reserve} = s_{C,i}^{uncert}\left(\frac{i_{Risk,P}}{LR_D} \cdot L_{i,y} + \gamma_i \cdot \mathrm{RN}_y\left[L\right] + i_{Risk,RD} \cdot RD_y\left[L\right] + \sum_{ToE} i_{Risk,Exp}^{ToE} \cdot \rho_{i,y}^{\mathrm{ToE}}\right) \tag{29}$$

Total EVM Capital pre statutory capital correction

The total EVM capital stream is given by the sum of all the components that were calculated:

$$K_{i,y}^{EVM,1} = \frac{\left(c_{Reg}K_{i,y}^{Reg,1} + c_{Ra}K_{i,y}^{Ra} + c_{Risk}K_{i,y}^{Risk}\right)}{\left(c_{Reg} + c_{Ra} + c_{Risk}\right)} \tag{30}$$

6.2.2 Capital Cost pre statutory capital correction

Capital Cost post tax for breakdown i in year y are denoted by $\mathrm{CoC}_{i,y}$. Note that capital cost pre tax is is "legacy" term which might lead to confusion, so we've got it here only for historic reasons (denoted by $\tilde{C}_{i,y}$ if applicable). The capital cost post tax are calculated by applying the EVM intensity $i^{\mathrm{EVM}}_{\mathrm{CoC}}$ to the EVM Capital $K^{EVM}_{i,y}$ on a breakdown level.

For the cost of capital, we need to introduce the weights of Regulatory, Rating Agency, and Risk capital as follows.

$$w_{\mathsf{Ra}} = \frac{c_{Ra}}{c_{Ra} + c_{Risk} + c_{Reg}} \tag{31}$$

$$w_{\mathsf{Reg}} = \frac{c_{Reg}}{c_{Ra} + c_{Risk} + c_{Reg}} \tag{32}$$

$$w_{\mathsf{Risk}} = \frac{c_{Risk}}{c_{Ra} + c_{Risk} + c_{Reg}} \tag{33}$$

Regulatory Capital Cost, Volume part, pre statutory capital correction

$$\mathrm{CoC}^{Reg,1,Vol}_{i,y} = i^{\mathrm{EVM}}_{\mathrm{CoC}} \cdot w_{\mathsf{Reg}} \cdot K^{Reg,1,Vol}_{i,y} \tag{34}$$

Regulatory Capital Cost, overall part

$$\mathrm{CoC}^{Reg,Ovr}_{i,y} = i^{\mathrm{EVM}}_{\mathrm{CoC}} \cdot w_{\mathsf{Reg}} \cdot K^{Reg,Ovr}_{i,y} \tag{35}$$

Regulatory Capital Cost, Scenario part

$$\mathrm{CoC}^{Reg,Cat}_{i,y} = i^{\mathrm{EVM}}_{\mathrm{CoC}} \cdot w_{\mathsf{Reg}} \cdot K^{Reg,Cat}_{i,y} \tag{36}$$

Rating agency Capital Cost volume part

$$\mathrm{CoC}^{Ra,Vol}_{i,y} = i^{\mathrm{EVM}}_{\mathrm{CoC}} \cdot w_{\mathsf{Ra}} \cdot K^{Ra,Vol}_{i,y} \tag{37}$$

Rating agency Capital Cost Overall part

$$\mathrm{CoC}^{Ra,Ovr}_{i,y} = i^{\mathrm{EVM}}_{\mathrm{CoC}} \cdot w_{\mathsf{Ra}} \cdot K^{Ra,Ovr}_{i,y} \tag{38}$$

Rating agency Capital Cost Scenario part

$$\mathrm{CoC}_{i,y}^{RaScen} = i_{\mathrm{CoC}}^{\mathrm{EVM}} \cdot w_{\mathsf{Ra}} \cdot K_{i,y}^{RaScen} \tag{39}$$

Risk Capital Cost Overall

$$\mathrm{CoC}_{i,y}^{RiskOvr} = i_{\mathrm{CoC}}^{\mathrm{EVM}} \cdot w_{\mathsf{Risk}} \cdot K_{i,y}^{RiskOvr} \tag{40}$$

Risk Capital Cost Scenario

$$\mathrm{CoC}_{i,y}^{RiskScen} = i_{\mathrm{CoC}}^{\mathrm{EVM}} \cdot w_{\mathsf{Risk}} \cdot K_{i,y}^{RiskScen} \tag{41}$$

Risk Capital Cost Volume

$$\mathrm{CoC}_{i,y}^{RiskVolume} = i_{\mathrm{CoC}}^{\mathrm{EVM}} \cdot w_{\mathsf{Risk}} \cdot K_{i,y}^{RiskVolume} \tag{42}$$

Capital Cost Overall

$$\mathrm{CoC}_{i,y}^{Ovr} = \mathrm{CoC}_{i,y}^{Risk,Ovr} + \mathrm{CoC}_{i,y}^{Ra,Ovr} + \mathrm{CoC}_{i,y}^{Reg,Ovr} \tag{43}$$

Capital Cost Volume, uncorrected

$$\mathrm{CoC}_{i,y}^{Vol,1} = \mathrm{CoC}_{i,y}^{Risk,Vol} + \mathrm{CoC}_{i,y}^{Reg,1,Vol} + \mathrm{CoC}_{i,y}^{Ra,Vol} \tag{44}$$

Capital Cost Scenario

$$\mathrm{CoC}_{i,y}^{Scen} = \mathrm{CoC}_{i,y}^{Risk,Scen} + \mathrm{CoC}_{i,y}^{Ra,Scen} + \mathrm{CoC}_{i,y}^{Reg,Scen} \tag{45}$$

Total Capital Cost, uncorrected

$$\mathrm{CoC}_{y}^{1} = \mathrm{CoC}_{y}^{Scen} + \mathrm{CoC}_{y}^{Vol,1} + \mathrm{CoC}_{y}^{Ovr} \tag{46}$$

Note: as a double check, this need to be equal to $i_{\mathrm{CoC}}^{\mathrm{EVM}} \cdot K_{i,y}^{EVM,1}$.

6.2.3 Double taxation pre correction

$$\mathrm{DT}_{i,y} = K_{i,y}^{EVM,1} \cdot \vartheta_i \cdot \delta \tag{47}$$

where ϑ_i is the tax rate for breakdown i and δ is the carrier yield.

6.2.4 Market Risk Charge, uncorrected

$$MRC_{i,y}^1 = i_{HC,EL} \cdot L_{i,y} + i_{HC,Risk} \cdot K_{i,y}^{Risk} + i_{HC,RA} \cdot K_{i,y}^{Ra} + i_{HC,Reg} \cdot K_{i,y}^{Reg,1} \tag{48}$$

6.2.5 Internal cost, uncorrected

Note that backwards compatibility has been dropped: pre 2008 calculation methods have beed removed in this version.

Internal Expenses :

For an overview of the tables from which these numbers have to be taken see the corresponding section.

$$
\begin{aligned}
I_{i,y}^{Vol,Sr} &= \lambda_A \cdot \left(\frac{EL_1 + EL_0}{EL_1 + \sum_{i \in \text{Accpt}} L_{D,i}} \right)^{1-\rho} \zeta \sum_i (c^{Vol} \cdot f_{BU}^{Vol} \cdot h_{Prod}^{Vol} \cdot g_{LoB}^{Vol}) \cdot L_{D,i,y} && (49) \\
I_{i,y}^{OH,Sr,1} &= \tilde{c}^{OH,Capital} \zeta \frac{\mathrm{CoC}_{i,y}^1}{1-\theta_i} + c^{OH,Capital} \zeta \mathrm{CoC}_{i,y}^1
\end{aligned}
$$

(51)

$$I_{i,y}^{Fix,Sr} = \begin{cases} \frac{L_{N,i}}{\sum_{i \in \text{Accpt}} L_{N,i}} I_{BU,Prod}^{Fix} & y = 0 \\ 0 & y > 0 \end{cases} \tag{52}$$

The sum of the internal cost is, then, given by

$$I_{i,y}^{Sr,1} = I_{i,y}^{Vol,Sr} + I_{i,y}^{OH,Sr,1} + I_{i,y}^{Fix,Sr}. \tag{53}$$

6.2.6 Correction of capital cost

Now, knowing Internal Cost, Capital Cost and Double Taxation, we correct the Statutory Capital (and thus, the Regulatory Capital such that is newly is

$$
\begin{aligned}
K_{i,y}^{Reg} = \; & s_{C,i}^{uncert} \Bigg(\frac{s_{P,i}}{LR_{D,i}} \cdot L_{i,y} + s_{R,N} \cdot \mathrm{RN}_y [L_i] + s_{R,D} \cdot RD_y [L_i] + s_\rho \cdot SF_{i,y} \\
& + \alpha_i SP_i^{inc} \cdot SF_{i,y}^{\text{Scenario}} + s_{\rho,\text{ESB,Cat}} \cdot SF_{i,y}^{ESB} + \sum_{ToE} i_{Reg,ToE} \cdot \rho_{i,y}^{\text{ToE}} \Bigg) \\
& + w_r \max (0; \mathrm{RN}_y [L_i] - RD_y [L_i] - RD_y \left[I_i^1 + \mathrm{CoC}_i^1 + DT_i^1 + MRC_i^1 \right]
\end{aligned}
$$

$$(55)$$

Note that with this regulatory capital, now all dependent quantities (EVM Capital, capital cost, internal cost, double taxation, market risk charge) have to be recalculated.

Regulatory Capital Cost, Volume part, post statutory capital correction

$$\mathrm{CoC}_{i,y}^{Reg,Vol} = i_{\mathrm{CoC}}^{\mathrm{EVM}} \cdot w_{\mathsf{Reg}} \cdot K_{i,y}^{Reg,Vol} \tag{56}$$

Capital Cost Volume, post statutory capital correction

$$\mathrm{CoC}_{i,y}^{Vol} = \mathrm{CoC}_{i,y}^{Risk,Vol} + \mathrm{CoC}_{i,y}^{Reg,Vol} + \mathrm{CoC}_{i,y}^{Ra,Vol} \tag{57}$$

EVM Capital, post statutory capital correction

$$K_{i,y}^{EVM} = \frac{\left(c_{Reg} K_{i,y}^{Reg} + c_{Ra} K_{i,y}^{Ra} + c_{Risk} K_{i,y}^{Risk}\right)}{(c_{Reg} + c_{Ra} + c_{Risk})} \tag{58}$$

Total Capital Cost, post statutory capital correction

$$\mathrm{CoC}_{y} = \mathrm{CoC}_{y}^{Scen} + \mathrm{CoC}_{y}^{Vol} + \mathrm{CoC}_{y}^{Ovr} \tag{59}$$

6.2.7 Double taxation, post statutory capital correction

$$\mathrm{DT}_{i,y} = K_{i,y}^{EVM} \cdot \vartheta_i \cdot \delta \tag{60}$$

6.2.8 Market Risk Charge, post statutory capital correction

$$MRC_{i,y} = i_{HC,EL} \cdot L_{i,y} + i_{HC,Risk} \cdot K_{i,y}^{Risk} + i_{HC,RA} \cdot K_{i,y}^{Ra} + i_{HC,Reg} \cdot K_{i,y}^{Reg} \tag{61}$$

6.2.9 Internal cost, post statutory capital correction

$$I_{i,y}^{OH,Sr} = \tilde{c}^{OH,Capital} \zeta \frac{\mathrm{CoC}_{i,y}}{1-\vartheta_i} + c^{OH,Capital} \zeta \mathrm{CoC}_{i,y}$$

$$(63)$$

The sum of the internal expense load is, then, given by

$$I_{i,y}^{Sr} = I_{i,y}^{Vol,Sr} + I_{i,y}^{OH,Sr} + I_{i,y}^{Fix,Sr}. \tag{64}$$

6.2.10 Internal cost, split into fixed and variable

The internal cost are split (for marginal contribution purposes, thus for being able to properly define CMR) into fixed and variable cost. We consider the fixed ones to be "there" anyways in the company and the variable ones directly caused by the contract. In order to split them, we use the parameter w_{fix}.

$$I_{i,y}^{Sr,\mathsf{fix}} = w_{\mathsf{fix}} \cdot I_{i,y}^{Sr} \tag{65}$$

$$I_{i,y}^{Sr,\mathsf{var}} = (1 - w_{\mathsf{fix}}) \cdot I_{i,y}^{Sr} \tag{66}$$

6.2.11 Economic Return and economic profit

The economic Profit post tax at breakdown level is

$$EP_{i,y} = p_i \cdot \mathrm{CoC}_{i,y} \tag{67}$$

where the profit factors p_i is determined by the price level:

- At Underwriting cost, $p_{UC} = -1$.
- At Production cost , $p_i = p_{PC} = 0$.
- At Cycle Reference cost, $p_i = p_{CR,i}$
- At Renewal Target cost, $p_i = p_{RT,i}$

Both the Cycle Reference profit factors and the Renewal Target profit factors are provided by RDS. Note that $p_{CR,i}$ is a constant (typically around 1) while $p_{RT,i}$ depends on the Line of Business, the Market and the Product.

The economic return stream post tax at breakdown level is equal to

$$R_{i,y} = \mathrm{CoC}_{i,y} + EP_{i,y} \tag{68}$$

6.3 Tax calculation

A given piece of business has to pay for taxes on the underwriting results as well as the taxes on the investment income generated by the replicating portfolio which is assumed to be invested in risk-free zero coupon bonds which produce exactly the cash flows needed to pay all liabilities generated or allocated to the piece of business we look at.

In this section, we describe the calculation of underwriting tax and the tax on investment income on the replicating portfolio for a reinsurance transaction. The model is based on an "open book" assumption, which recognizes immediately any tax credits. It assumes that the legal entity the business is written on is profitable – otherwise, no taxes on profit at all would have to be charged – and that the tax credit due to a loss generating transaction can be compensated by the taxes on profitable business: this means that the allocated taxes have the same sign as the profit.

6.3.1 Underwriting Tax

Top-down

Define the nominal underwriting result Res^{UW} for the whole contract as

$$\begin{aligned}\text{Res}_y^{\text{UW}} &= \sum_{y=0}^{N} \big\{ P_y - Q_y - L_y - I_y - \text{HC}_y \\ &\quad - \left(\text{RN}_y \left[P - Q - L - I - \text{HC}\right] - \text{RN}_{y-1} \left[P - Q - L - I - \text{HC}\right]\right) \big\}. \end{aligned} \tag{69}$$

Bottom-up

Define the nominal bottom-up underwriting result after tax to be achieved in year y for breakdown i as

$$\text{Res}_{i,y}^{\text{UW}AT} = \text{CoC}_{i,y} + \text{DT}_{i,y} + \text{EP}_{i,y} \tag{70}$$

Now, we assume – for prospective business – an underwriting tax payment pattern which is equal to the premium payment pattern.

$$p_{i,y}^{\text{Tax, UW}} = p_{\text{Accpt},y}^{\text{Prem}} \tag{71}$$

Note that we might use the following components later on.

$$\text{Res}_{i,y}^{\text{UW CoC}} = p_{\text{Accpt},y}^{\text{Prem}} \cdot \sum_y \frac{\text{CoC}_{i,y}}{1 - \vartheta} \tag{72}$$

$$\text{Res}_{i,y}^{\text{UW DT}} = p_{\text{Accpt},y}^{\text{Prem}} \cdot \sum_y \frac{\text{DT}_{i,y}}{1 - \vartheta} \tag{73}$$

$$\text{Res}_{i,y}^{\text{UW EP}} = p_{\text{Accpt},y}^{\text{Prem}} \cdot \sum_y \frac{\text{EP}_{i,y}}{1 - \vartheta} \tag{74}$$

The nominal taxes to be paid on that amount are then, given an effective tax rate in year y of ϑ_y

$$T_{\text{Nom}}^{\text{UW}} = \sum_{y=0}^{N} \vartheta \text{Res}_y^{\text{UW}}. \tag{75}$$

The components of this stream are

$$T_{i,y}^{\text{UW}} = T_{i,y}^{\text{UW,CoC}} + T_{i,y}^{\text{UW,DT}} + T_{i,y}^{\text{UW,EP}} = \vartheta \cdot \left[\text{Res}_{i,y}^{\text{UW CoC}} + \text{Res}_{i,y}^{\text{UW DT}} + \text{Res}_{i,y}^{\text{UW EP}}\right]. \tag{76}$$

The underwriting tax to be covered by the piece of business we're looking at is then given by the net present value of the cash flow T_y^{UW}

$$PV_d\left[T^{\text{UW}}\right] = \sum_{y=0}^{N} \frac{T_y^{\text{UW}}}{(1 + d_y)^y} \tag{77}$$

6.3.2 Tax on investment return on the replicating portfolio

It can easily be seen that the replicating portfolio RP_y in every cash flow year y is given by

$$\mathrm{RP}_y = P_y - Q_y - L_y - I_y - T_y^{\mathrm{UW}} - T_y^{\mathrm{II}} - \mathrm{CoC}_y - \mathrm{DT}_y - \mathrm{HC}_y - \mathrm{FC}_y. \tag{78}$$

In order to interpret RP_y, a positive amount means that we can actually buy zero-coupon bonds for that money, and a negative amount means that we've got to have a zero-coupon bond in that year paying out exactly this amount of money.

When we estimate this quantity bottom-up, then we see that it's equivalent to the economic profit.

$$\mathrm{RP}_{i,y} = \mathrm{EP}_{i,y} \tag{79}$$

Now, the interest earned on the replicating portfolio in year y with a transfer-price of fund interest rate curve d is

$$\mathrm{RP}_y \cdot (1 - \mathrm{DF}_y\,[d]) = \mathrm{RP}_y \cdot \left(1 - \frac{1}{(1-d_y)^y}\right). \tag{80}$$

This amount is taxable, thus the equation to solve is

$$\begin{aligned} T_y^{\mathrm{II}} &= \vartheta \cdot \mathrm{RP}_y \cdot (1 - \mathrm{DF}_y\,[d]) \\ &= \vartheta \cdot \left(\mathrm{RP}_y - T^{\mathrm{II}}\right) \cdot (1 - \mathrm{DF}_y\,[d]) + \vartheta \cdot T^{\mathrm{II}} \cdot (1 - \mathrm{DF}_y\,[d])\,. \end{aligned} \tag{81}$$

One sees that therefore, the bottom-up tax on investment income estimate in year y is given by

$$T_y^{\mathrm{II}} = \frac{\mathrm{RP}_{i,y}\vartheta\,(1 - \mathrm{DF}_y[d])}{1 - \vartheta\,(1 - \mathrm{DF}_y[d])}. \tag{82}$$

6.4 External Expenses

The external expense cash flow Q_i is given by

$$Q = e_{\mathrm{tot}} \cdot P, \tag{83}$$

where

$$e_{\mathrm{tot}} = e_c^{\mathsf{INPUT}} + e_d^{\mathsf{INPUT}} \tag{84}$$

is the sum of commissions and deductions.

Note that the loading service should throw an exception if $e_{\mathrm{tot}} = e_c^{\mathsf{INPUT}} + e_d^{\mathsf{INPUT}} \geq 100\%$. The premium P which is needed for (83) will be defined in the next section.

6.4.1 Gross Premium, bottom-up

The premium we've got to ask for has to cover – in its present value – all cost introduced so far and is therefore defined as follows. Please note that the premium pattern is always taken from the input $p_{P,i,y}^{\mathsf{INPUT}}$.

$$\mathrm{PV}_d\,[P] = \mathrm{PV}_d\left[L + Q + I + \mathrm{CoC} + DT + EP + T^{\mathrm{UW}} + T^{\mathrm{II}} + HC + FC\right] \tag{85}$$

This means that the nominal premium is given by

$$P^{Nom} = \frac{1}{(1-e_{\mathrm{tot}})\sum_y \frac{p_{P,i,y}^{\mathsf{INPUT}}}{(1+d_{i,y})^y}\mathrm{PV}_d\,[L + I + \mathrm{CoC} + DT + EP + T^{\mathrm{UW}} + T^{\mathrm{II}} + HC + FC]}. \tag{86}$$

With that, the external expenses can be calculated as already described above.

6.5 EVM Items, KPIs and ODS-P Output

The EVM items which have to be added up in order to obtain the reference premiums are as follows. All of these quantities are present values of cash flows defined in the sections above.

- Expected Loss L_i is given by (5)
- Internal Expenses I_i are given by (64). Note that the ODS-P fields acquisition cost is to be filled with $I_{i,y}^{Sr,\mathsf{fix}}$ and the ODS-P field runoff cost with $I_{i,y}^{Sr,\mathsf{var}}$.
- Capital Cost CoC_i are given by (59)
- Double Taxation DT_i are given by (60)
- Underwriting Tax T_i^{UW} are given by (77)
- Investment Tax T_i^{II} are given by (82)
- Hedging Cost MRC_i are given by (61)
- Economic Profit EP_i is given by (67)
- Funding Cost (in the future, not yet part of concept)
- Gross Premium
- External Expenses

Note that based on these quantities, we define the following performance indicators for a given contract.

- CMR The CMR on a breakdown level is defined by

$$\mathrm{CMR}_{i,y} = I_{i,y}^{\mathsf{fix}} + \mathrm{EP}_{i,y}^{\mathrm{CR}} + T_{i,y}^{\mathrm{II}} + T_{i,y}^{\mathrm{UW}} - \vartheta \frac{\mathrm{CoC}_{i,y} + \mathrm{DT}_{i,y}}{1 - \vartheta}. \tag{87}$$

- CMRoC

Additional quantities of interest which will be given as reference output quantities are as follows.

- Loss discount factor
- Average payment time
- Average interest rate
- Nominal reserves-time, nominal
- Nominal reserves-time, discounted
- Discounted reserves-time, discounted
- Risk Capital
- Rating Agency Capital
- Regulatory Capital
- EVM Capital

6.5.1 Performance indicators

6.5.2 Additional quantities of interest

Average Settlement Time:

If a pattern has been delivered in the interface, then drop the average settlement time regardless if it's been specified as well or not and set τ_i to

$$\tau_i = \sum_{y=0}^{N} y\, p_{L,i,y}. \tag{88}$$

Note: The Loading Service throws an exception if $\tau_i < 0$.

Loss Discount Factor:

The discounted expected loss $L_{D,i}$ per breakdown i is defined by

$$L_{D,i} := \mathrm{PV}_d\,[L] = \sum_{j=0}^{N} \frac{L_{i,y}}{(1+d_y)^y} \tag{89}$$

and

$$f_{d,i} := L_{D,i}/L_{N,i}. \tag{90}$$

Average Interest Rate:

If $d_{i,M}$ is set in the input, ignore it and replace with the following calculation.

$$d_{i,M} = \frac{\ln\frac{1}{f_{d,i}}}{\ln f_{d,i} + \tau_i} \tag{91}$$

Note that from now on, we will use the cash flow operators from the appendix.

Nominal Reserves-Time, Nominal:

$$NRT_{N,i} := \sum_{y=0}^{N} \mathrm{RN}_y\,[L_i] \tag{92}$$

Nominal Reserves-Time, Discounted:

$$NRT_{D,i} := PV_d\,[\mathrm{RN}_y\,[L_i]] \tag{93}$$

Discounted Reserves-Time, Discounted:

$$DRT_{D,i} := PV_d\,[RD_y\,[L_i]] \tag{94}$$

6.6 Calculations at Acceptance Level

We consider an acceptance with N sections $i = 1, \dots, N$.

Proposed Risk Measure:

The proposed (overall) risk measure ρ^{PROP} is calculated as in (8).

Capital Cost, Scenario Part:

$$\mathrm{CoC}_y^{Scen} = \sum_{i=1}^{N} \mathrm{CoC}_{i,y}^{Scen} \tag{95}$$

where $\tilde{C}_i^{SF}$ and C_i^{SF} are given by (45).

Capital Cost, Volume Part:

$$\mathrm{CoC}_y^{VOL} = \sum_{i=1}^{N} \mathrm{CoC}_{i,y}^{VOL} \tag{96}$$

where CoC_i^{VOL} and CoC_i^{VOL} are given by (57).

Capital Cost, Overall Part:

$$\mathrm{CoC}_y^{OVR} = \sum_{i=1}^{N} \mathrm{CoC}_{i,y}^{OVR} \tag{97}$$

where $\mathrm{CoC}_{i,y}^{OVR}$ and $\mathrm{CoC}_{i,y}^{OVR}$ are given by (43).

Return:

The total economic return before tax $\tilde{R}$ and after tax R are calculated as follows:

$$R_y = \sum_{i=1}^{N} R_{i,y} \tag{98}$$

where the section return $R_{i,y}$ is given by (**??**).

The EROC is defined as

$$EROC := \delta + \frac{\mathrm{CoC} + EP}{K^{EVM}}. \tag{99}$$

Here

$$\mathrm{CoC} = \mathrm{CoC}^{Scen} + \mathrm{CoC}^{VOL} + \mathrm{CoC}^{OVR} = \sum_{i=1}^{N} \mathrm{CoC}_{i,y} \tag{100}$$

$$EP = \sum_{i=1}^{N} EP_i \tag{101}$$

$$K^{EVM} = \sum_{i=1}^{N} K_i^{EVM} \tag{102}$$

are the total capital cost and economic profit at acceptance level and δ is the carrier yield. Note that the EROC depends on the price level.

Upfront premium

The upfront nominal premium is, as there are no reinstatements modelled, equal to the nominal premium already calculated.

7 Calculations for the Realized Price Level

The calculations for the internal cost and for the capital cost are the same for the realized price levels as for the reference price levels. For the premium, economic return, economic profit and EROC there are a few differences which will be discussed in this section.

7.1 Calculations at Acceptance Level

The realized nominal premium is given on an acceptance level by $P_{real}^{\mathsf{INPUT}}$. To determine the premium apttern, we first set

$$\tau^{\mathsf{Prem}} = \begin{cases} \tau^{Prem,INPUT} & \text{if } \tau^{Prem,INPUT} \neq \texttt{null} \\ 0 & else \end{cases}. \tag{103}$$

Then, we derive a Poisson pattern in the following way

$$p_y^{\mathsf{Prem,\ Poi}} = e^{-\tau^{\mathsf{Prem}}} \frac{\left(\tau^{\mathsf{Prem}}\right)^y}{y!}. \tag{104}$$

Finally, we select the premium pattern to be used as follows.

$$p_y^{\mathsf{Prem}} = \begin{cases} p_y^{\mathsf{Prem,INPUT}} & \text{if } p_y^{\mathsf{Prem,INPUT}} \neq \texttt{null} \\ p_y^{\mathsf{Prem,\ Poi}} & \text{else} \end{cases} \tag{105}$$

Thus, the premium cash flow P (with components P_y) is given by

$$P_y^{\mathsf{real}} = p_y^{\mathsf{Prem}} \cdot P_{\mathsf{real}}^{\mathsf{INPUT}}. \tag{106}$$

We get the discounted realized gross premium

$$P_D = \mathrm{PV}_d \left[P^{\mathsf{real}}\right]. \tag{107}$$

The discounted realized net premium is then

$$\begin{aligned} P_D^{\mathsf{net}} &= \mathrm{PV}_d \left[P^{\mathsf{real}} - Q^{\mathsf{real}}\right] && (108) \\ &= \mathrm{PV}_d \left[P^{\mathsf{real}} - e_{tot} \cdot P^{\mathsf{real}}\right] && (109) \end{aligned}$$

where the external expense cash flow is given by

$$Q^{\mathsf{real}} = e_{tot} \cdot P^{\mathsf{real}}. \tag{110}$$

Economic Profit and Economic Return

The *realized* economic profit (CM3, also known as profit after tax) is calulcated by subtracting the costs and the expected loss from the discounted premium P_D:

$$EP_{\mathsf{real},D} = \mathrm{PV}_d \left[P - Q - L - I - \mathrm{CoC} - DT - T^{\mathrm{UW}} - T^{\mathrm{II}} - HC - FC \right]. \tag{111}$$

The (estimated) economic profit before tax is the given by

$$\tilde{EP}_D^{\mathsf{real}} = \frac{1}{1-\vartheta} \cdot \mathrm{PV}_d \left[EP_{\mathsf{real}} \right]. \tag{112}$$

In the same way, we define the realized economic return before tax $\tilde{EP}^{\mathrm{real}}$ as

$$\tilde{EP}^{\mathrm{real}} = \frac{1}{1-\vartheta} \cdot \mathrm{PV}_d \left[EP^{\mathrm{real}} + \mathrm{CoC} + DT \right]. \tag{113}$$

8 Nat Cat Price Guidance

In the first step to calculate the Nat Cat price guidance, the number of risks and the expected loss are summed for each main scenario. Then, the risk rate on line for every main scenario in the pricing is calculated using:

$$RROL = \frac{L_D}{SL} \tag{114}$$

where L_D is the expected loss summed for the specific scenario. The Nat cat price guidance is now the interpolation between the proportional and non-proportional treaty **and facultative** nat cat price guidance factors:

$$NCPG_{factor} = NCPG_{NP} \cdot x + NCPG_{Prop} \cdot (1-x) \tag{115}$$

where $NCPG_{NP}$ is the Nat cat price guidance factor for treaty or facultative non proporional at the given $RROL$, $NCPG_{Prop}$ is the Nat cat price guidance factor for treaty Prop at the given $RROL$ and x is calculated using the following formula:

$$x = \min(0.2572 \cdot \ln(\max(NR; 10)) - 0.592224886; 1) \tag{116}$$

where NR is the number of risks for the specific scenario.

The NCPG premium on breakdown level can now be calculated using:

$$NCPG = NCPG_{factor} \cdot P_{i,CR}. \tag{117}$$

$P_{i,CR}$ is the net premium cycle reference on breakdown level. The same factor is applied to all breakdowns with the same main scenario.

The NCPG Premium on acceptance level is then calculated using:

$$NCPG = \sum_i (NCPG_{factor} \cdot P_{i,CR}) \cdot \frac{P_N}{P_D} \tag{118}$$

A RDS tables

T_LOB_PARAMETER

Column Name	Data Type	Java Identifier
ALPHA_0	FLOAT	RiskOvrIntensity
GAMMA	FLOAT	RiskReserveIntensity
S_P	FLOAT	SolvencyPremiumIntensity
S_R	FLOAT	SolvencyLossIntensity
W_R	FLOAT	RegulatoryReserveIntensity
R_P	FLOAT	VolumePremiumIntensity
R_R	FLOAT	VolumeLossIntensity
LR_D	FLOAT	DiscountedLossRatio
S_R_D	FLOAT	SolvencyLossIntensityDiscounted
POID	NUMBER (9)	
PRODUCT_CODE	VARCHAR2 (50 Byte)	
LOB_CODE	VARCHAR2 (50 Byte)	
CARRIER_CODE	VARCHAR2 (50 Byte)	
SESSION_ID	NUMBER	
STATUS	VARCHAR2 (20 Byte)	
VALID_FROM_DATE	TIMESTAMP(6)	
VALID_UNTIL_DATE	TIMESTAMP(6)	

T_CONSTANT_PARAMETER

Column Name	Data Type	Java Identifier
P_CR_SF	FLOAT	
SURCH_SF	FLOAT	
P_CR_VOL	FLOAT	
P_CR	FLOAT	CRProfitFactor (before tax)
P_CR_ON_PURE_CAC	FLOAT	CRProfitFactor (after tax)
VERSION_NUMBER	FLOAT	
POID	NUMBER (9)	
SESSION_ID	NUMBER	
STATUS	VARCHAR2 (20 Byte)	
VALID_FROM_DATE	TIMESTAMP(6)	
VALID_UNTIL_DATE	TIMESTAMP(6)	
CSGP	FLOAT	ESBIntensity
SP_CAP_ADJ_FACTOR	FLOAT	RaAdjustmentFactor
SP_CAT_INCREASE_FACTOR	FLOAT	RaIncreaseFactor

T_MARKET_PARAMETER

Column Name	Data Type	Java Identifier
POID	NUMBER (9)	
LOB_CODE	VARCHAR2 (50 Byte)	
PRODUCT_CODE	VARCHAR2 (50 Byte)	
MARKET_CODE	VARCHAR2 (50 Byte)	
SESSION_ID	NUMBER	
STATUS	VARCHAR2 (20 Byte)	
VALID_FROM_DATE	TIMESTAMP(6)	
VALID_UNTIL_DATE	TIMESTAMP(6)	
P_RT	FLOAT	RTProfitFactor (before tax)
P_RT_ON_PURE_CAC	FLOAT	RTProfitFactor (after tax)

T_Carrier_PARAMETER

Column Name	Data Type	Java Identifier
C_RA	FLOAT	RaCapitalCharge
C_REG	FLOAT	RegCapitalCharge
C_RAC	FLOAT	RiskCapitalCharge
POID	NUMBER (9)	
CARRIER_CODE	VARCHAR2 (50 Byte)	
SESSION_ID	NUMBER	
STATUS	VARCHAR2 (20 Byte)	
VALID_FROM_DATE	TIMESTAMP(6)	
VALID_UNTIL_DATE	TIMESTAMP(6)	
RISK_FREE	FLOAT	RiskFreeRate

T_Scenario_PARAMETER

Column Name	Data Type	Java Identifier
ALPHA	FLOAT	ScenarioIntensity
K	FLOAT	Scenario Shortfall parameter
SSF_QUANTILE	FLOAT	
SSF_WEIGHT	FLOAT	
ESBRAC	CHAR (1 Byte)	
ESBCAPACITY	CHAR (1 Byte)	
SSFCAPACITY	CHAR (1 Byte)	
POID	NUMBER (9)	
SESSION_ID	NUMBER	
PERIL_CODE	VARCHAR2 (50 Byte)	
MAIN_SCENARIO_CODE	VARCHAR2 (50 Byte)	
SCENARIO_CODE	VARCHAR2 (50 Byte)	
STATUS	VARCHAR2 (20 Byte)	
VALID_FROM_DATE	TIMESTAMP(6)	
VALID_UNTIL_DATE	TIMESTAMP(6)	

T_WEIGHTED_INTERNAL_COST

Column Name	Data Type	Java Identifier
POID	VARCHAR2 (50 Byte)	
SESSION_ID	VARCHAR2 (50 Byte)	
PRODUCT_CODE	VARCHAR2 (50 Byte)	
LOB_CODE	VARCHAR2 (50 Byte)	
BUS_UNIT_CODE	VARCHAR2 (50 Byte)	
STATUS	VARCHAR2 (20 Byte)	
VALID_FROM_DATE	TIMESTAMP(6)	
VALID_UNTIL_DATE	TIMESTAMP(6)	
I_ACQ_FIX_PROG	FLOAT	FixedCostProg
I_ACQ_FIX_BUSINESS	FLOAT	FixedCostAccept
I_ACQ_BETA	FLOAT	LossInt
I_ACQ_GAMMA	FLOAT	GammaPow
I_ACQ_RHO	FLOAT	PowerTerm
R_RUN_OFF_ALPHA	FLOATLossIntRunOff	
I_CAP_FIX_PROG	FLOAT	Cap
I_ACQ_PCT_EL	FLOAT	AquisVolumeInt
I_RUN_OFF_PCT_EL	FLOAT	RunoffVolumeInt
I_OVERHEAD_PCT_COC	FLOAT	OverheadPctBT
I_OVERHEAD_PCT_COC_PURE	FLOAT	OverheadPctAT
I_EXPENSE_EXPONENT	FLOAT	NonLinFactor
I_AVG_LOSS_SIZE_100P	FLOAT	AverageLossSize
I_SHIFT_INTENSITY	FLOAT	LossShift
I_BU_FACTOR	FLOAT	VolumeIntBU
I_LOB_FACTOR	FLOAT	VolumeIntLOB
I_PROD_FACTOR	FLOAT	VolumeIntProduct
I_ACQ_OF_100P	FLOAT	
I_VOLUME_INTENSITY	FLOAT	VolumeInt

B Cash flow operators

Take an expected loss cash flow L in the years $y \in \{0, \dots, N\}$. We denote its components by $\{L_y | 0 \leq y \leq N\}$. This cash flow is to be interpreted as punctual payments at the following points in time:

L_0 Loss to be paid immediately at inception of the contract

L_1 Loss to be paid (as a point payment) one year after contract inception

$\vdots$

All quantities defined until now are to be interpreted in the very same way: for example, the cost of capital CoC has $N+1$ components which are given by $\{\mathrm{CoC}_y | 0 \leq y \leq N\}$.

Note that a ”cumulative” payment as it is often used in reserving is defined as follows

$L_0^\star$ Cumulative amount paid up to the inception date of the contract (always equal to zero)

$L_1^\star$ Cumulative amount of lossed paid in the first year

$$\vdots$$

Simple conversion example: if we assume a homogeneous payment across the year, then a mapping from the payment pattern " $\star$ " to the "standard" one use in the loading concept would look as follows.

$$L_0 = \frac{1}{2}L_1^\star \tag{119}$$

$$L_1 = \frac{1}{2}L_1^\star + \dots \tag{120}$$

$$L_2 = \dots \tag{121}$$

With TPF interest rates d (having again elements $\{d_y | 0 \leq y \leq N\}$ we obtain a discount factor

$$\mathrm{DF}_y\left[d\right] = \frac{1}{(1+d_y)^y} \tag{122}$$

for every year. This leads to a discounted expected loss $\mathrm{PV}\left[L\right]$ of

$$\mathrm{PV}_d\left[L\right] = \sum_{j=0}^{N} \mathrm{DF}_j\left[d\right] L_j = \sum_{j=0}^{N} \frac{L_j}{(1+d_j)^j}. \tag{123}$$

Further on in the document, we will as well use forward rates which are defined via the TPF discount factors.

$$f_y\left[D\right] = f_y = \frac{\mathrm{DF}_{y+1}\left[d\right]}{\mathrm{DF}_y\left[d\right]} - 1 = \frac{(1+d_y)^y}{(1+d_{y+1})^{y+1}} - 1 \tag{124}$$

Based on that interpretation of the yearly cash flows, we

Further on, we define the nominal reserves RN_k needed at the end of the year k as

$$\mathrm{RN}_k^+\left[L\right] = \begin{cases} \sum_{y=k+1}^{N} L_y, & k \geq 0 \\ 0, & \text{otherwise.} \end{cases} \tag{125}$$

For discounted reserves, we define RD_k for the discounted reserves at the end of the year. We will use the forward rates f_y defined above for these quantities which are defined recursively in the following manner. Note that the range of years y for which the loss cash flow is defined still is $0 \leq y \leq N$.

$$RD_N\left[L\right] = 0, \tag{126}$$

$$RD_y\left[L\right] = \frac{RD_{y+1}\left[L\right] + L_{y+1}}{1+f_y}. \tag{127}$$

a) **Poisson approximation**

A Poisson model can often be used as a realistic approximation for modeling the pattern of expected loss payments:

$$L_{N}^{j} = L_{N} \cdot e^{-\tau} \cdot \frac{\tau^j}{j\,!}, \qquad j \geq 0 \tag{128}$$

We refer to the unknown parameter τ as the average settlement time as

$$\begin{aligned}\sum_{j=0}^{\infty} j \cdot \frac{L_{_N}^j}{L_{_N}} &= e^{-\tau} \sum_{j=1}^{\infty} j \cdot \frac{\tau^j}{j\,!} \\ &= e^{-\tau} \cdot \tau \sum_{j=1}^{\infty} \cdot \frac{\tau^{(j-1)}}{(j-1)\,!} \\ &= e^{-\tau} \cdot \tau \cdot e^{\tau} = \tau.\end{aligned} \tag{129}$$

τ can further be derived from the discount factor f_d. The Poisson model then permits to express the total discounted loss with the help of a simple closed formula:

$$\begin{aligned} L_{_D} &= \sum_{j=0}^{\infty} \frac{L_j}{(1+d)^j} = \mathrm{RN}\,[L] \cdot e^{-\frac{d \cdot \tau}{1+d}} \\ \Longrightarrow &\quad \mathrm{df} = e^{-\frac{d \cdot \tau}{1+d}}, \qquad \tau = \frac{1+d}{d} \cdot \log(1/f_d)\end{aligned}$$