

ADAPTING NEURON COUNT DURING TRAINING

A BAYESIAN NONPARAMETRIC VIEW

Mark van der Wilk
Invited Talk

14th International Conference on Bayesian Nonparametrics



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SCIENCE

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Filippi
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Based on a True Story

Adjusting Model Size in Continual Gaussian Processes: How Big is Big Enough?

Guiomar Pescador-Barrios¹ Sarah Filippi¹ Mark van der Wilk²

Spotlight at ICML 2025.

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Adjusting Model Size in Continual Gaussian Processes: How Big is Big Enough?

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A Bayesian Nonparametric View on Adapting Neuron Count During Training

In submission, soon to be on arxiv.

Thesis of the Talk

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Ideas from Bayesian Nonparametrics may help with new capabilities in deep learning

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Can we grow a NN's size as we see more data?

To avoid poor performance, from constant/restricted model size.



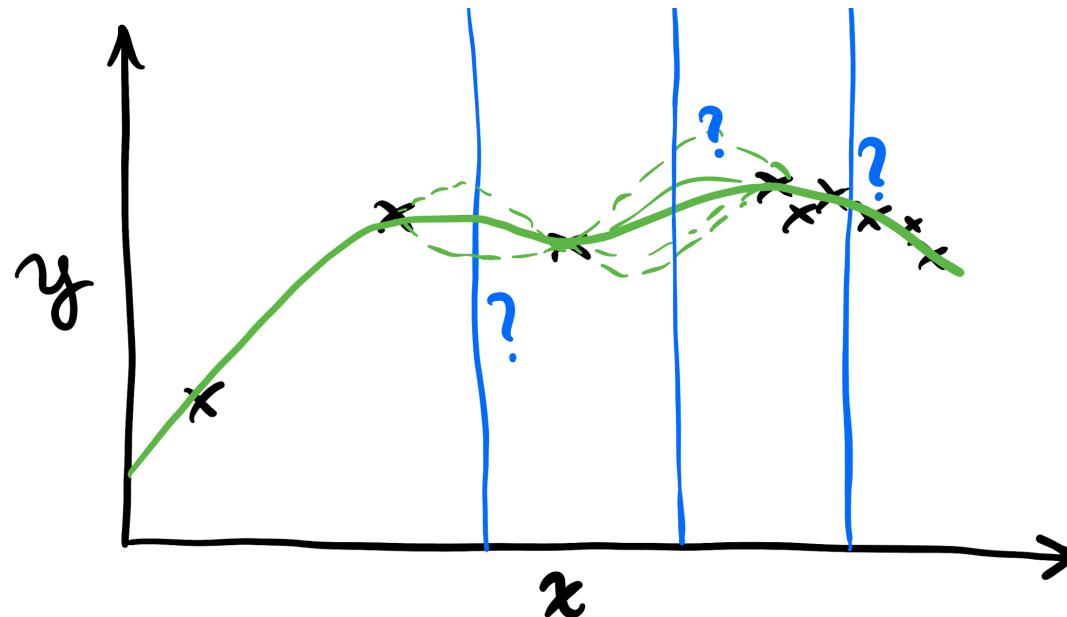
**Minimise model size, while predictions are
near-optimal**

Most of Machine Learning is just *Curve Fitting*

Dataset: $(x_n, y_n)_{n=1}^N$.

Inputs $x_n \in \mathcal{X}$, outputs $y_n \in \mathcal{Y}$.

Goal: Find $f : \mathcal{X} \rightarrow \mathcal{Y}$, that predicts well for new x .



Neural networks just parameterise functions $f_w(x)$.

Designing a Neural Network

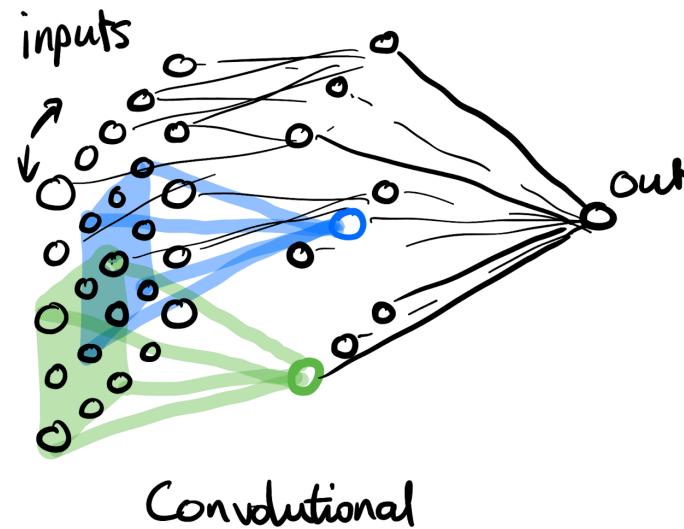
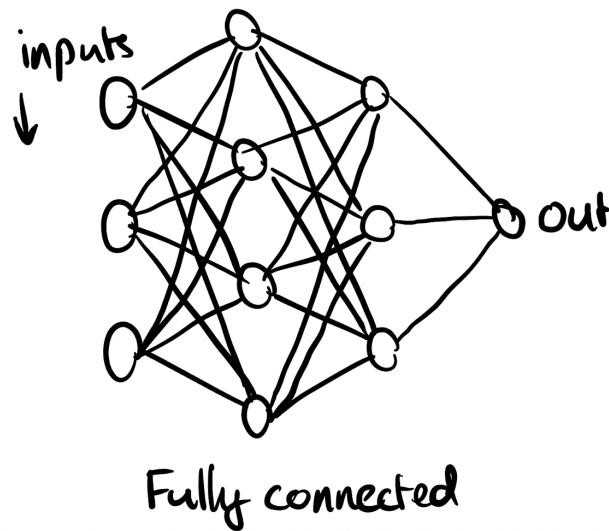
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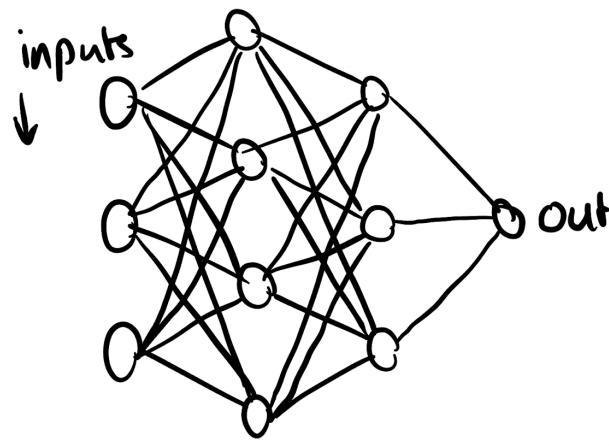
- Inductive bias: **connectivity structure** (architecture)



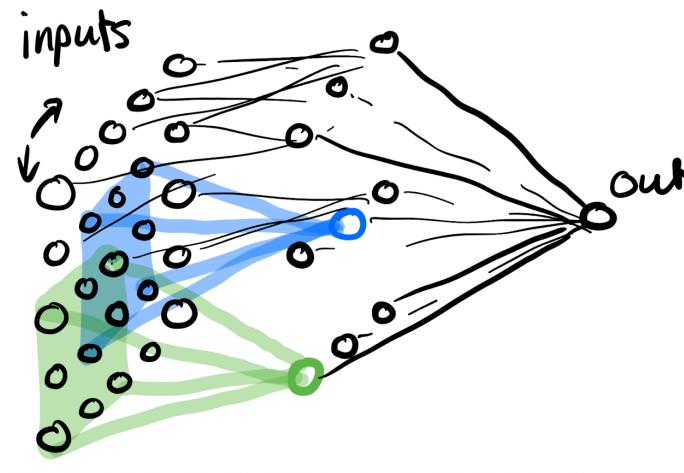
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Designing a Neural Network

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Fully connected

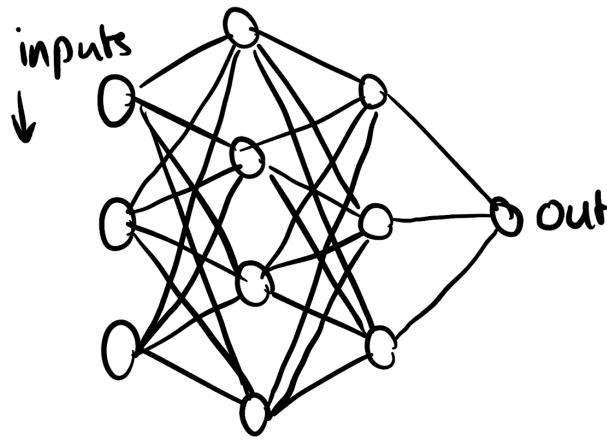


Convolutional

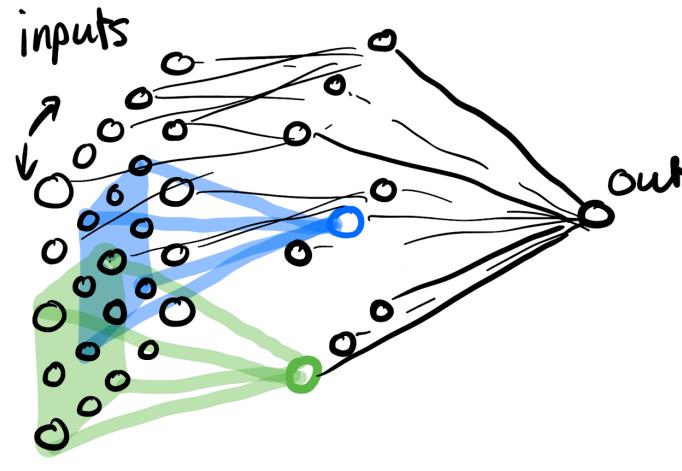
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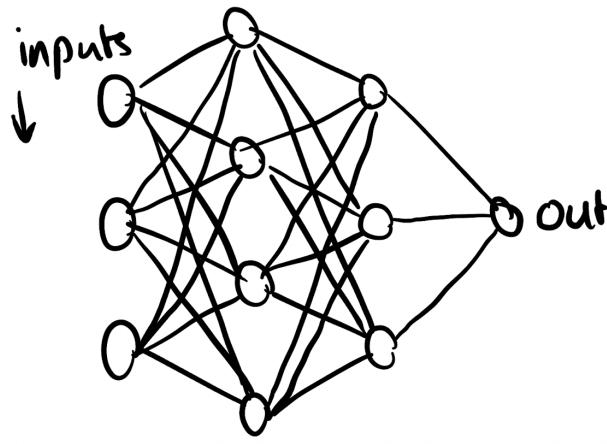
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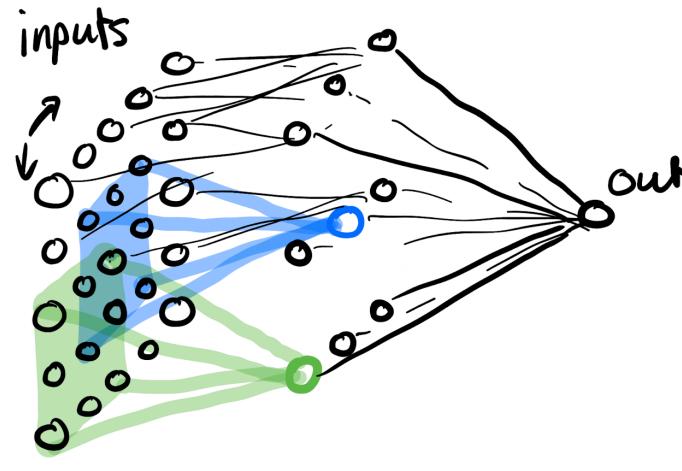
$$w_{t+1} \leftarrow w_t + \nabla_w \ell(f_w(x_t), y_t)$$

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These problems should be tackled *together*.

Problem Formulation (let's walk before we run)

Predictor is a *single layer* neural network:

$$f(x) = \sum_{m=1}^M \varphi(x; Z_m, \theta) w_m$$

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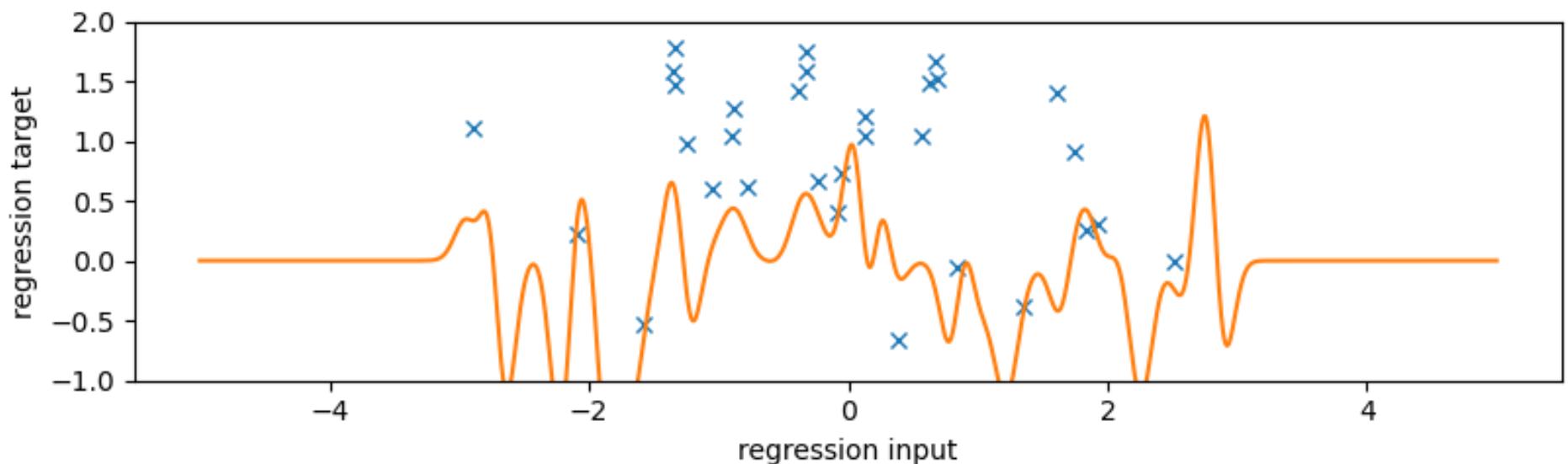
Start by finding *clear* answers for single-layer NNs.

1. **What is wrong with minimising losses?**
2. Bayesian Model Selection
2. Model Selection over Model Size? Or Nonparametrics?
3. A principle for selecting size

Training Loss / MaxLik is not sufficient

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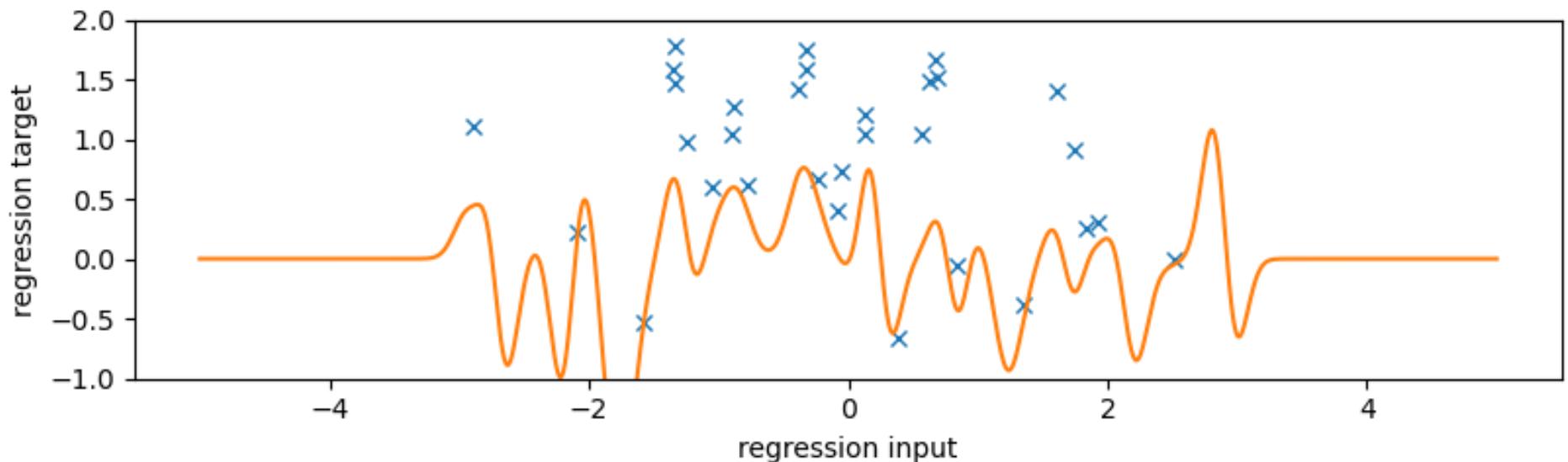
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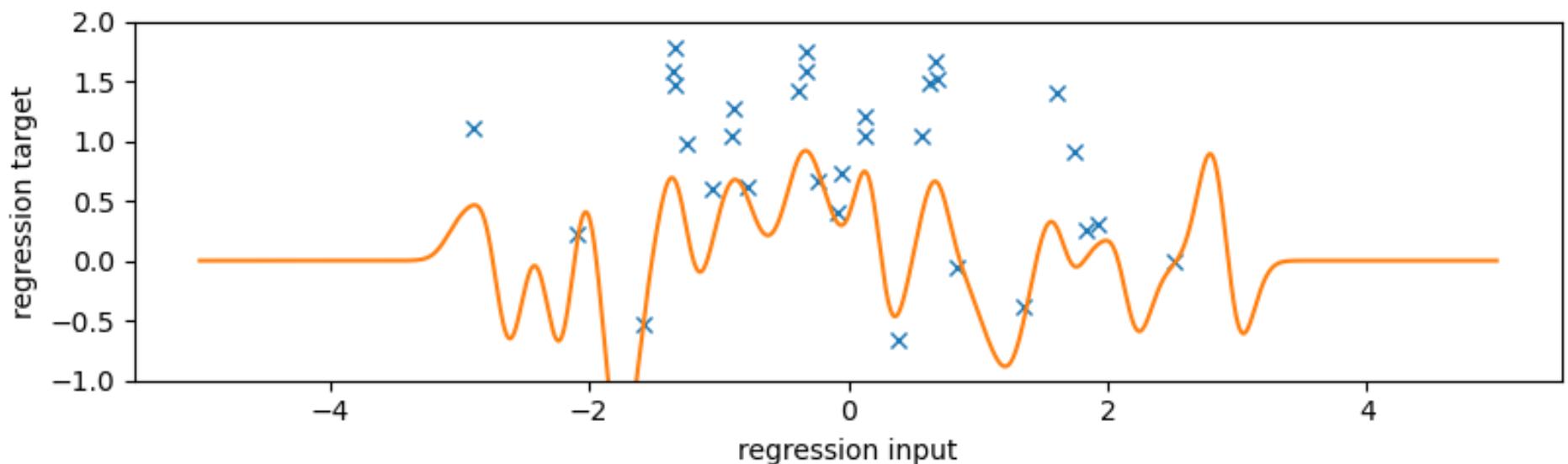
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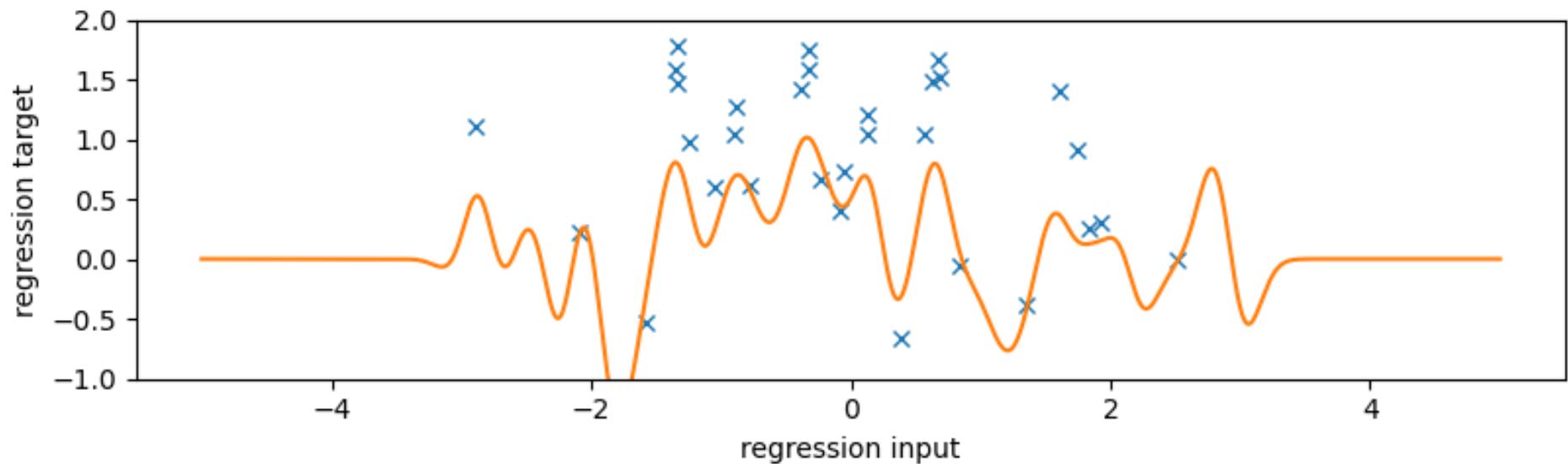
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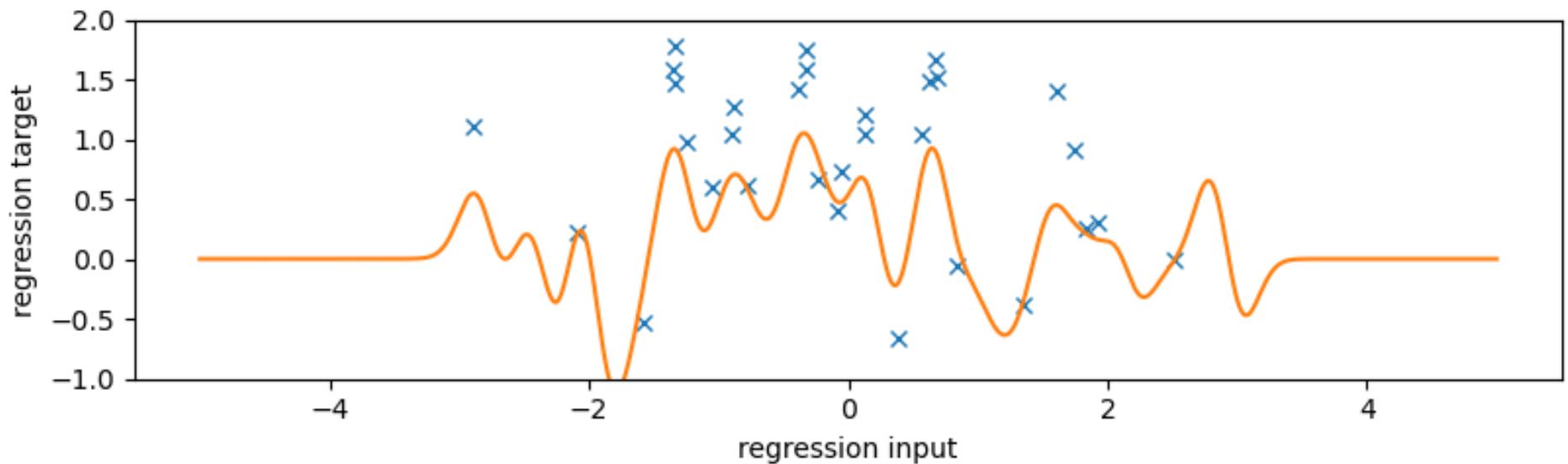
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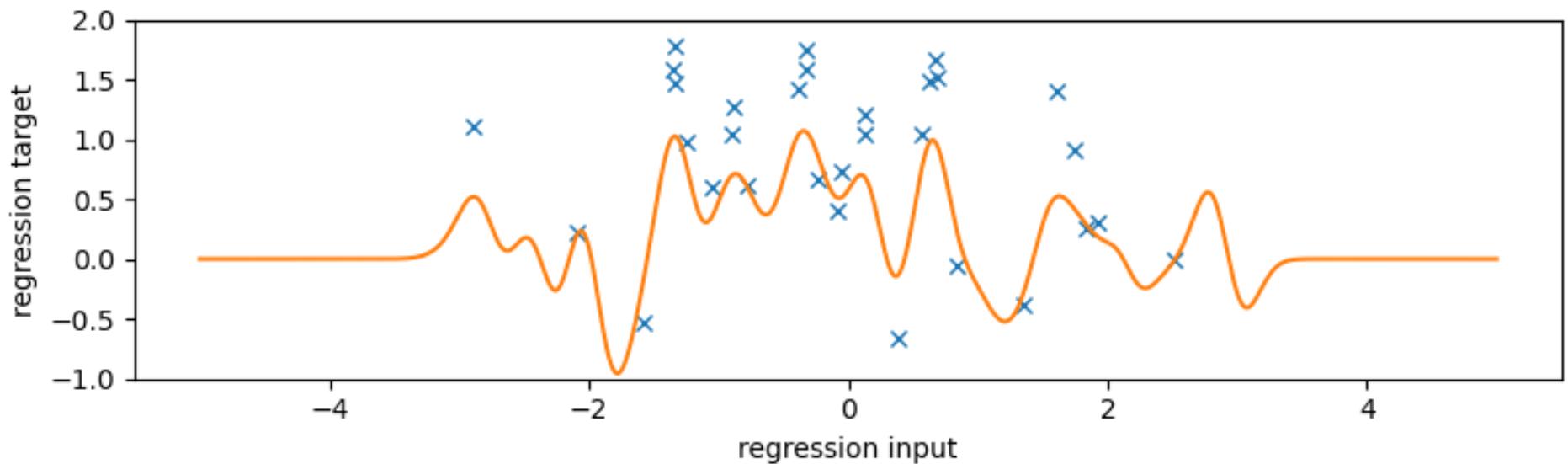
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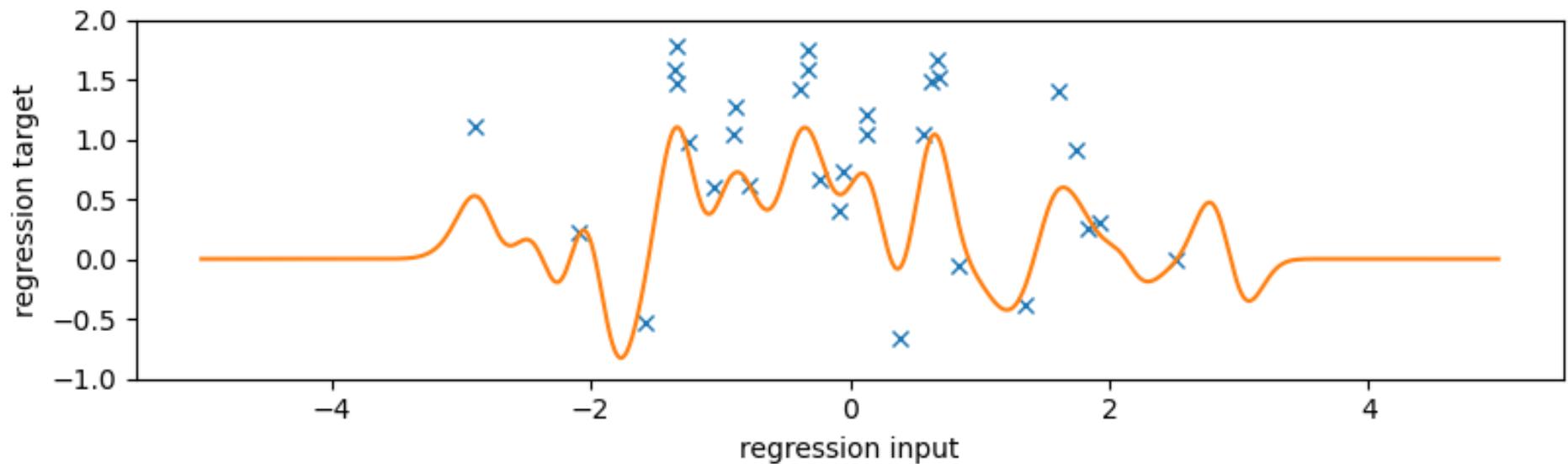
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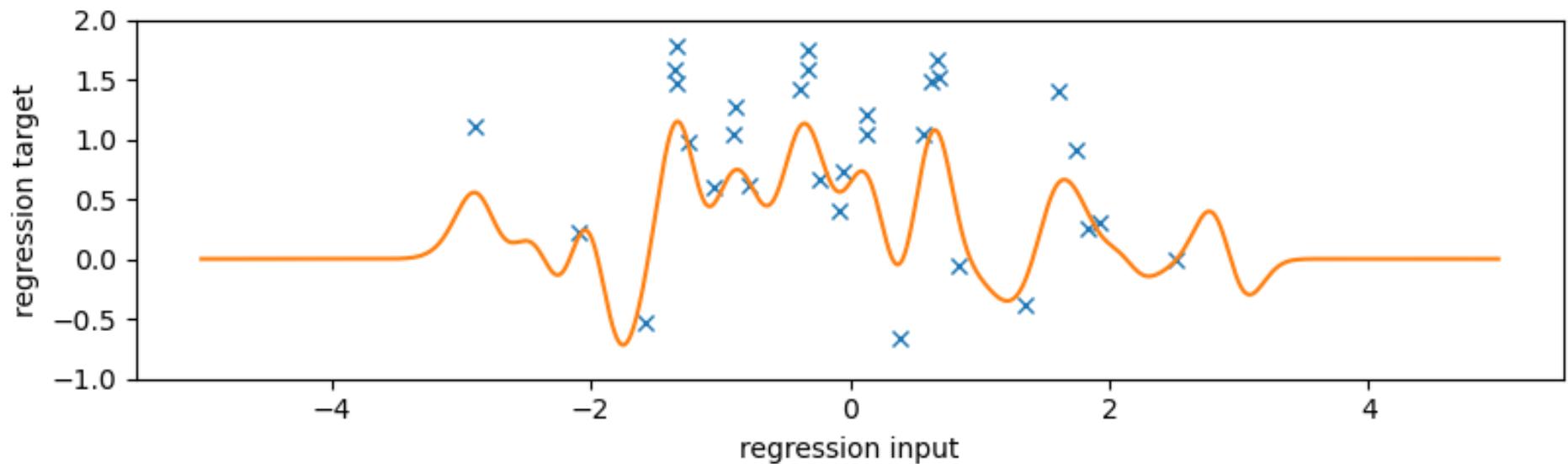
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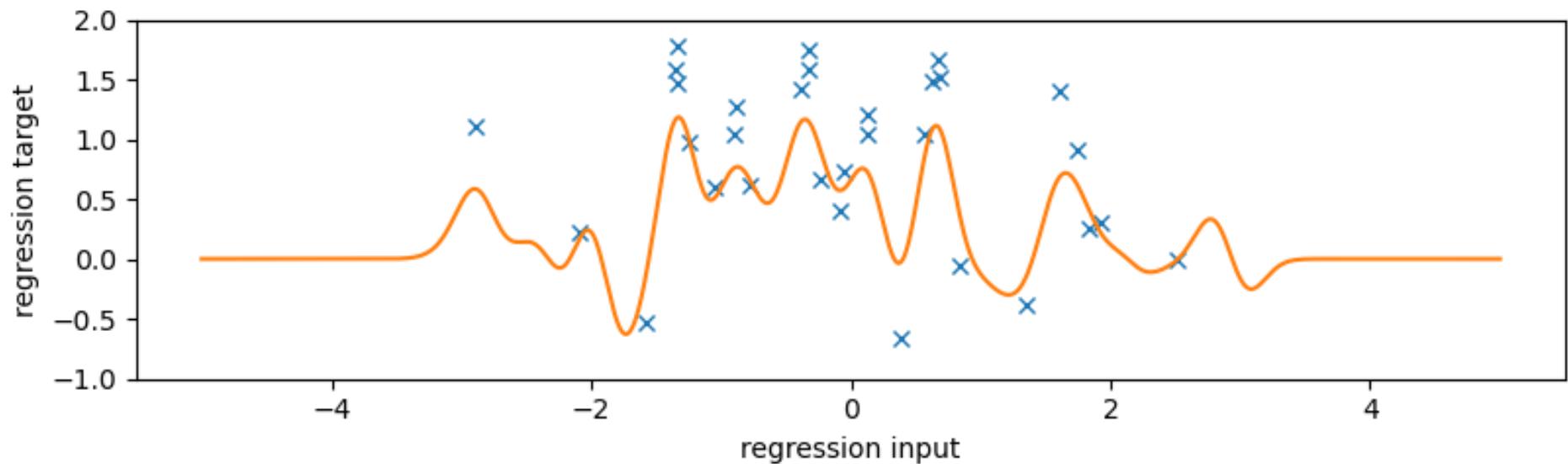
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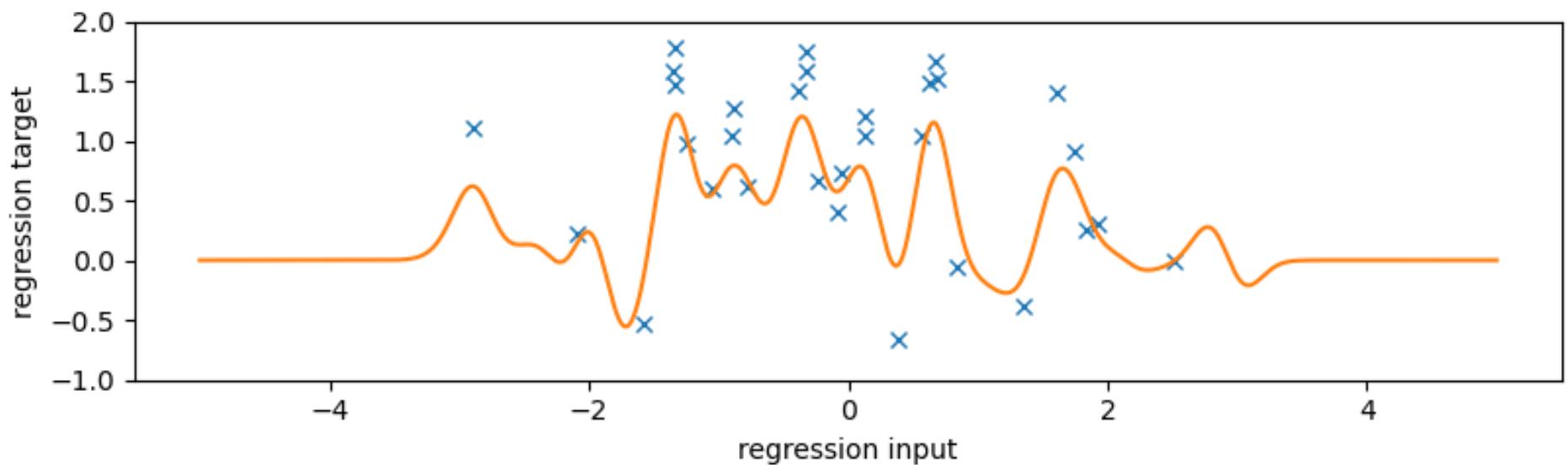
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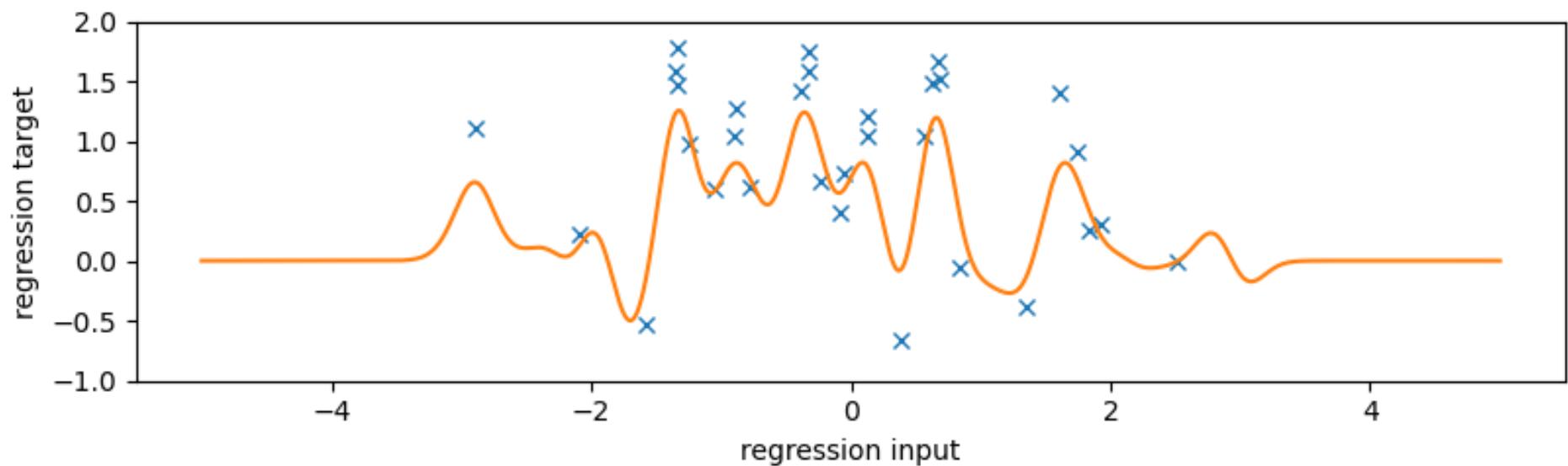
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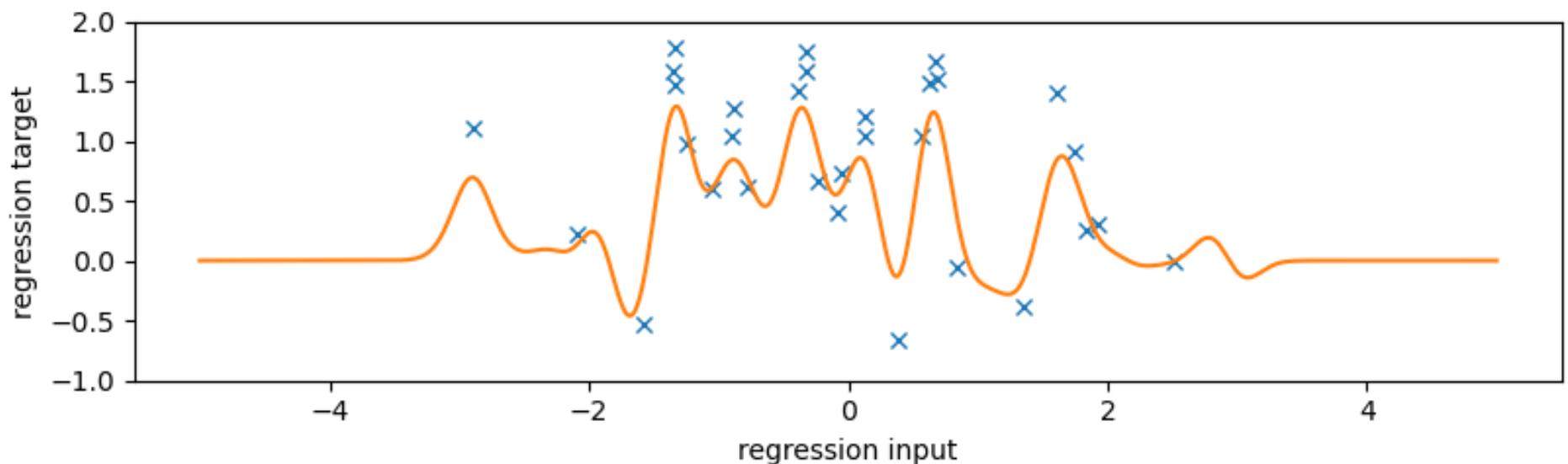
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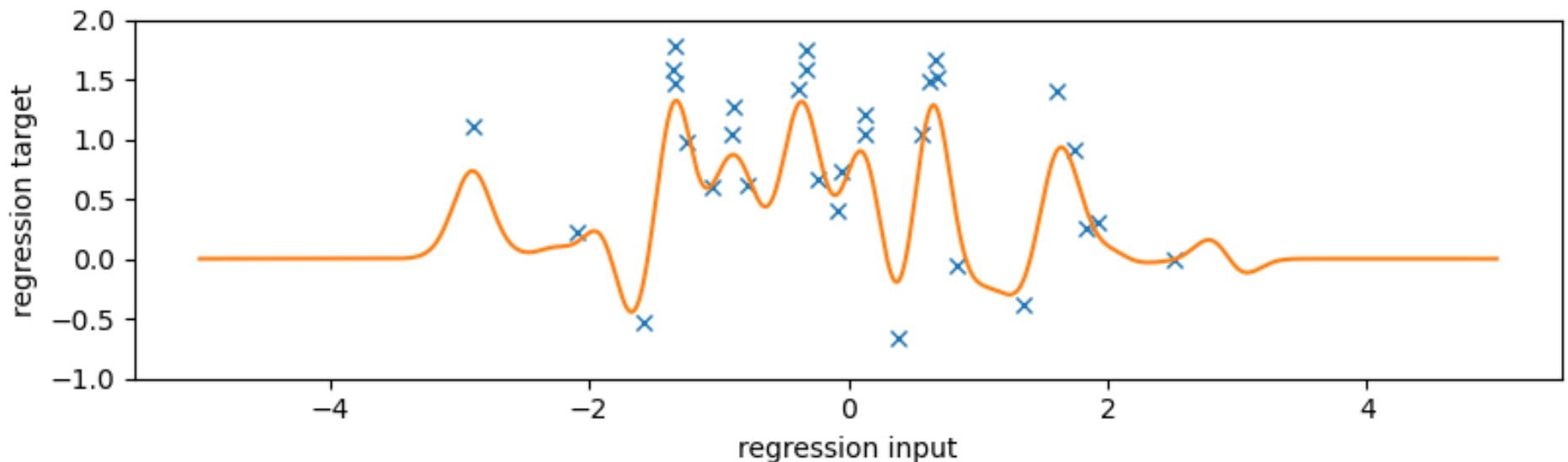
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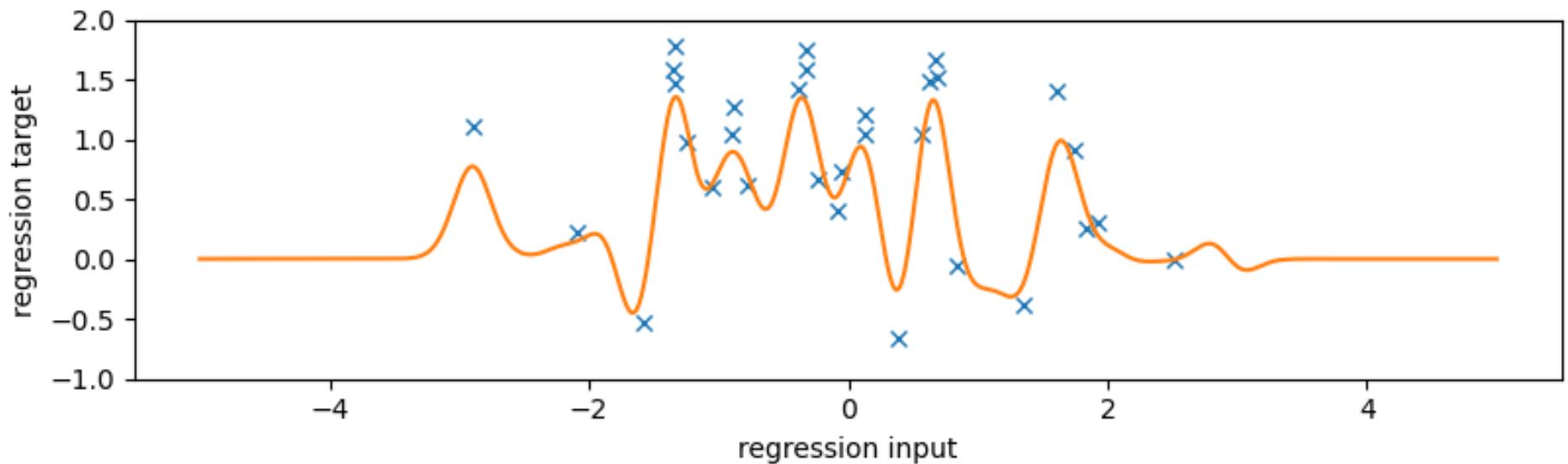
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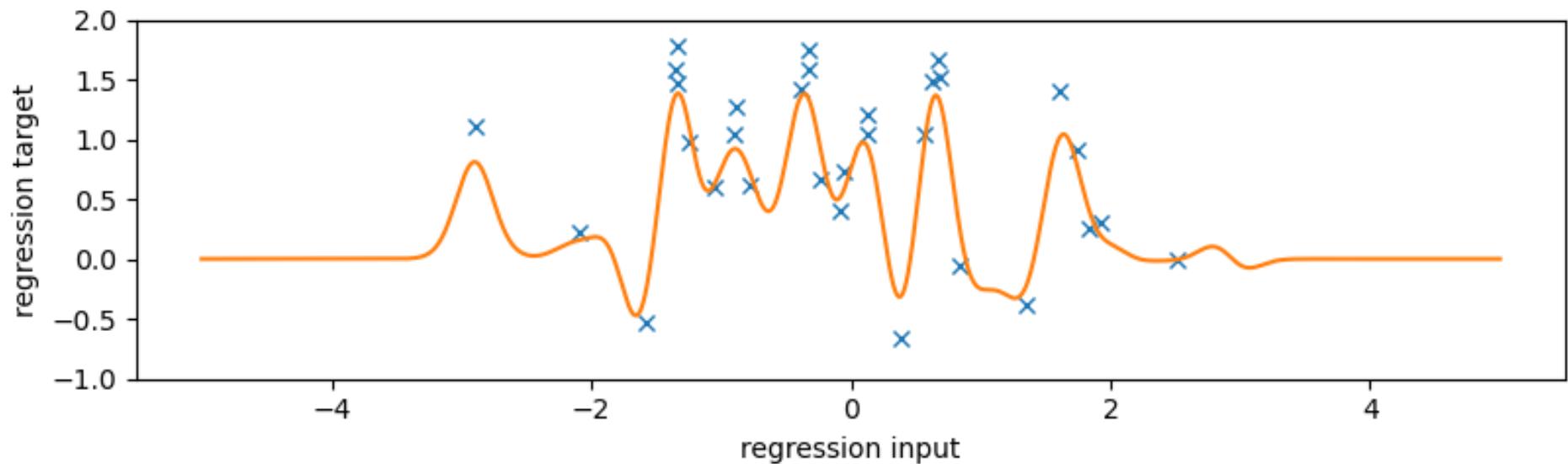
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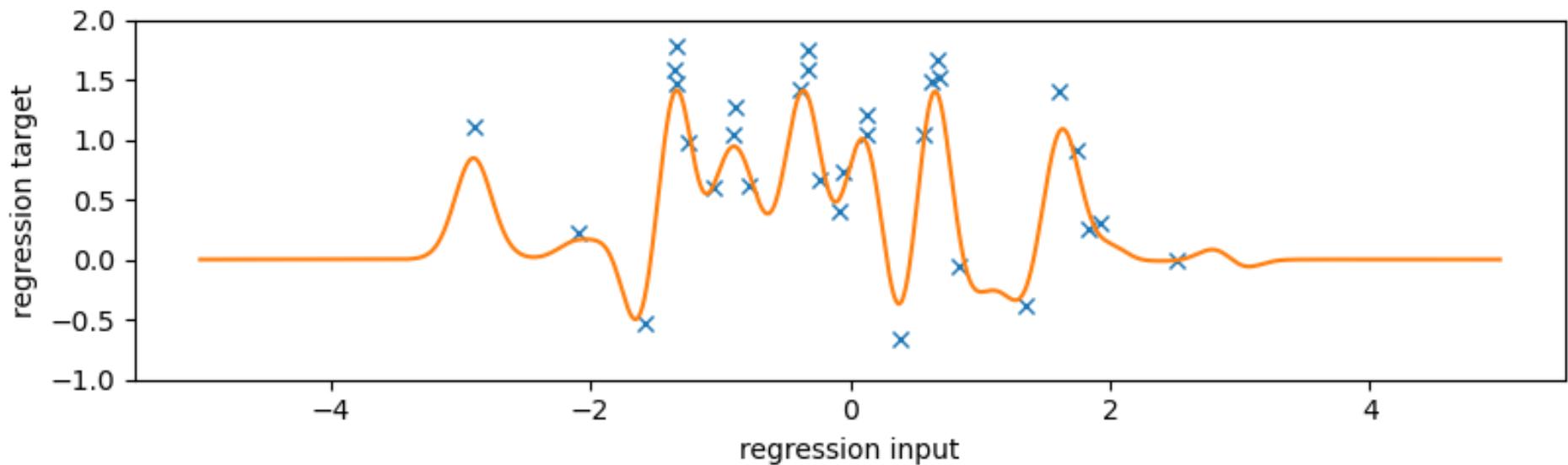
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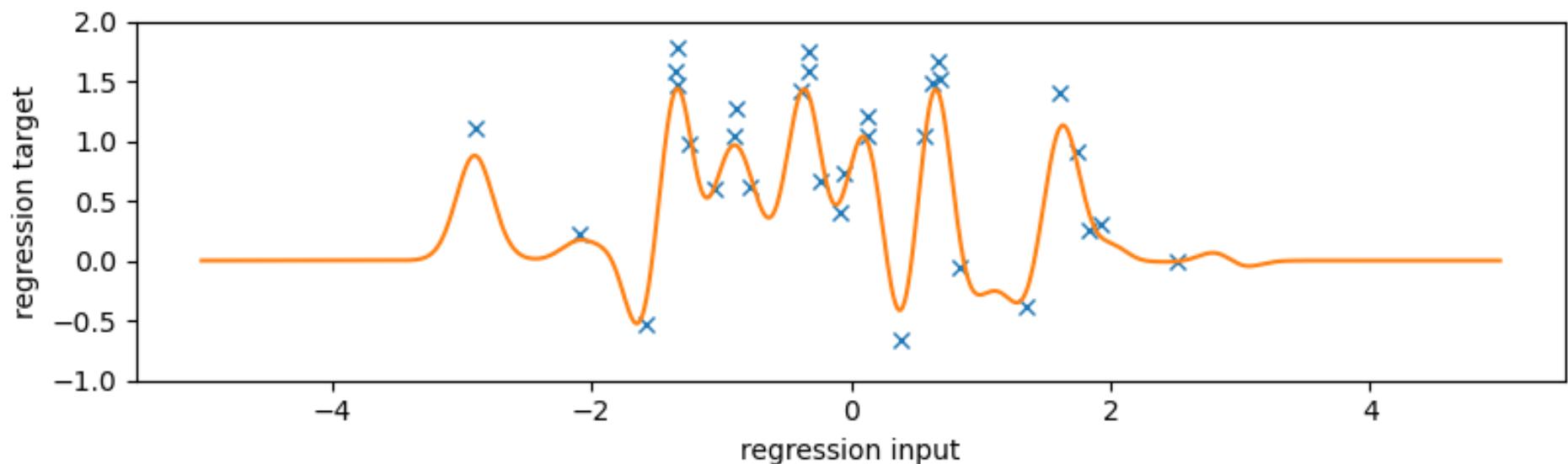
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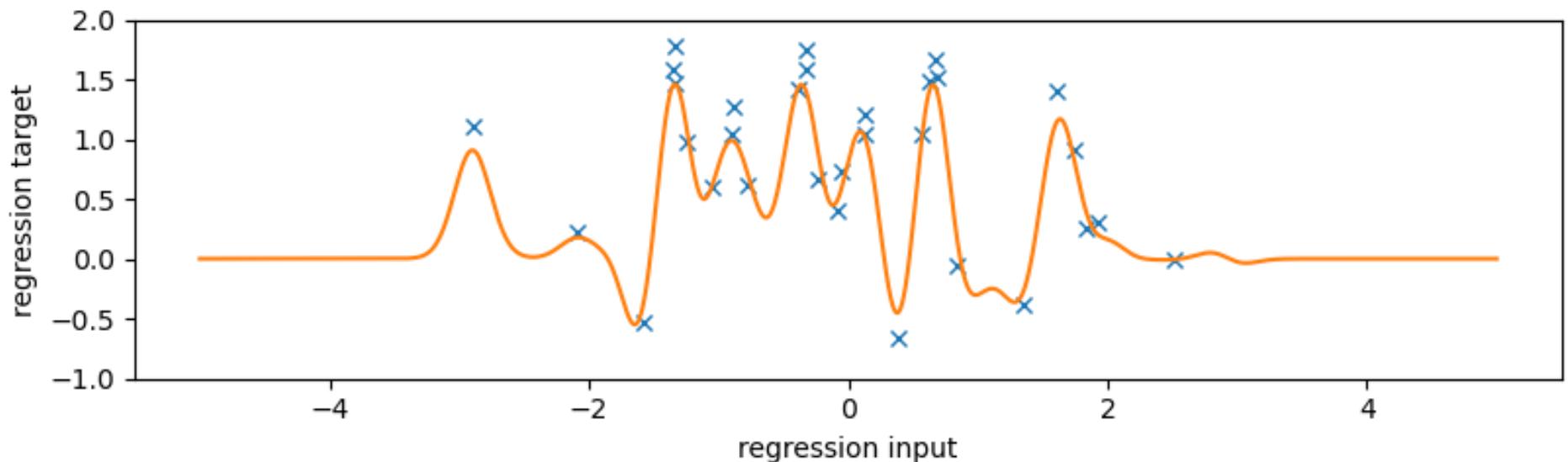
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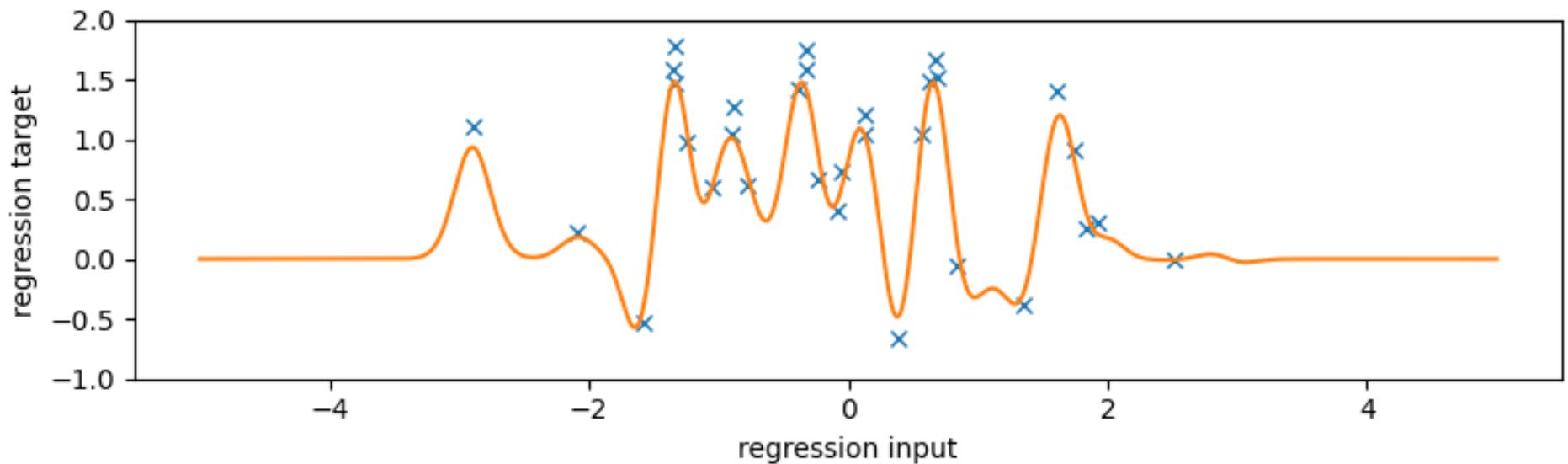
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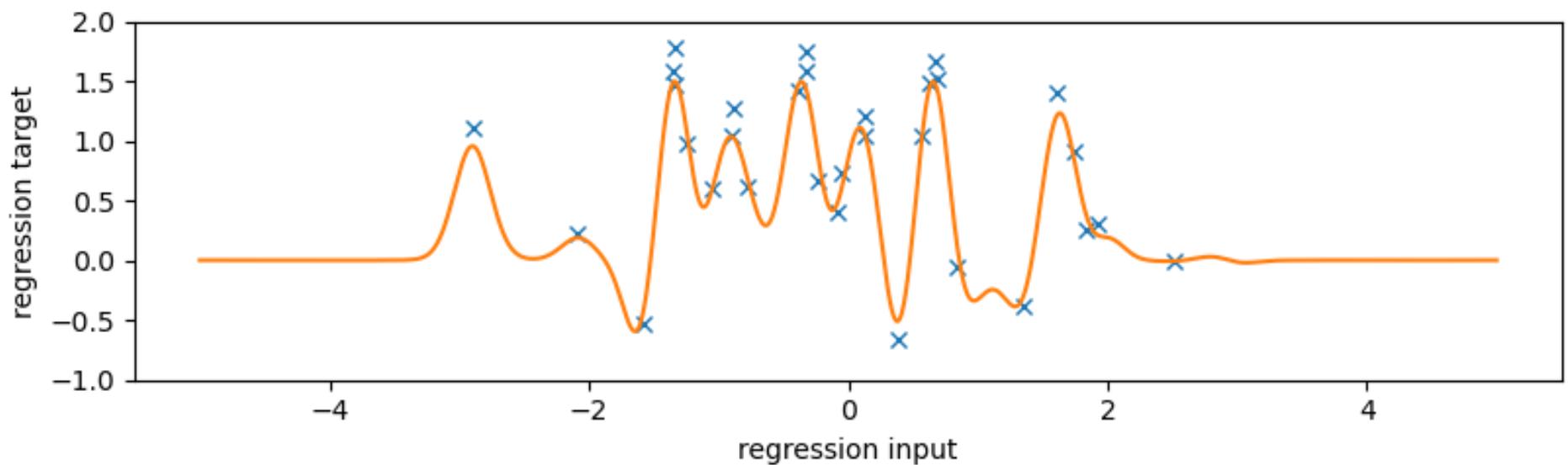
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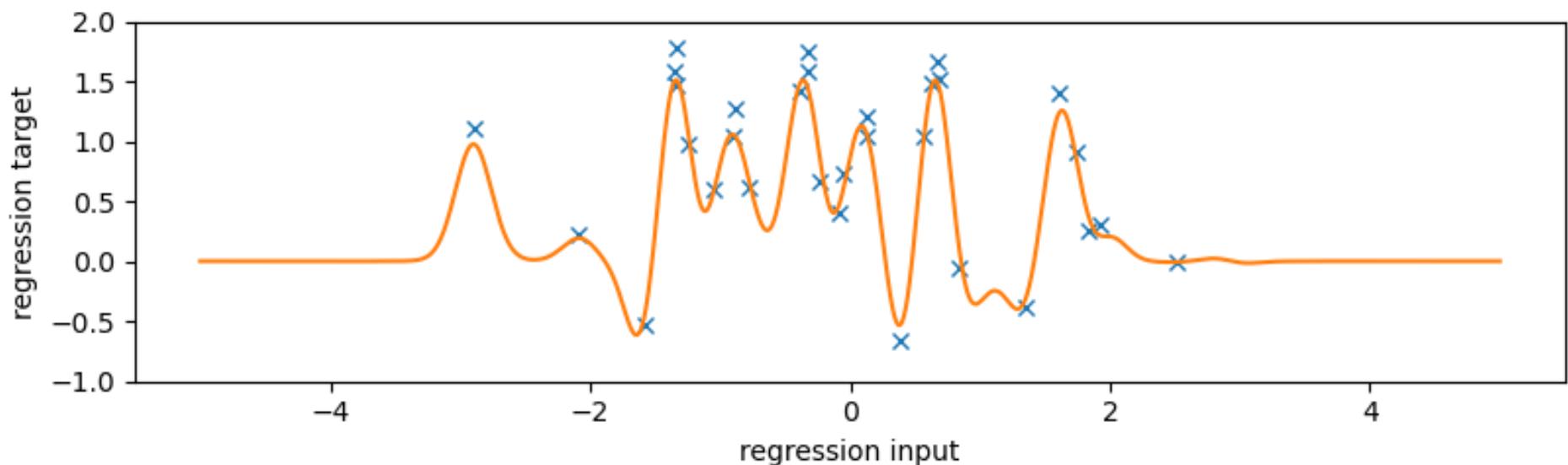
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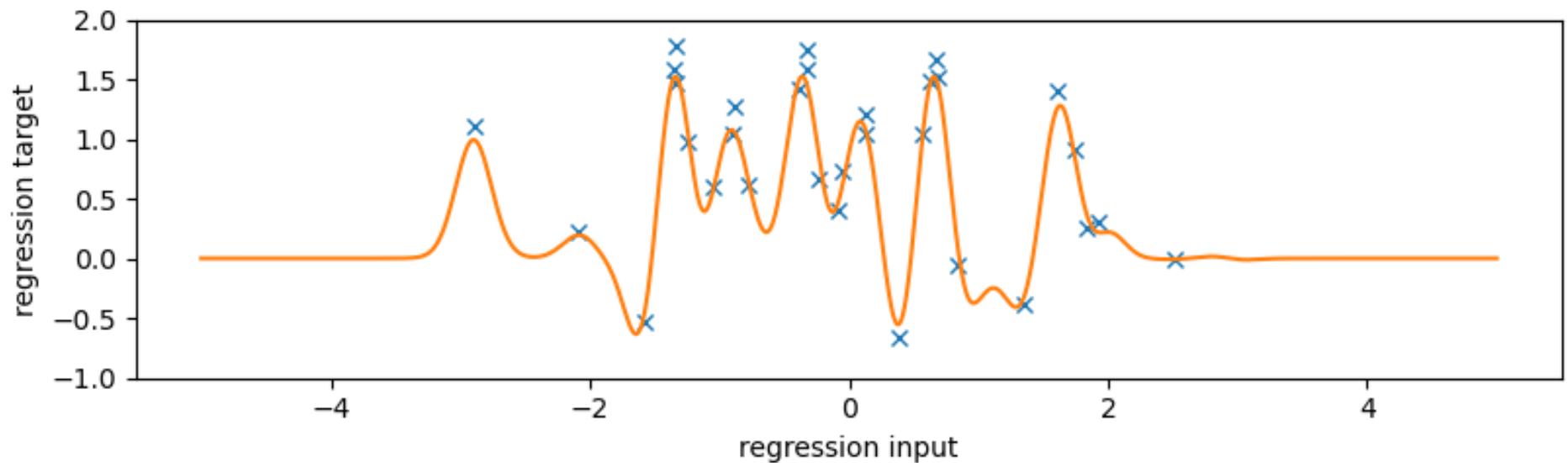
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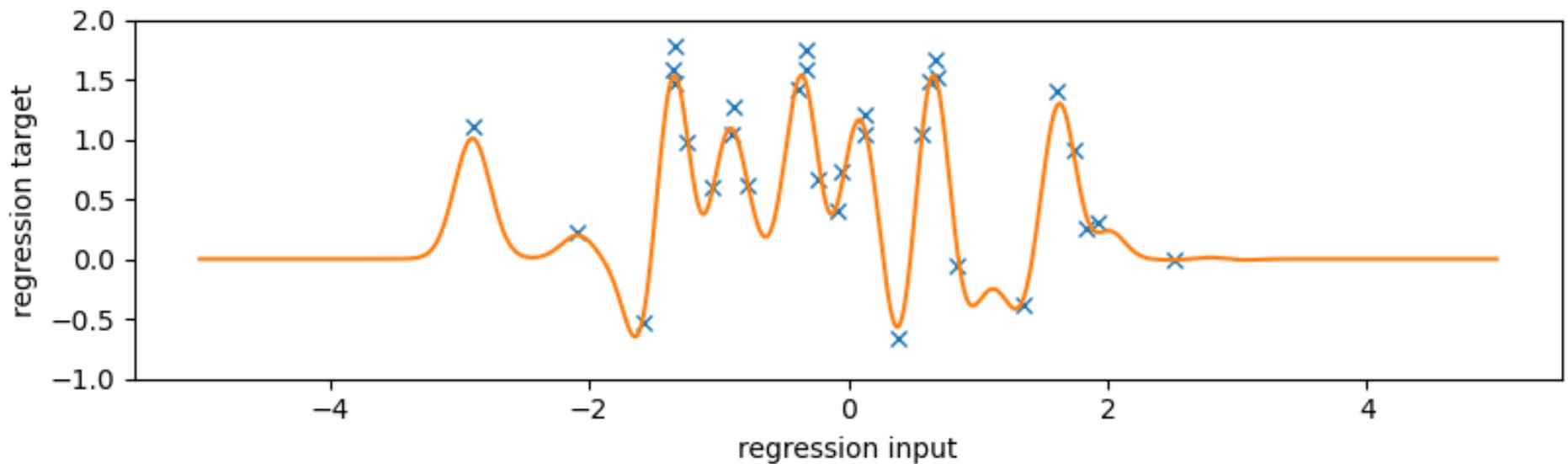
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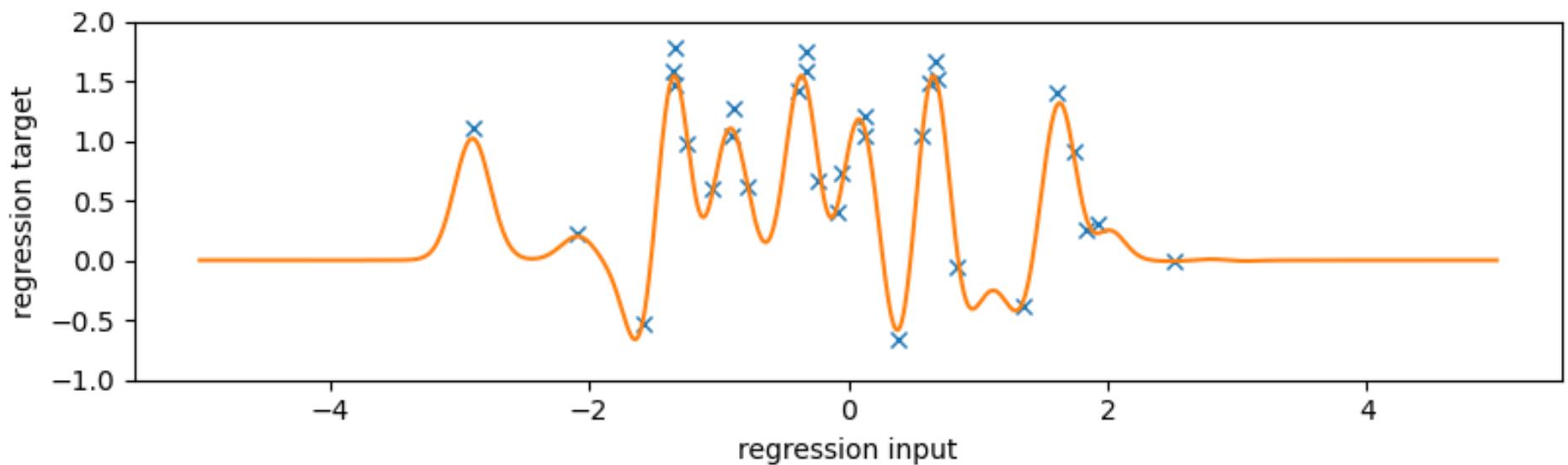
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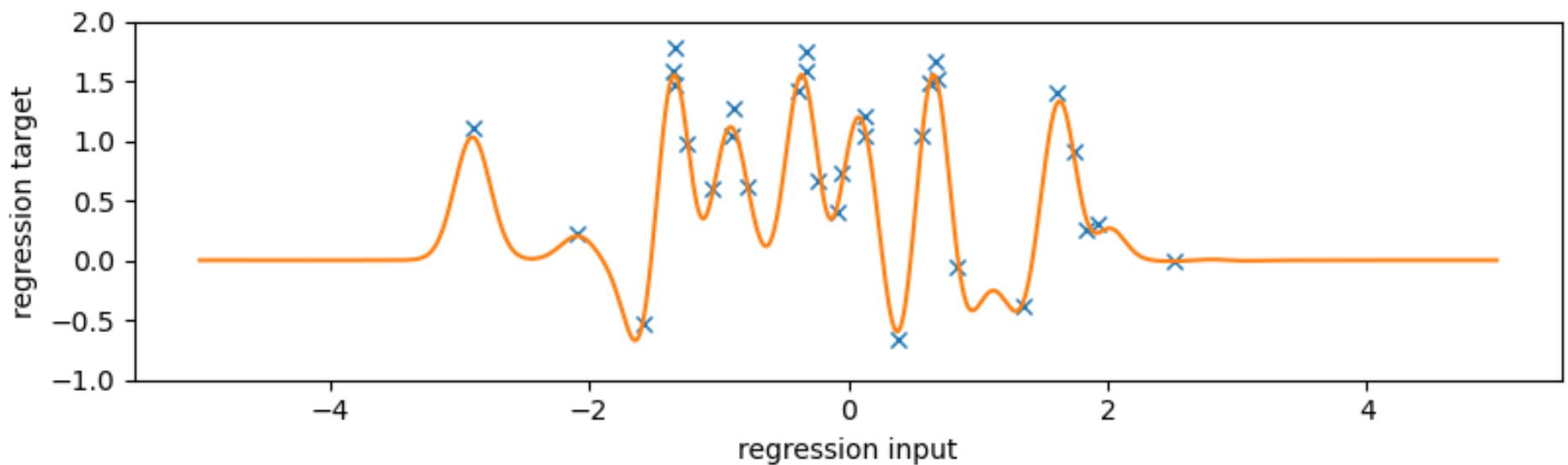
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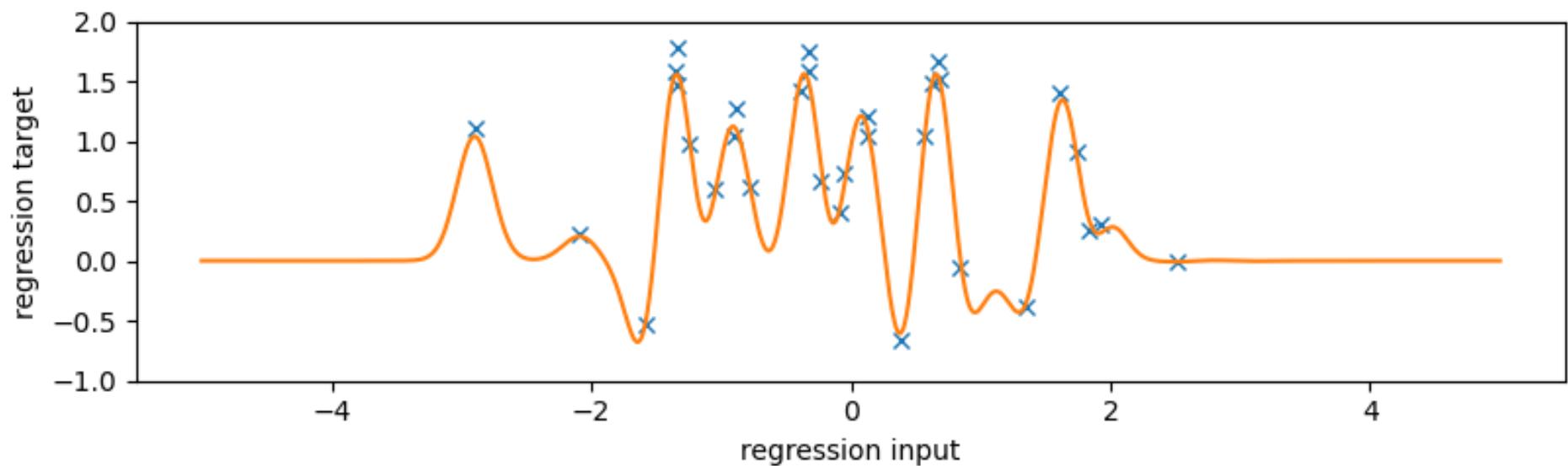
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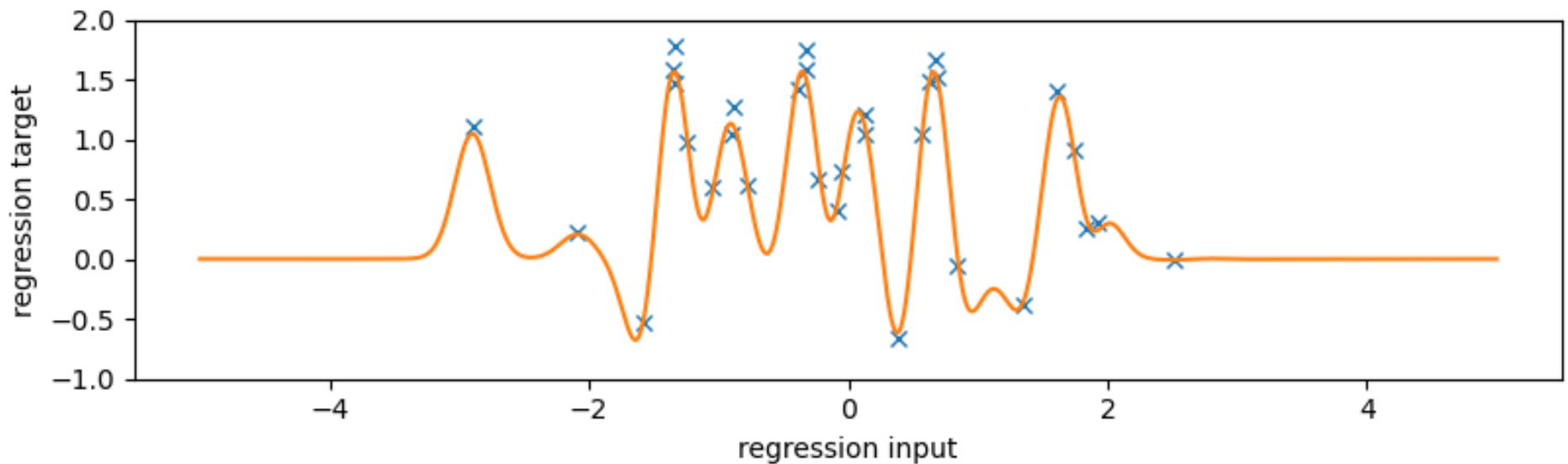
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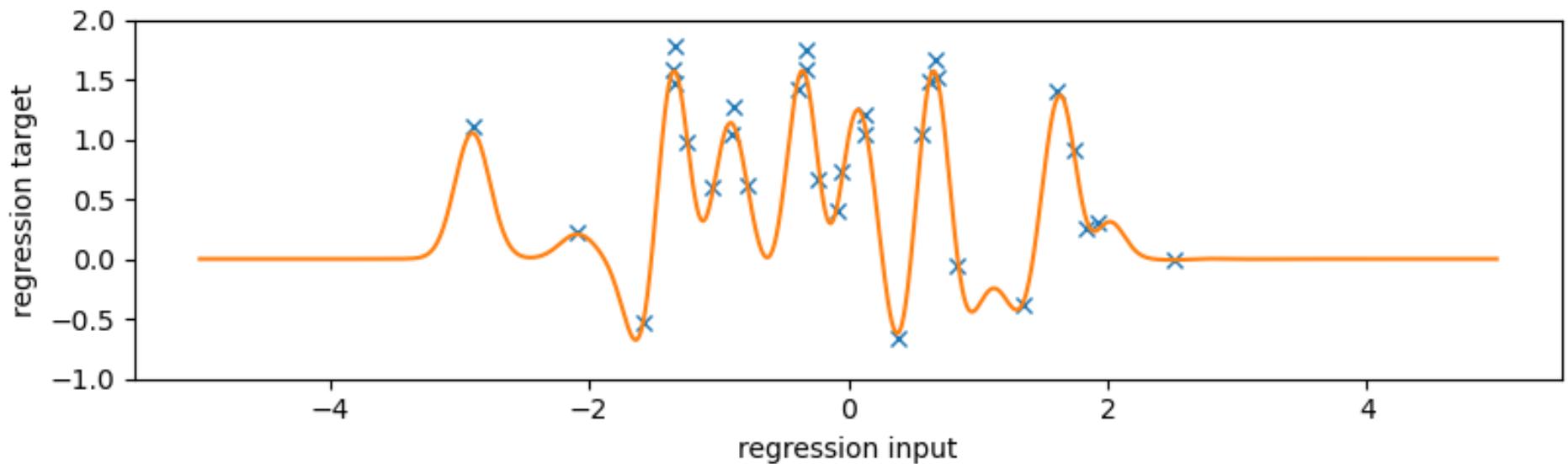
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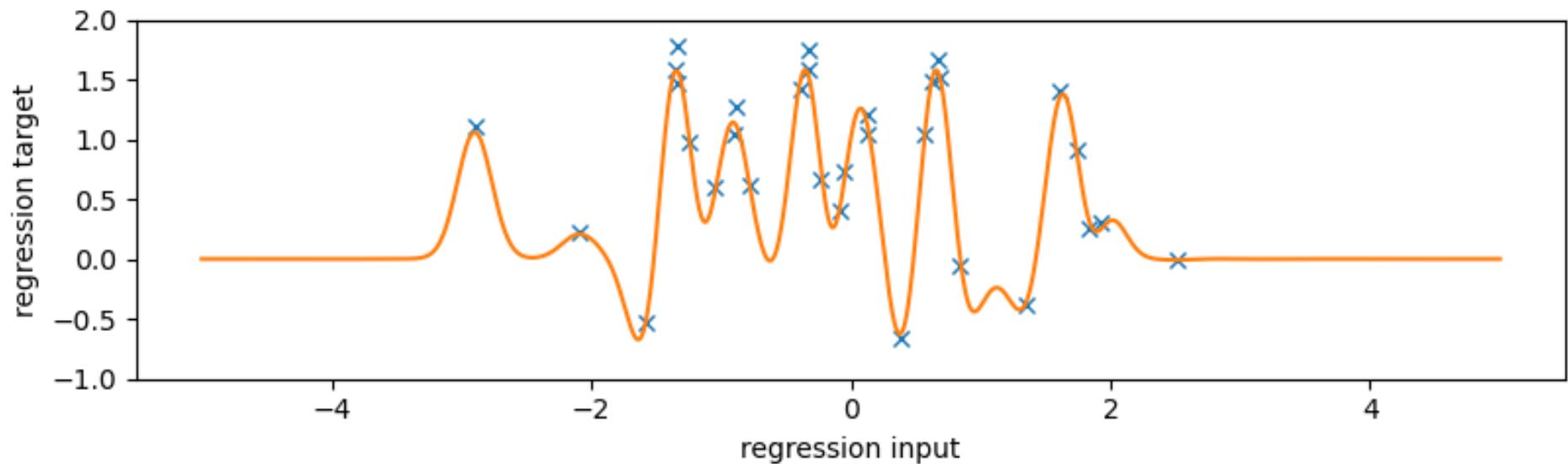
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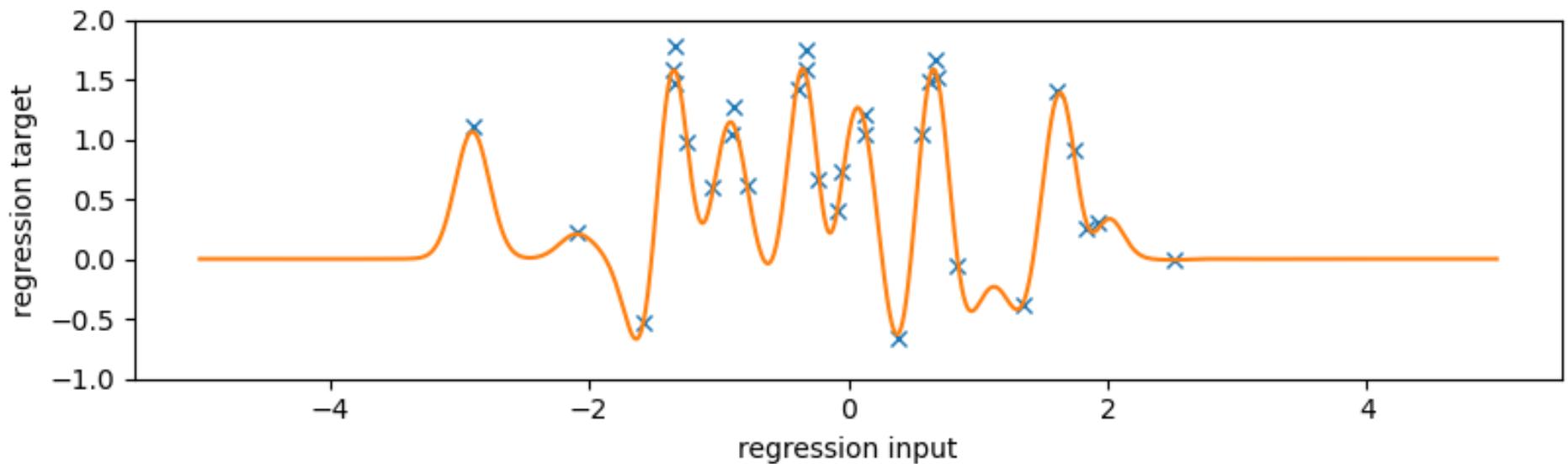
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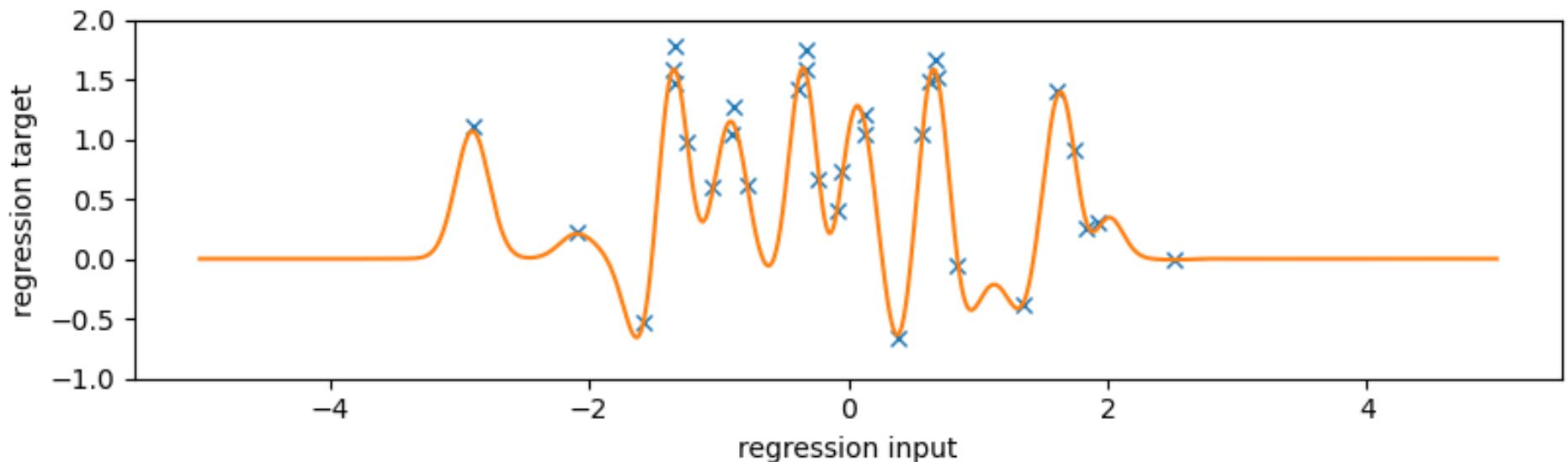
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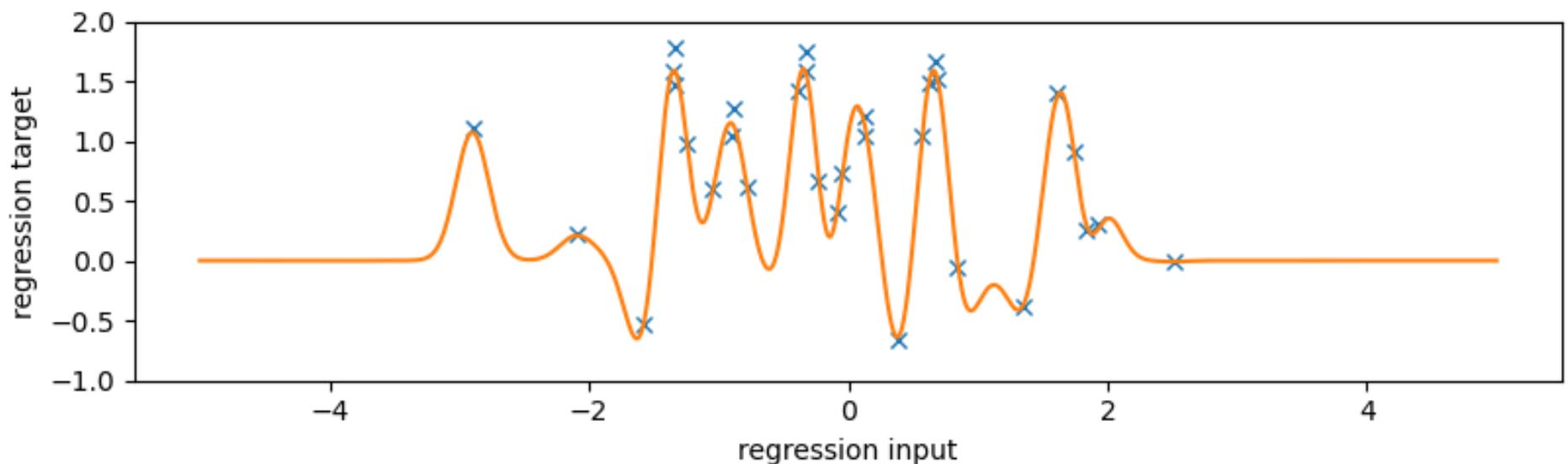
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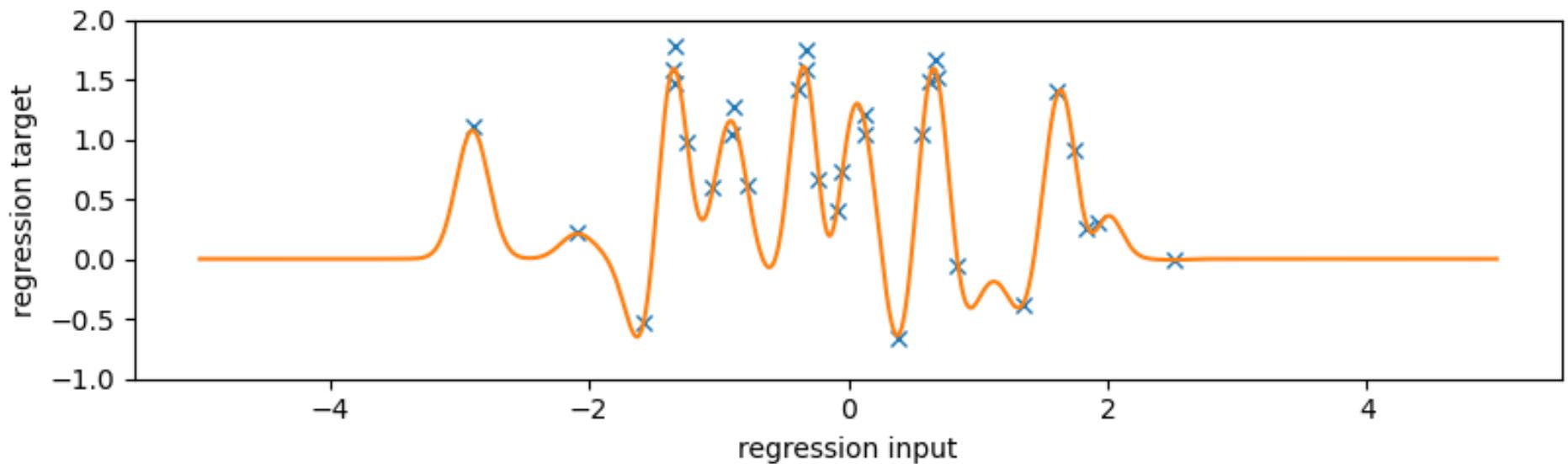
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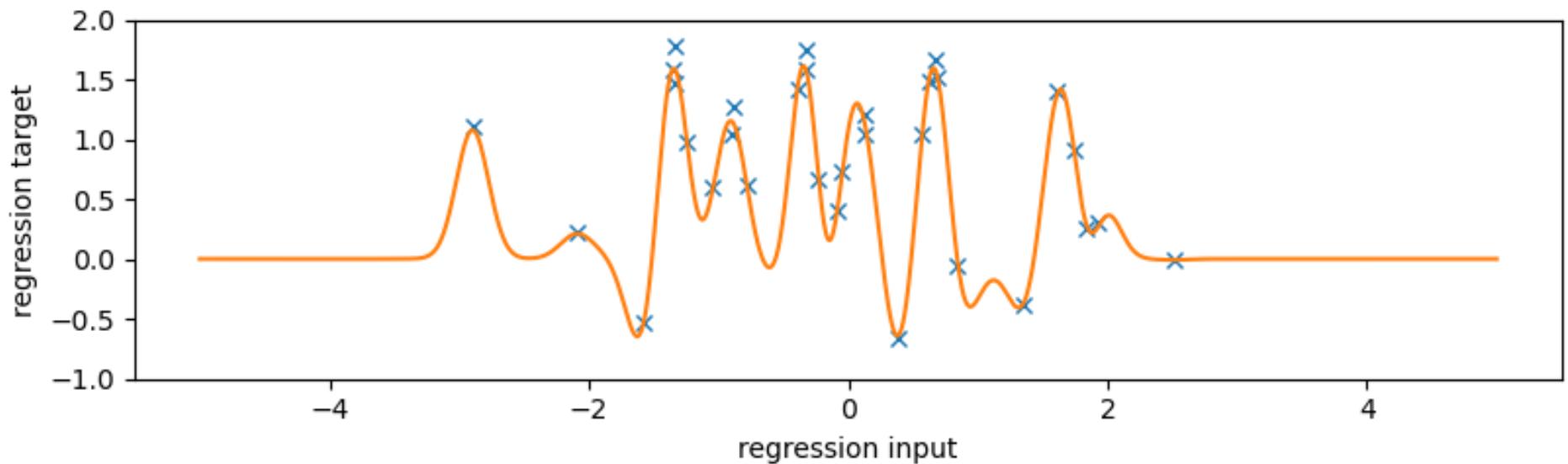
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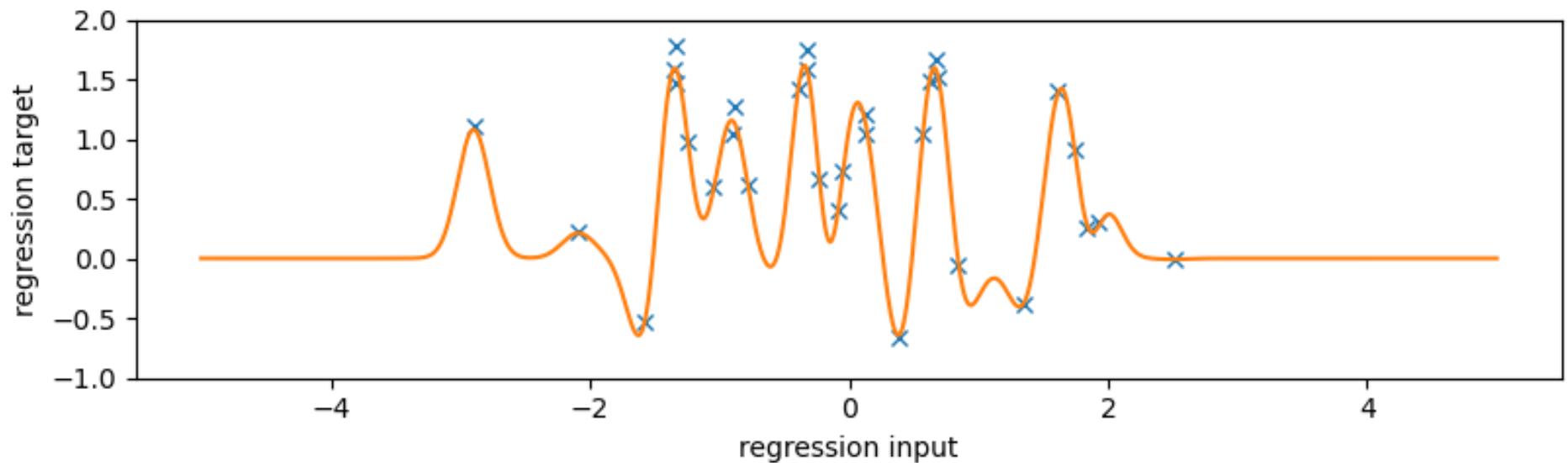
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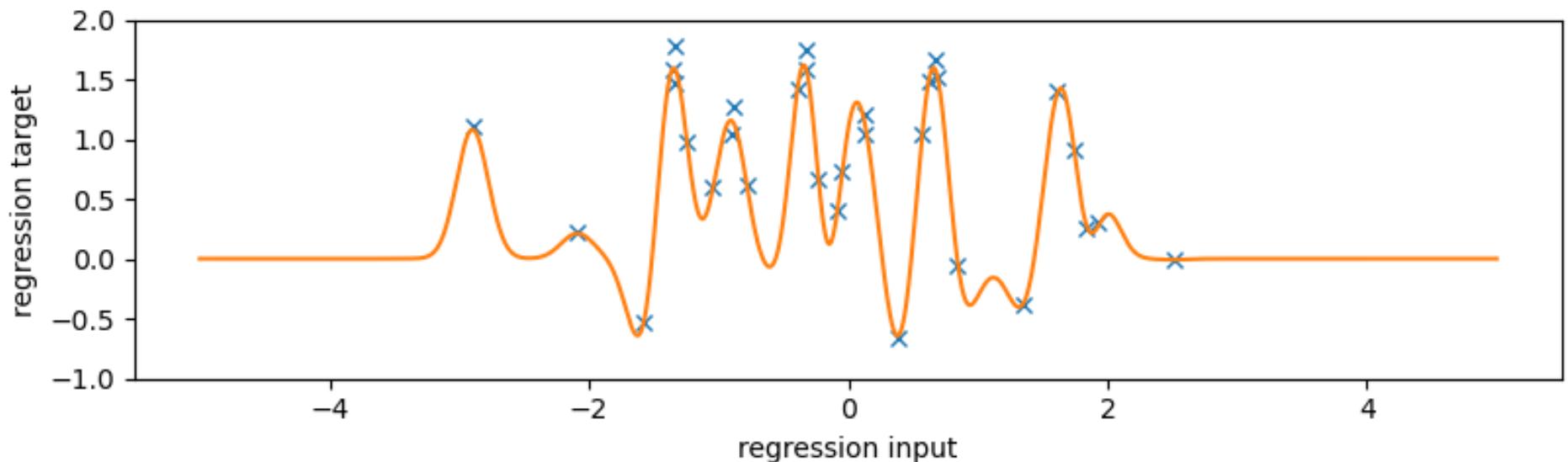
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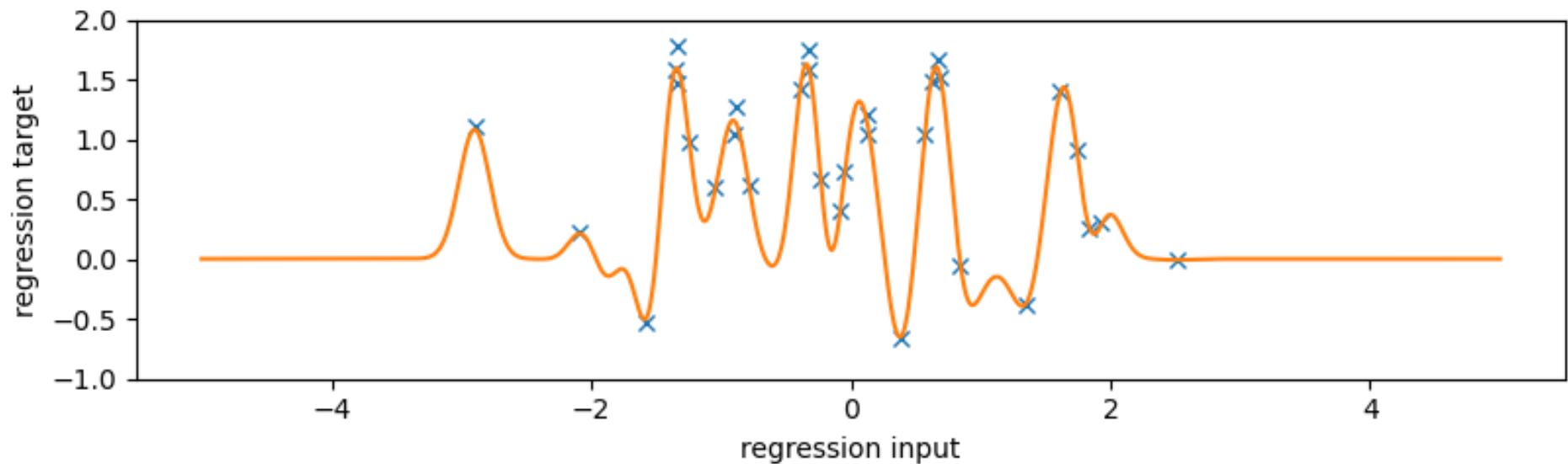
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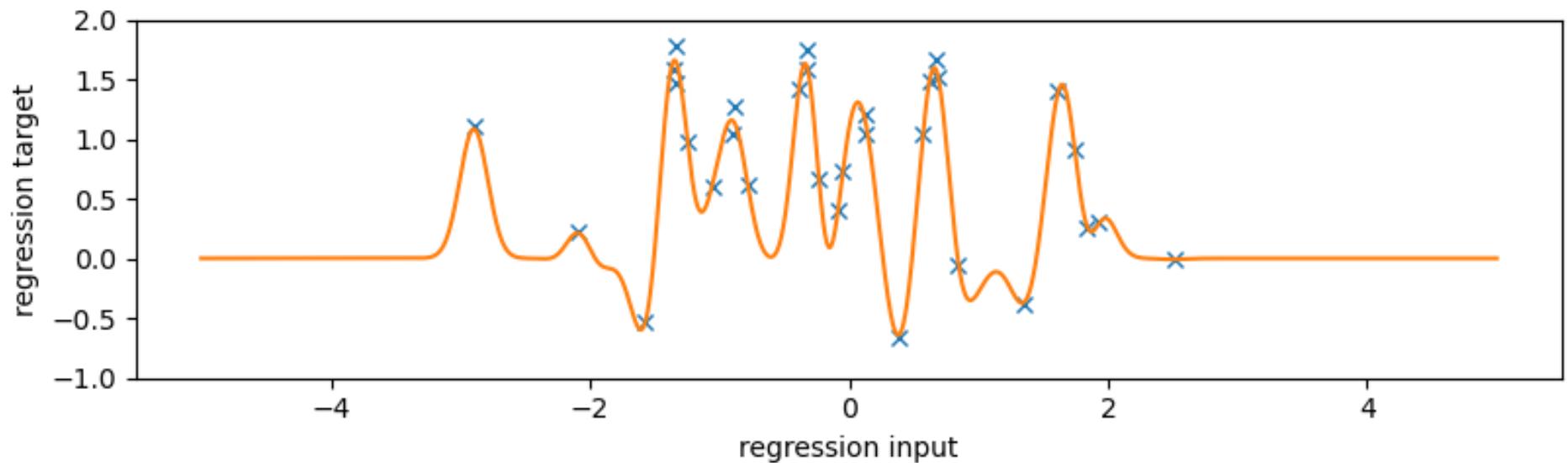
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Training Loss / MaxLik is not sufficient

If we train weights W only, given (θ, M) by

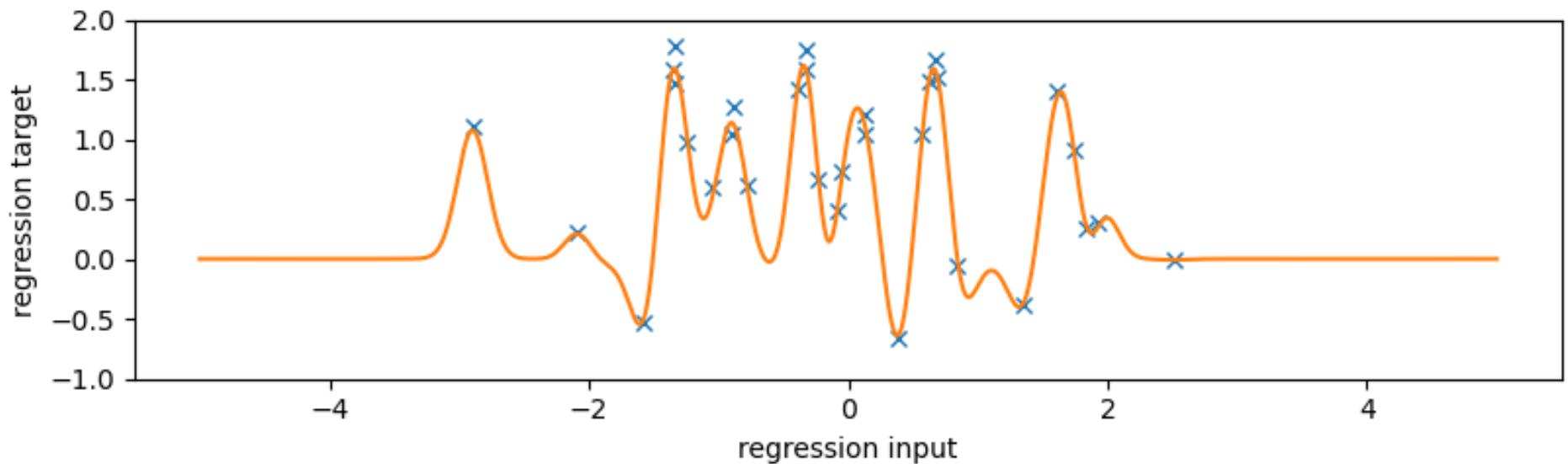
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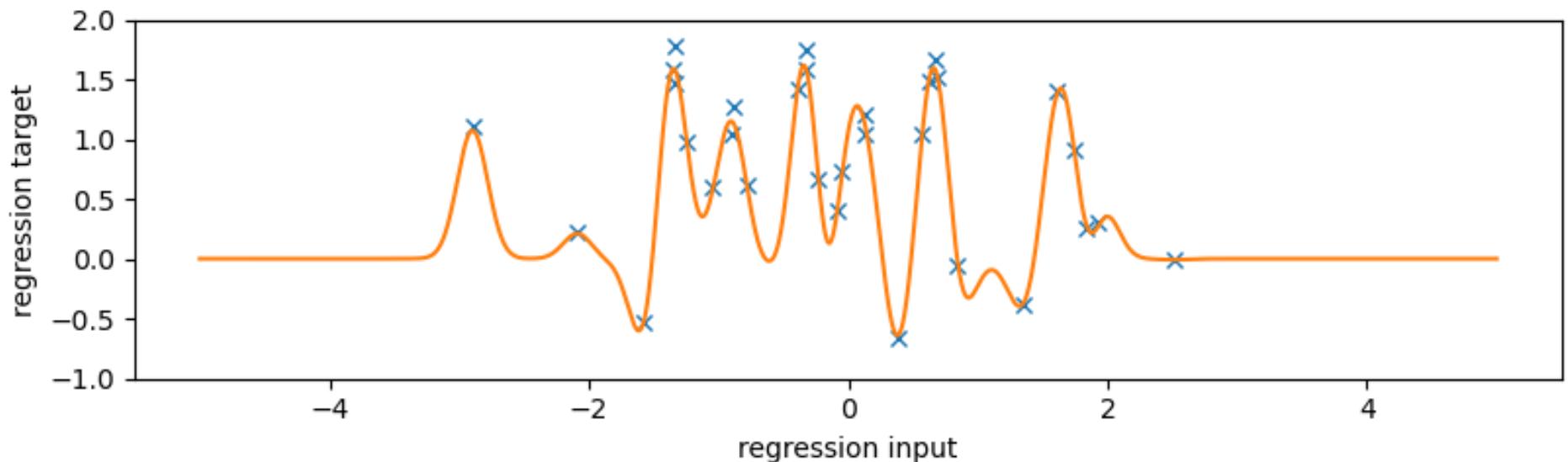
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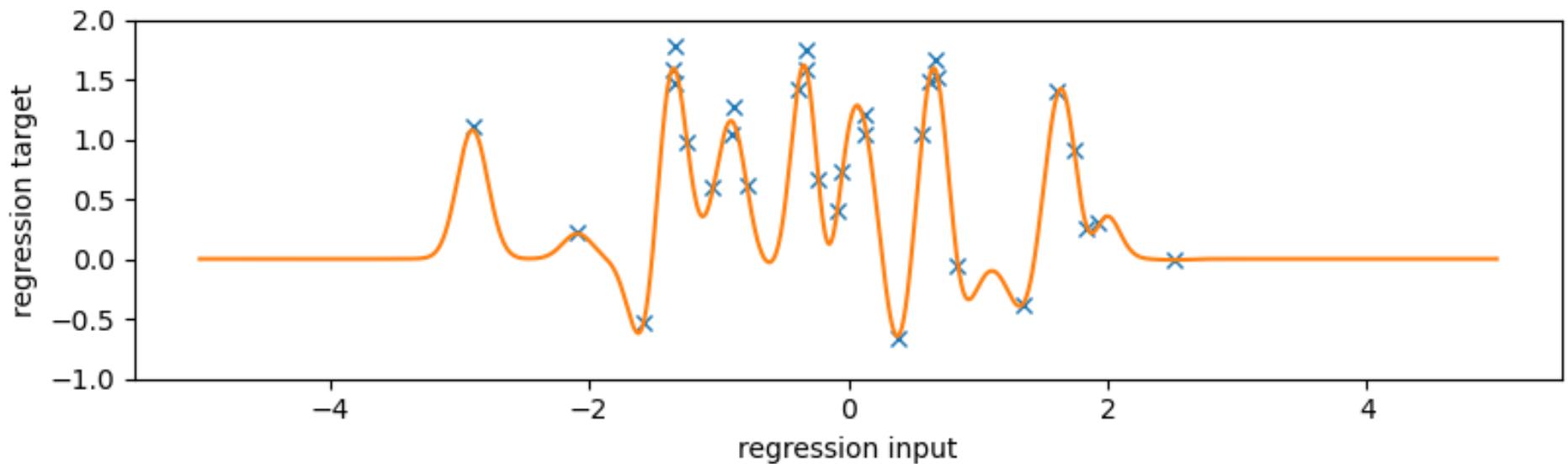
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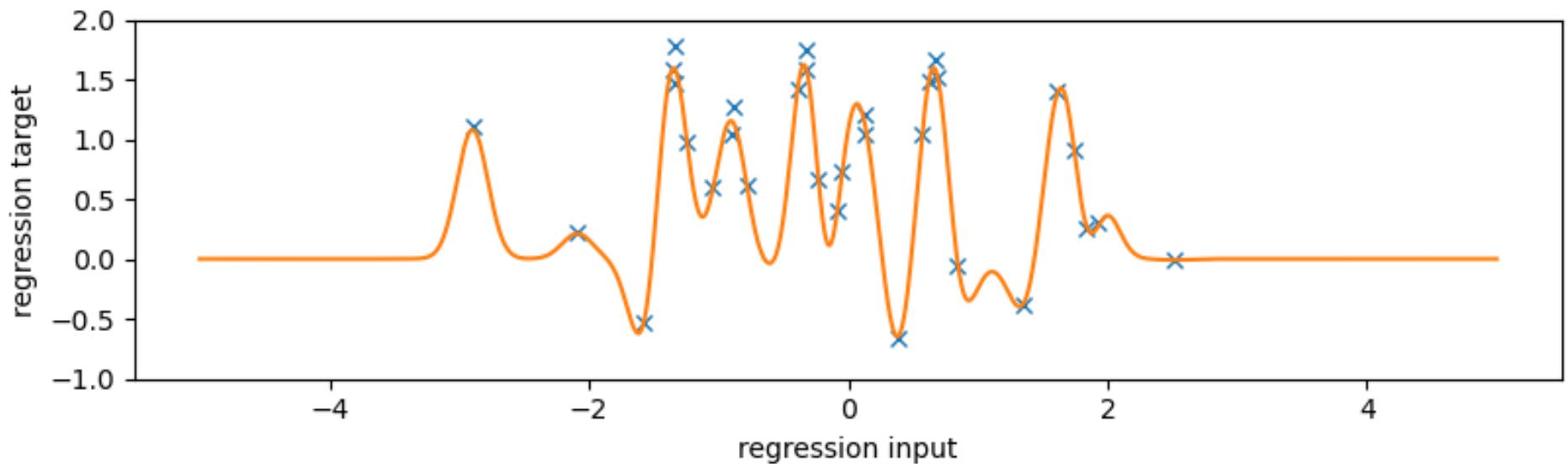
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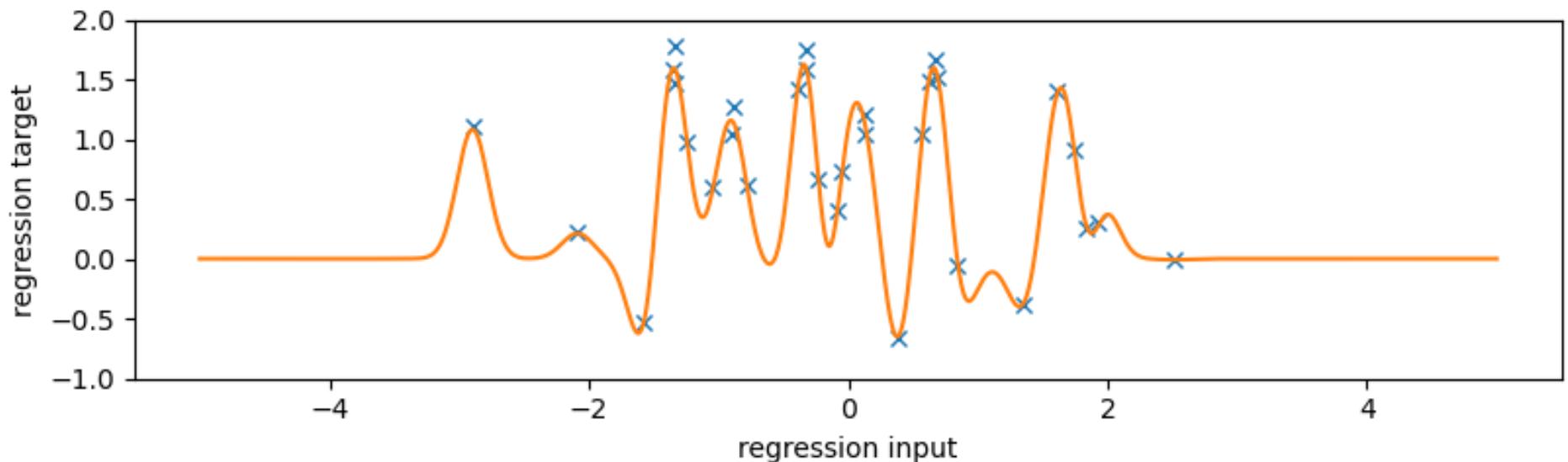
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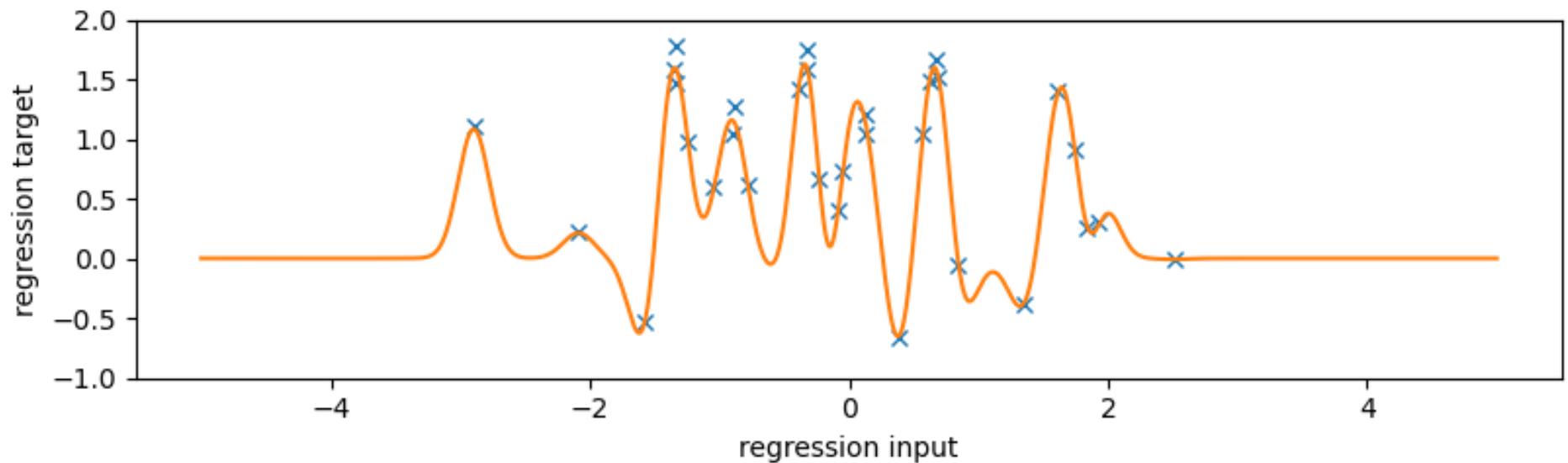
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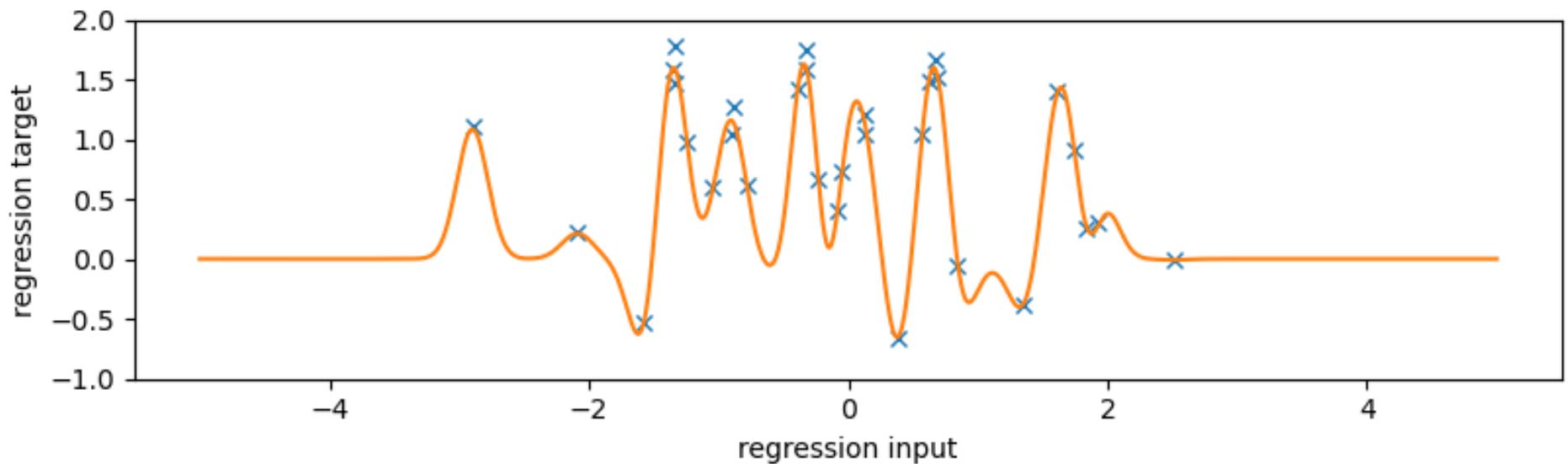
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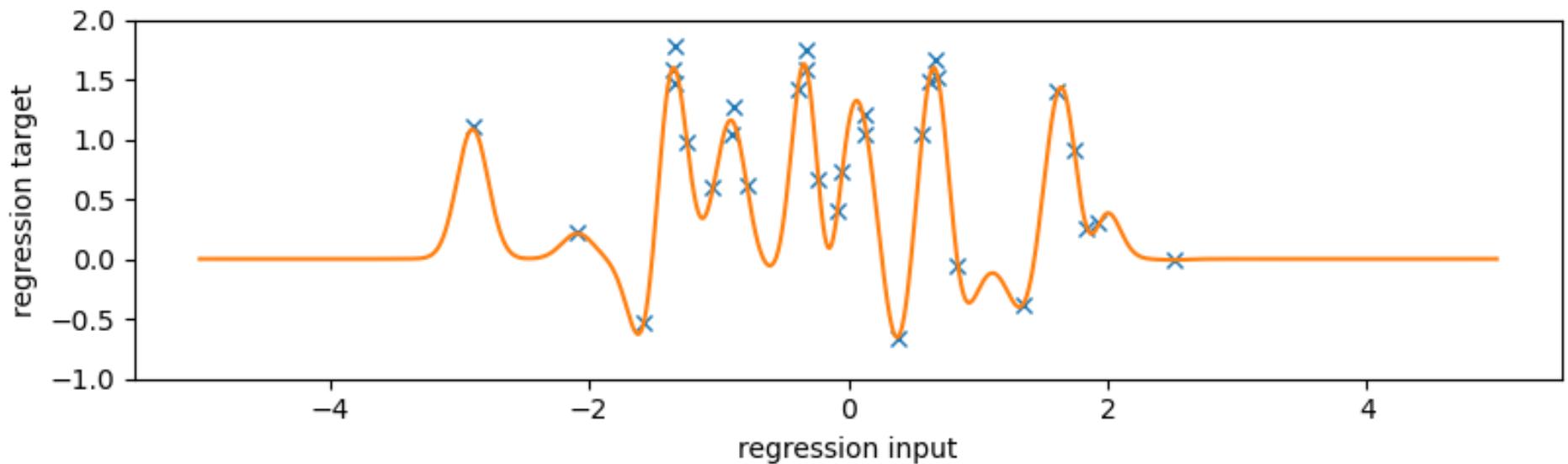
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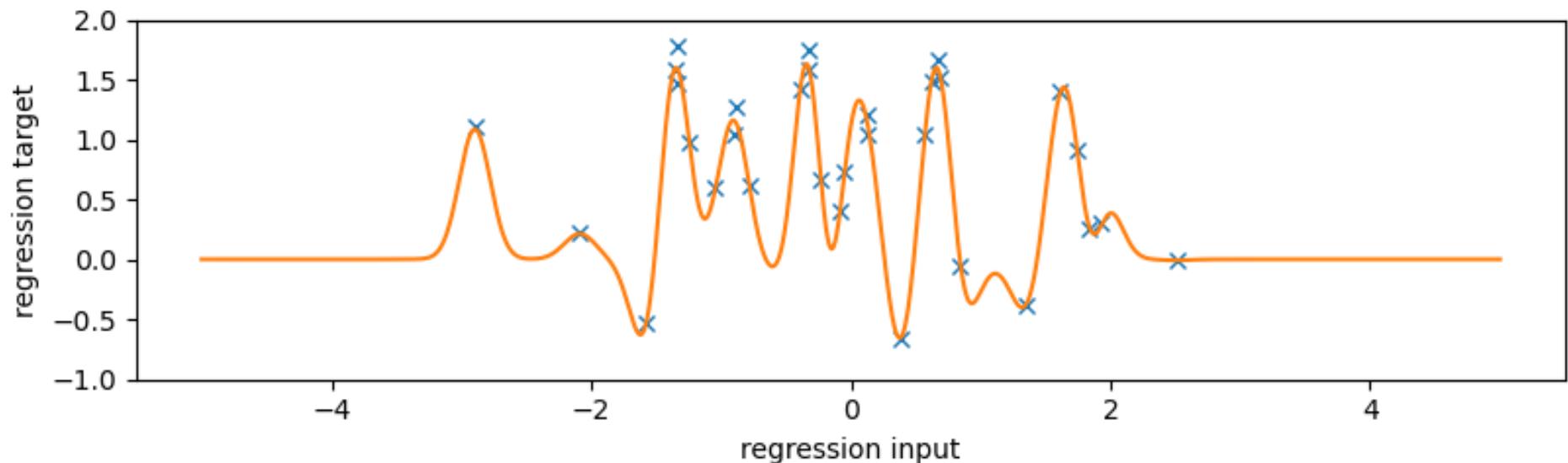
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Training Loss / MaxLik is not sufficient

If we train weights W only, *given* (θ, M) by

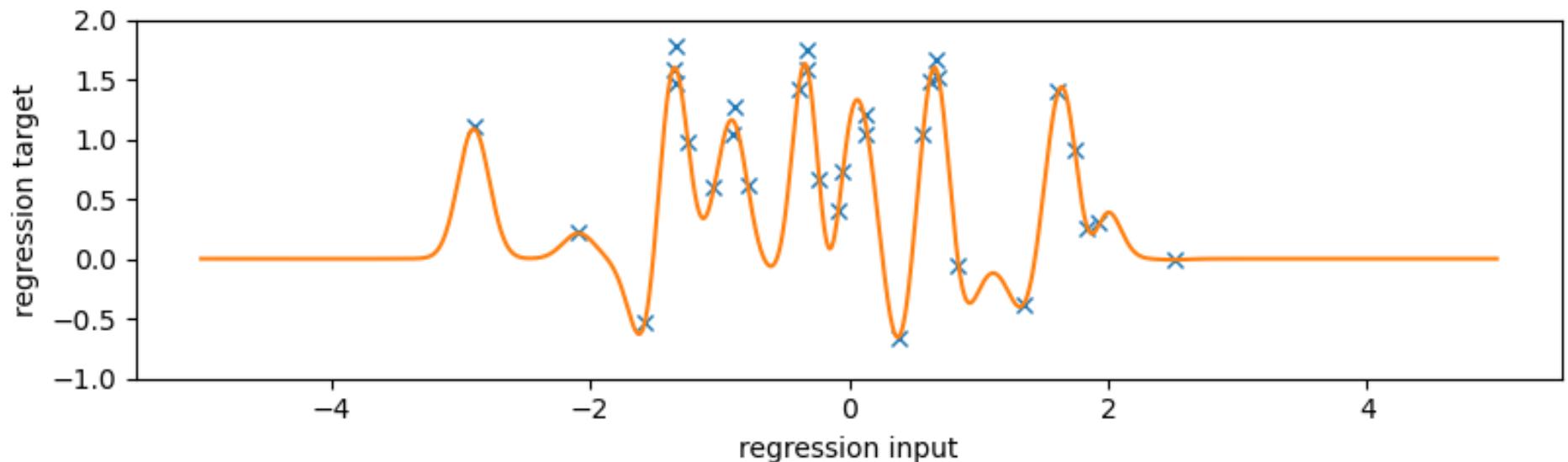
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Training Loss / MaxLik is not sufficient

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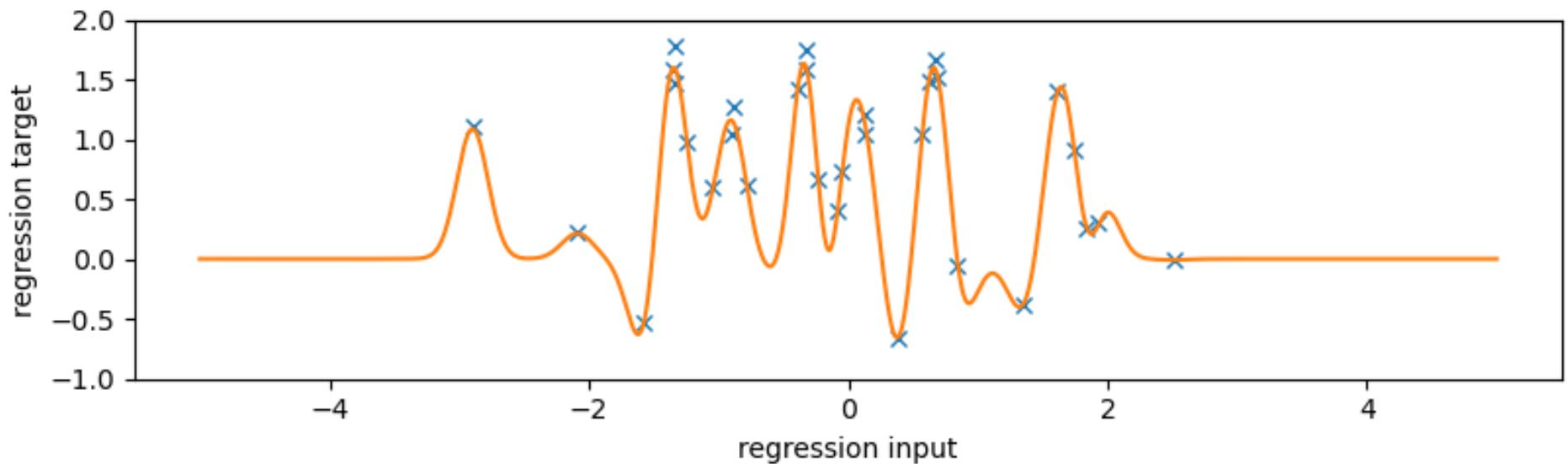
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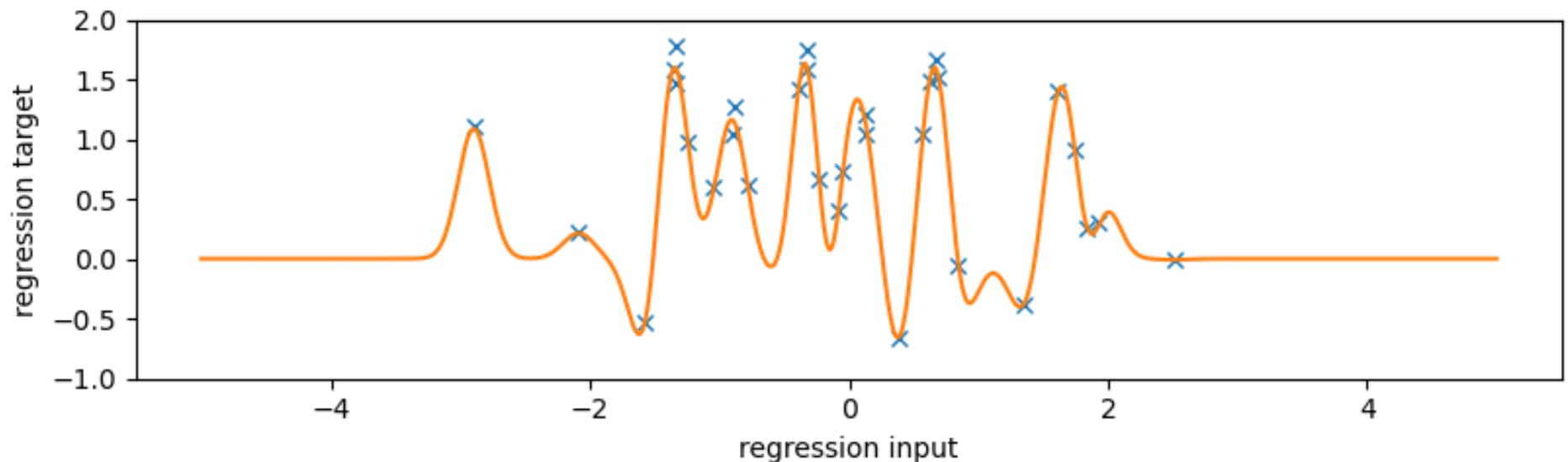
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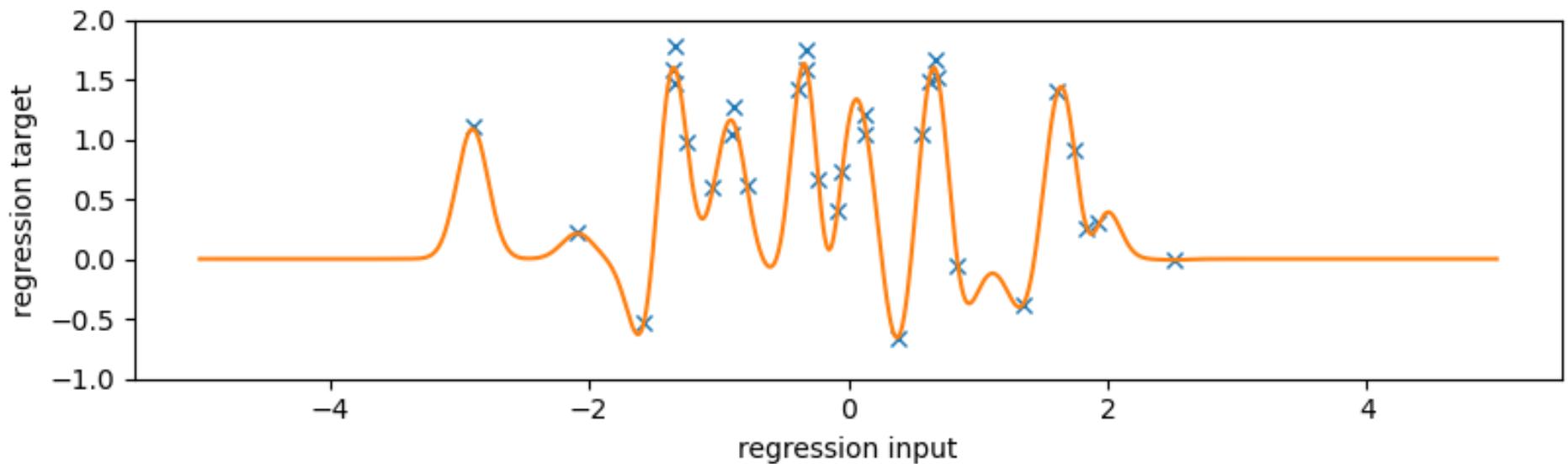
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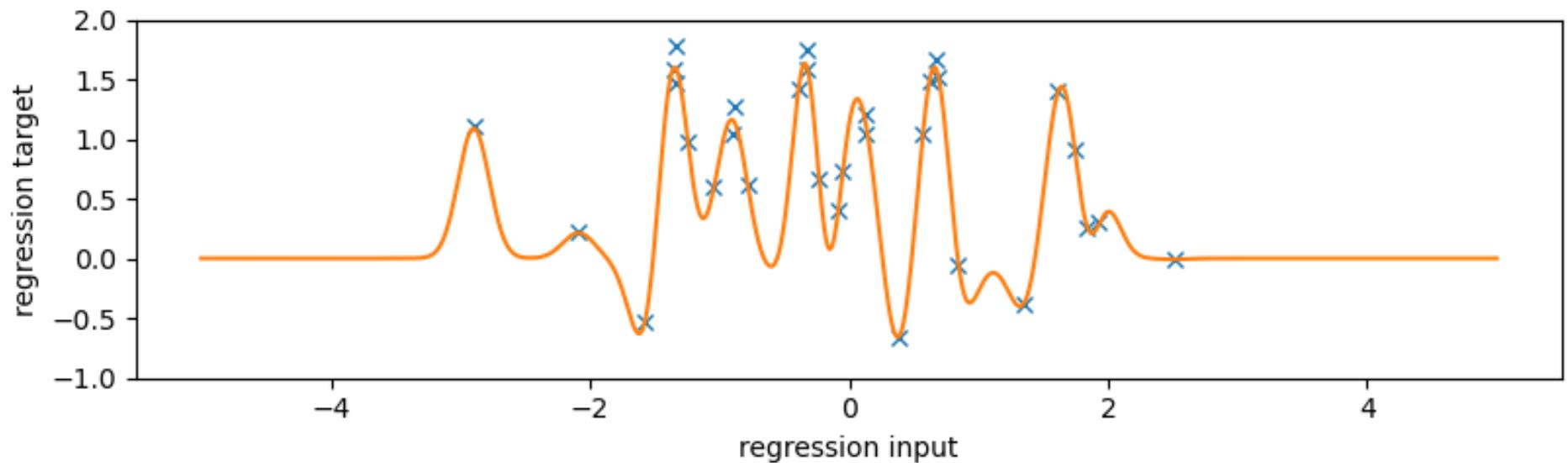
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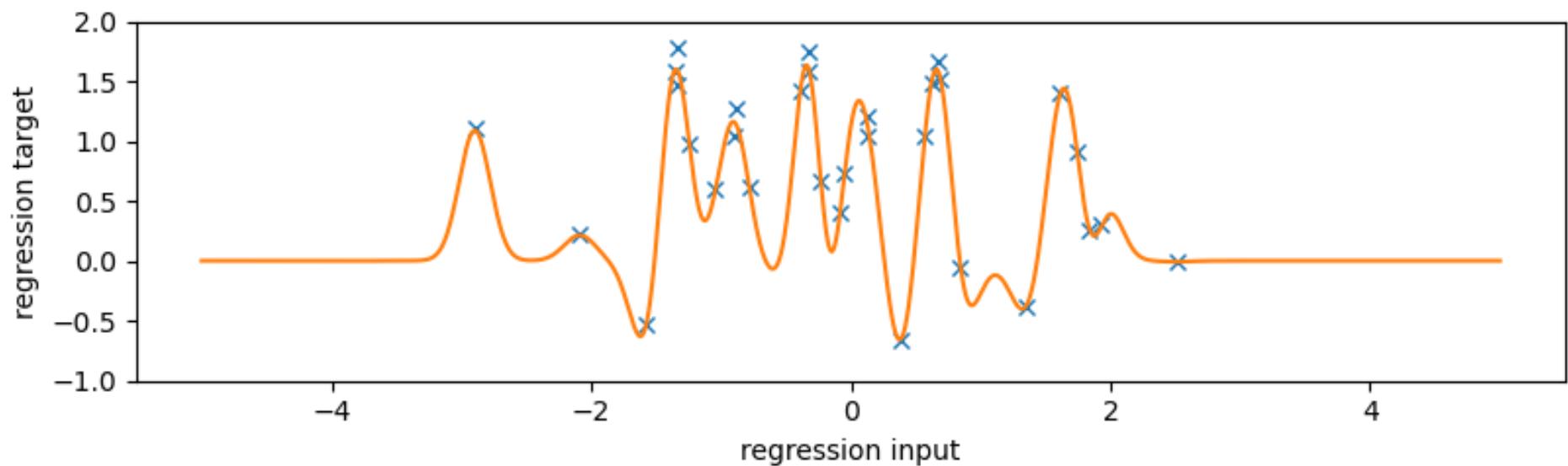
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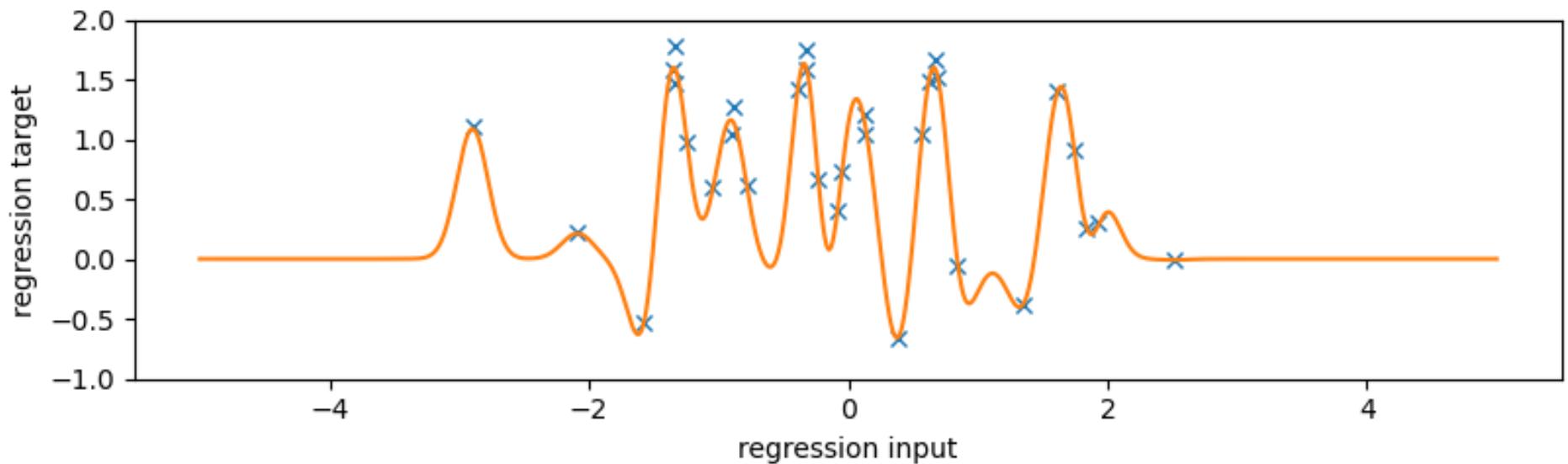
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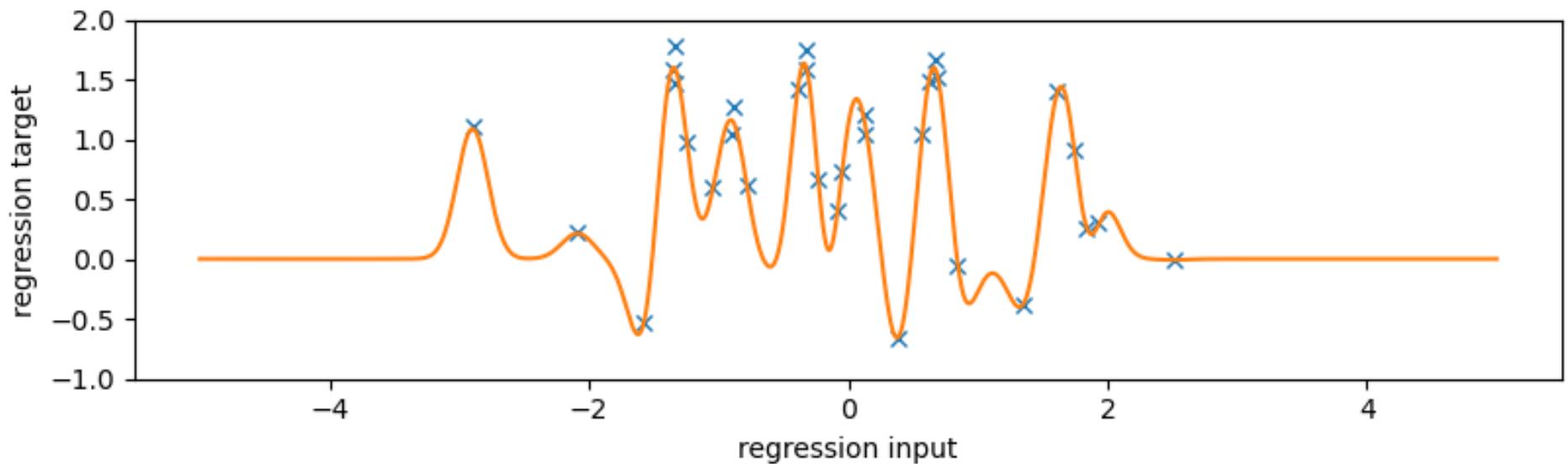
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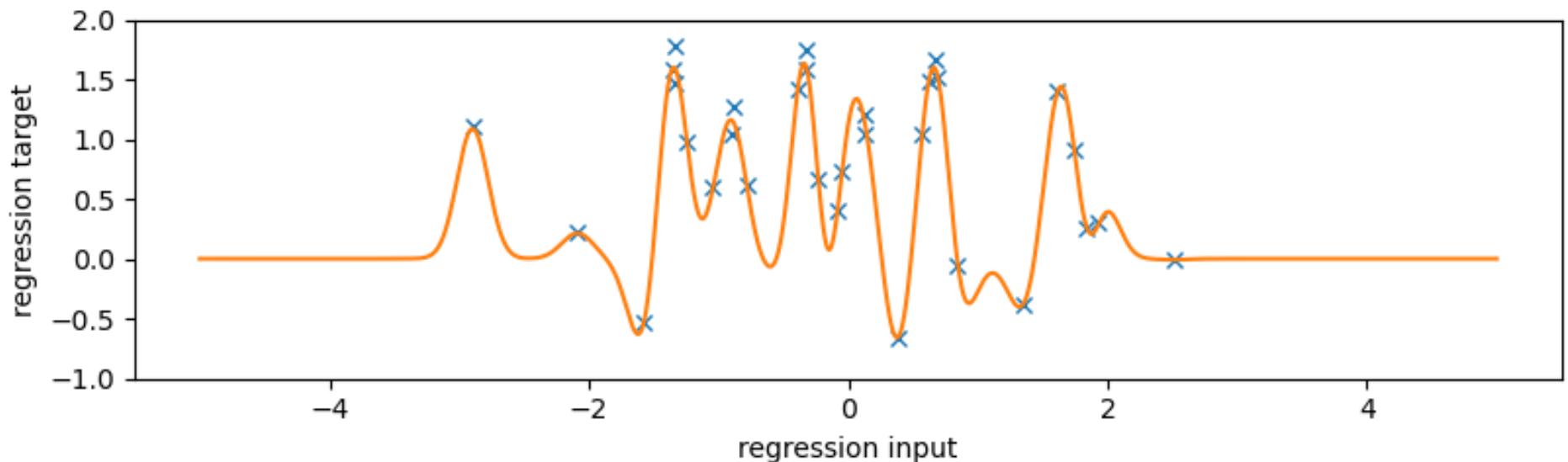
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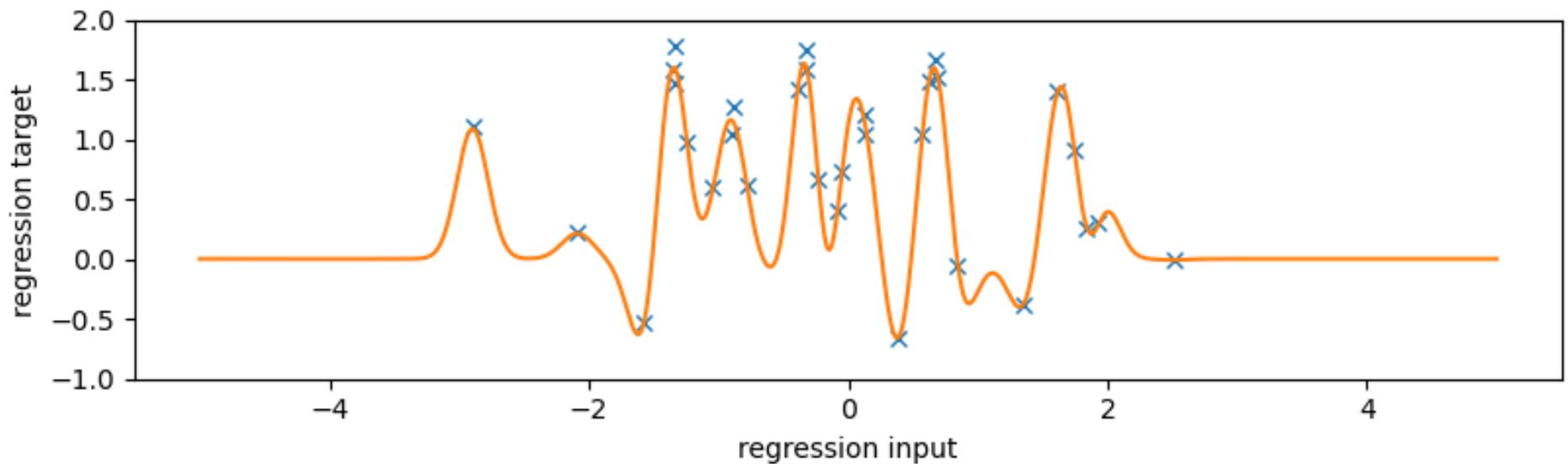
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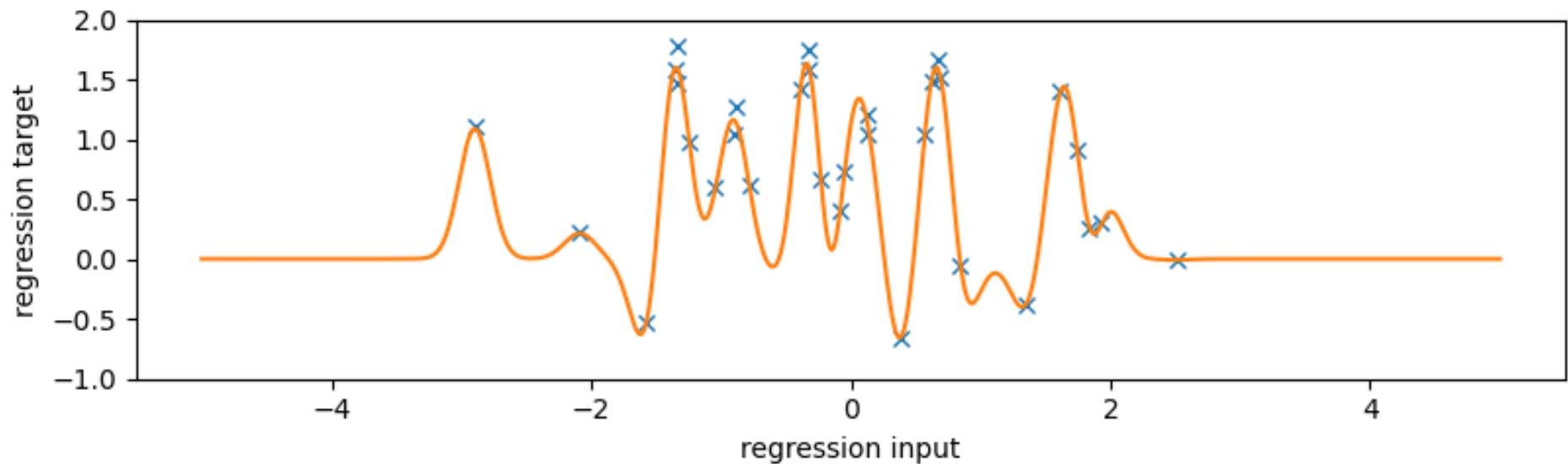
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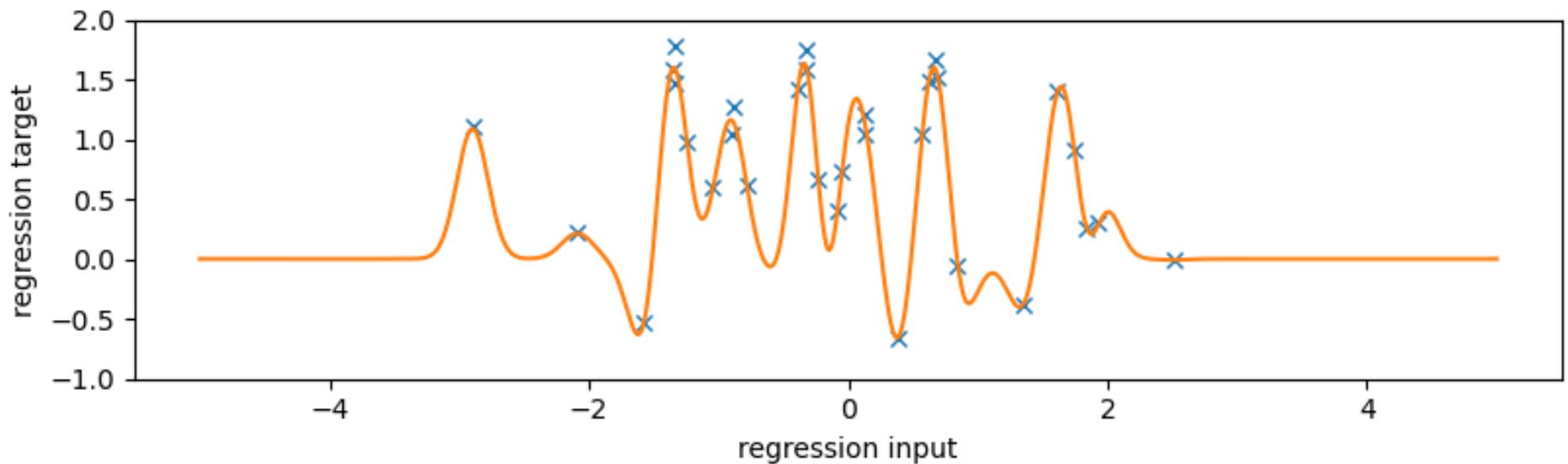
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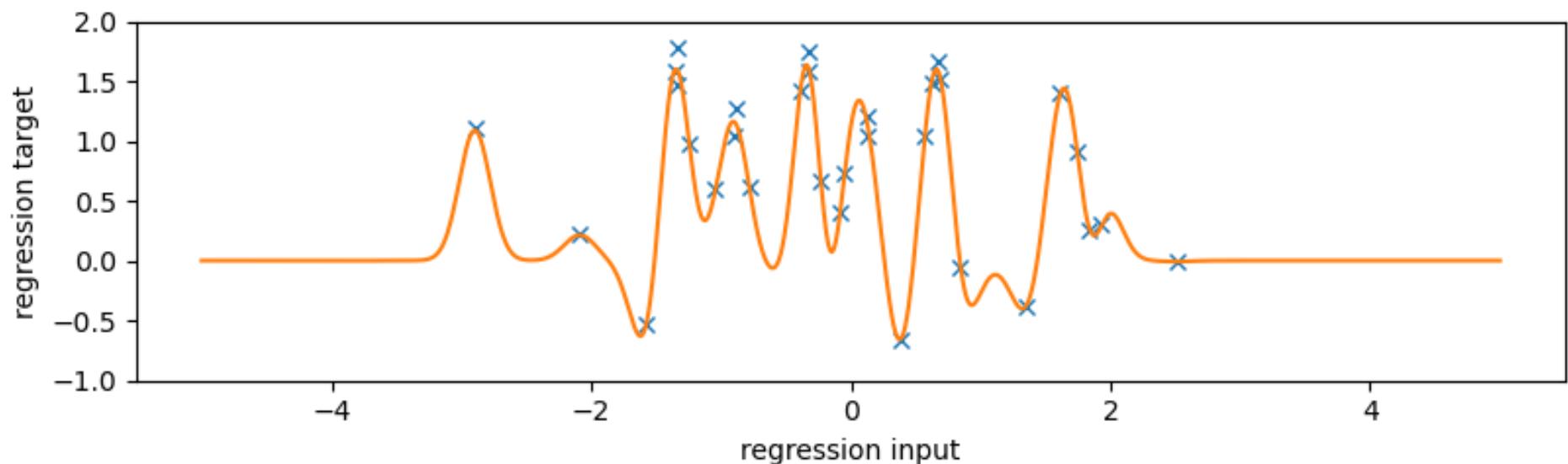
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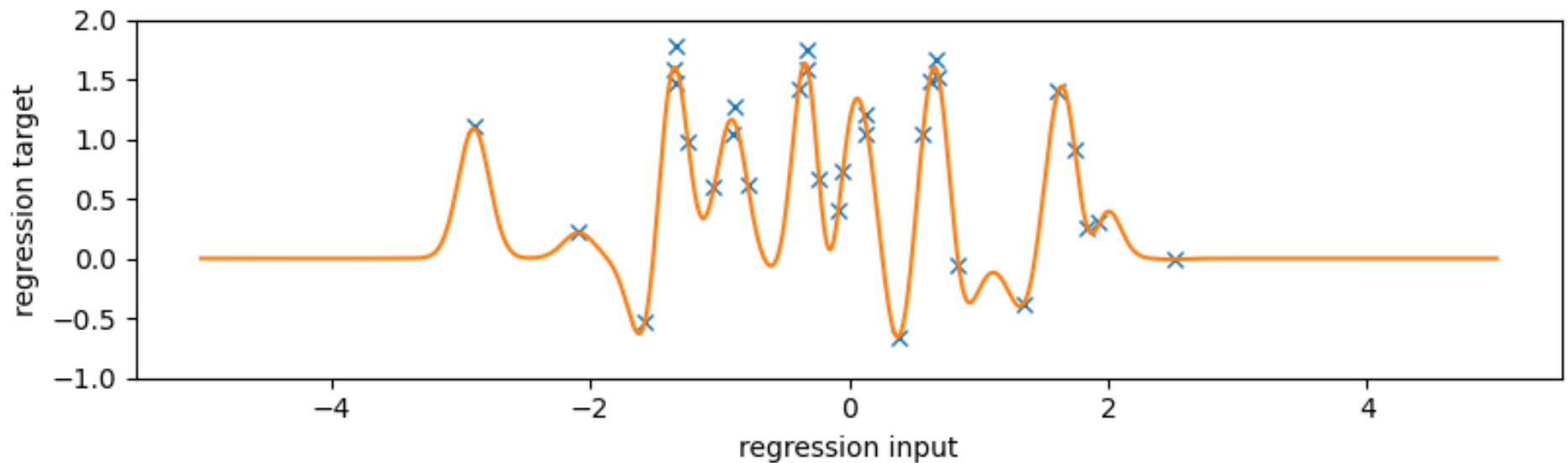
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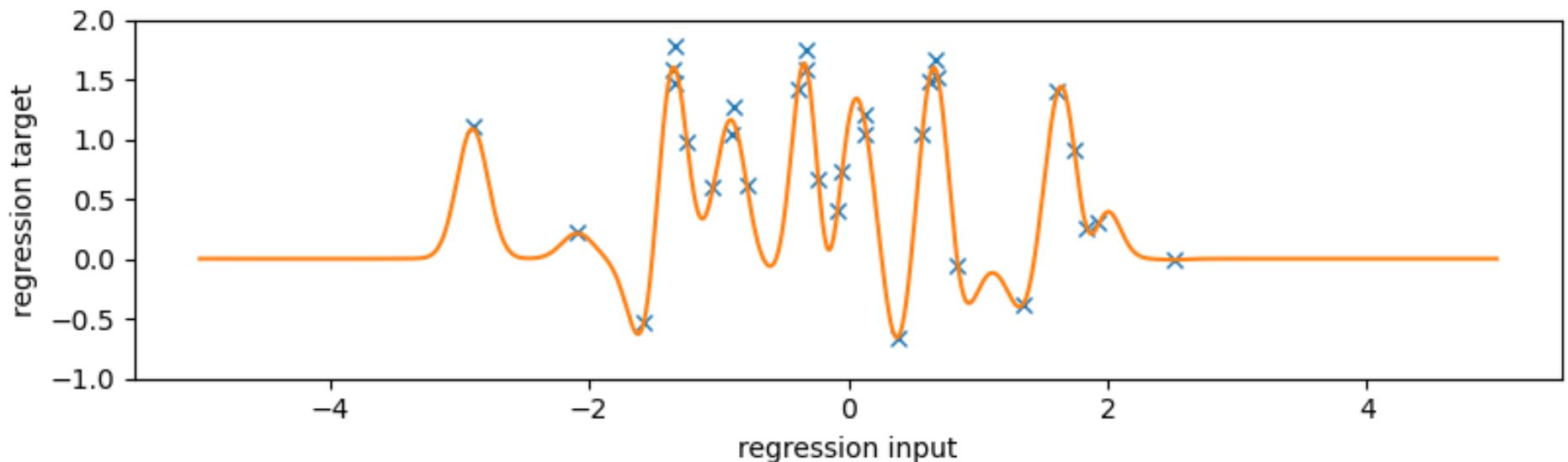
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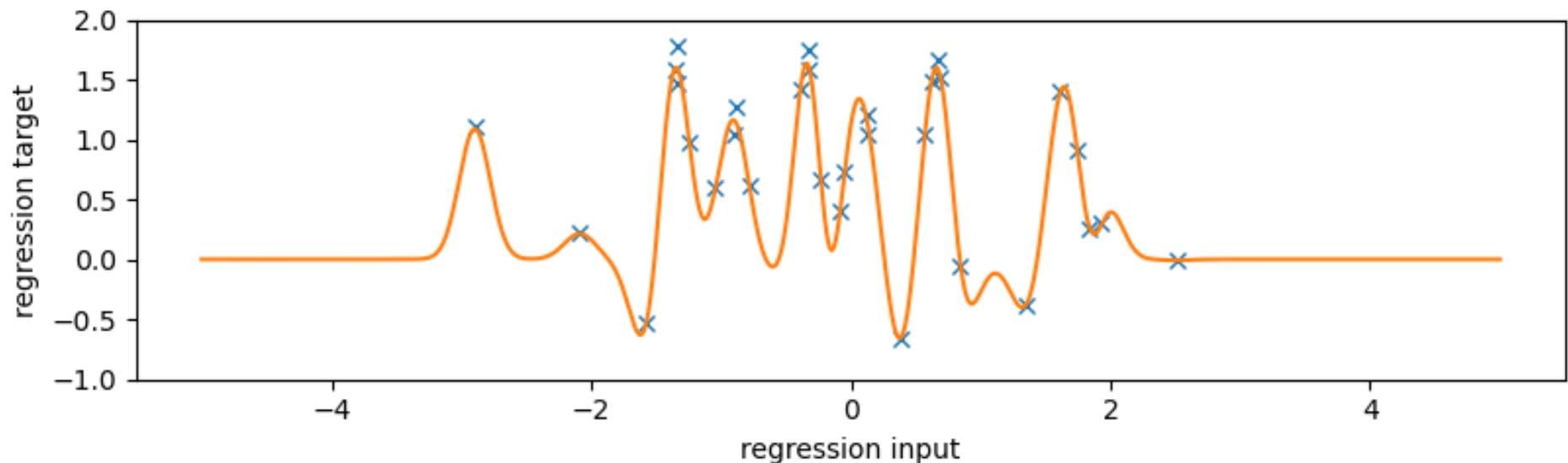
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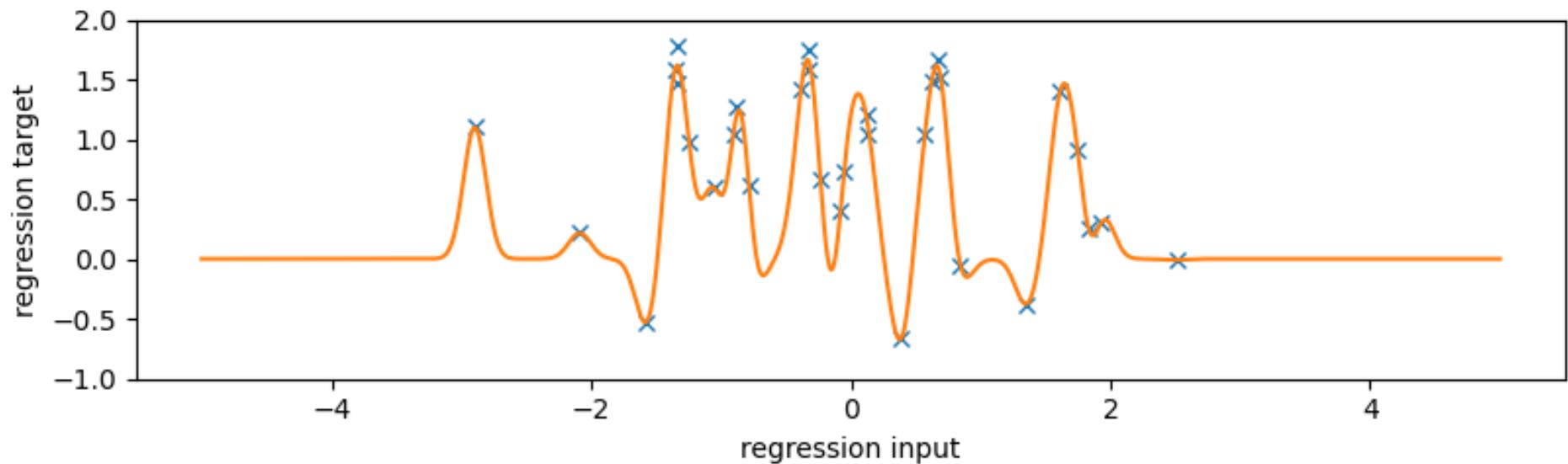
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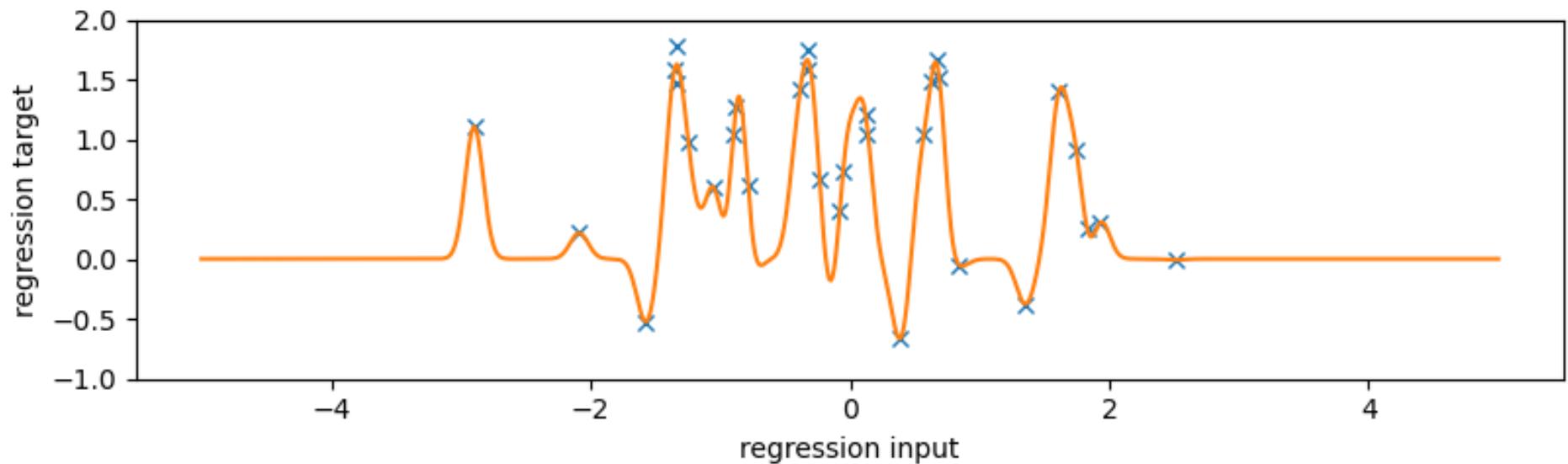
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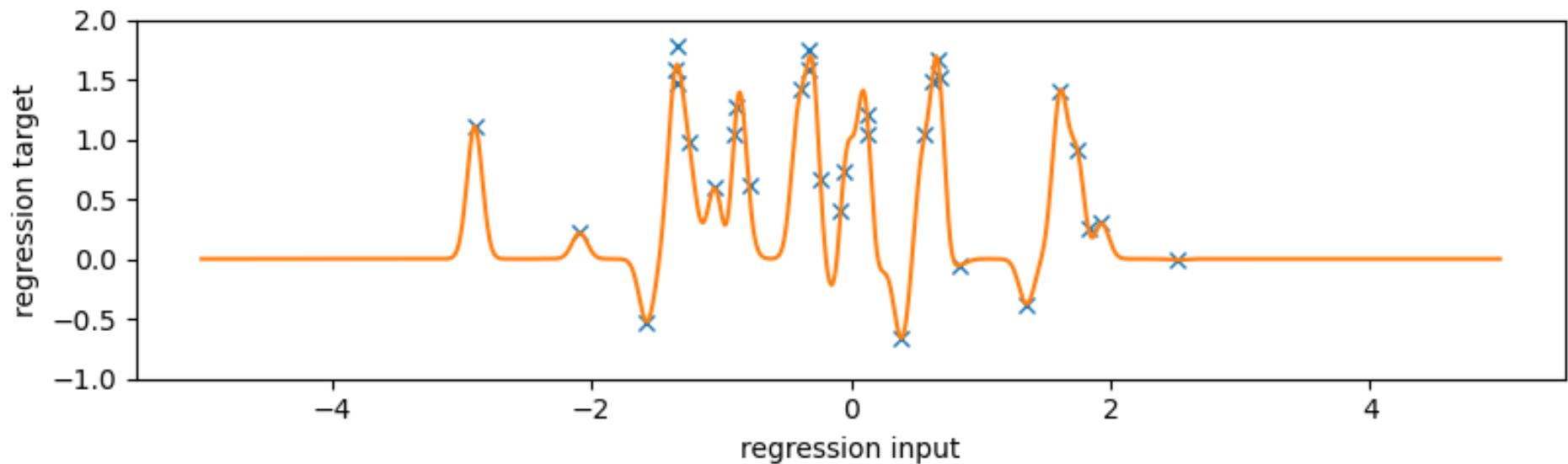
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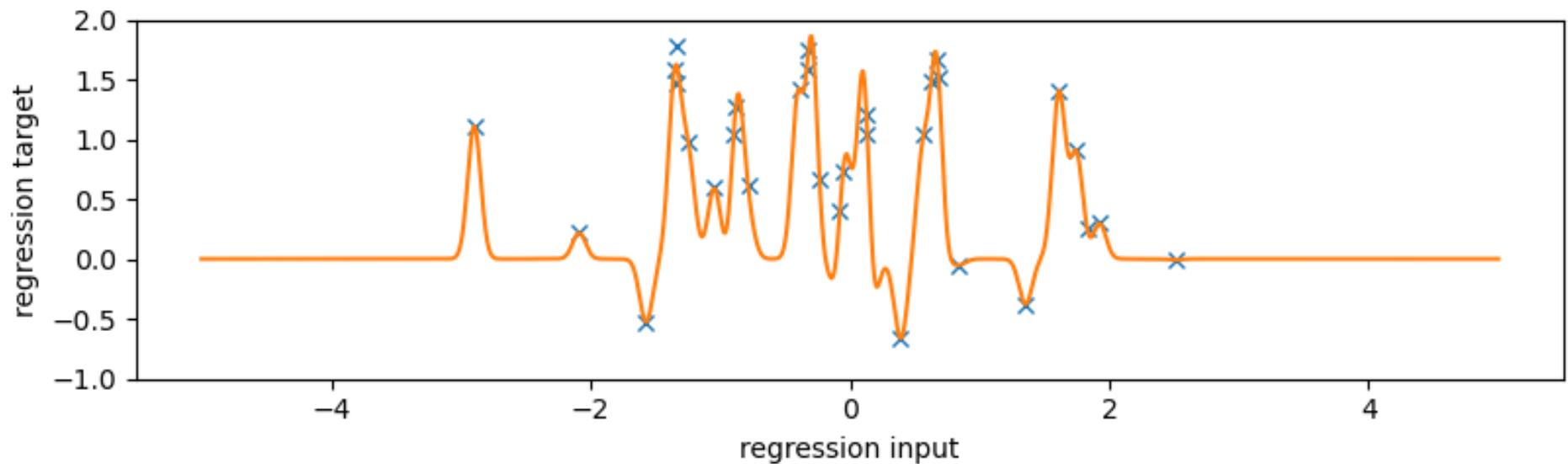
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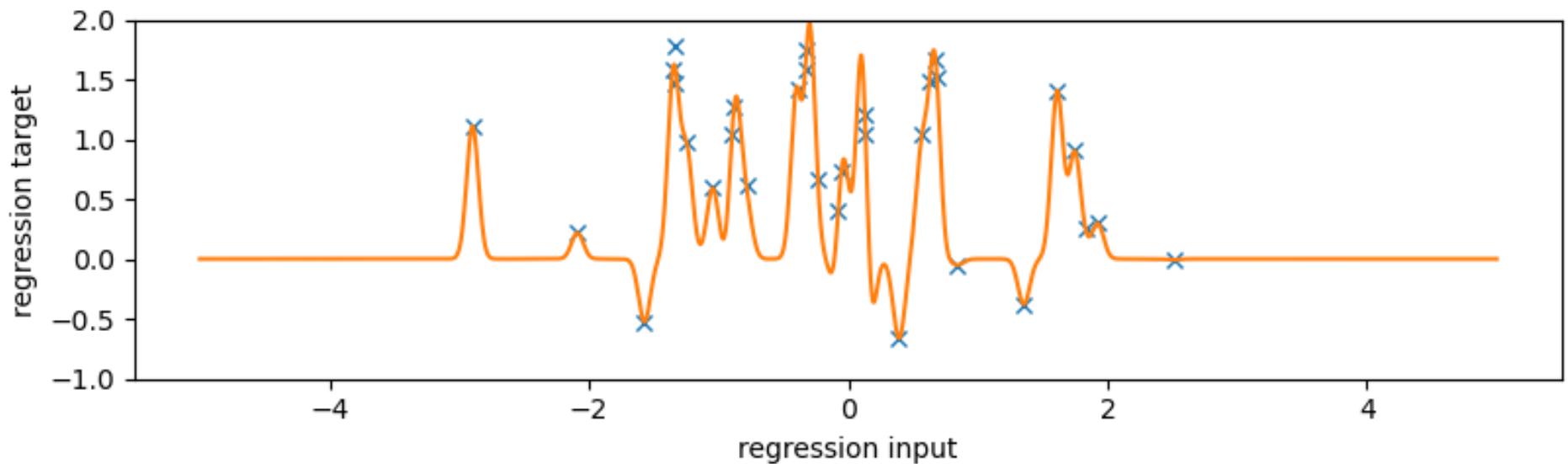
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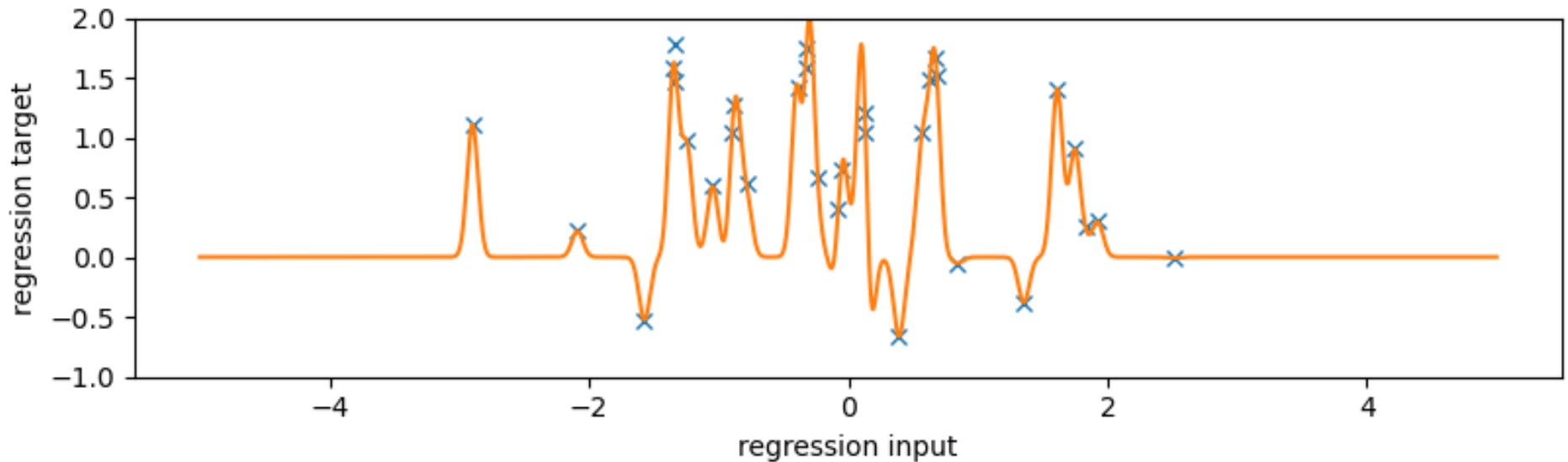
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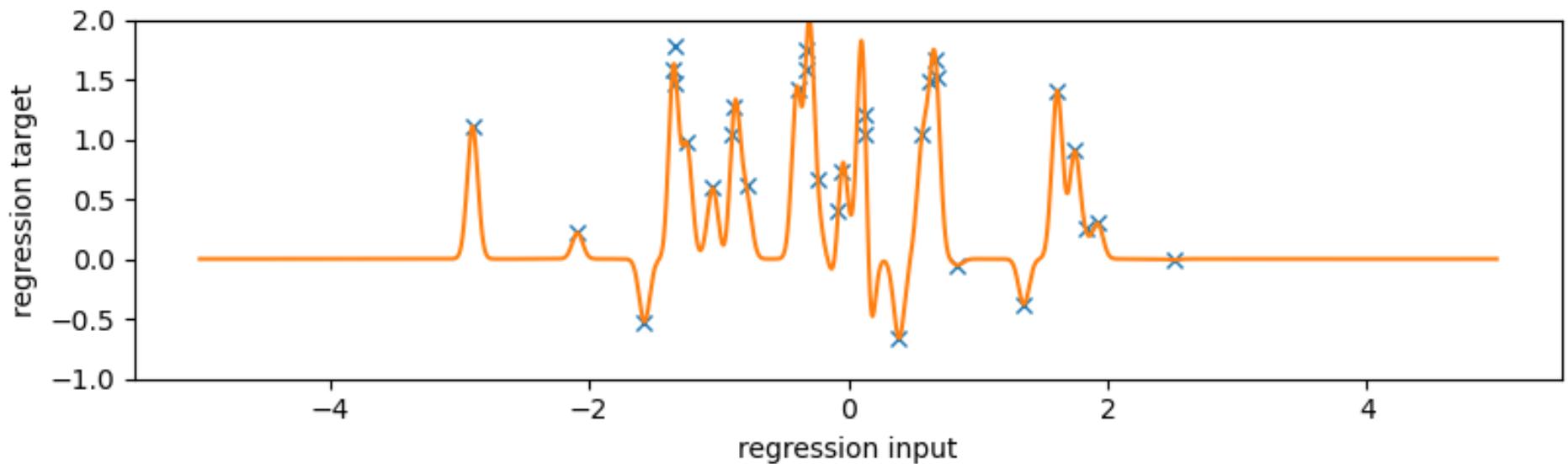
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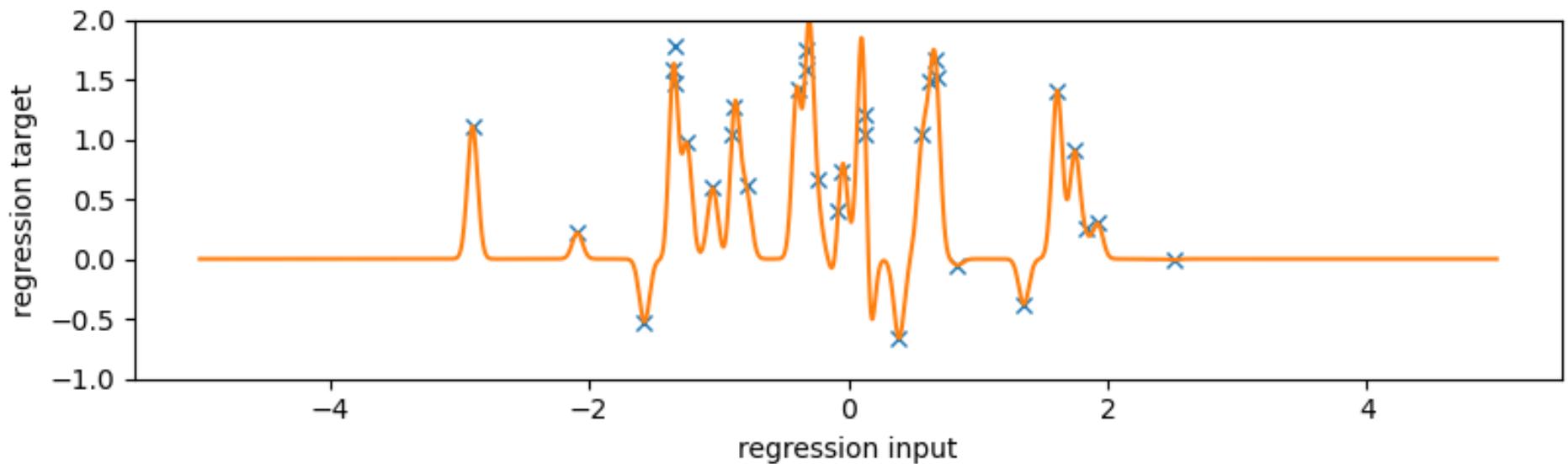
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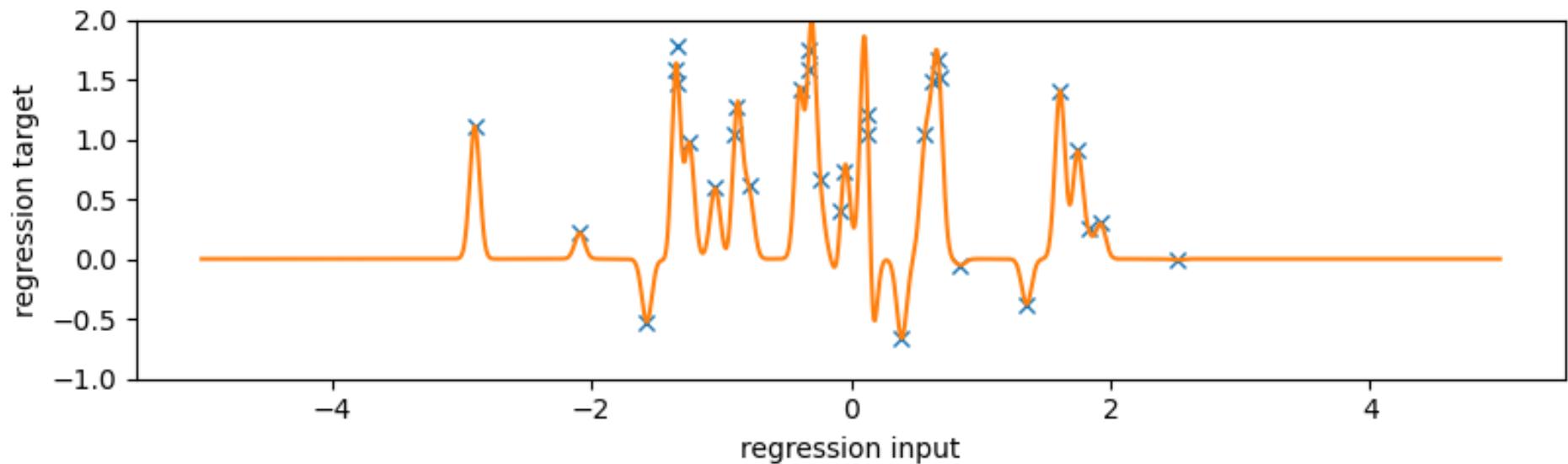
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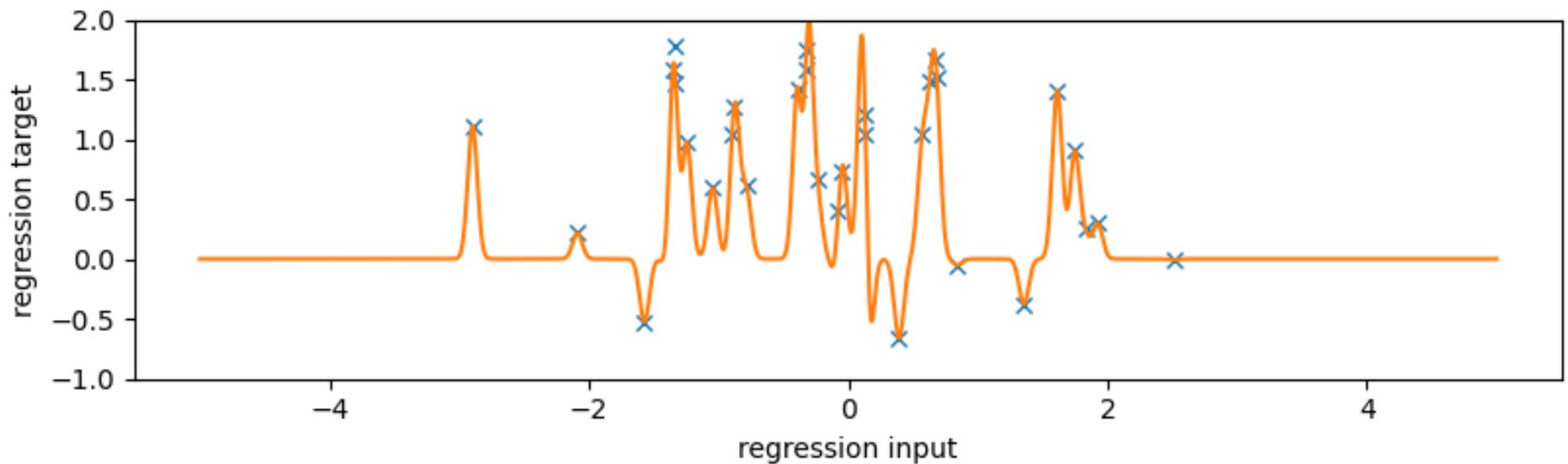
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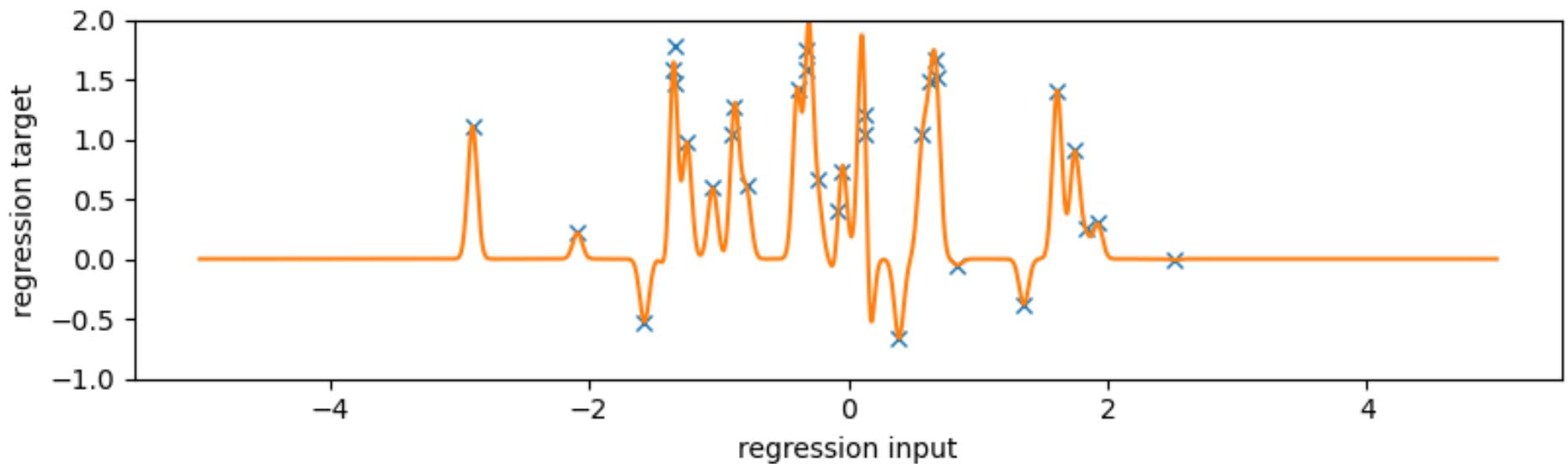
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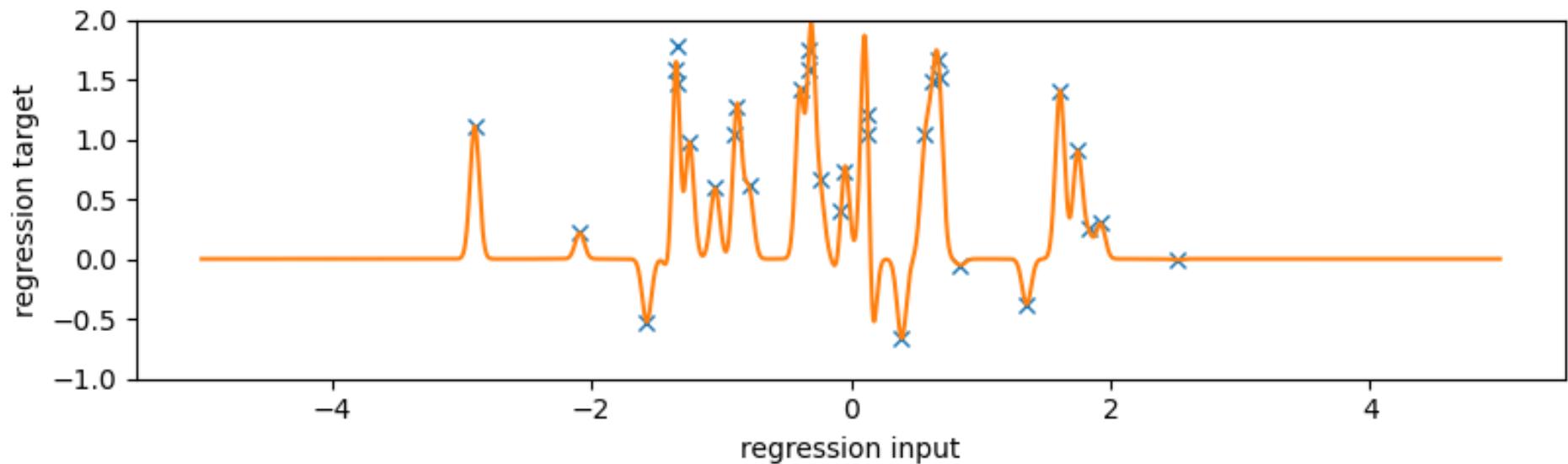
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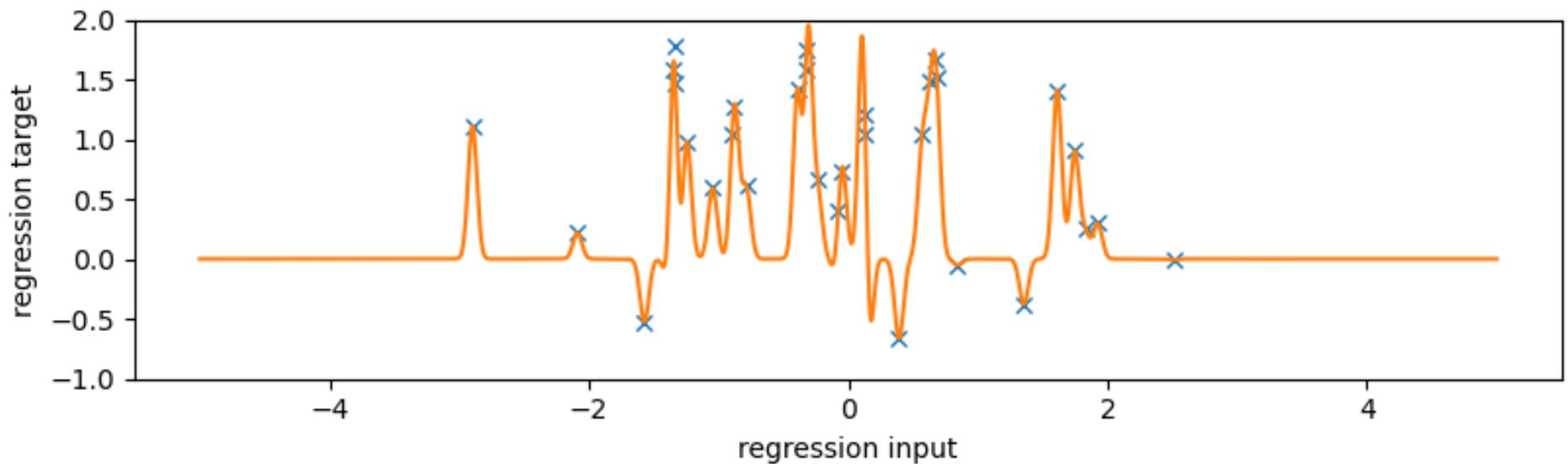
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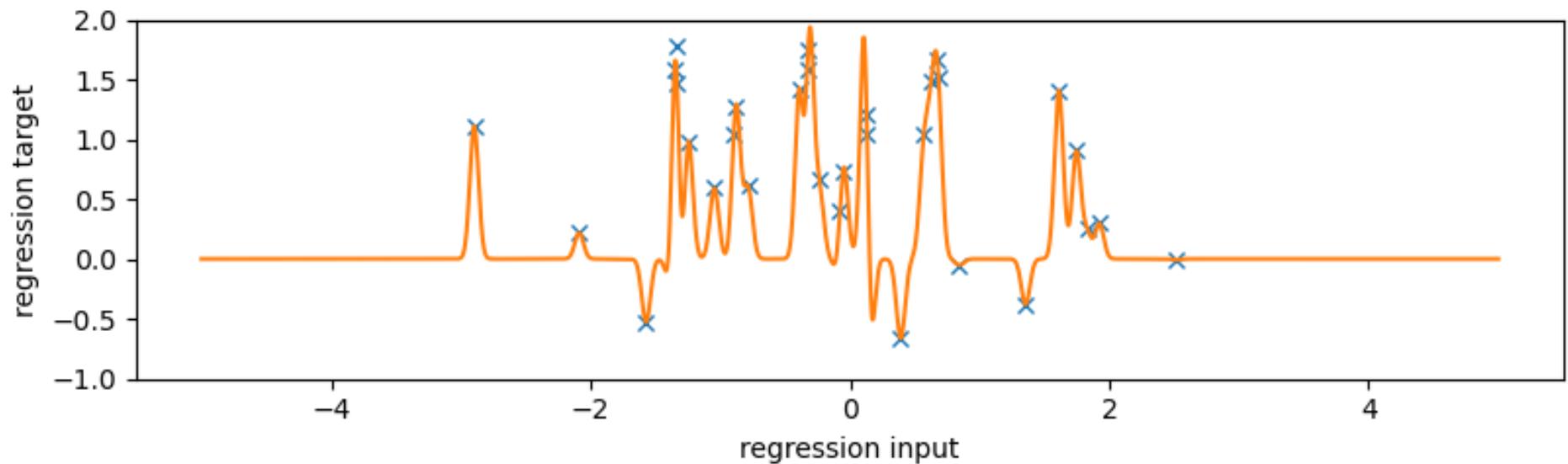
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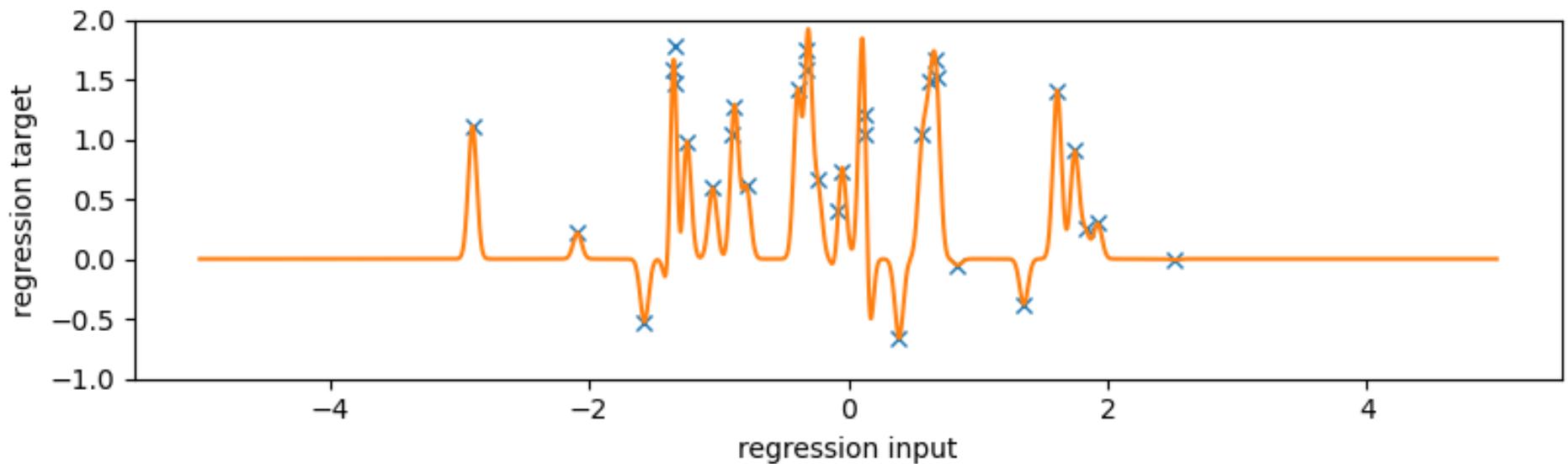
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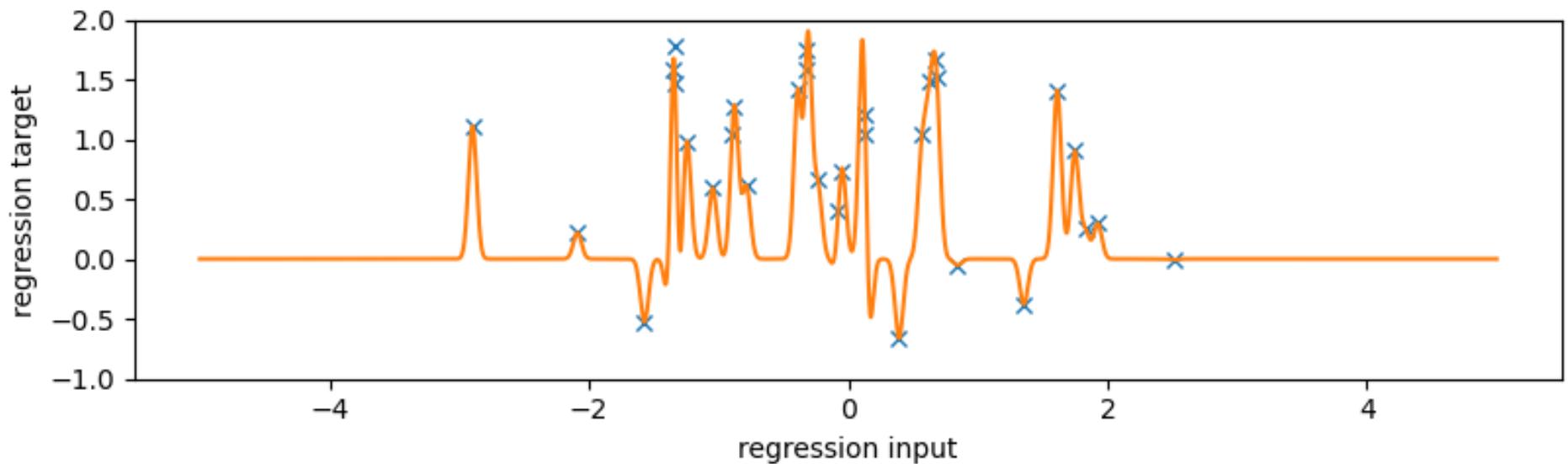
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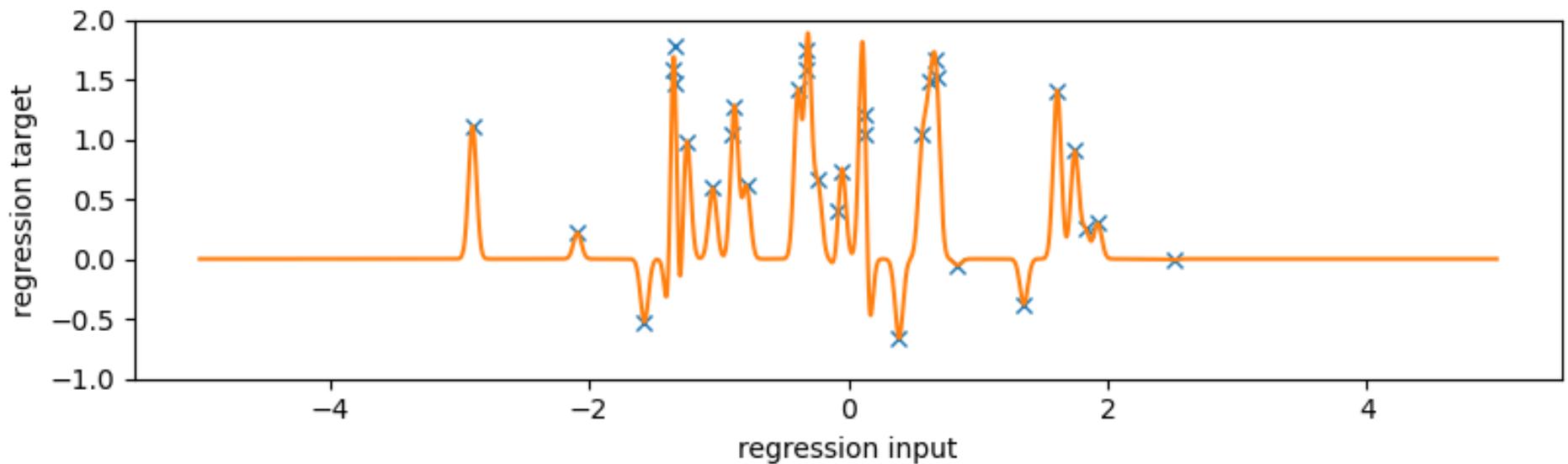
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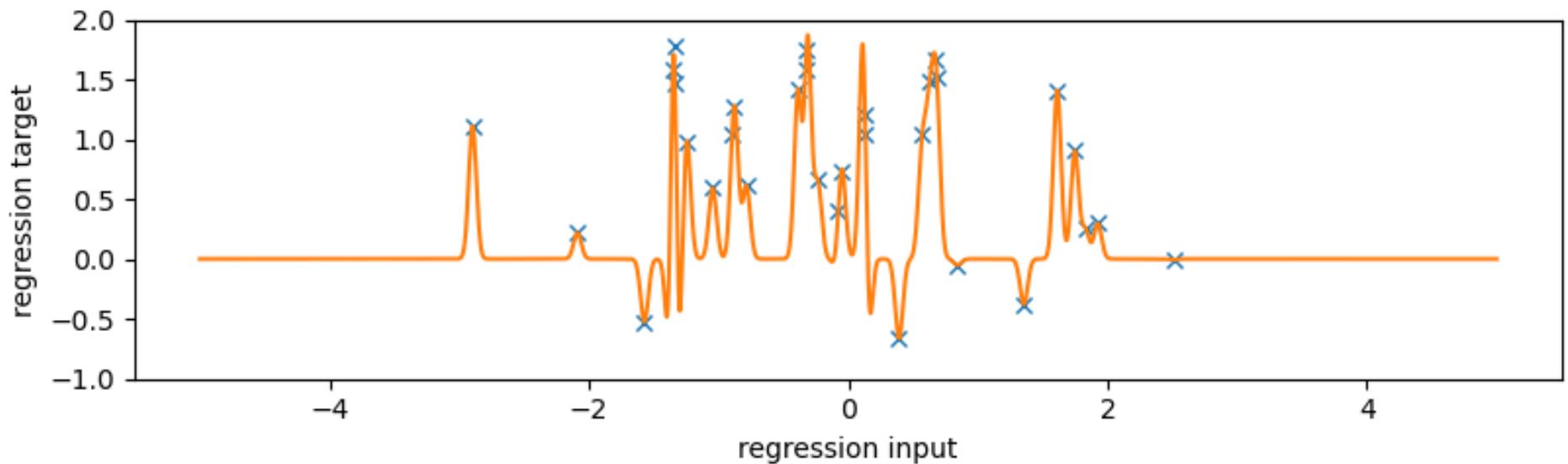
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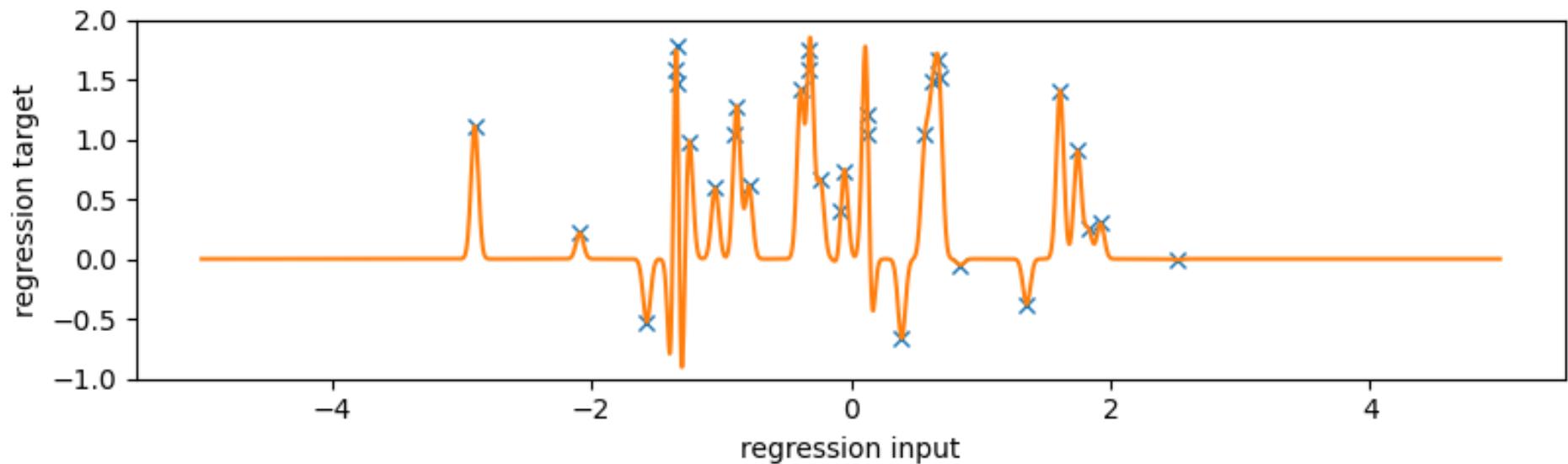
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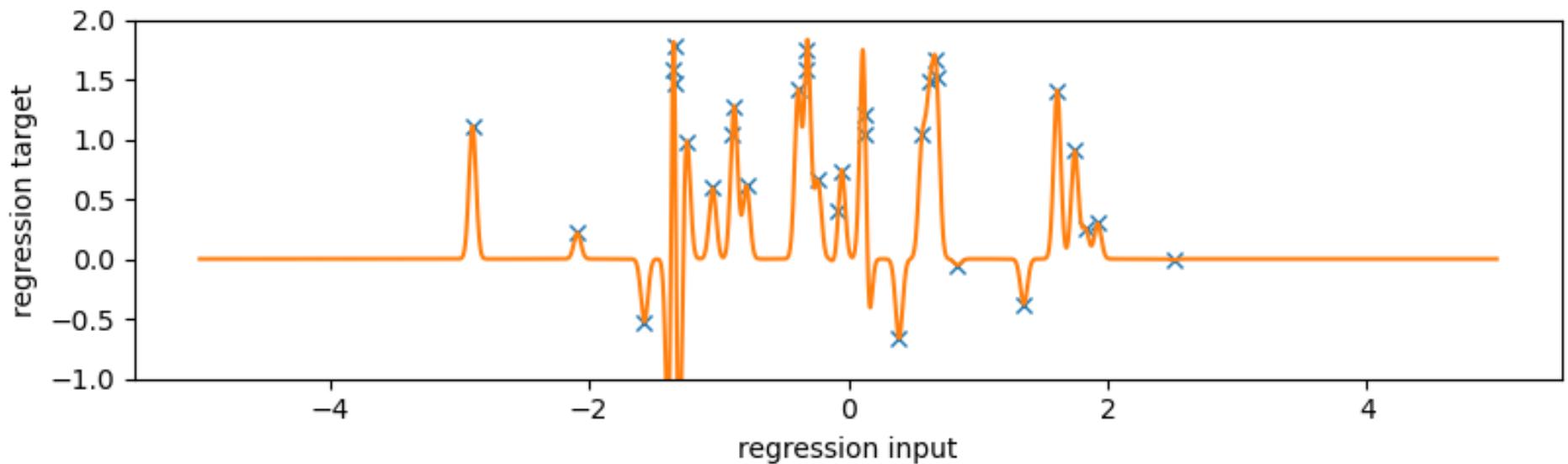
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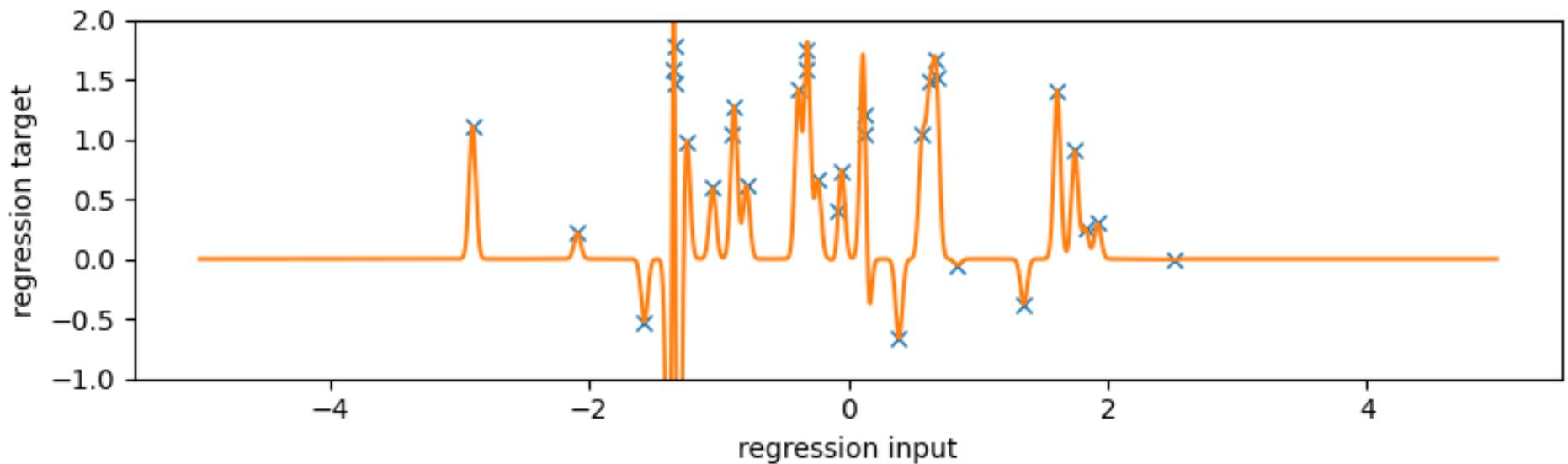
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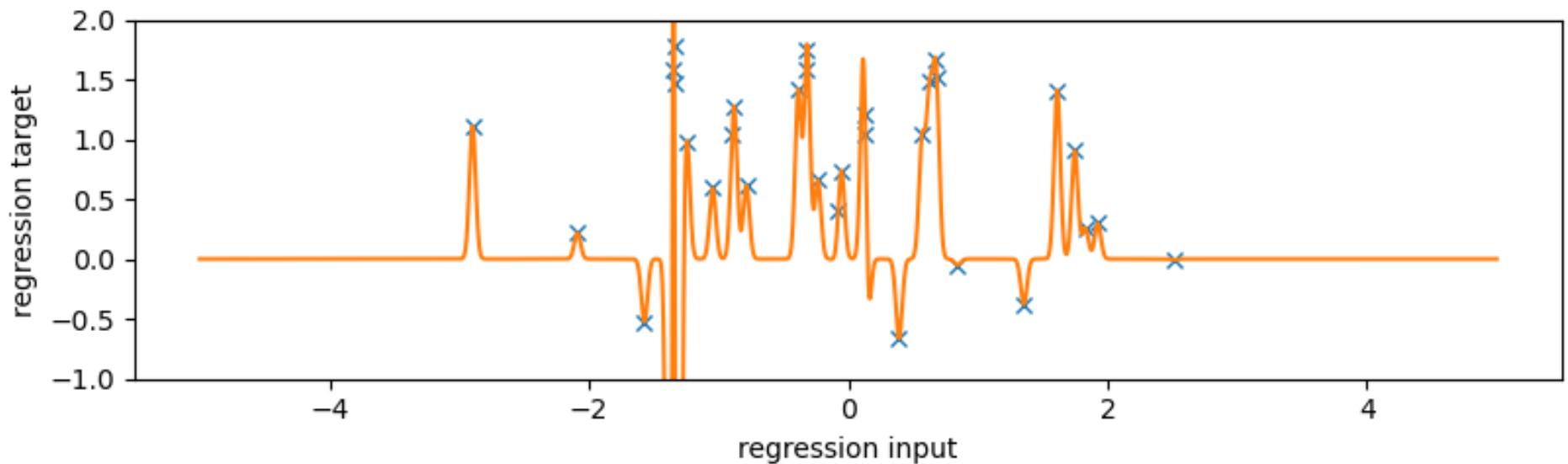
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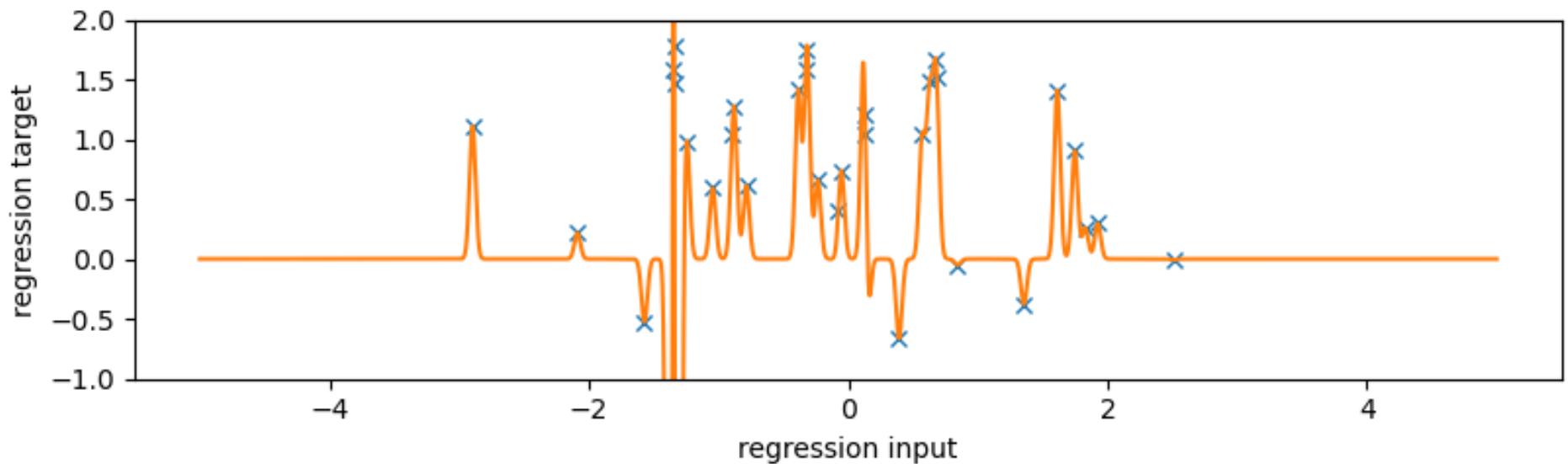
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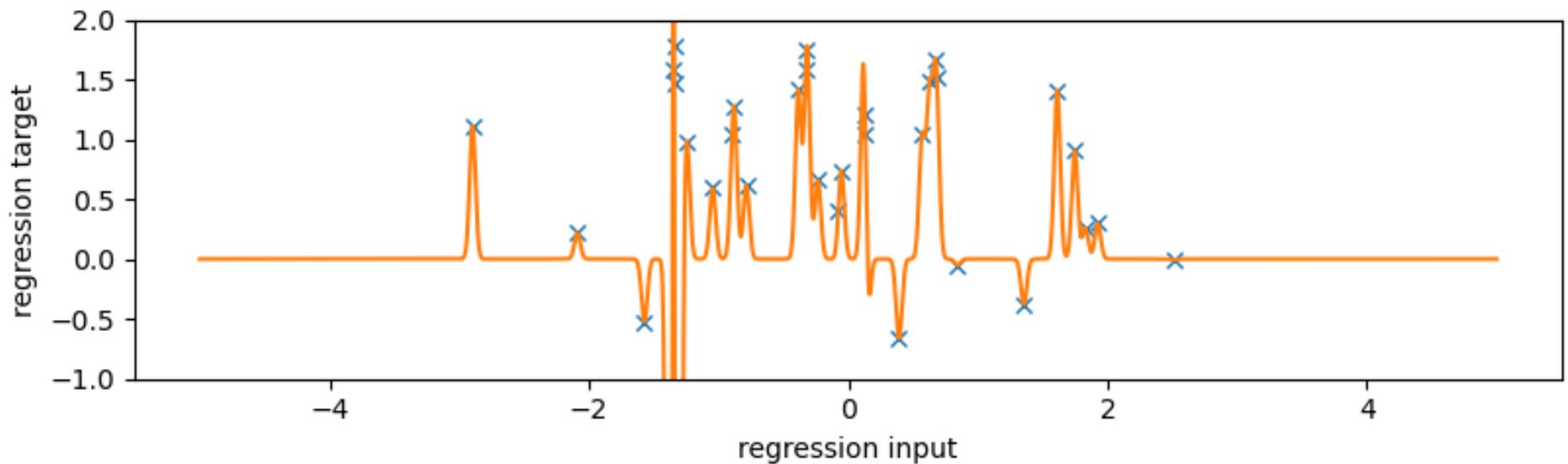
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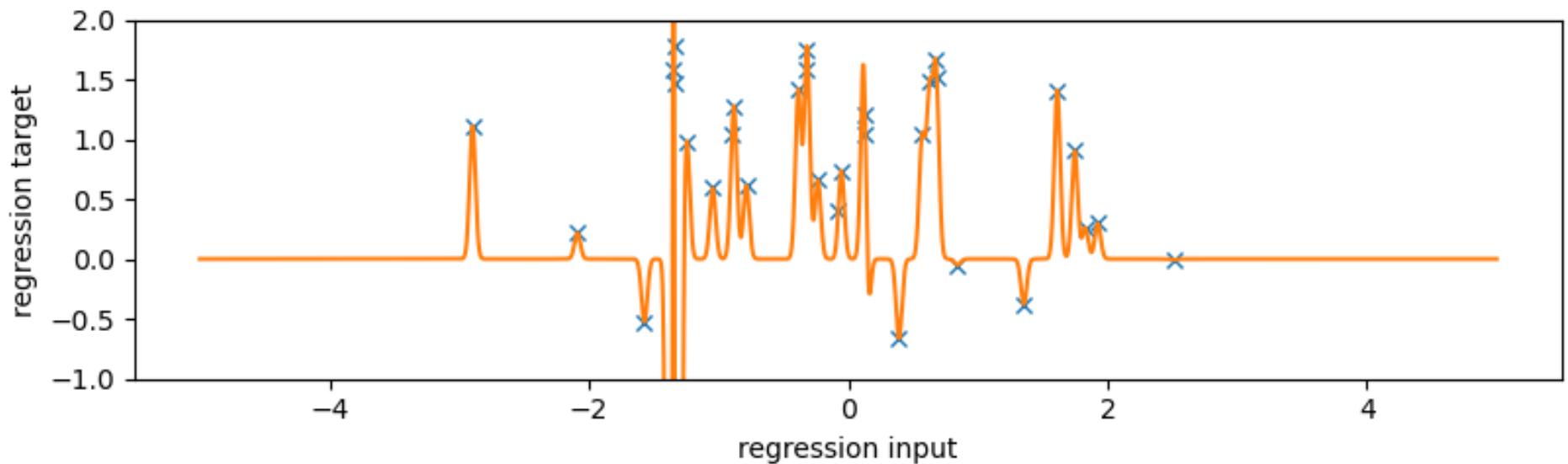
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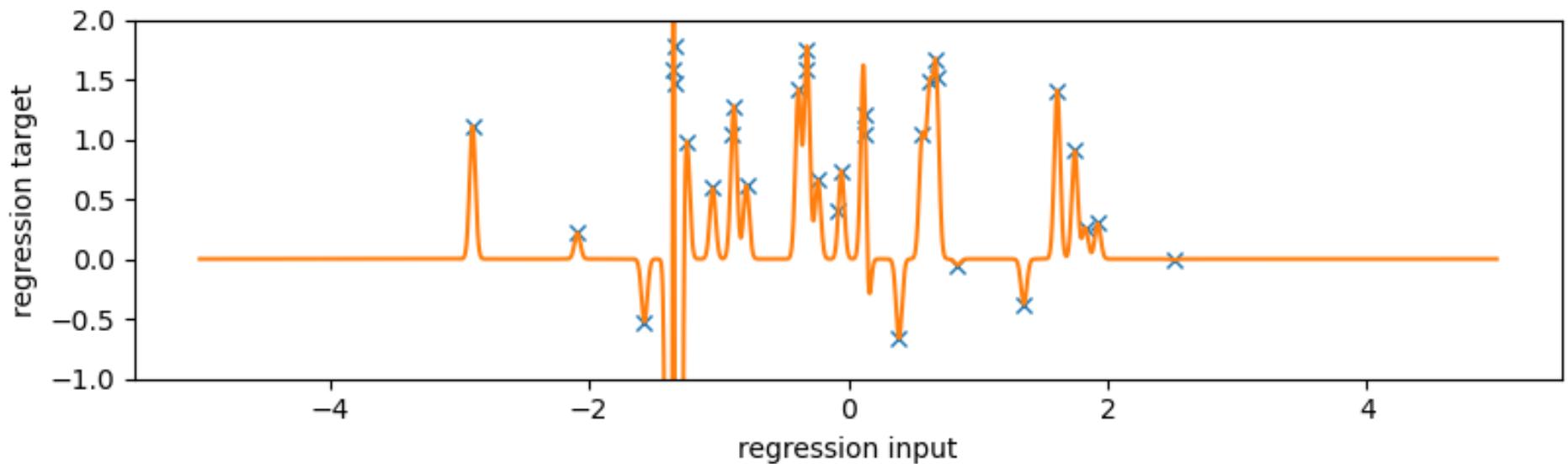
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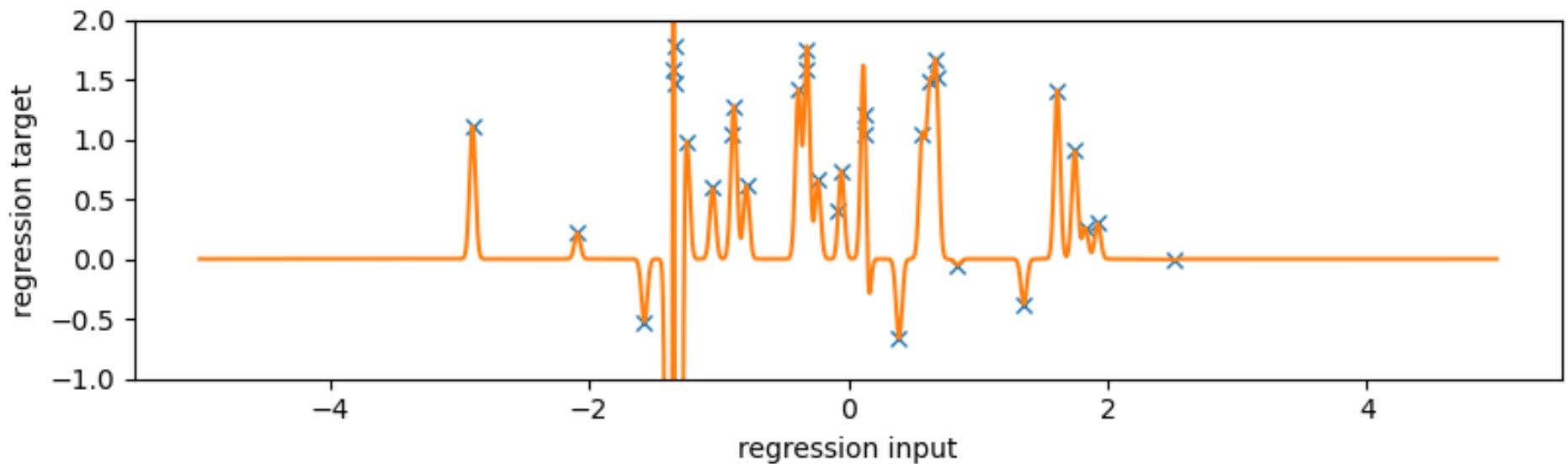
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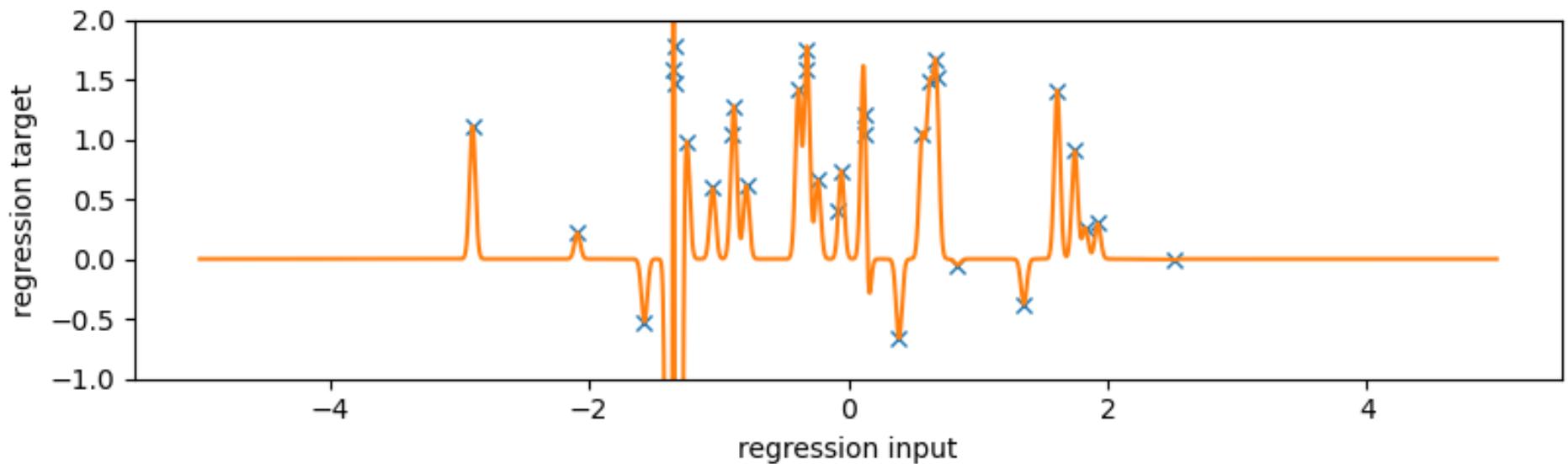
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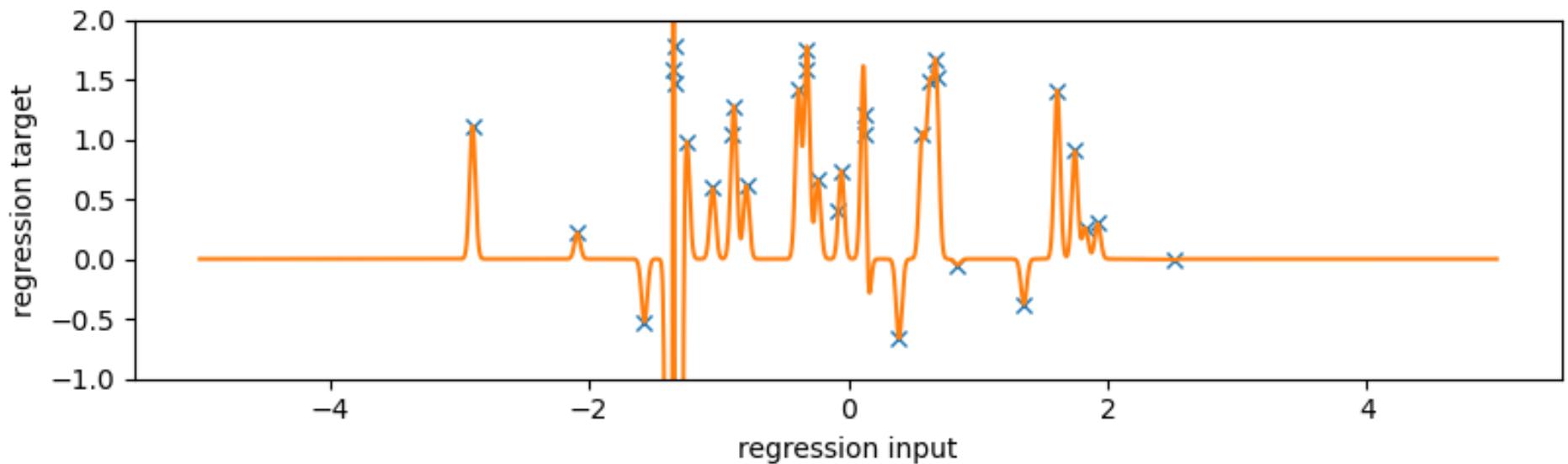
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Training Loss / MaxLik is not sufficient

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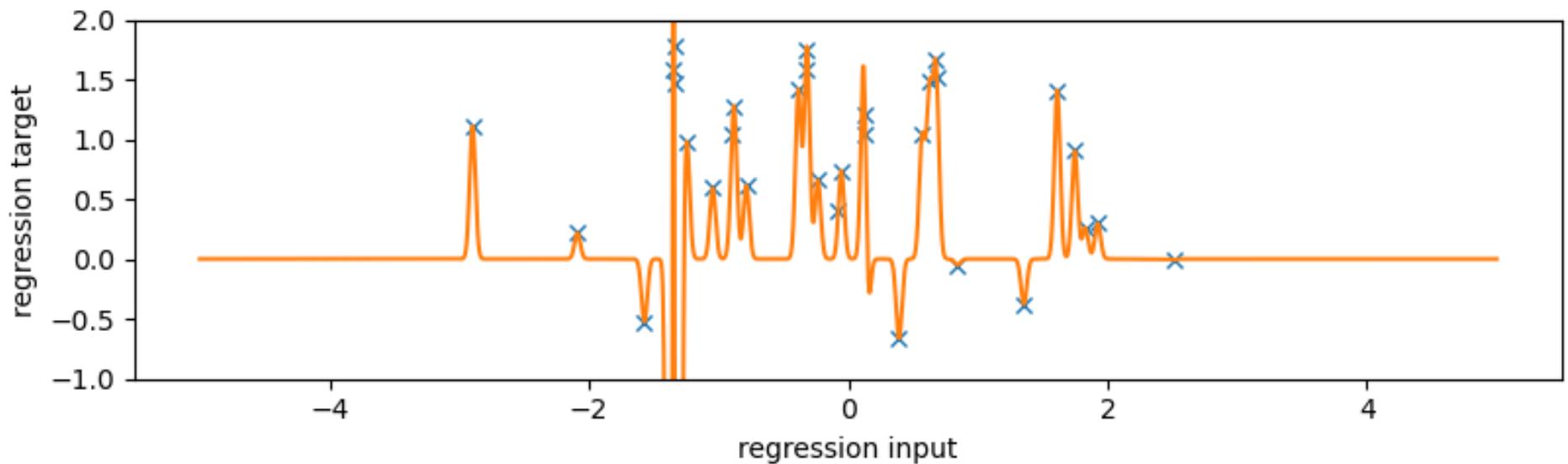
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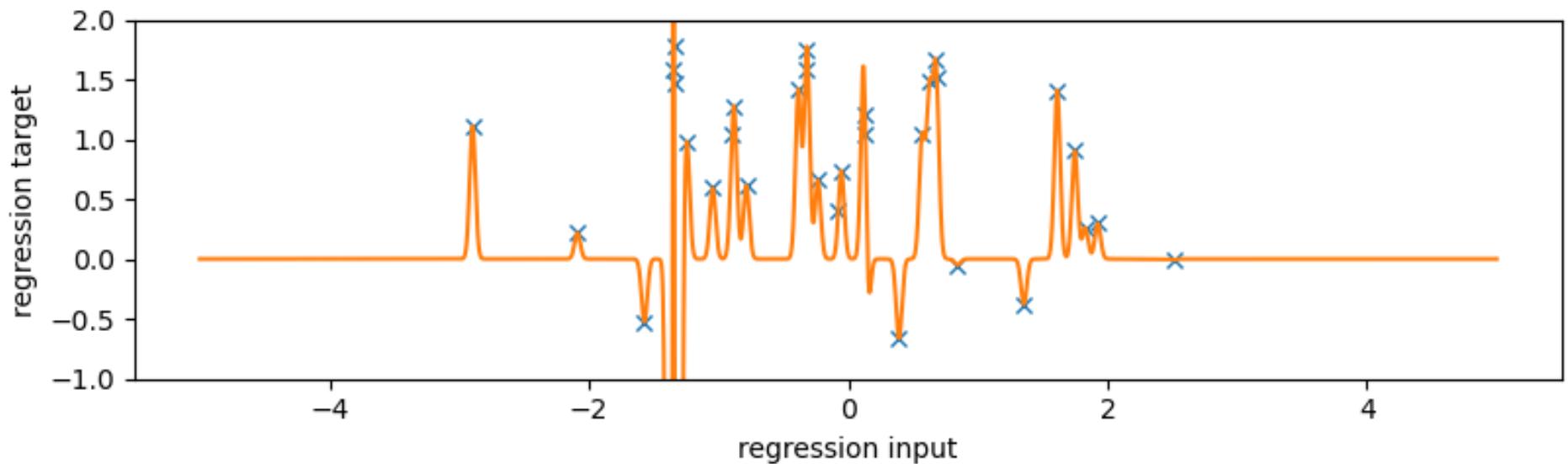
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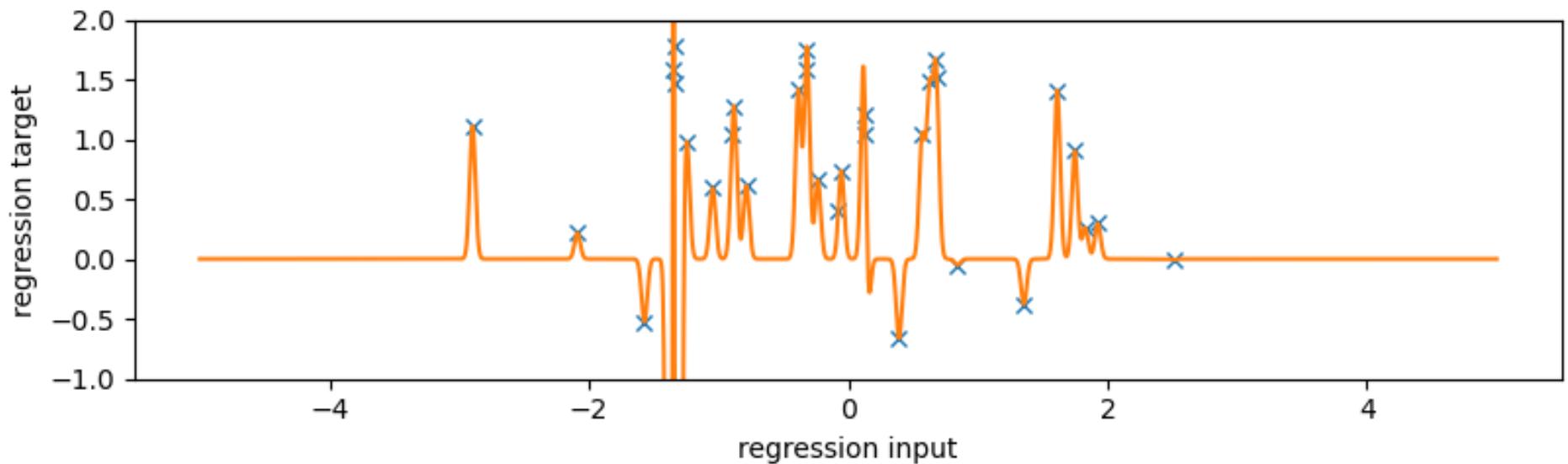
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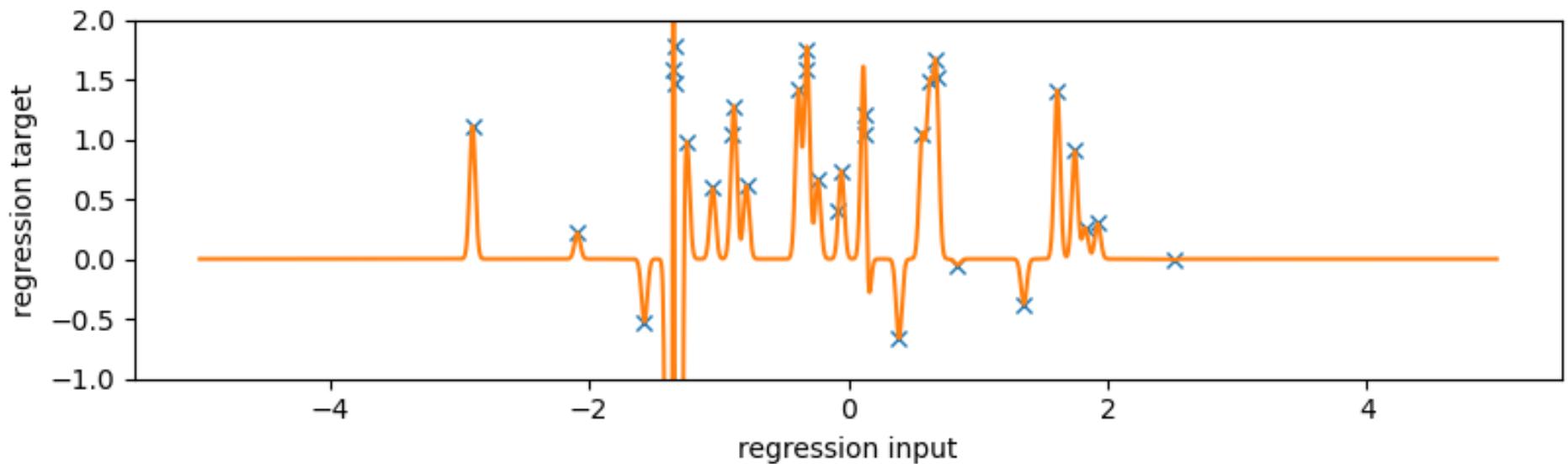
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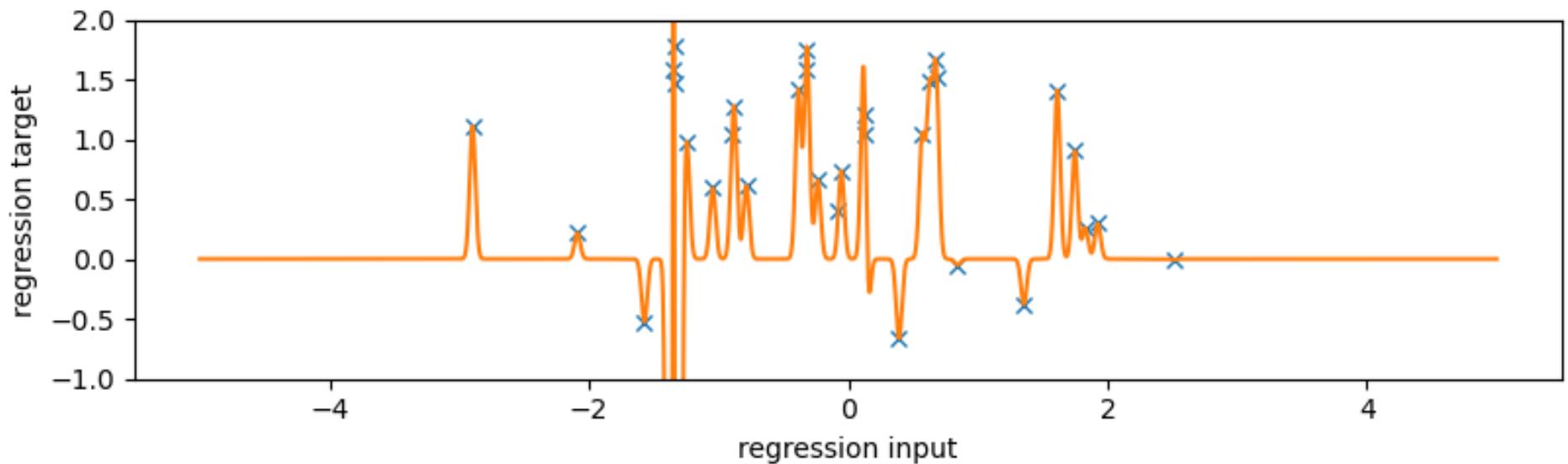
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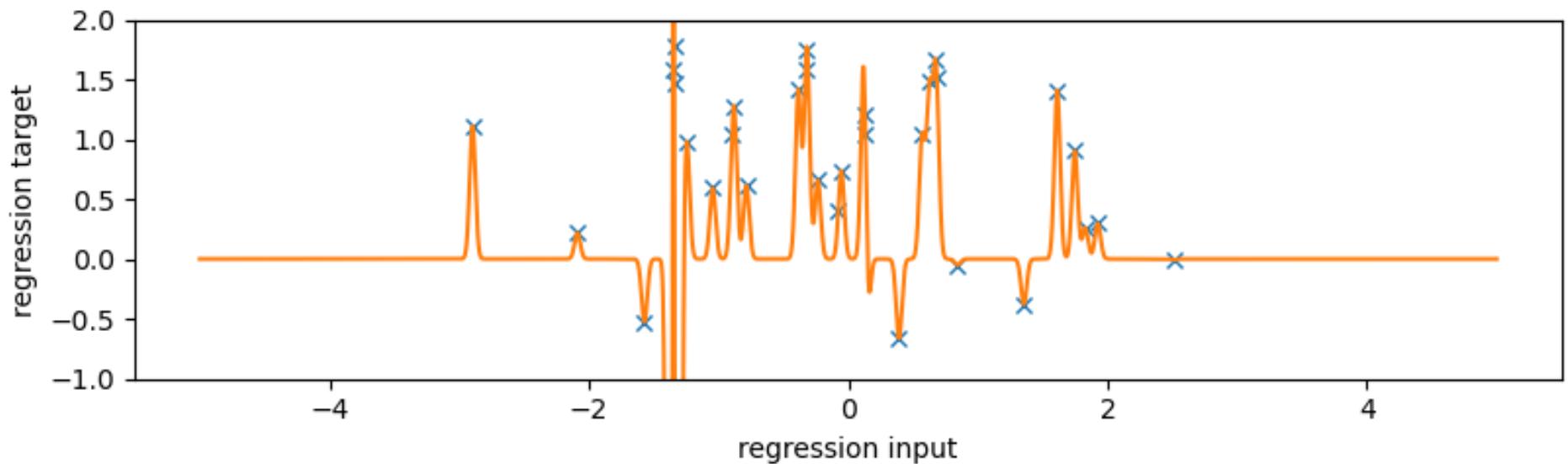
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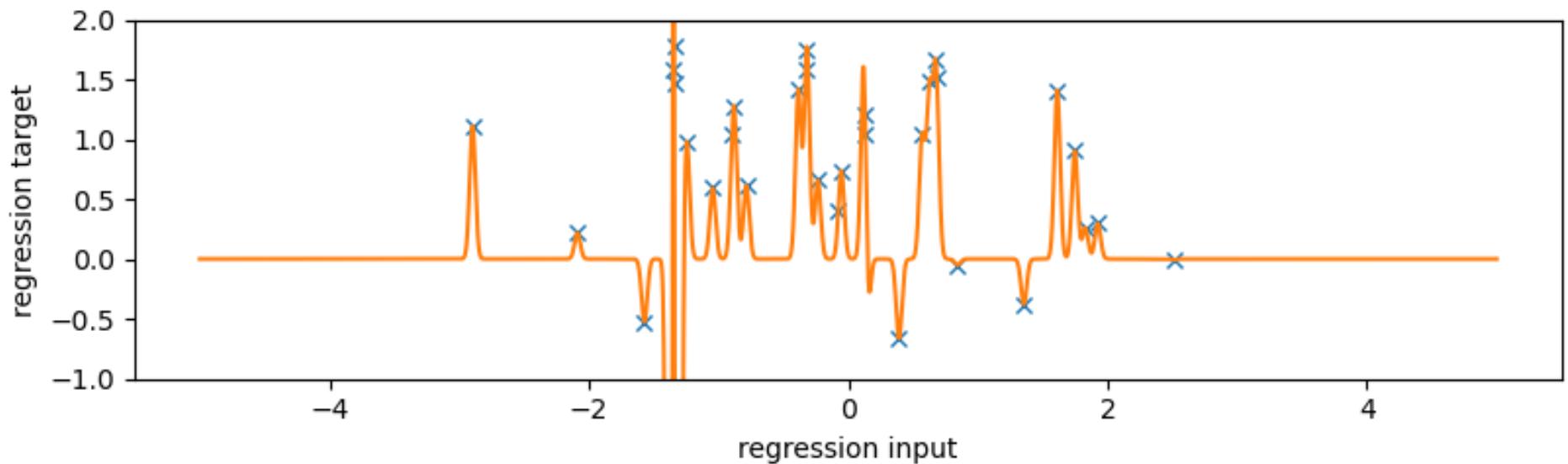
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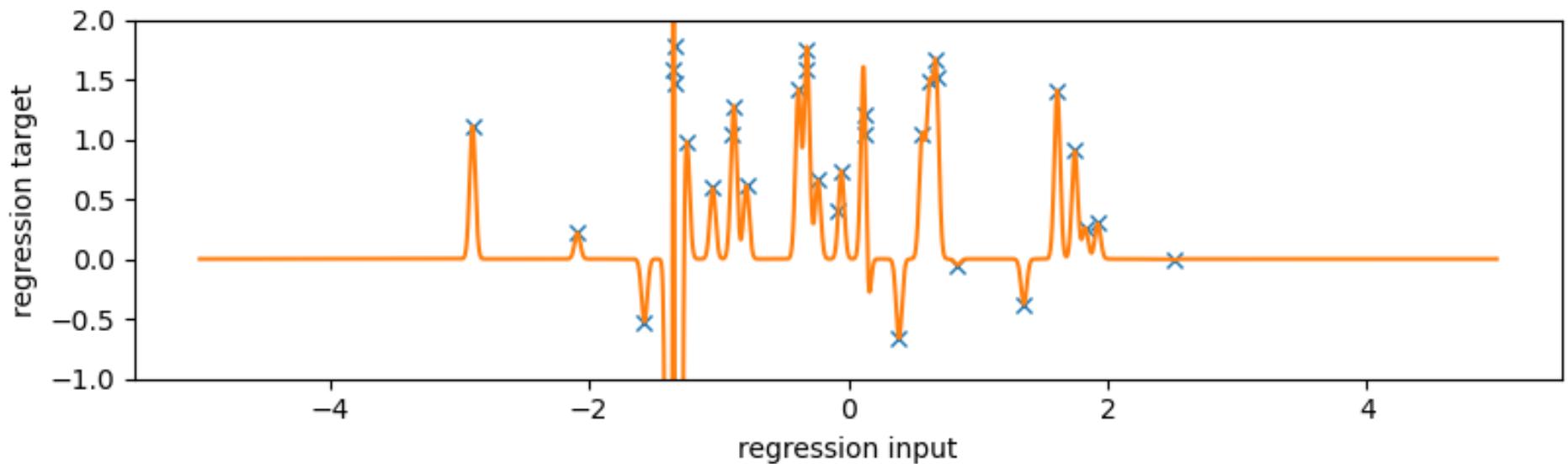
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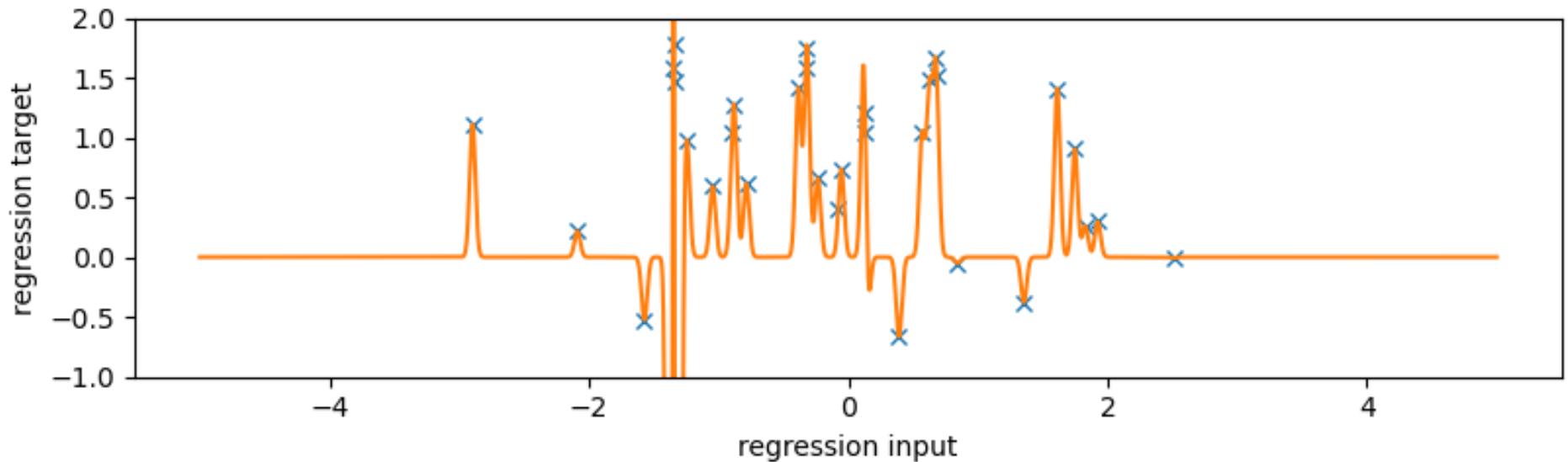
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- Restricting model size never improves loss $\Rightarrow M \rightarrow N$
- Narrower basis functions to allow more flexible functions
- “Overfitting”

1. What is wrong with minimising losses.
2. **Bayesian Model Selection?**
3. The Bayesian answer to model size: Nonparametrics.
3. A principle for selecting size

The Bayesian Answer

Let's accept the “large” number of basis functions for now, and solve the overfitting problem.

Bayesian inference is rumoured to be “robust to overfitting”.

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$$p(W, \theta | \mathcal{D}) = \frac{p(\mathcal{D} | W, \theta) p(W | \theta)}{p(\mathcal{D} | \theta)} \frac{p(\mathcal{D} | \theta) p(\theta)}{p(\mathcal{D})}$$

$$p(\mathcal{D} | \theta) = \int p(\mathcal{D} | W, \theta) p(W | \theta) dW$$

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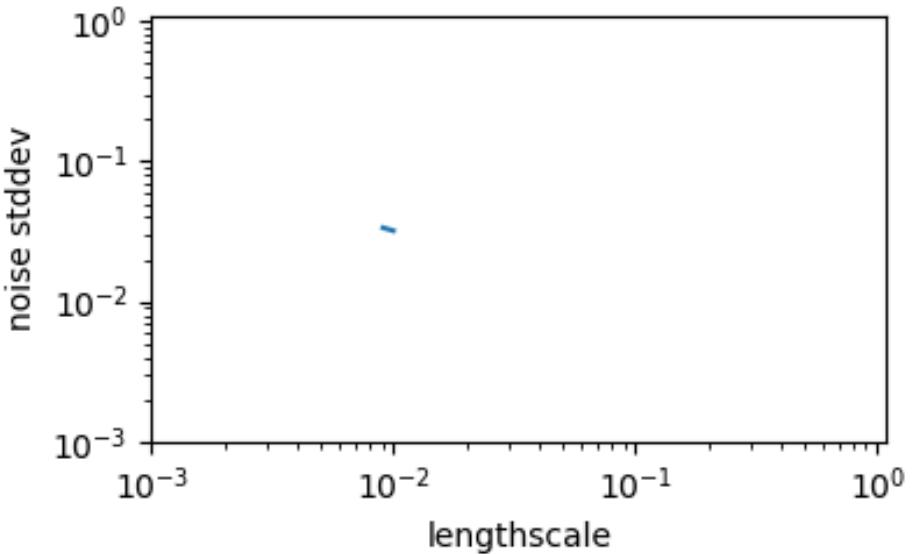
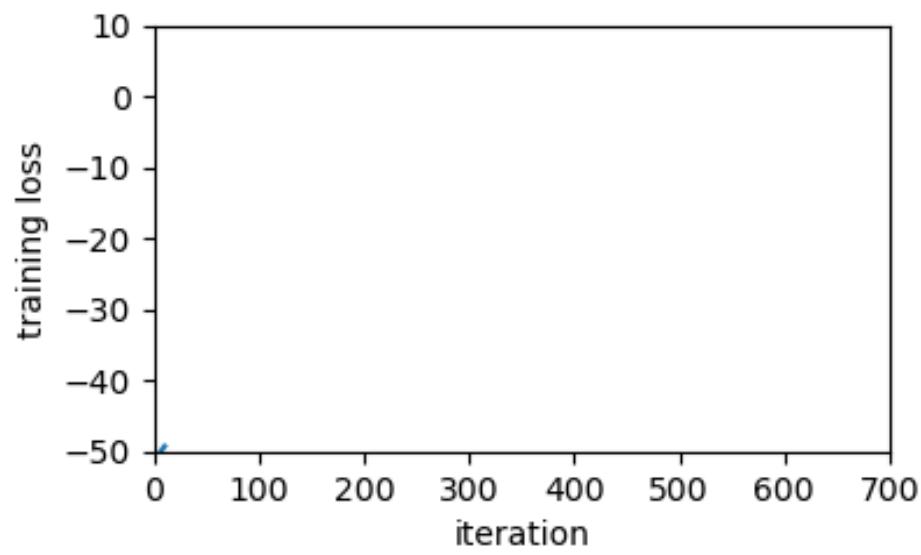
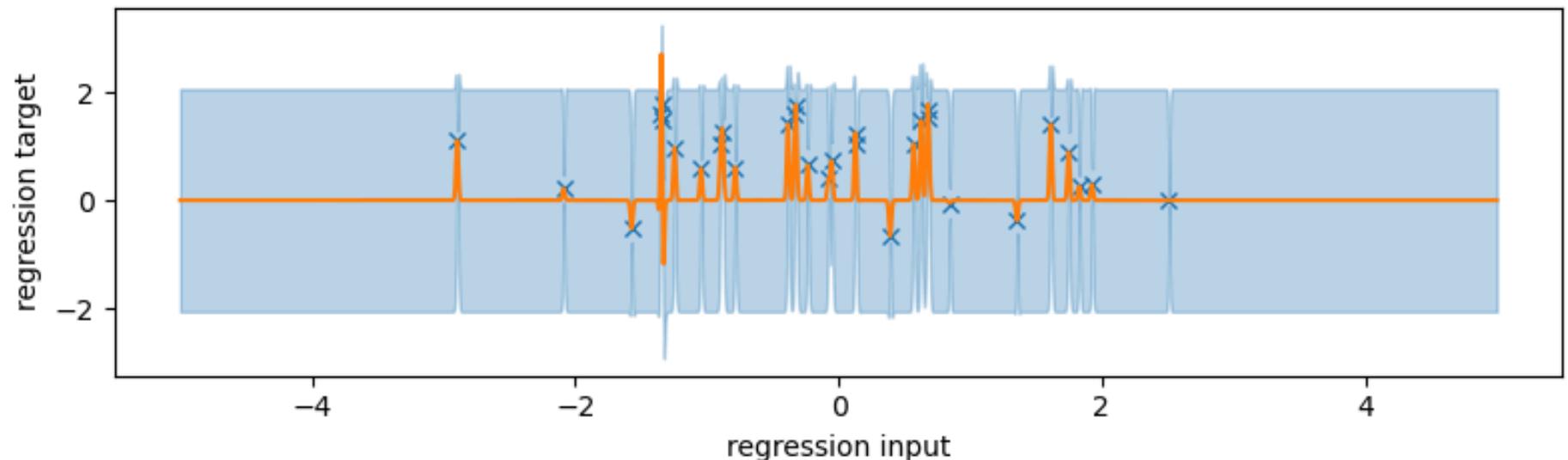
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Bayesian computations are often intractable. \Rightarrow Approximating $p(\mathcal{D}|\theta)$ is hard enough, let alone for many different values of θ !

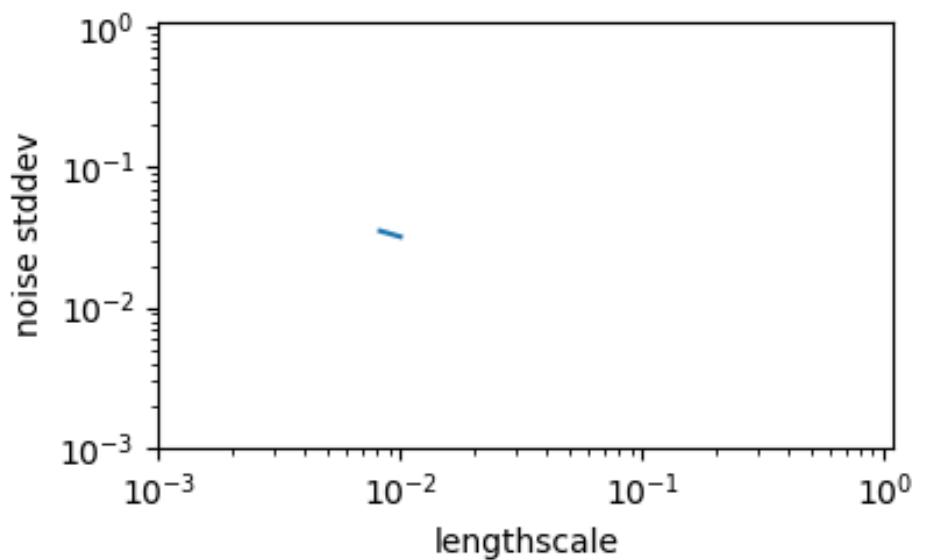
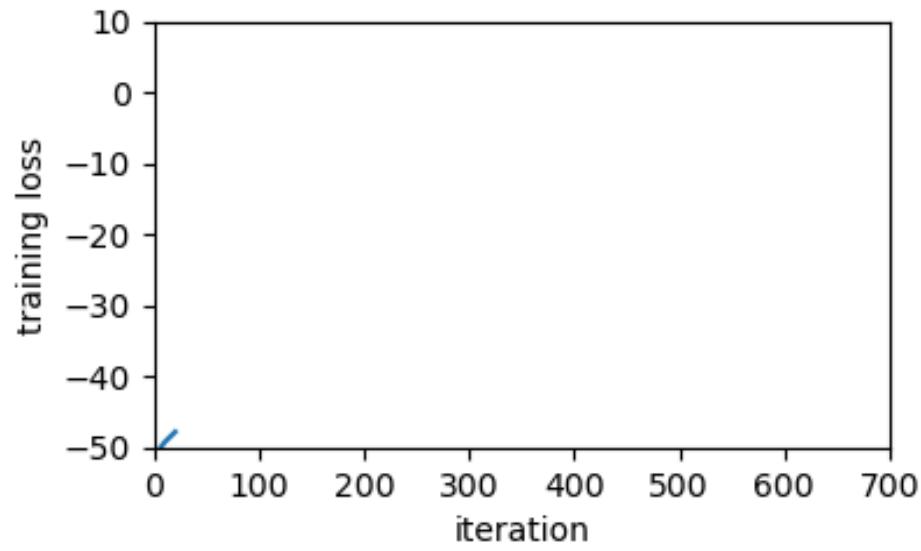
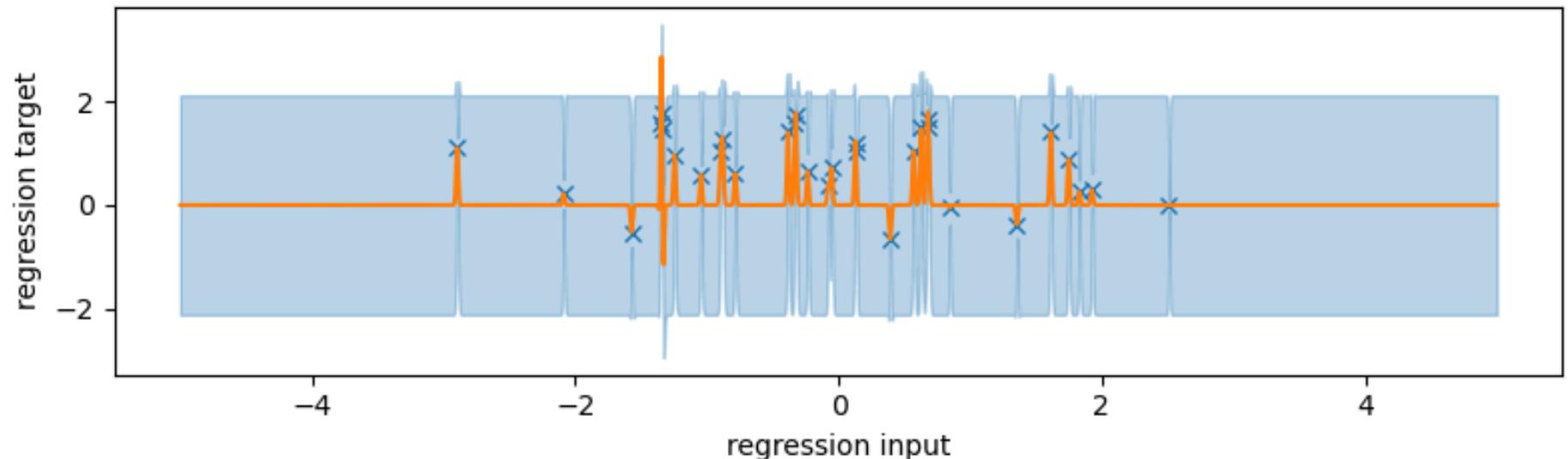
$$\theta^* = \operatorname{argmin}_{\theta} \log p(\mathcal{D} \mid \theta)$$

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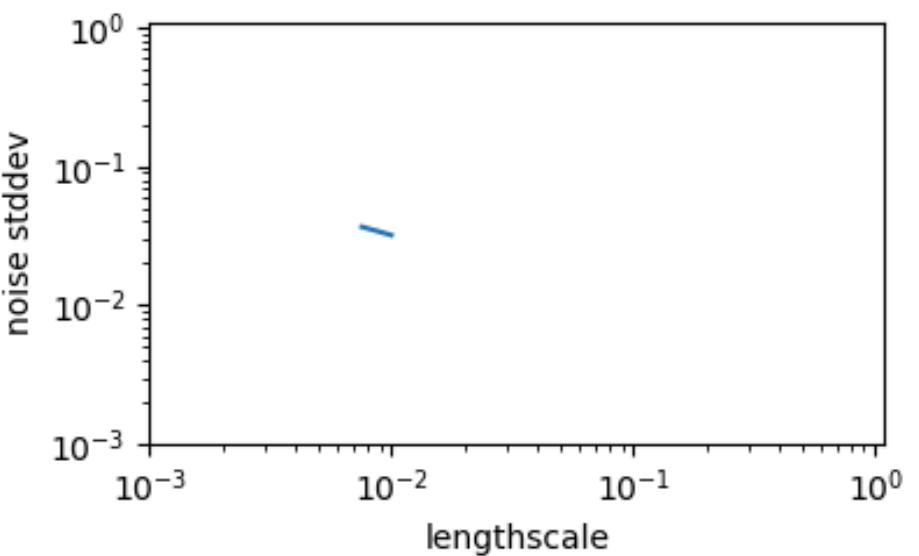
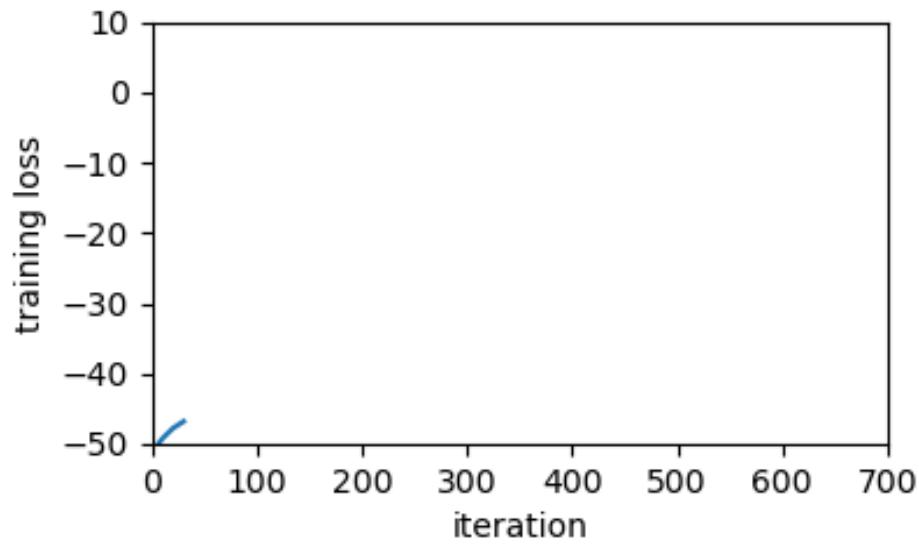
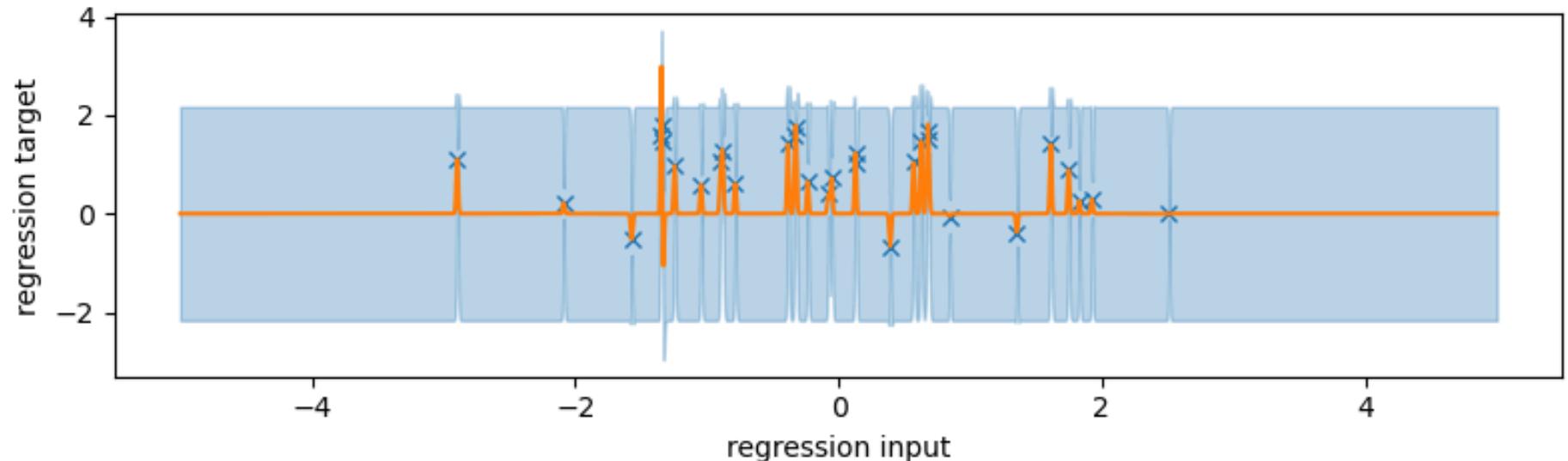
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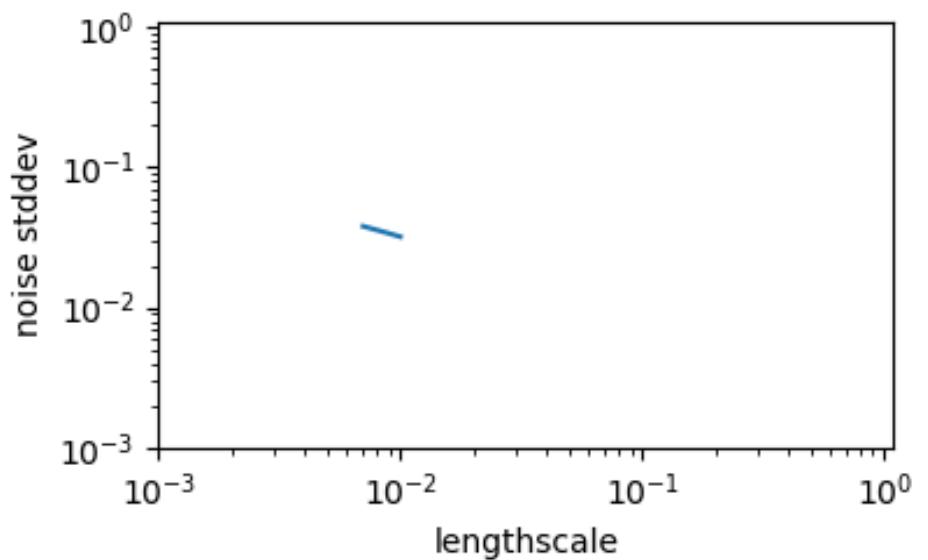
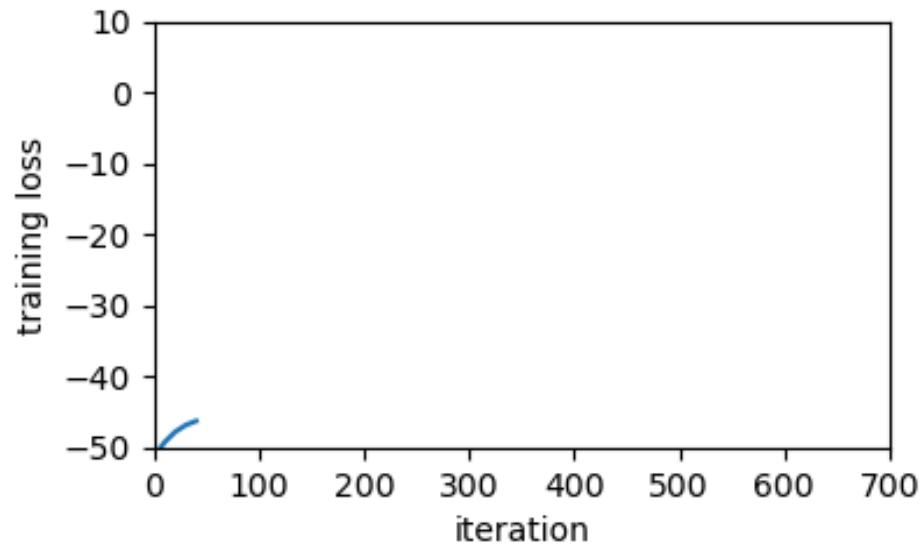
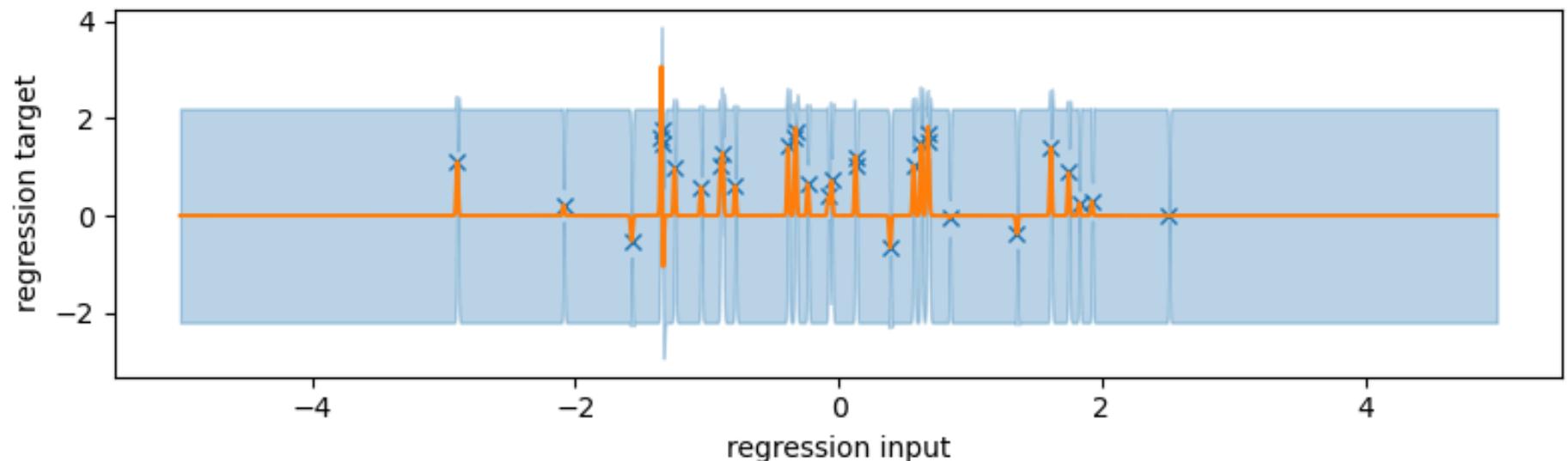
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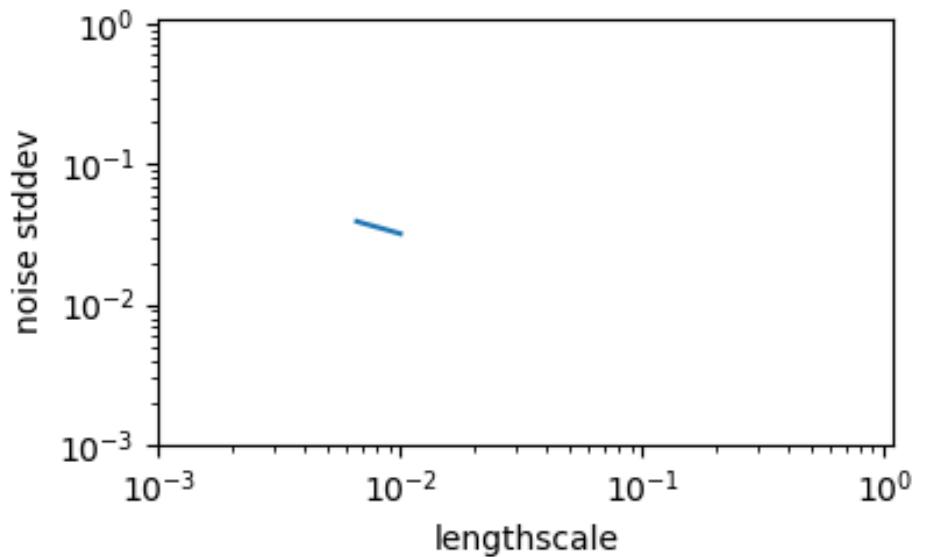
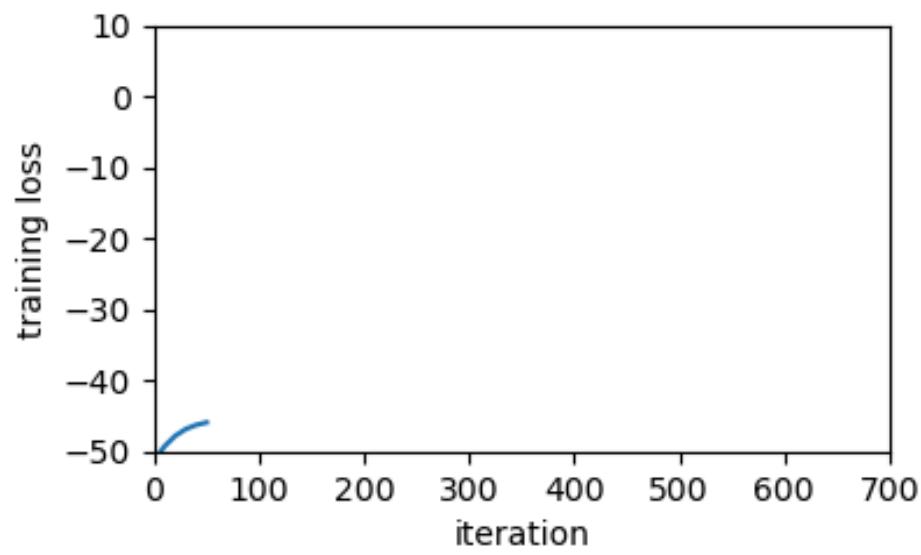
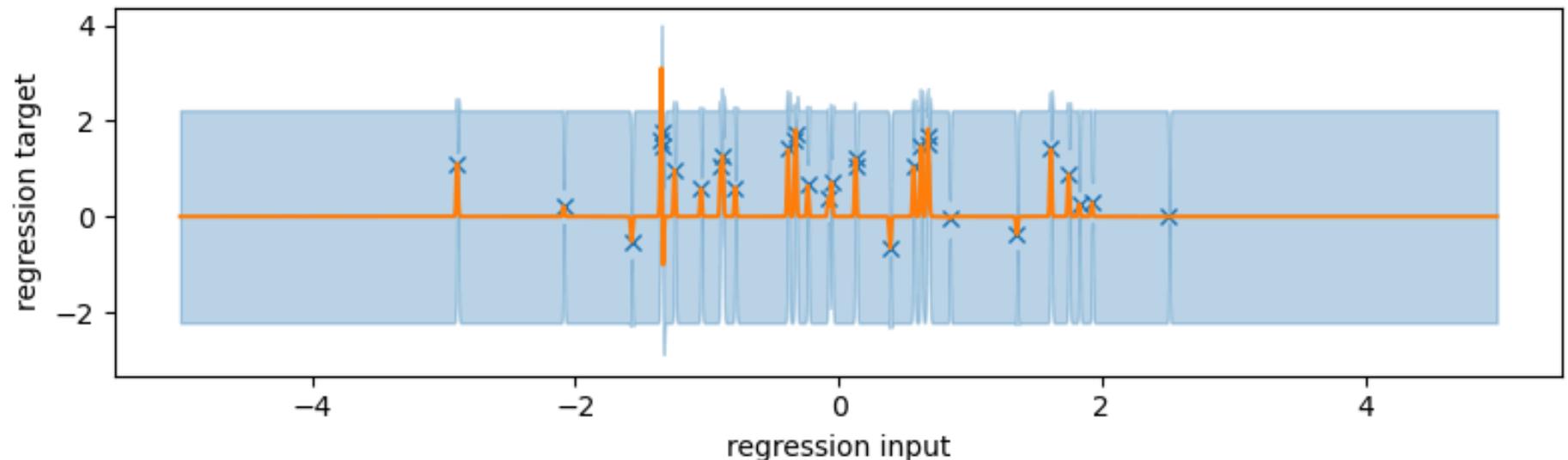
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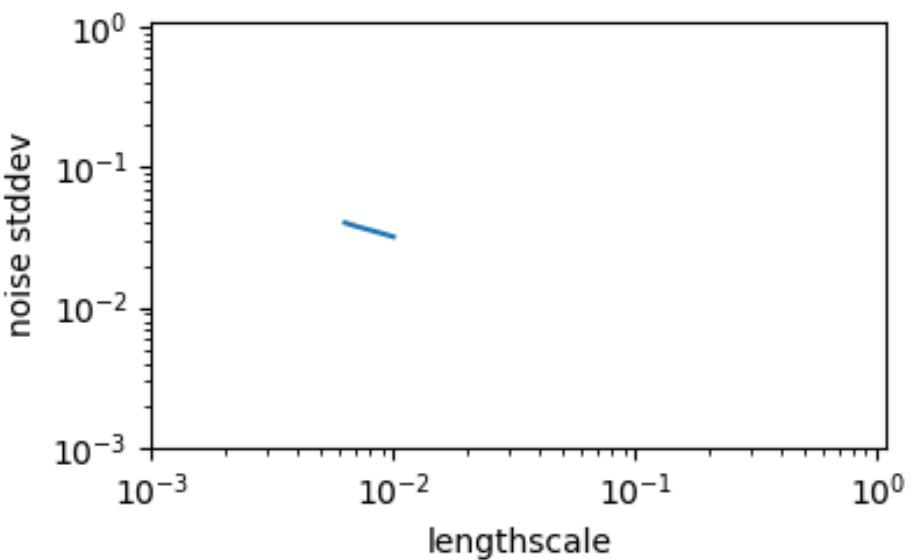
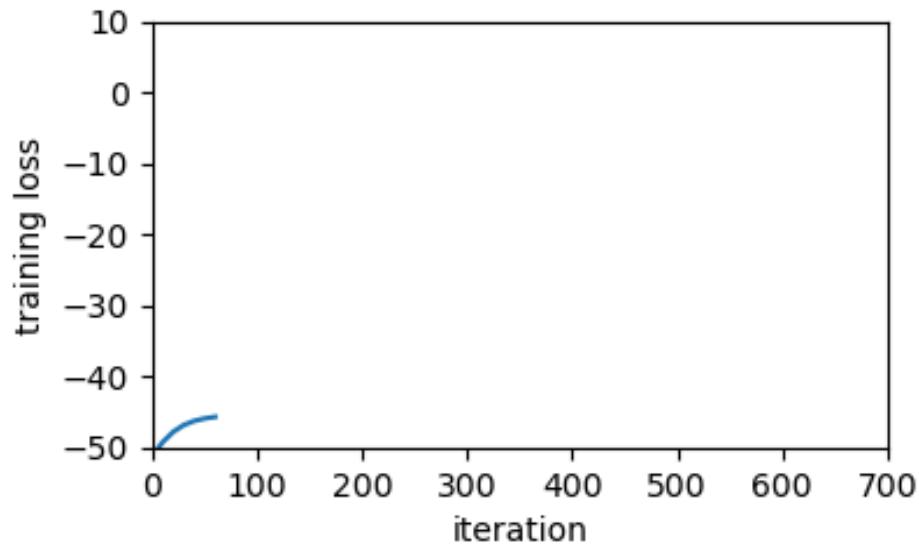
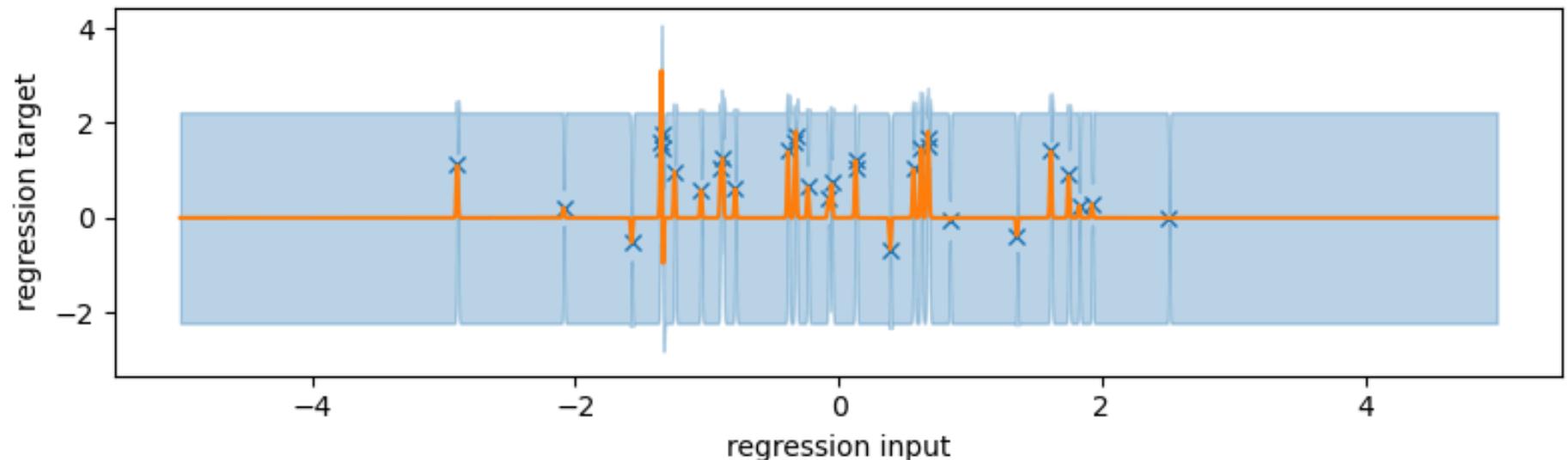
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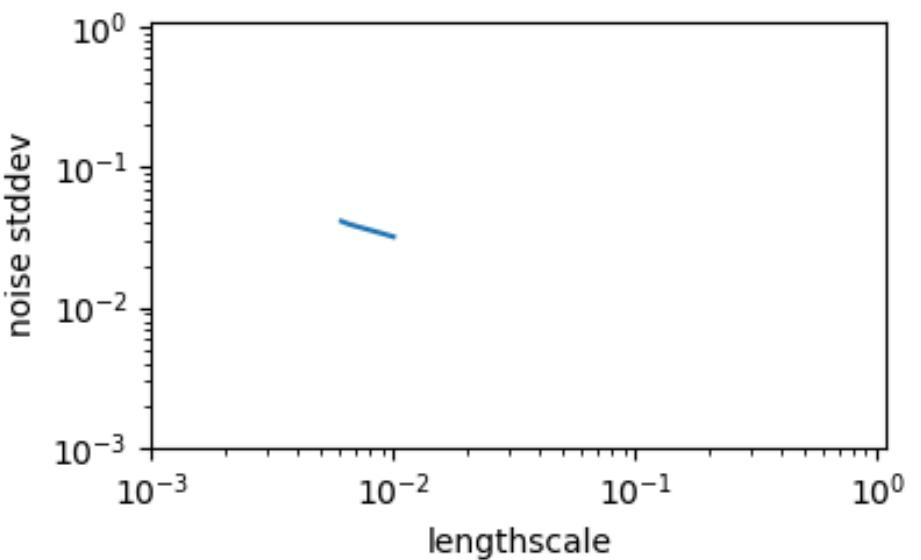
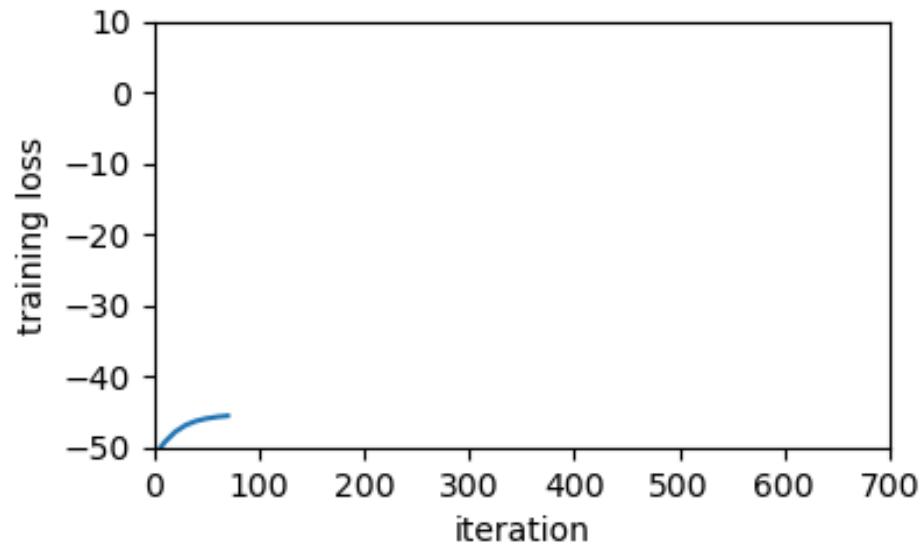
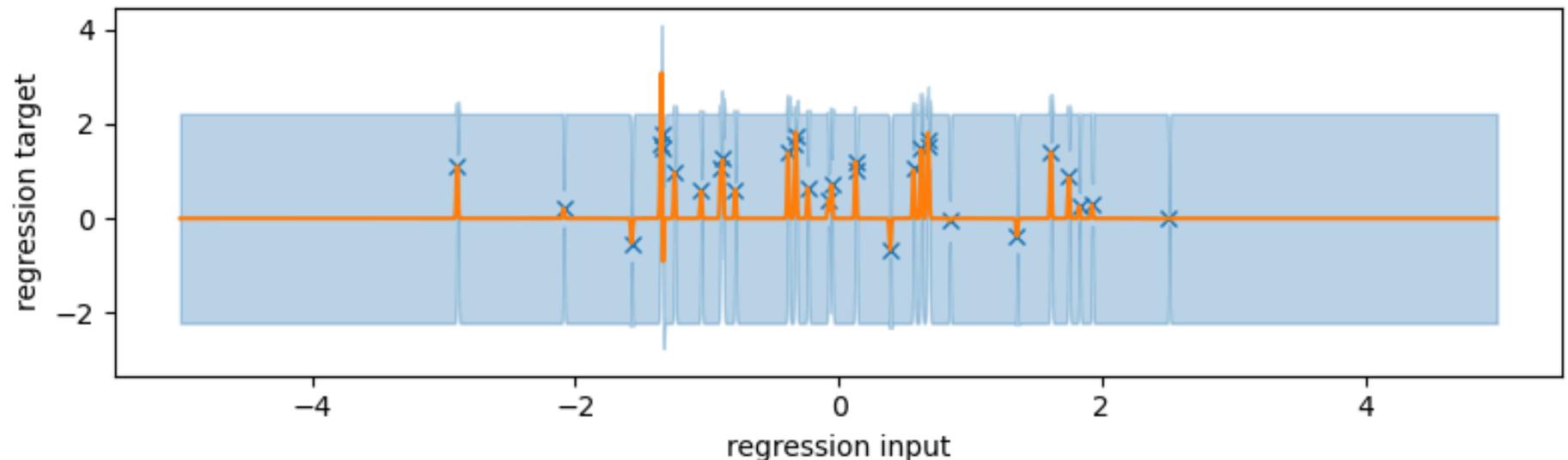
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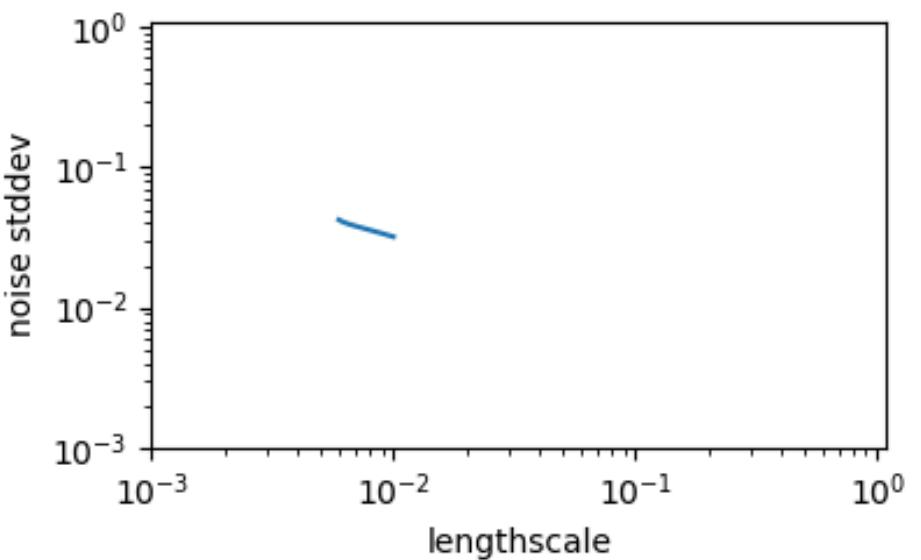
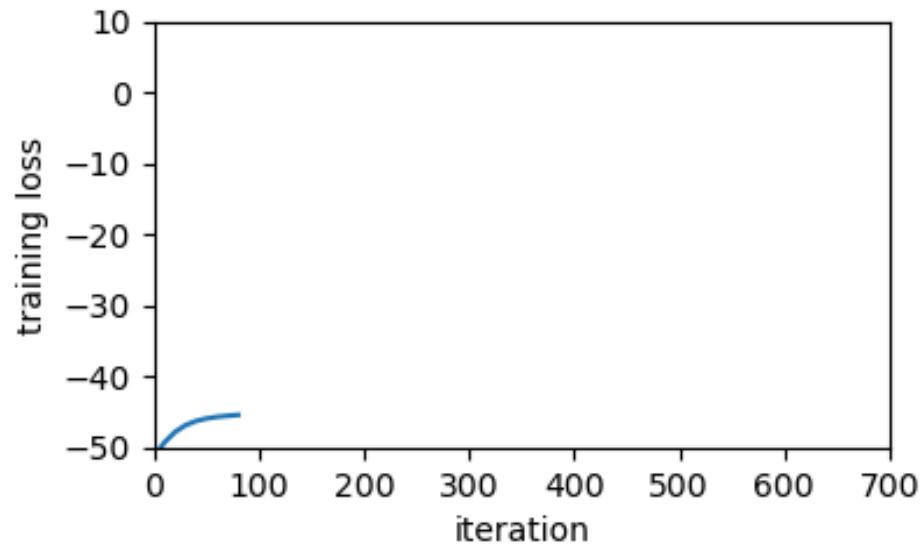
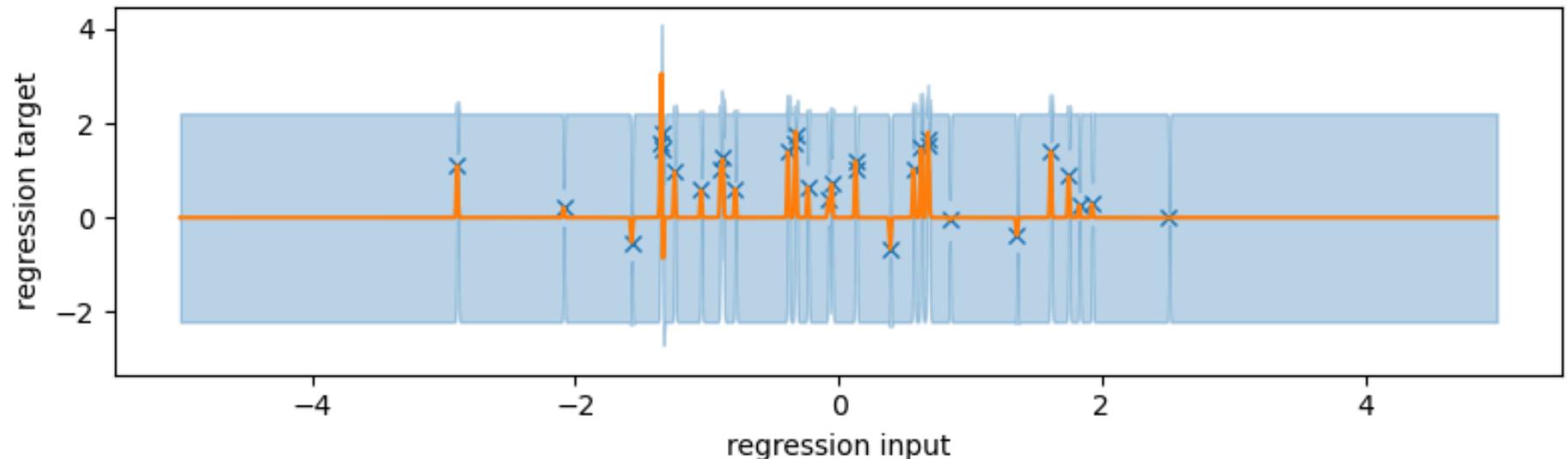
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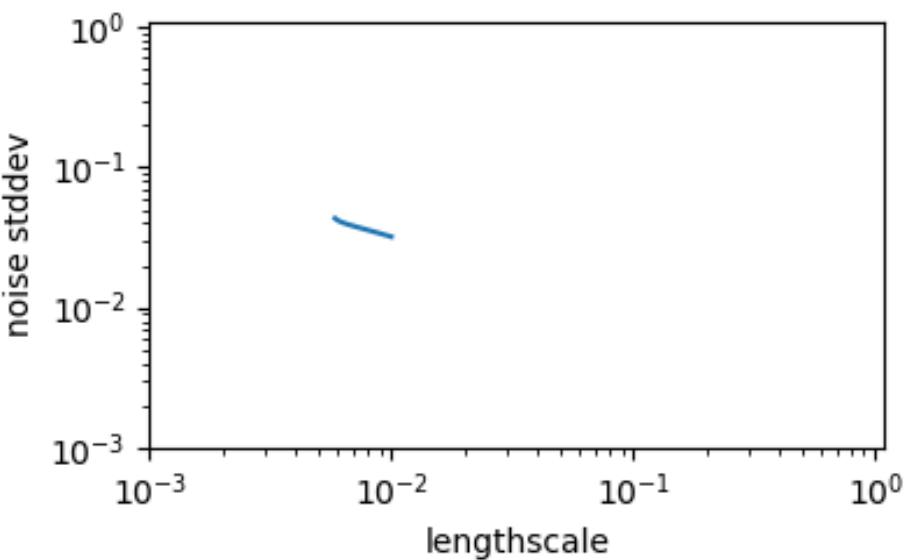
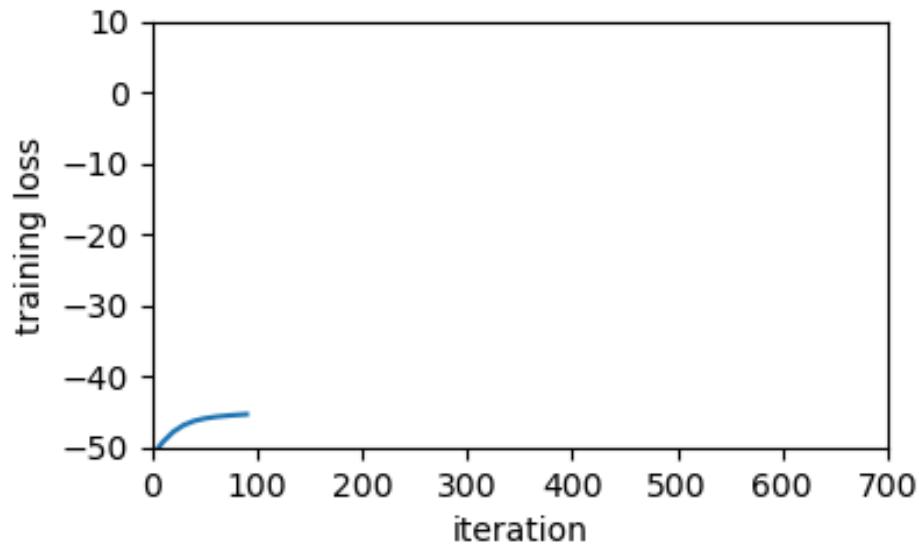
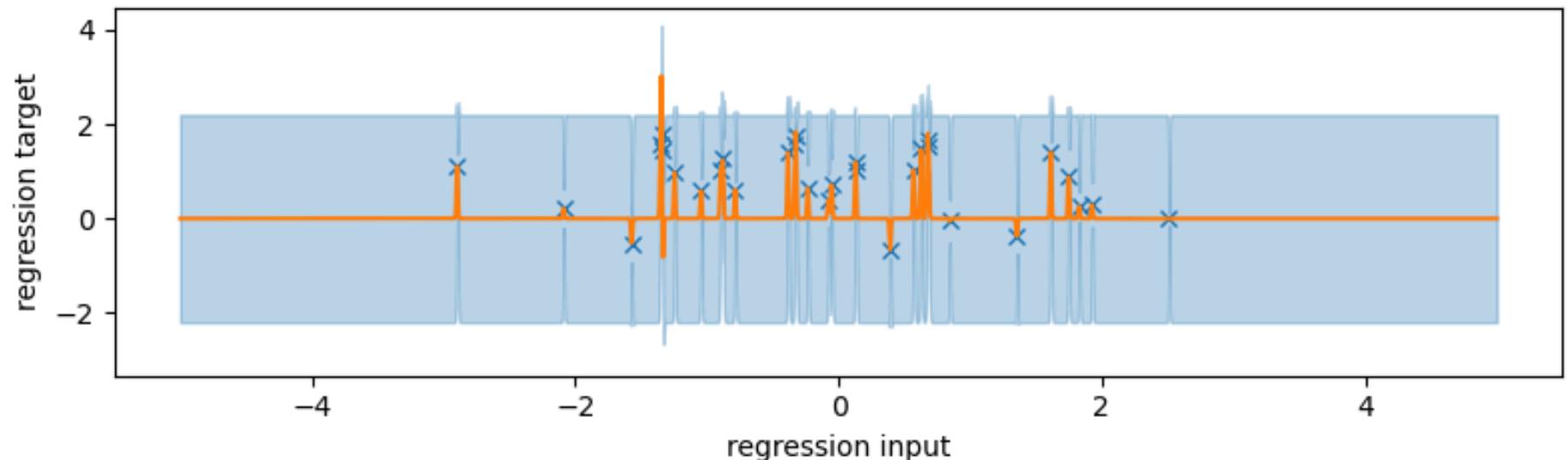
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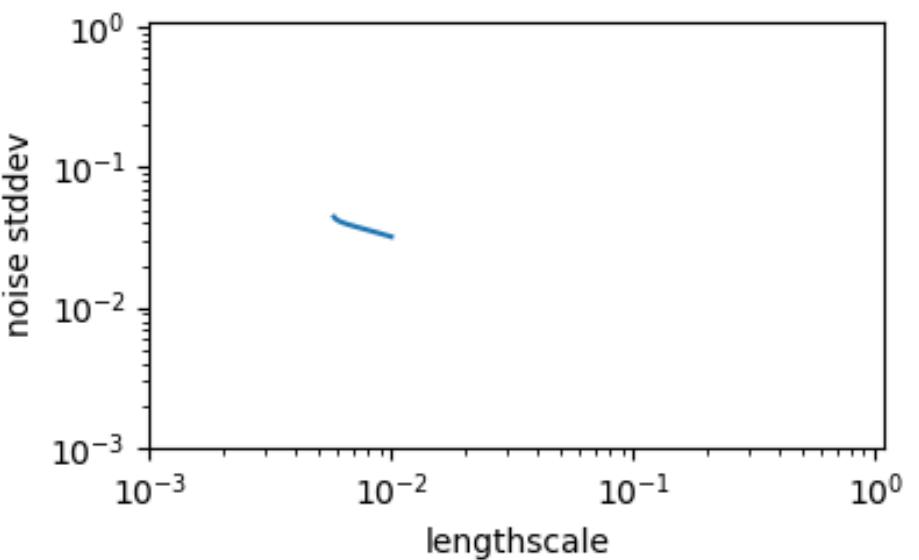
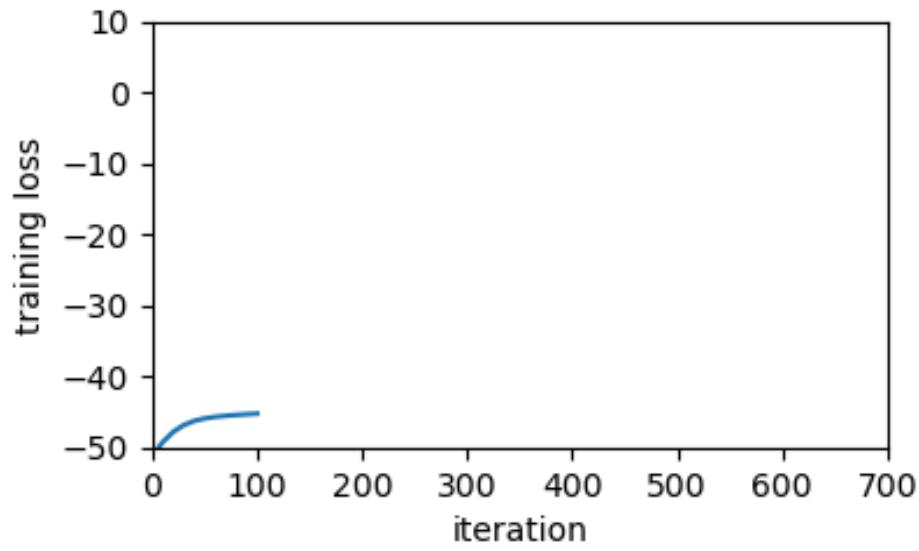
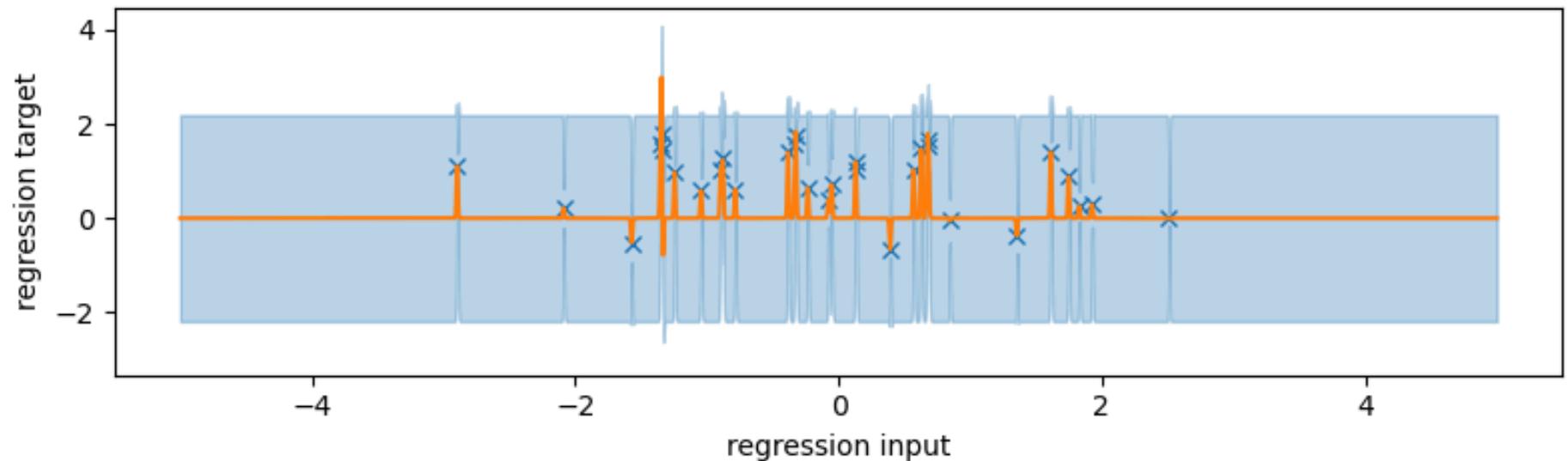
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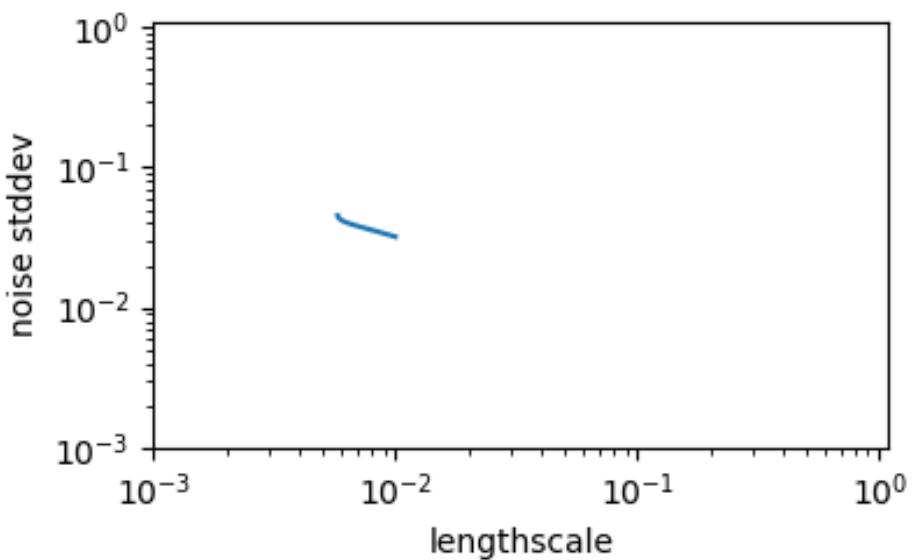
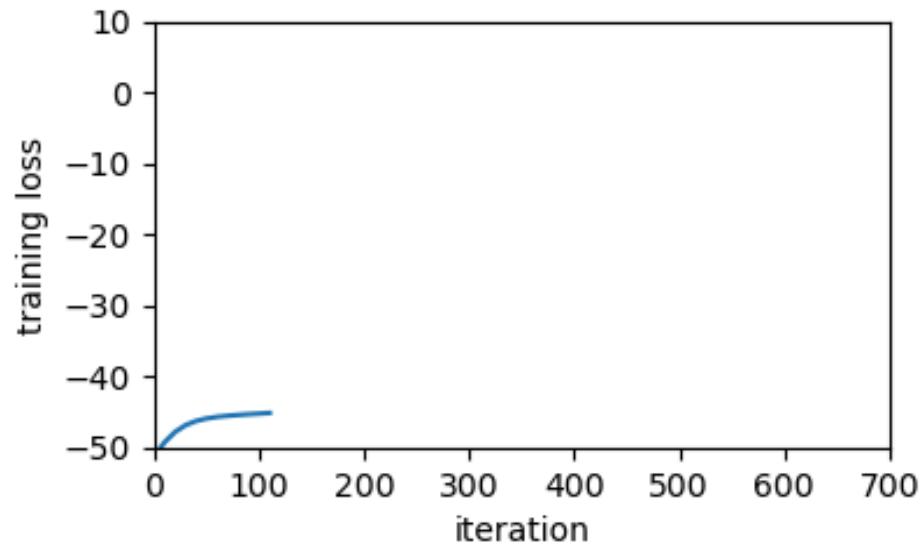
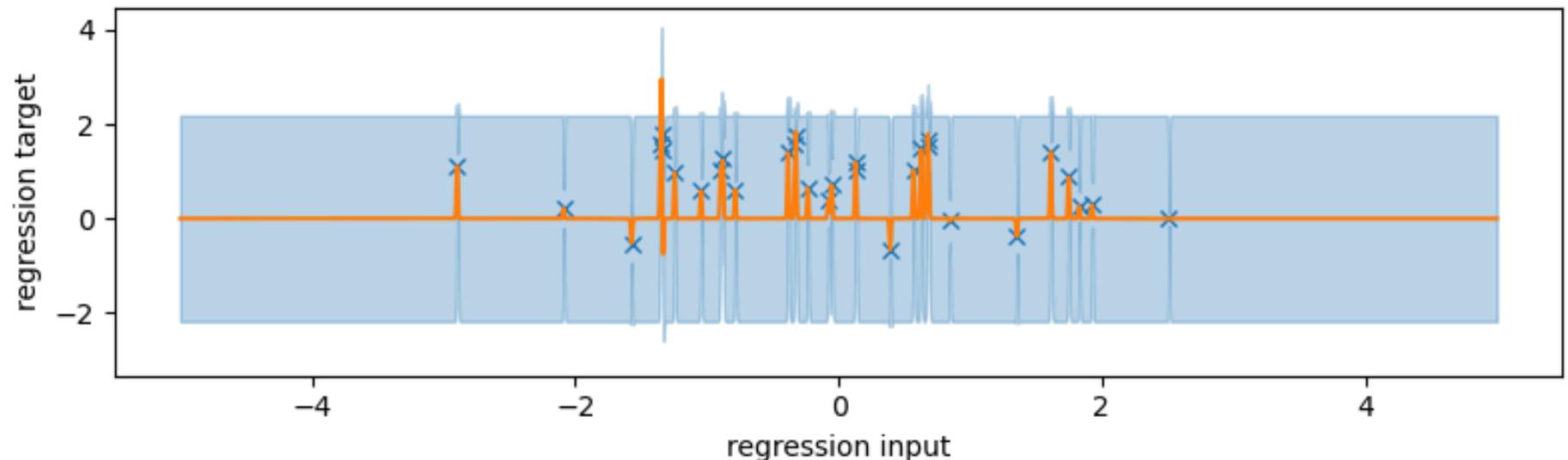
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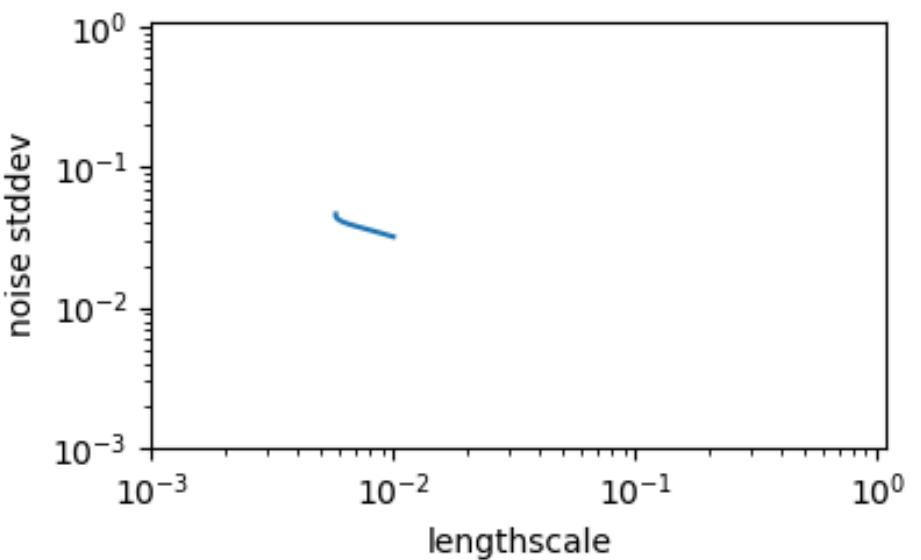
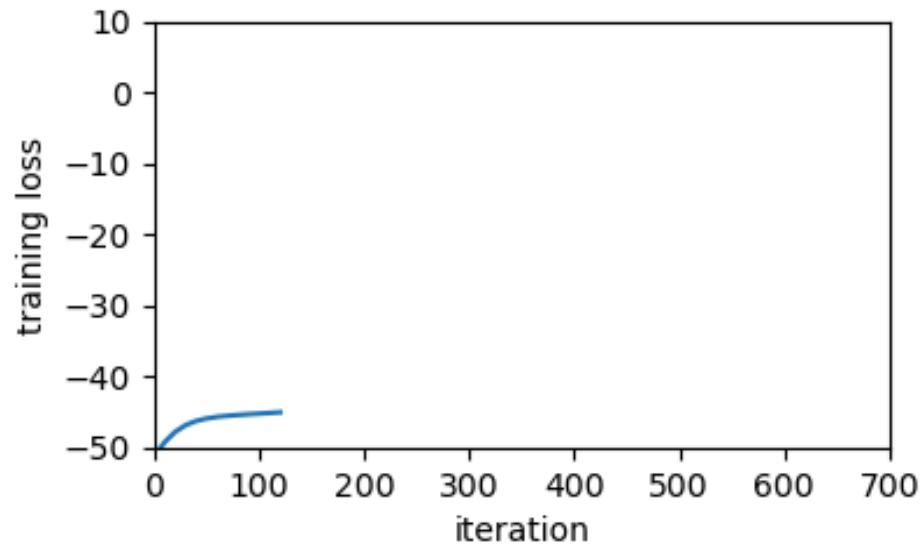
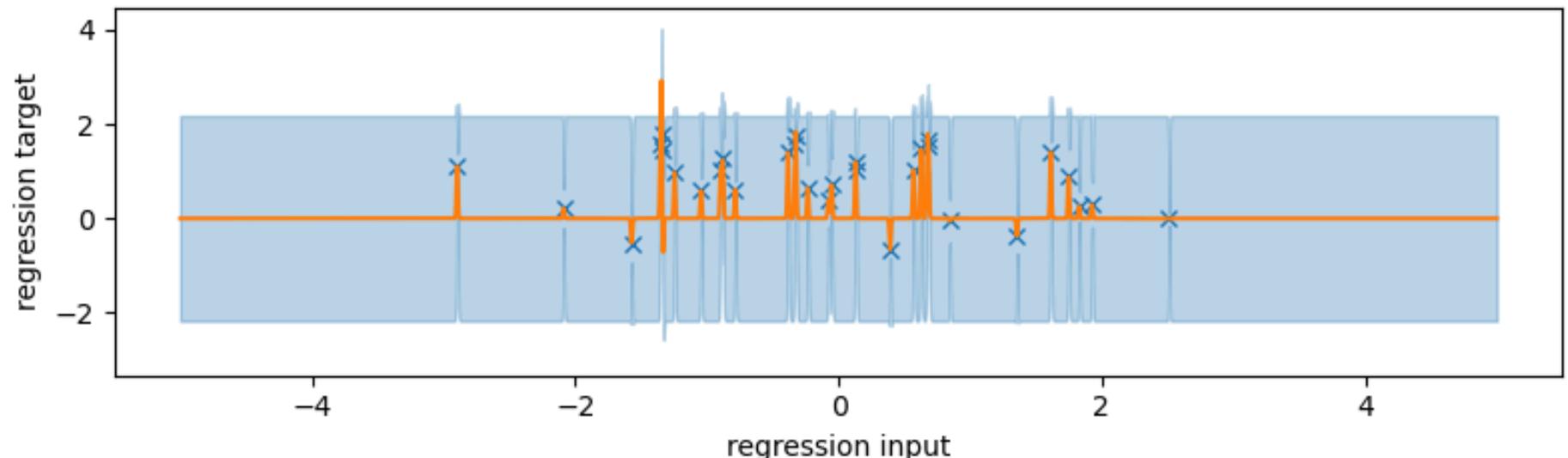
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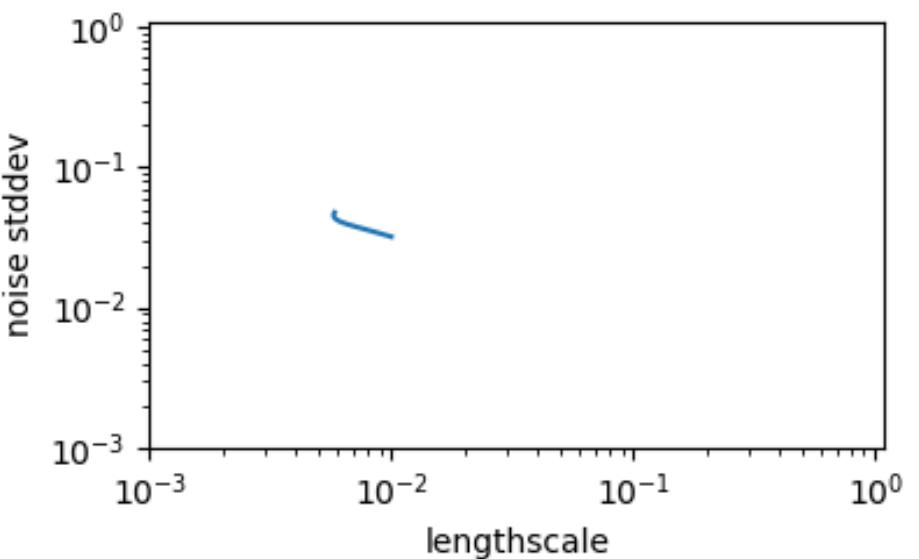
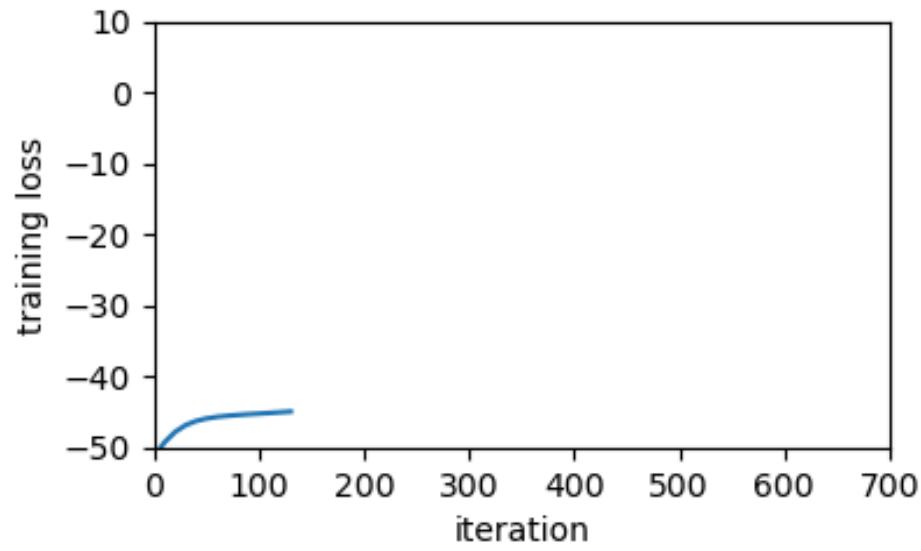
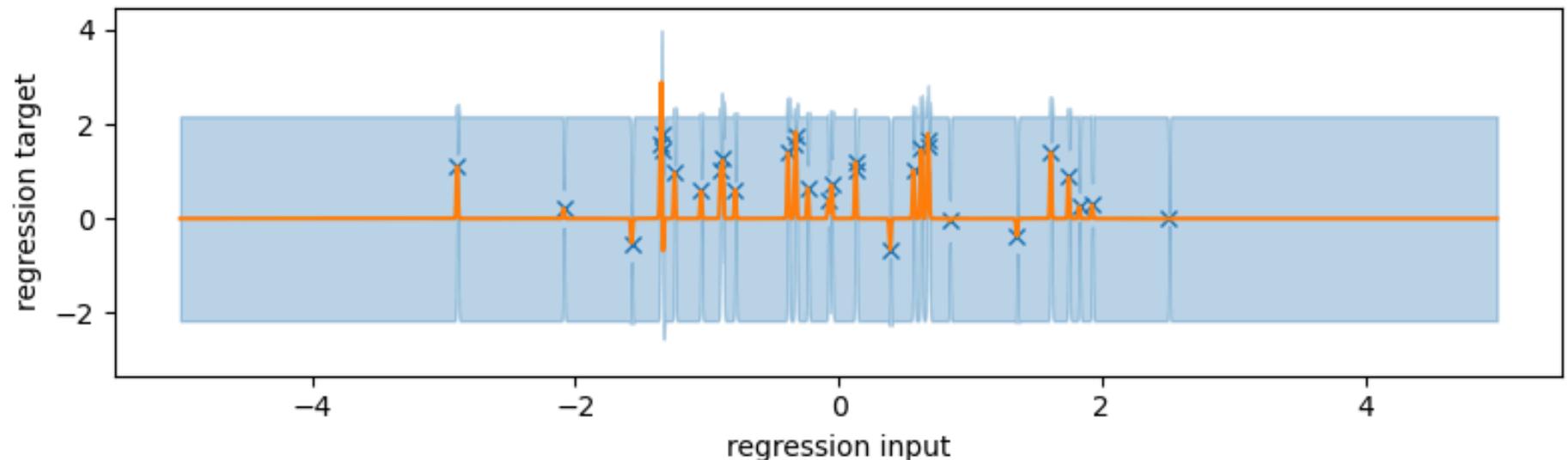
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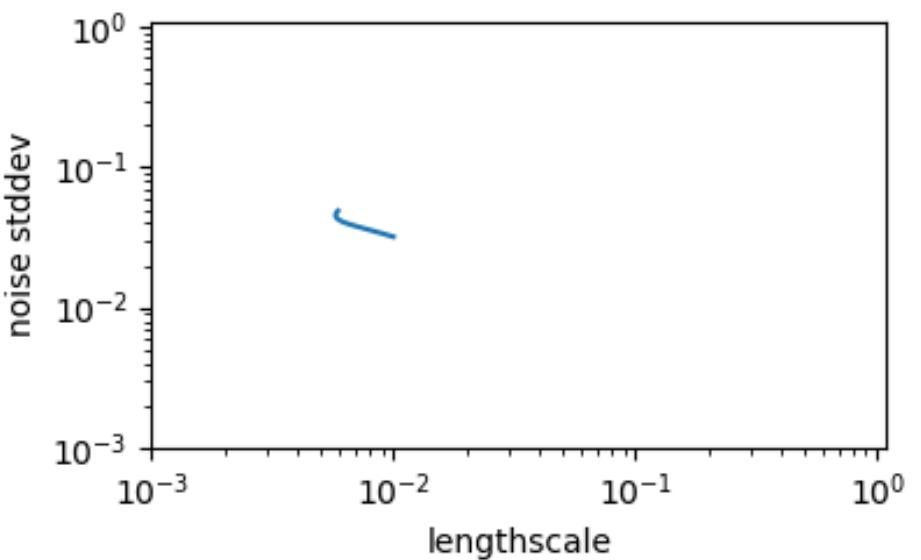
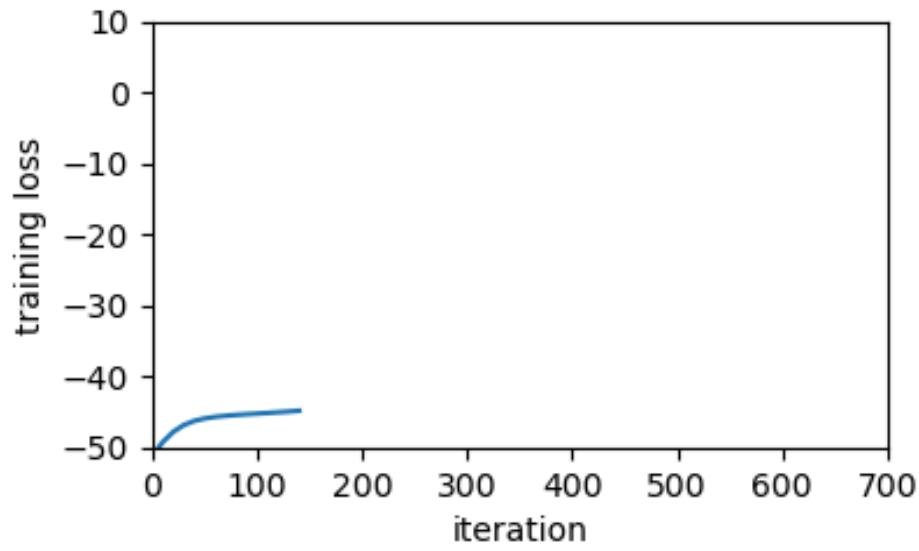
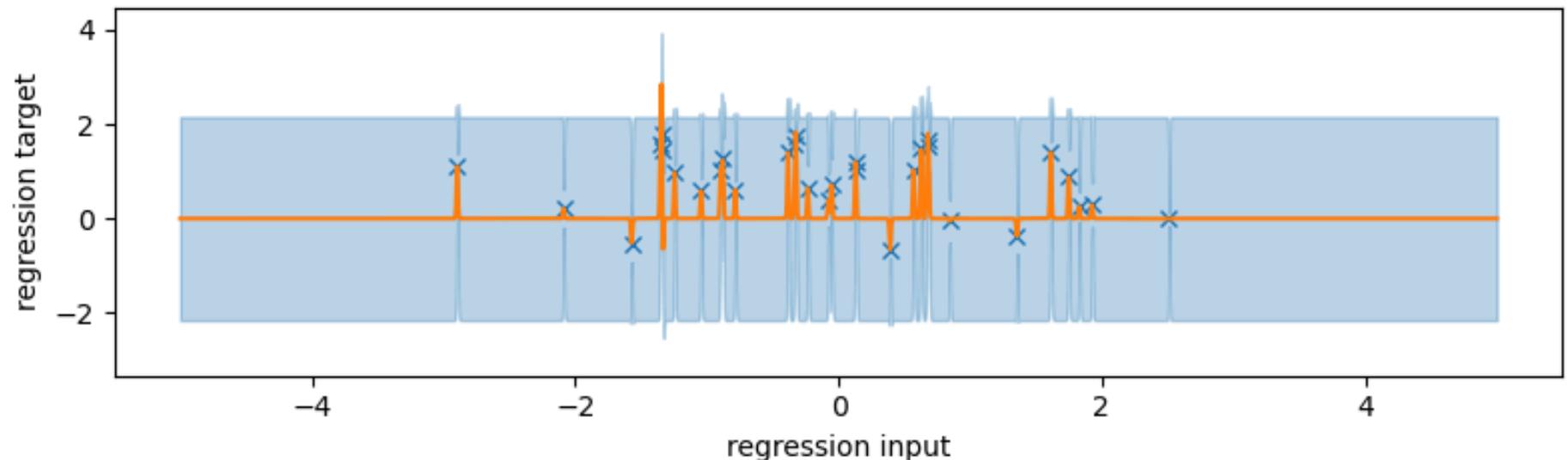
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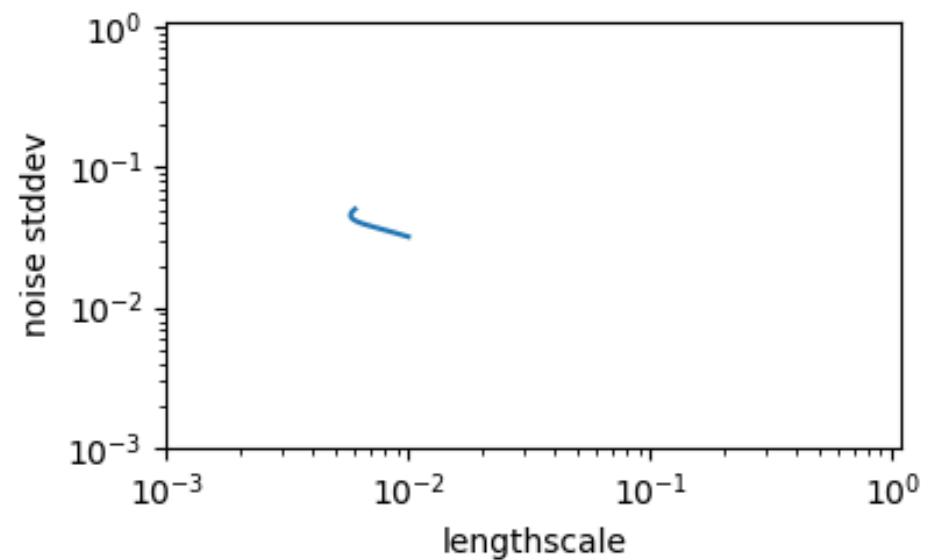
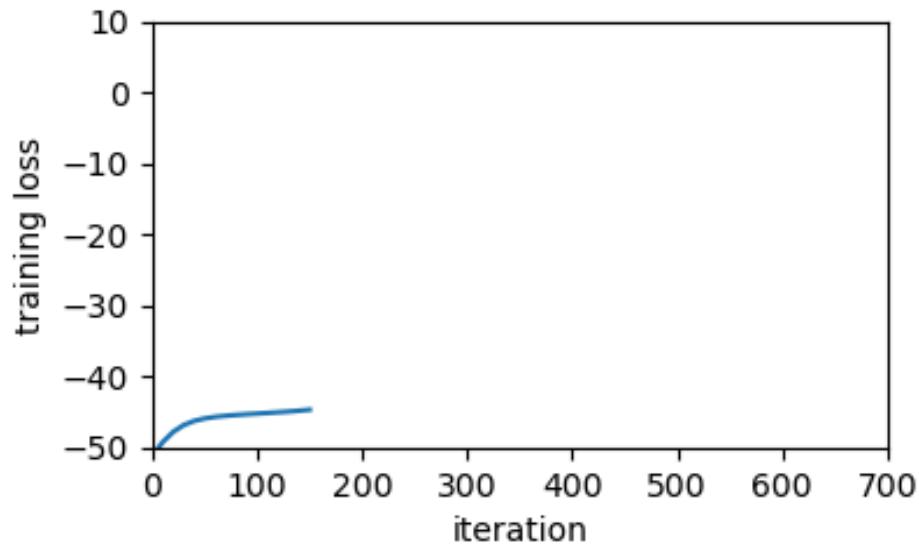
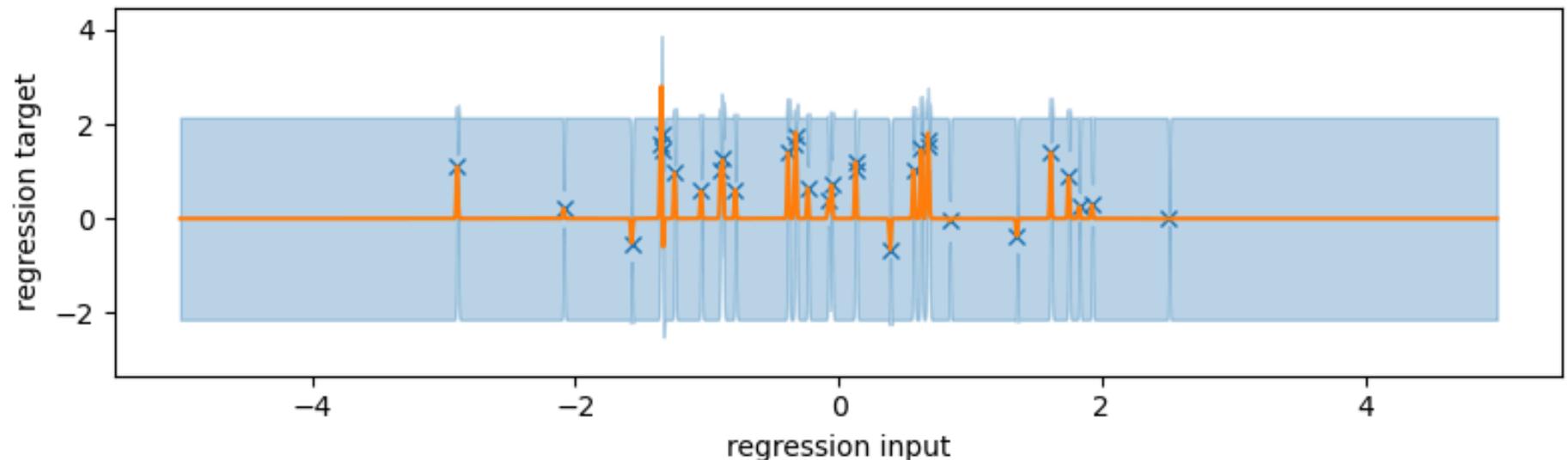
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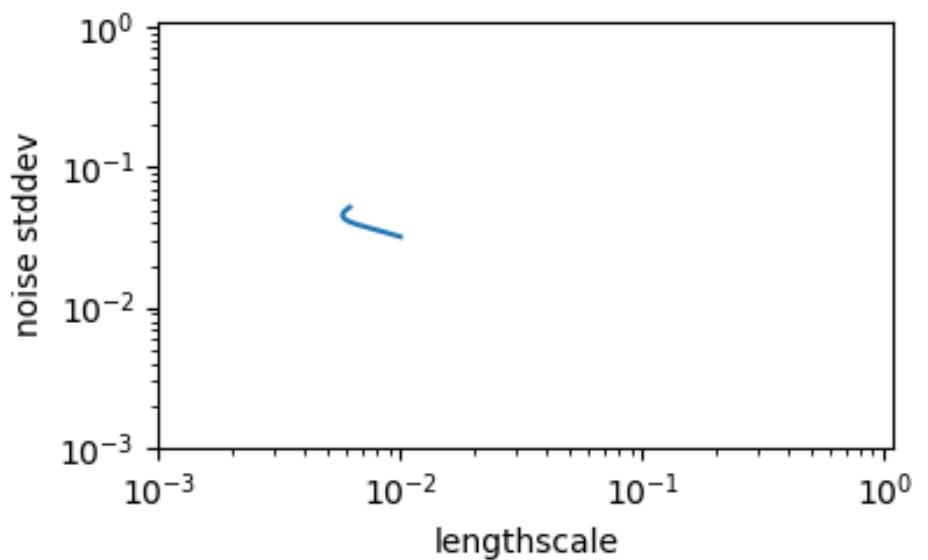
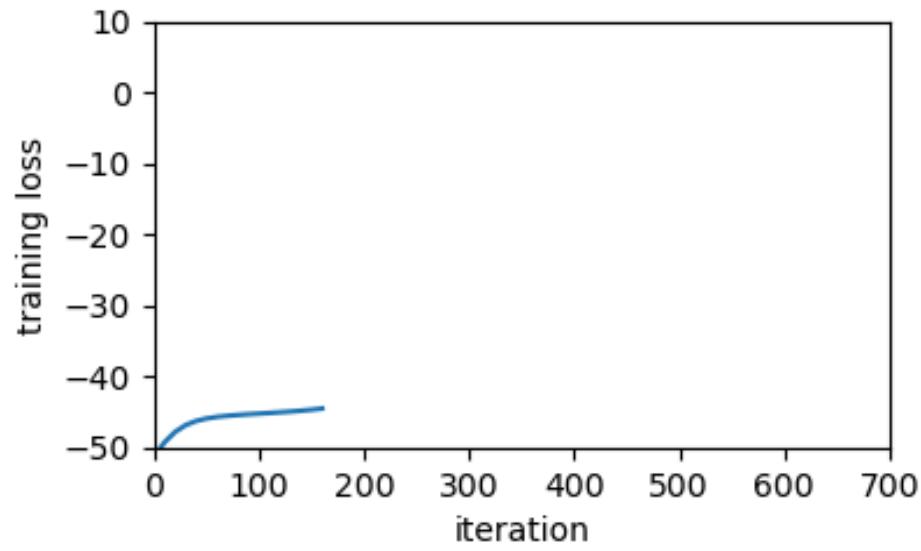
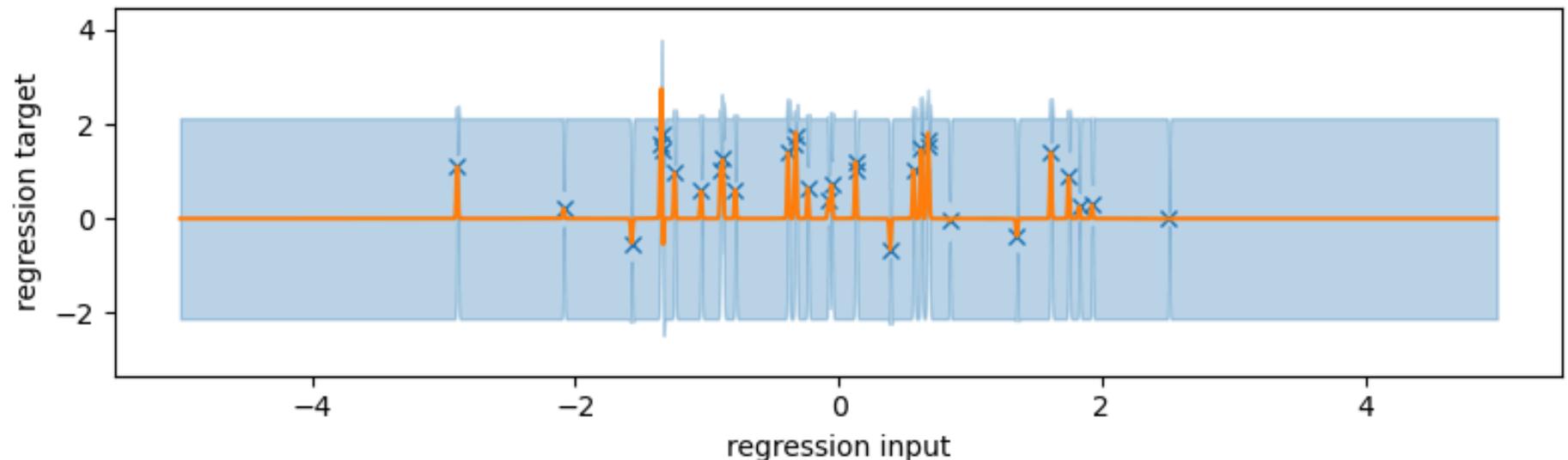
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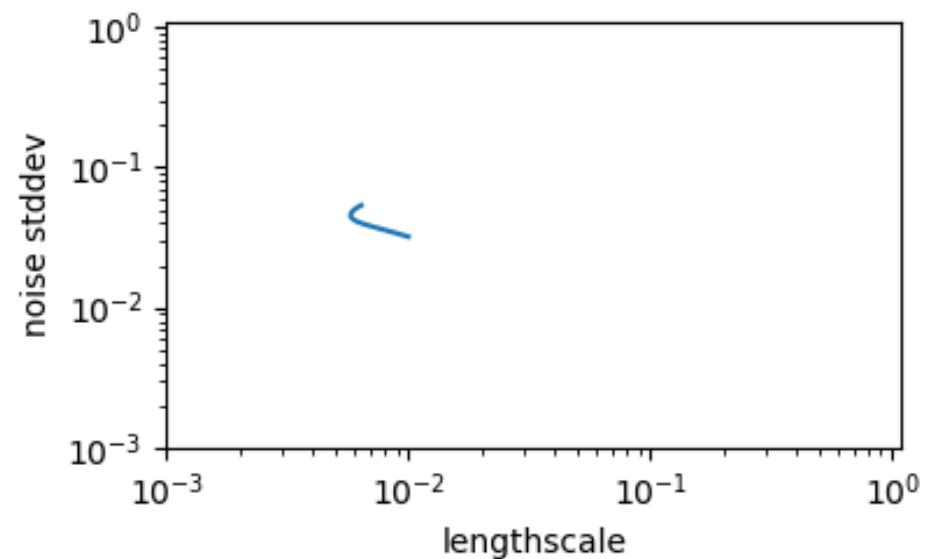
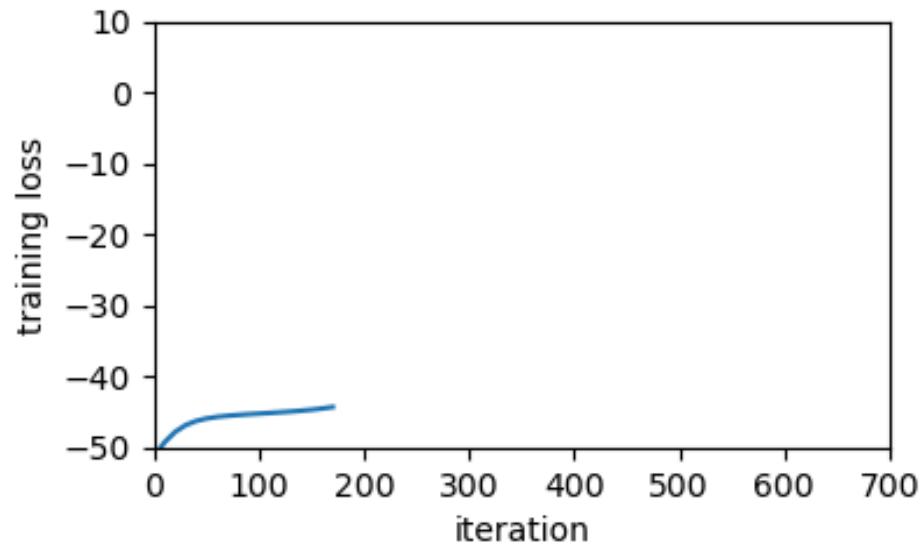
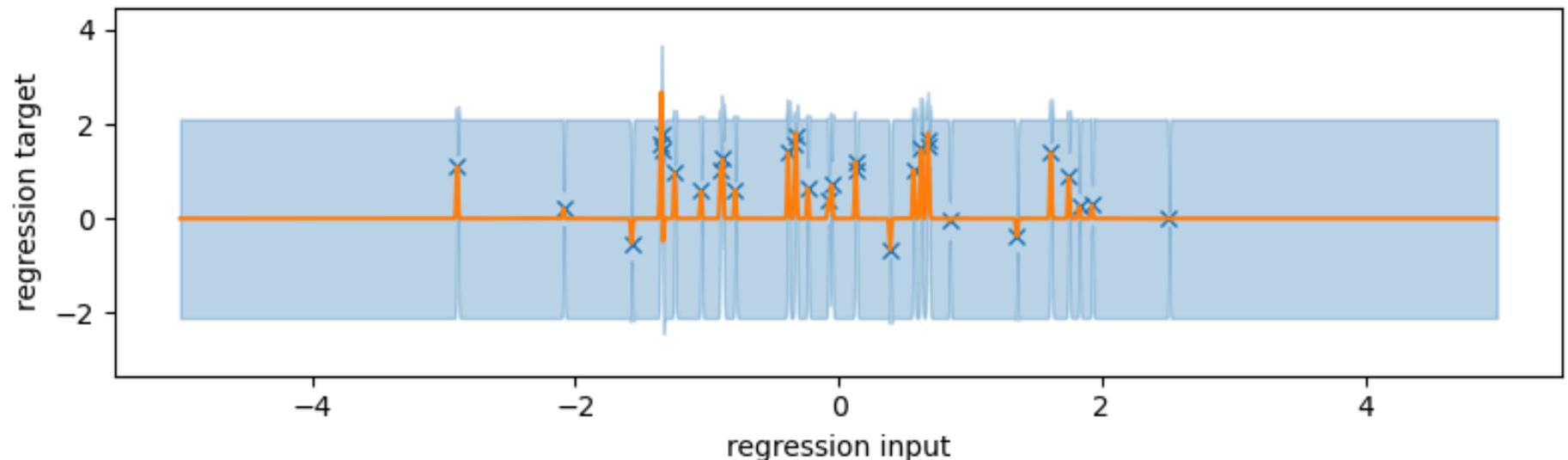
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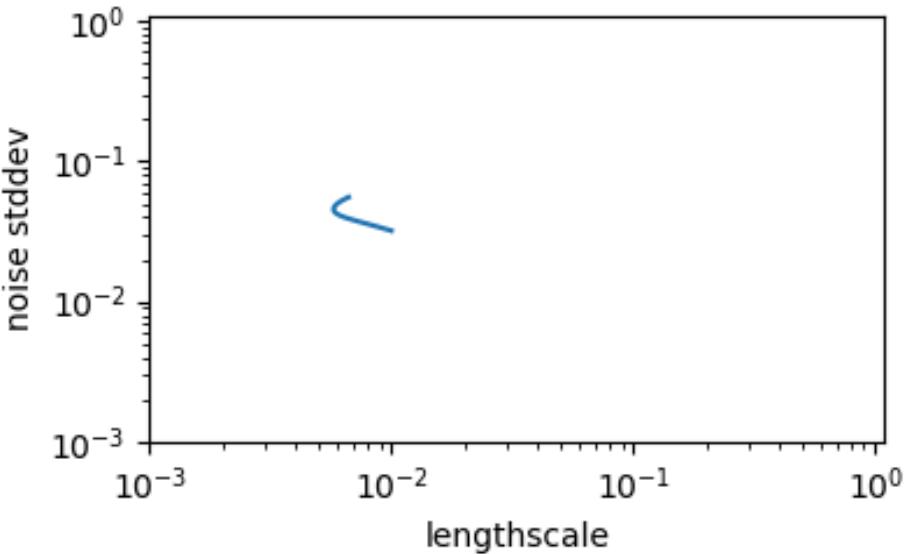
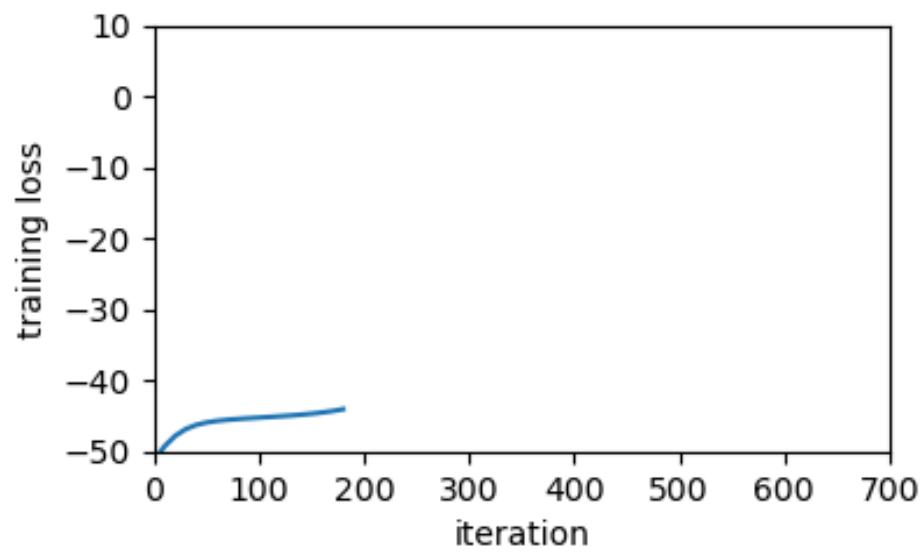
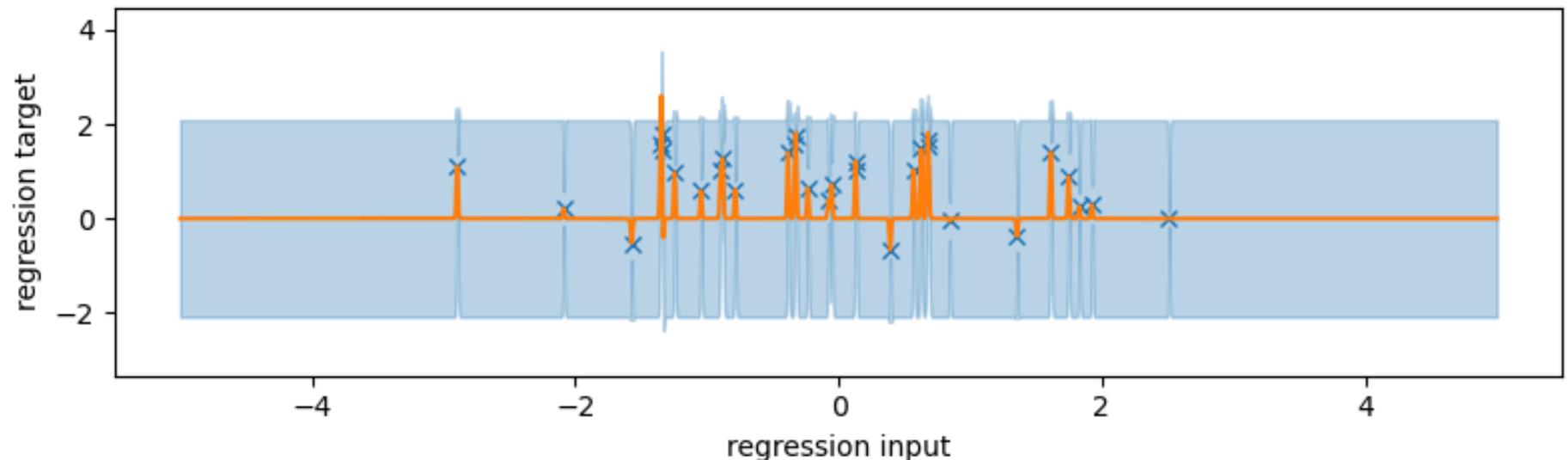
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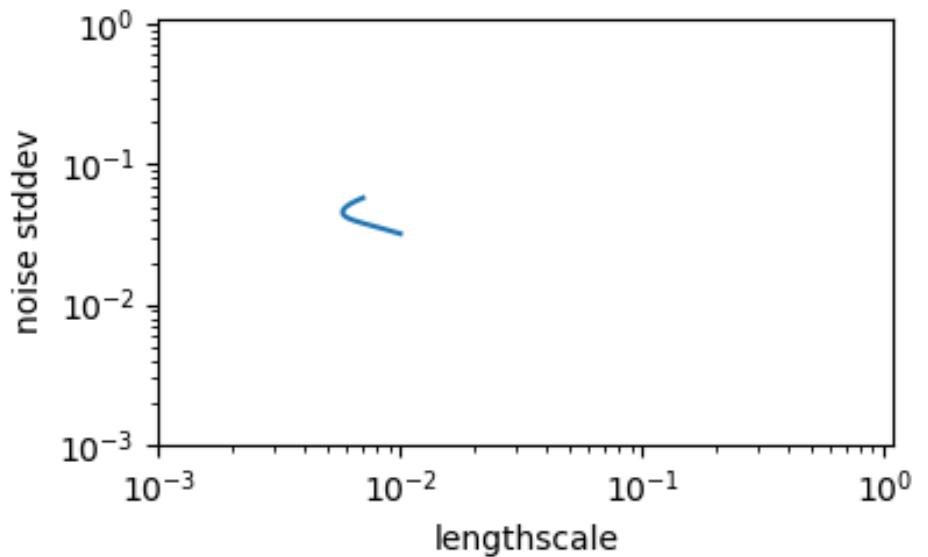
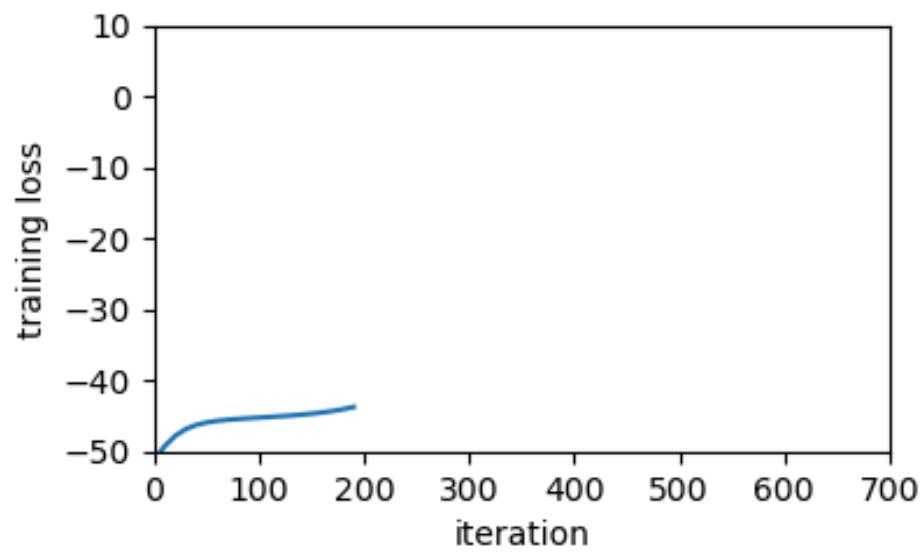
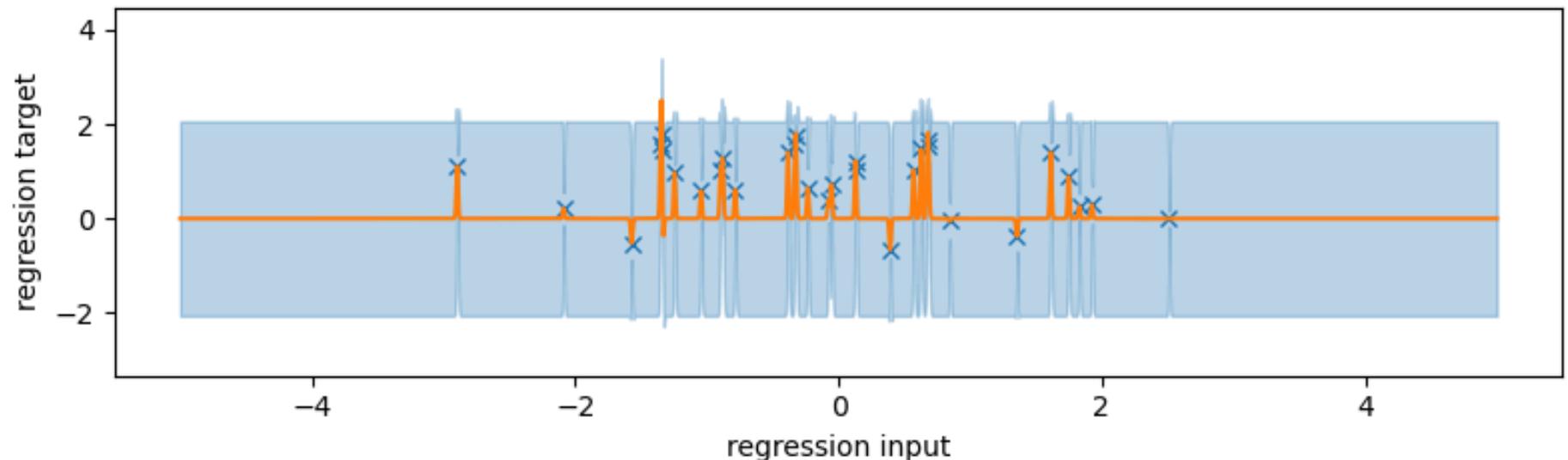
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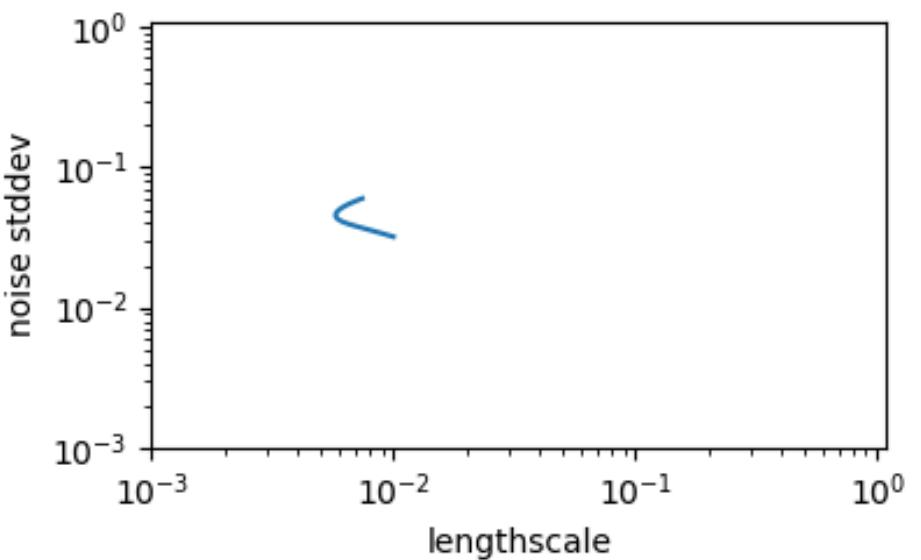
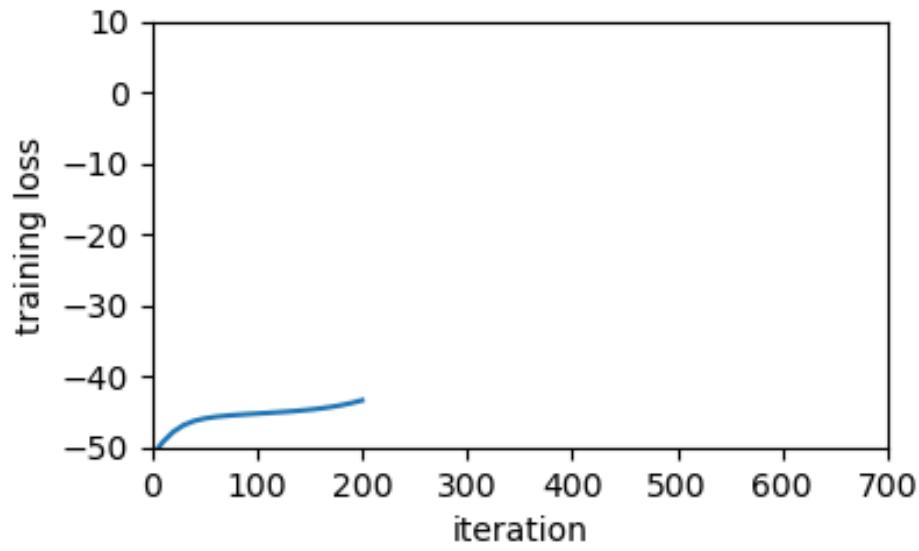
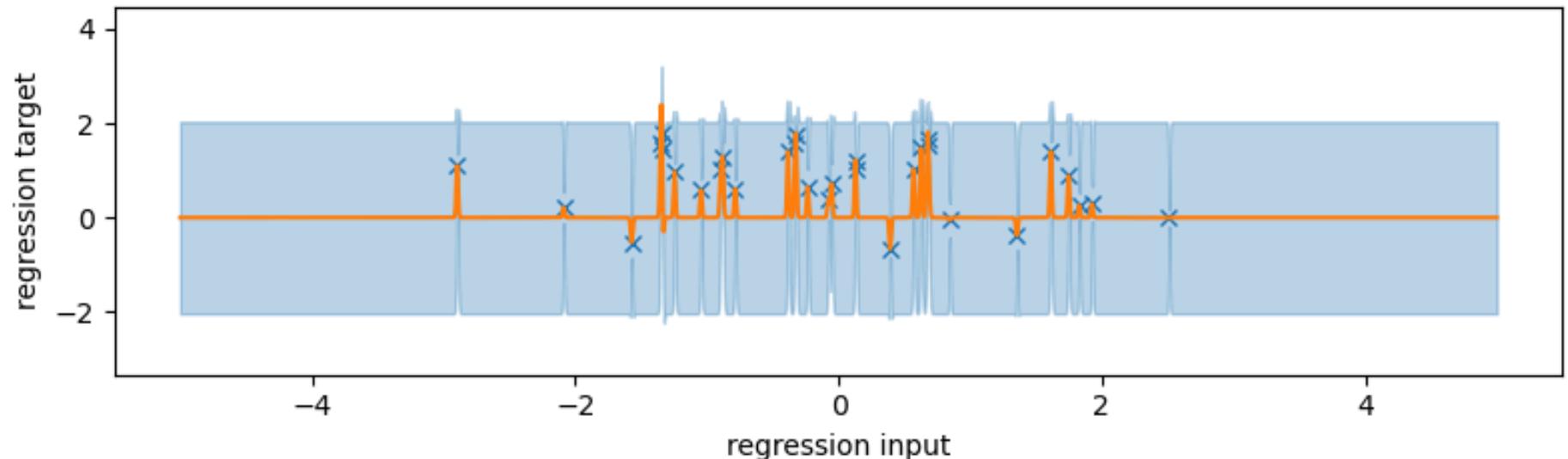
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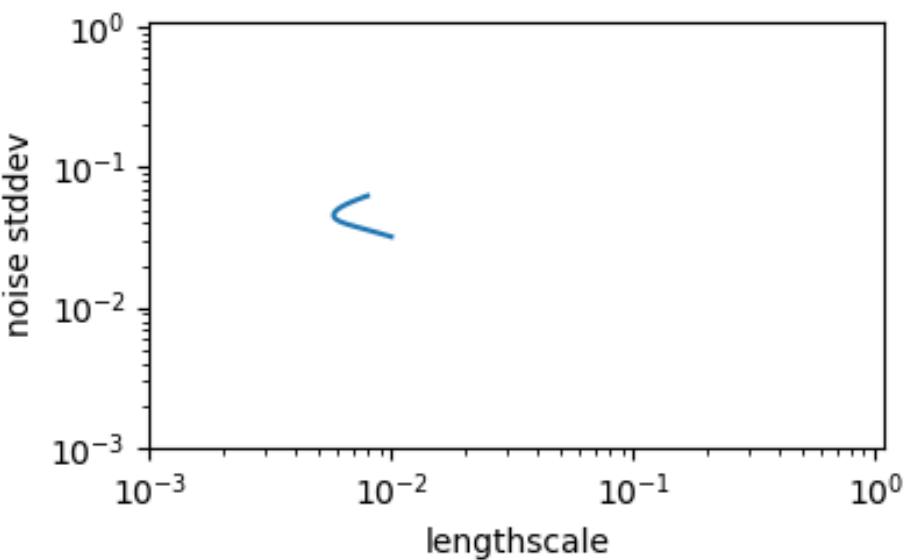
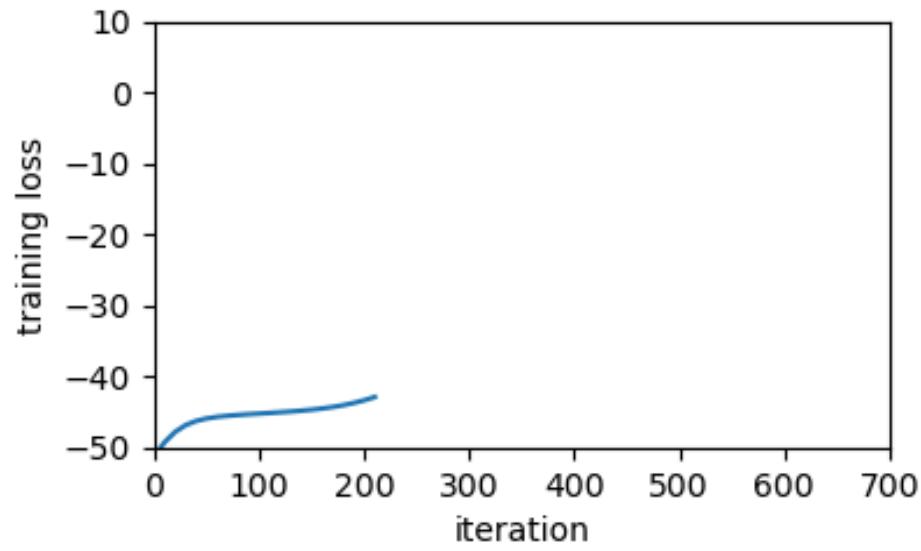
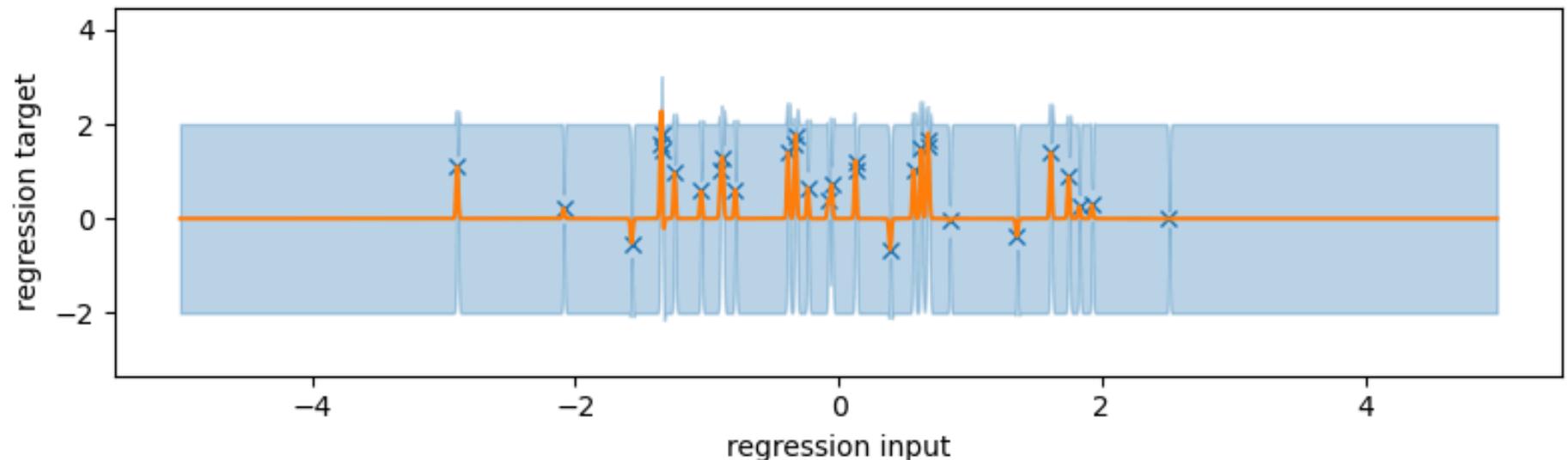
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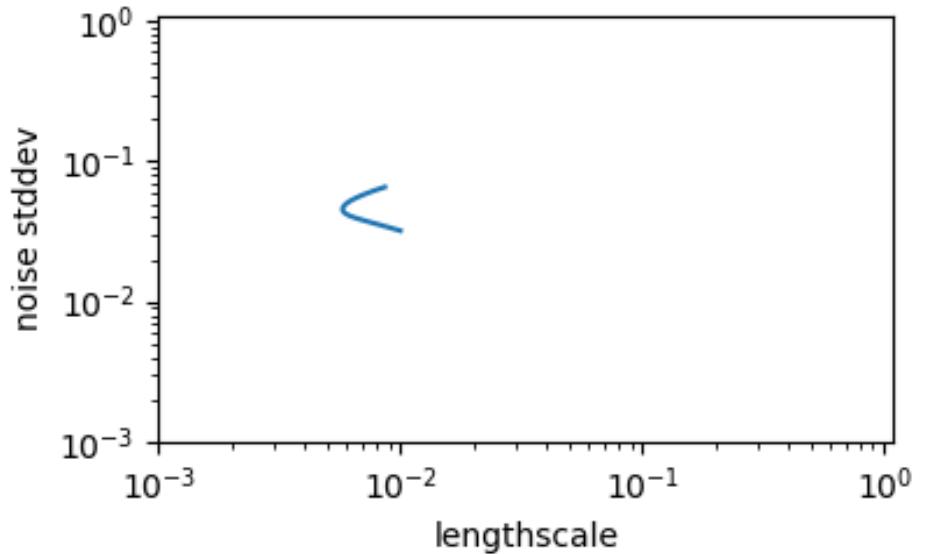
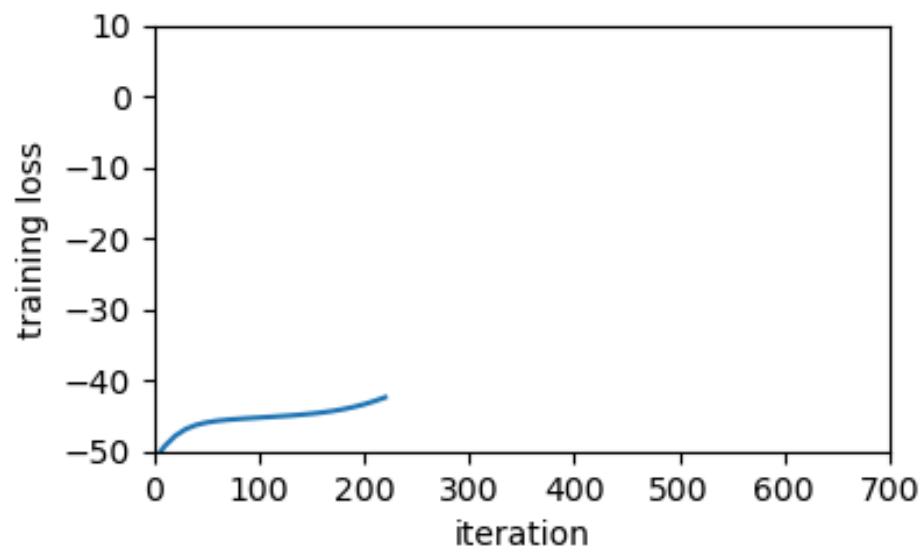
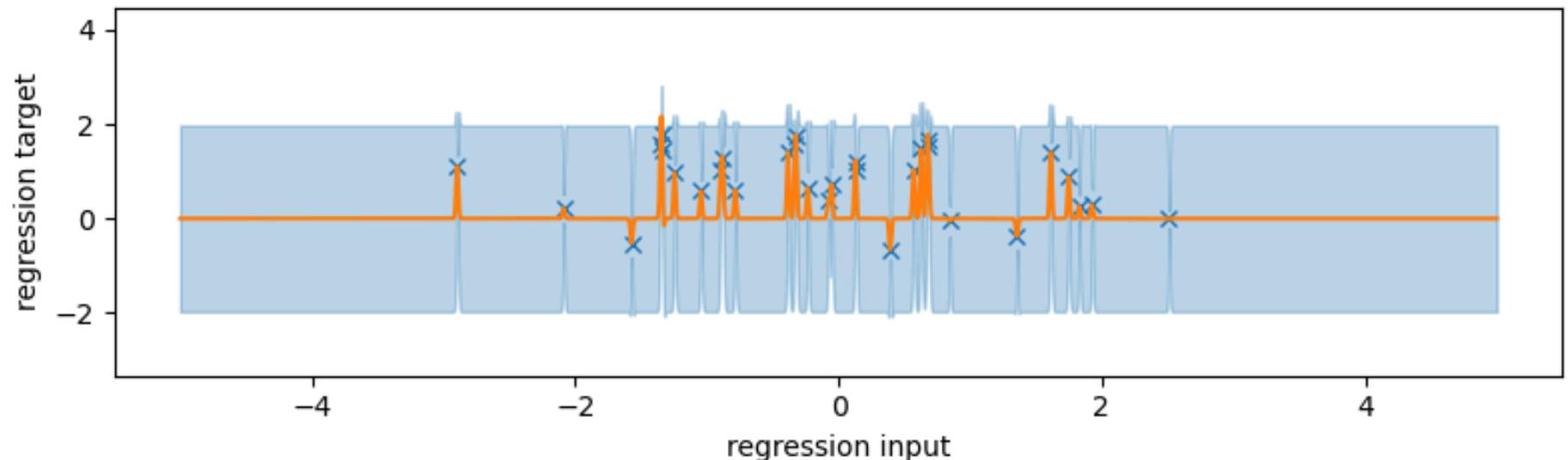
The Pragmatic Bayesian Answer



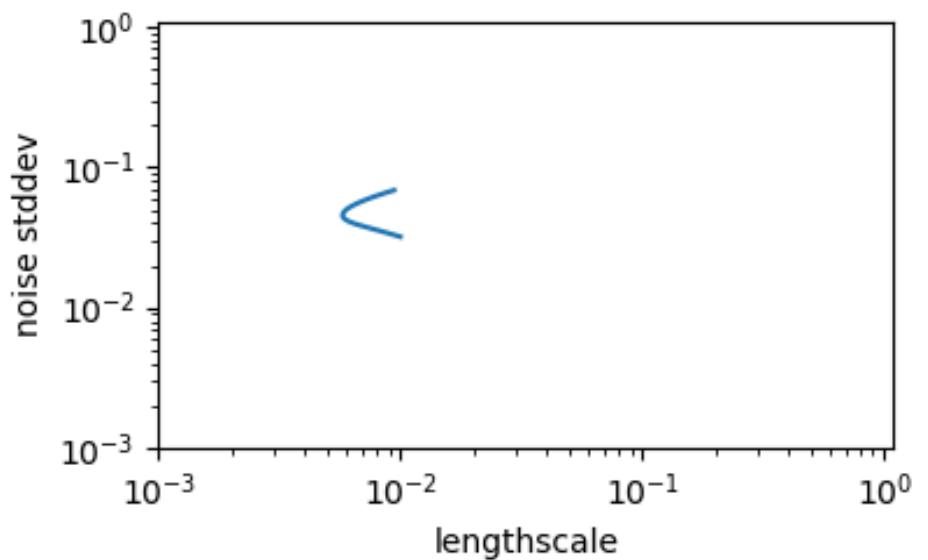
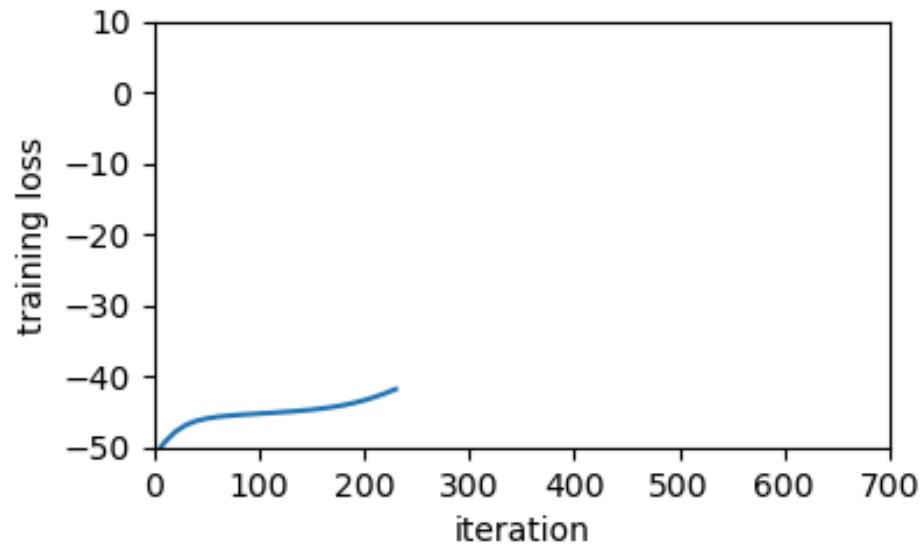
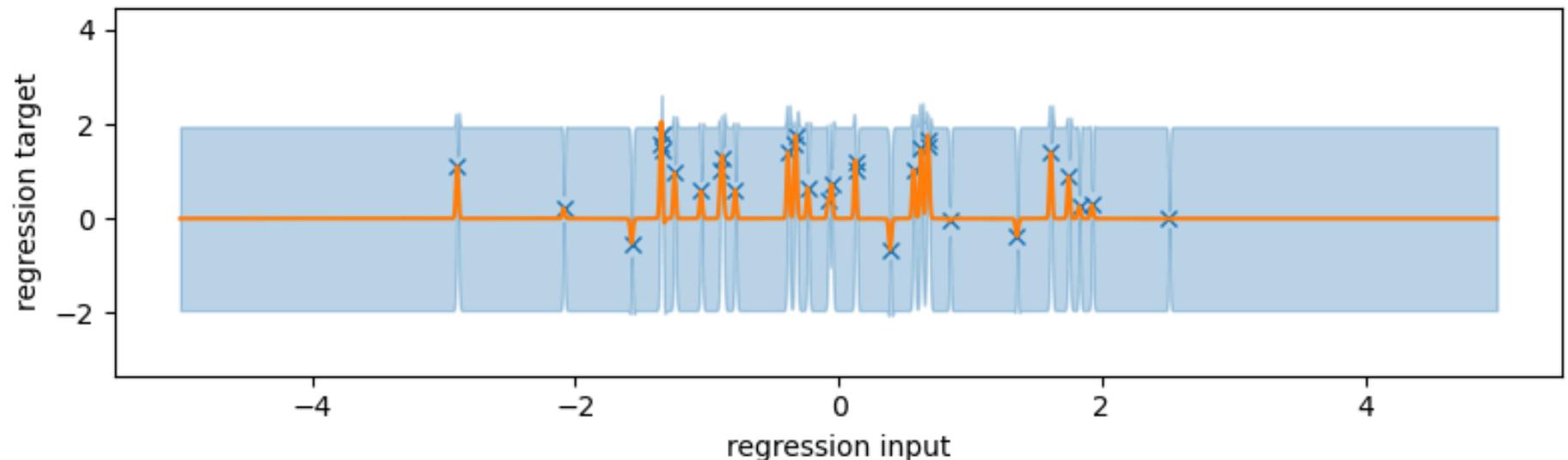
The Pragmatic Bayesian Answer



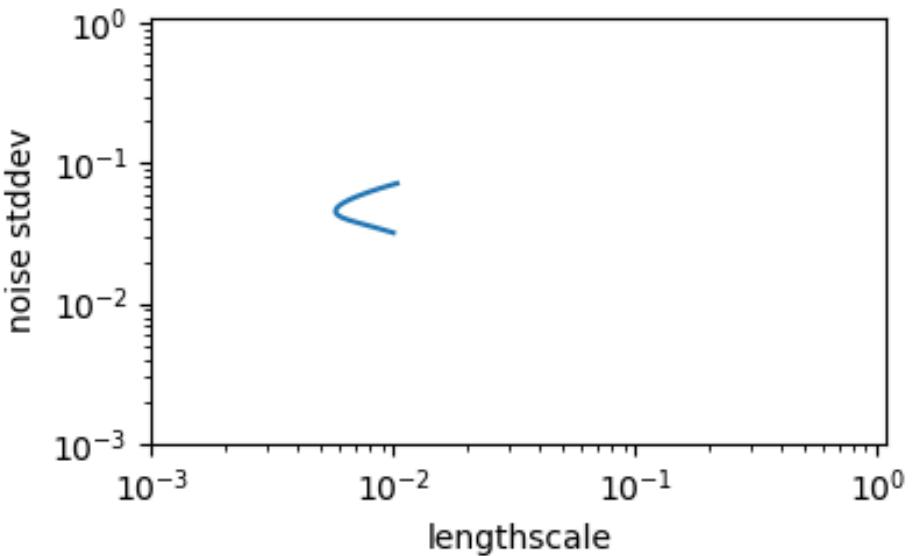
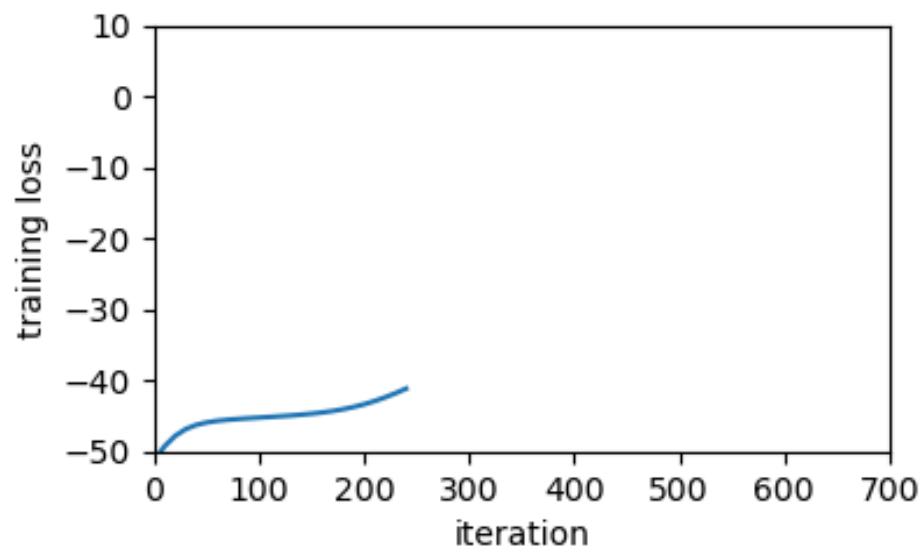
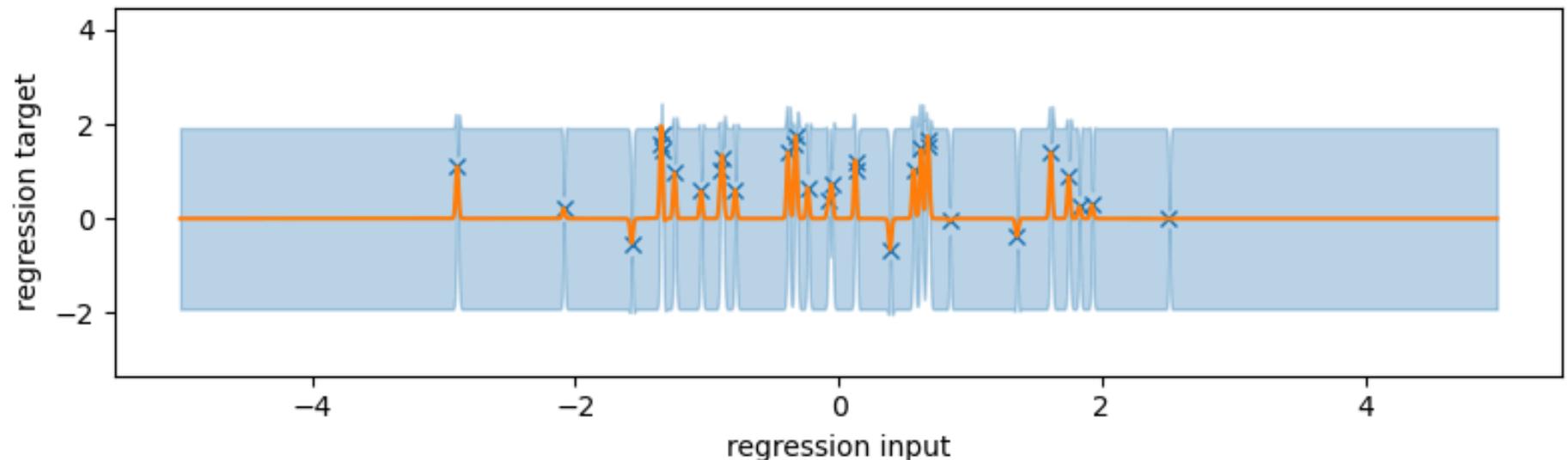
The Pragmatic Bayesian Answer



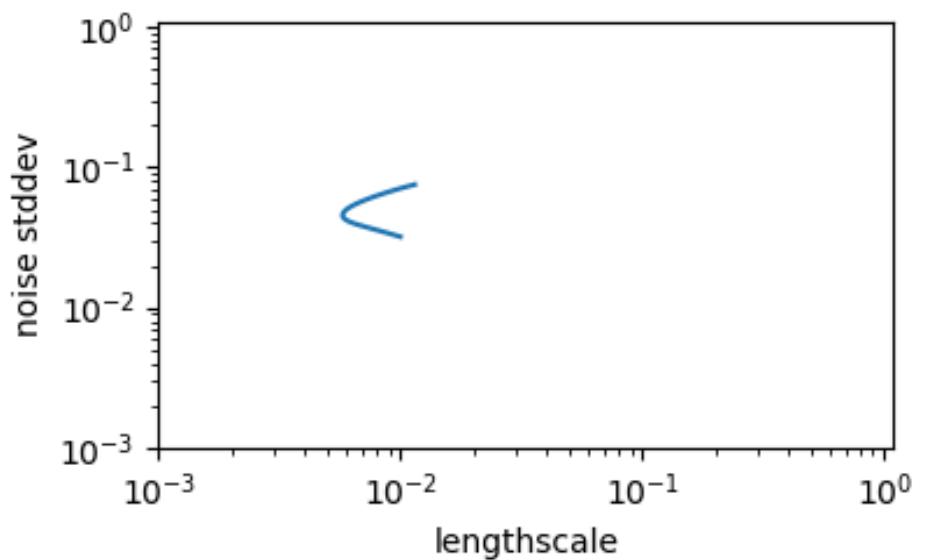
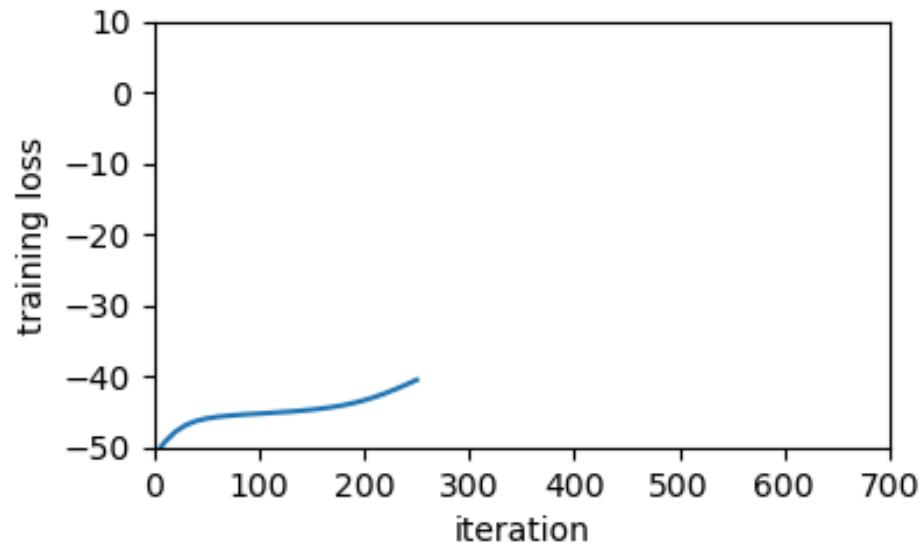
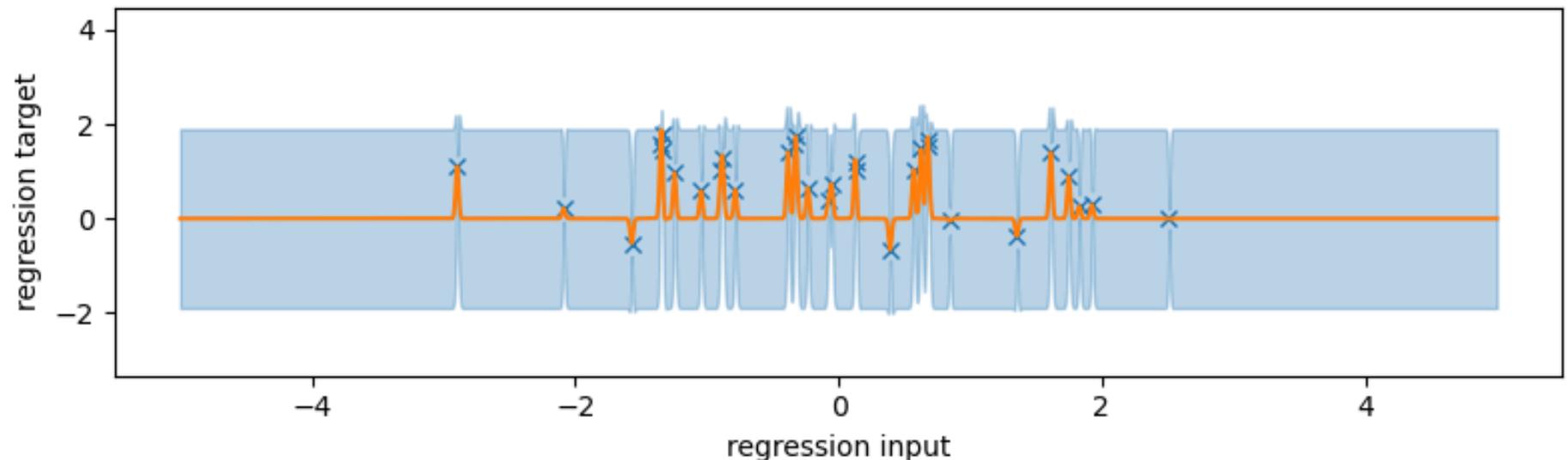
The Pragmatic Bayesian Answer



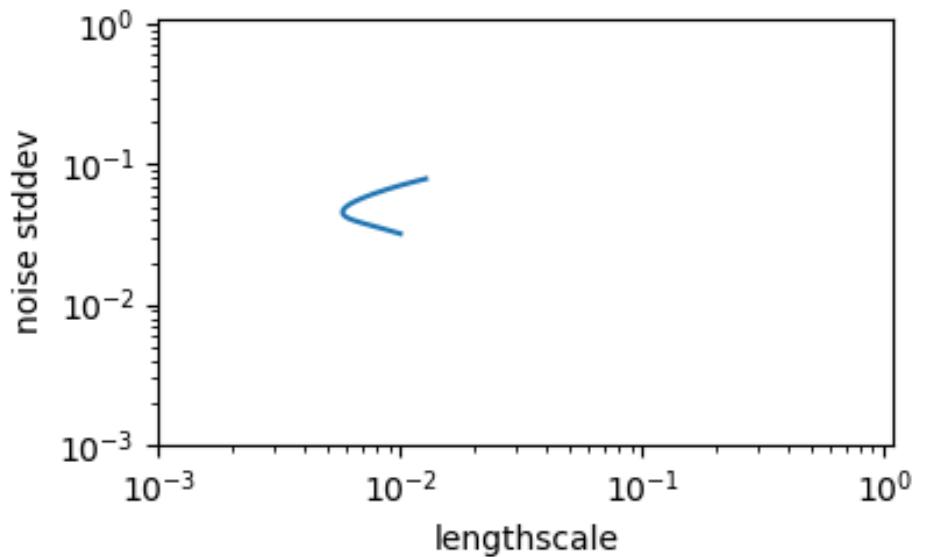
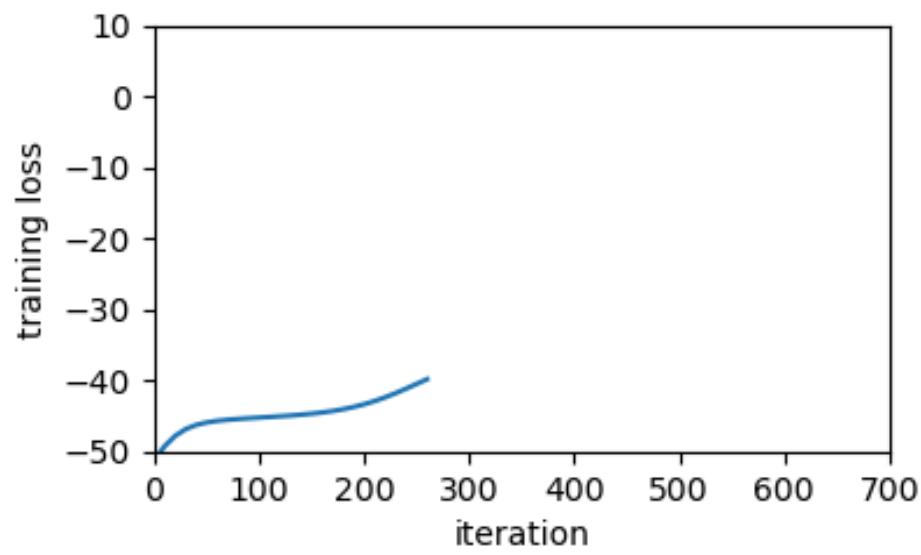
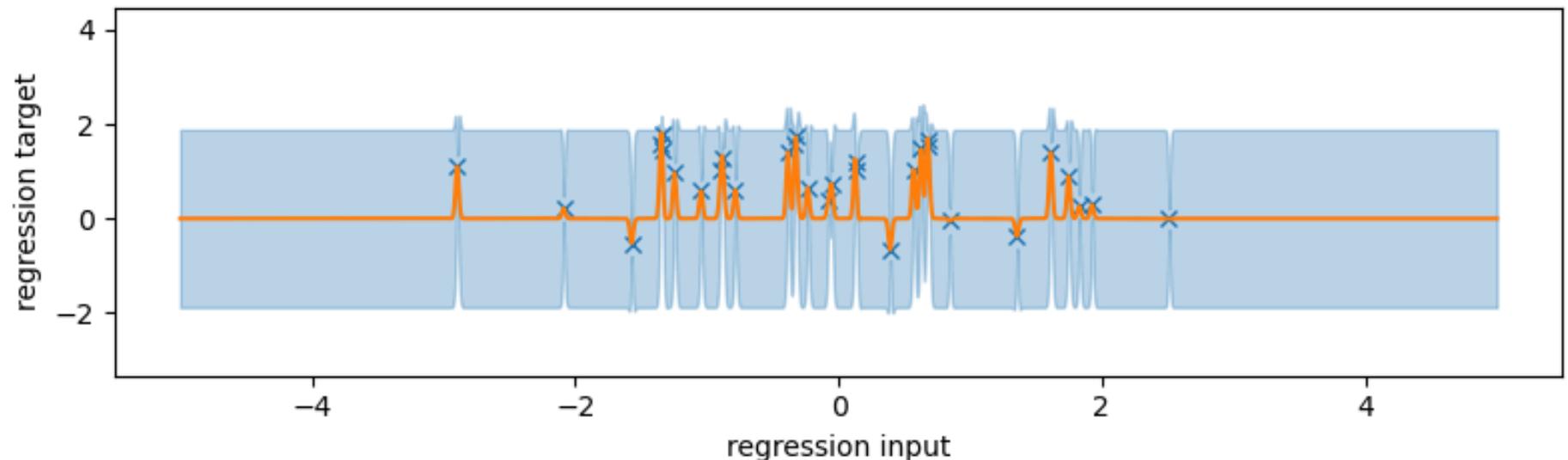
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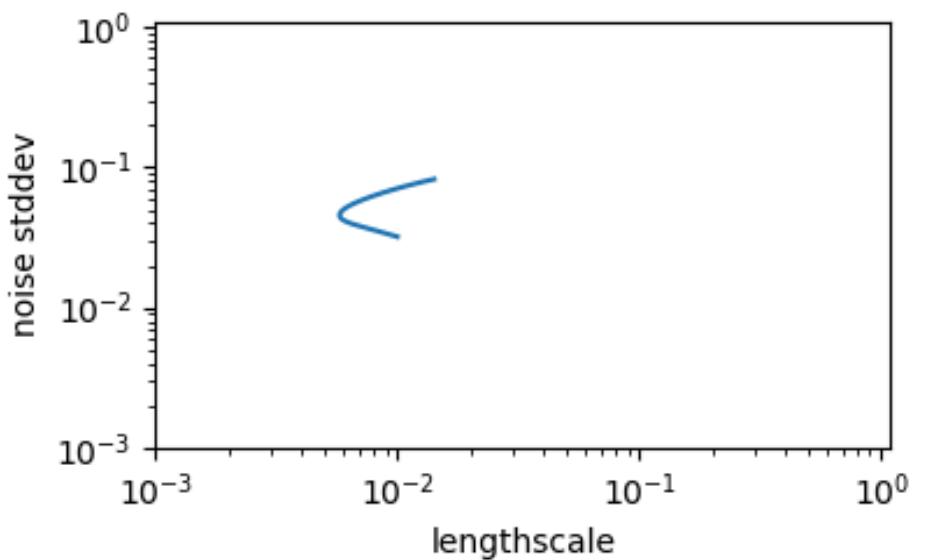
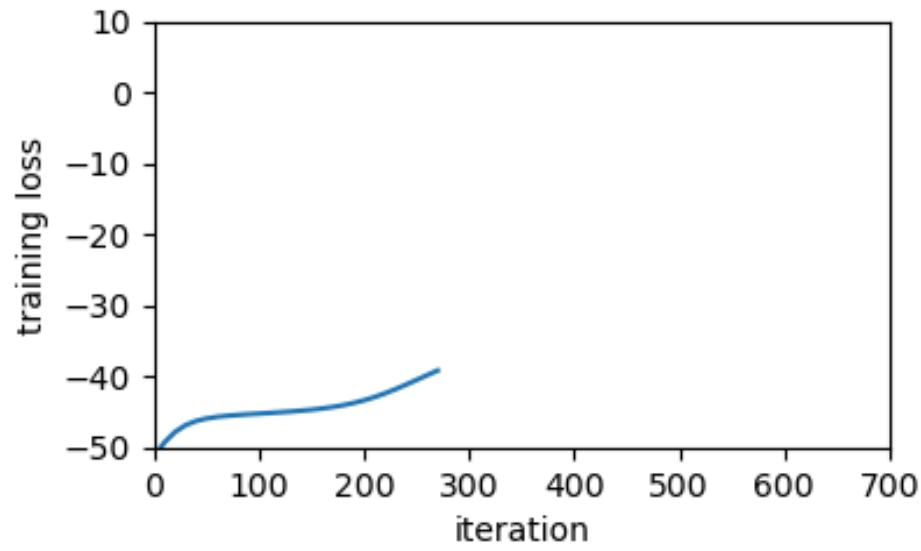
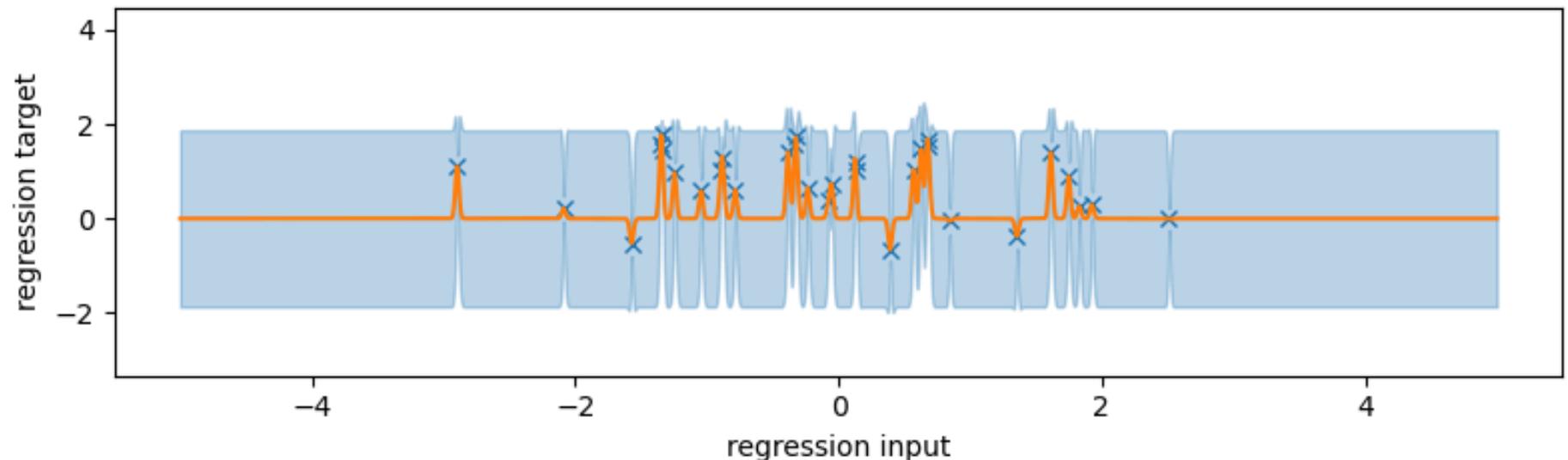
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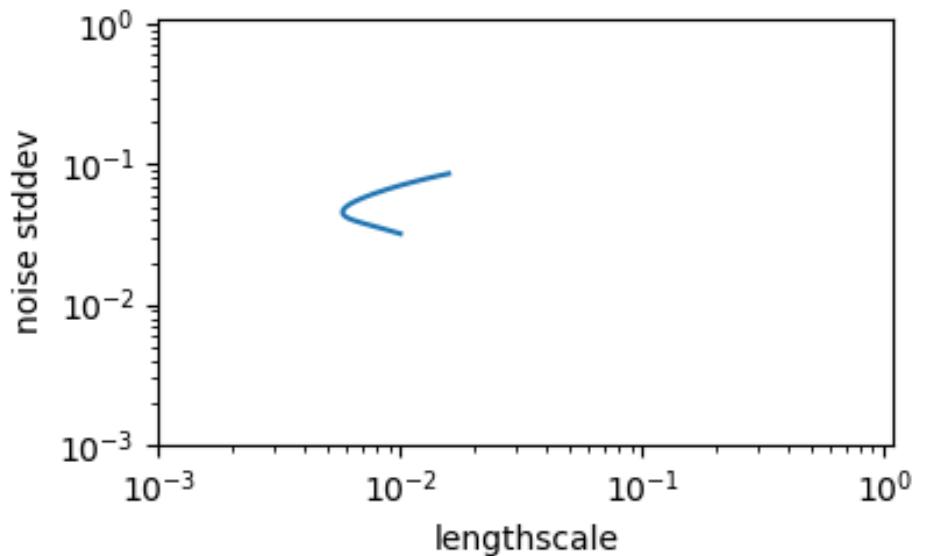
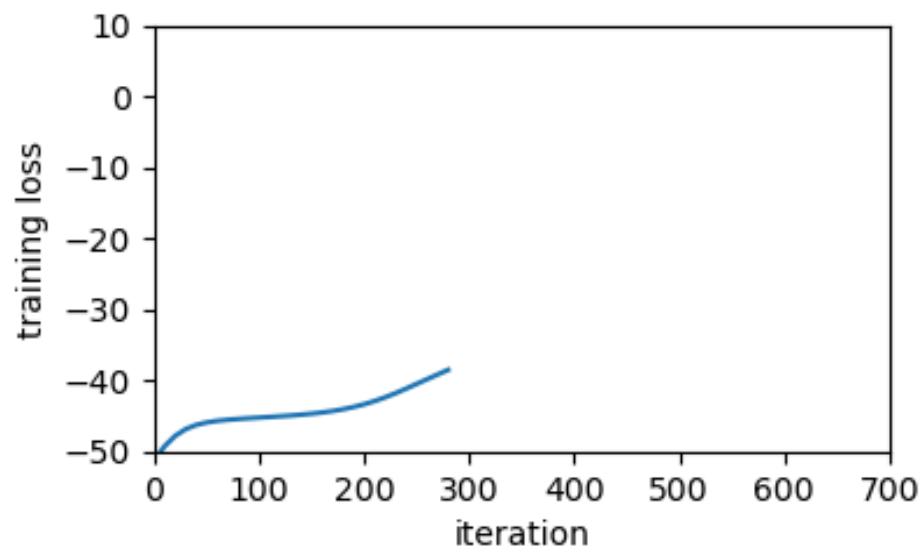
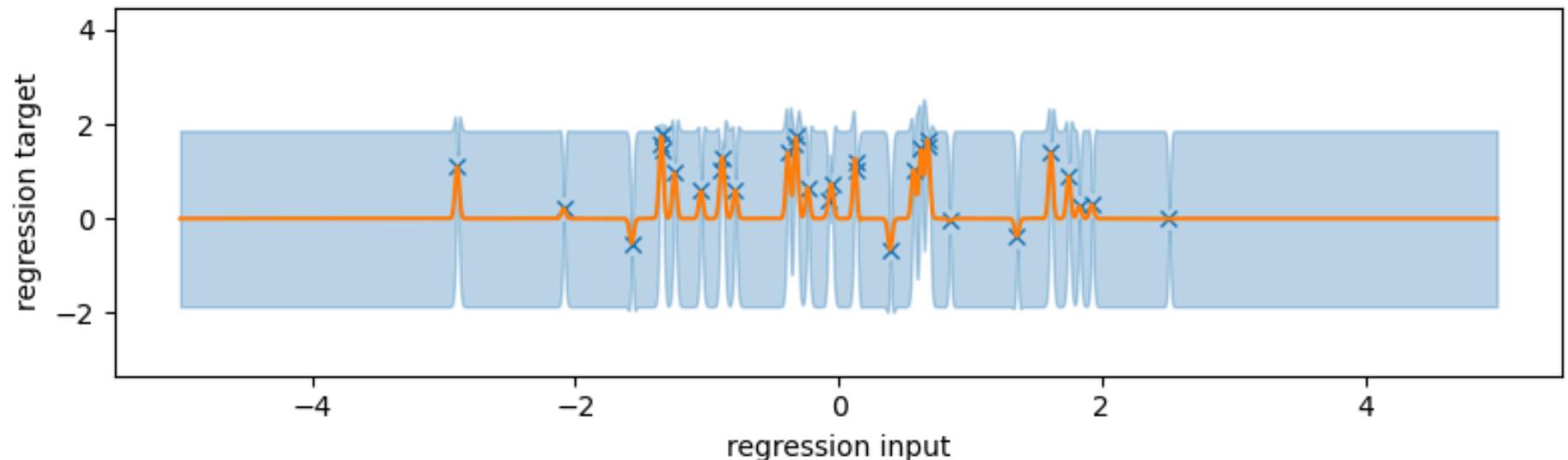
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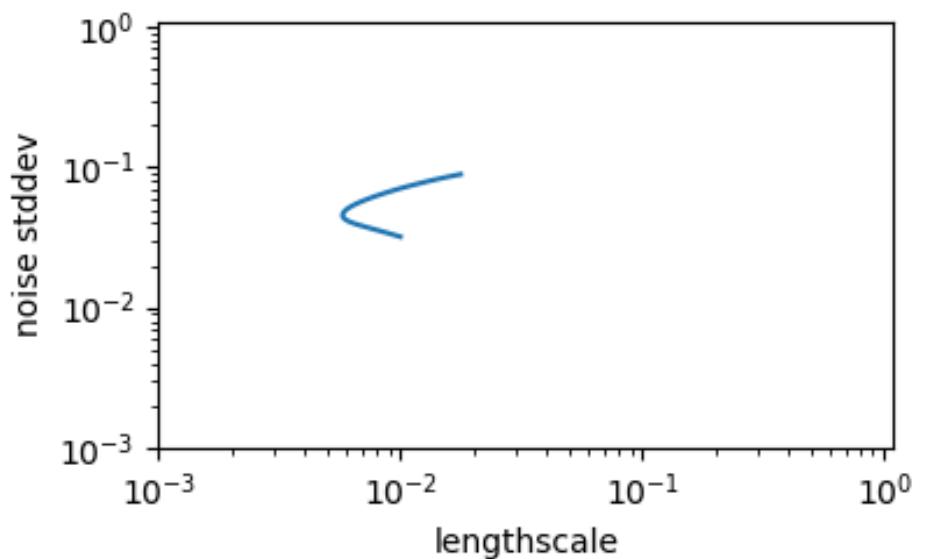
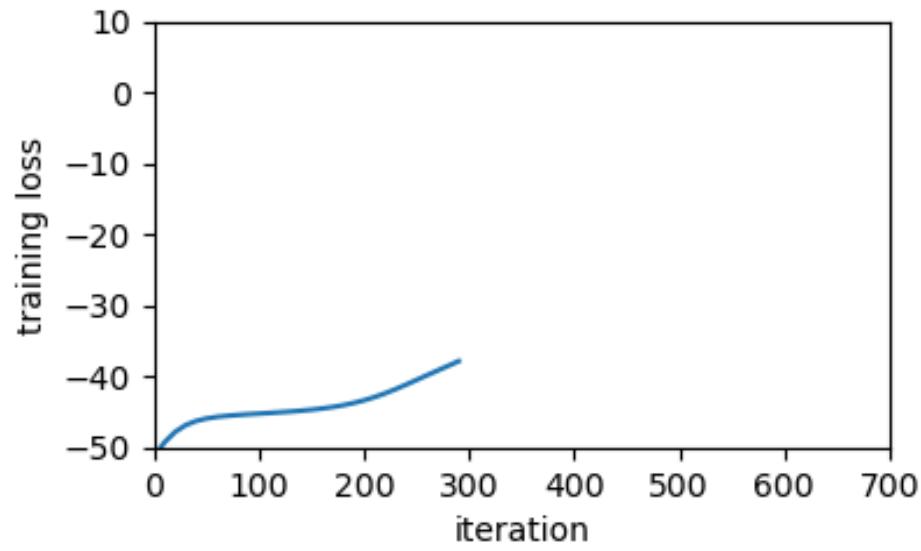
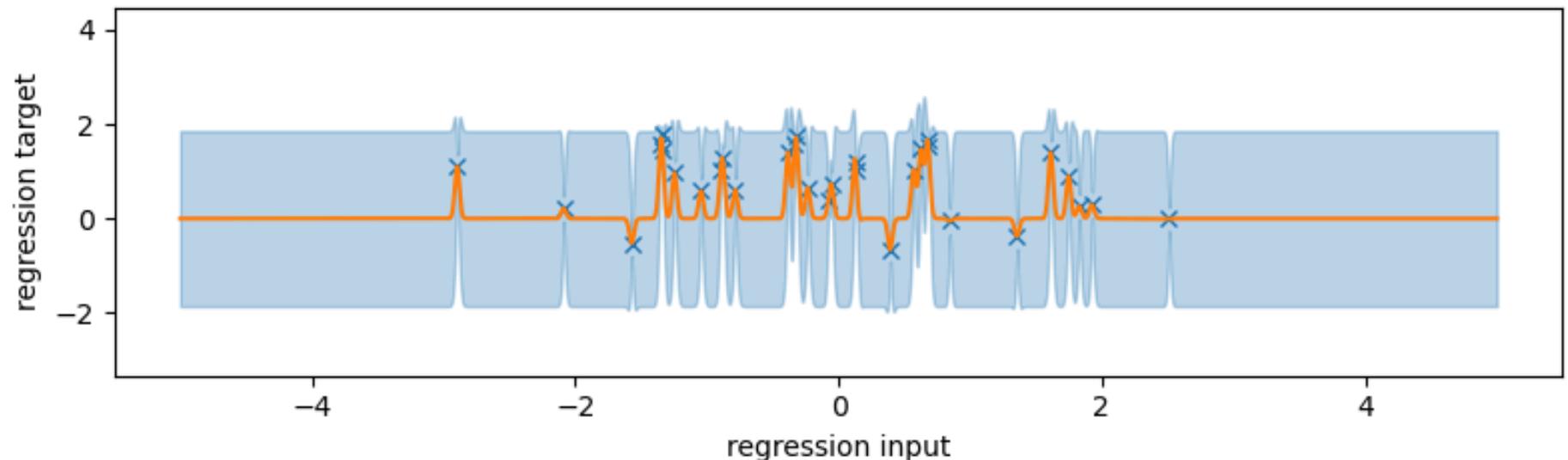
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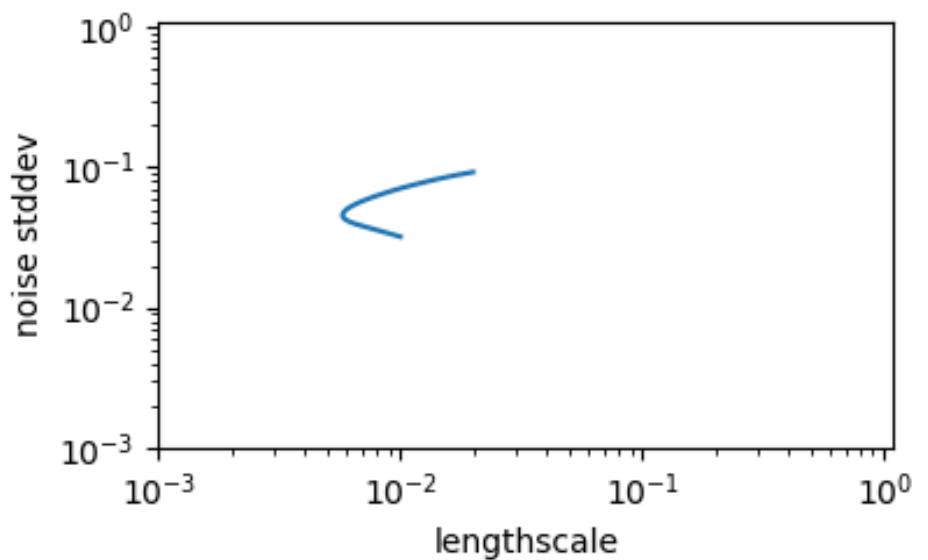
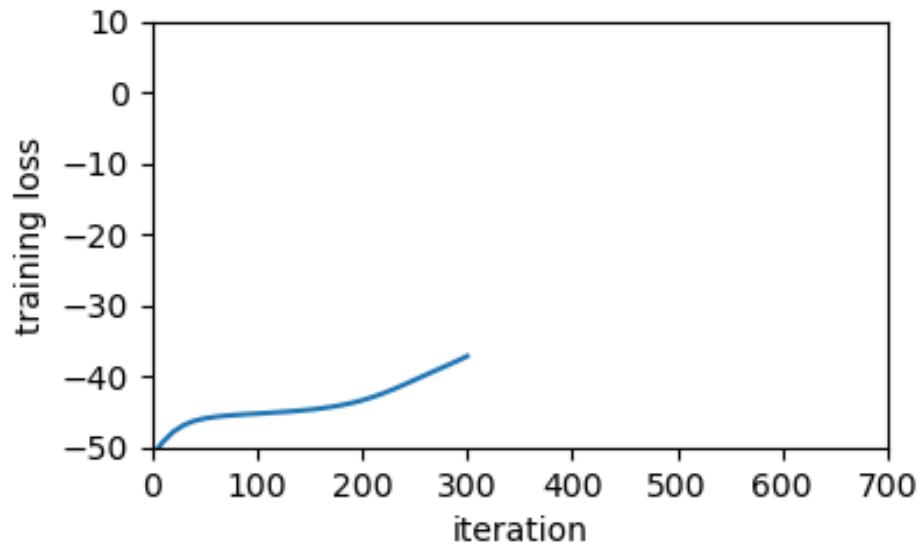
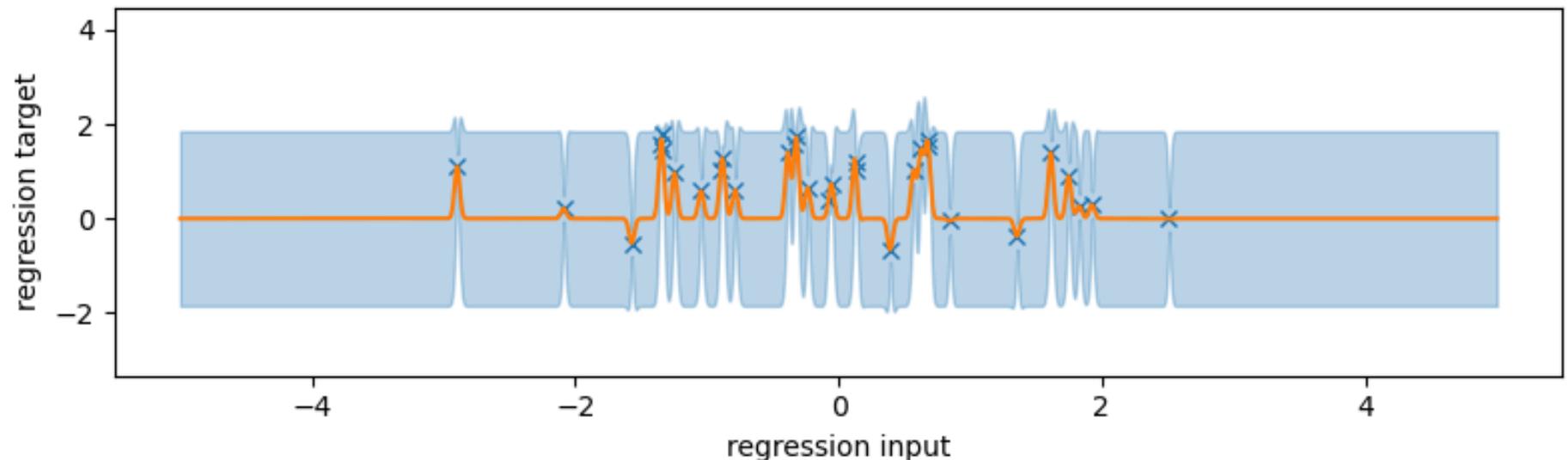
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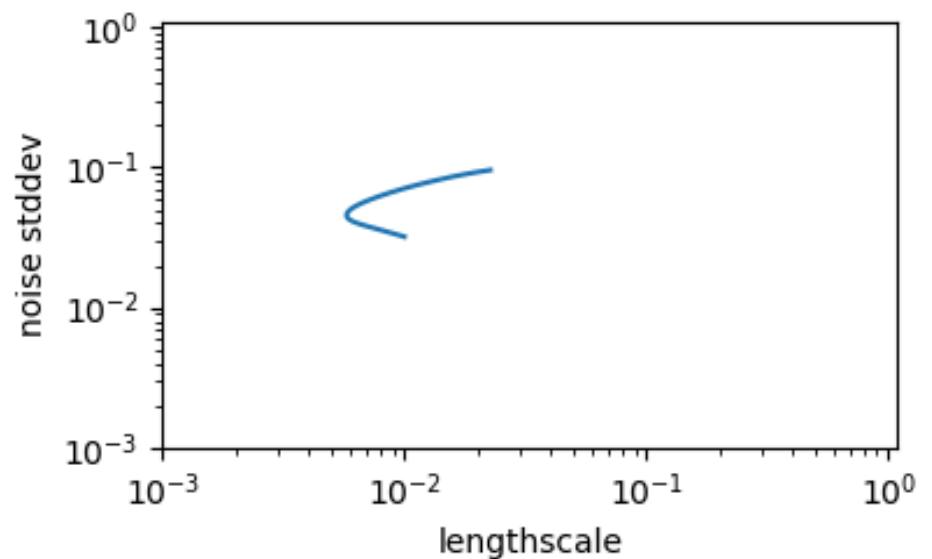
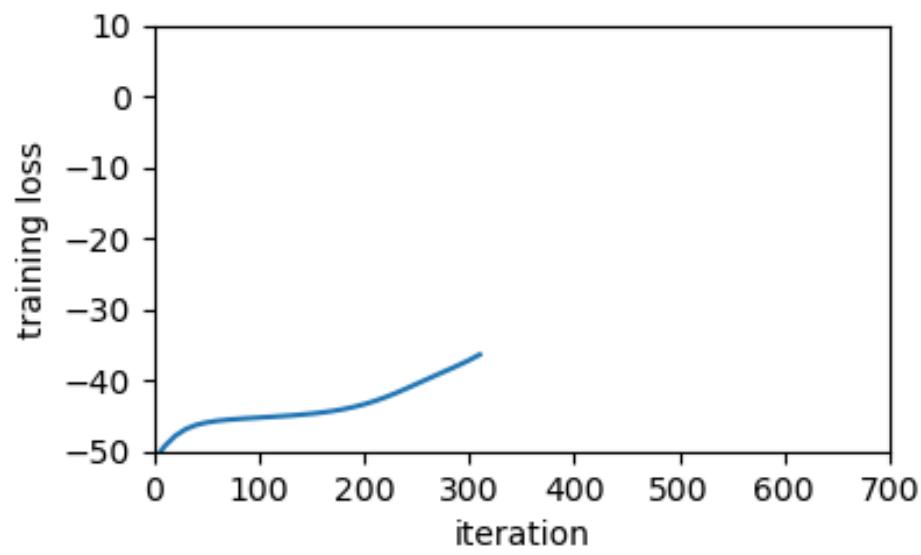
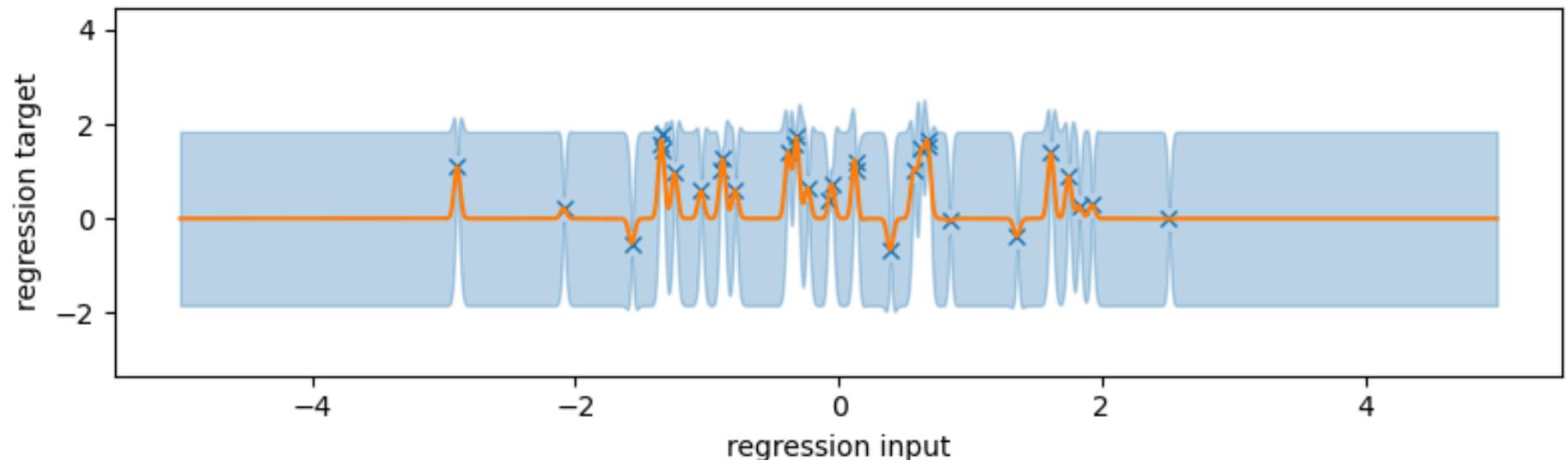
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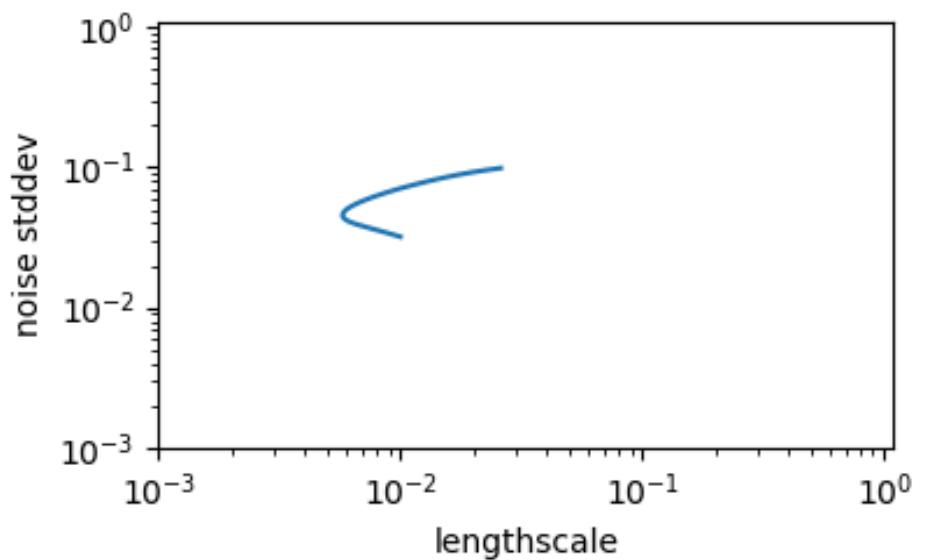
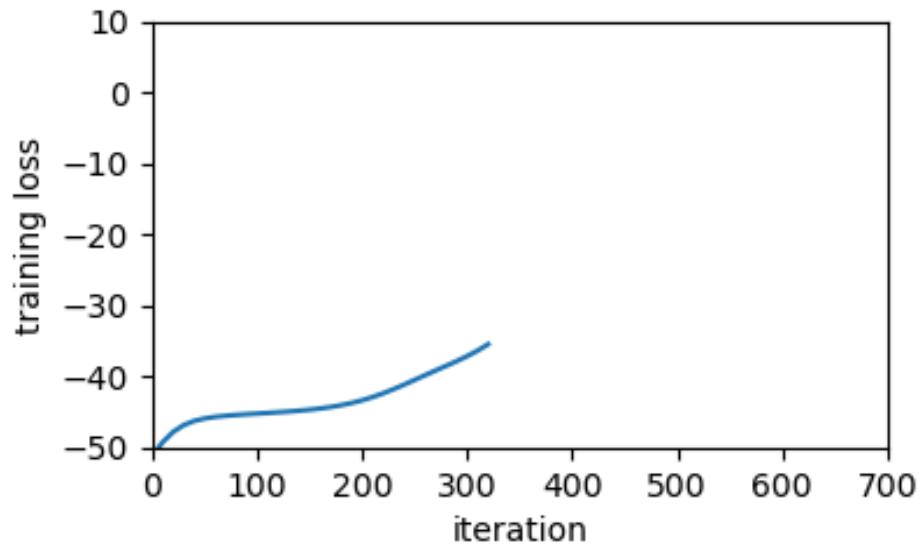
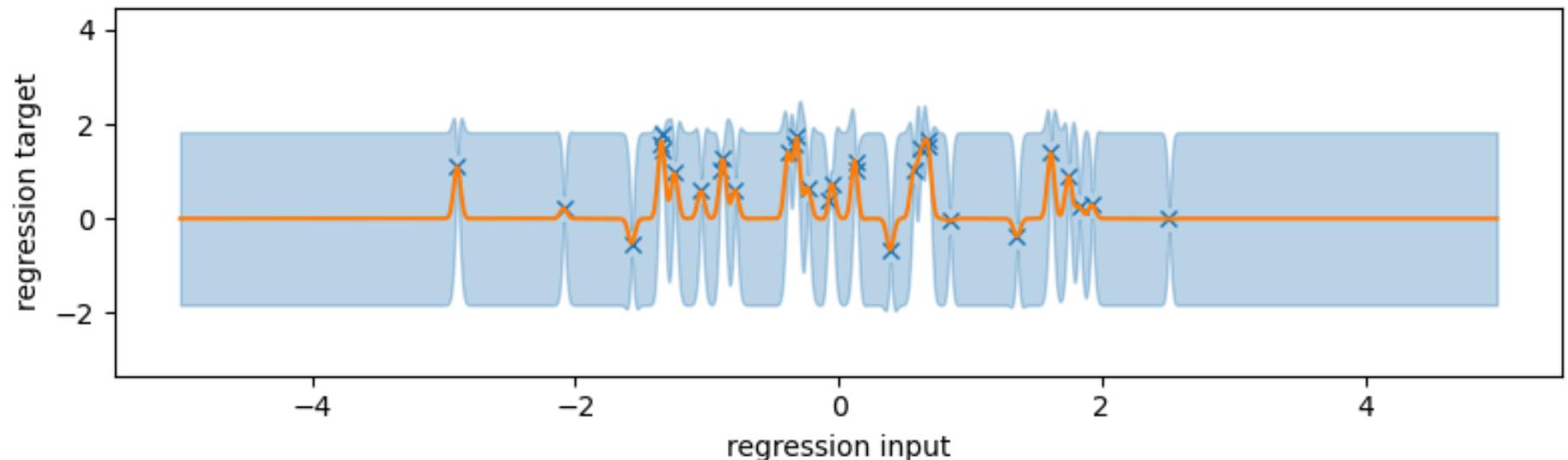
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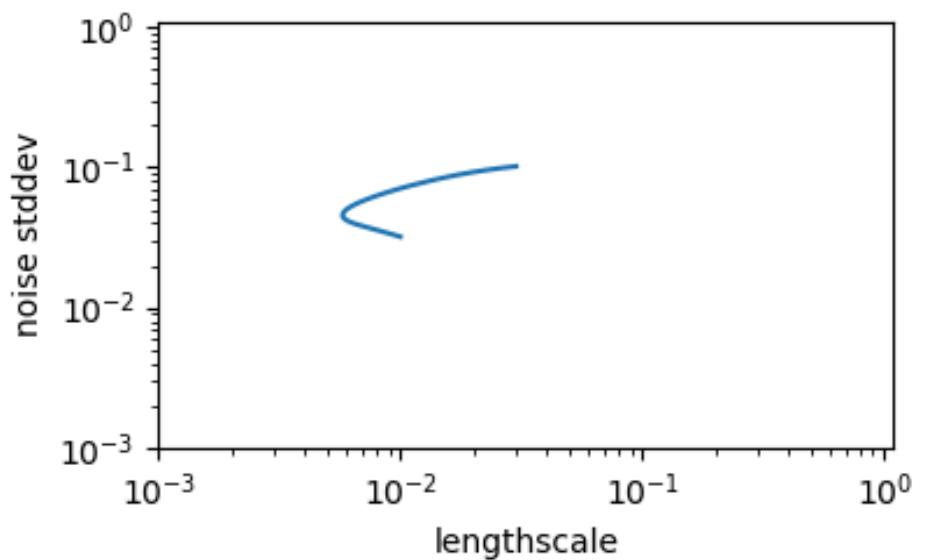
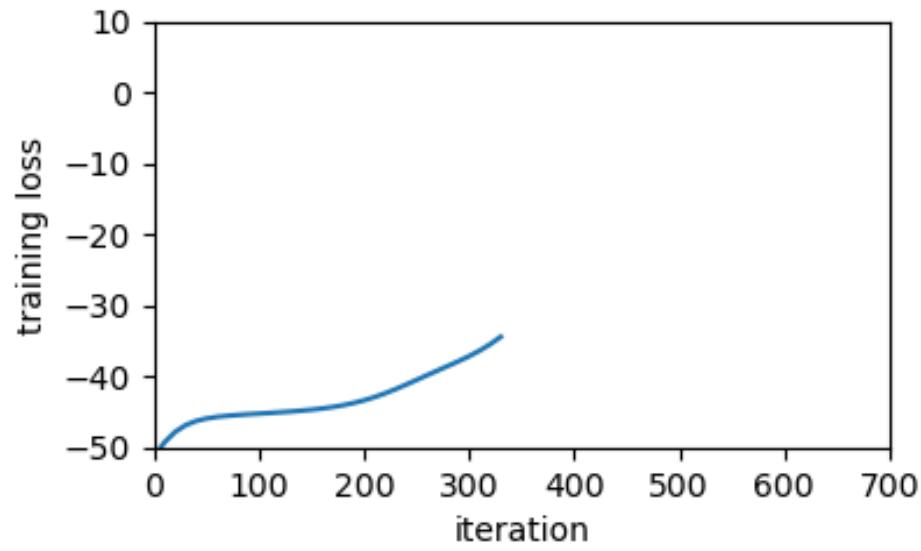
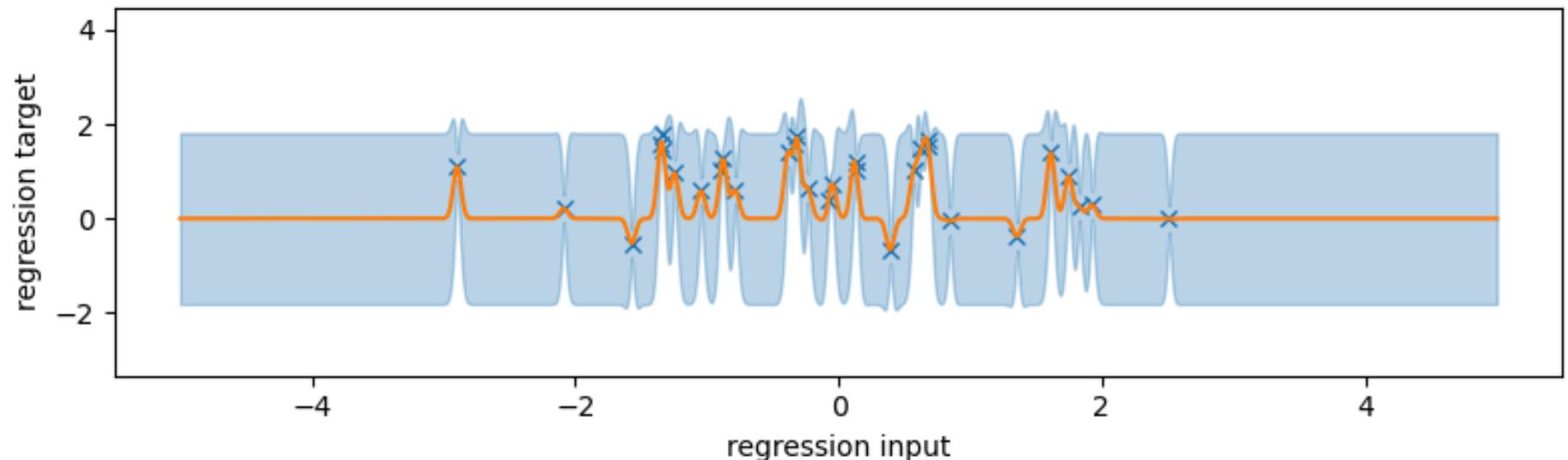
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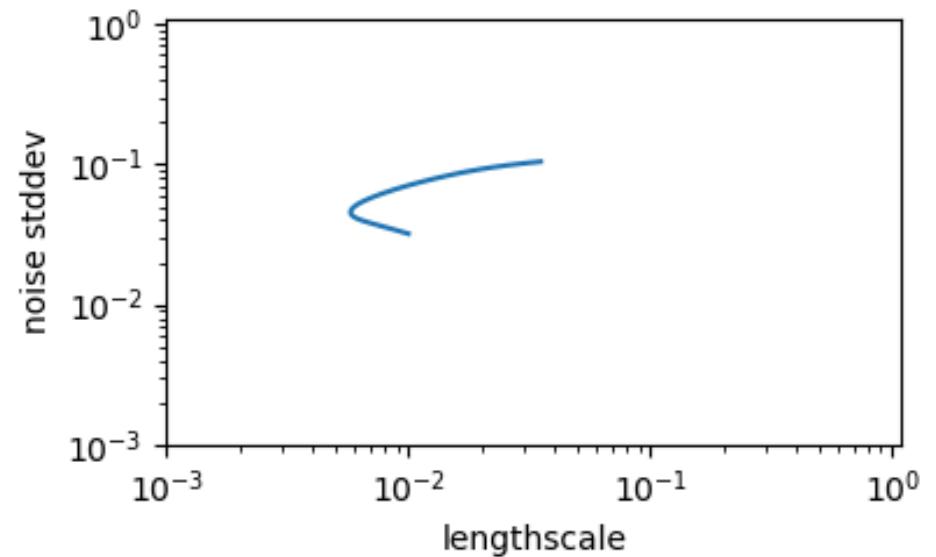
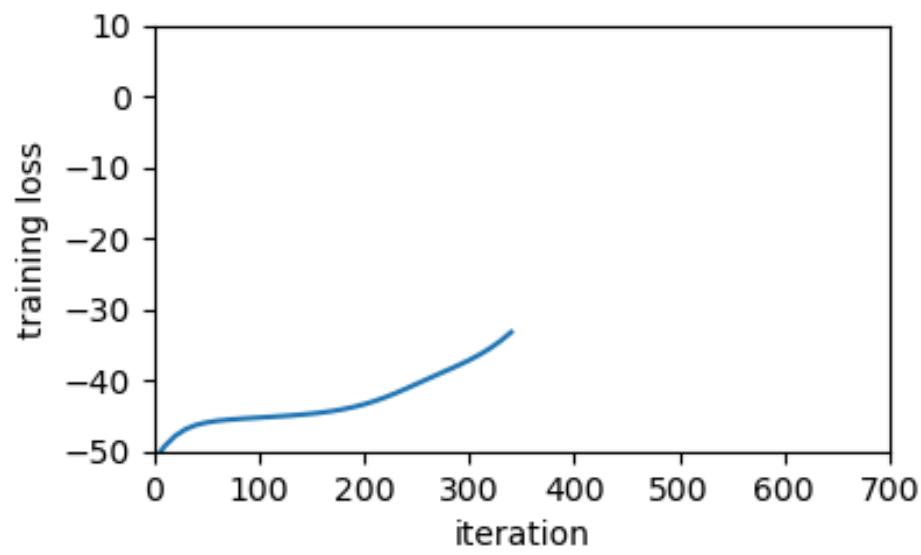
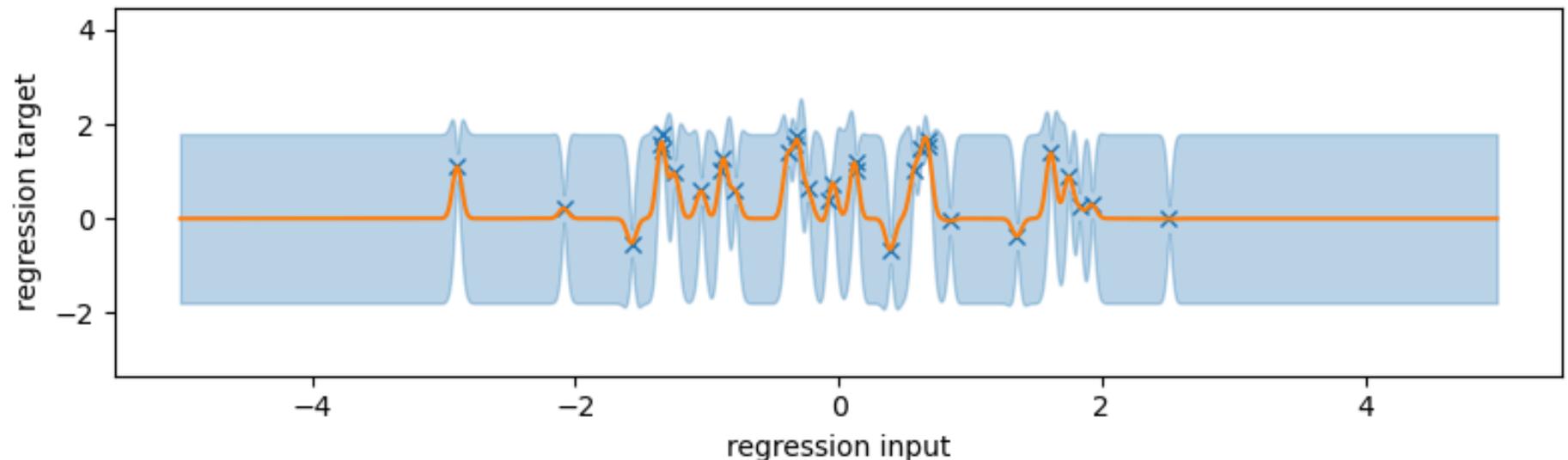
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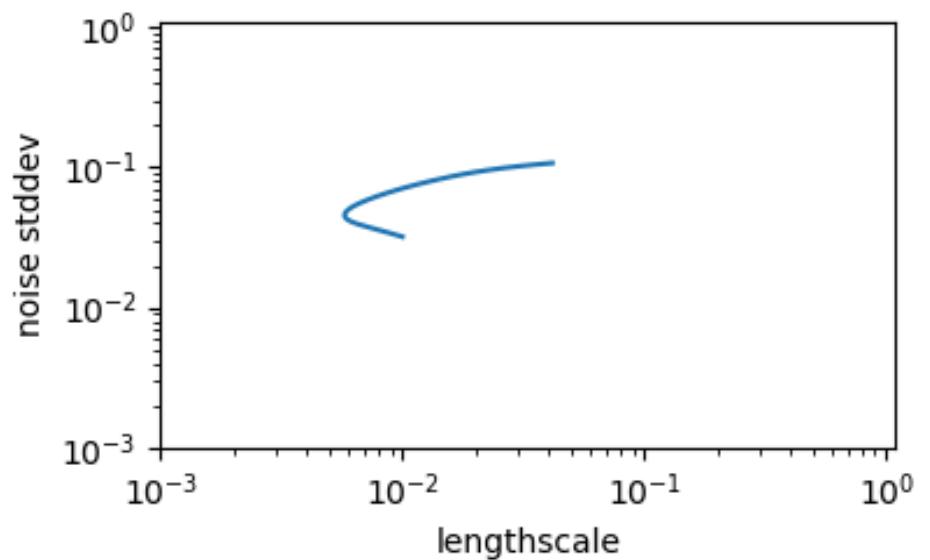
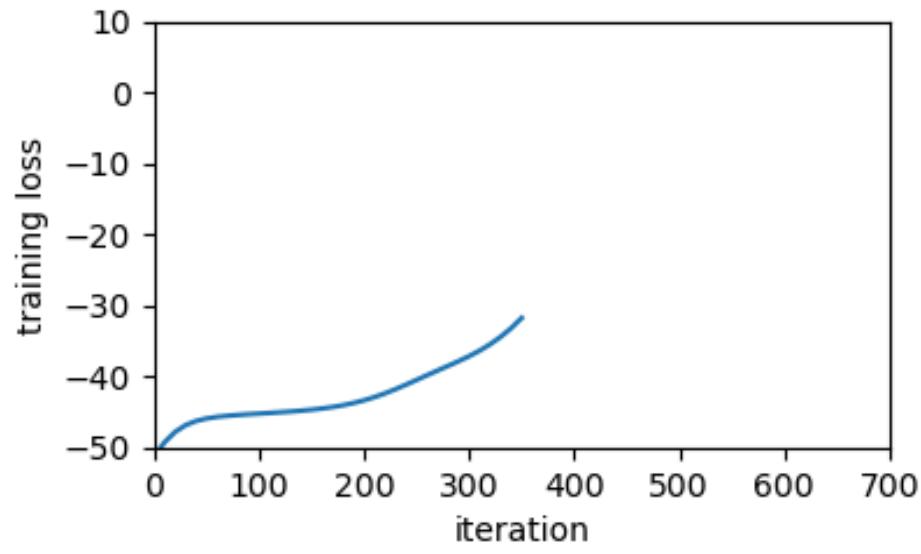
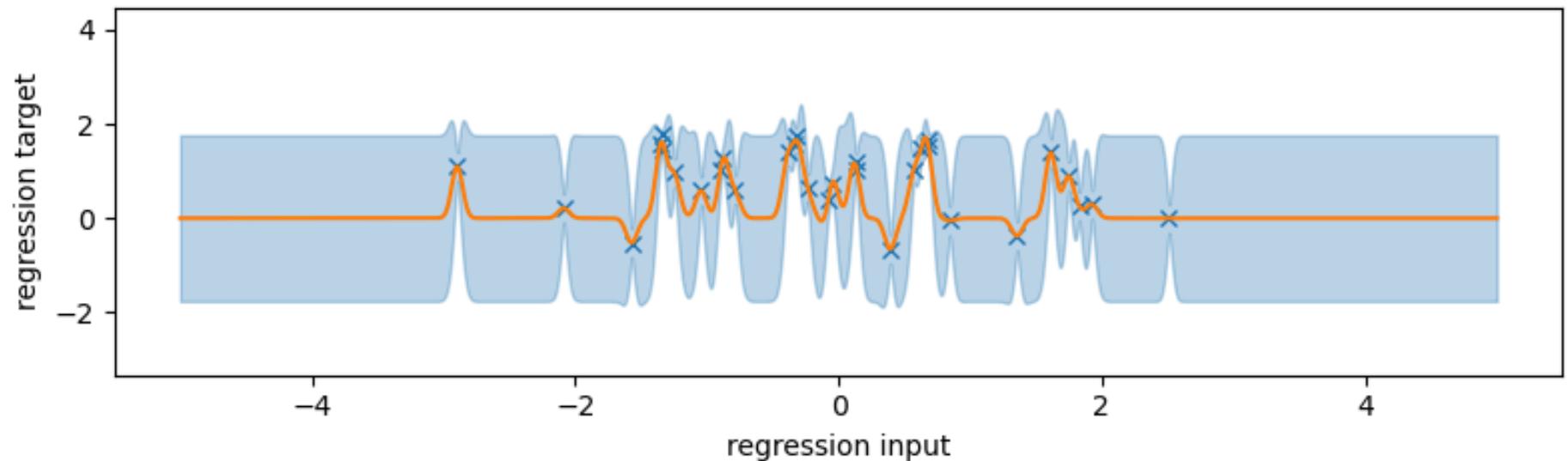
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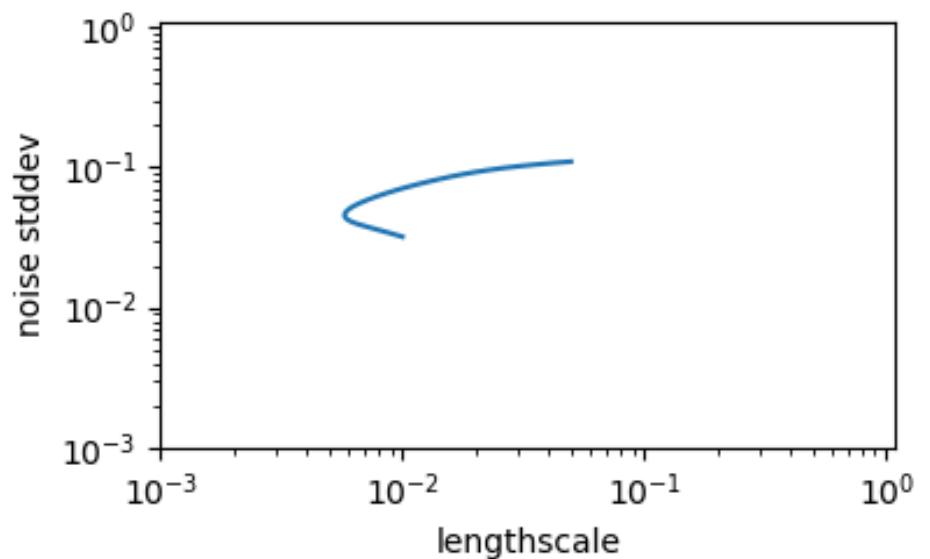
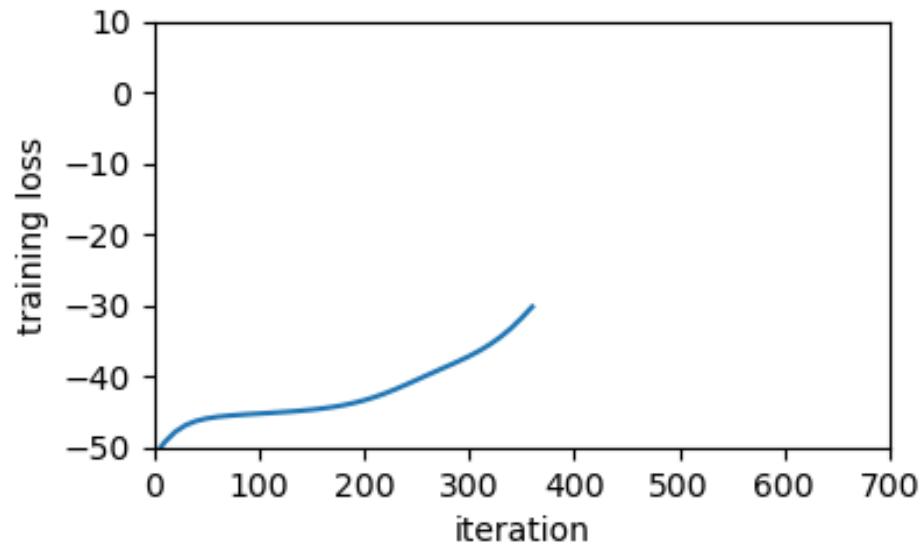
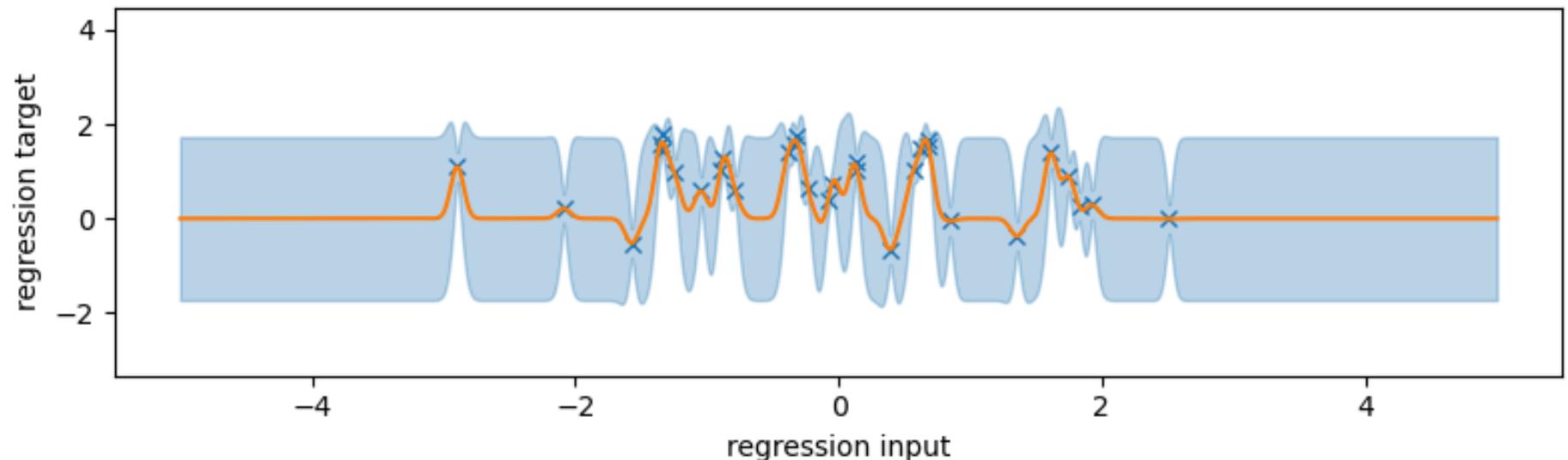
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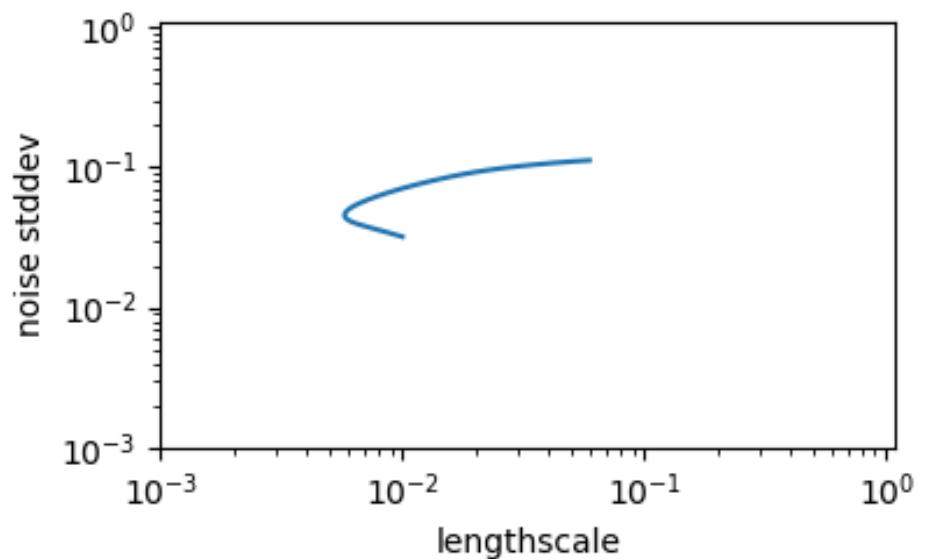
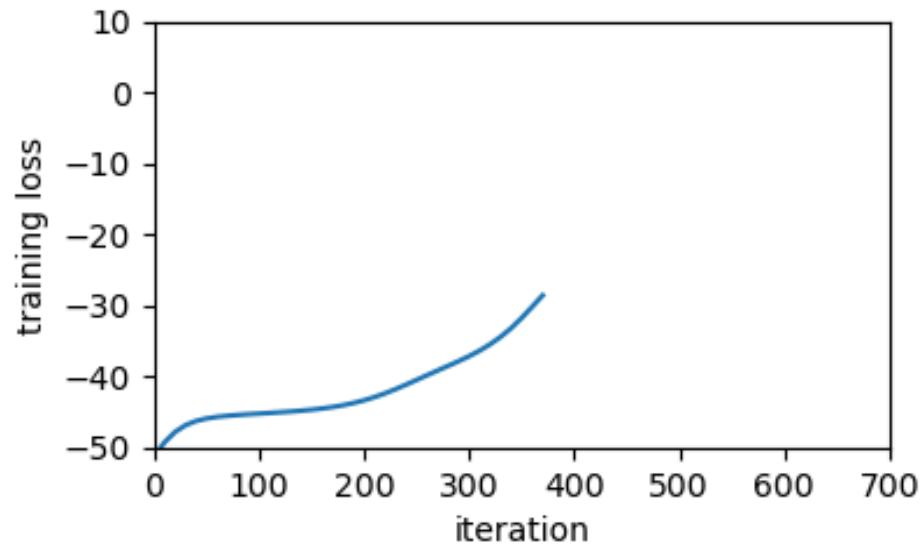
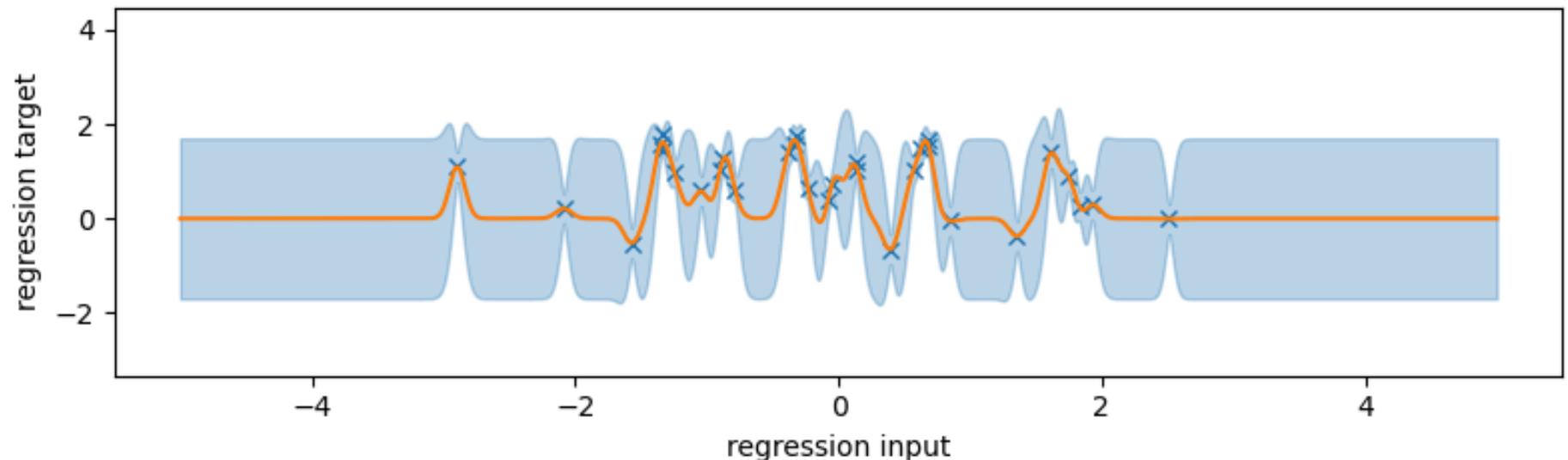
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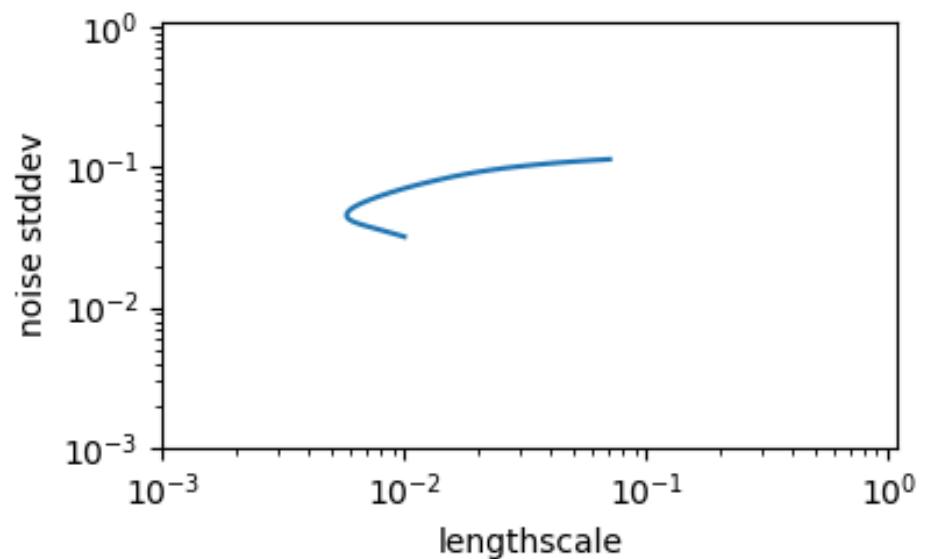
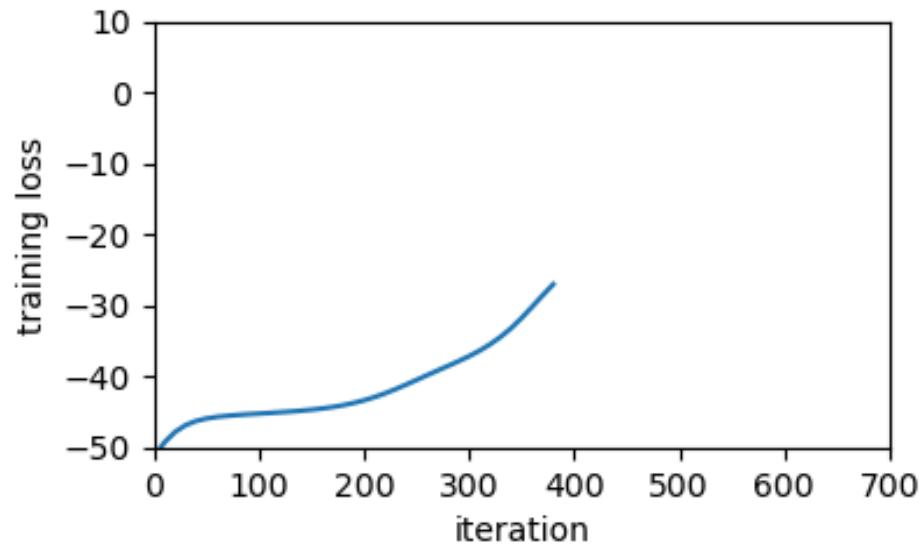
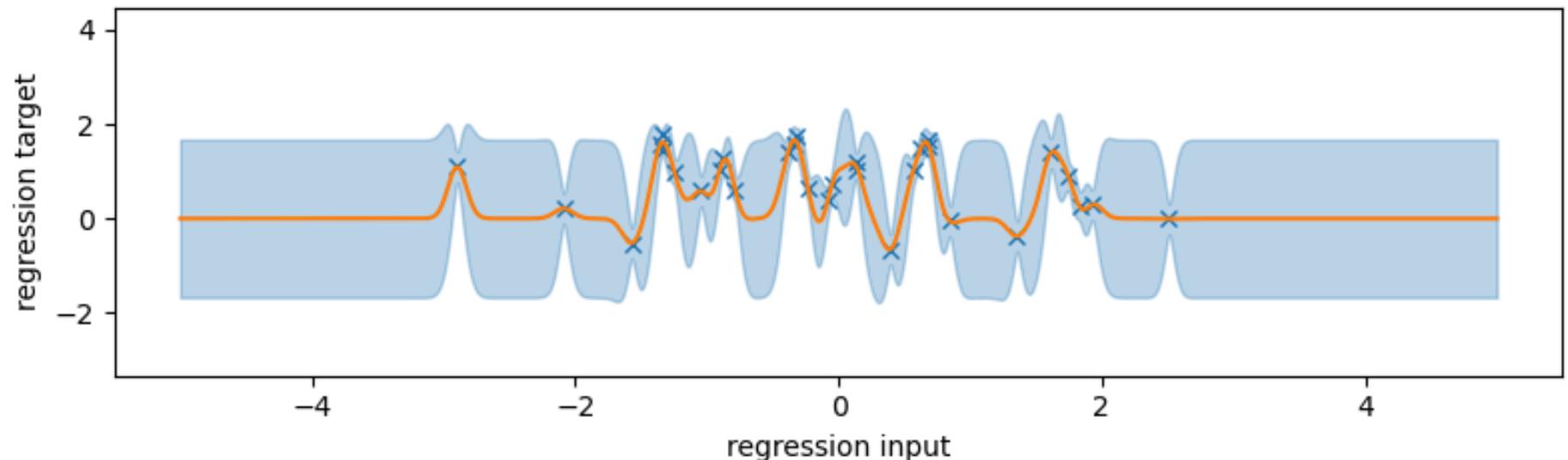
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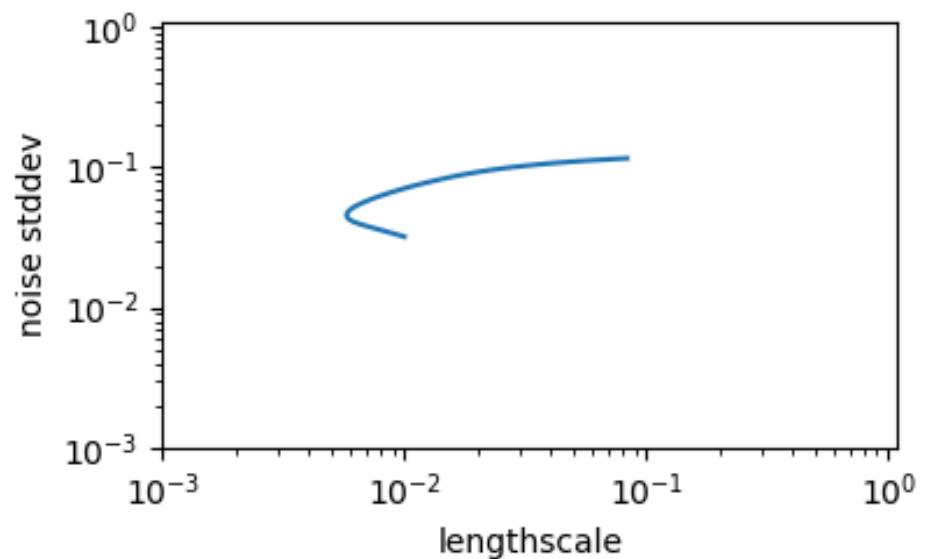
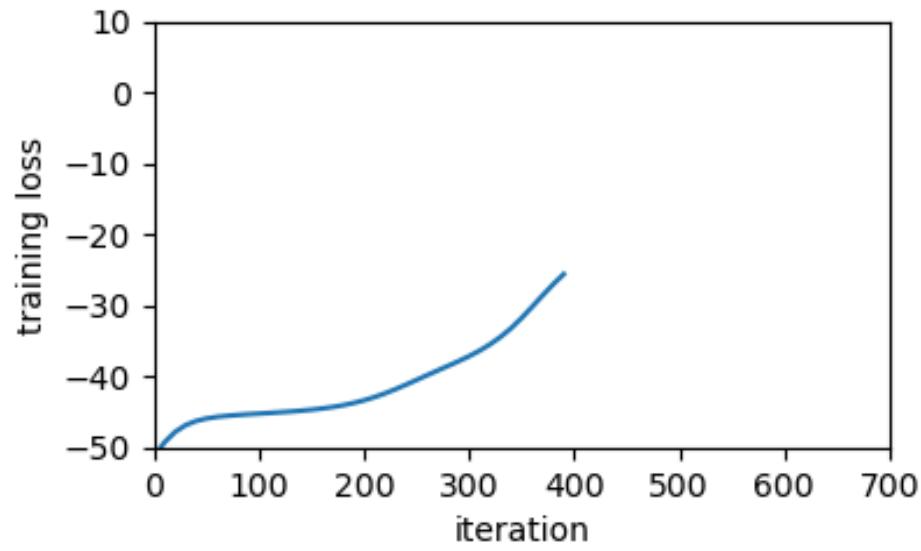
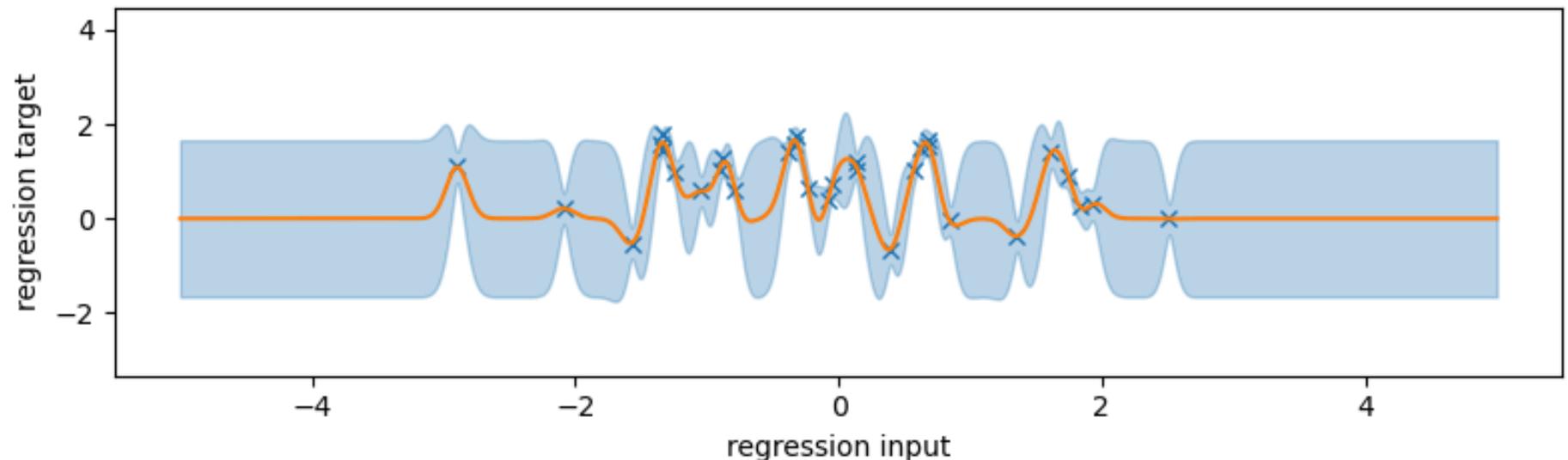
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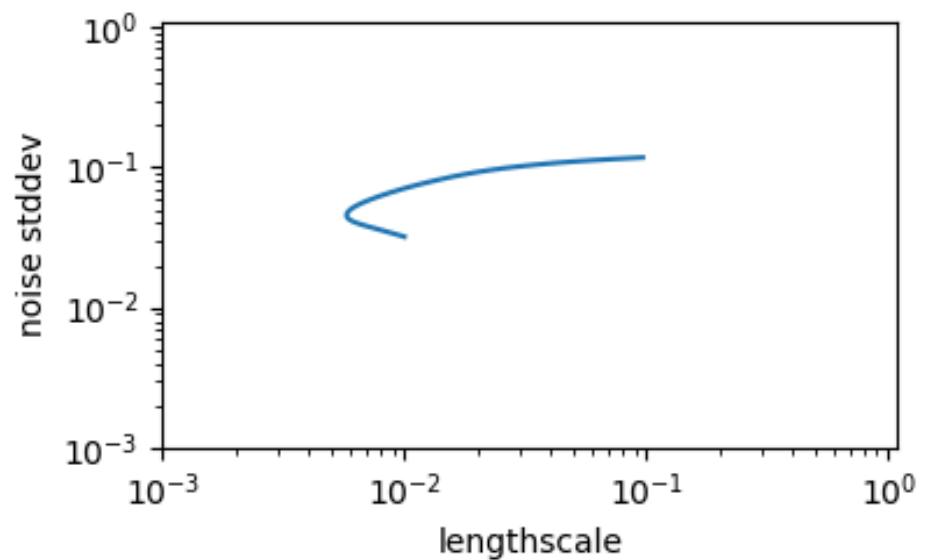
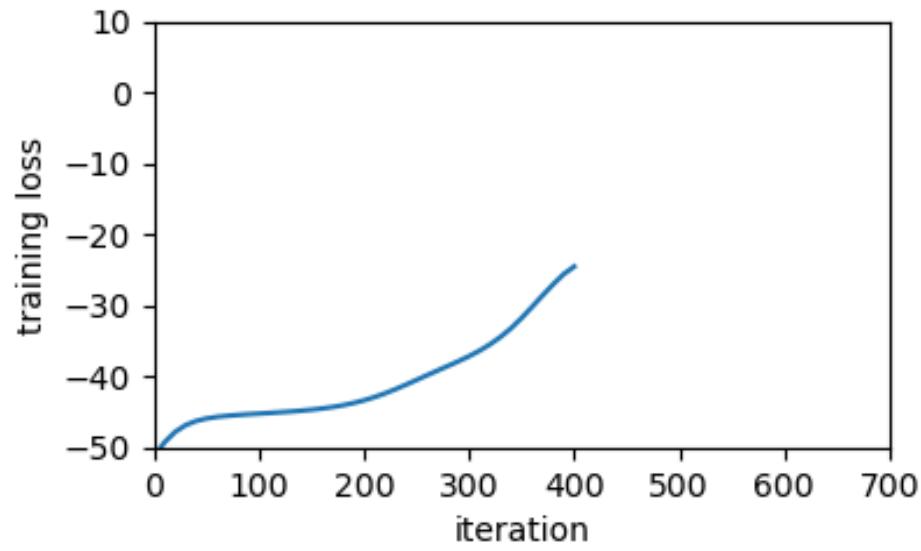
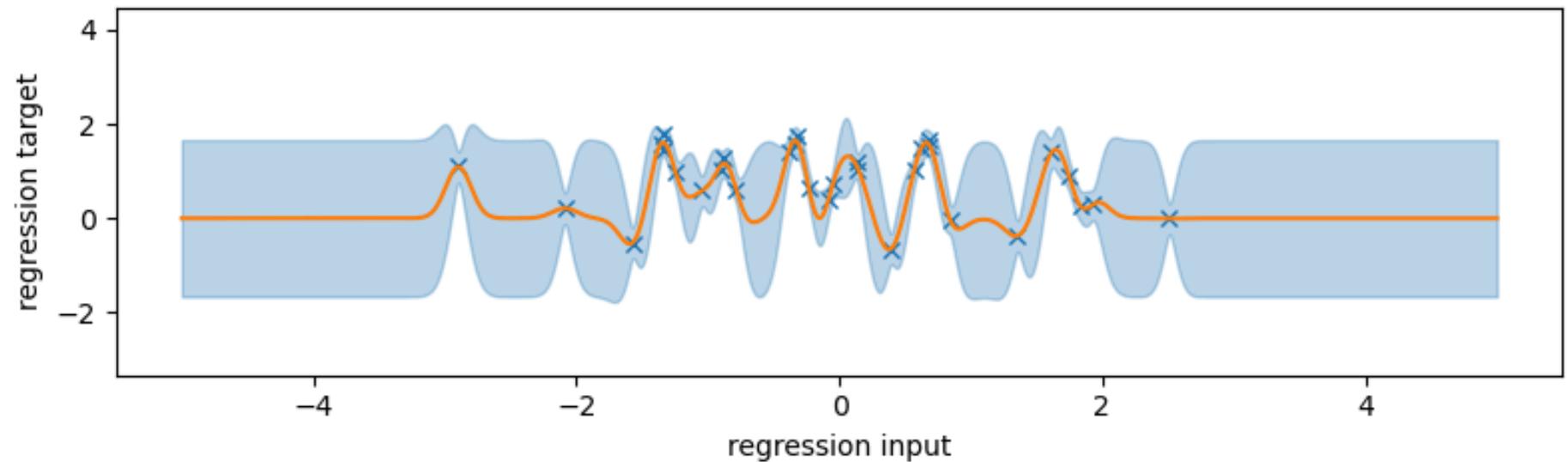
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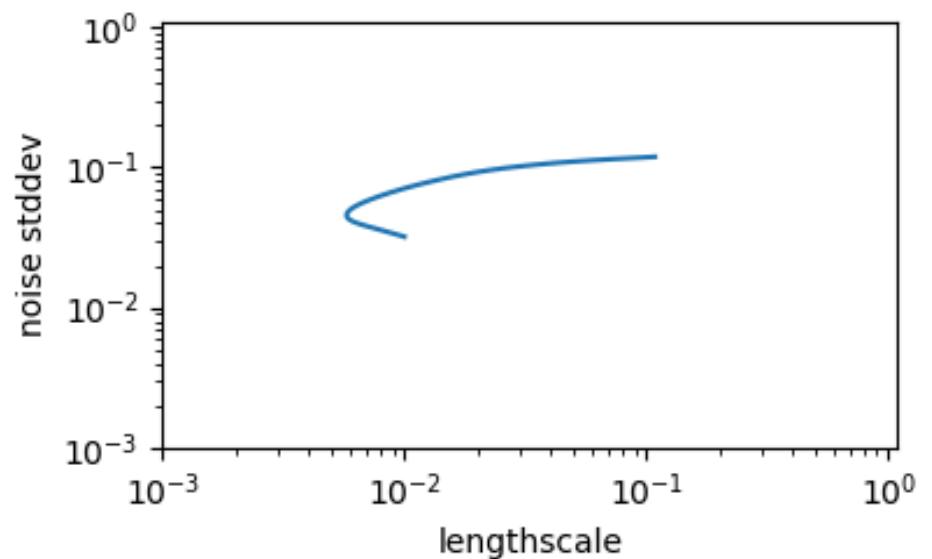
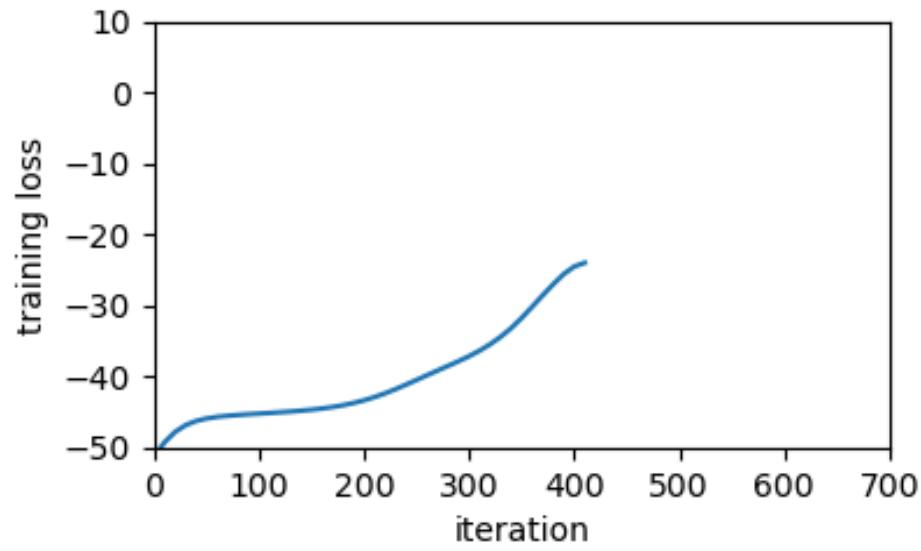
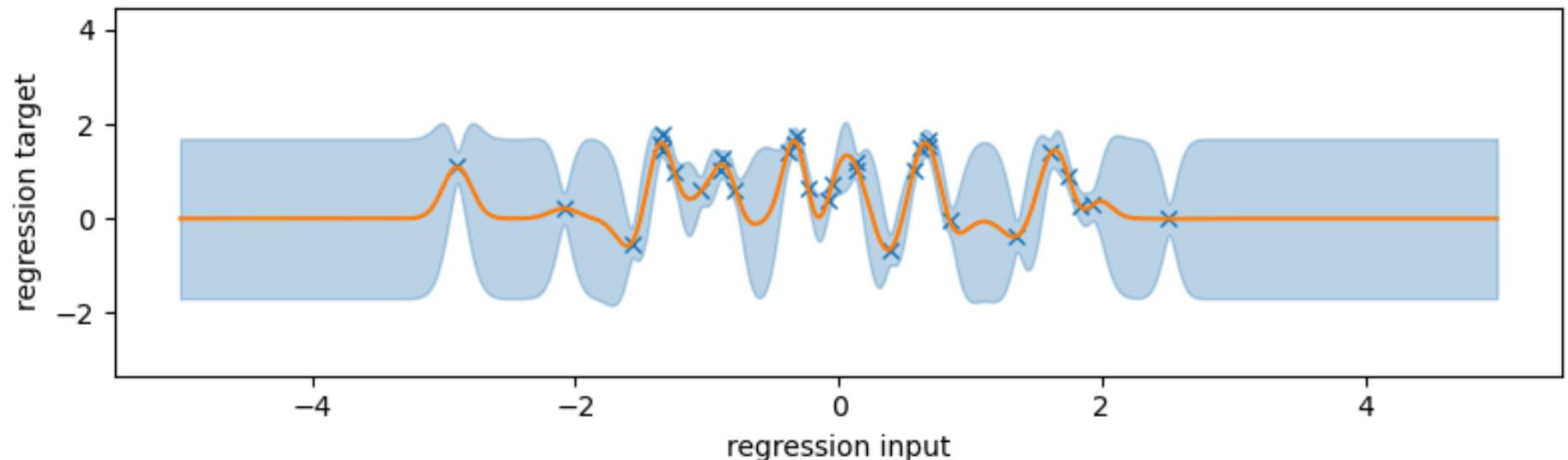
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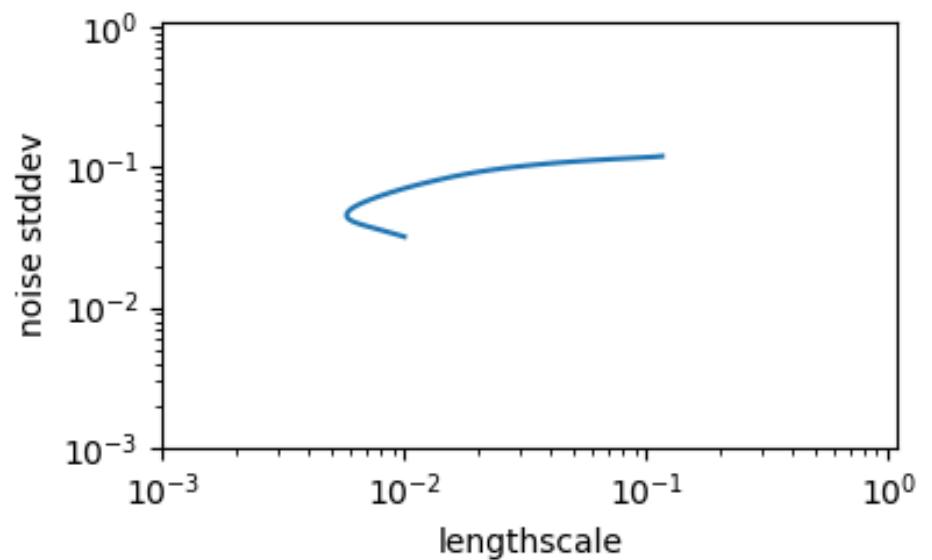
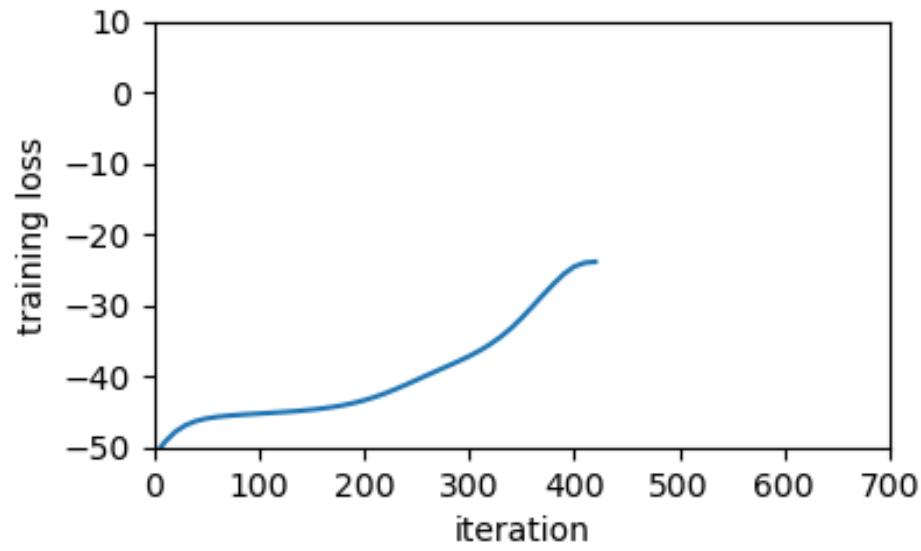
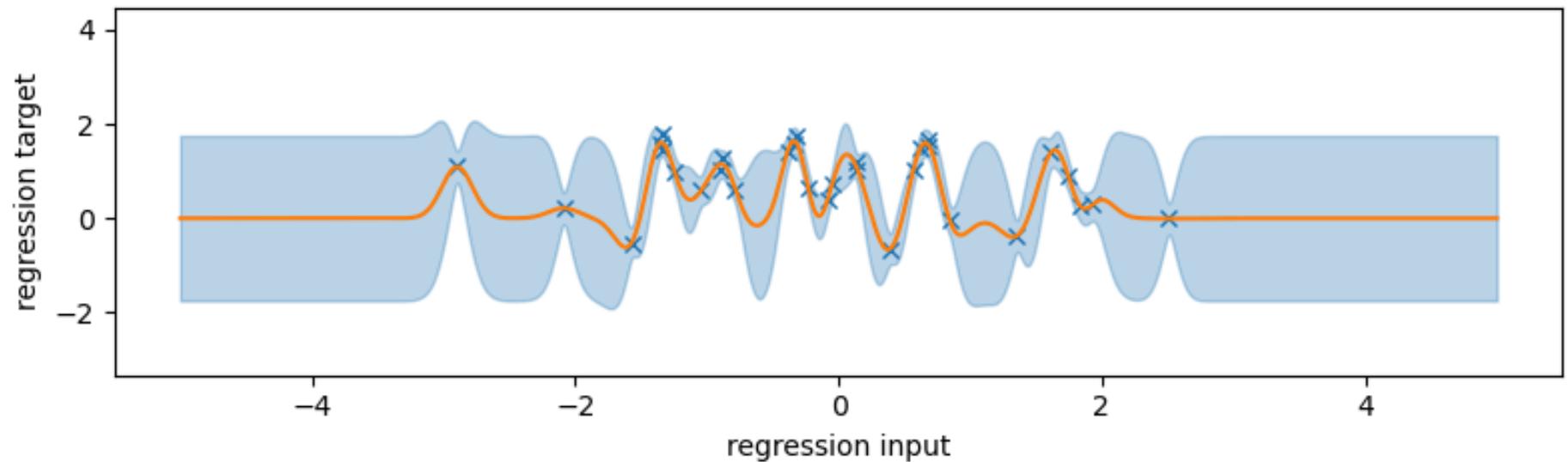
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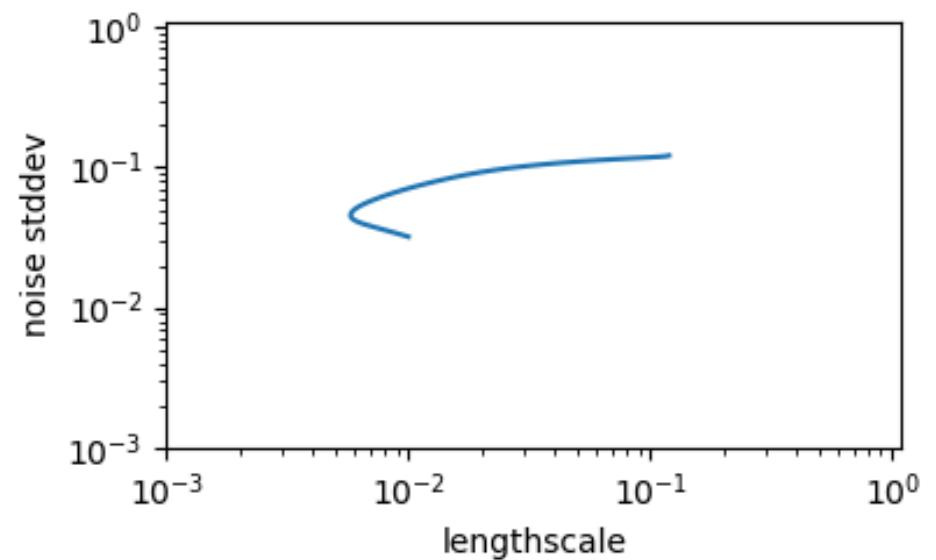
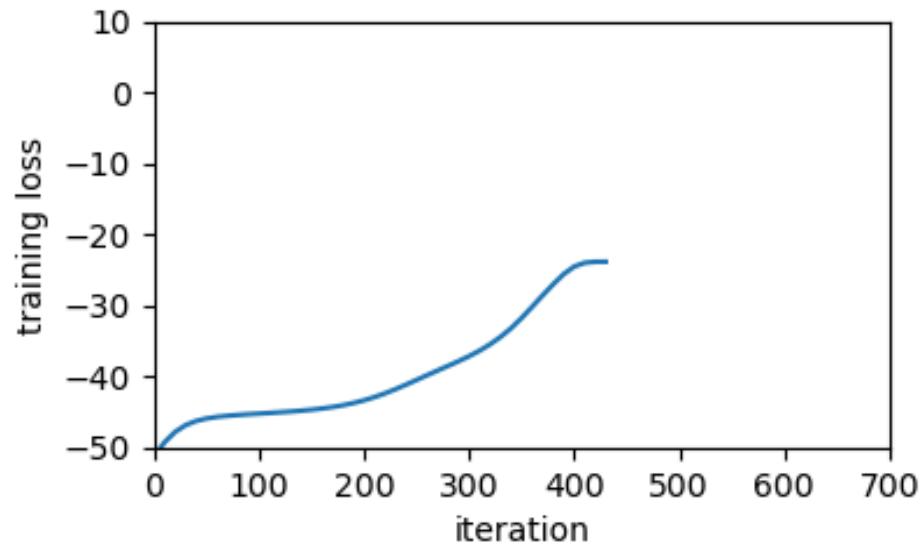
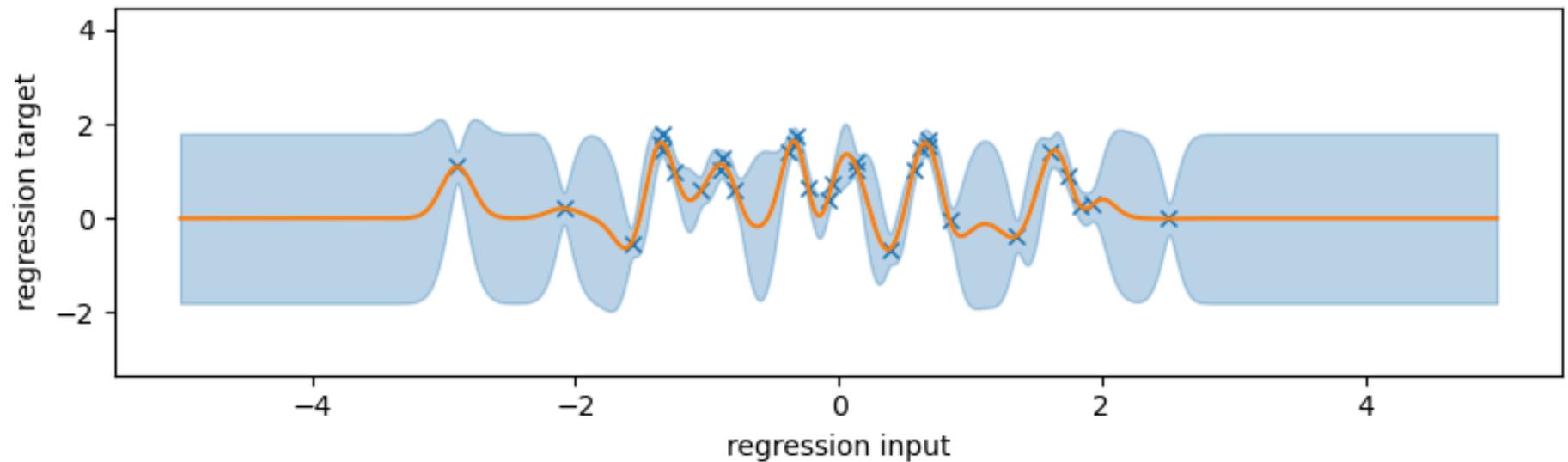
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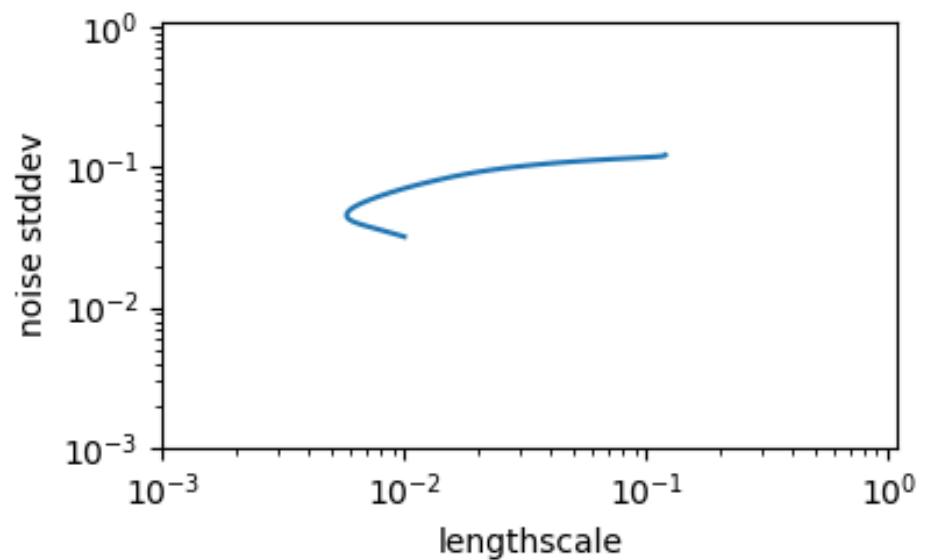
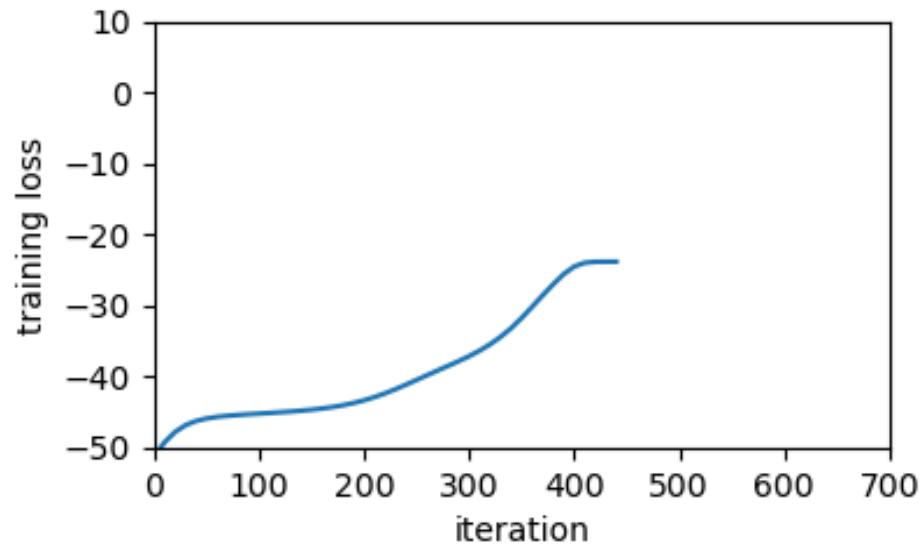
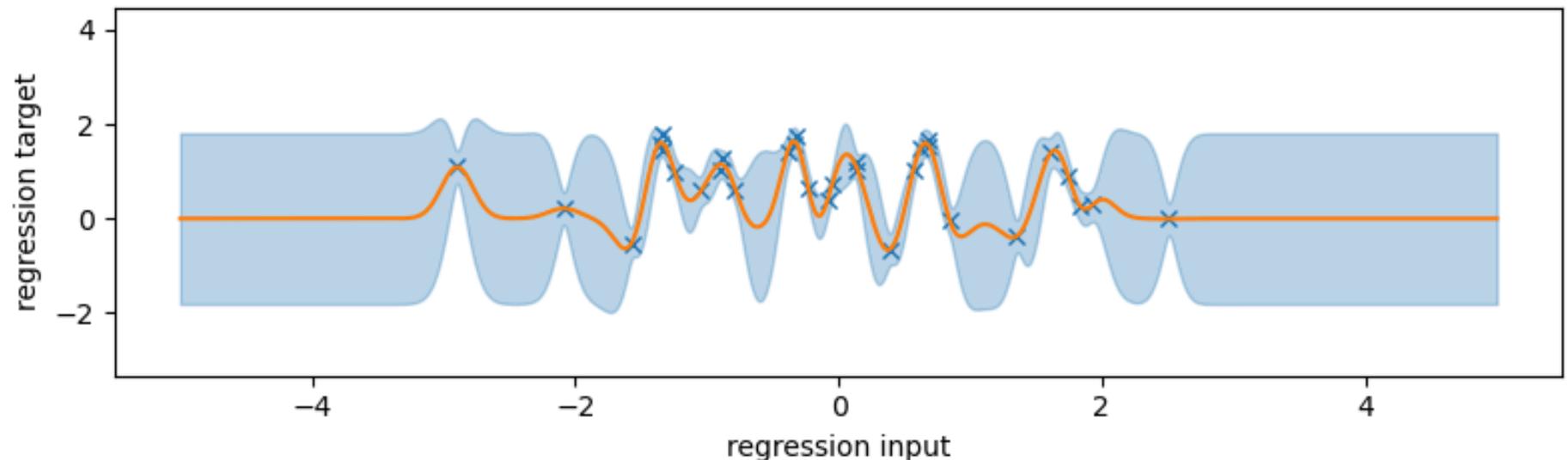
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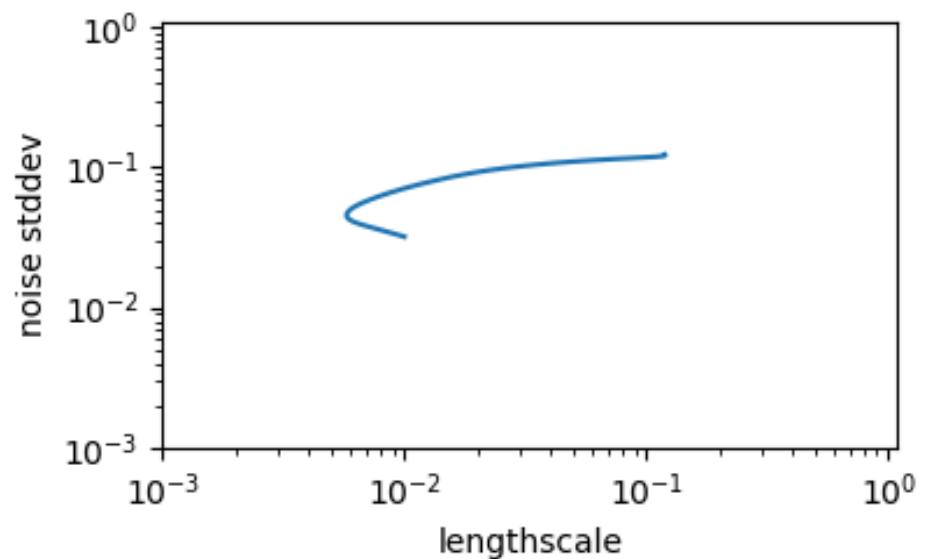
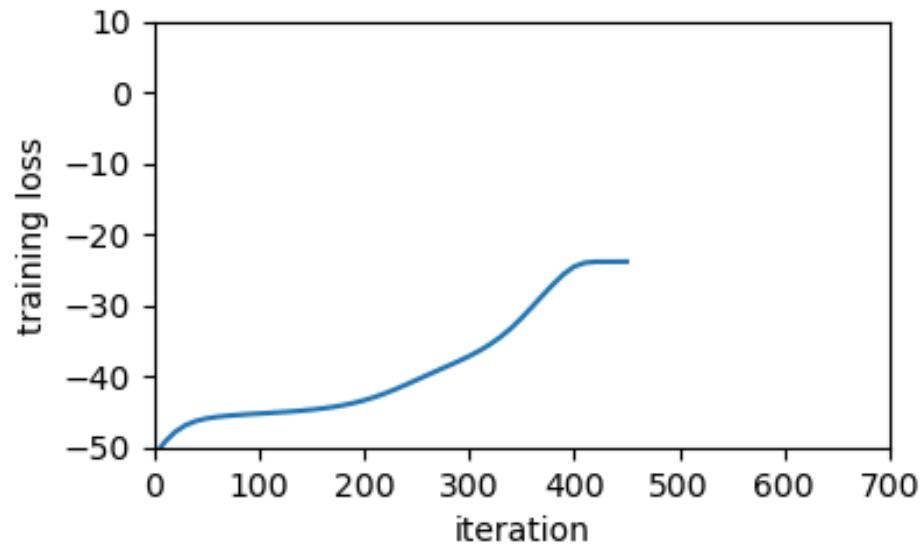
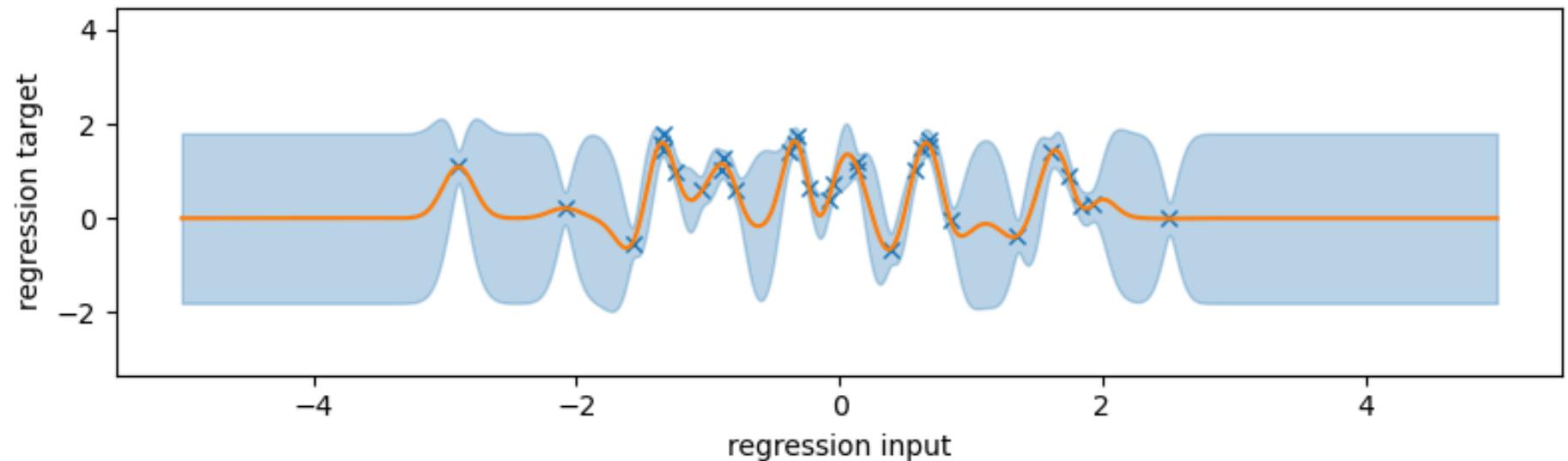
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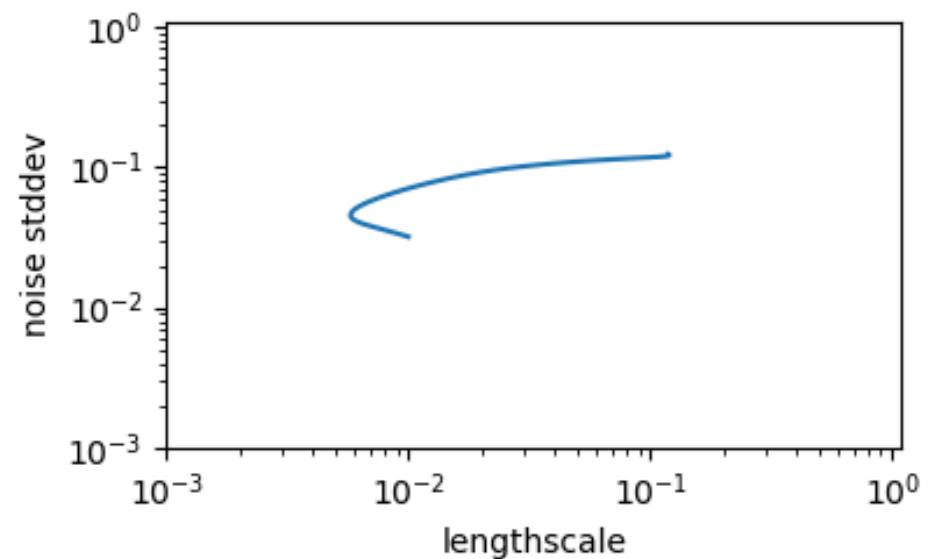
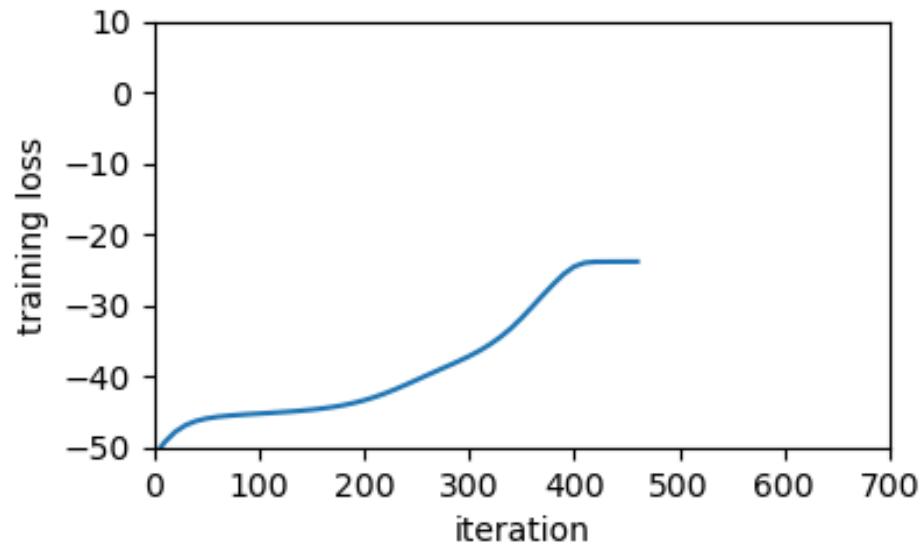
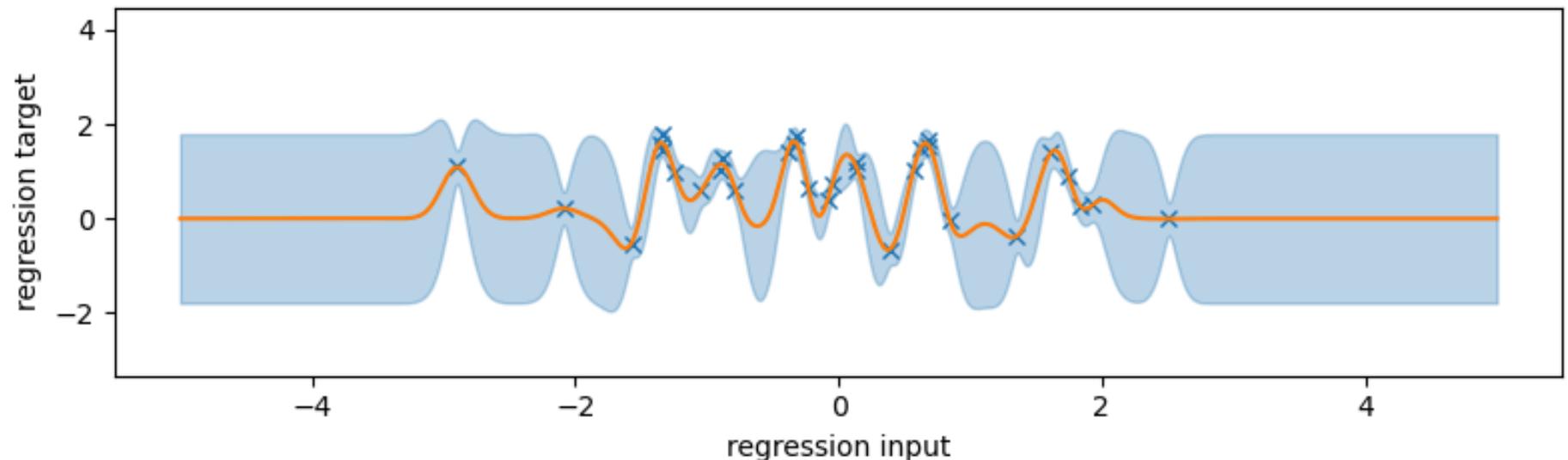
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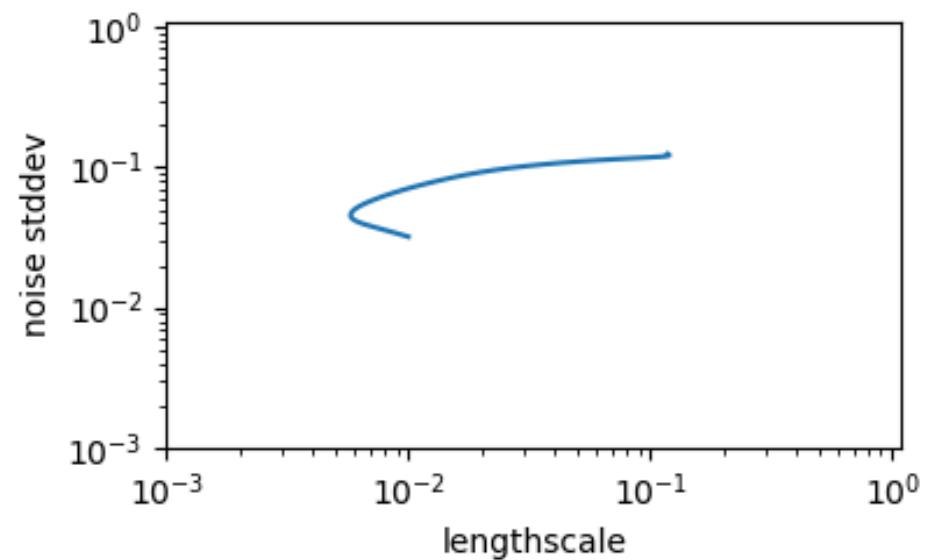
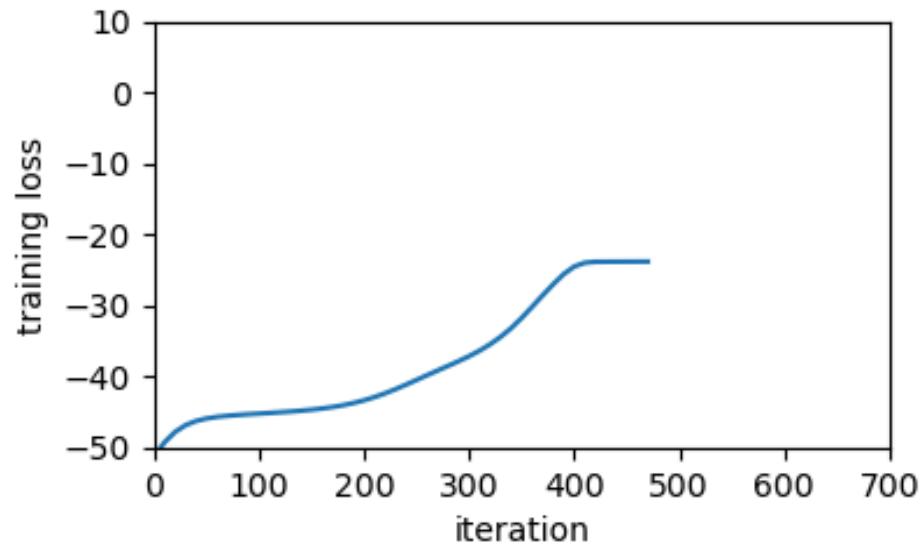
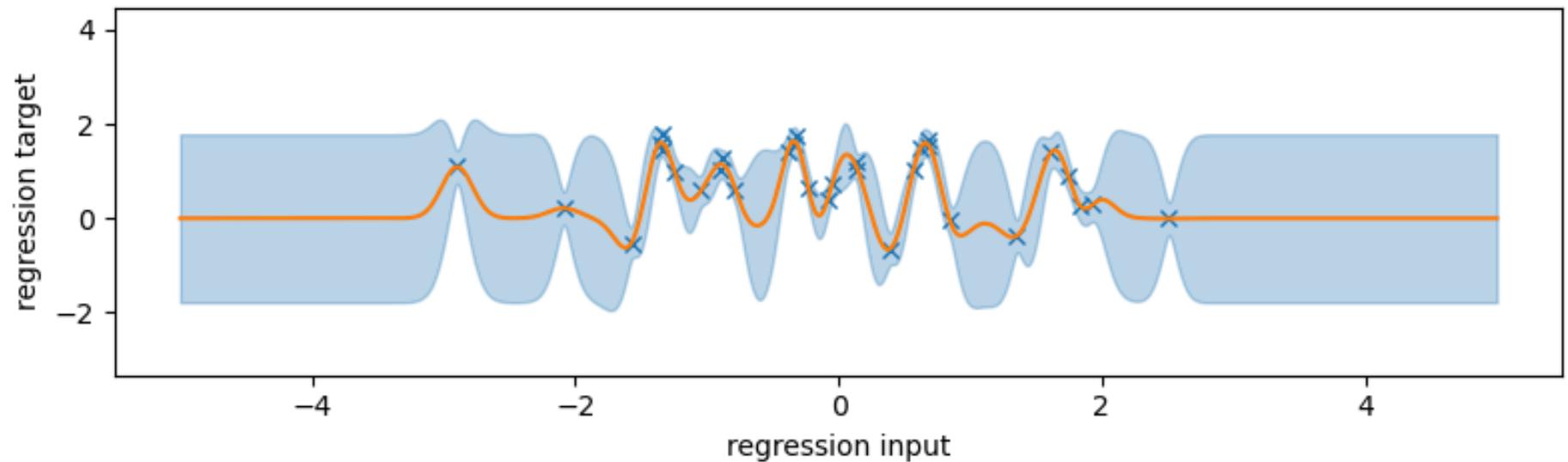
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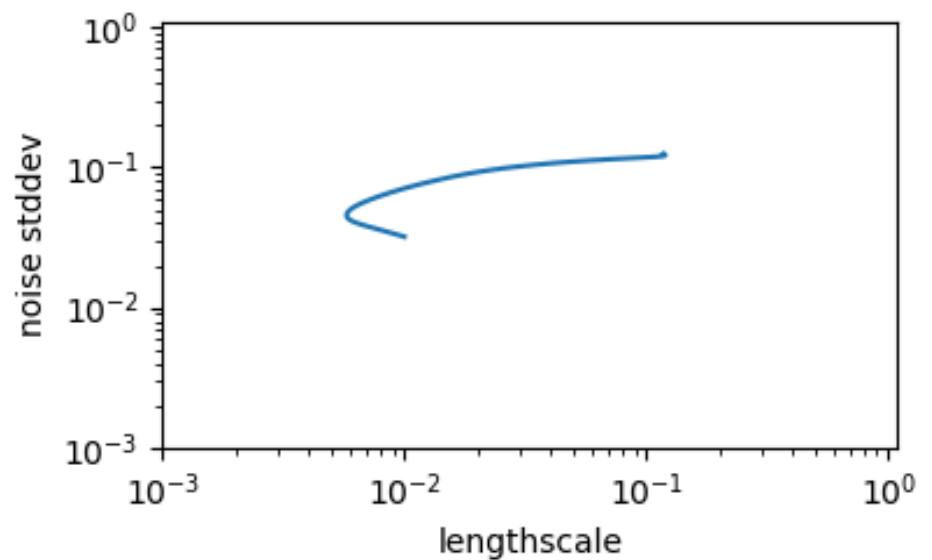
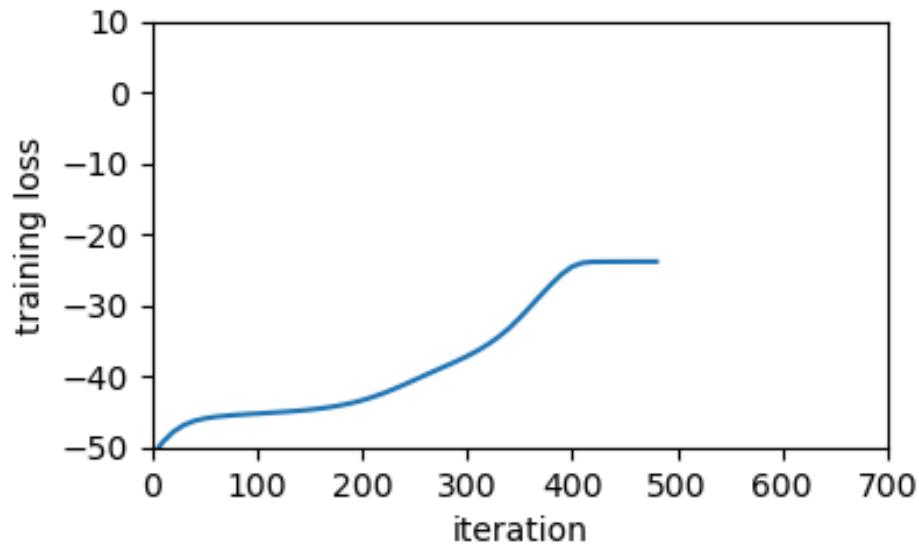
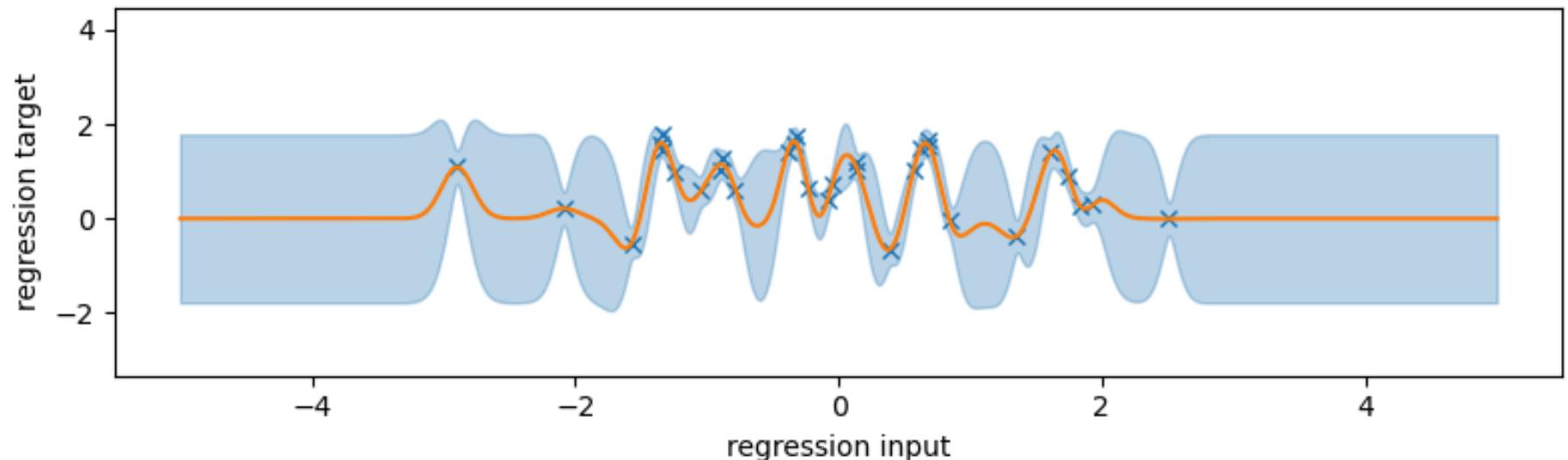
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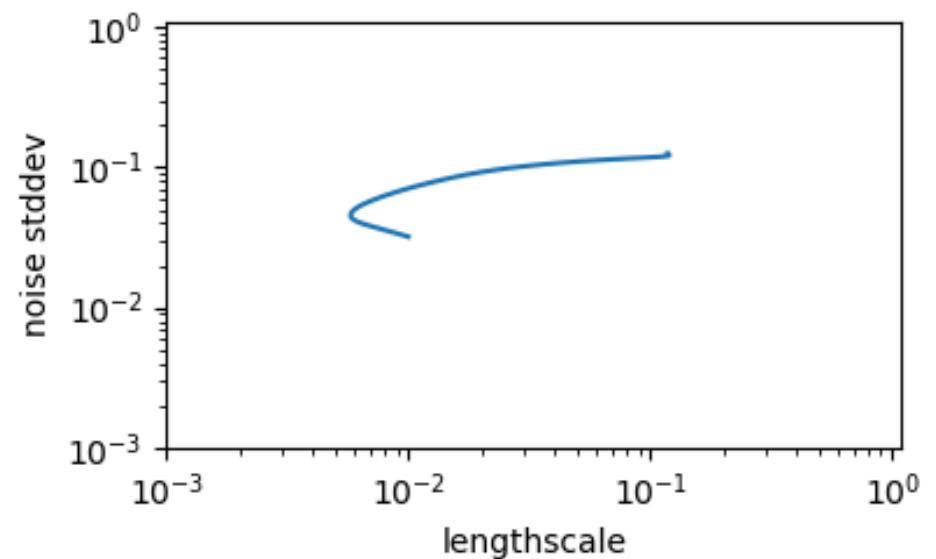
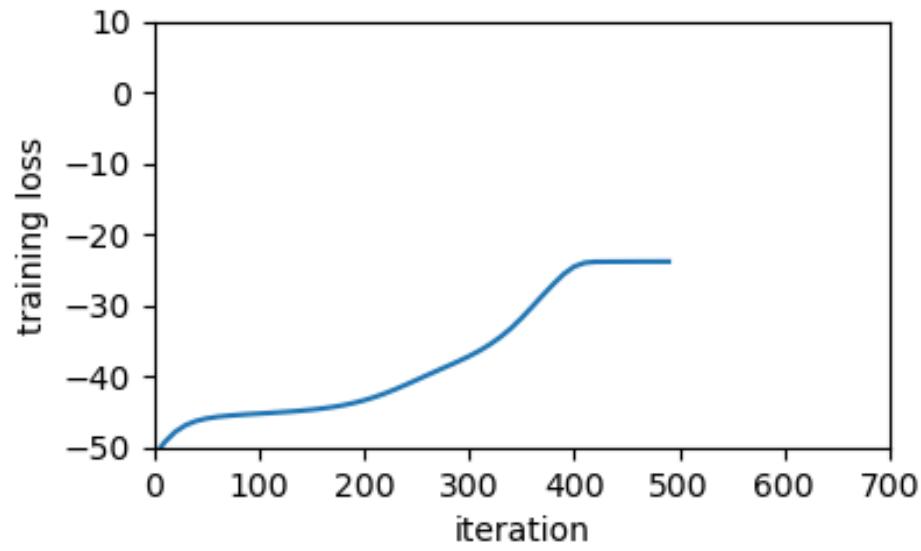
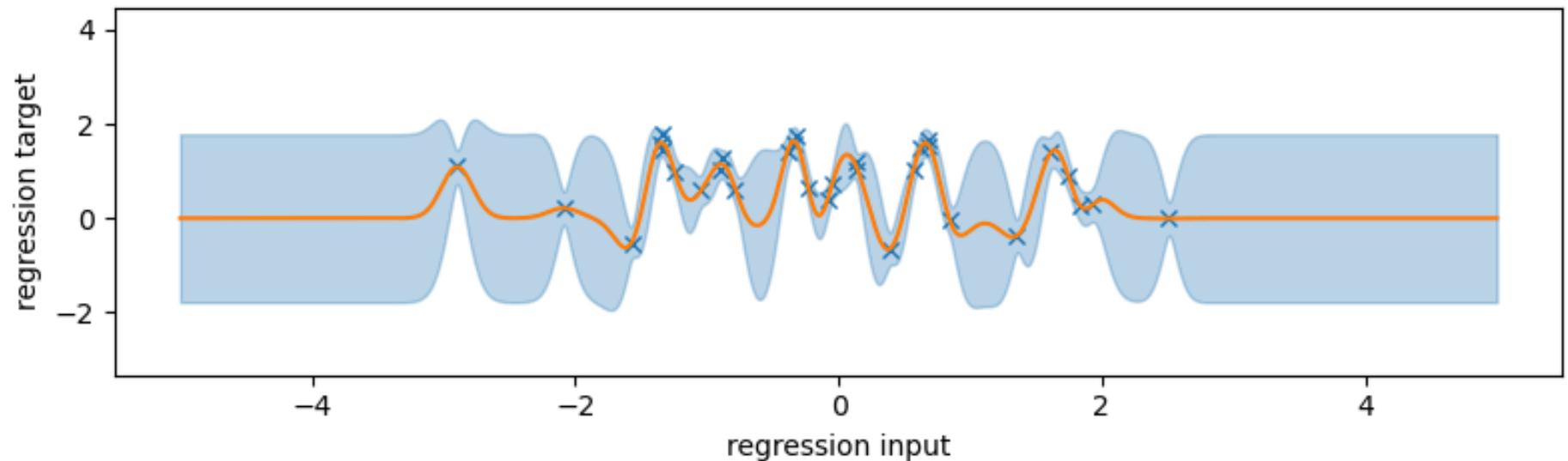
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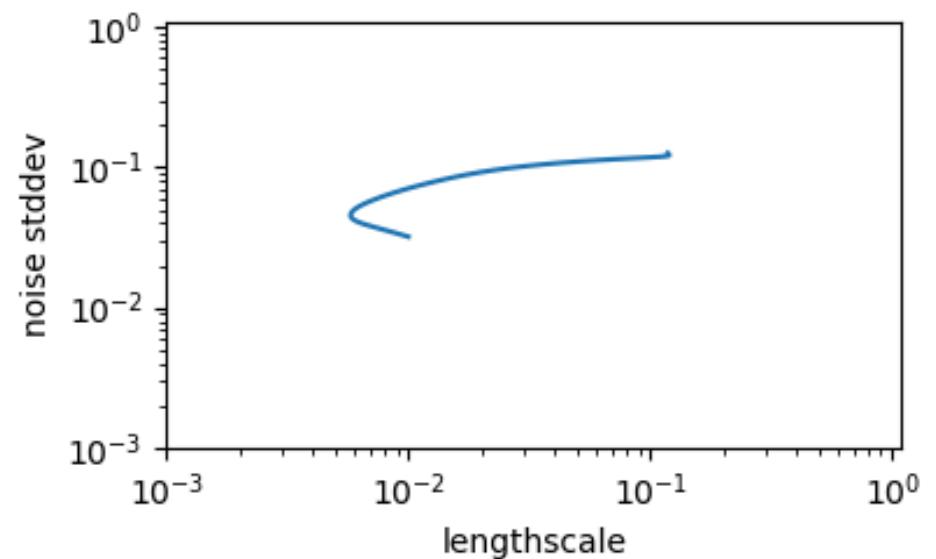
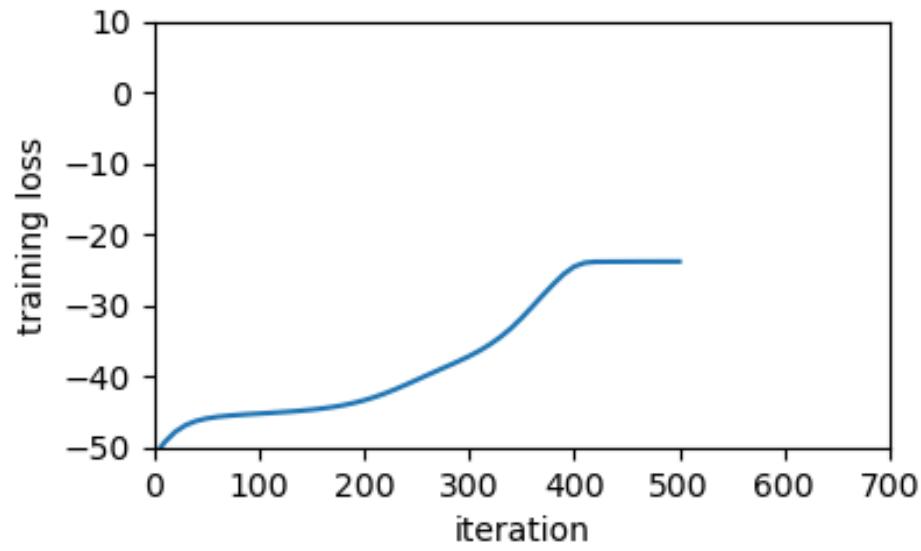
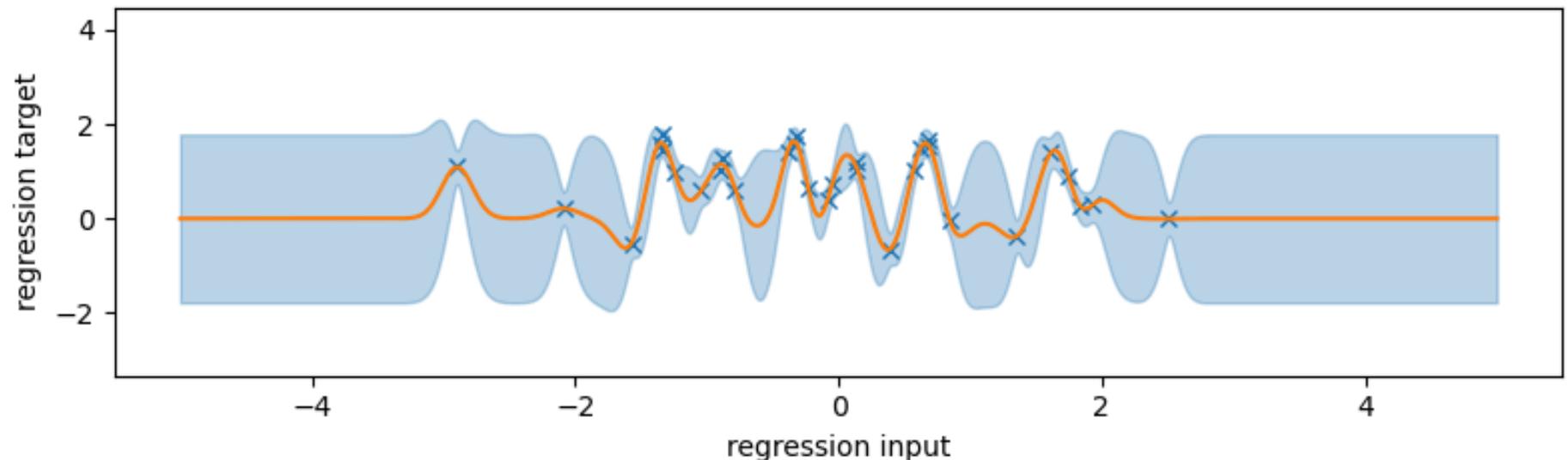
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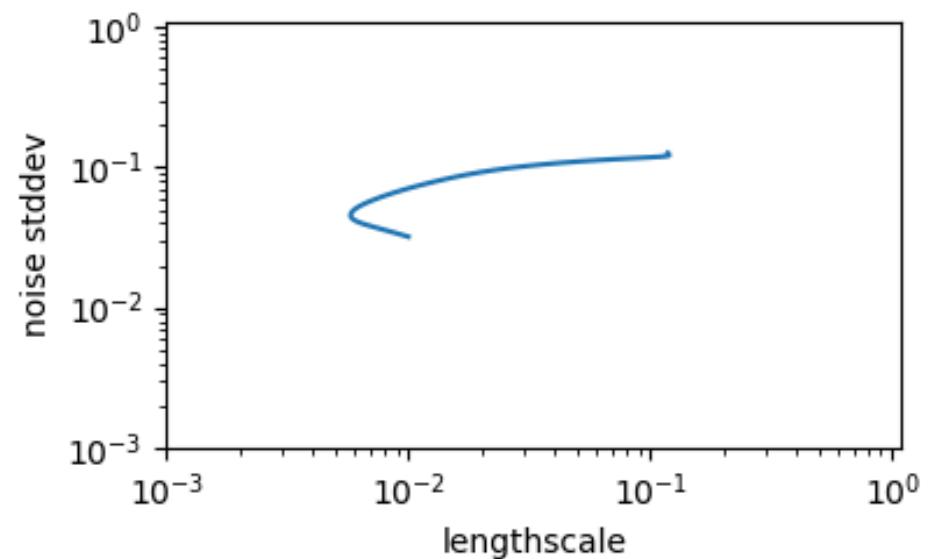
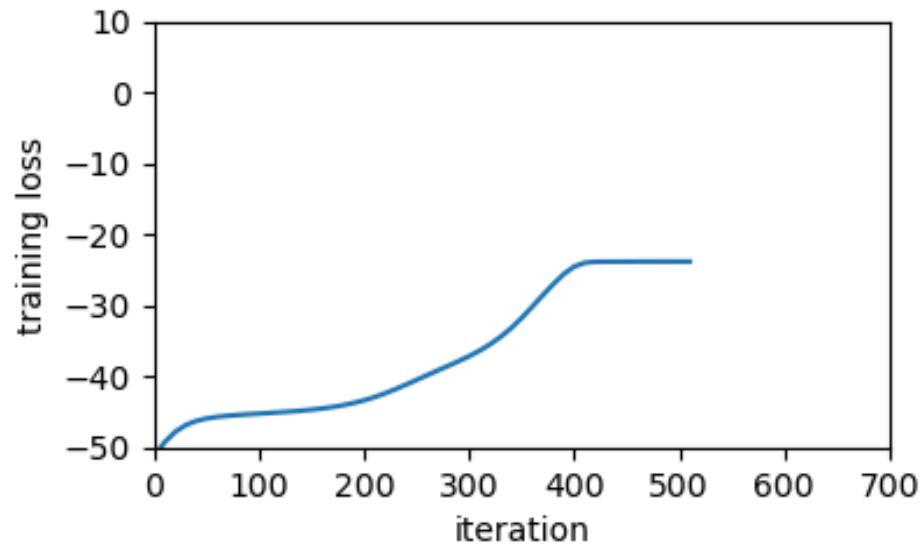
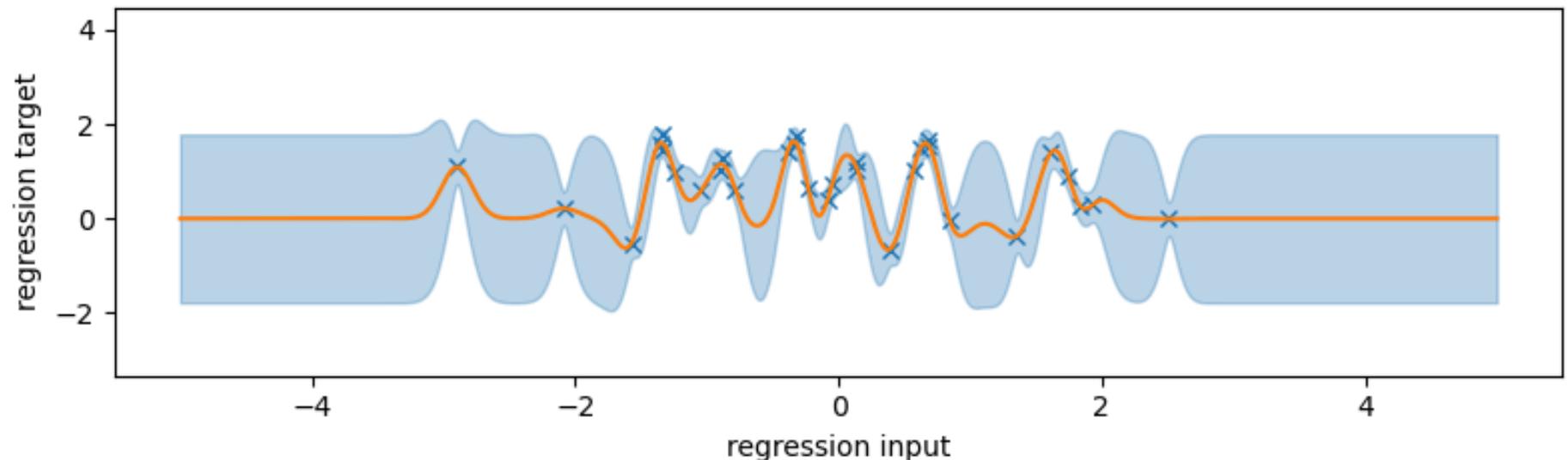
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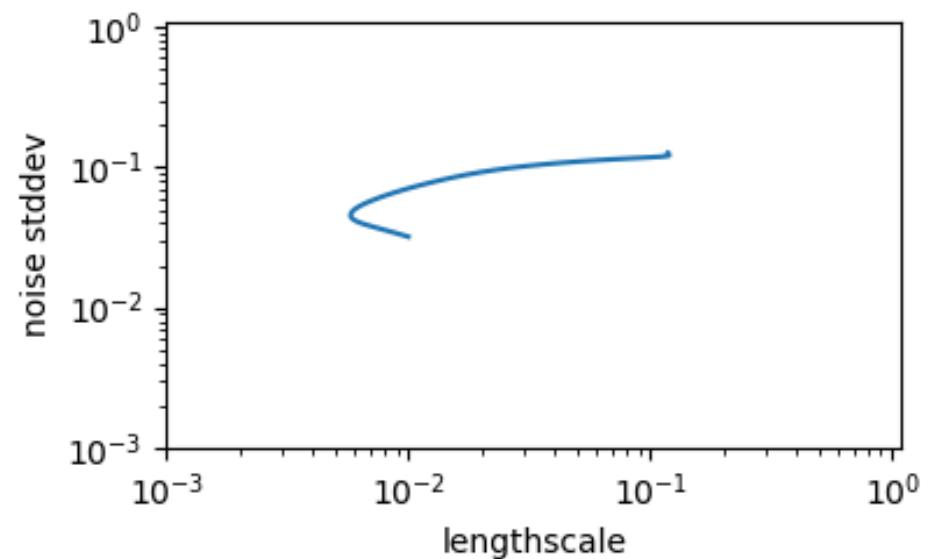
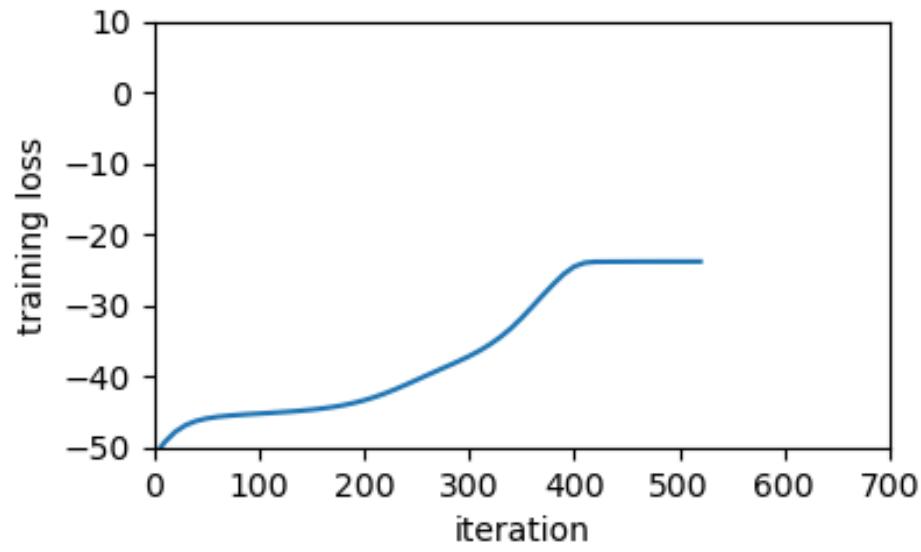
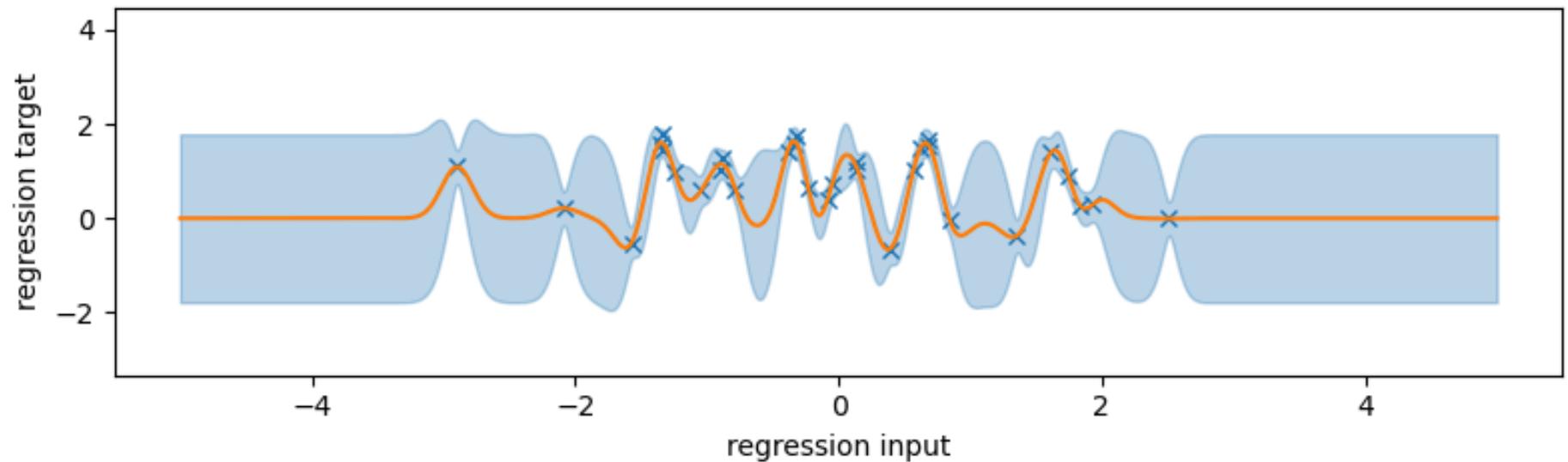
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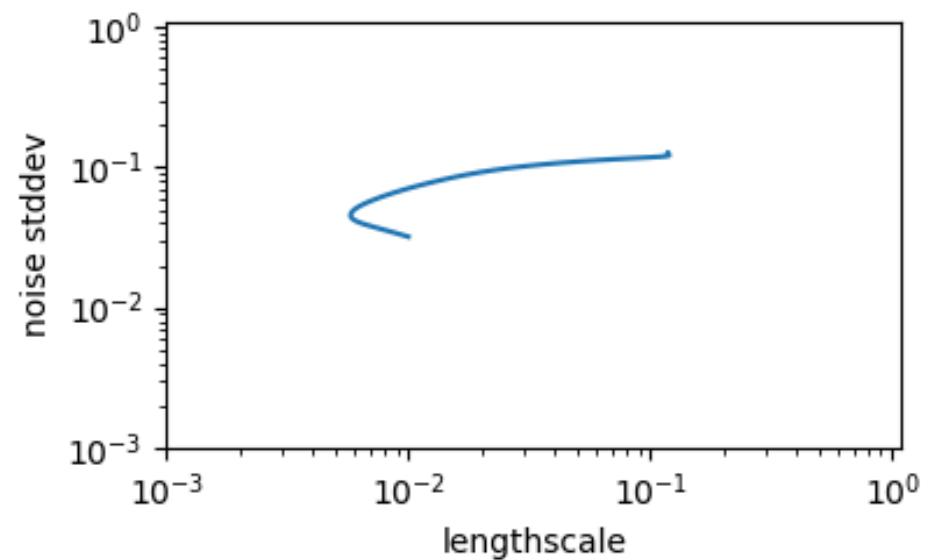
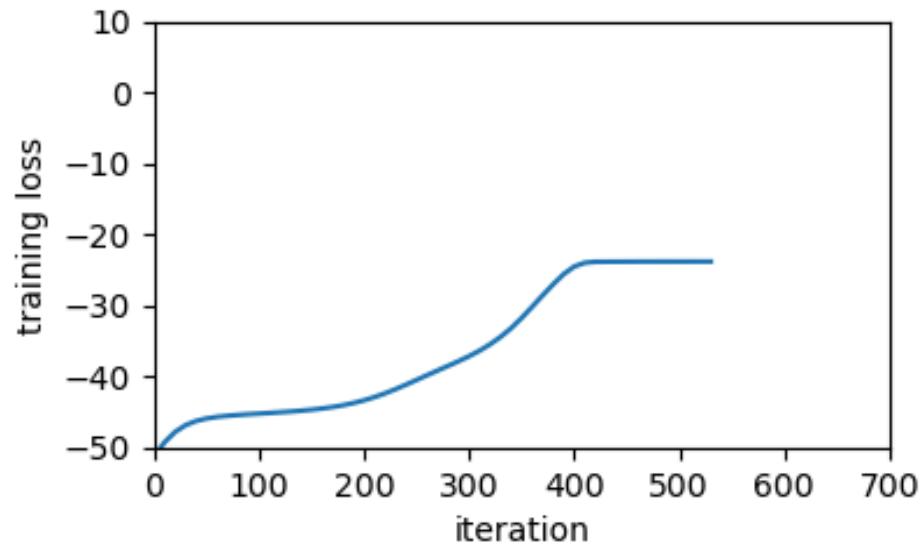
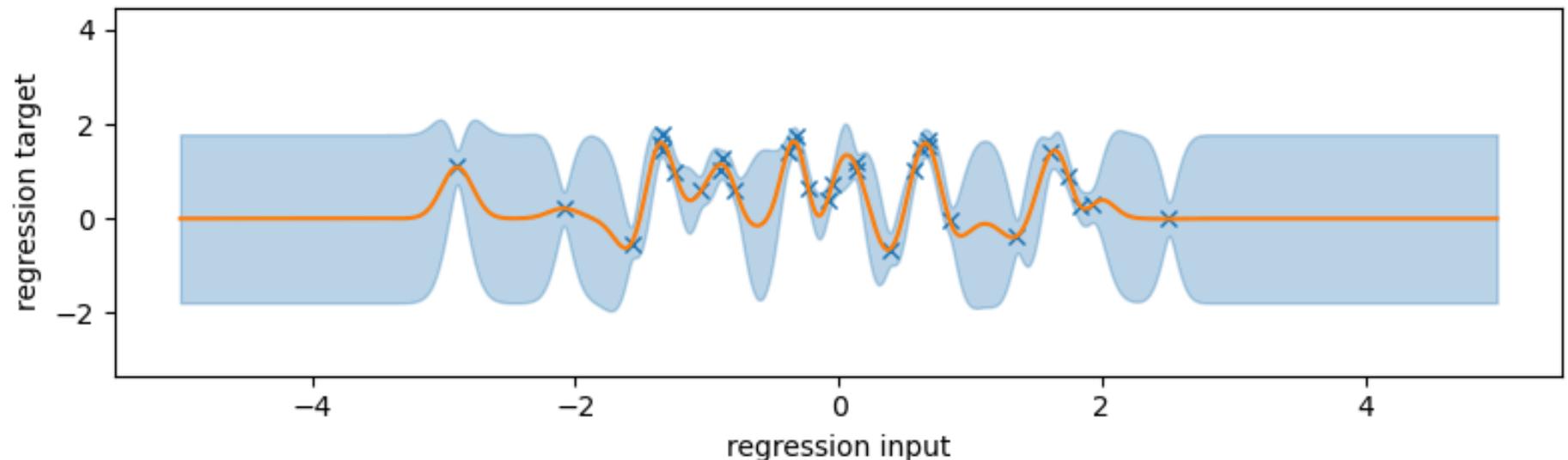
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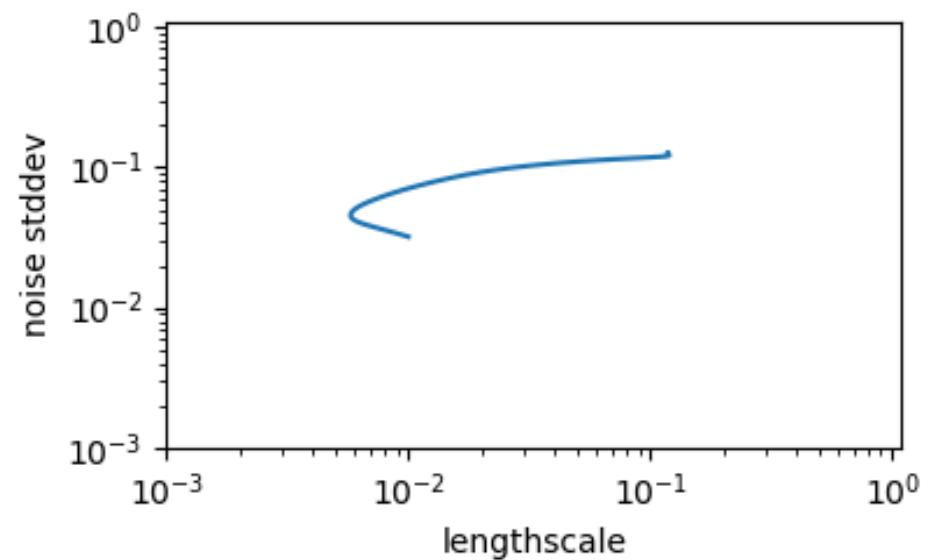
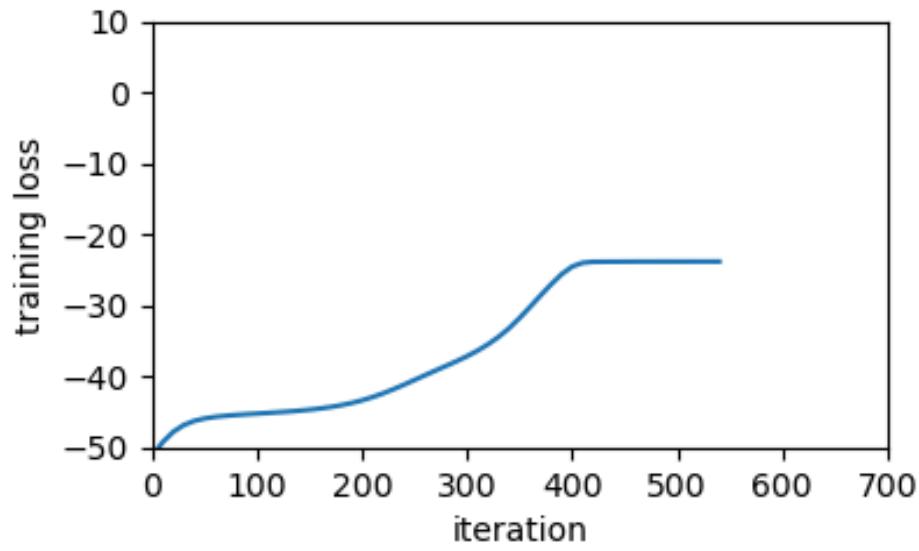
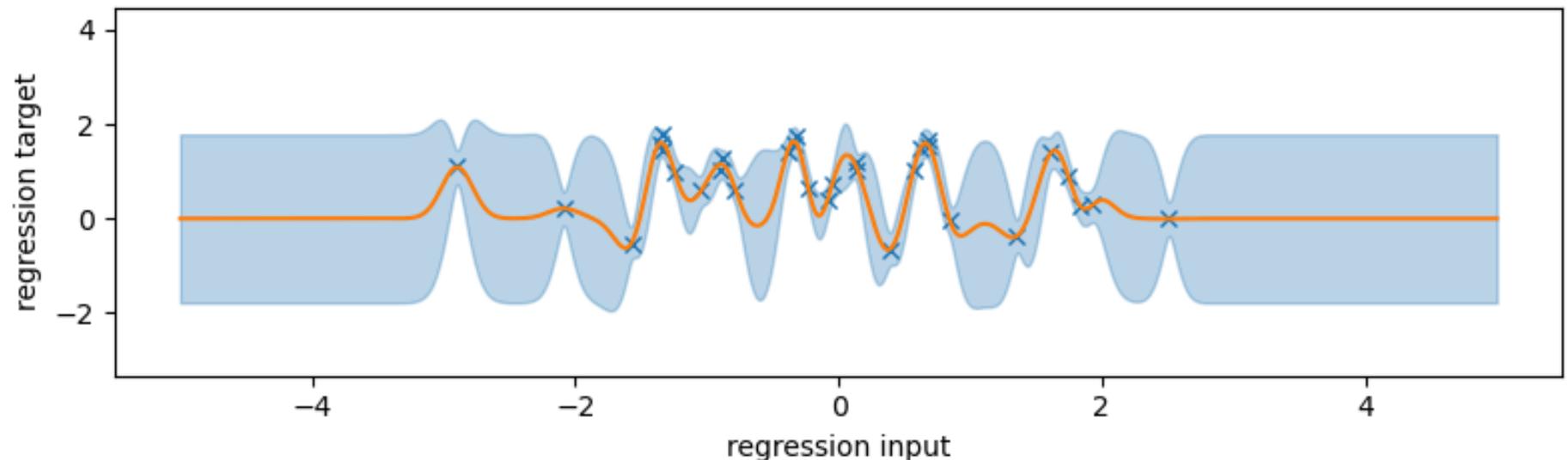
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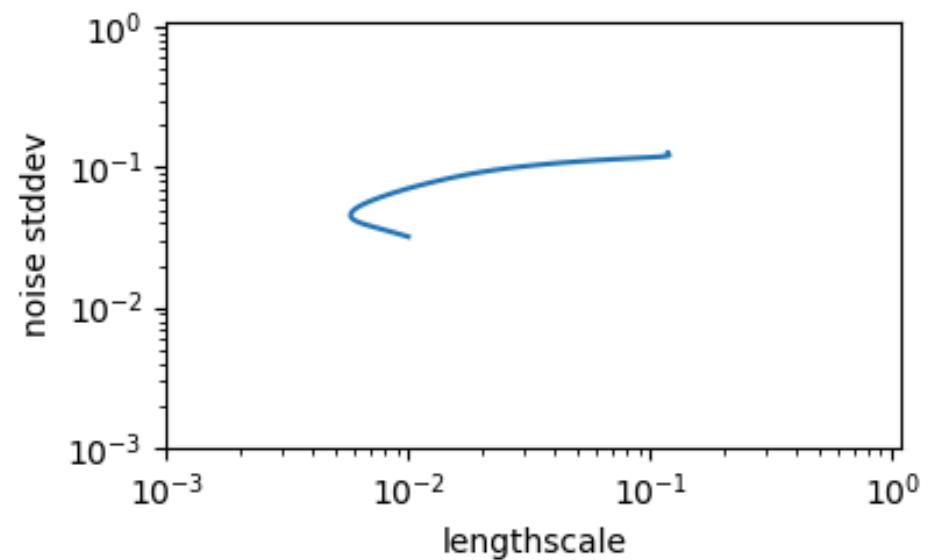
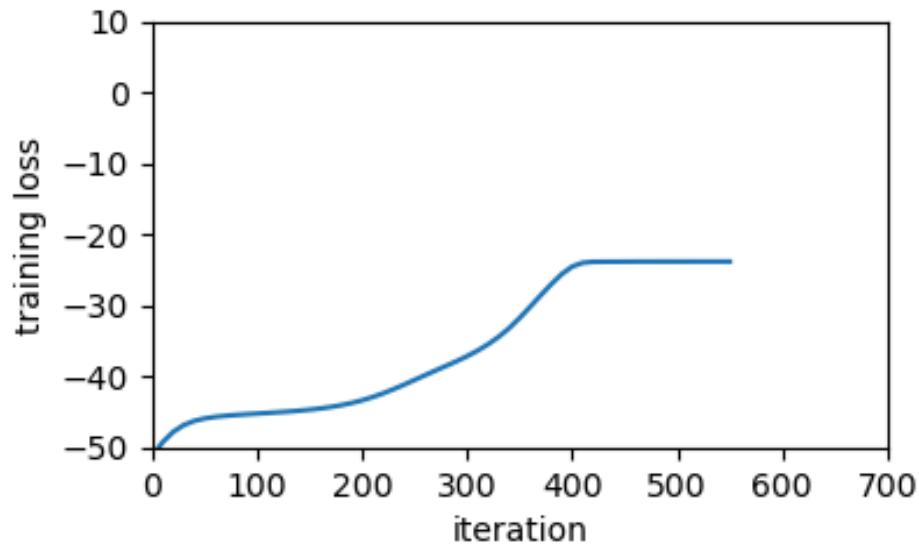
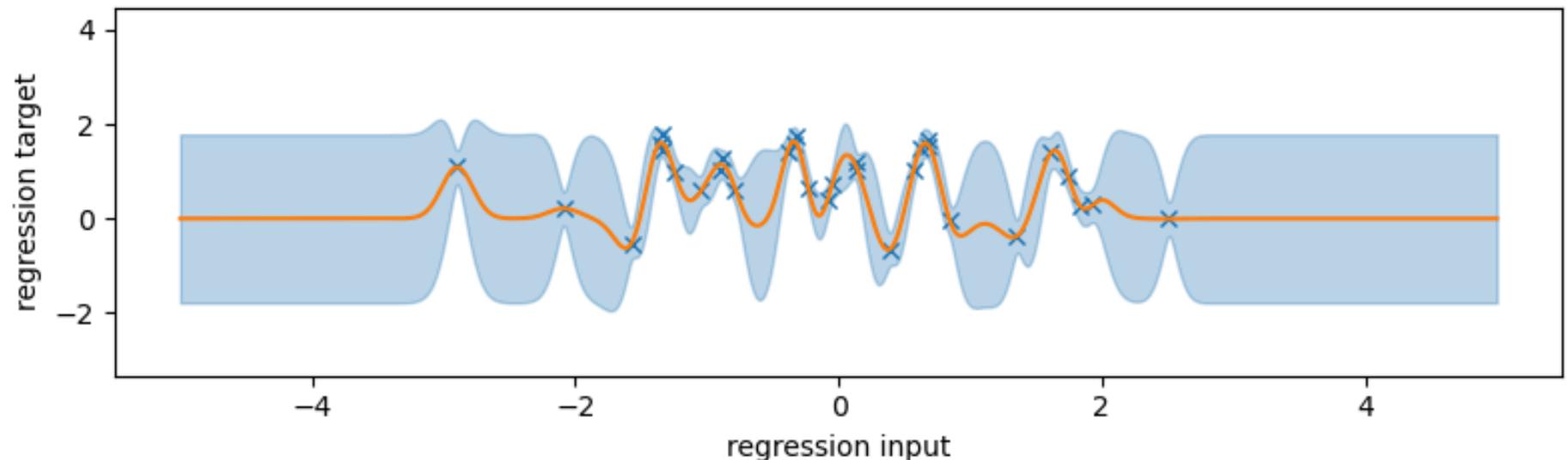
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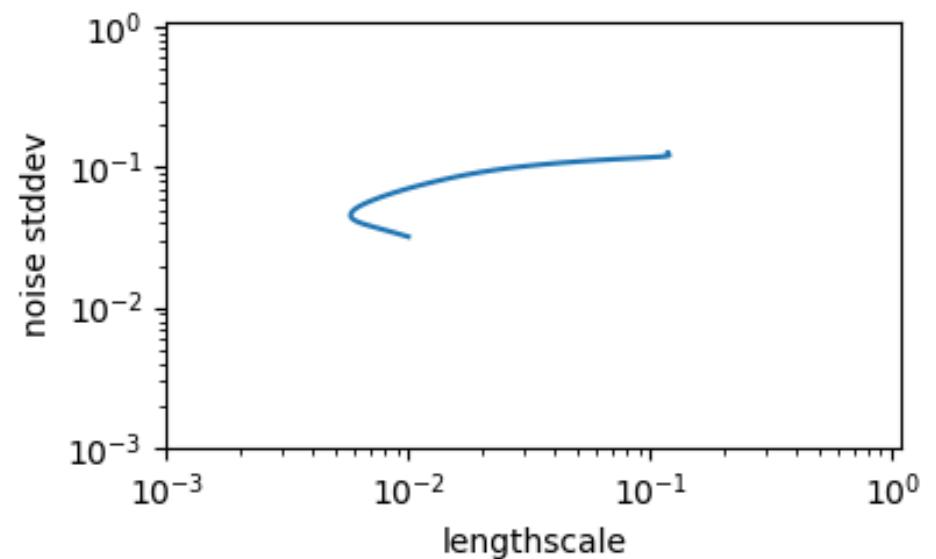
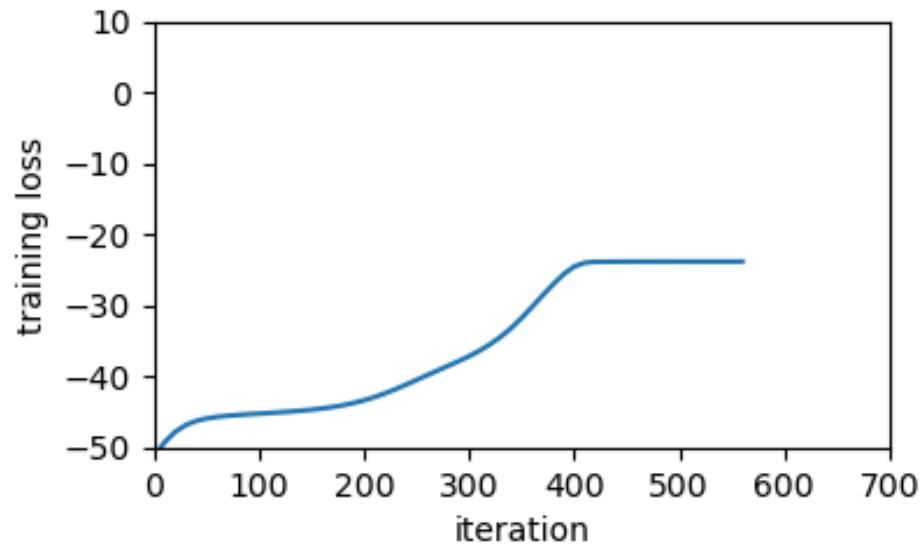
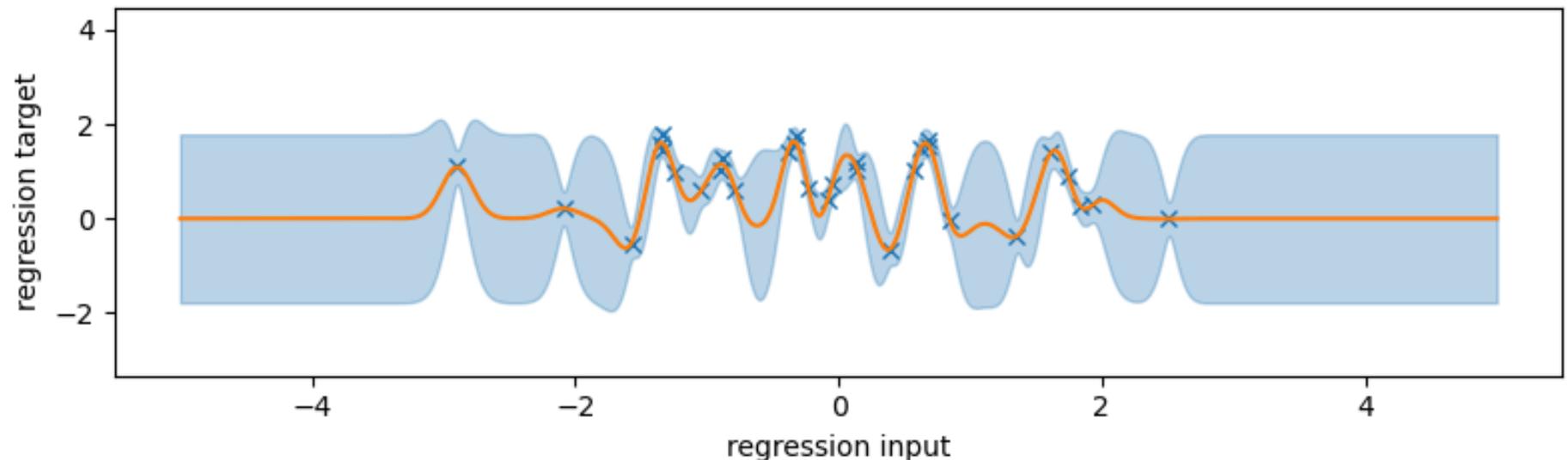
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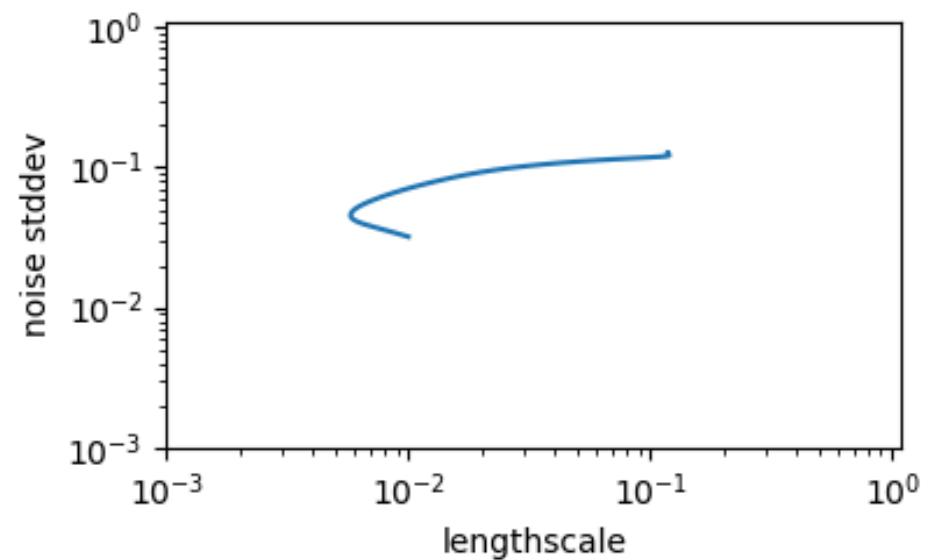
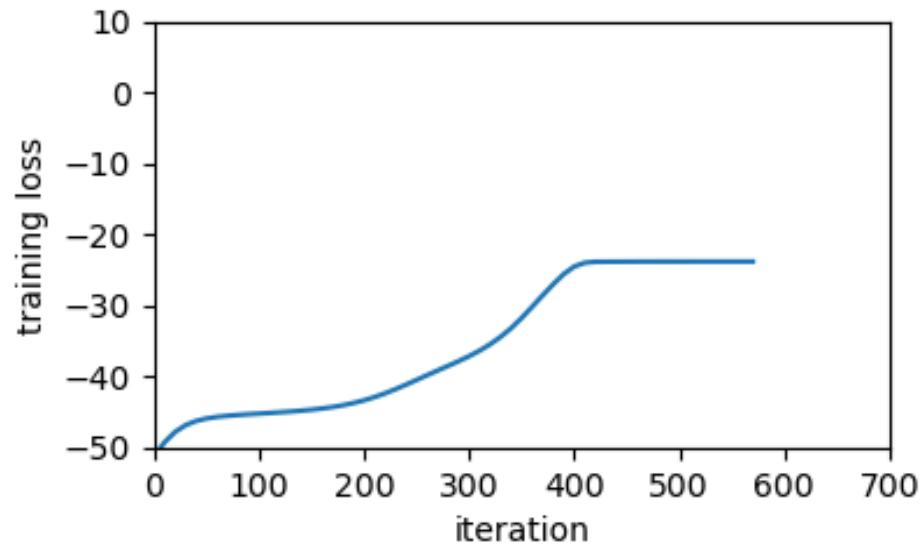
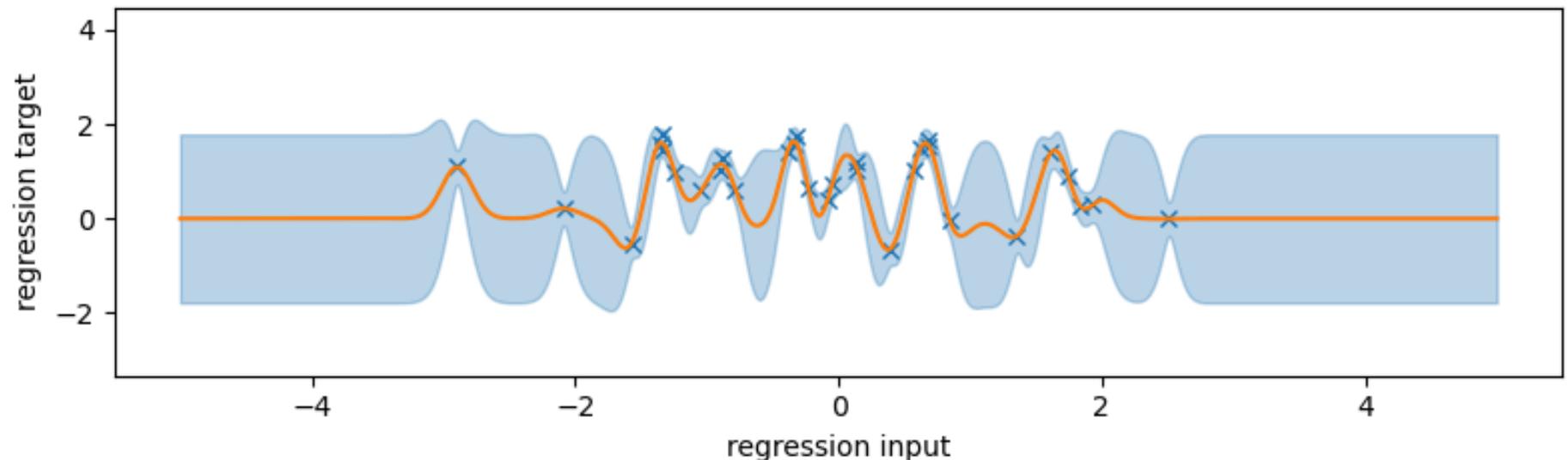
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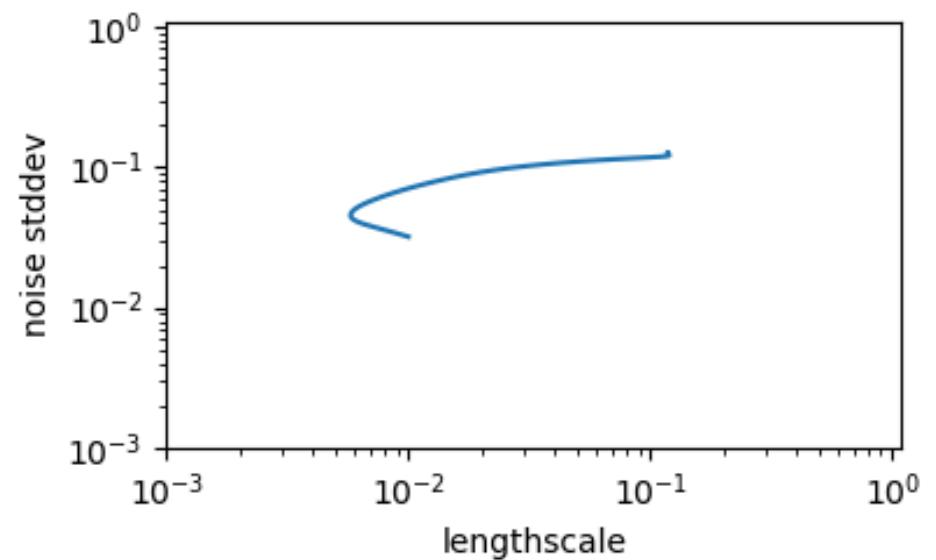
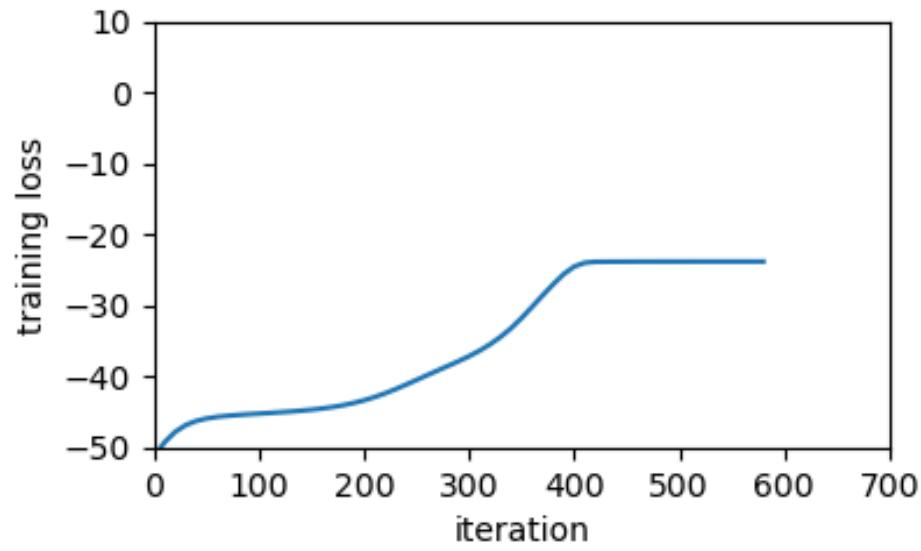
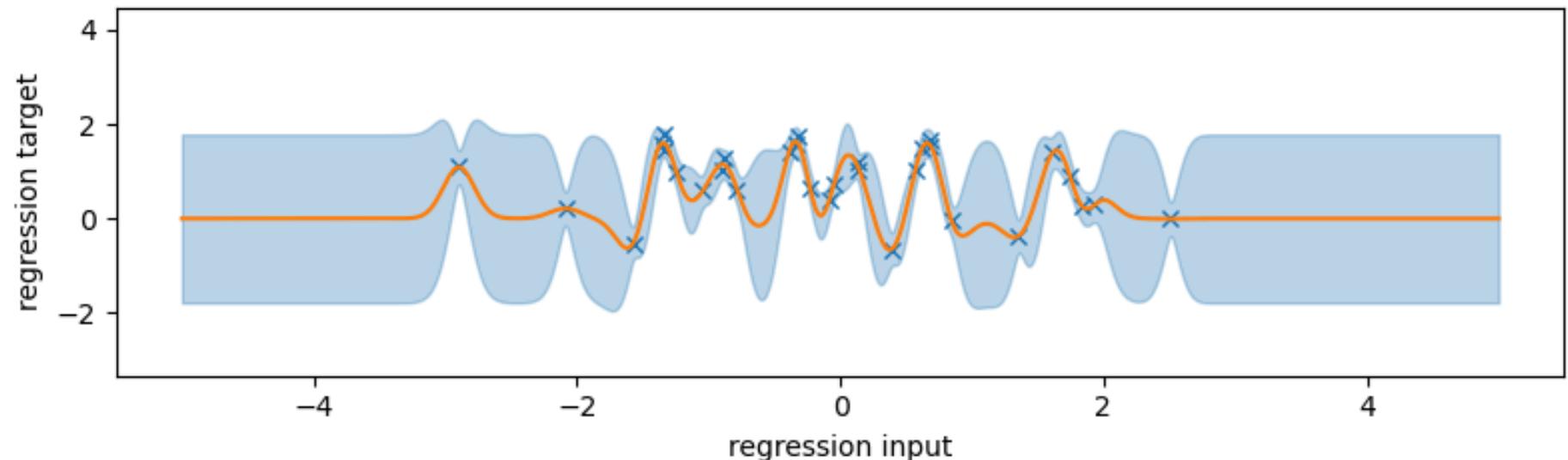
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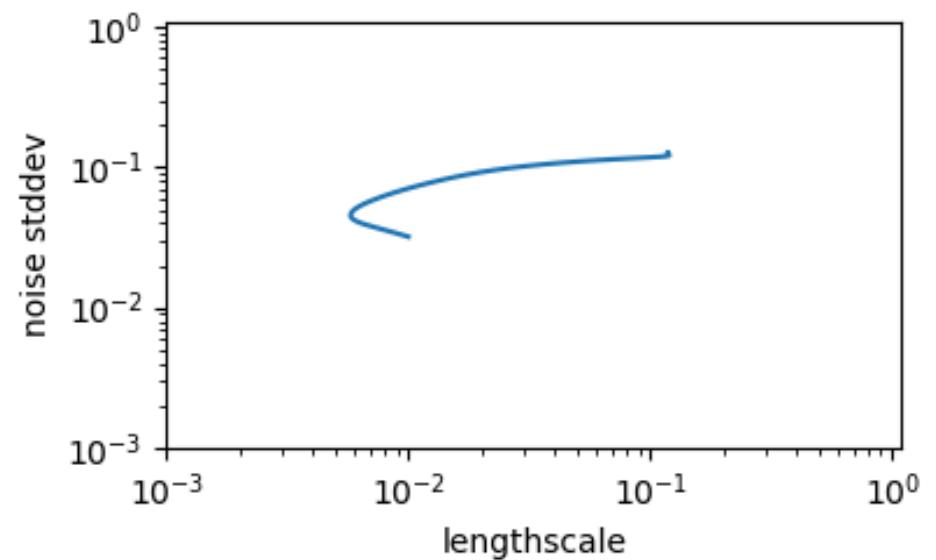
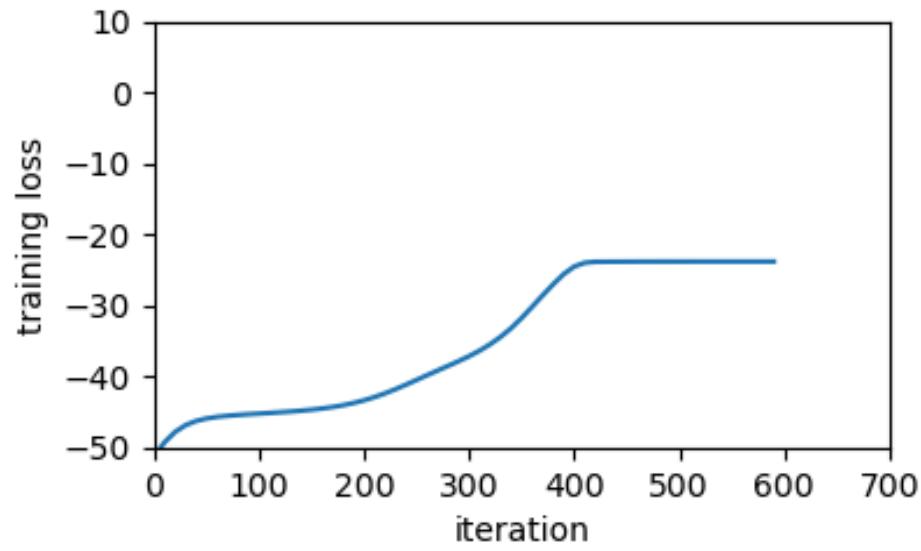
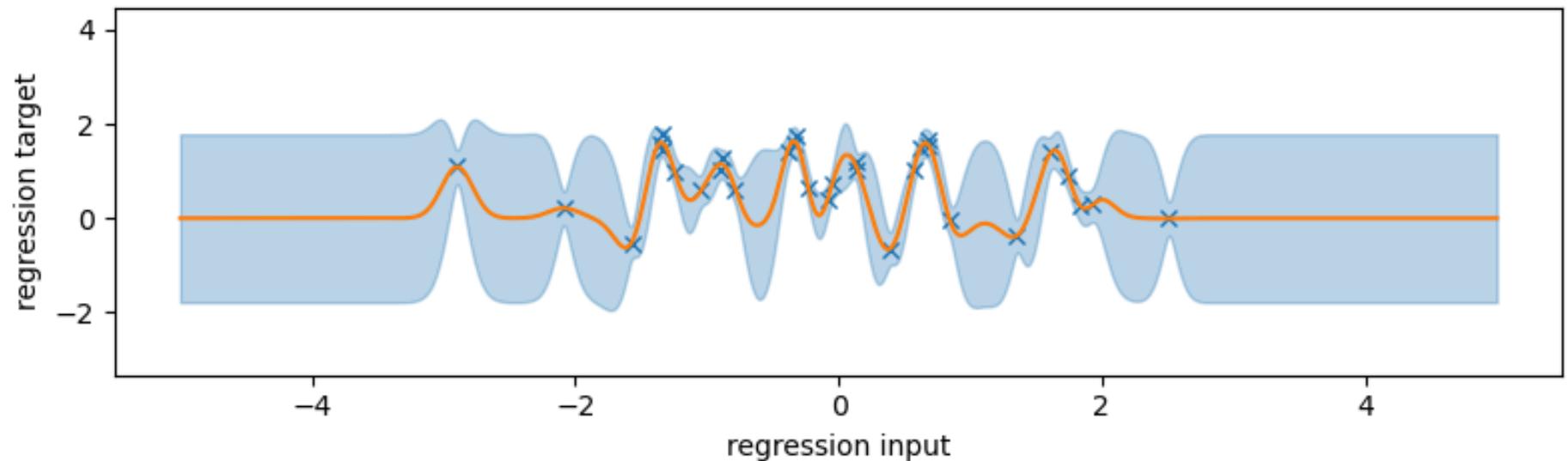
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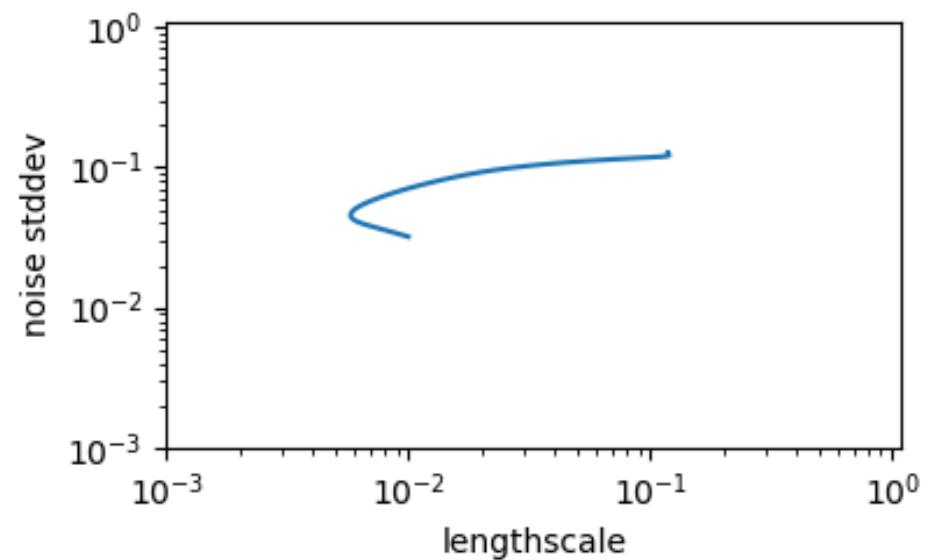
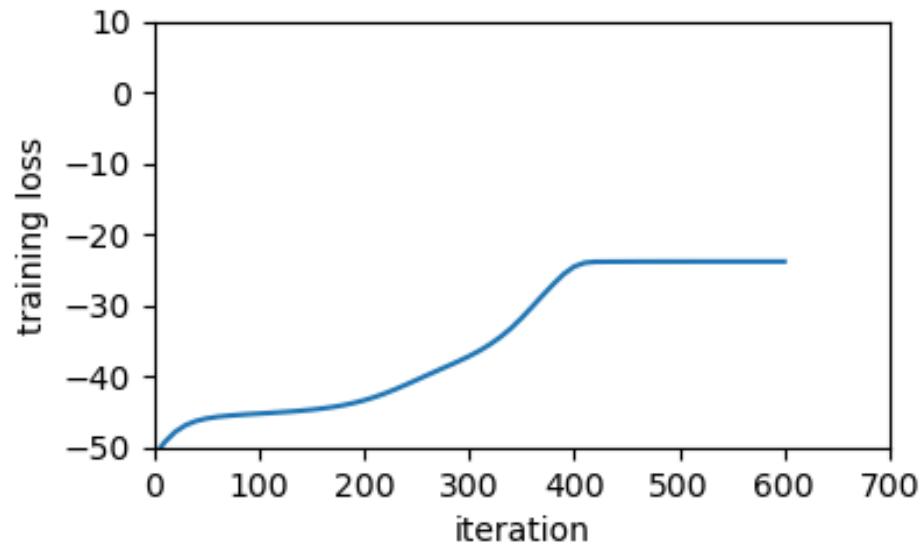
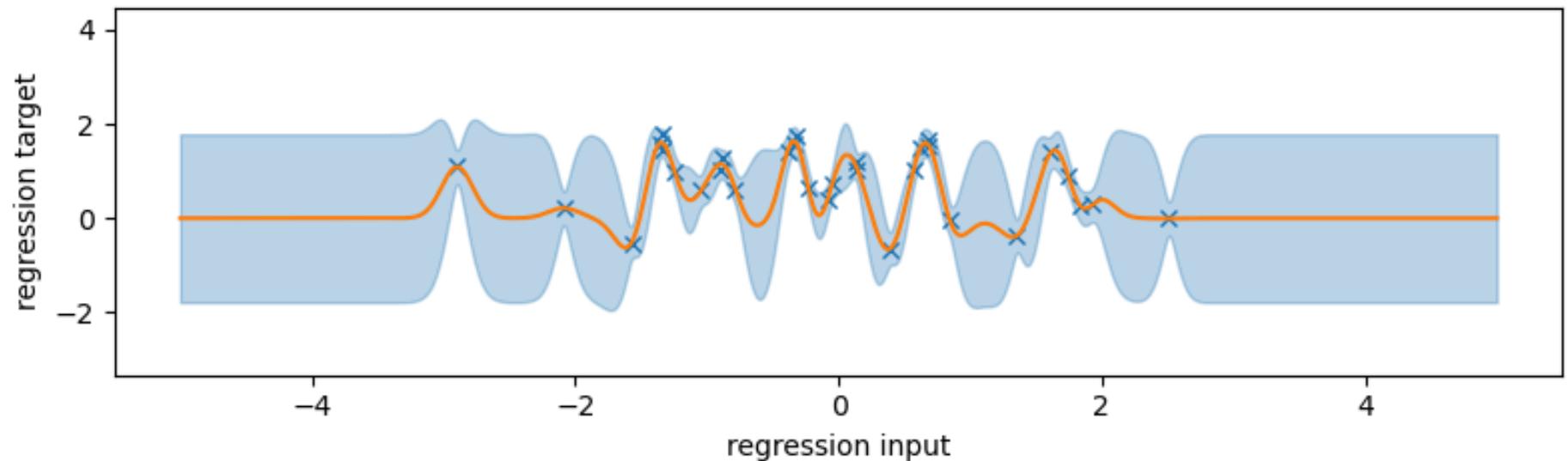
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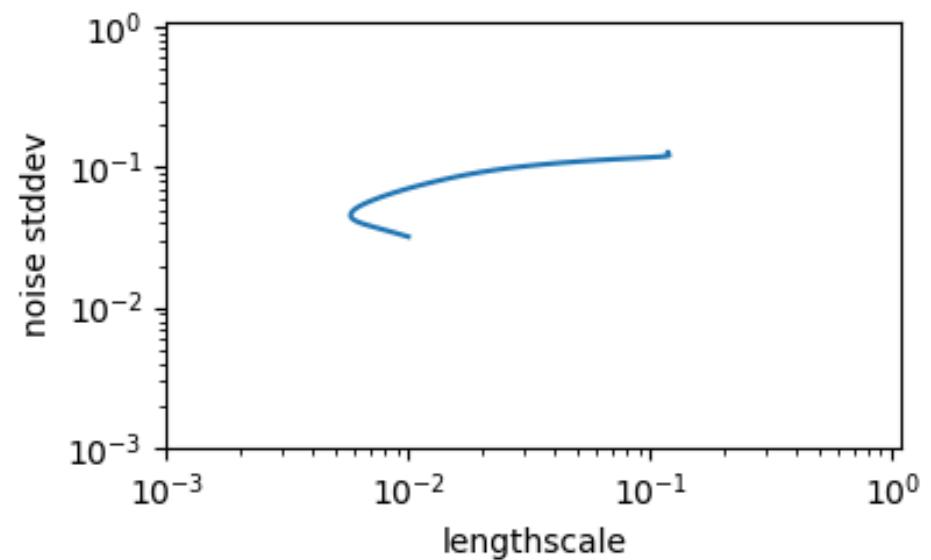
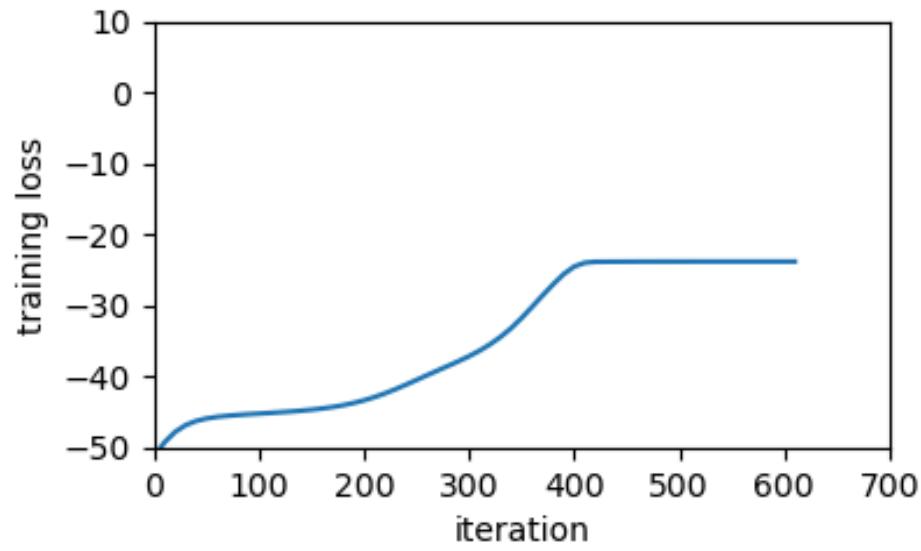
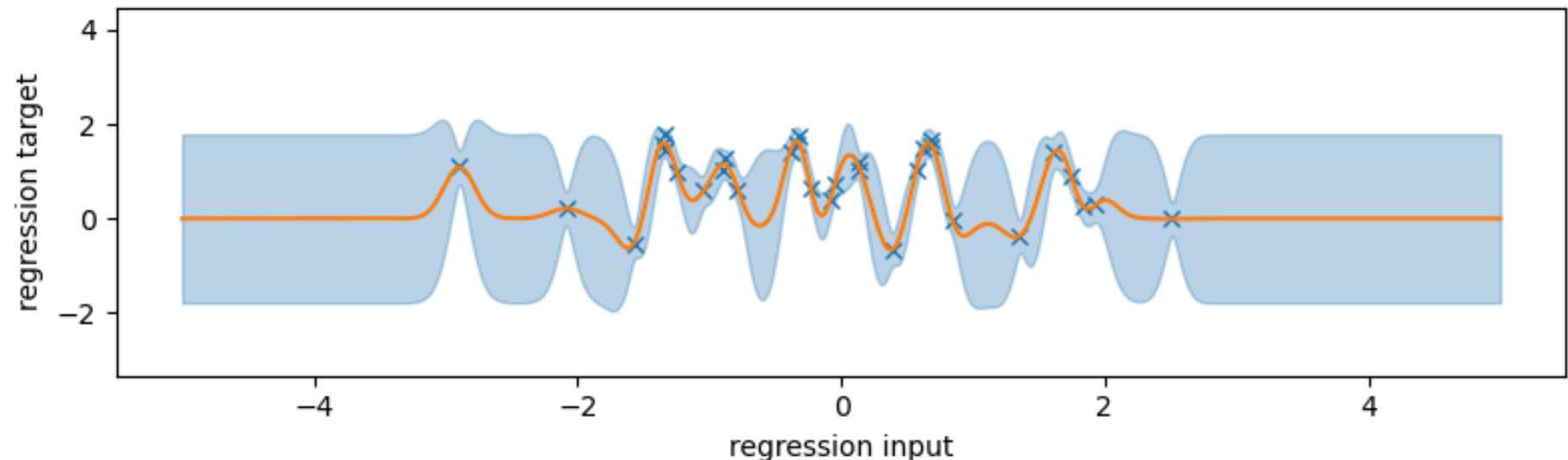
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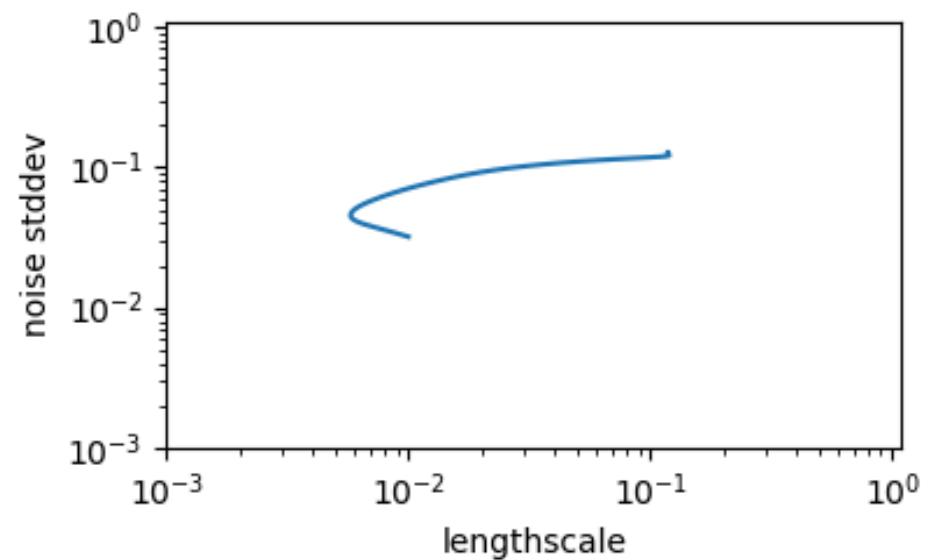
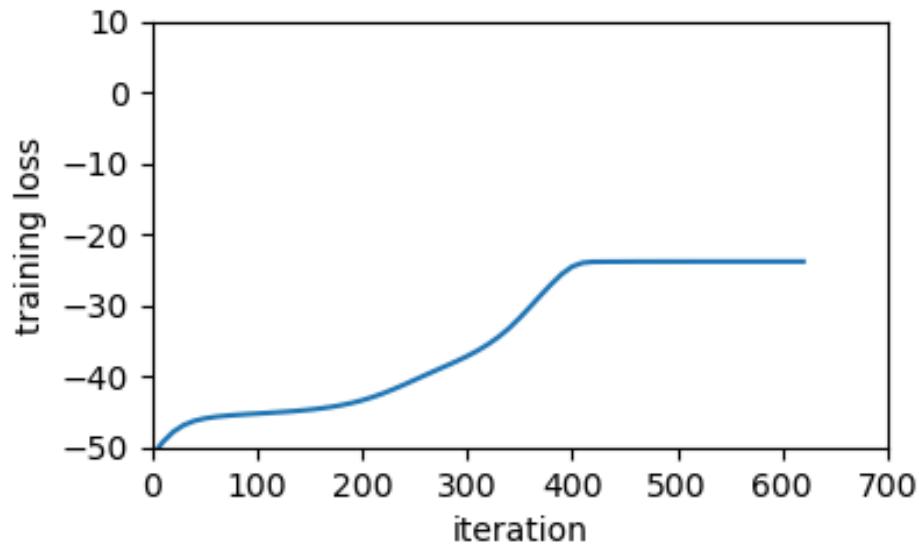
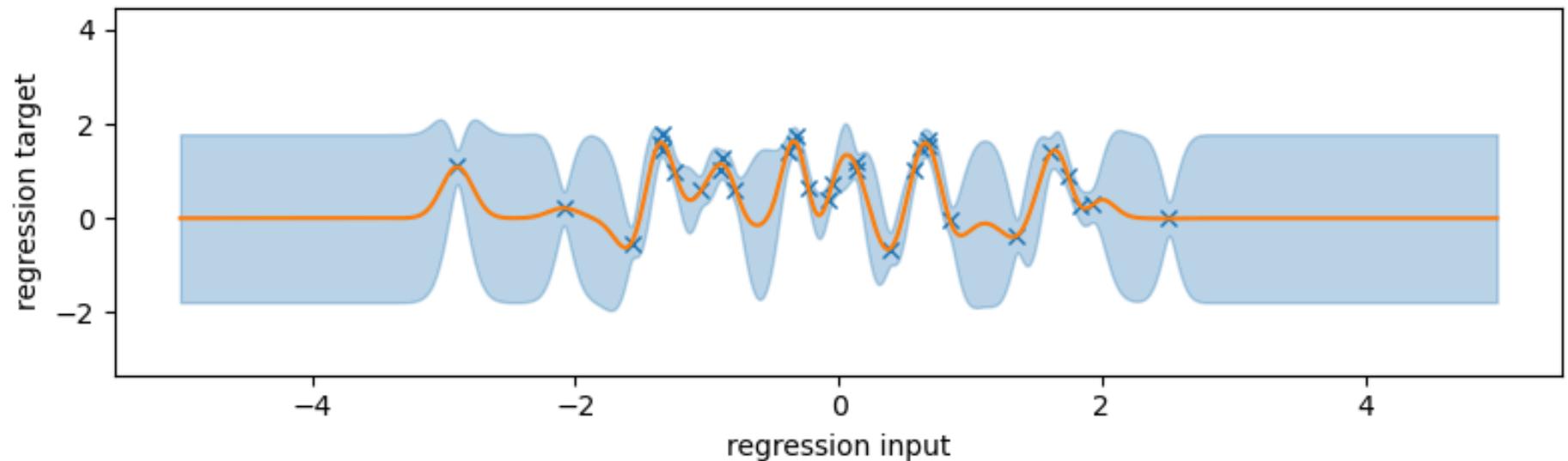
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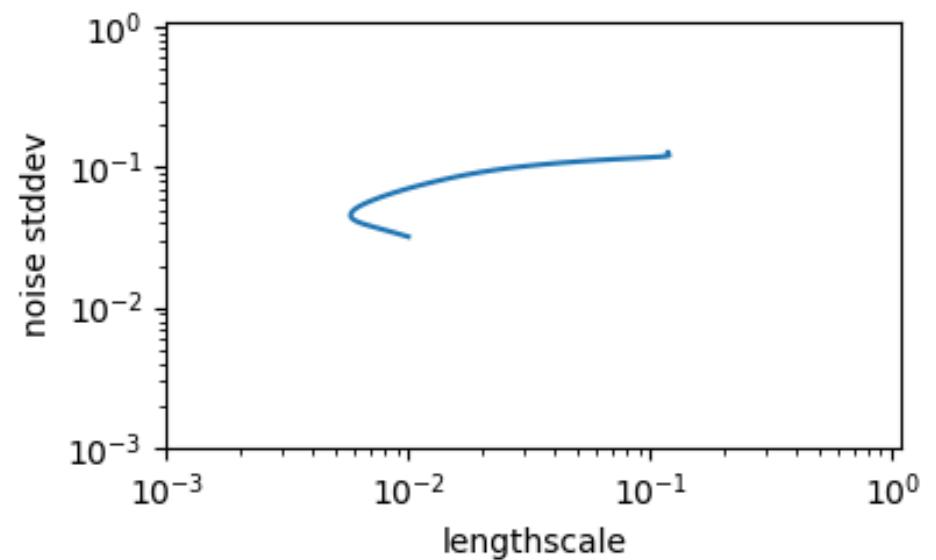
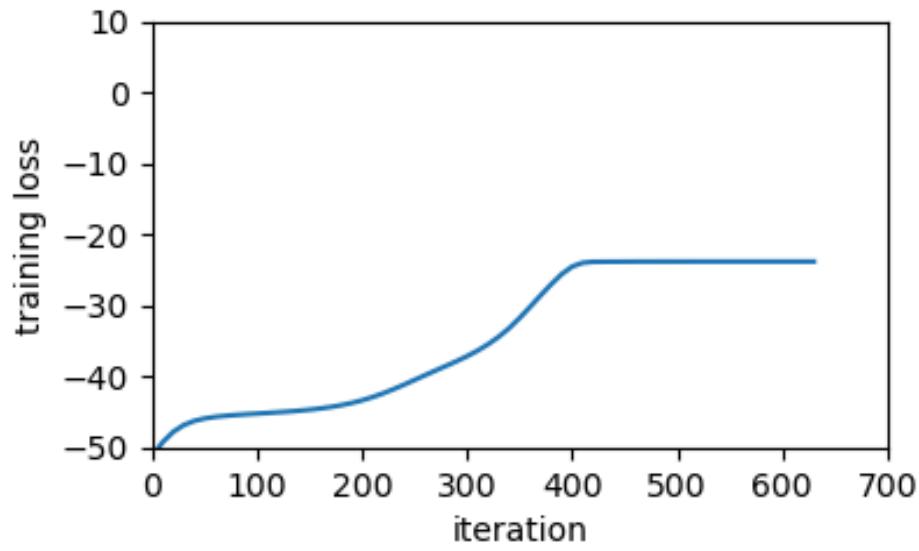
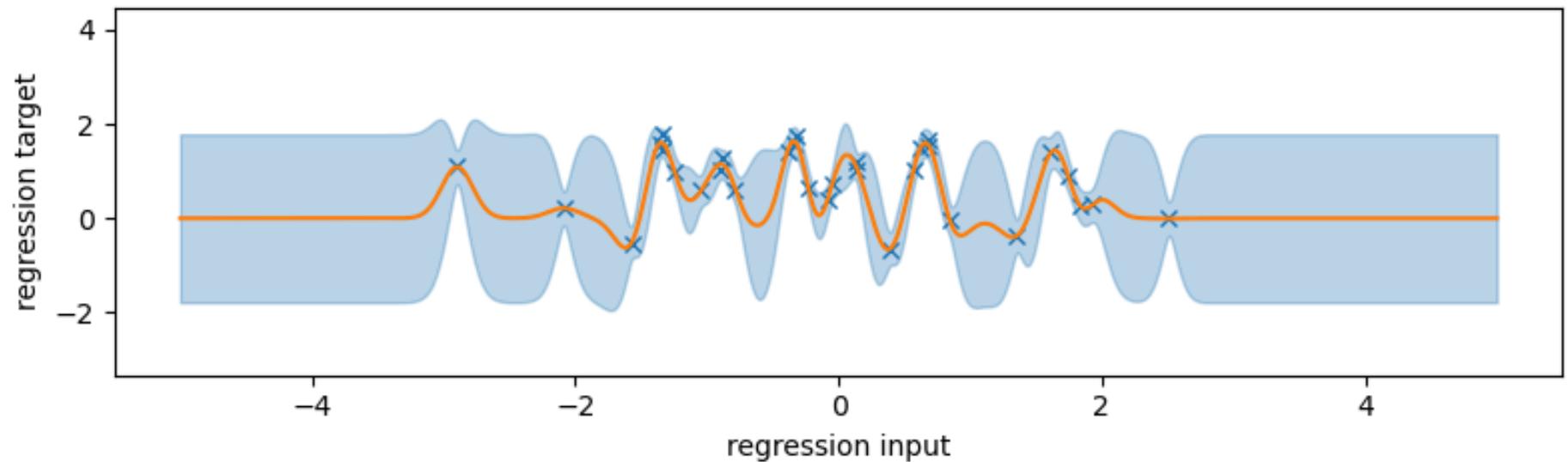
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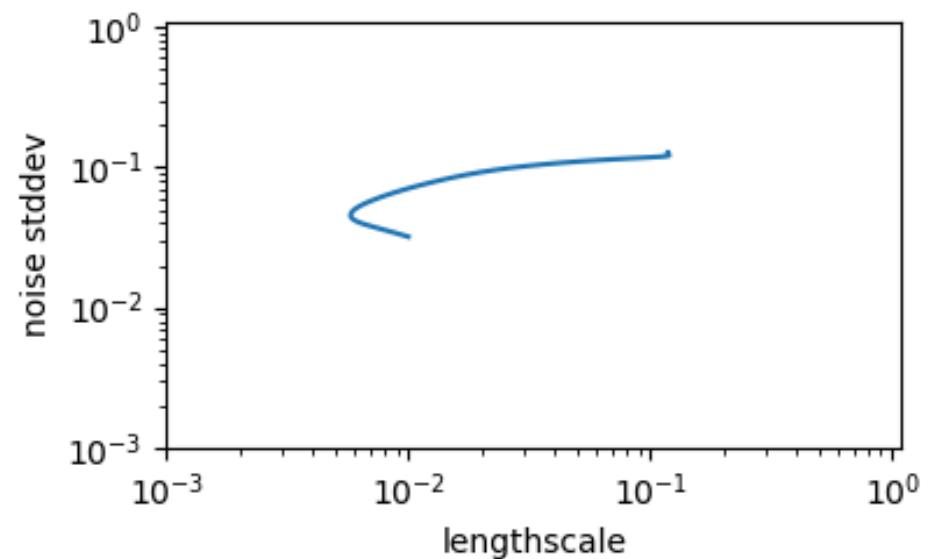
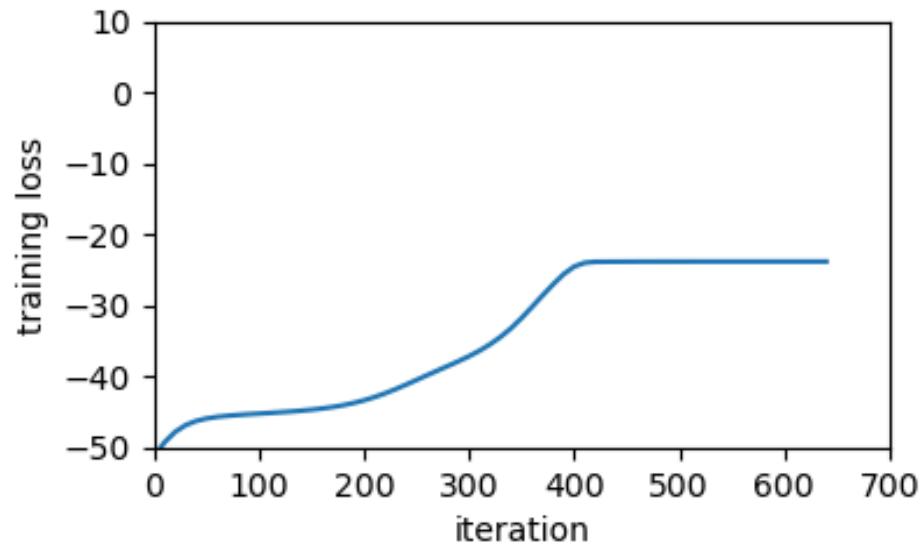
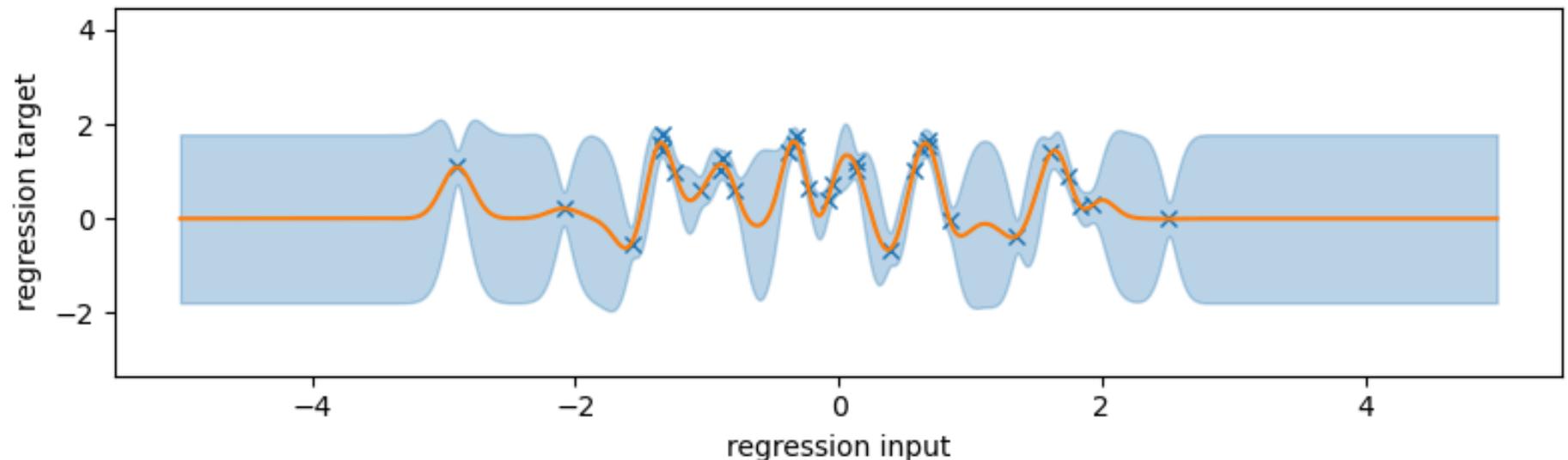
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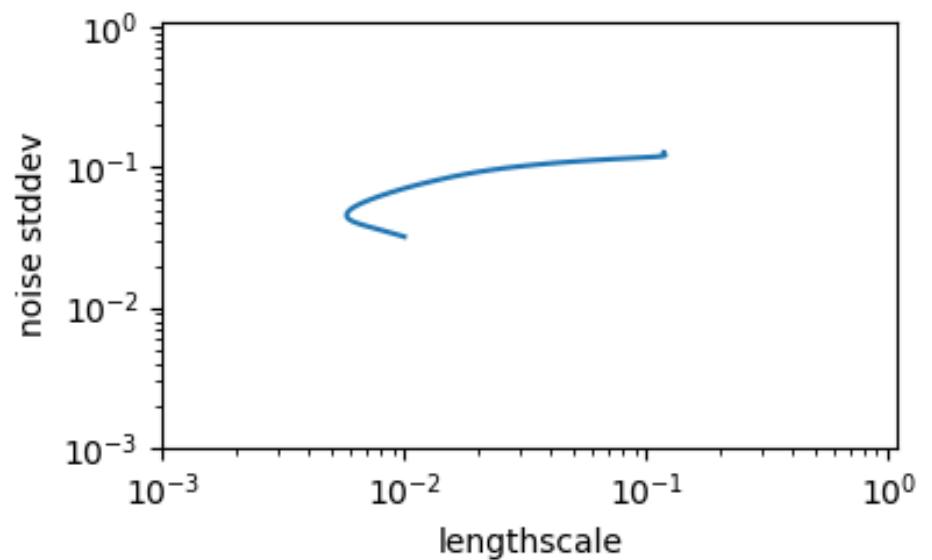
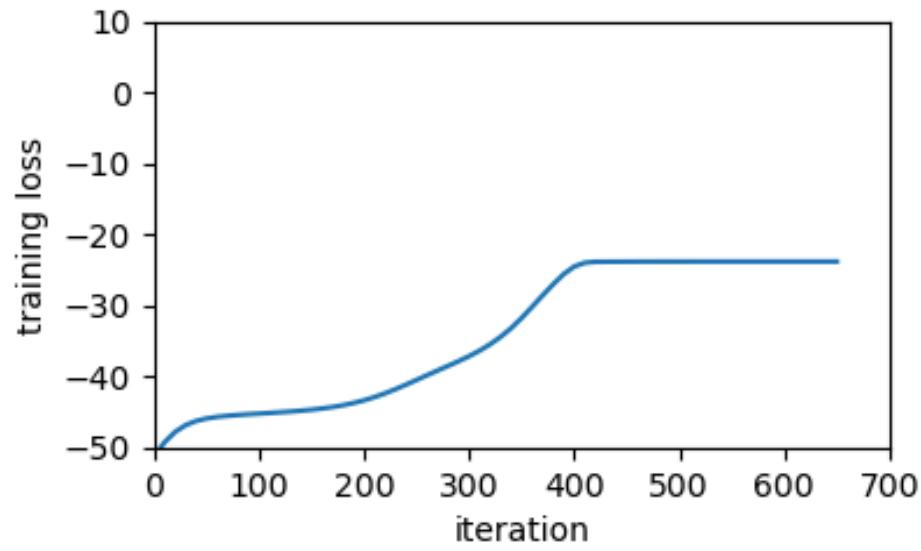
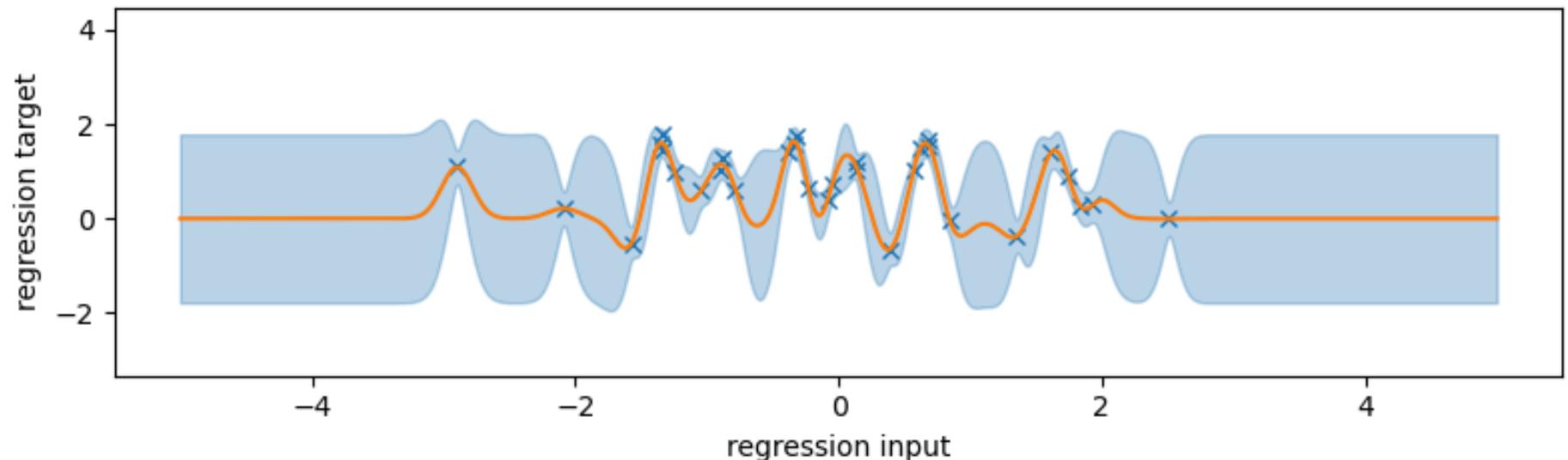
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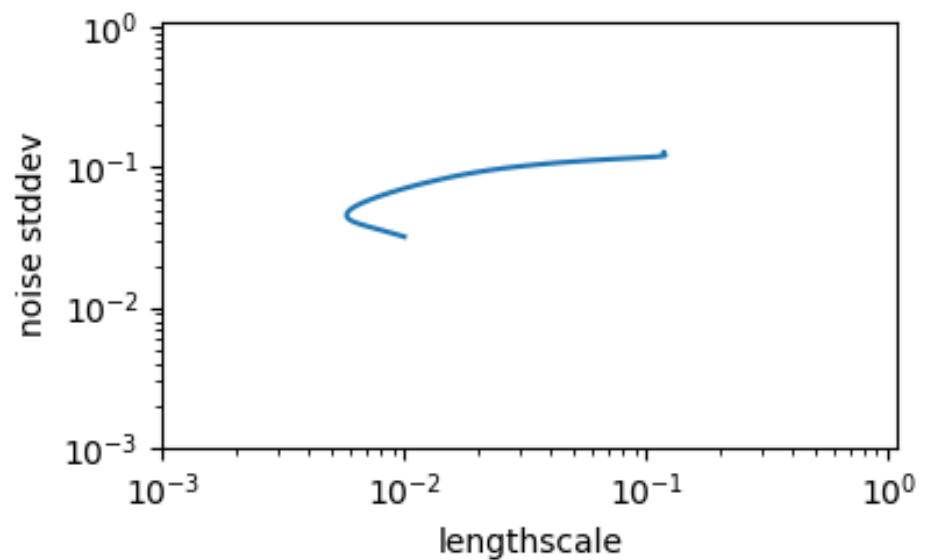
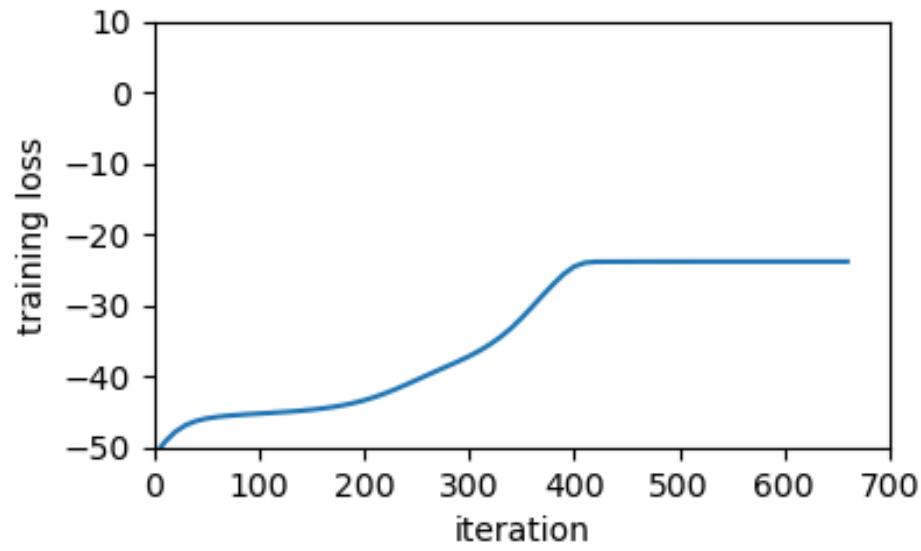
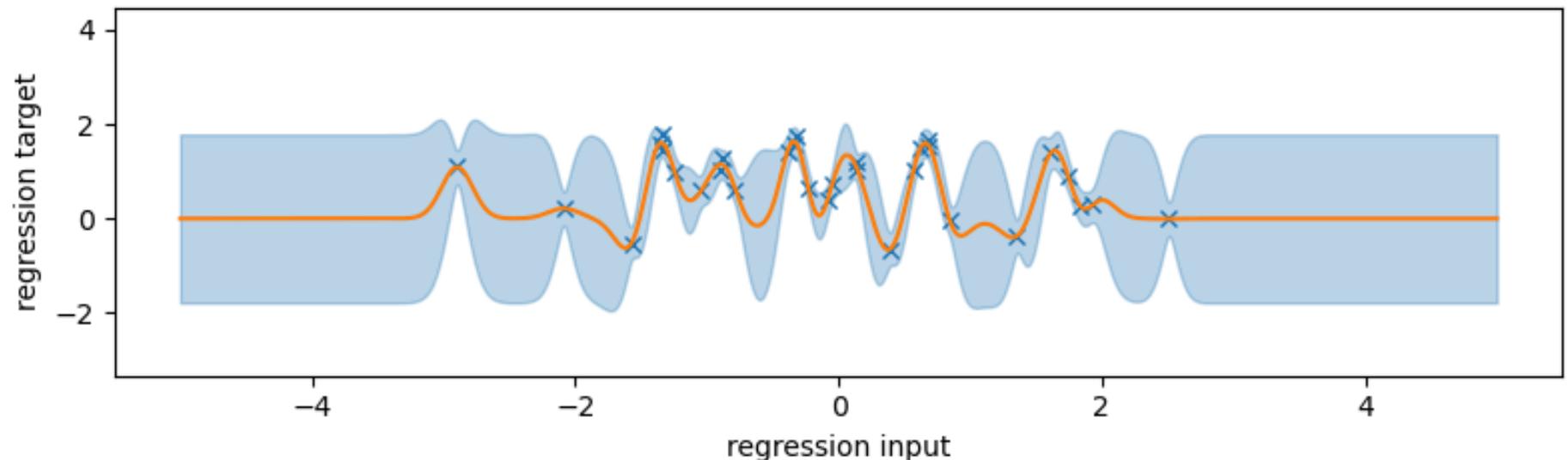
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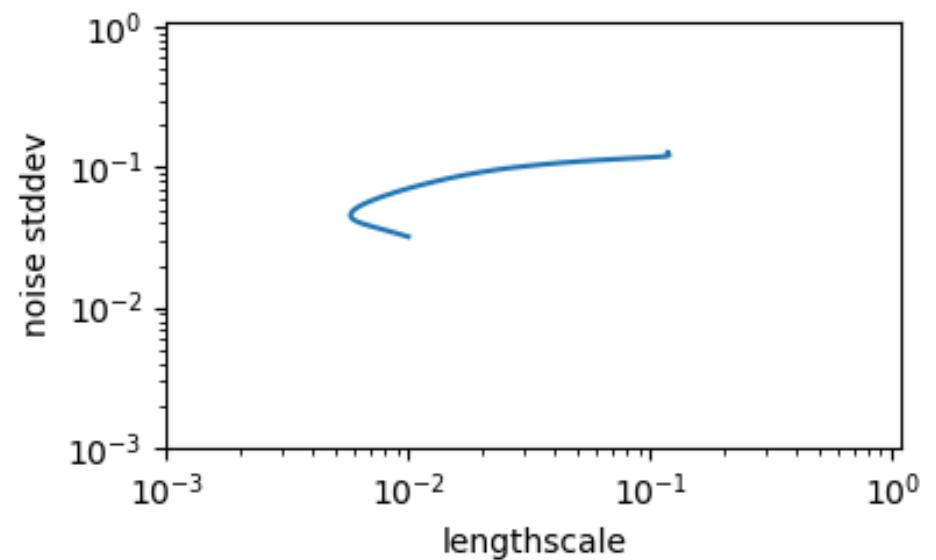
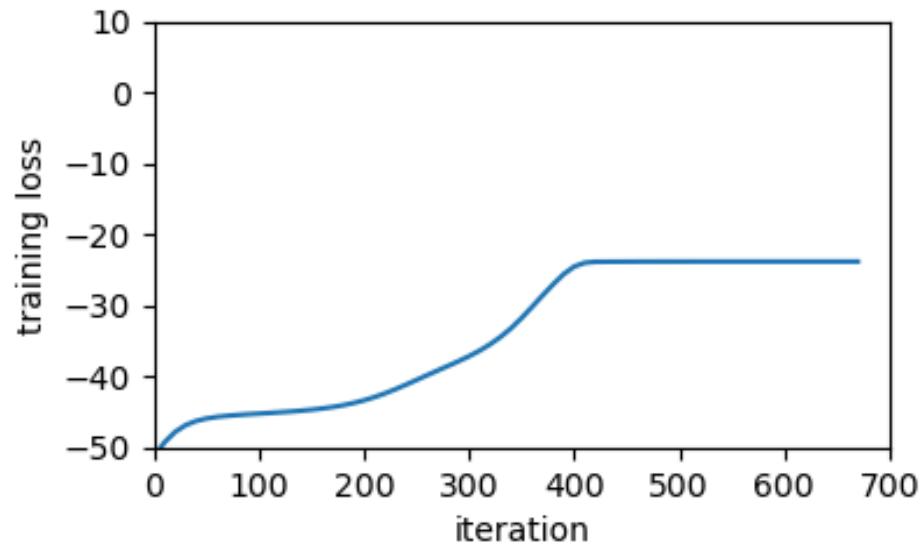
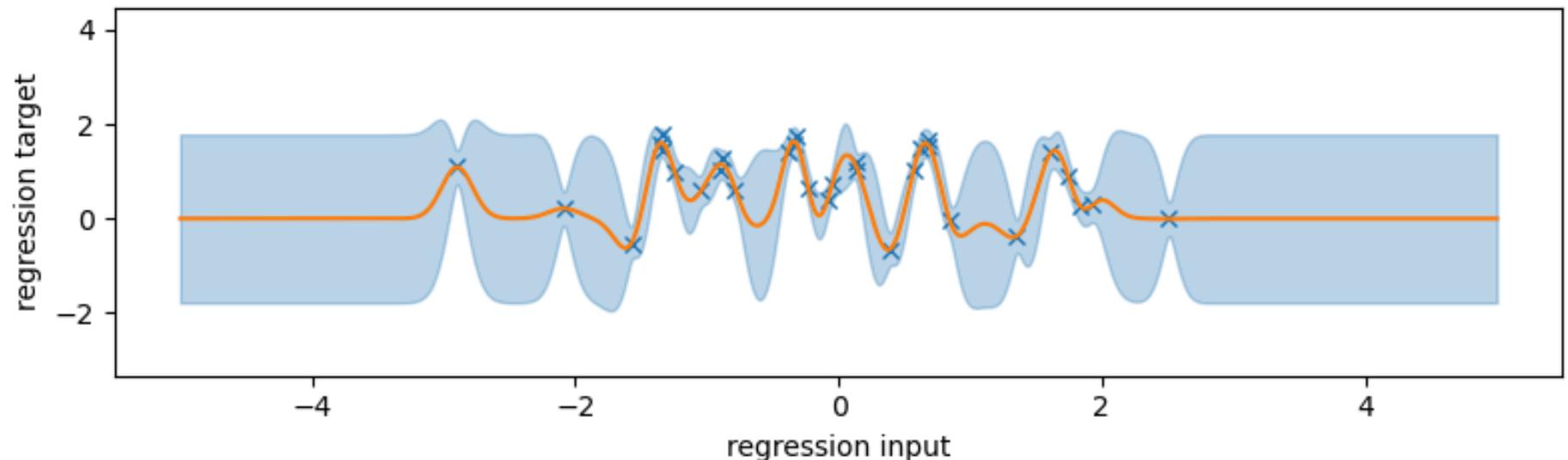
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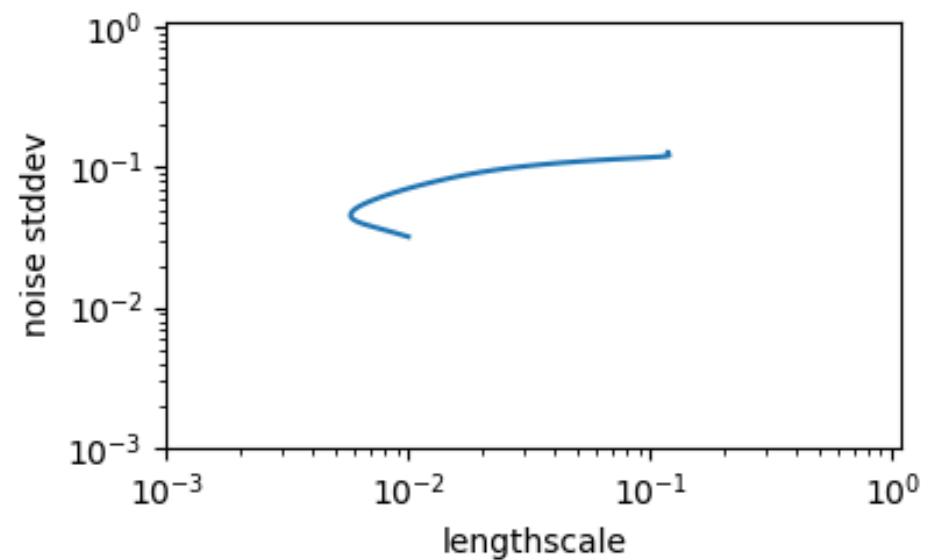
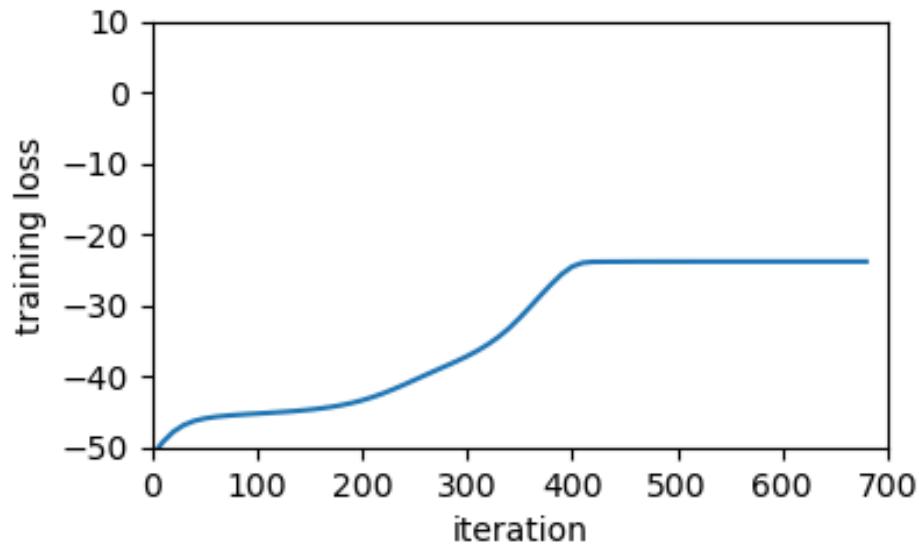
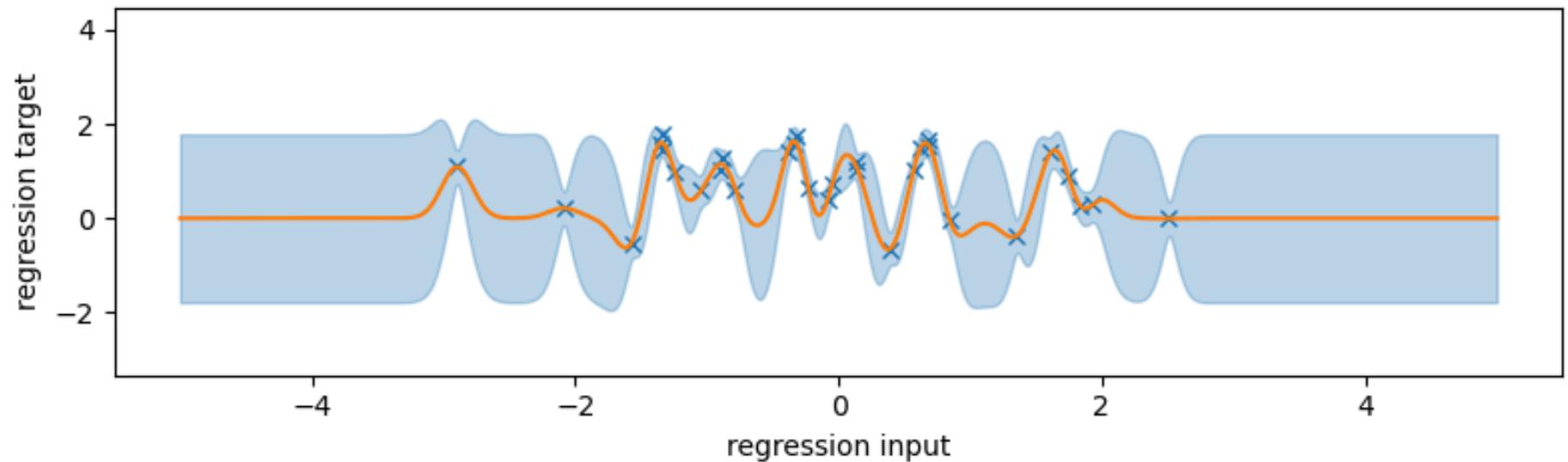
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- Interestingly, form of predictor is still single-layer NN:

$$f(x) = \sum_{m=0}^N \varphi(x; \theta, Z_m) w_m$$

$$\varphi(x; \theta, Z_m) = k_\theta(x, X_m) \quad \boldsymbol{w} = (K(X, X) + \sigma^2 I)^{-1} \boldsymbol{y}$$

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Our model grows, but by memorising *all* data!

1. What is wrong with minimising losses.
2. Bayesian Model Selection?
2. **The Bayesian answer to model size: Nonparametrics.**
3. A principle for selecting size

Why use Nonparametric models?

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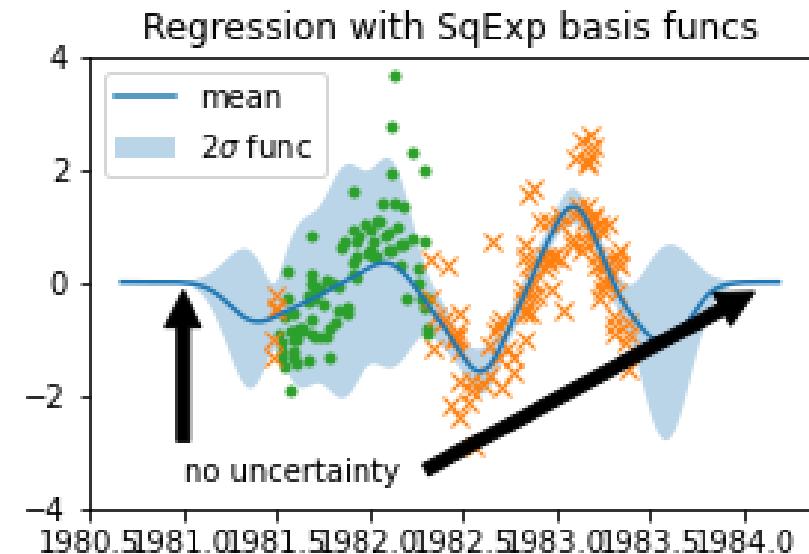
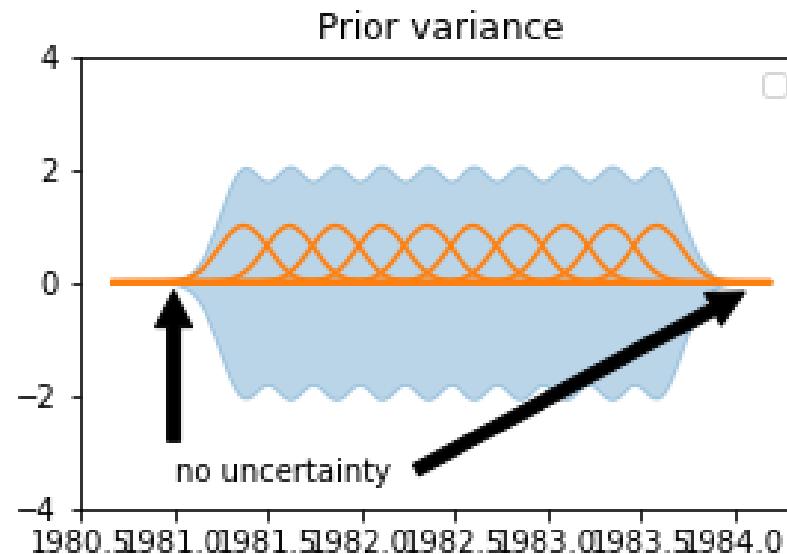
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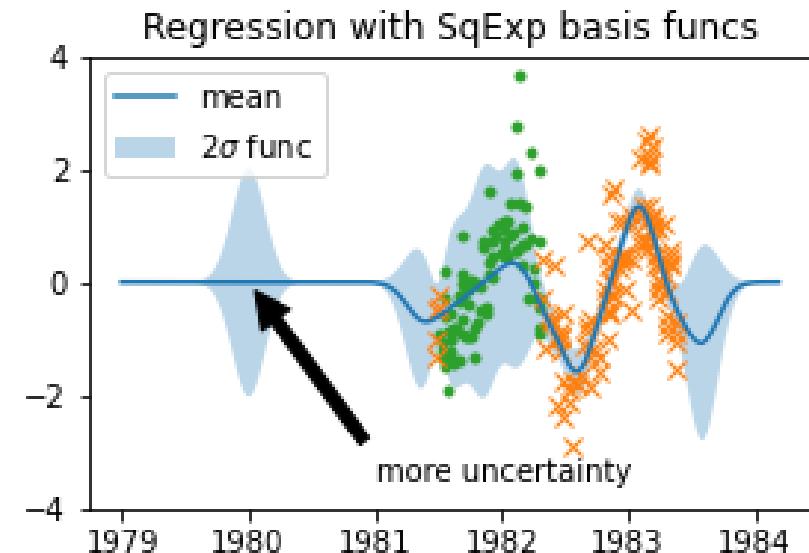
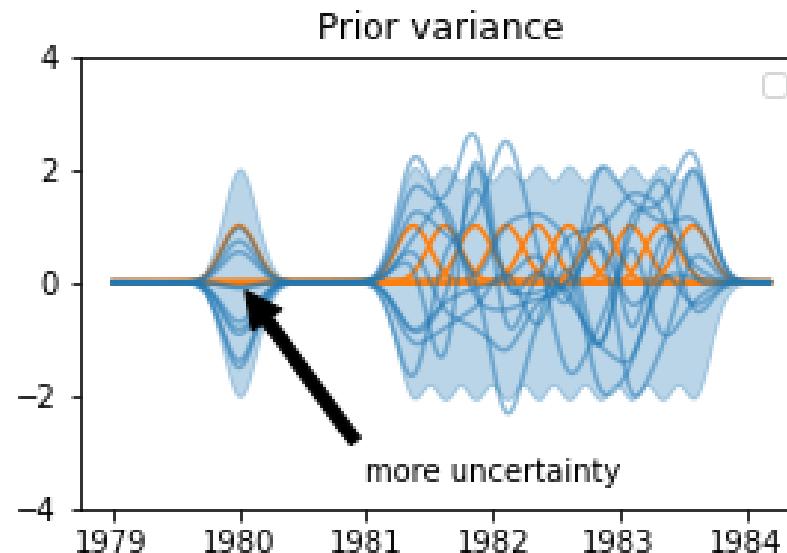


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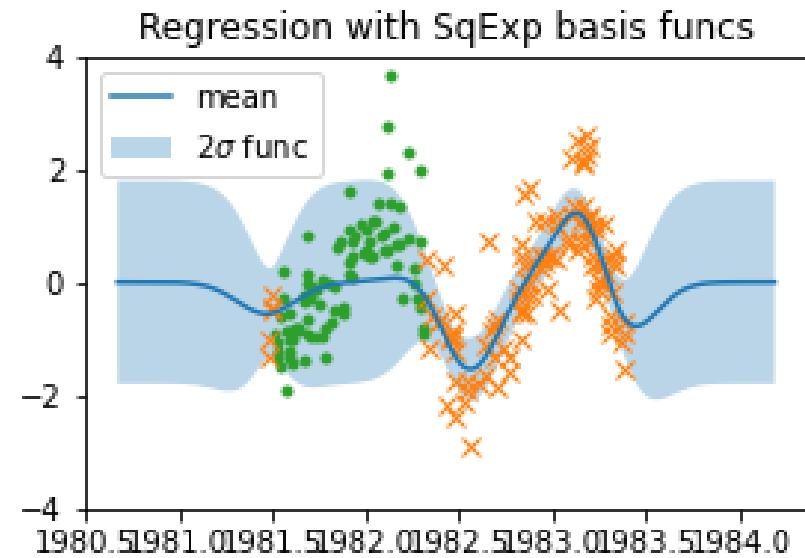
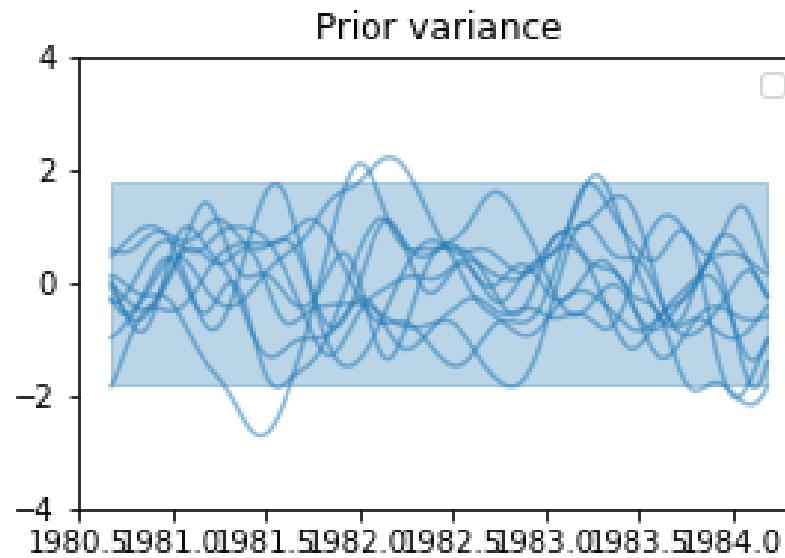


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Or are we stuck with memorising all the data?

We could do model selection over the model size...

$$p(W, \theta, M | \mathcal{D}) = \frac{p(\mathcal{D} | W, \theta, M) p(W | \theta, M)}{p(\mathcal{D} | \theta, M)} \frac{p(\mathcal{D} | \theta, M) p(\theta)}{p(\mathcal{D})}$$

$$\theta^*, M^* = \operatorname{argmax}_{\theta, M} \log p(\mathcal{D} | \theta, M)$$

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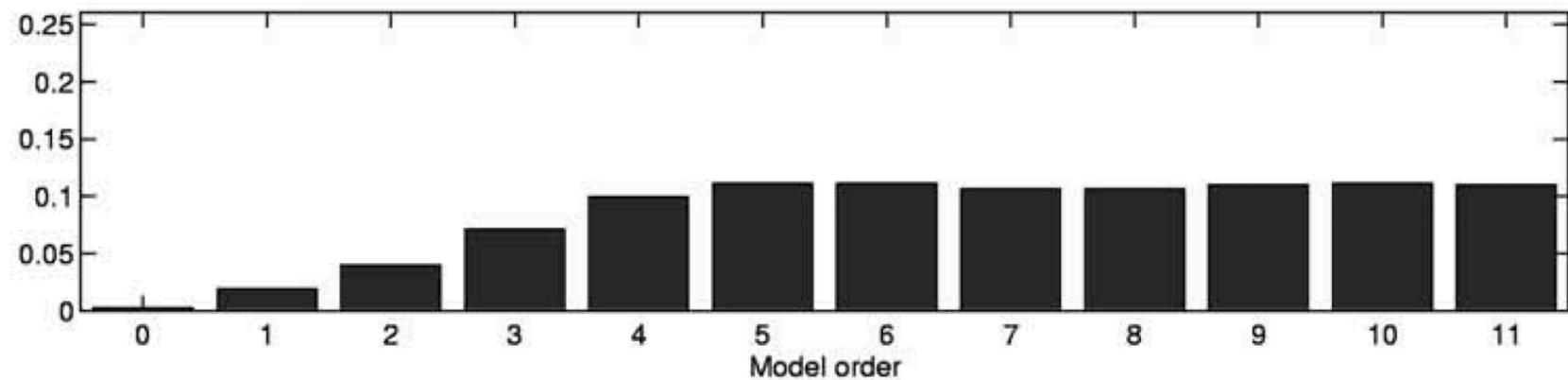
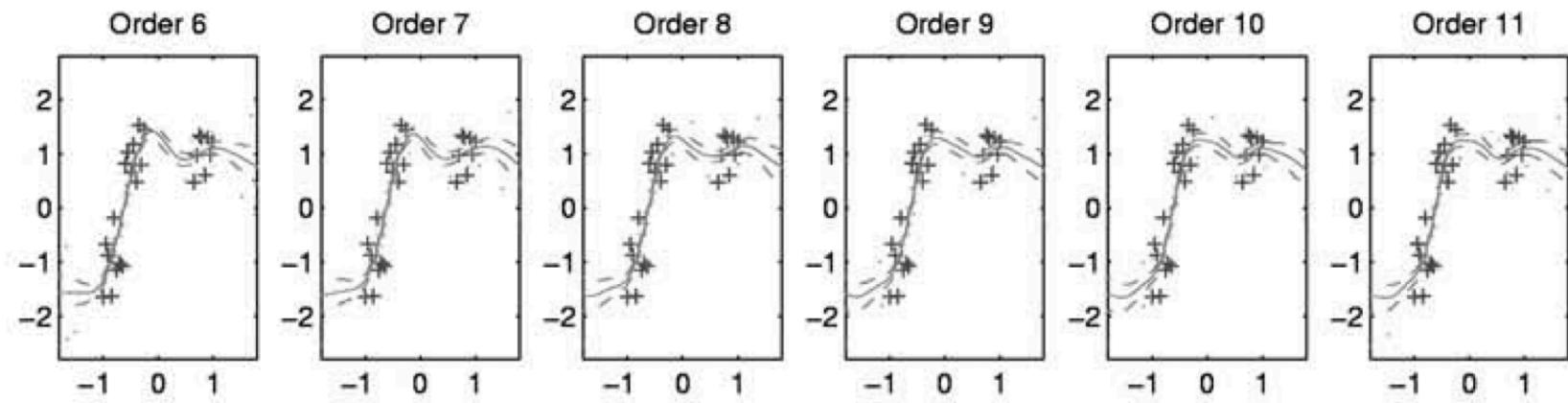
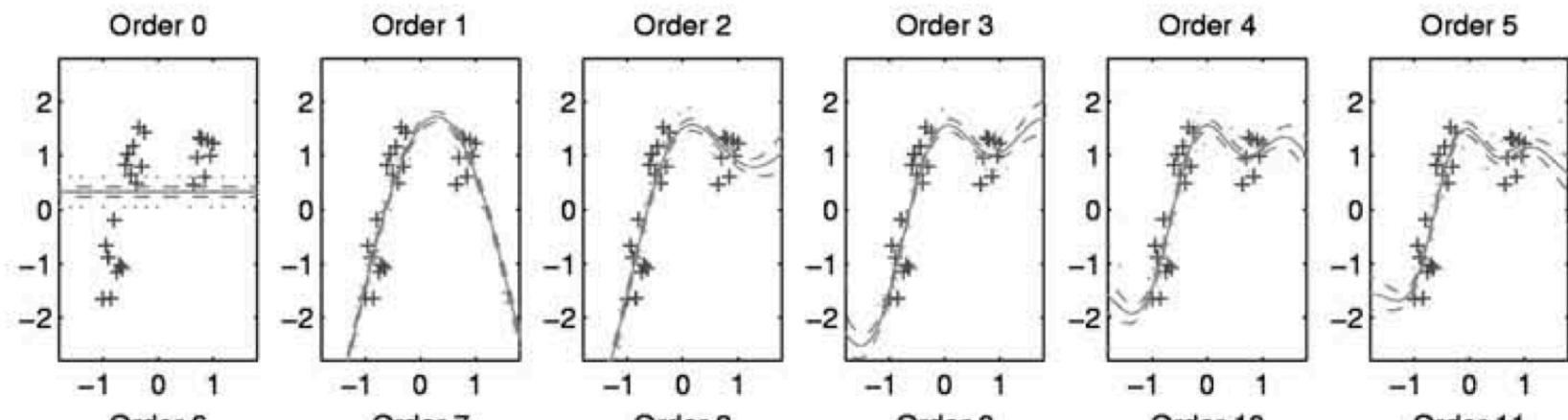
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See *Occam's Razor* (Rasmussen & Ghahramani, 2000). One of my favourite papers.





Bayes selects a nonparametric model!

- Bayes itself is pushing us to use “large” nonparametric models!
- Cannot rely on Bayes to choose a “small” model!

1. What is wrong with minimising losses?
2. Bayesian Model Selection
2. Model Selection over Model Size? Or Nonparametrics?
3. **A Principle for Selecting Model Size**



**What principle can determine a compressed model size,
without removing the benefits of
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**Define a nonparametric model,
then approximate it with $M < N$ basis funcs.**

Variational GP approximation

Step 1: Introduce family of approximate predictors

$$q(f(x)) = \mathcal{N} \left(f(x); \sum_{m=1}^M \varphi(x; Z_m, \theta) w_m, \dots \right)$$

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Since $\text{KL} > 0$... $\text{ELBO} \leq \log p(\mathcal{D}|\theta)$.

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② ELBO is a *unified objective* for all our questions!

- Optimising w.r.t. \mathbf{w}, Z : finds weights (min KL)
- Optimising w.r.t. θ : finds hyperparameters (max $\log p(\mathcal{D}|\theta)$)
- Select M large enough, that more gives diminishing returns!

When Should we Stop Adding Basis Functions?

More basis functions is always better:

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- We select M such that

$$\text{EUBO} - \text{ELBO} \leq \text{tolerance}$$

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- In single-layer models, we can also compute an upper bound to the marginal likelihood

$$\text{ELBO} \leq \log p(D|\theta) \leq \text{EUBO}$$

$$\therefore \text{KL}[q(f) \parallel p(f|\mathcal{D}, \theta)] \leq \text{EUBO} - \text{ELBO}$$

- We select M such that

$$\text{EUBO} - \text{ELBO} \leq \text{tolerance}$$

- Can achieve arbitrarily exact approximation with $M \ll N!$
(Burt et al., 2019; 2020)

When Should we Stop Adding Basis Functions?

More basis functions is always better:

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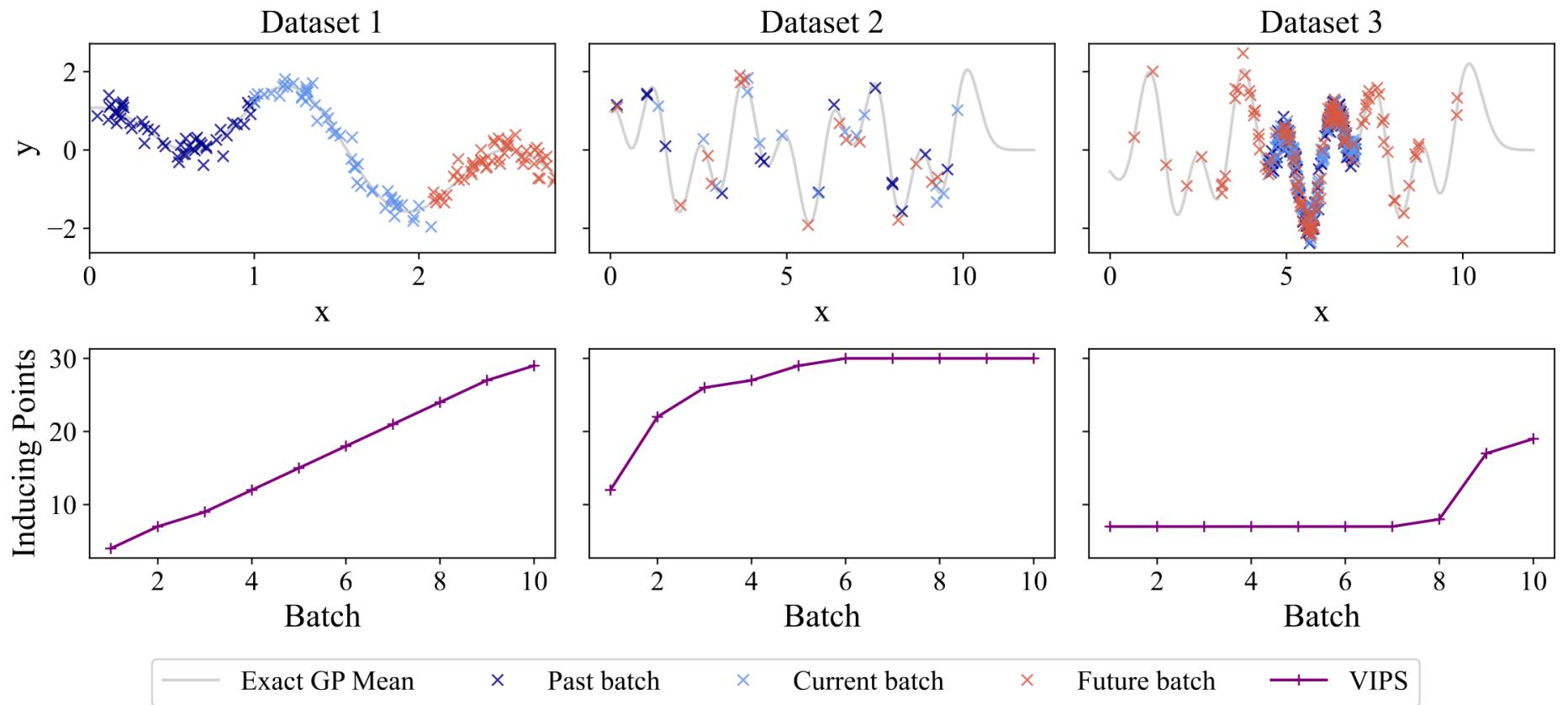
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 **Simple Rule, Interesting Adaptive Behaviour!**

Continual Learning (Pescador-Barrios et al., 2024)



Growth of neurons depends on *novelty* in data.

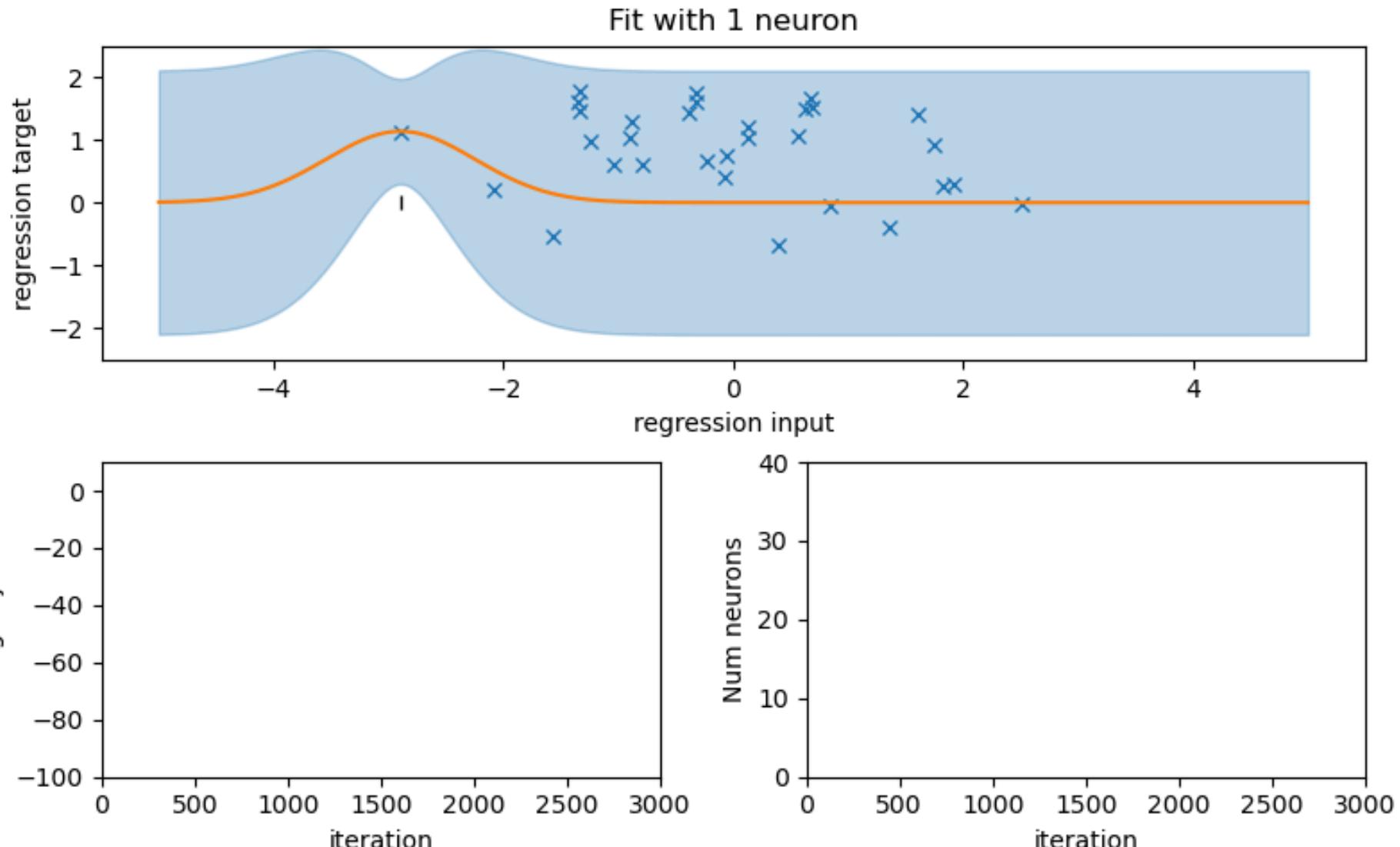
- Input range grows with N (constant novelty)
- Input range constant (diminishing novelty)
- Heavy tailed inputs (occasional novelty)

Growing Neurons, Grokking, Pruning

Number of neurons depends on inductive bias!

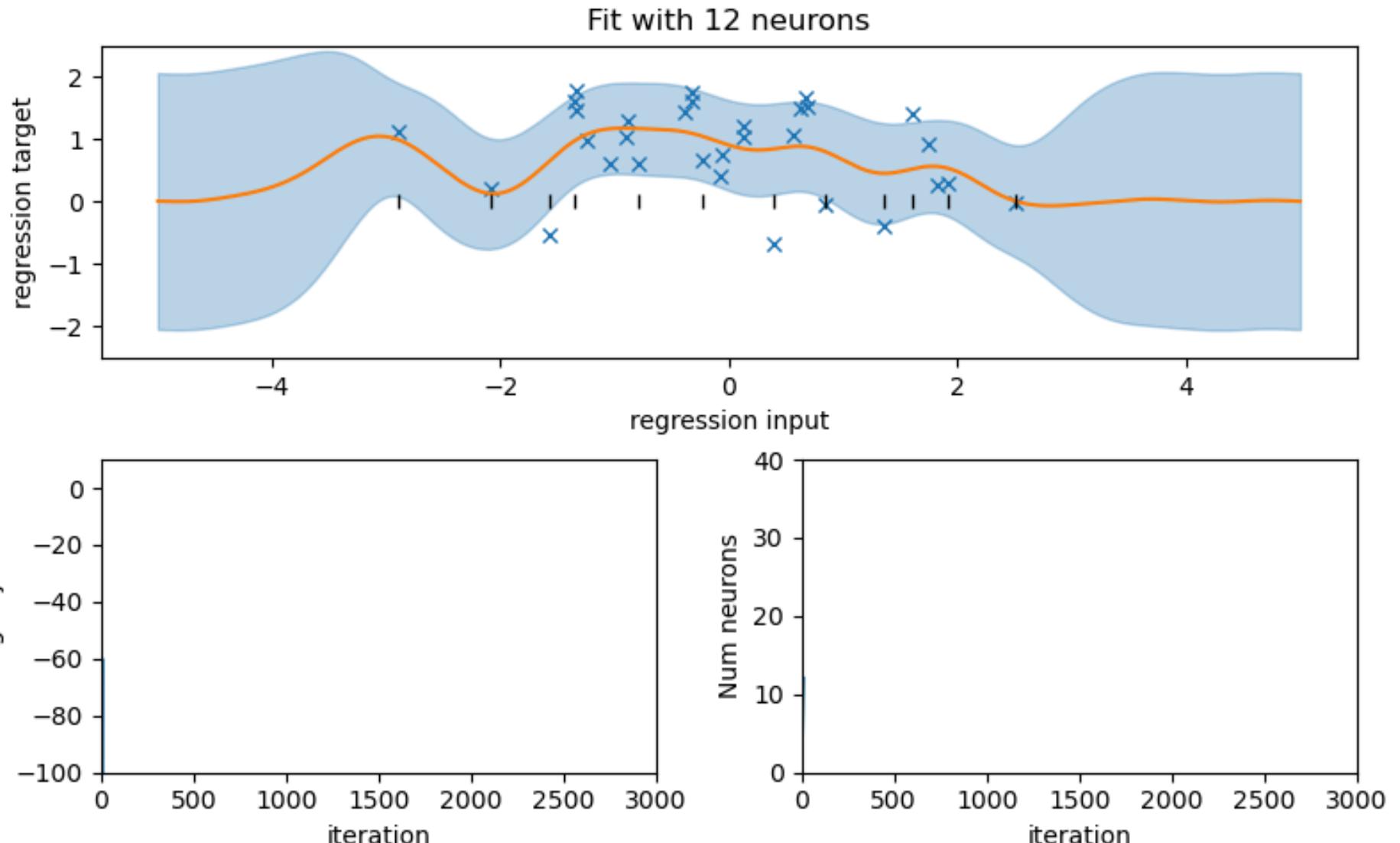
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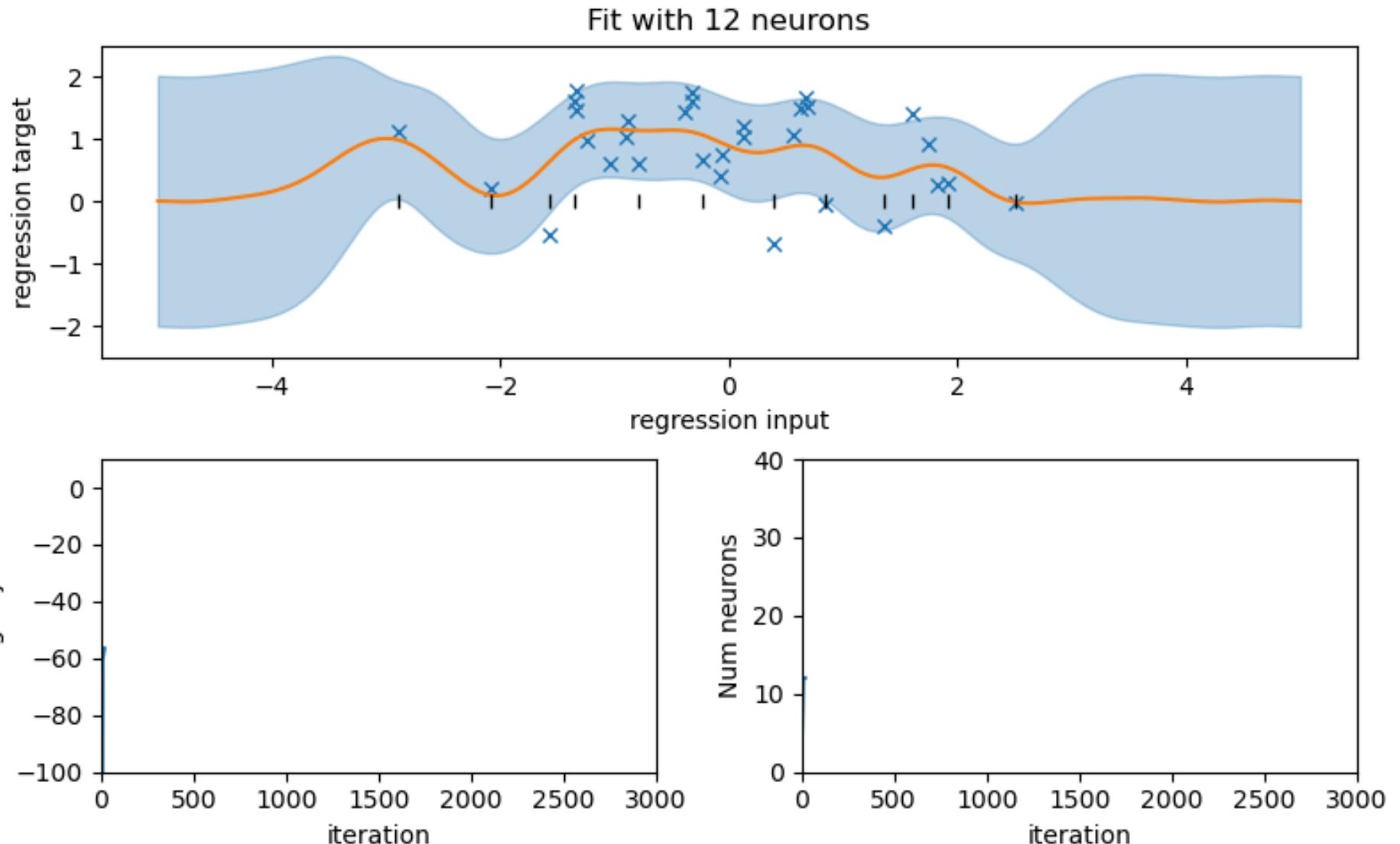
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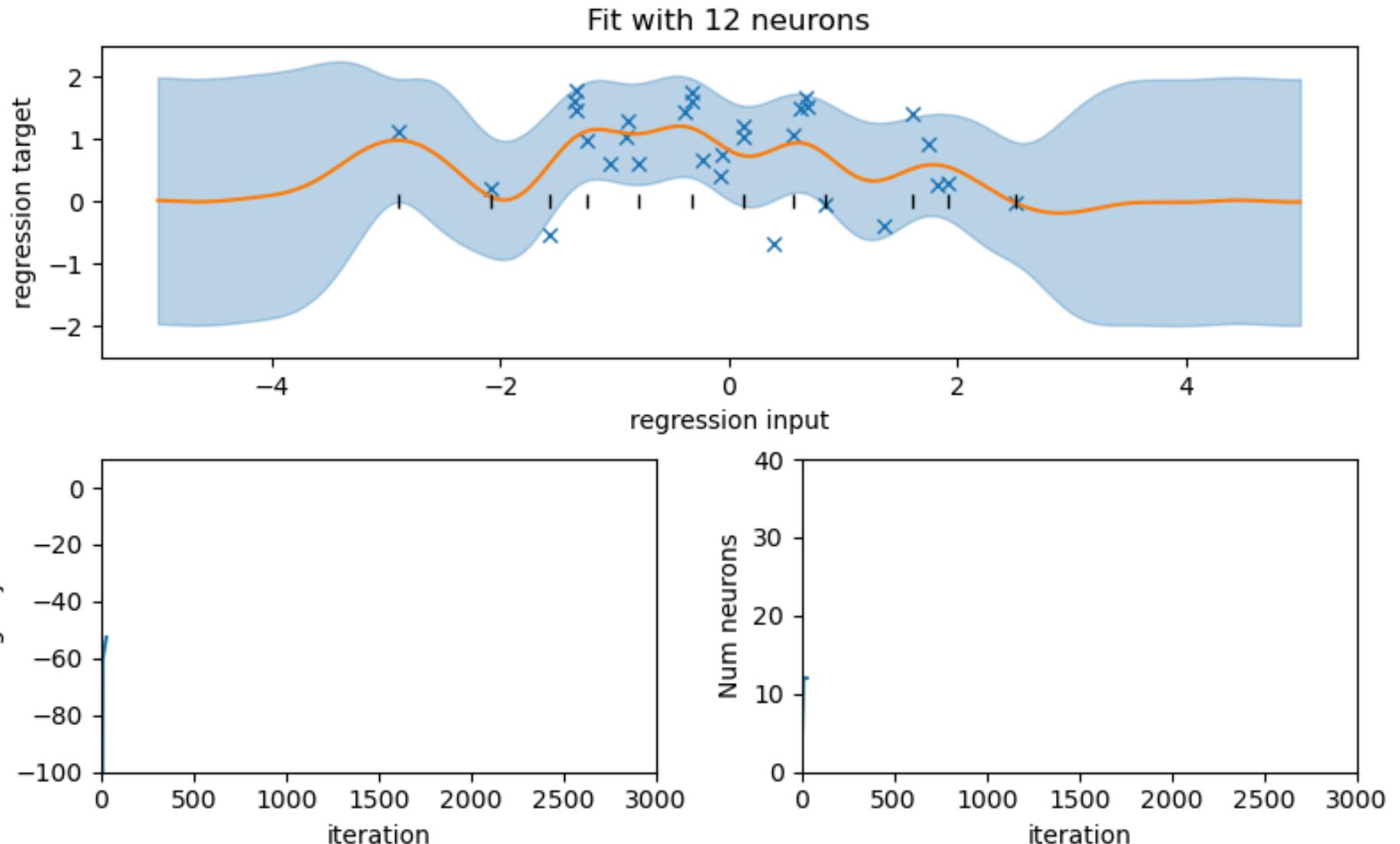
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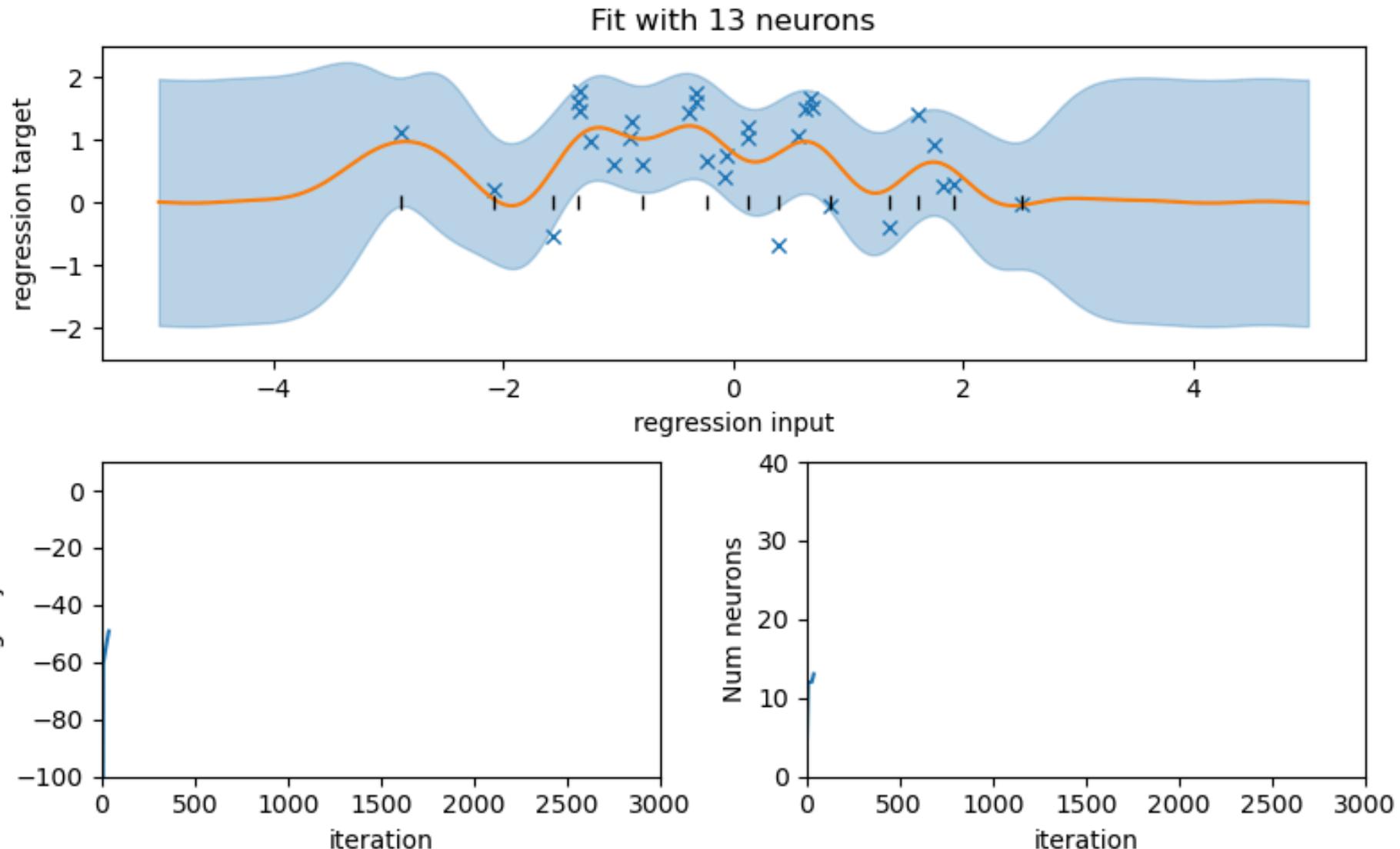
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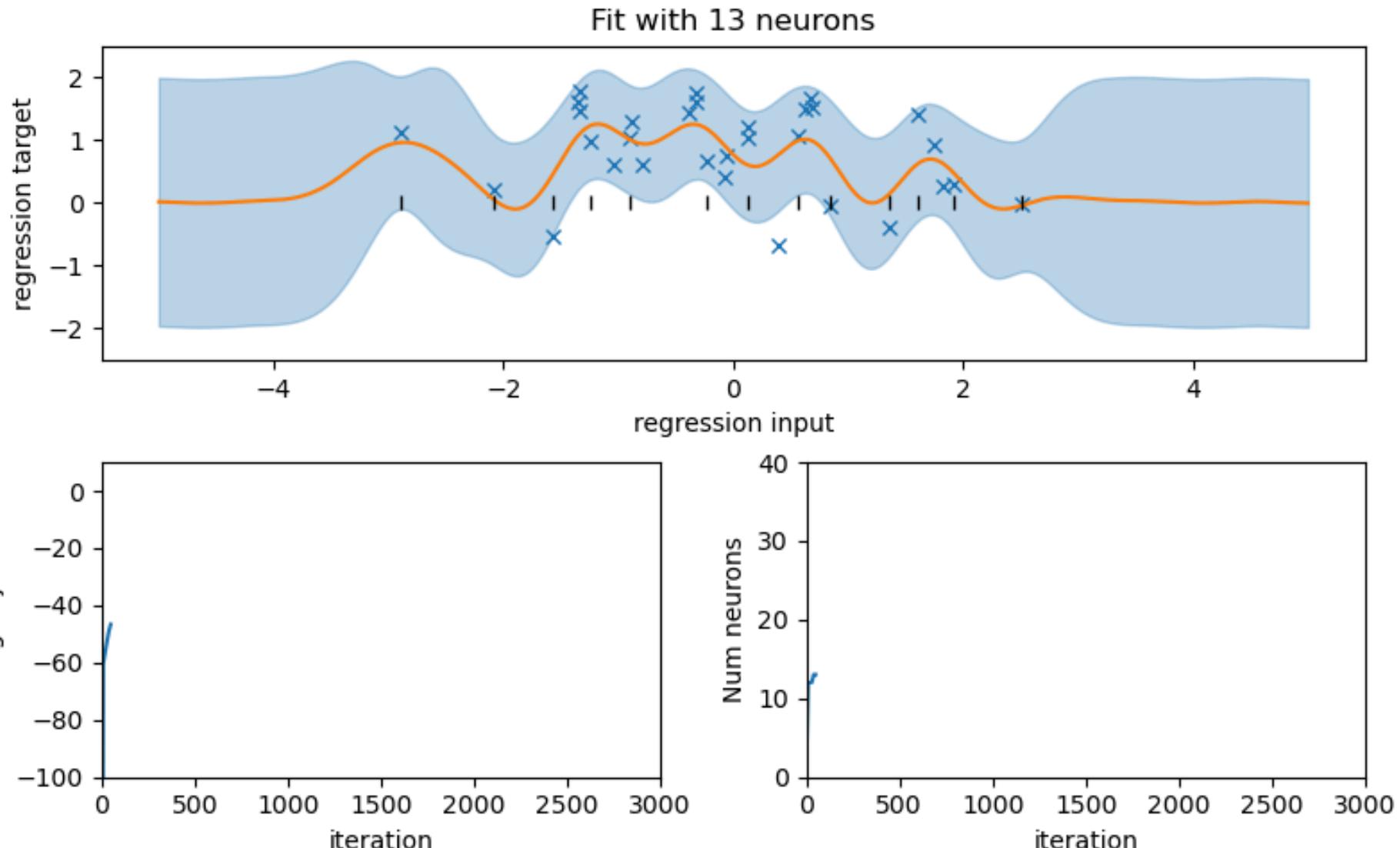
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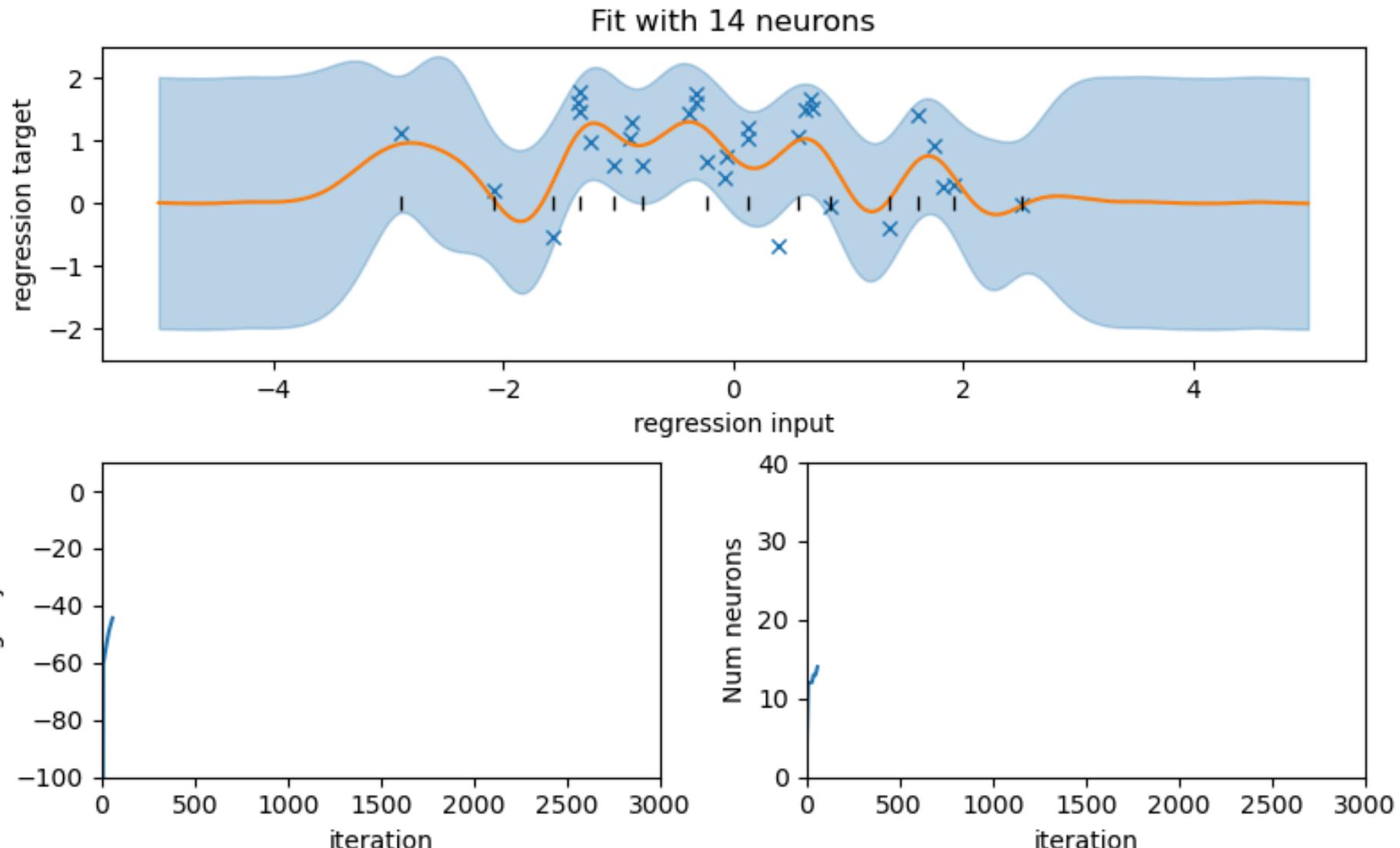
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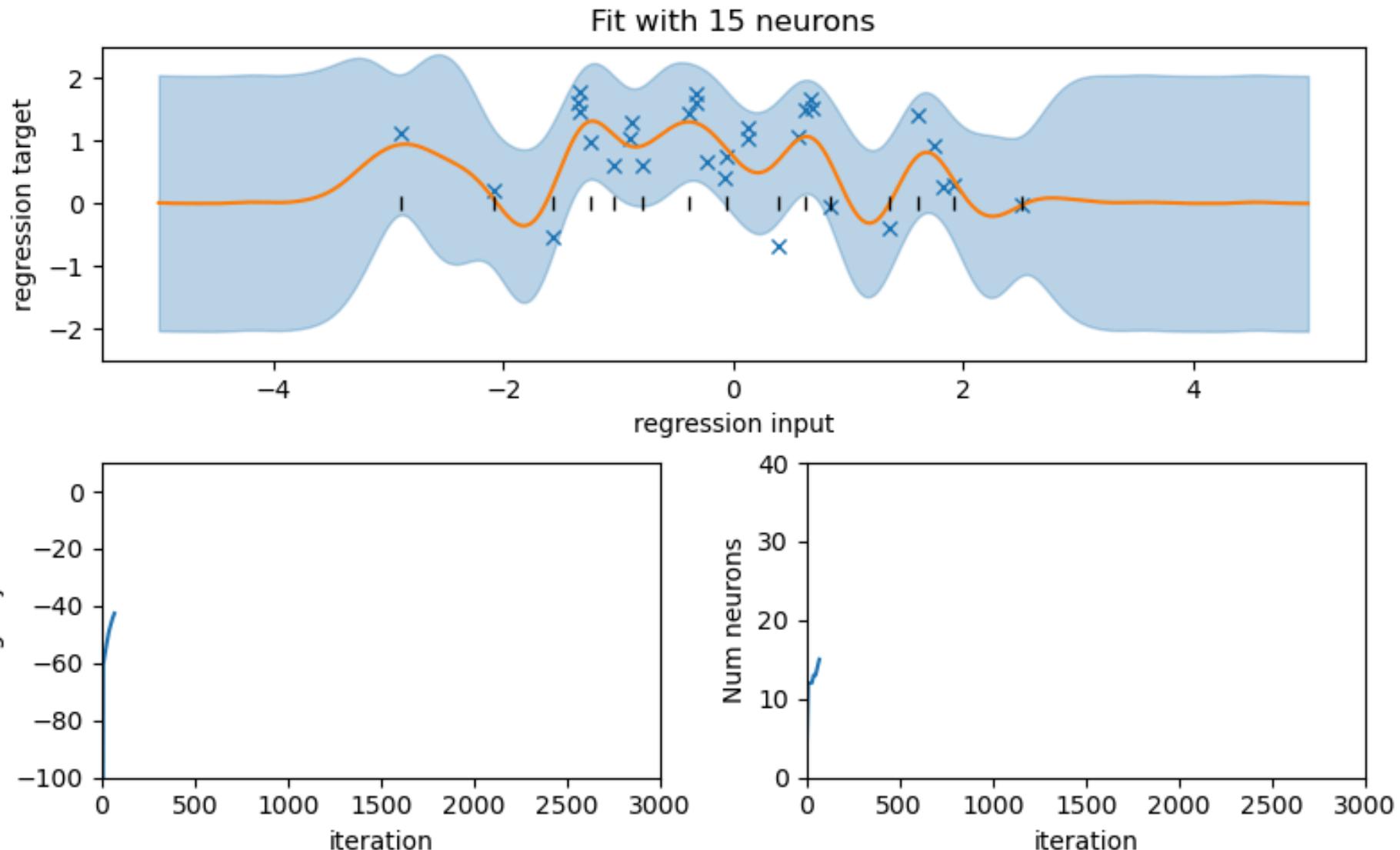
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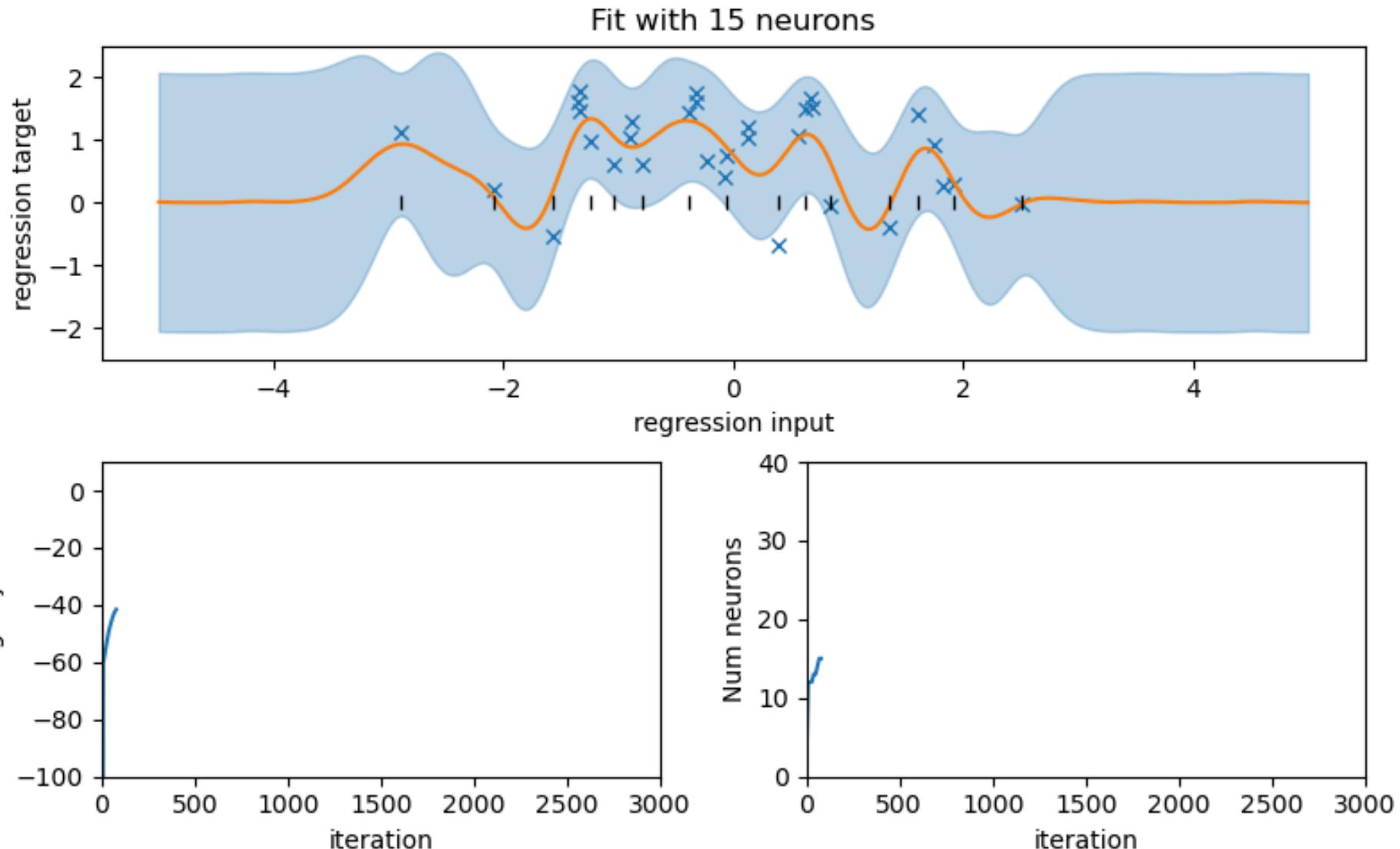
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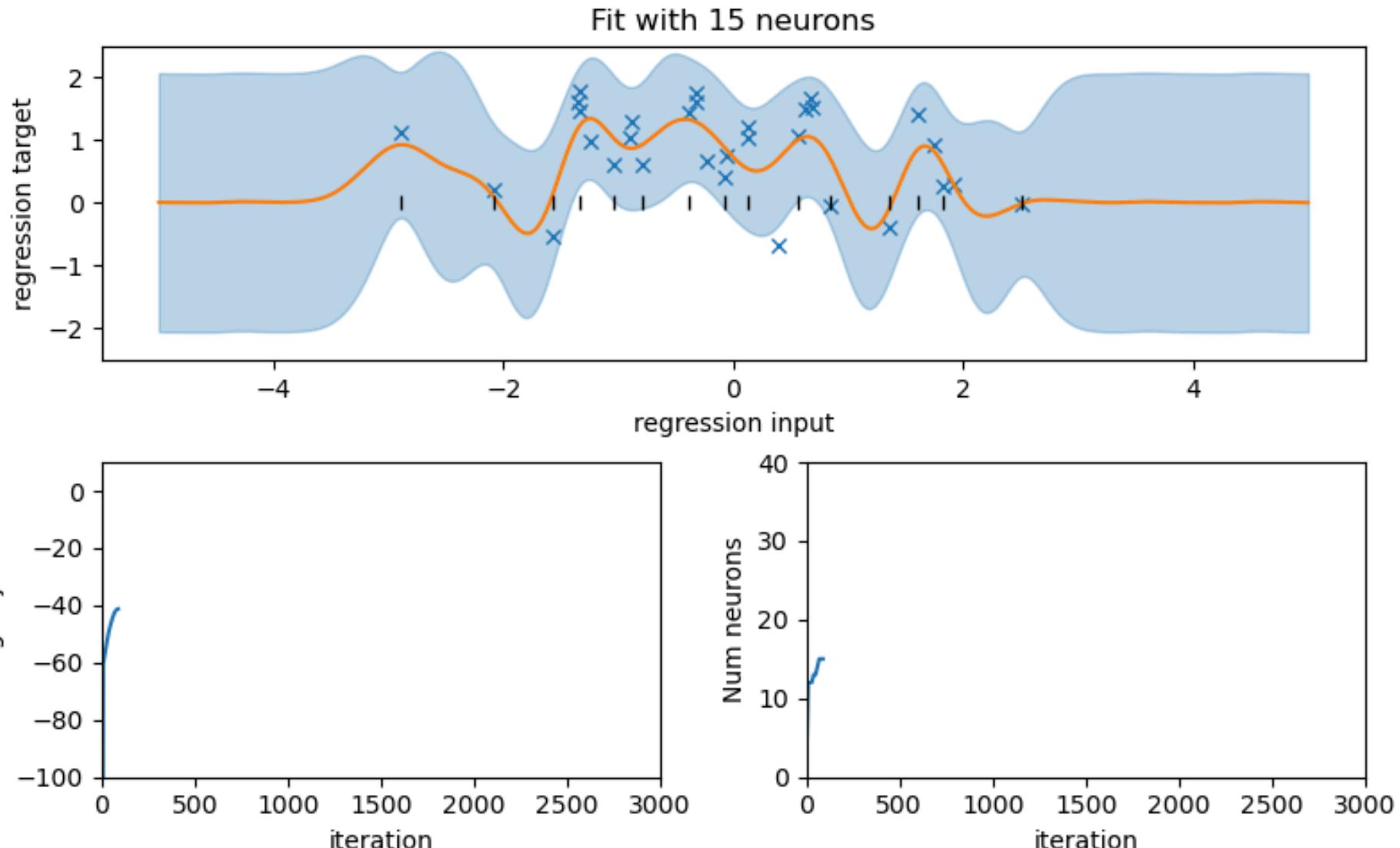
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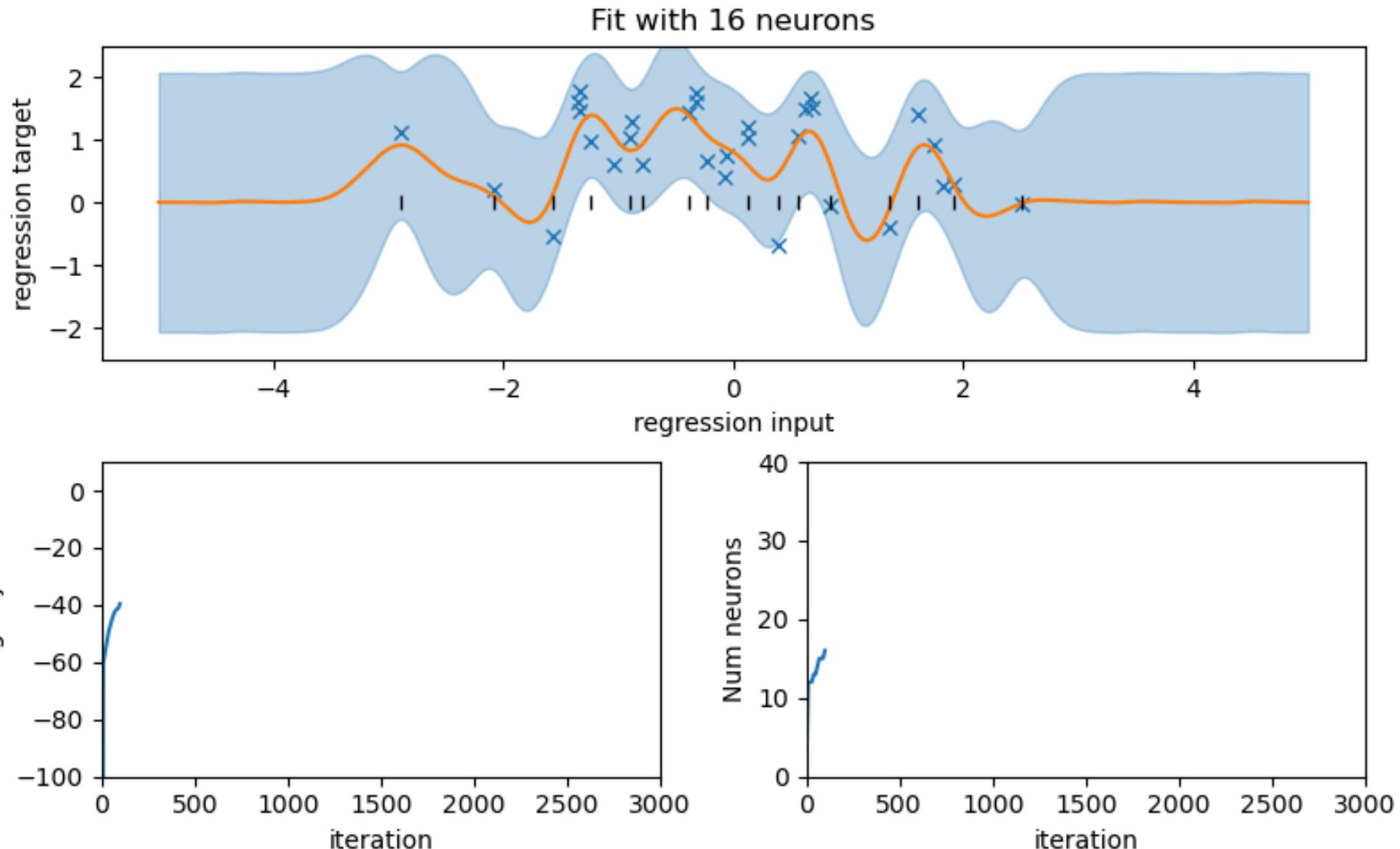
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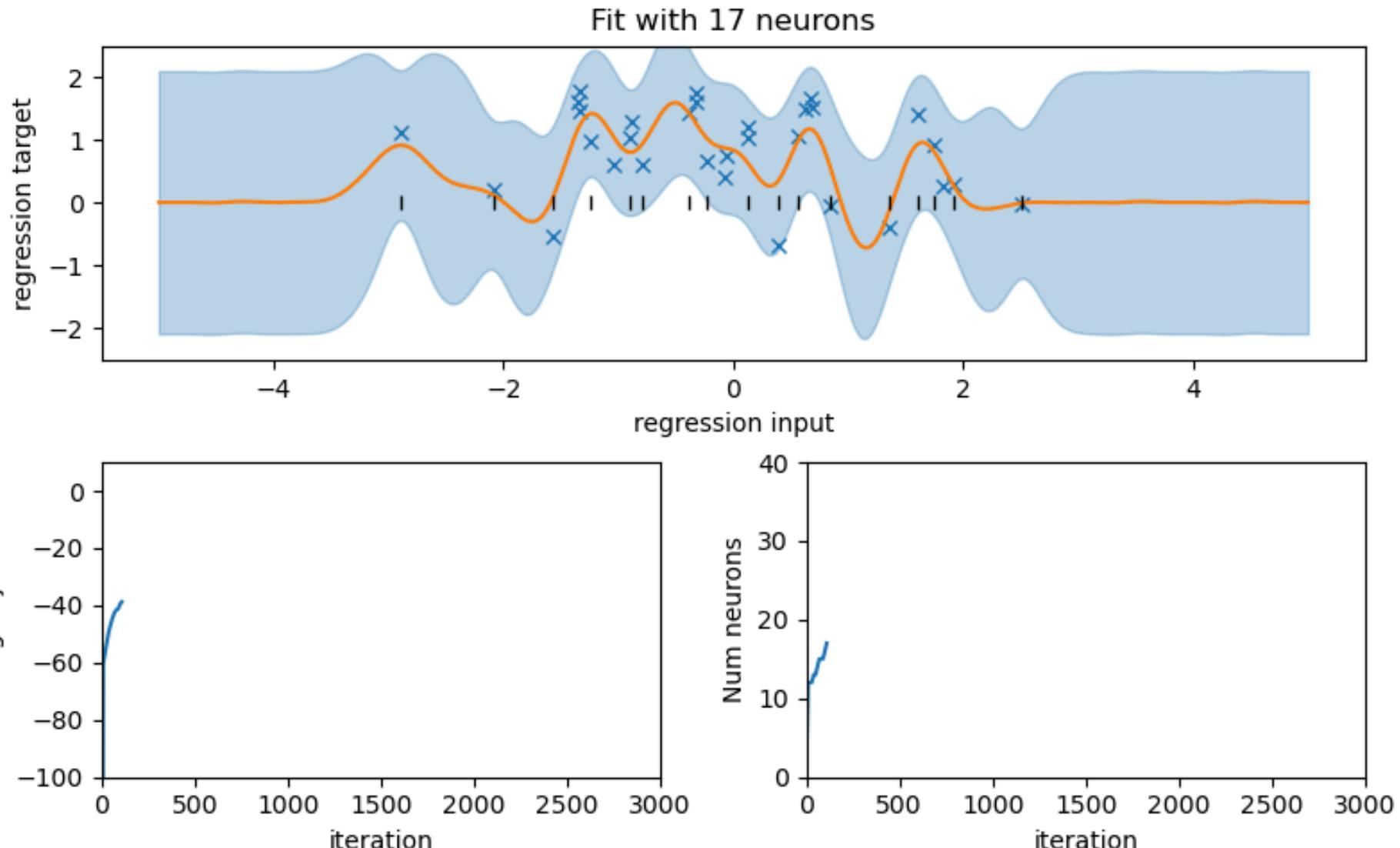
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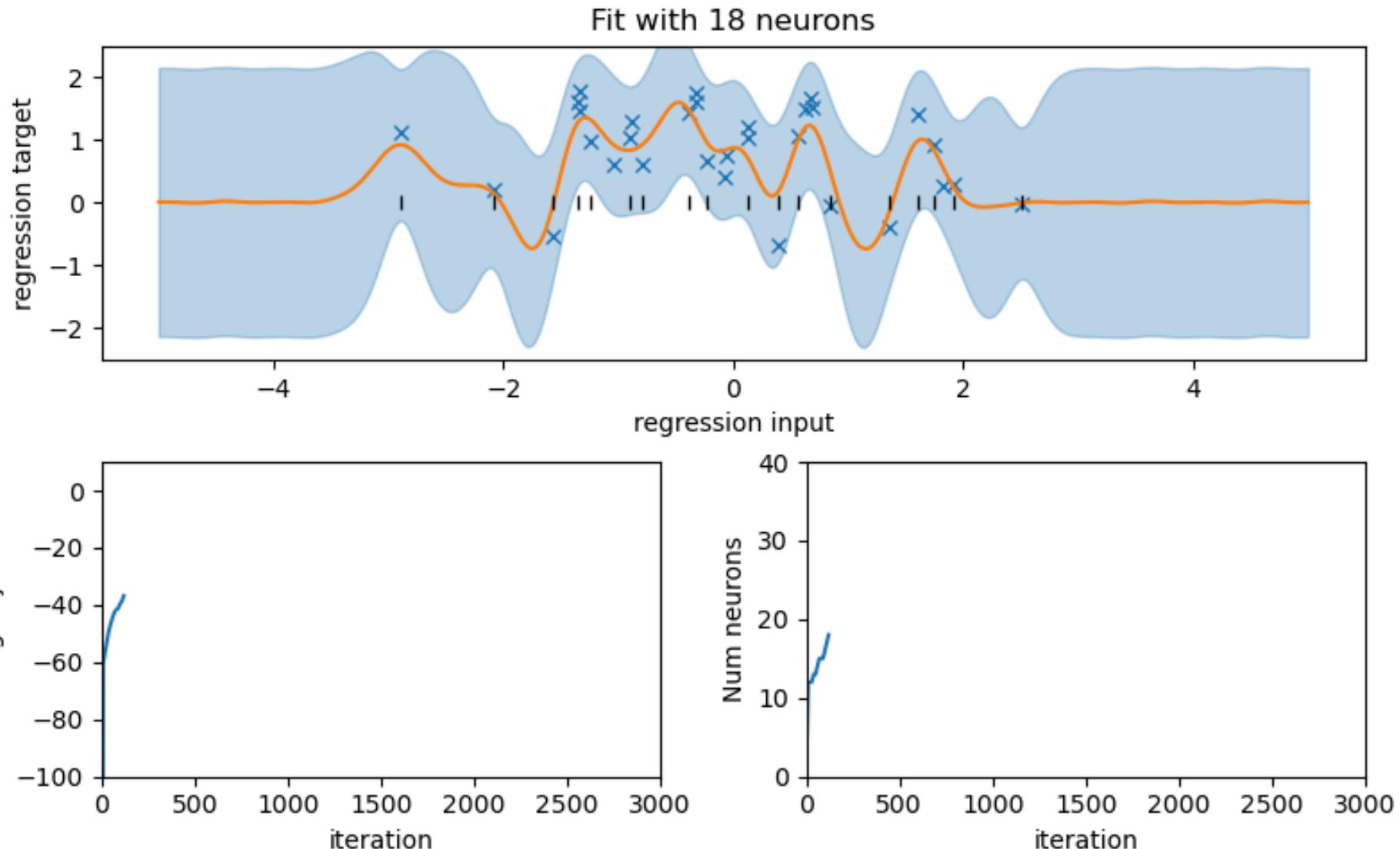
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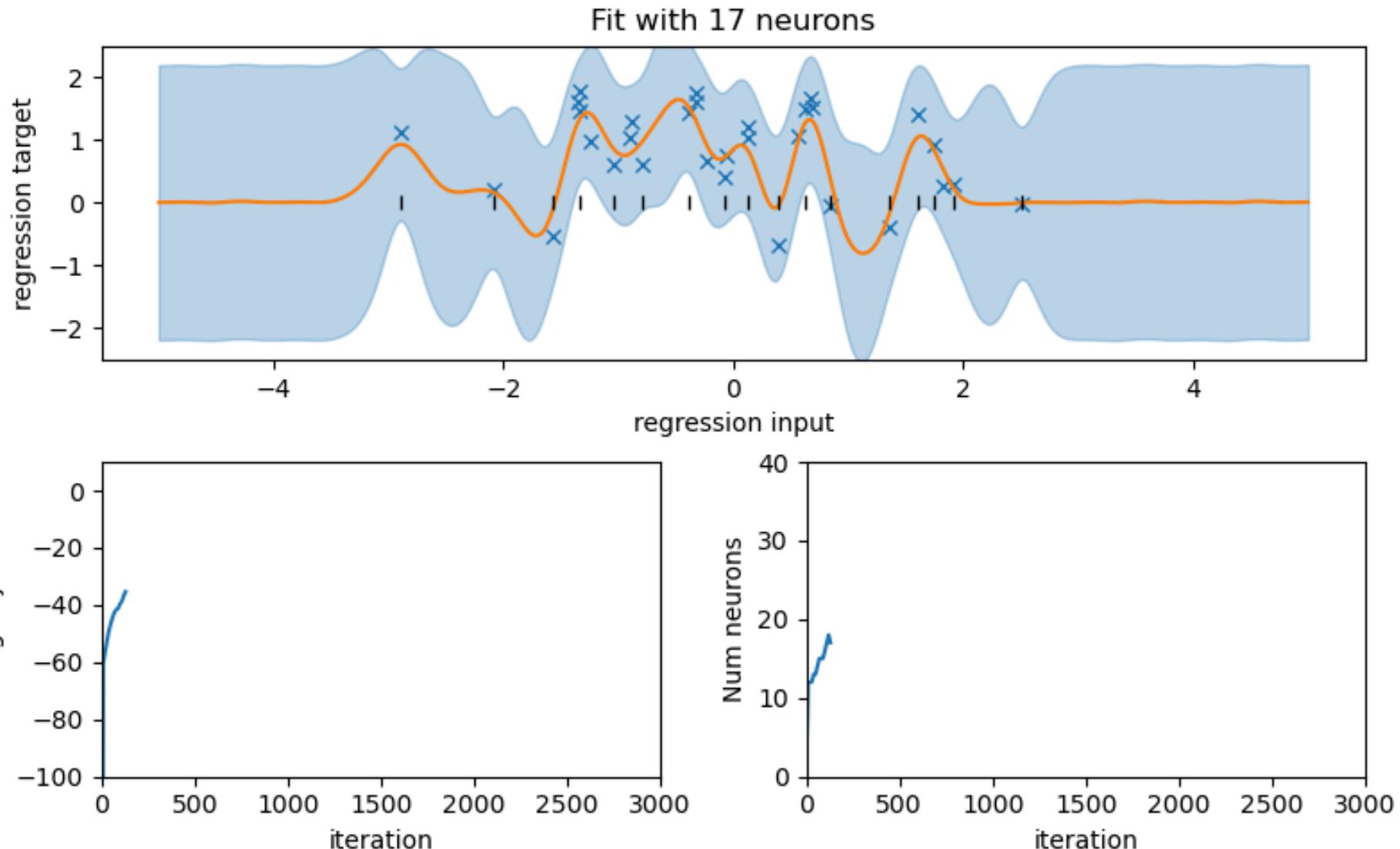
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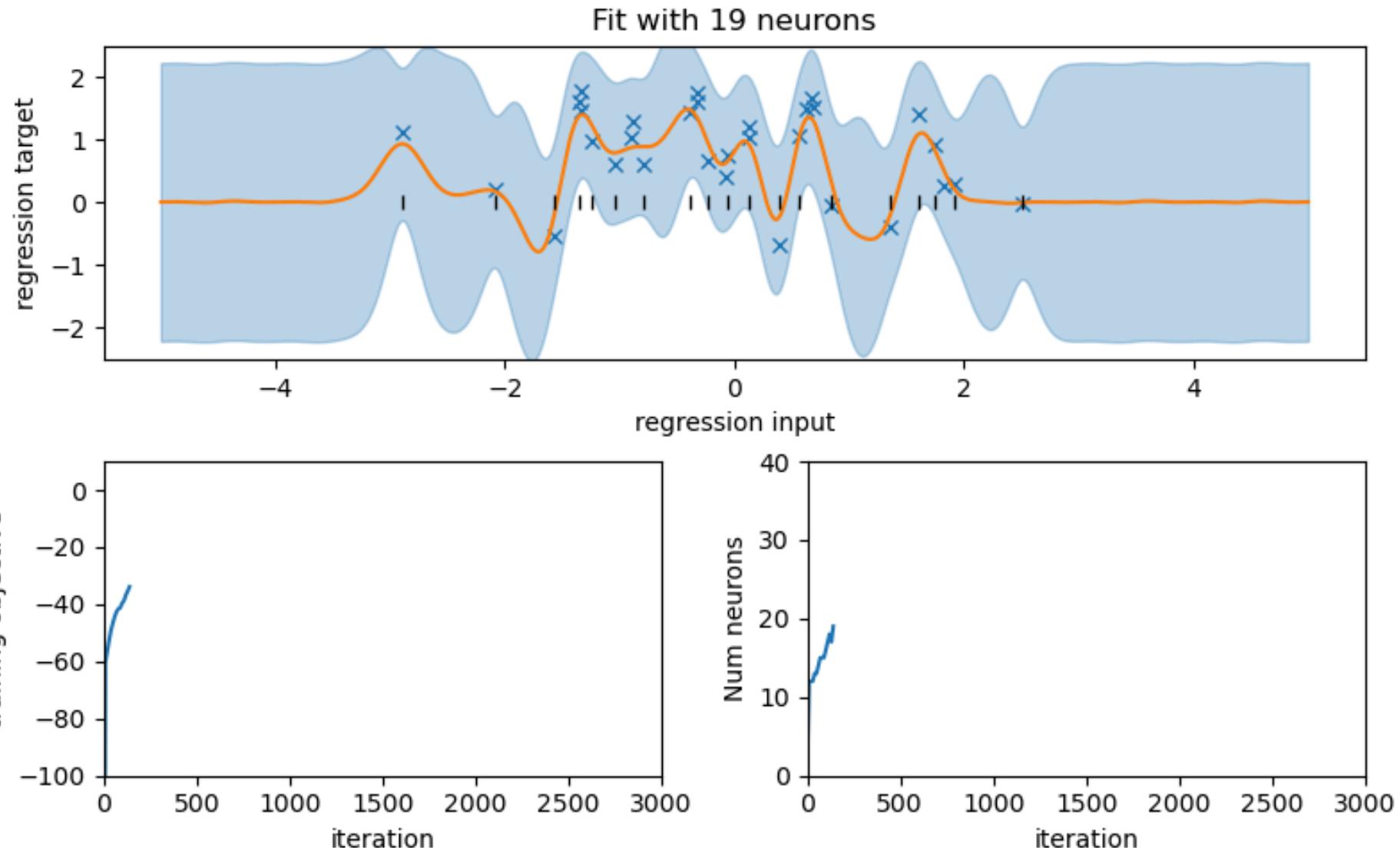
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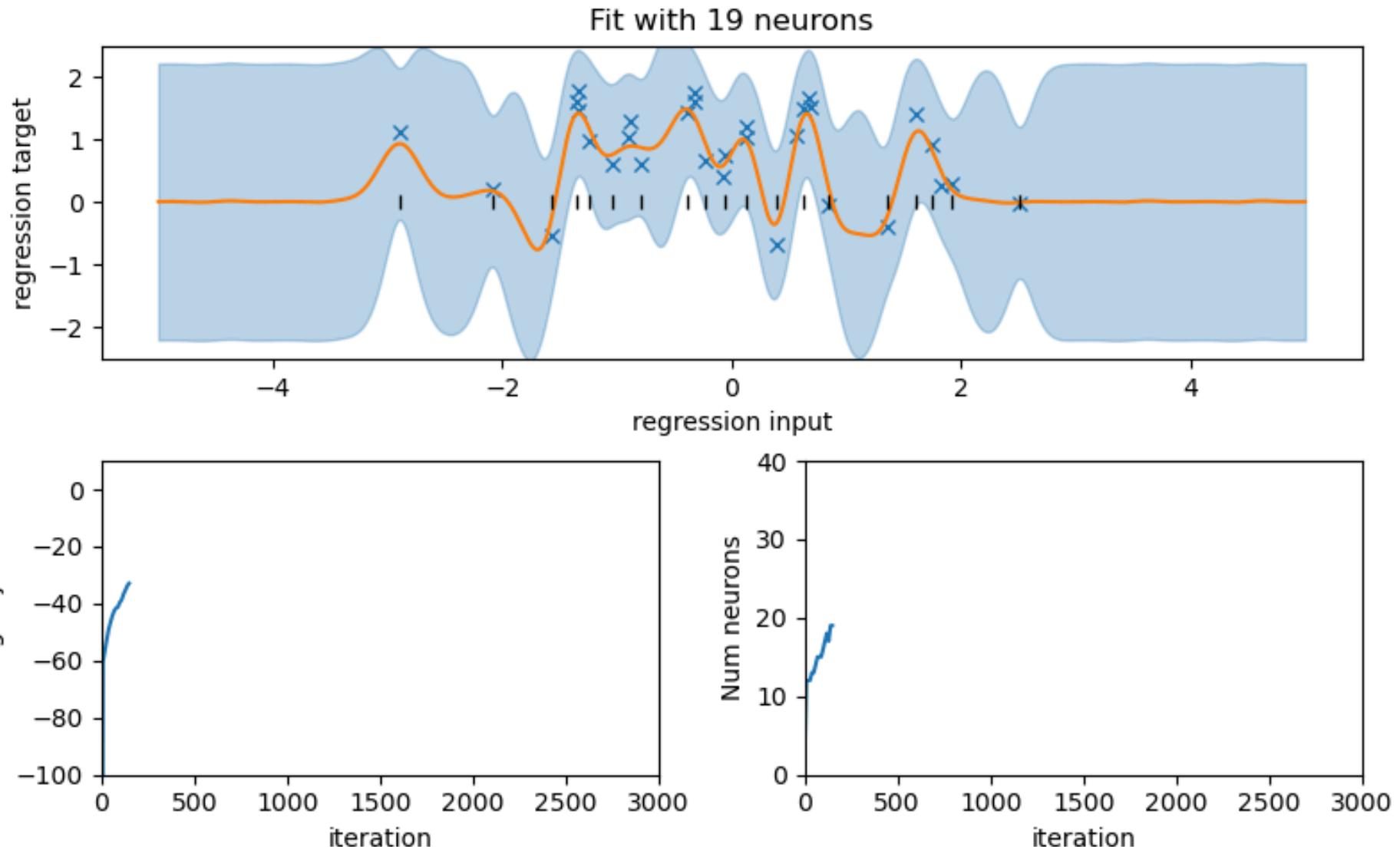
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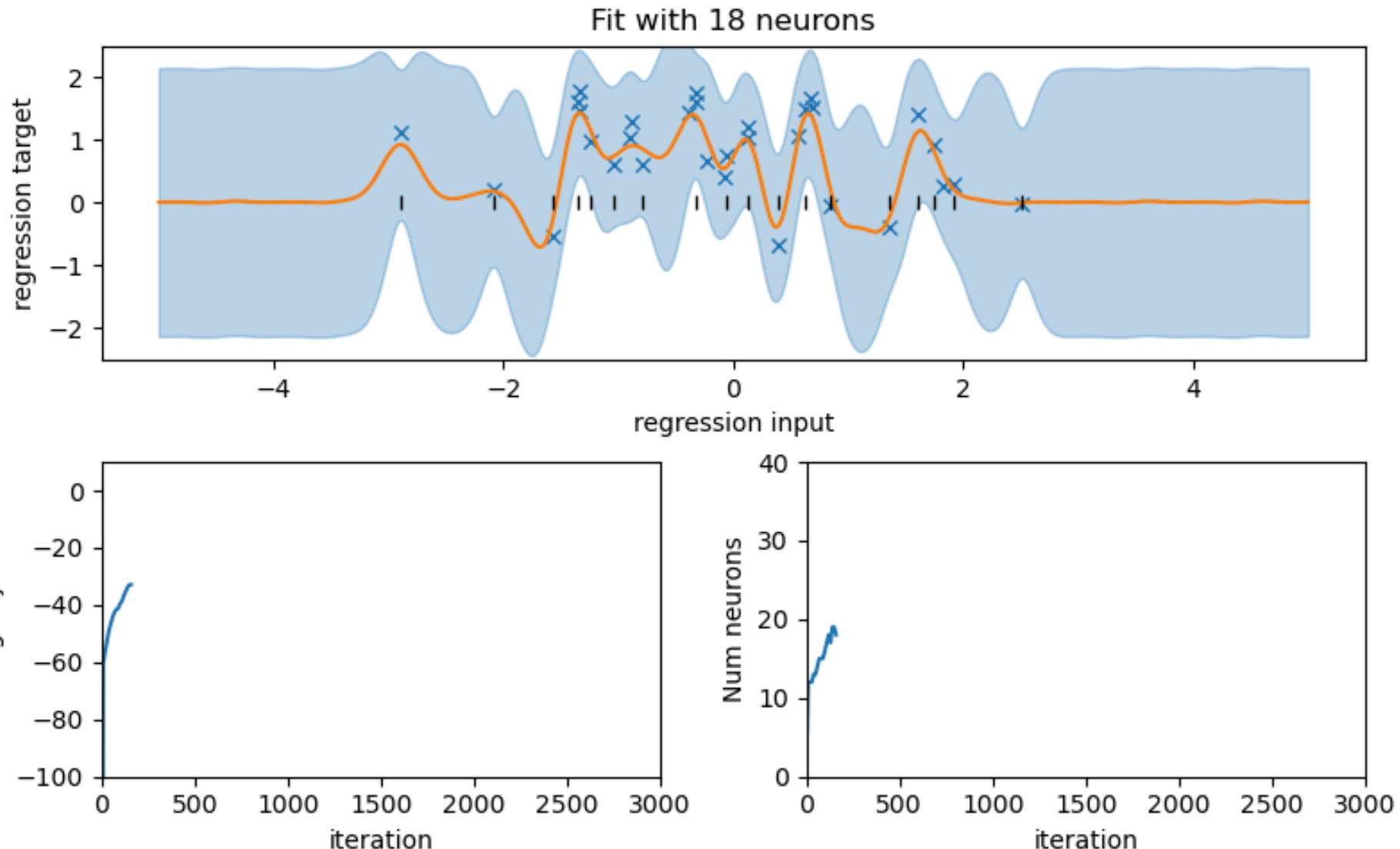
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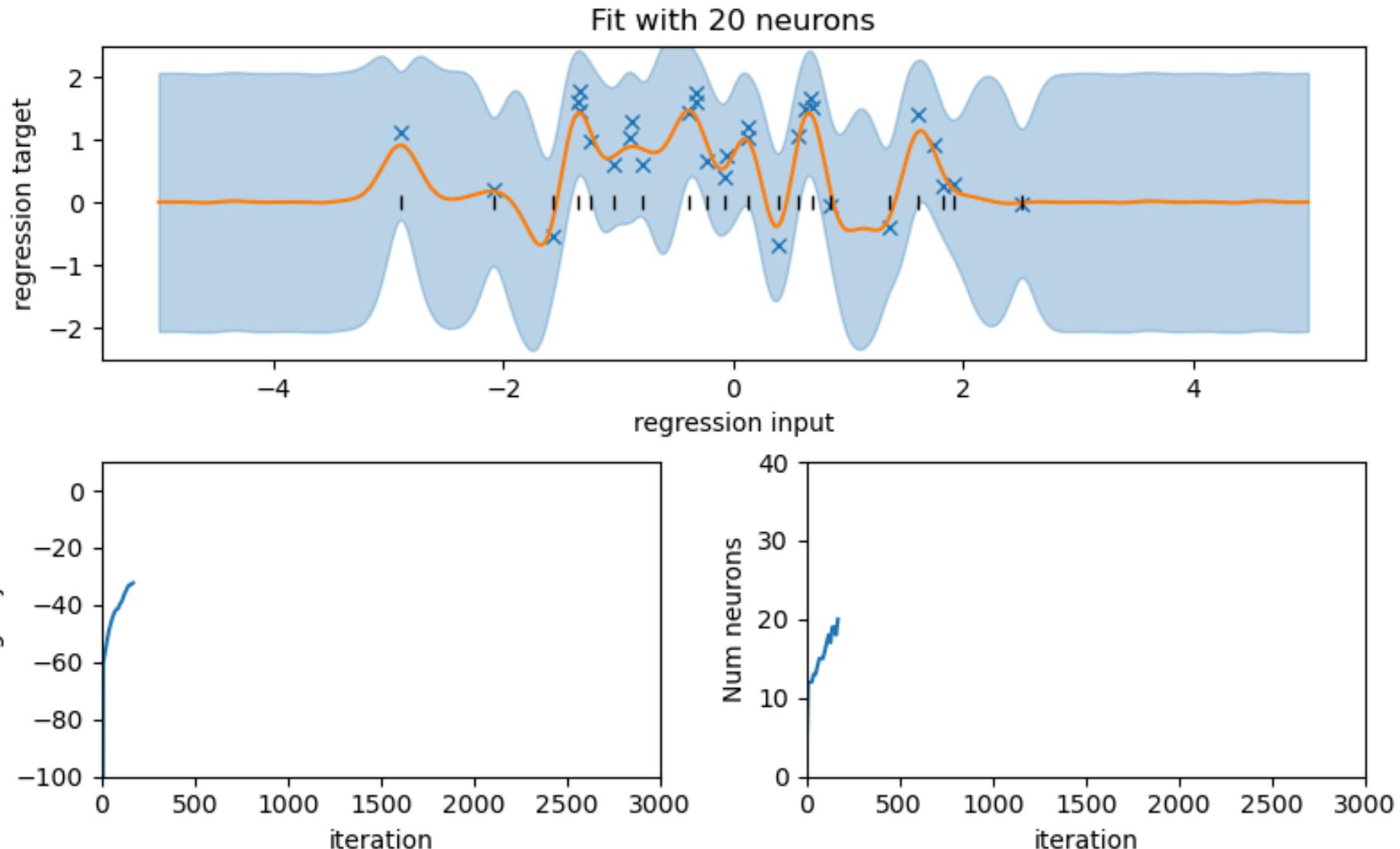
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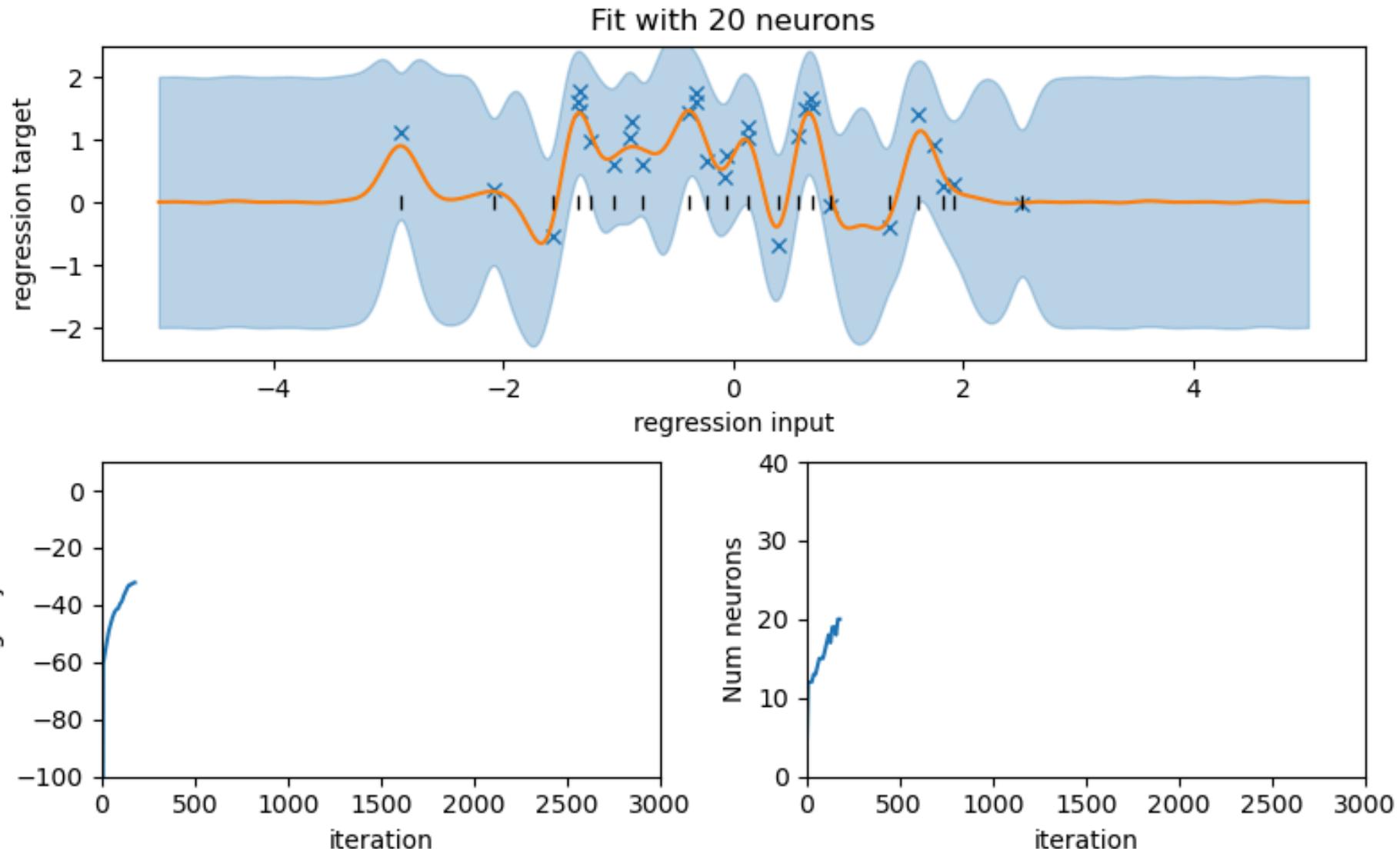
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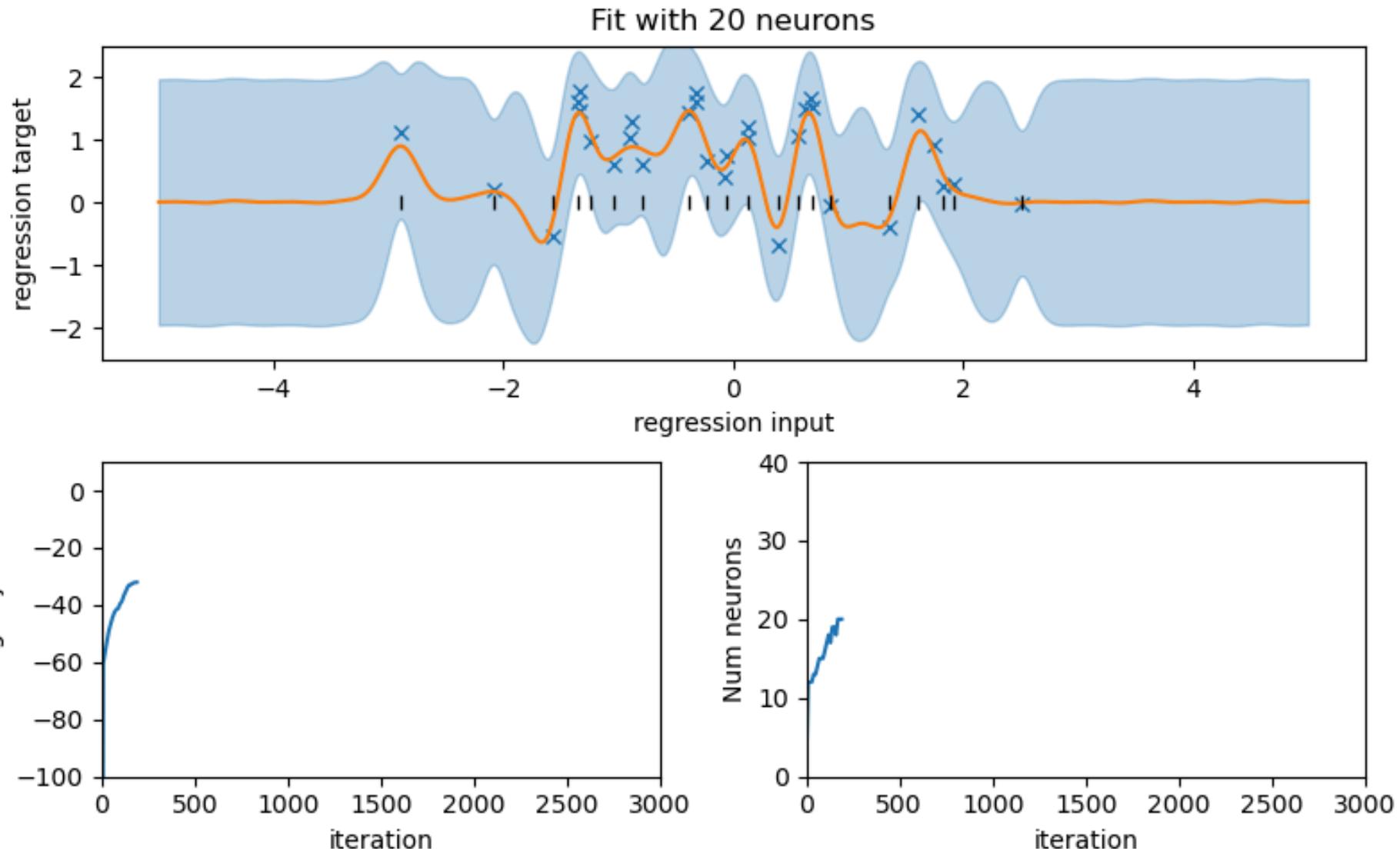
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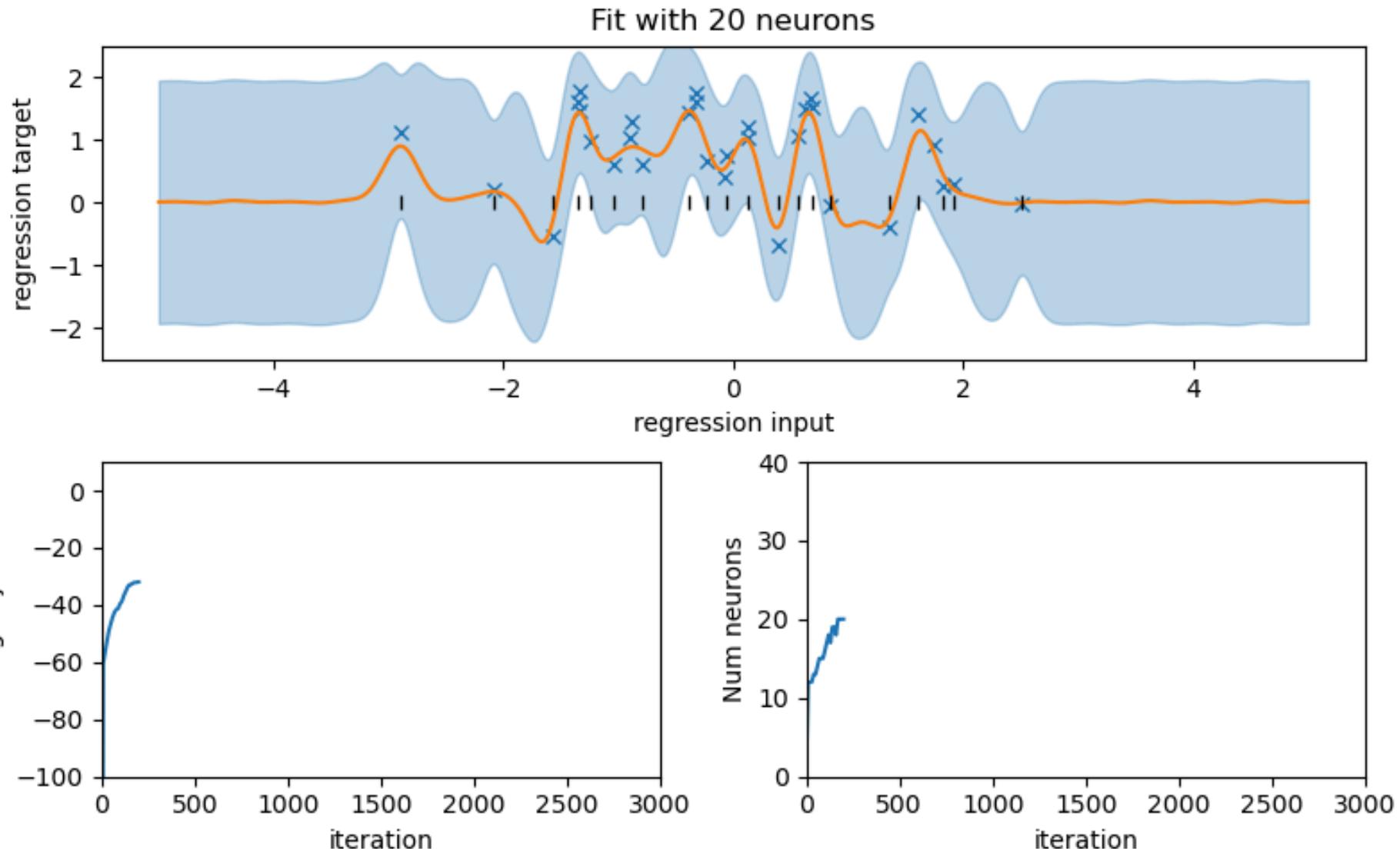
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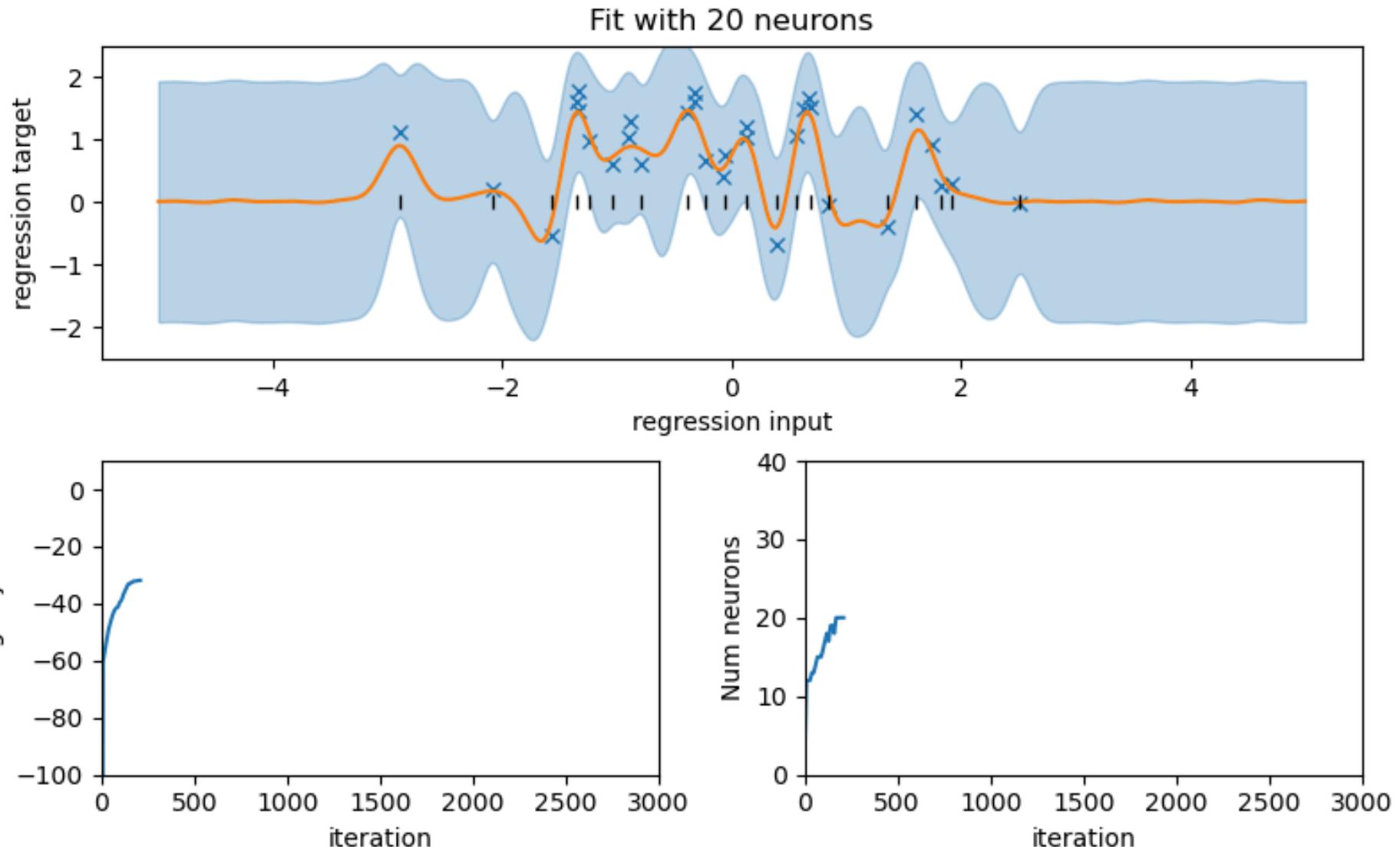
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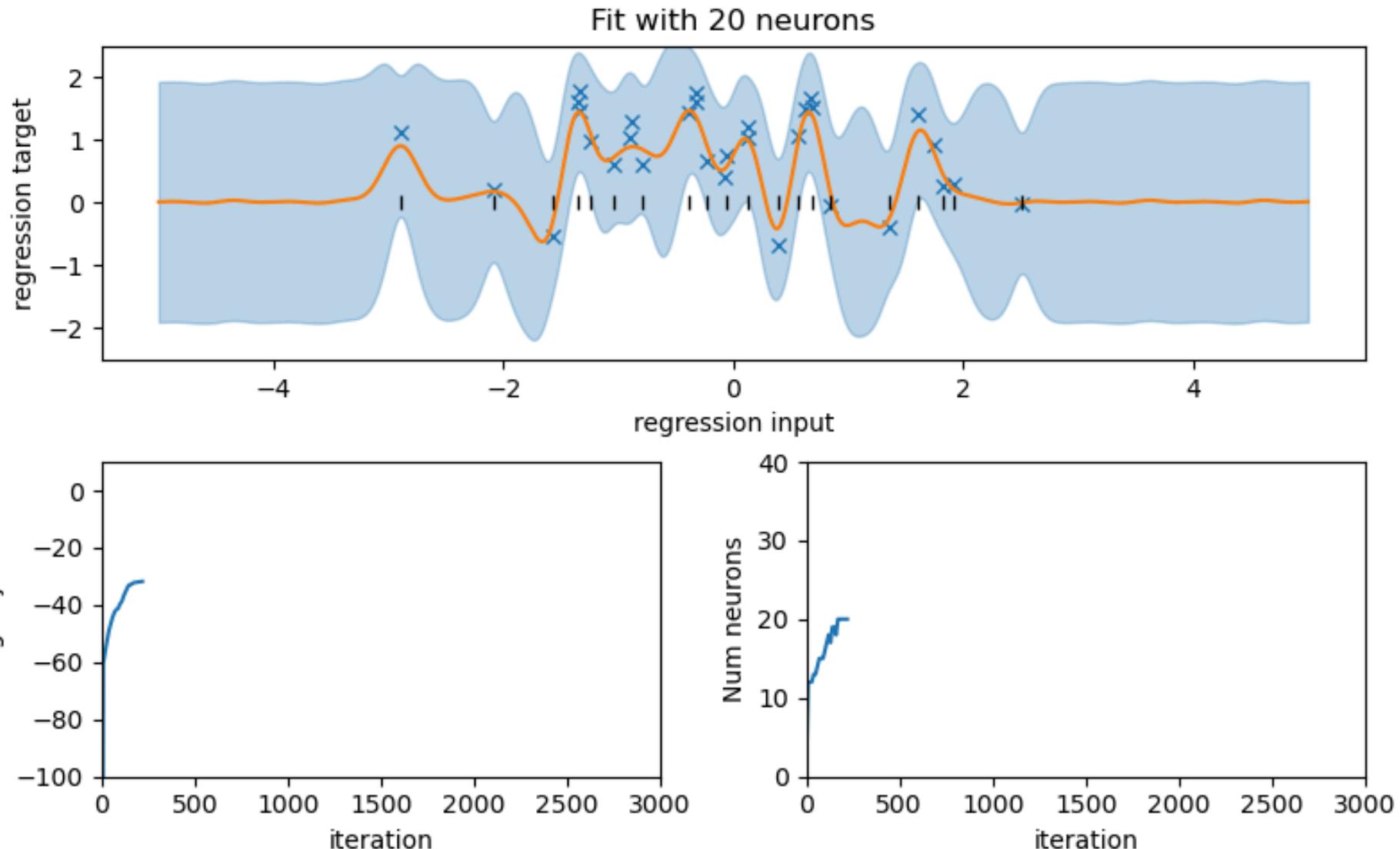
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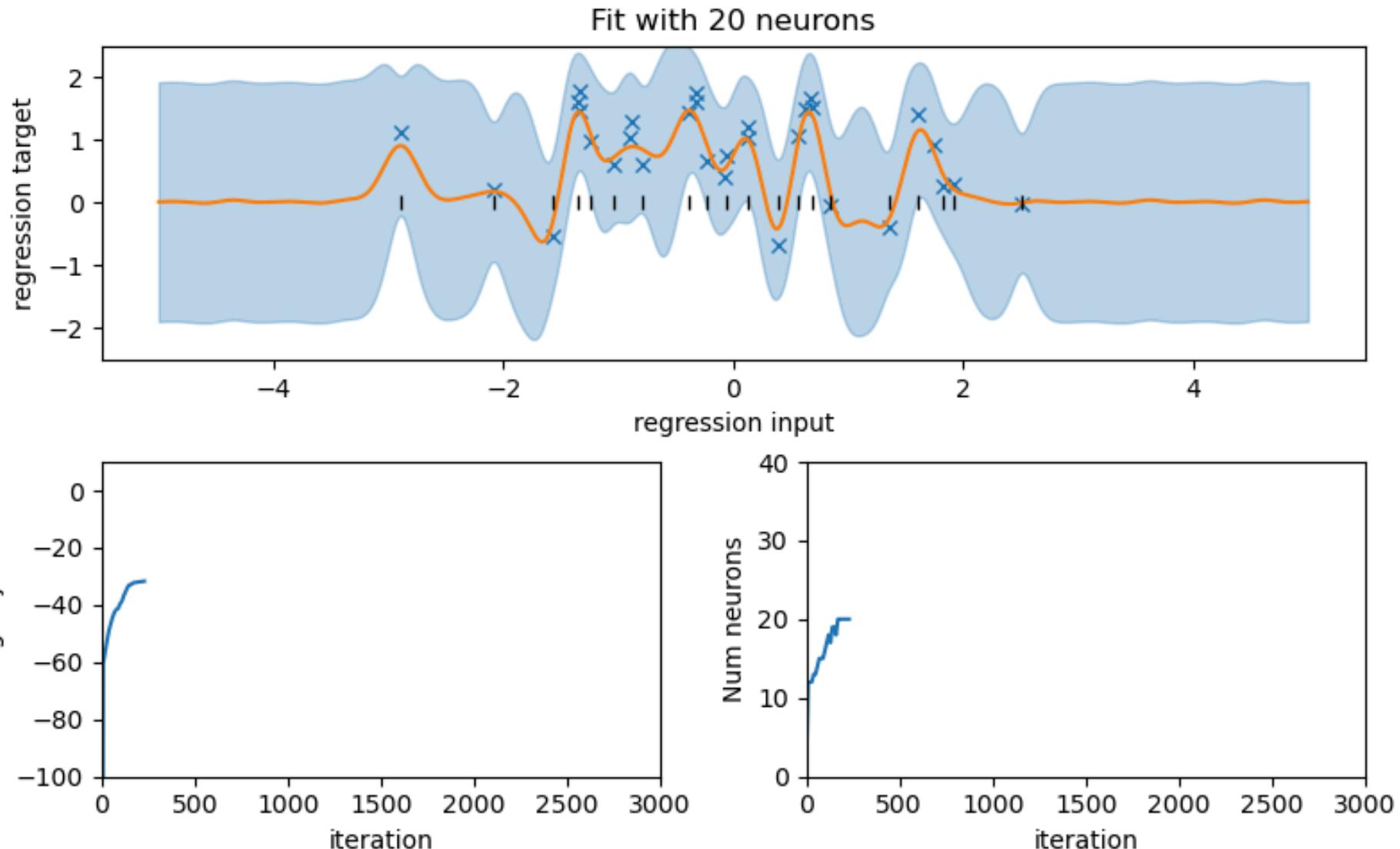
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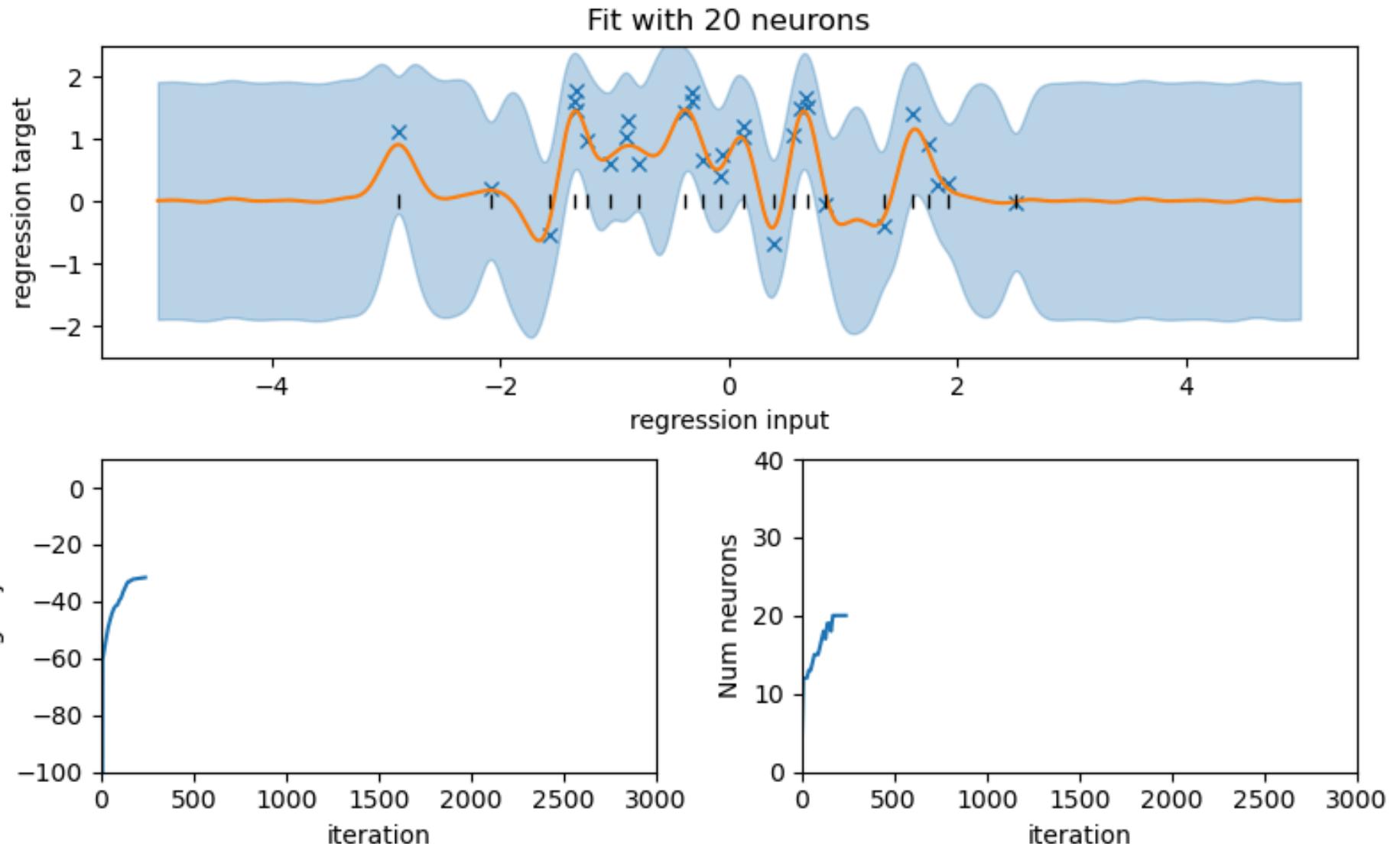
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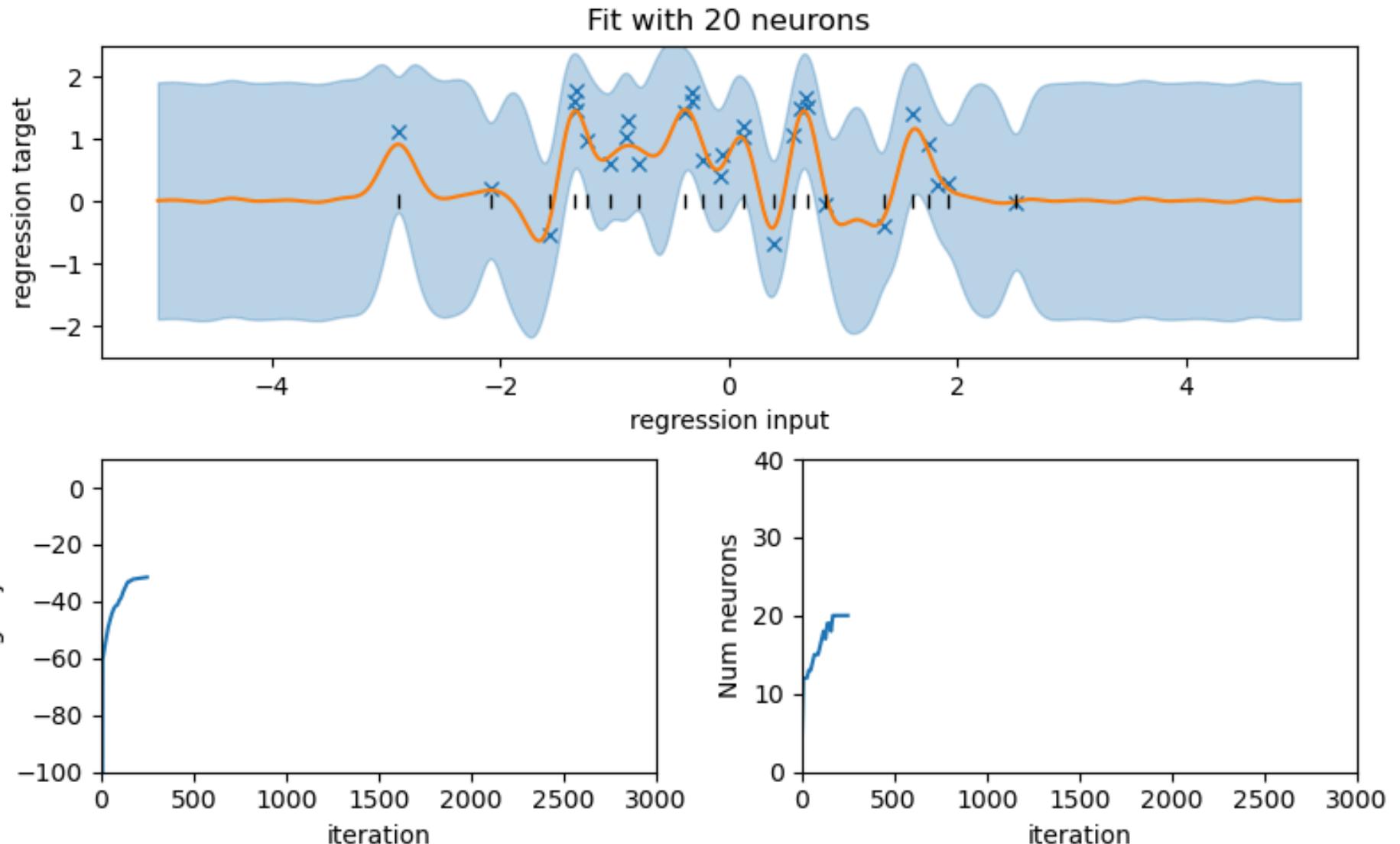
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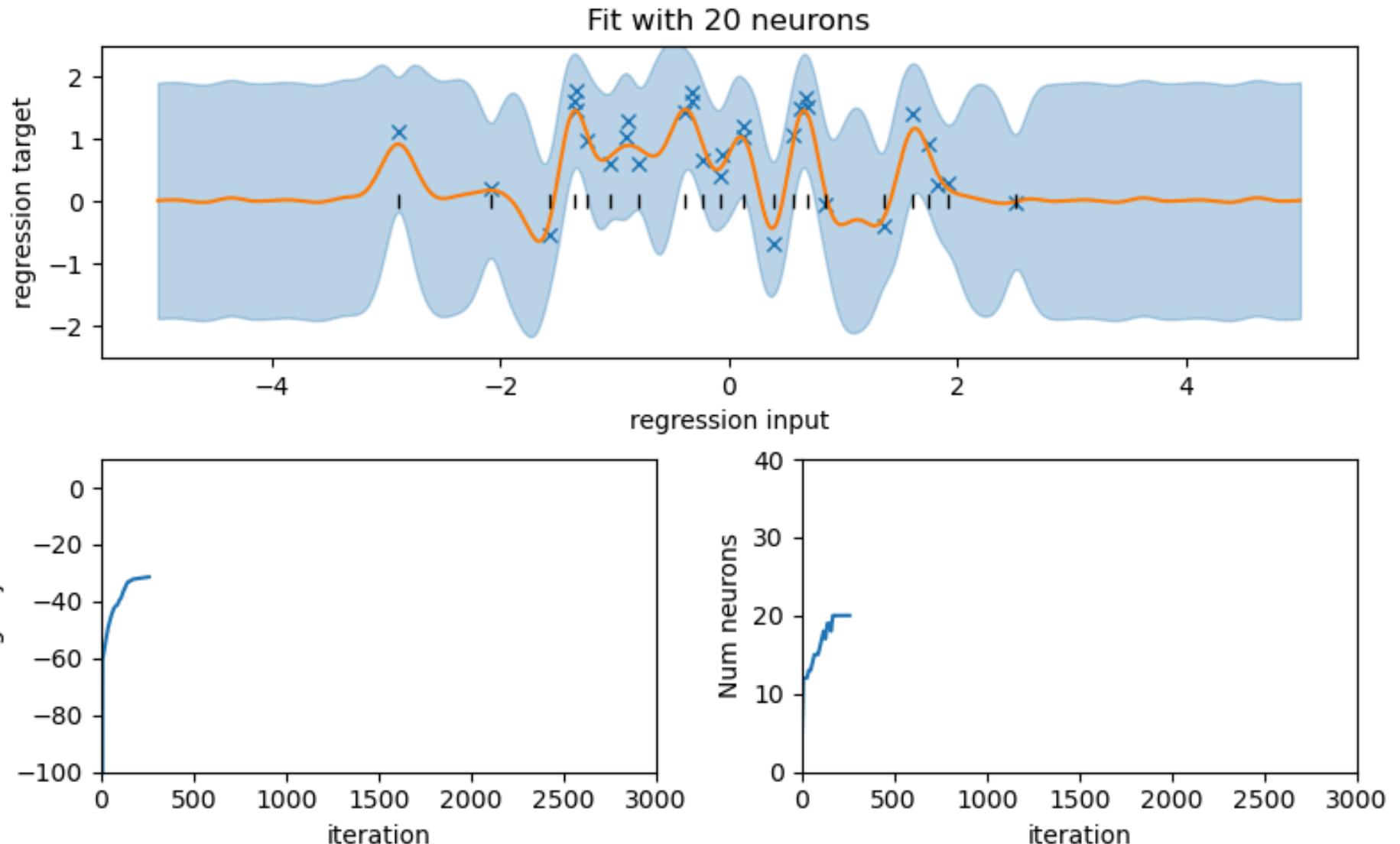
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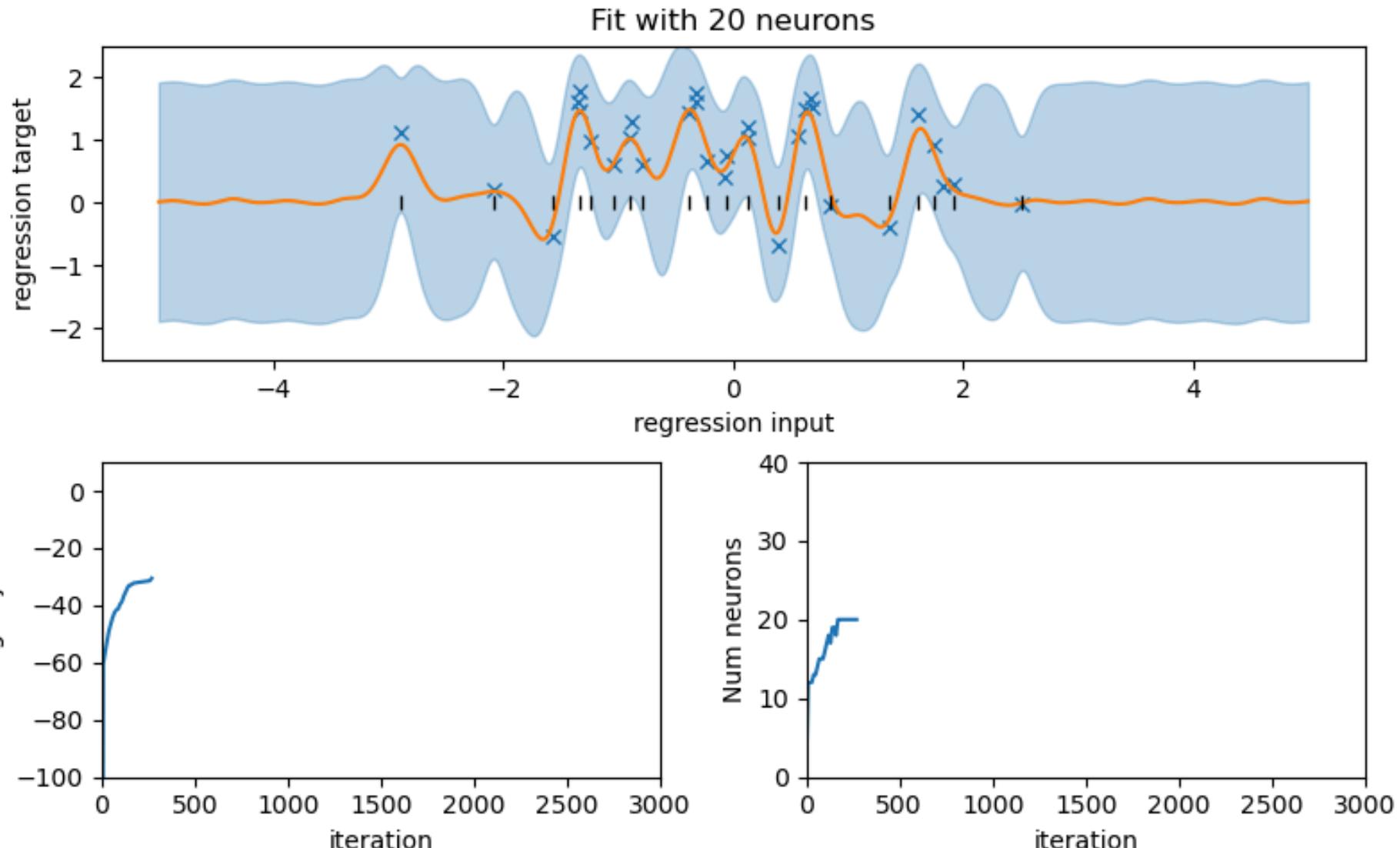
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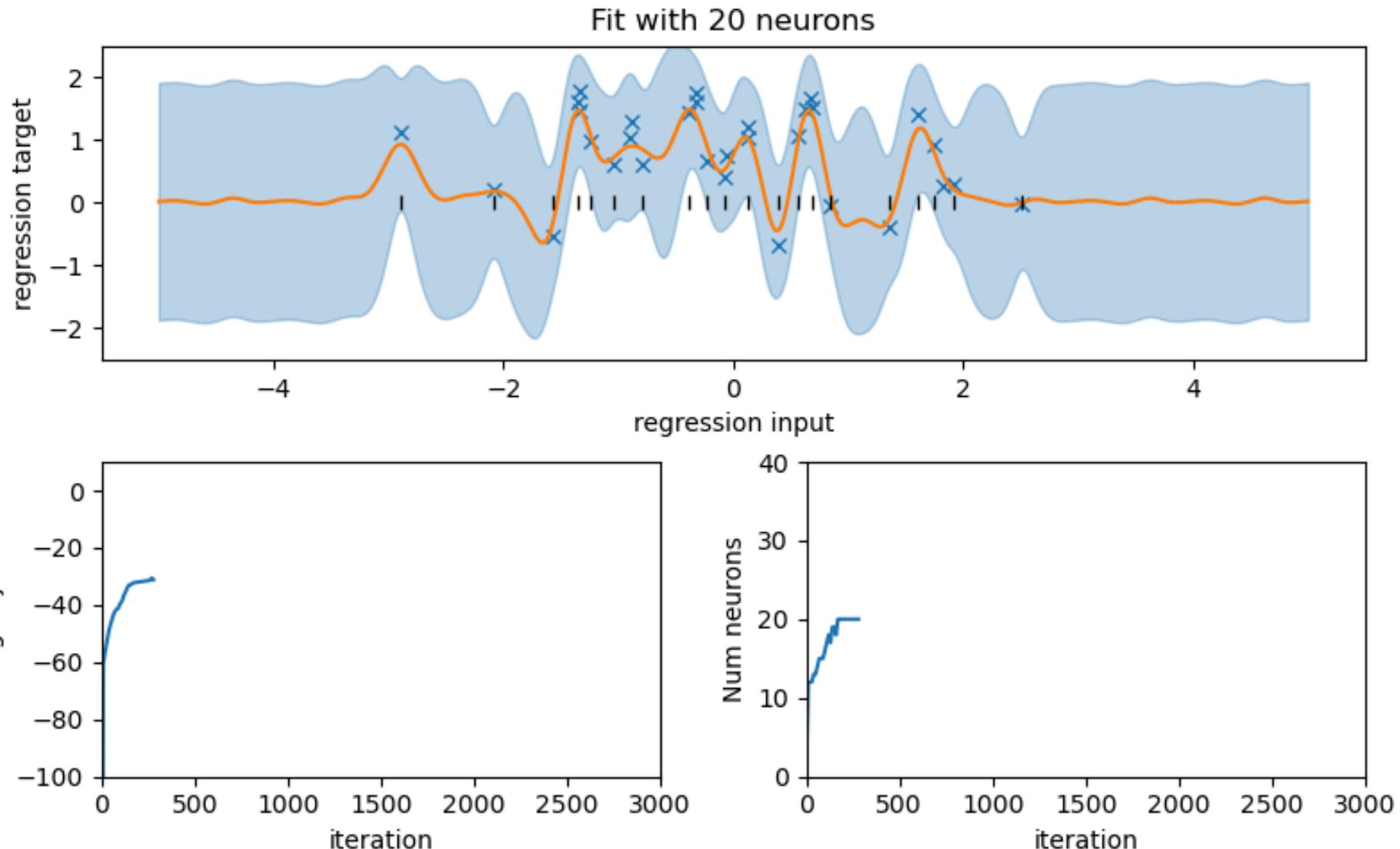
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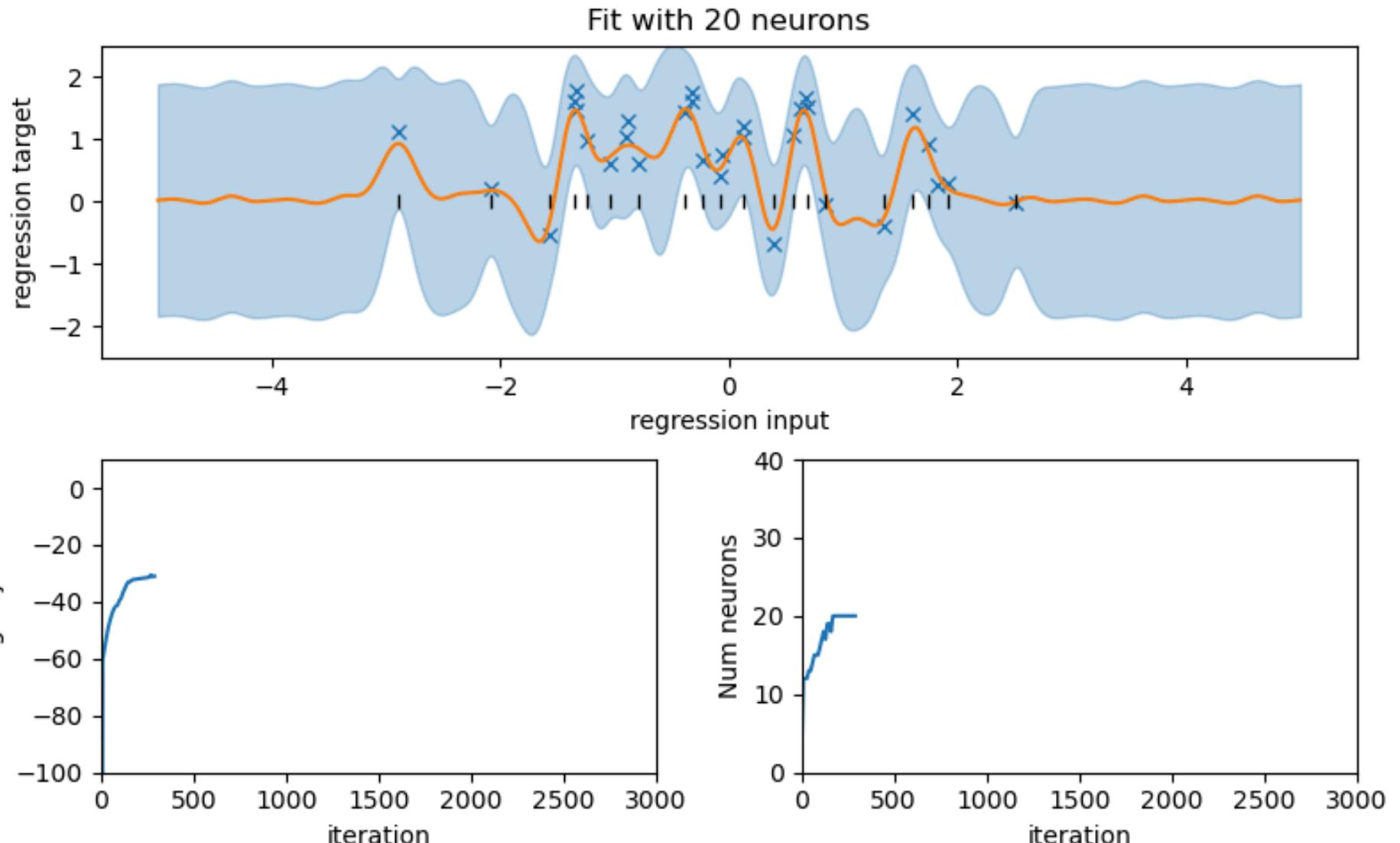
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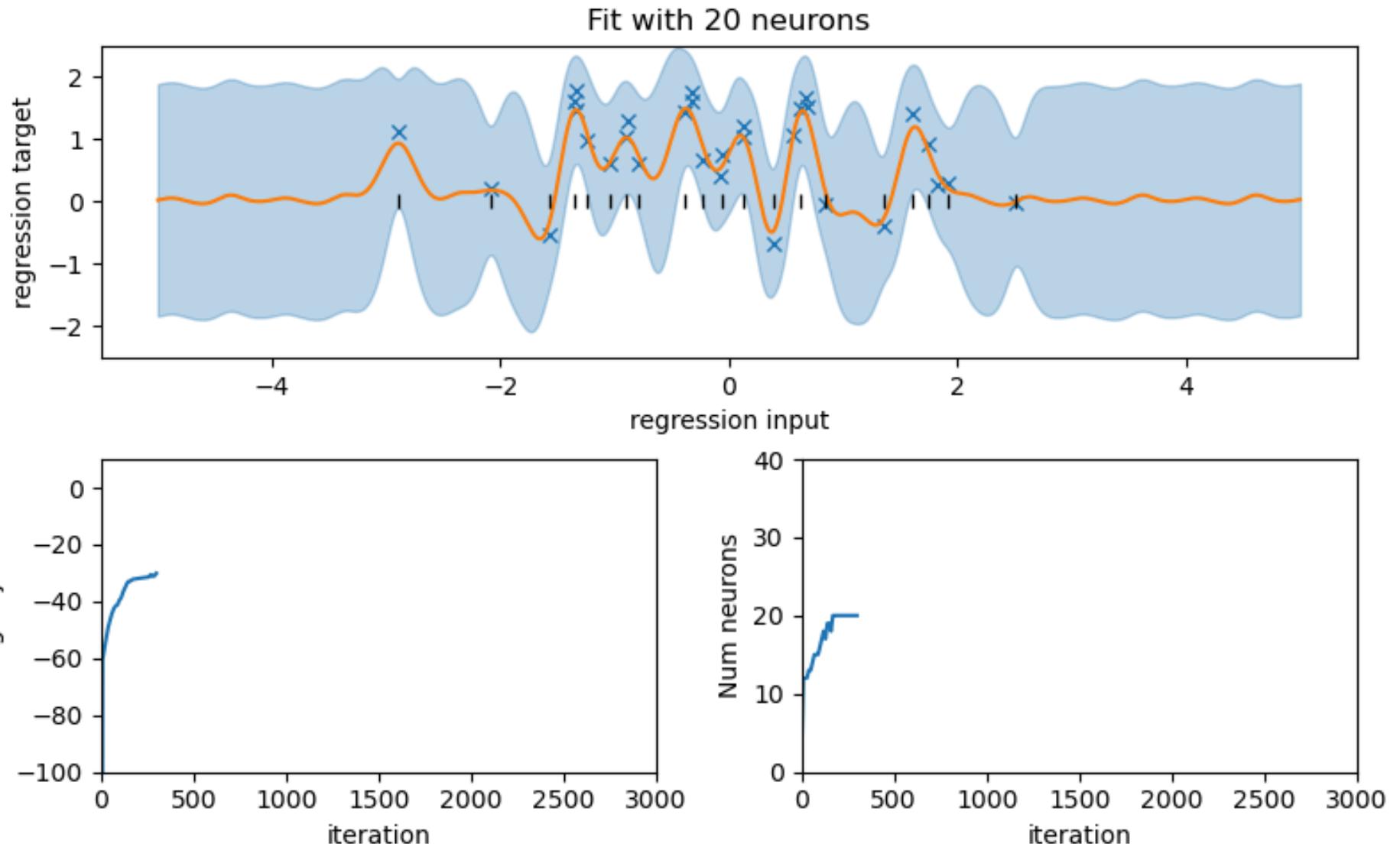
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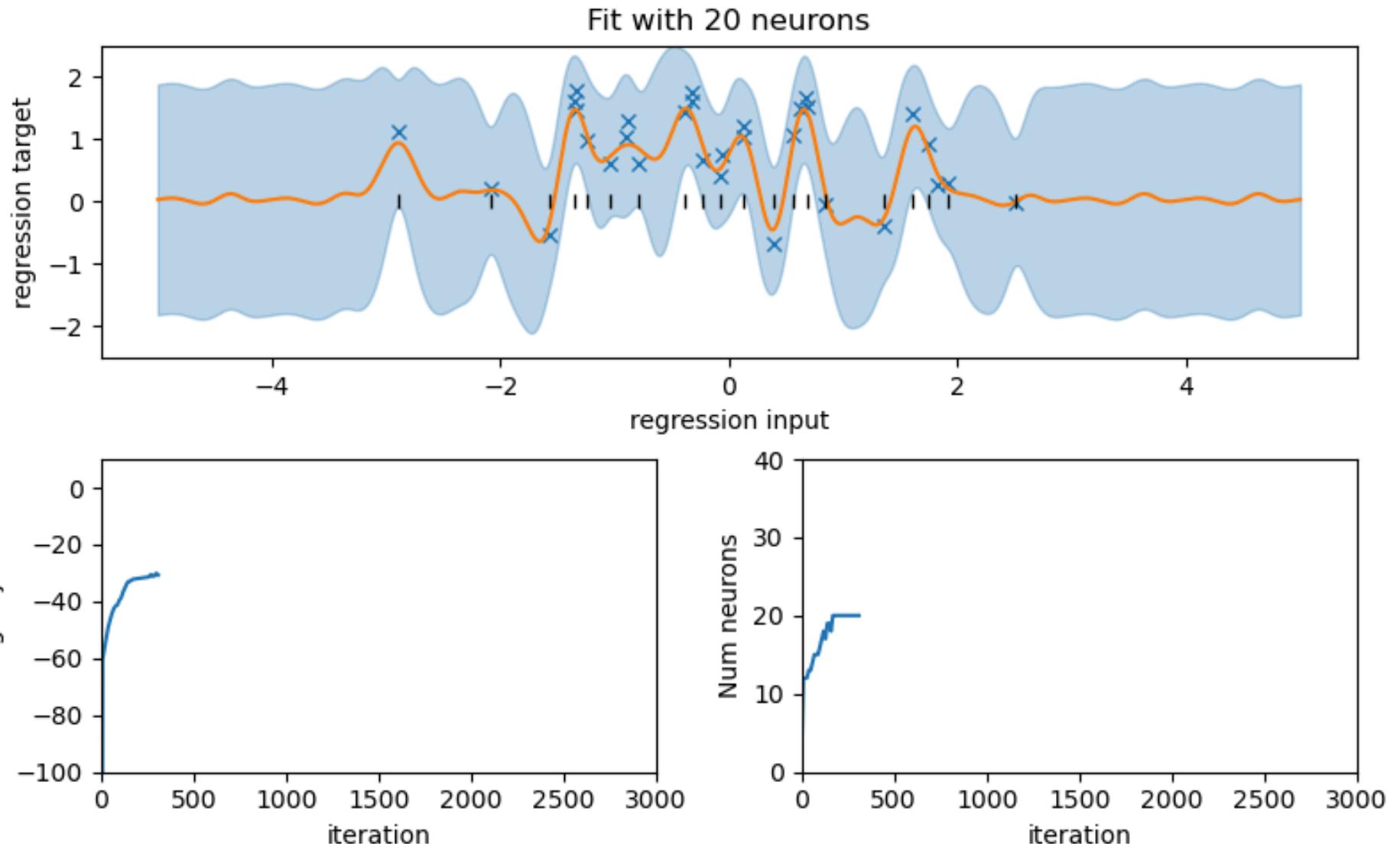
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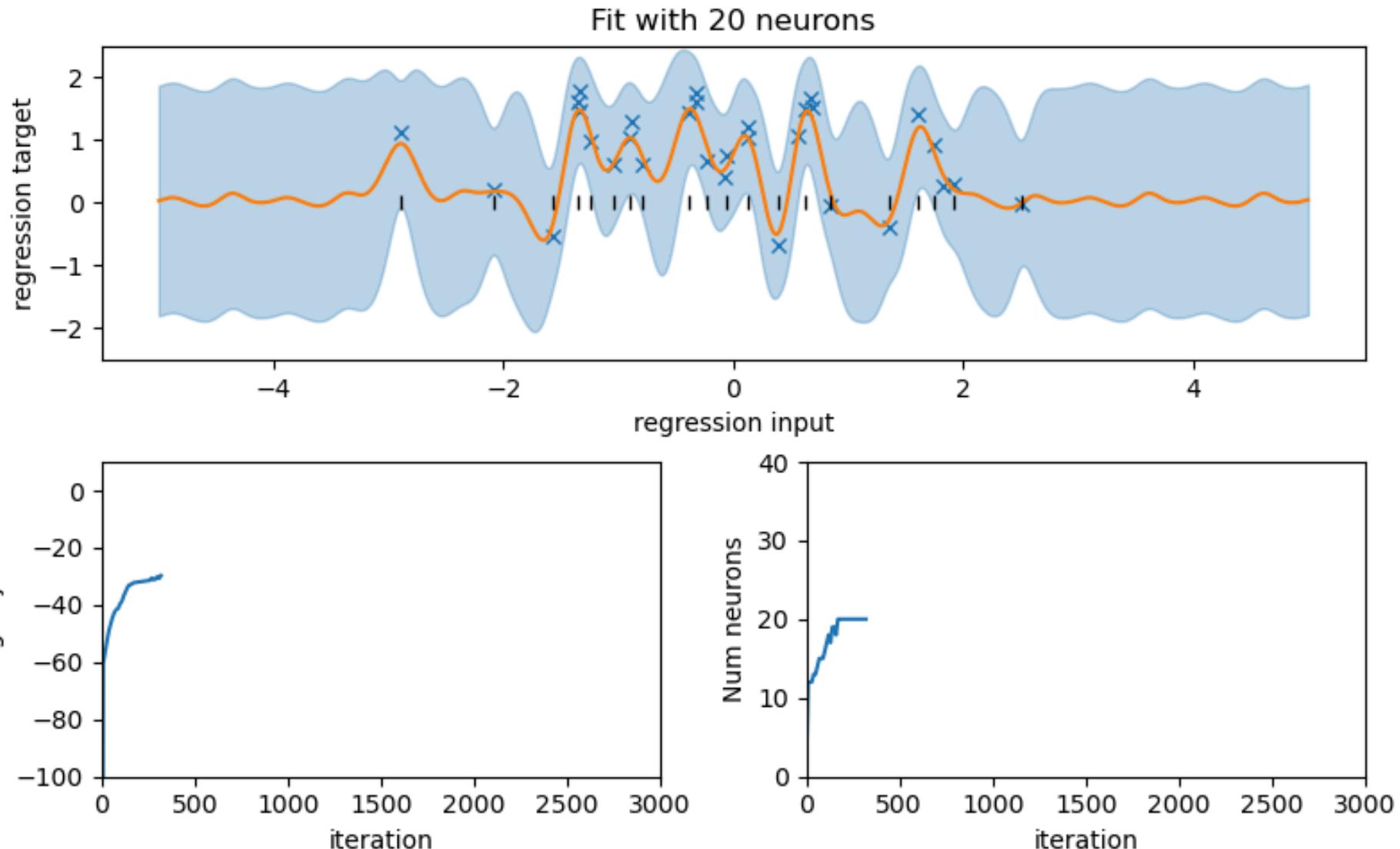
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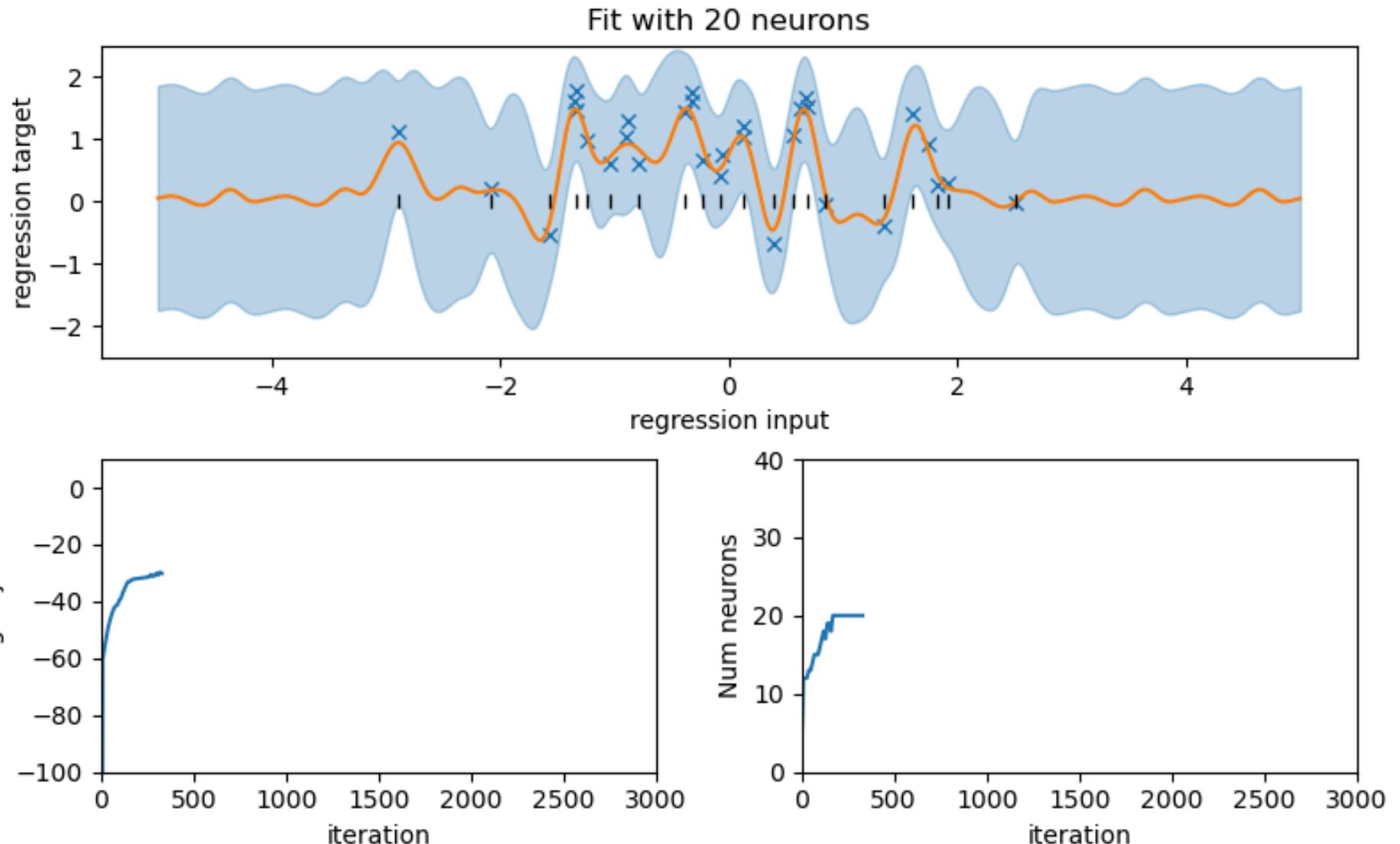
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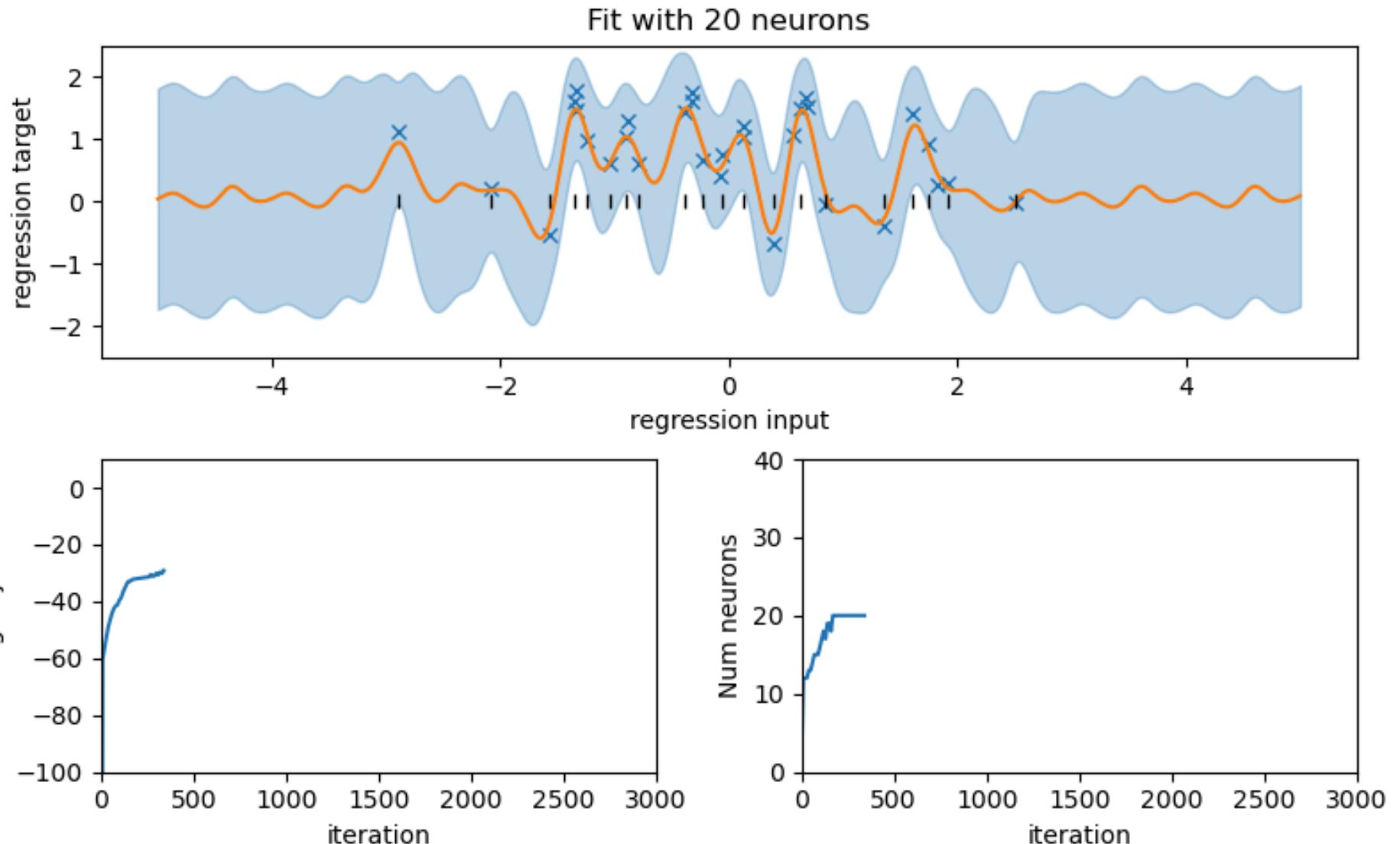
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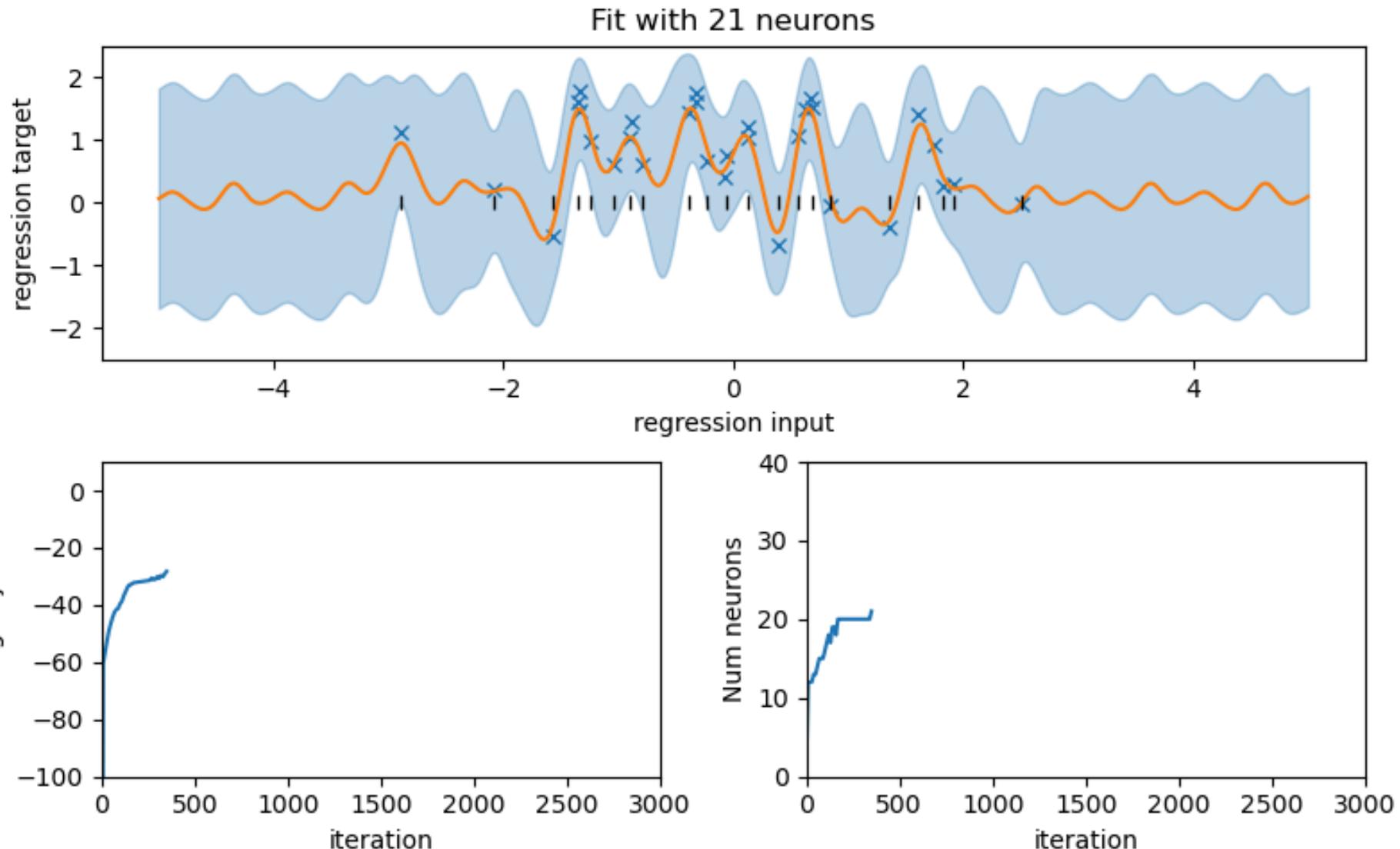
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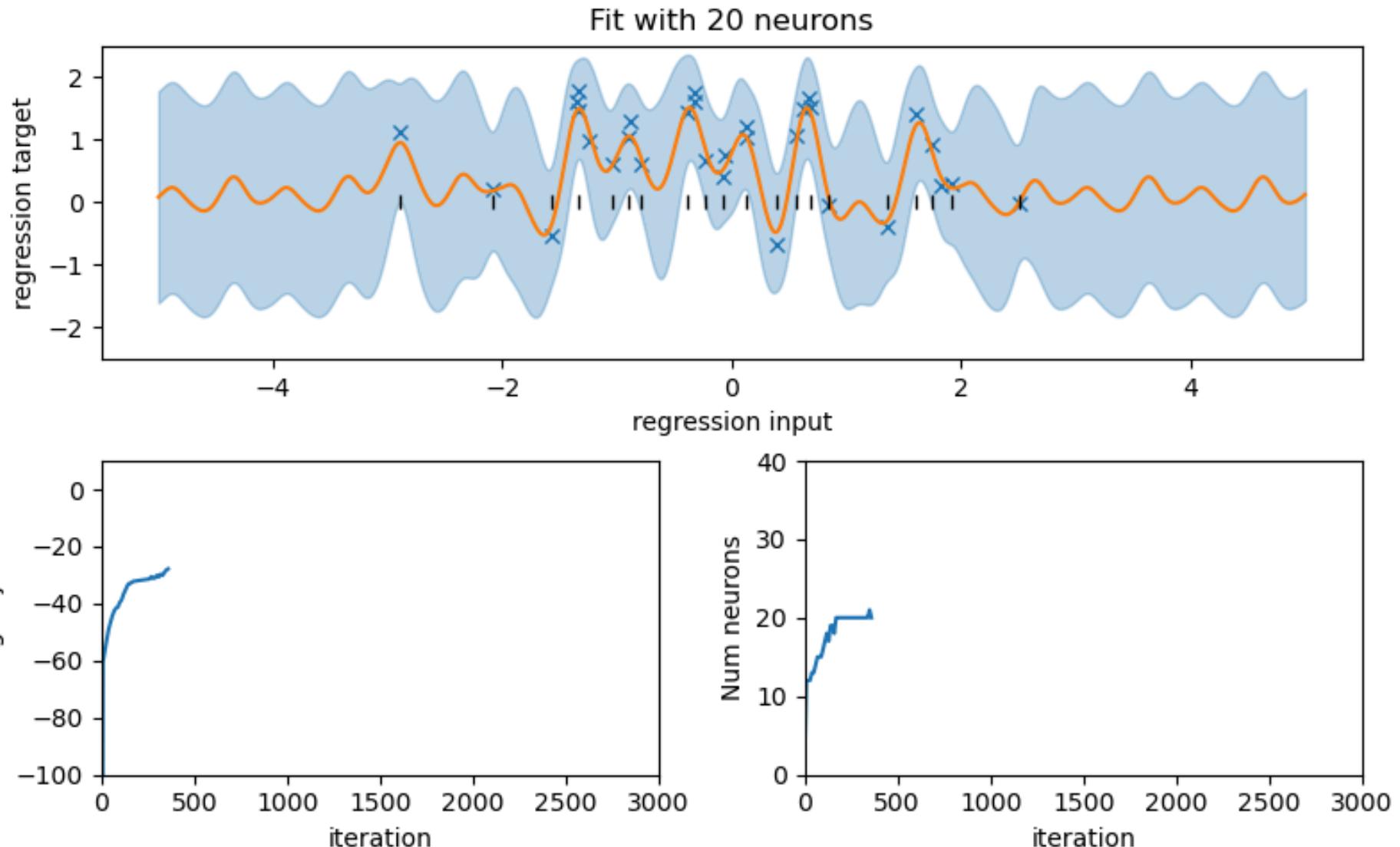
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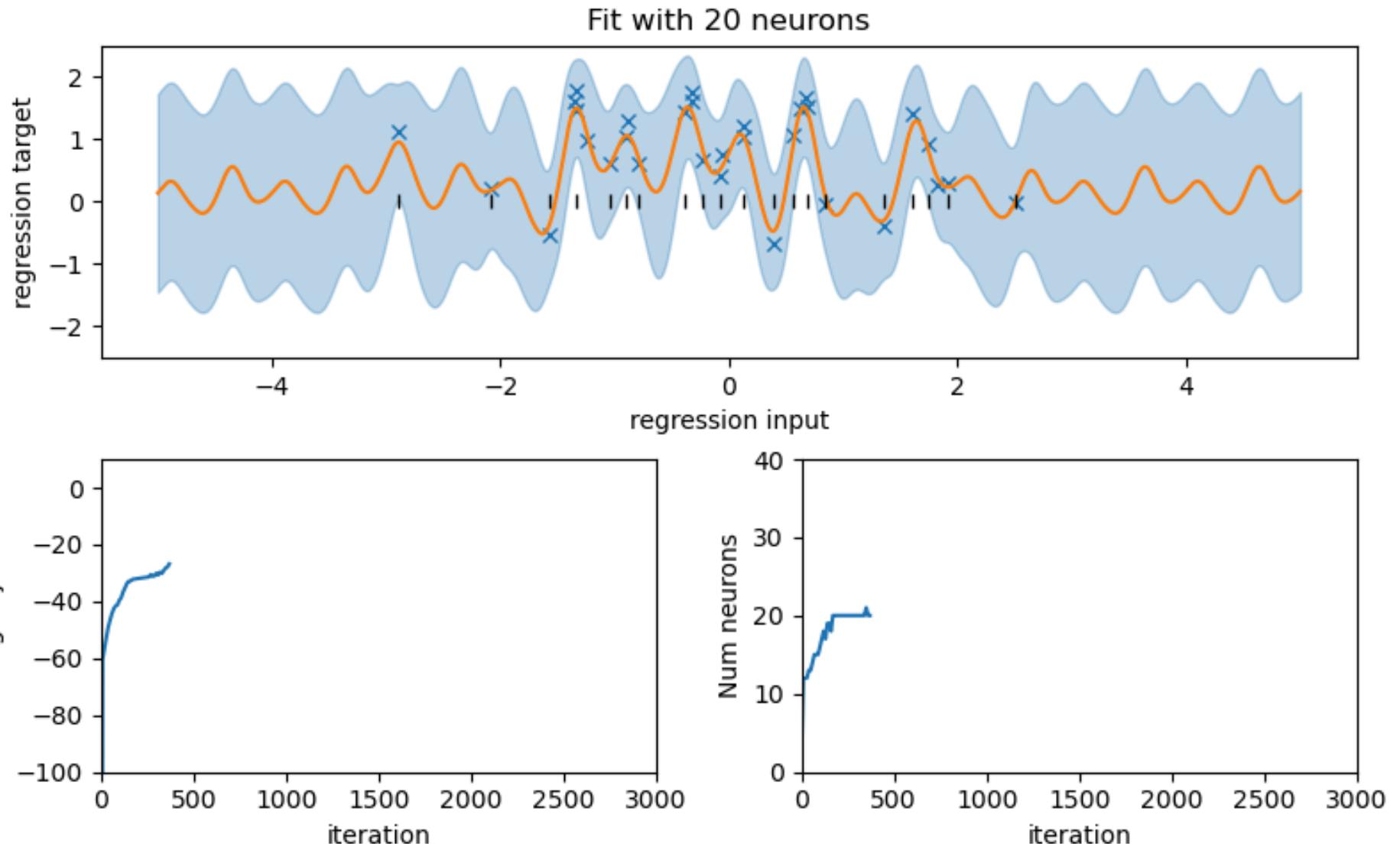
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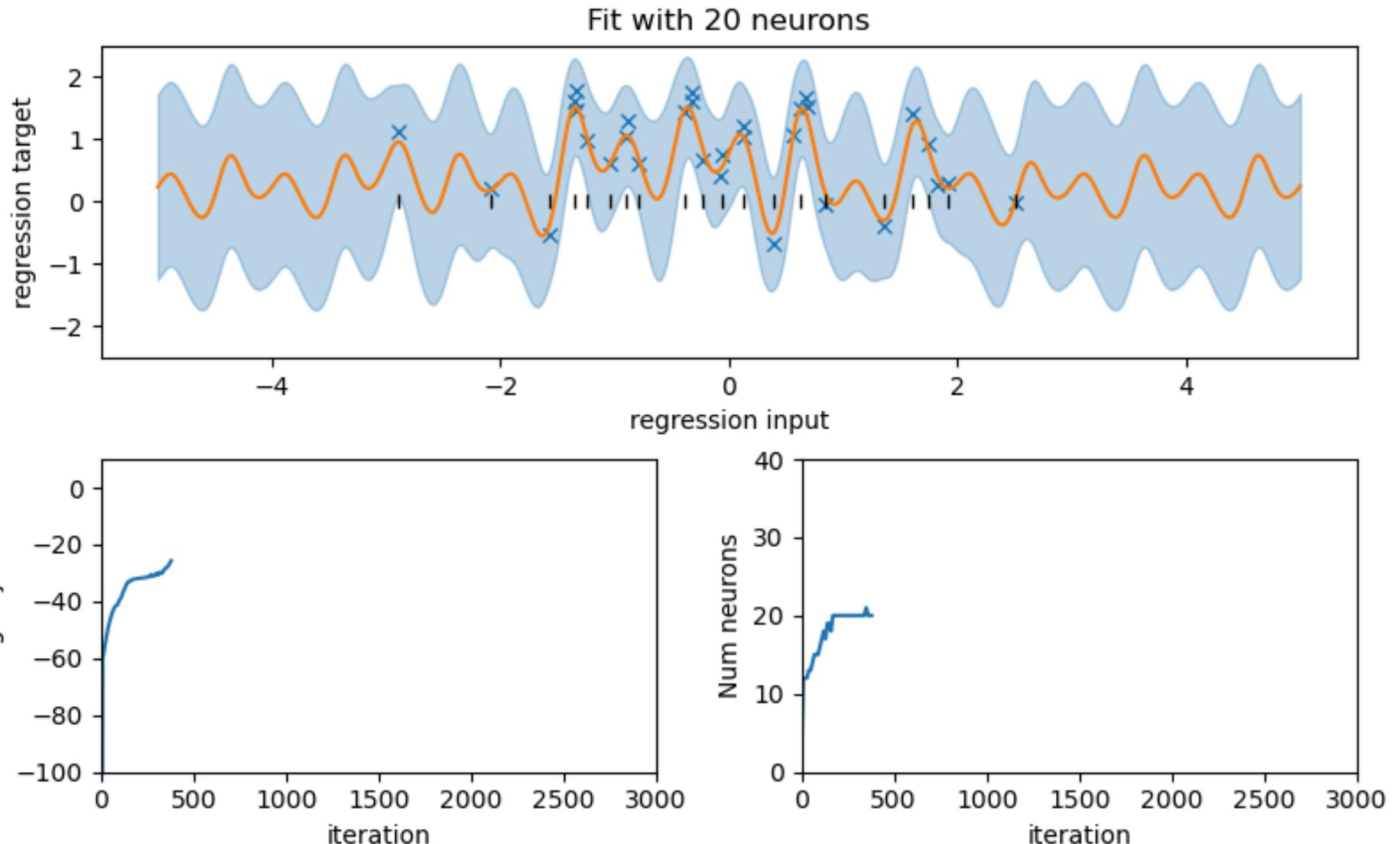
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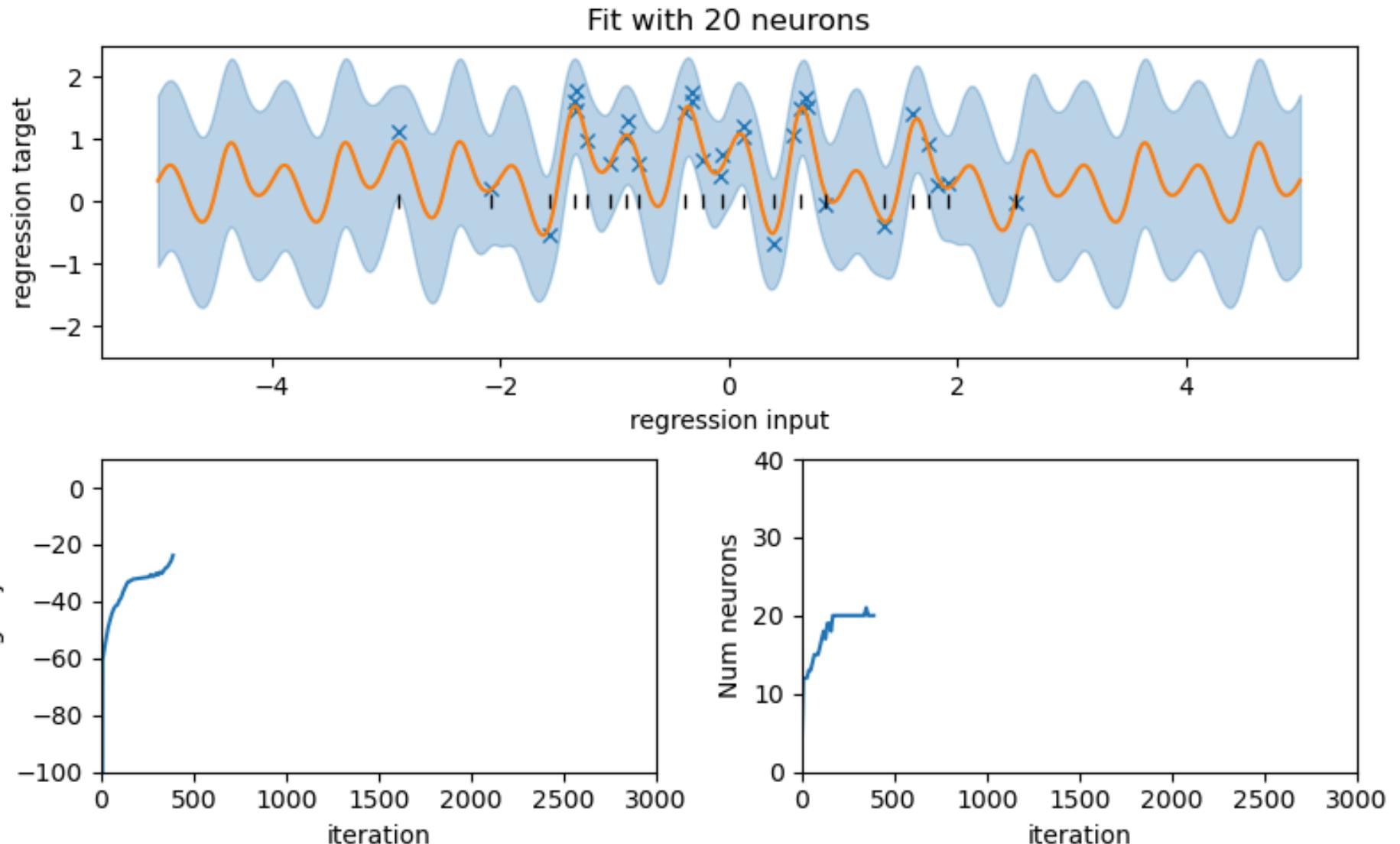
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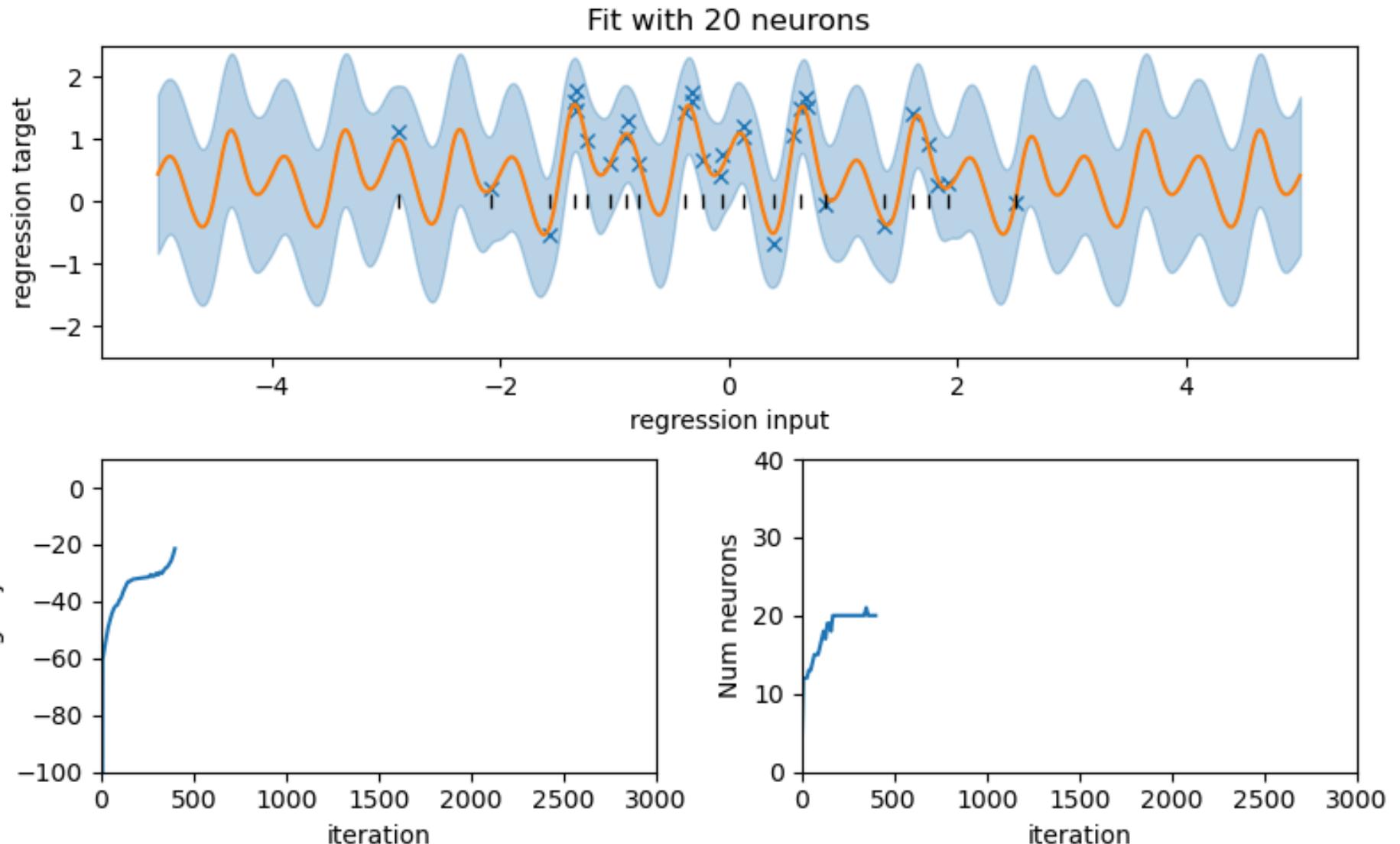
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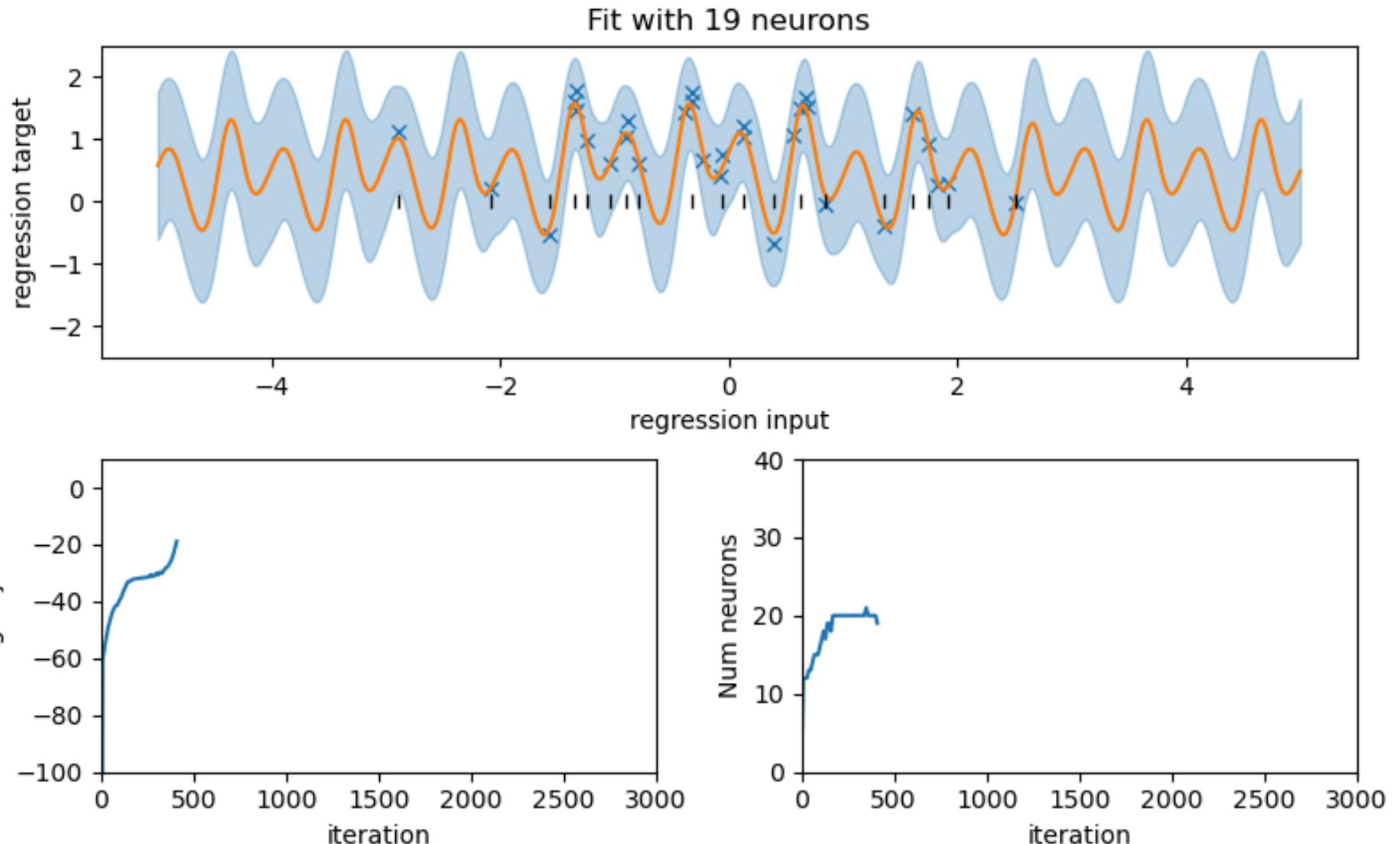
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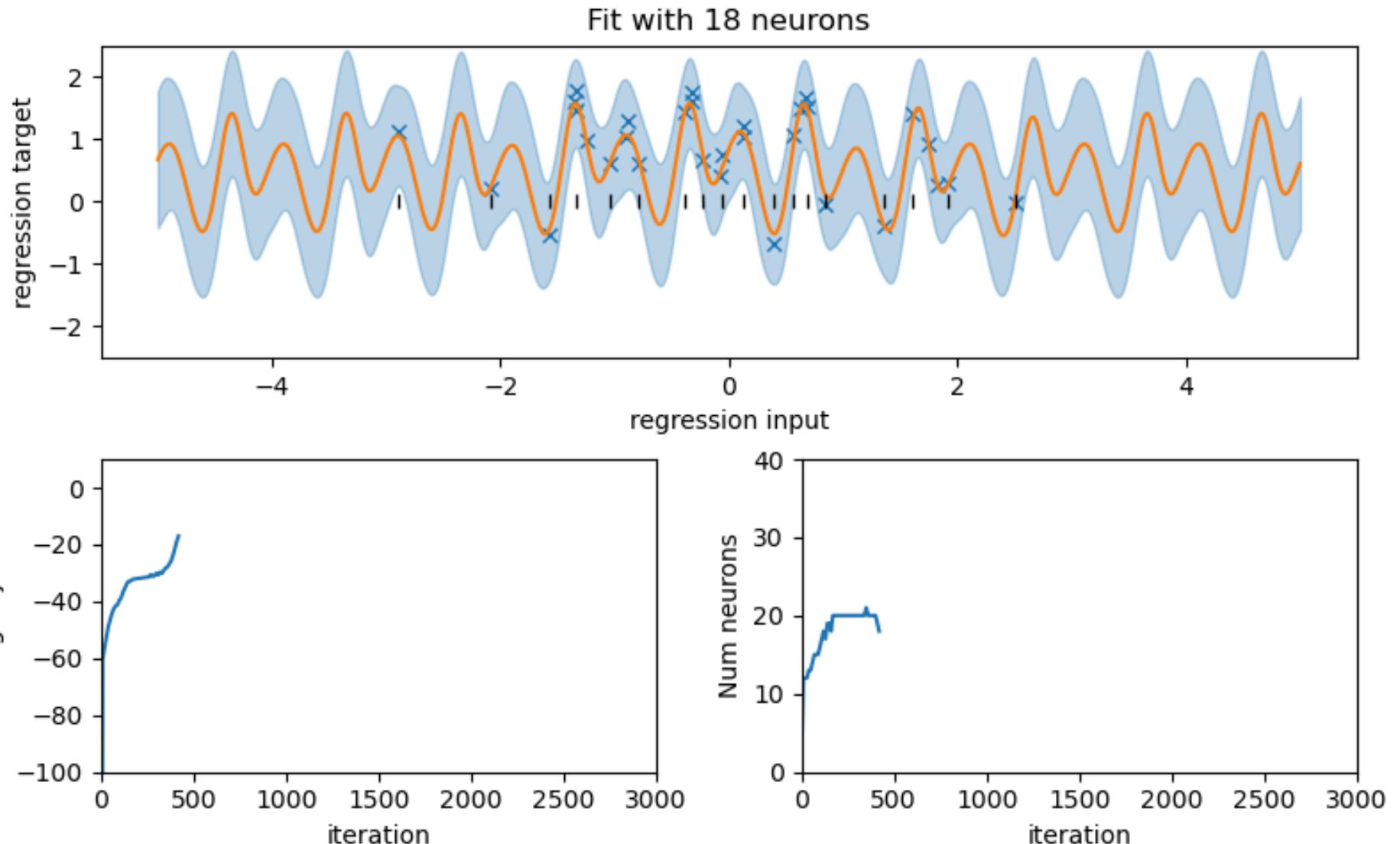
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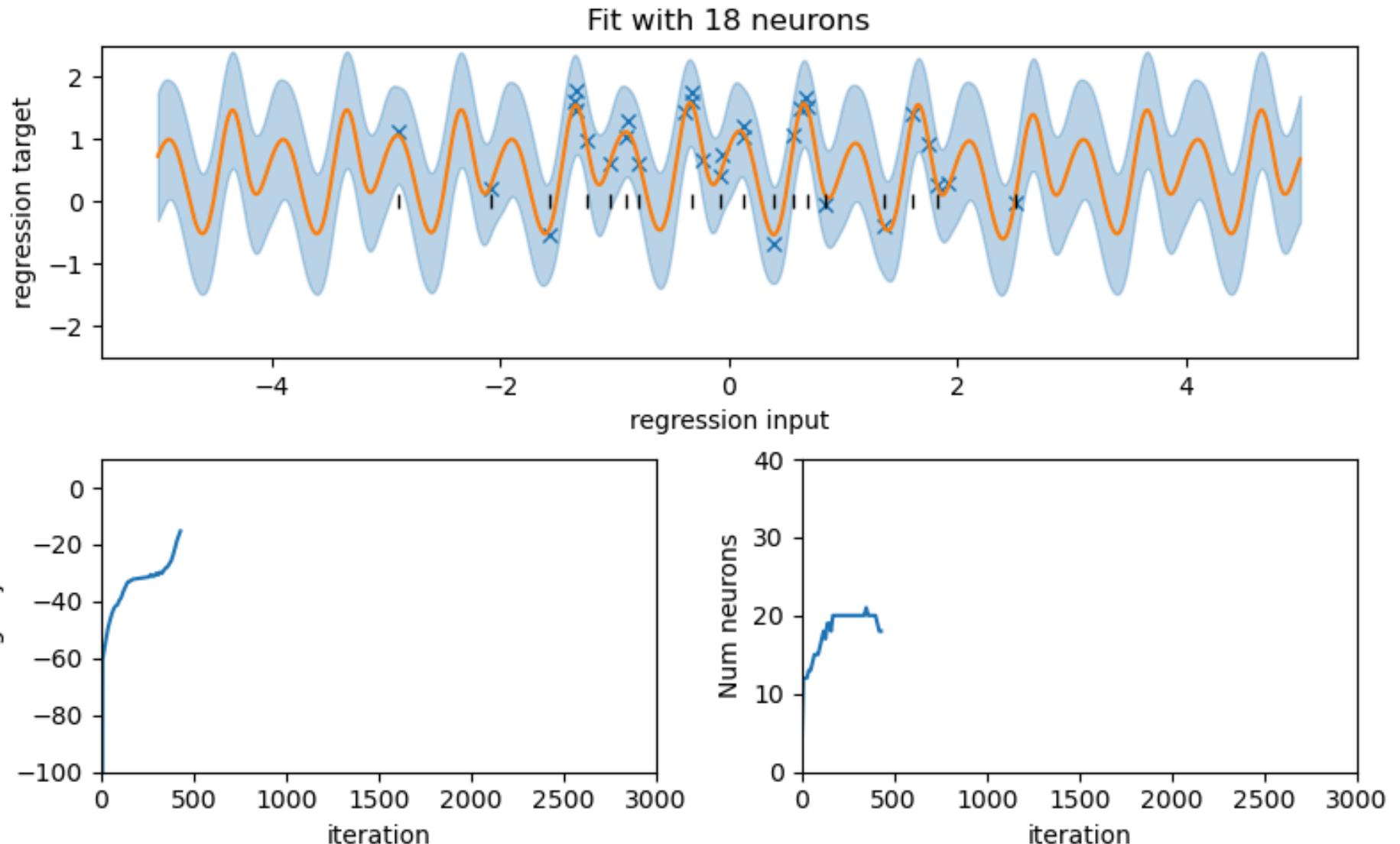
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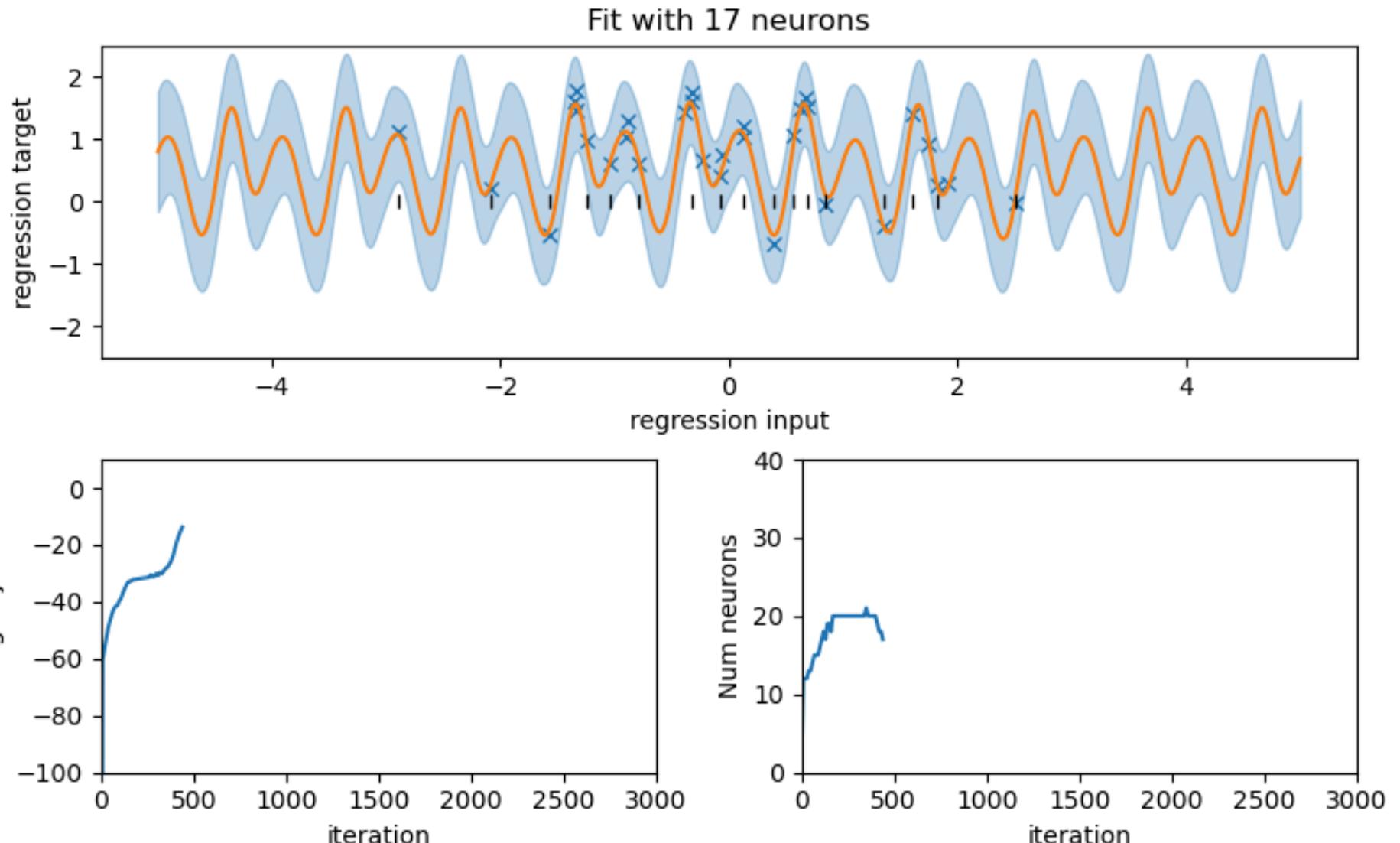
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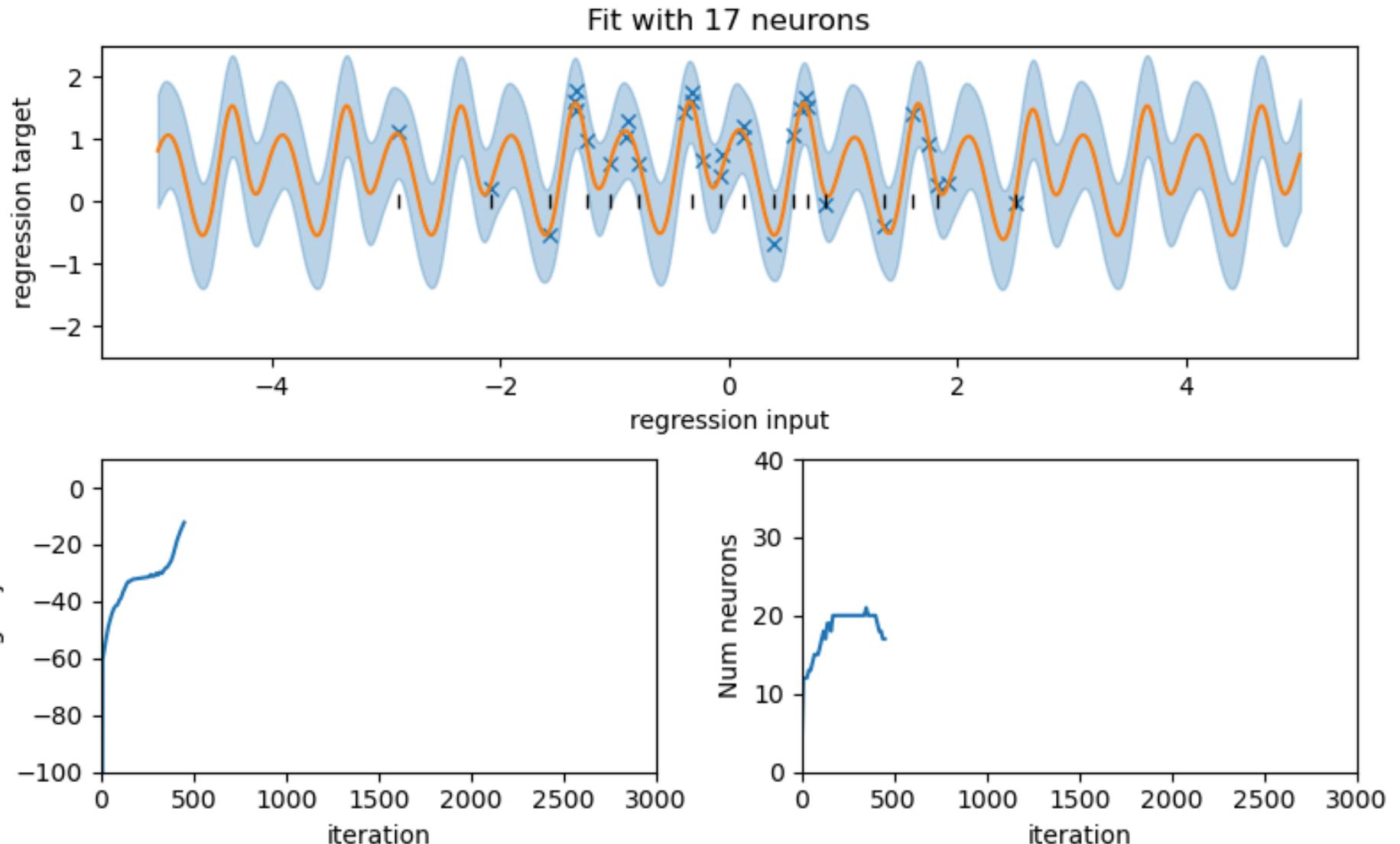
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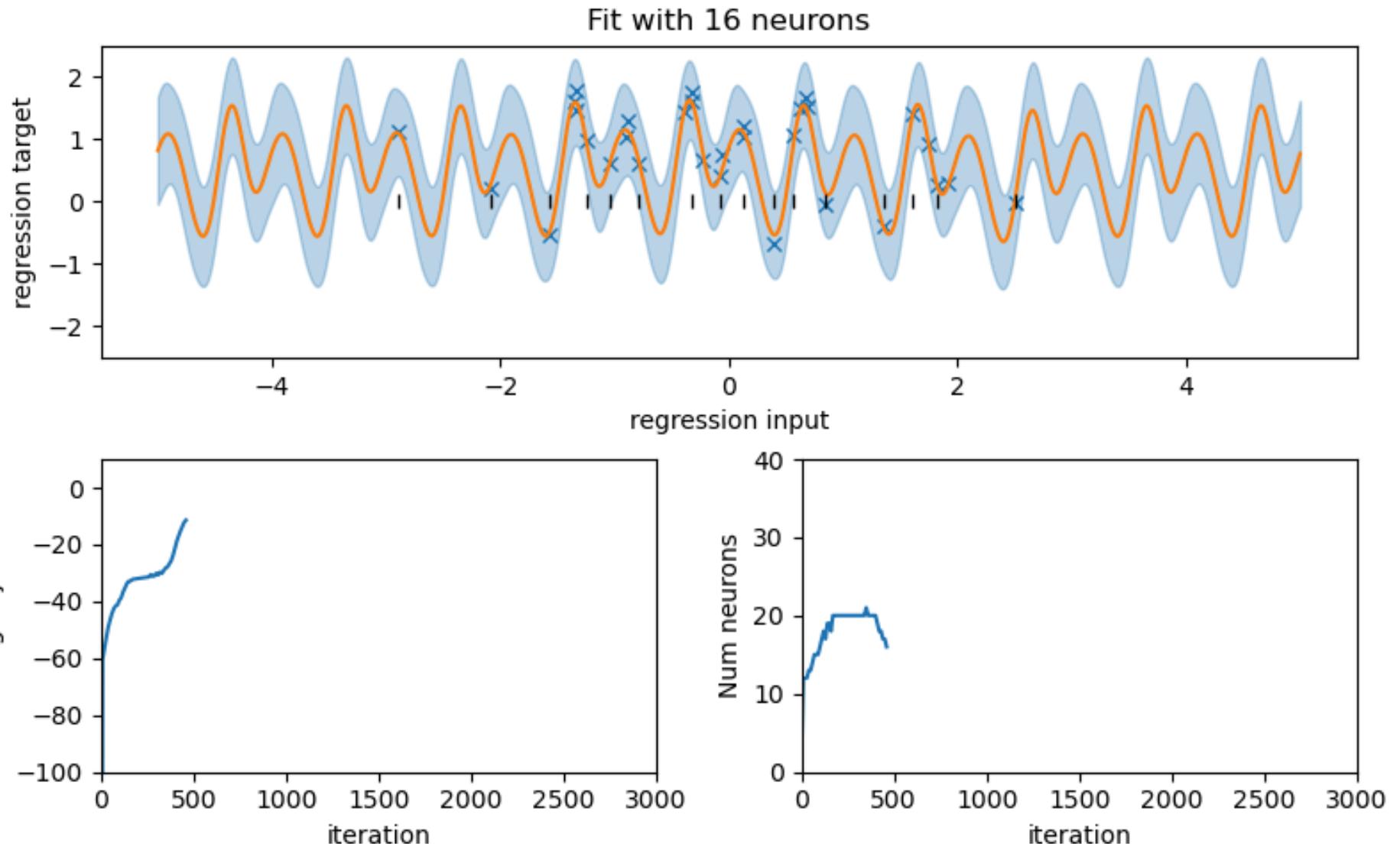
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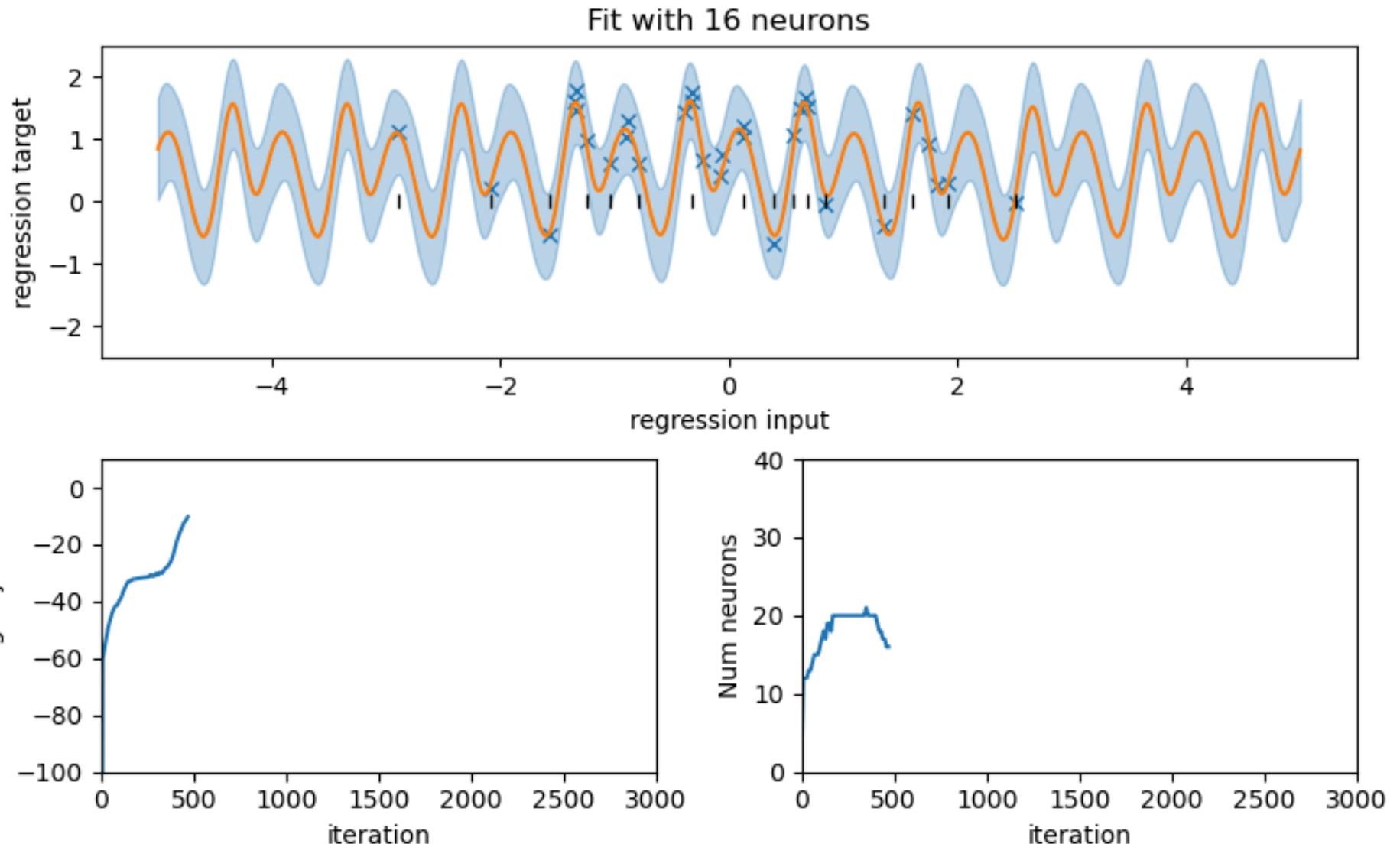
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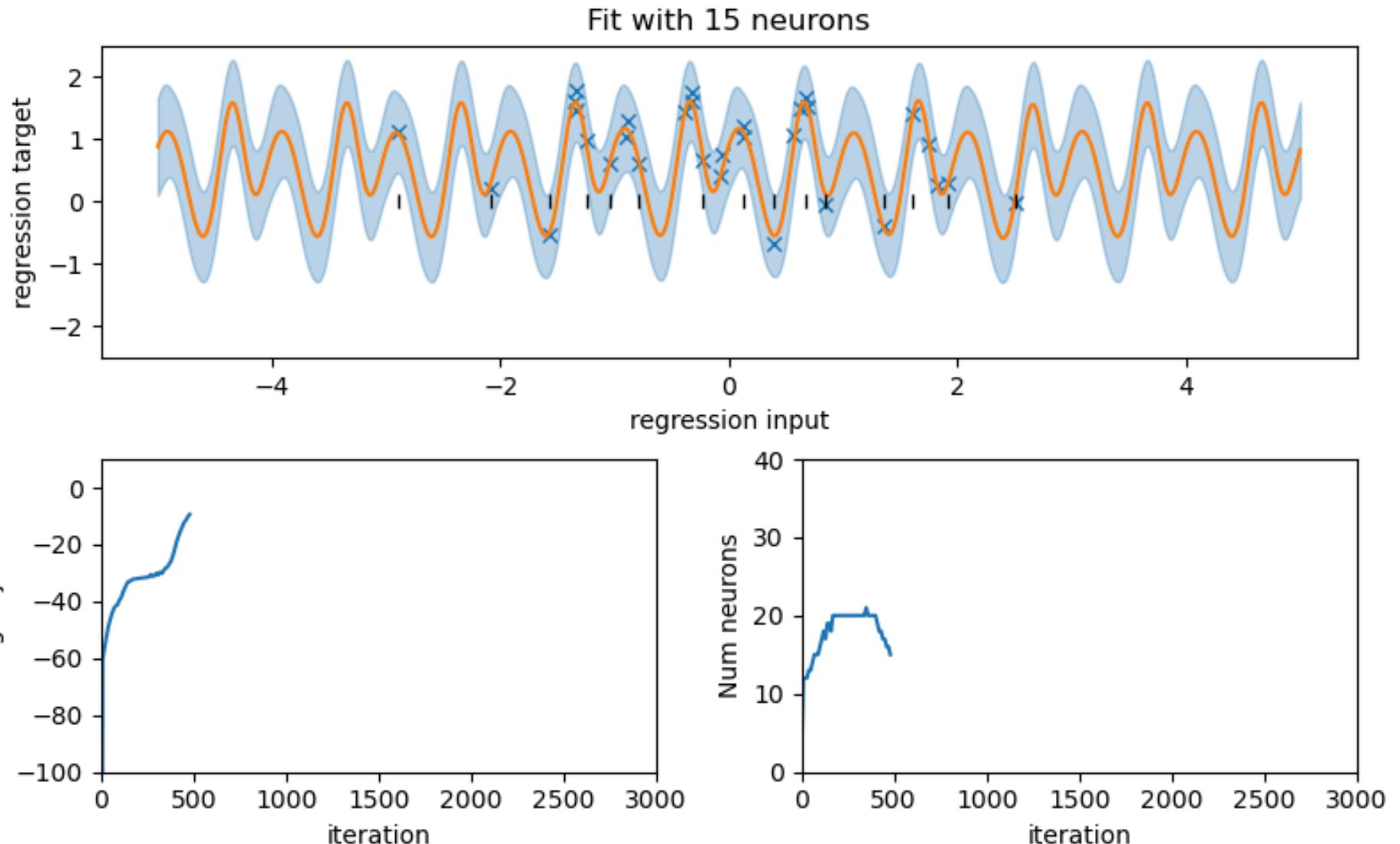
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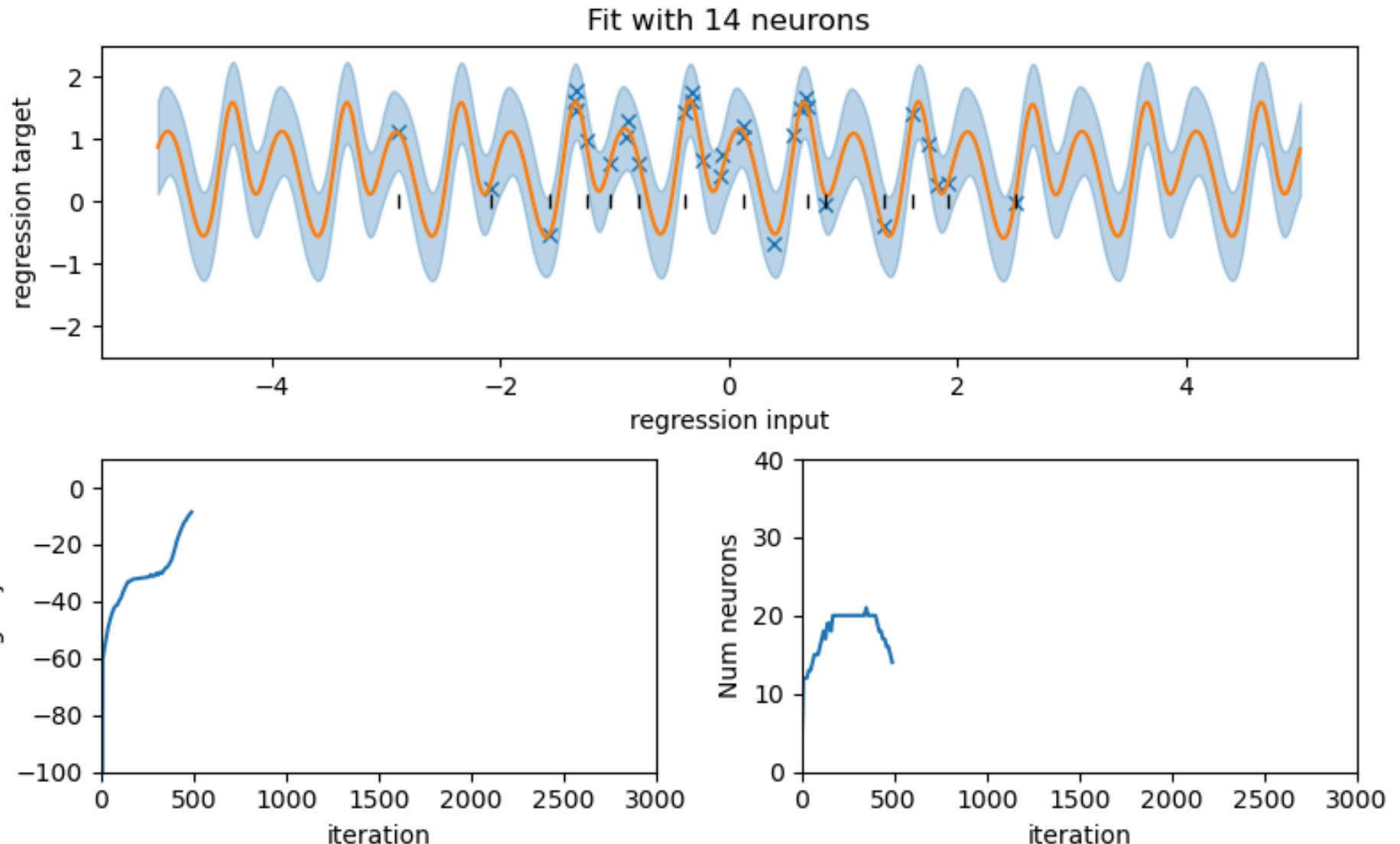
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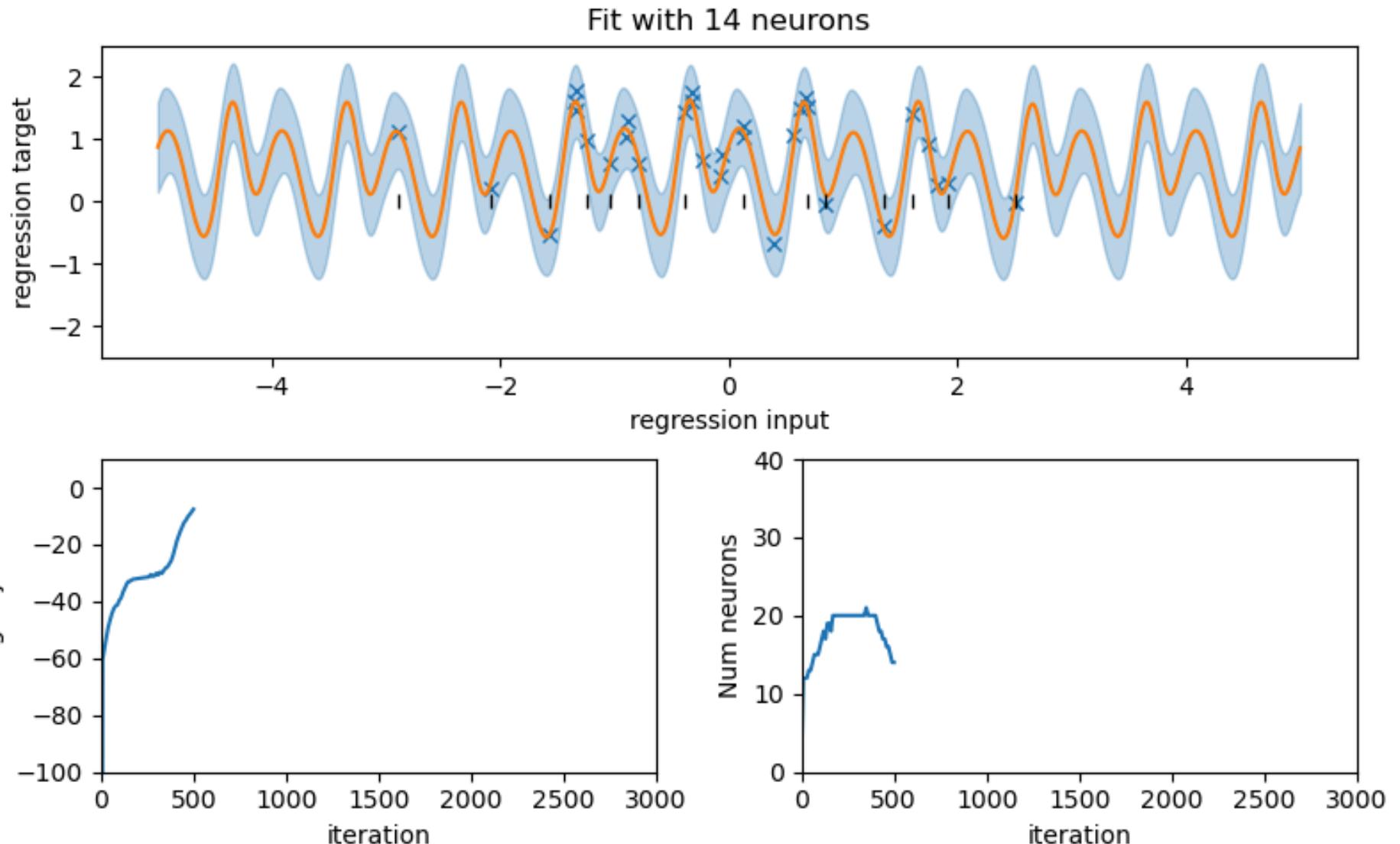
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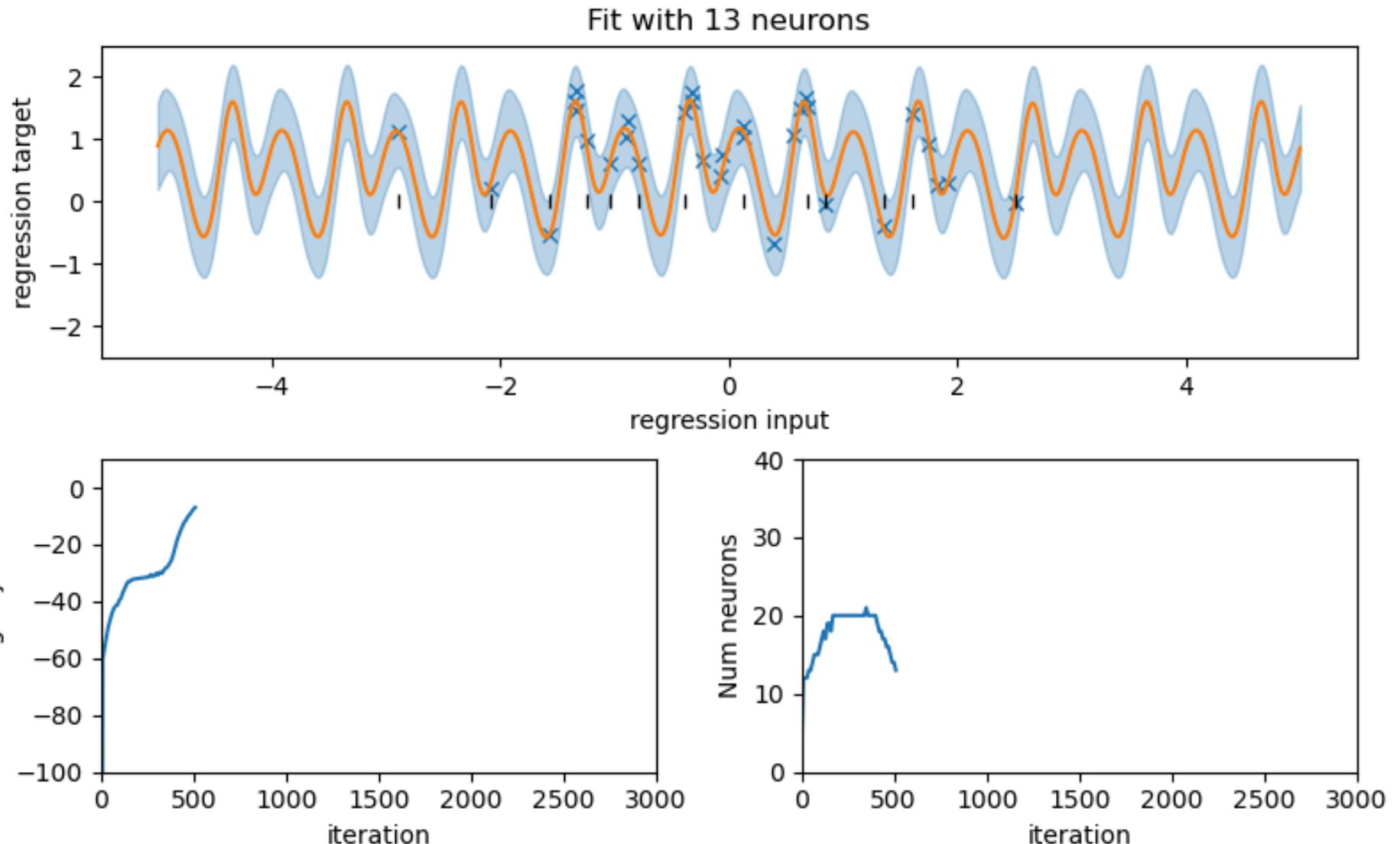
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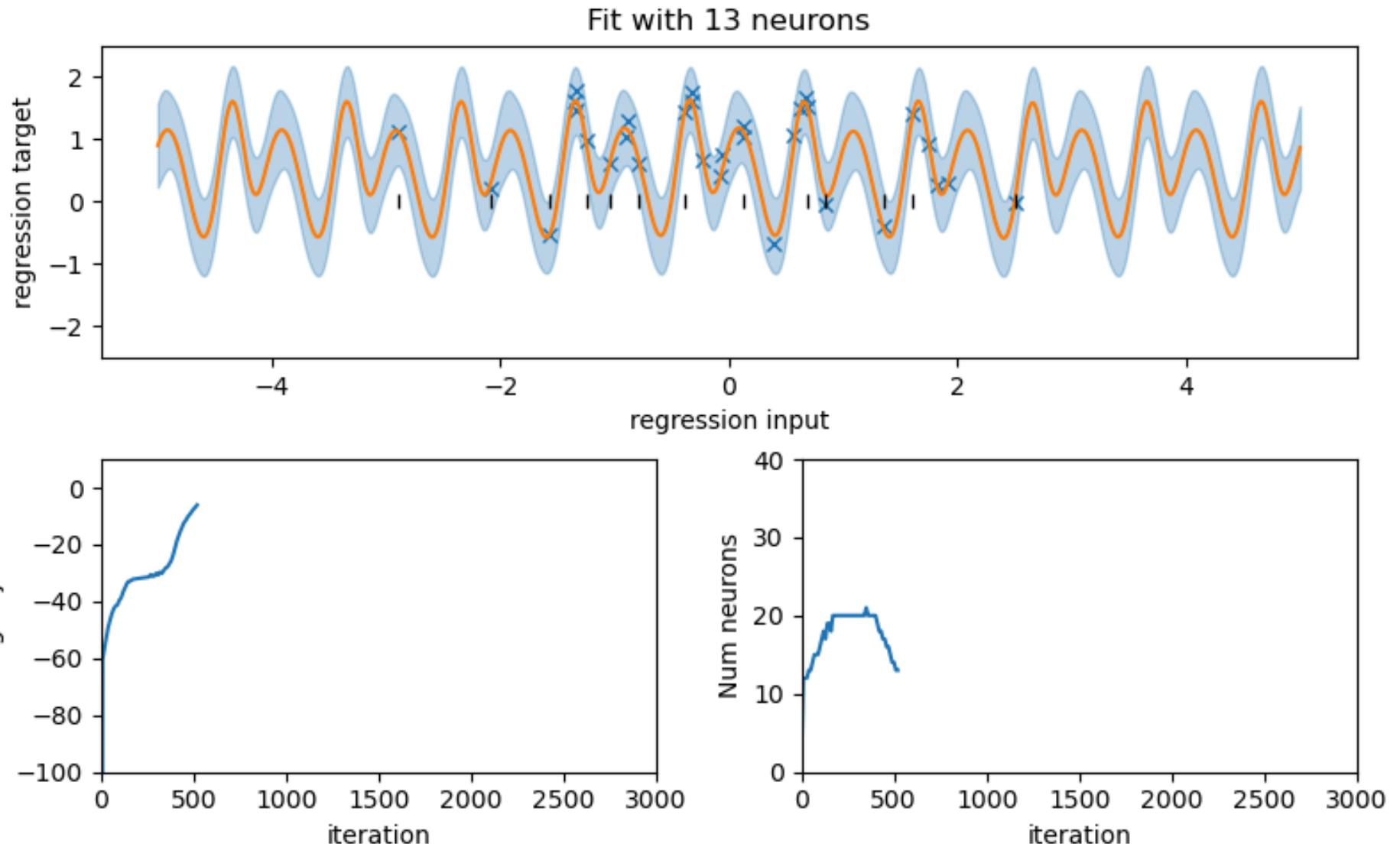
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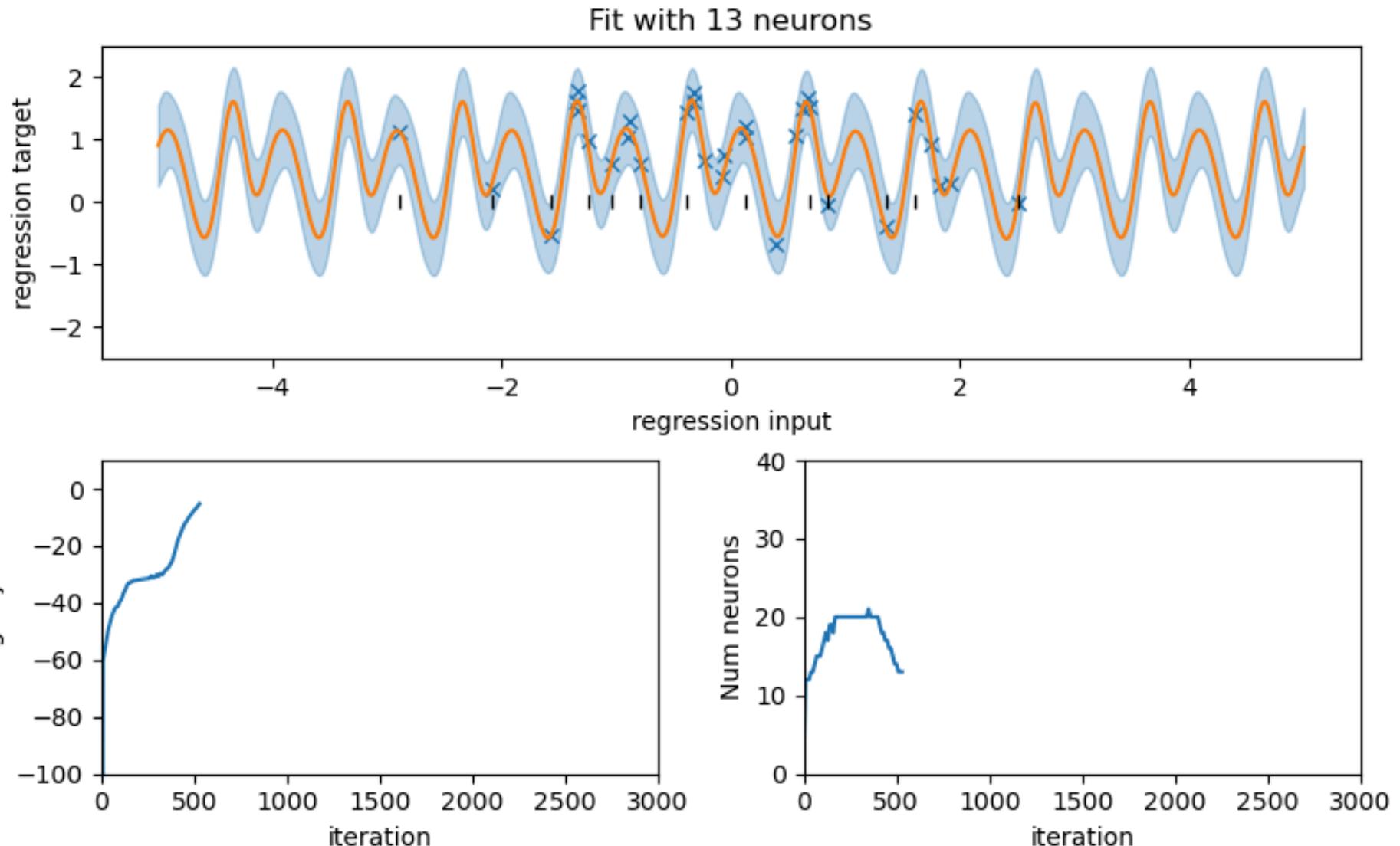
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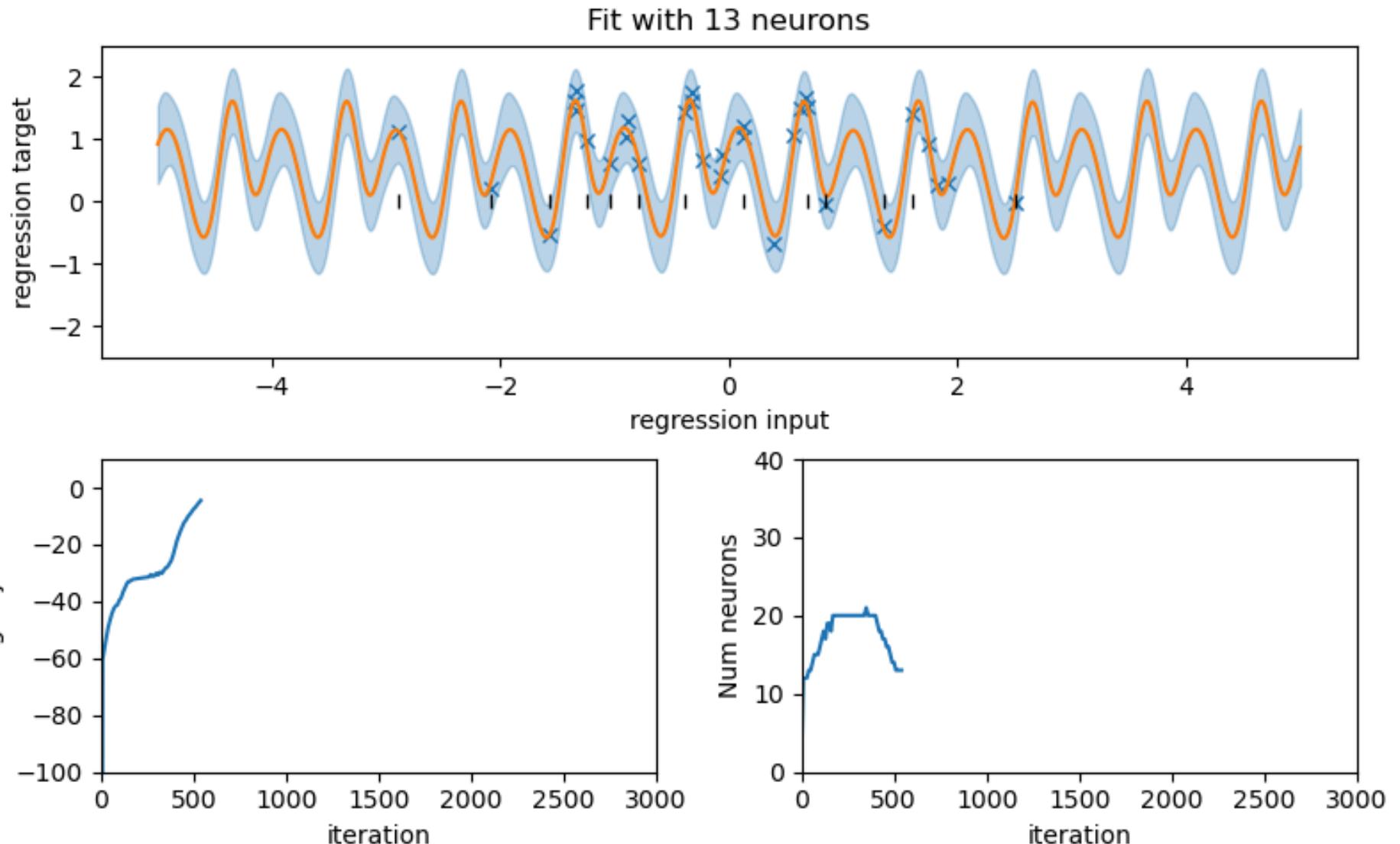
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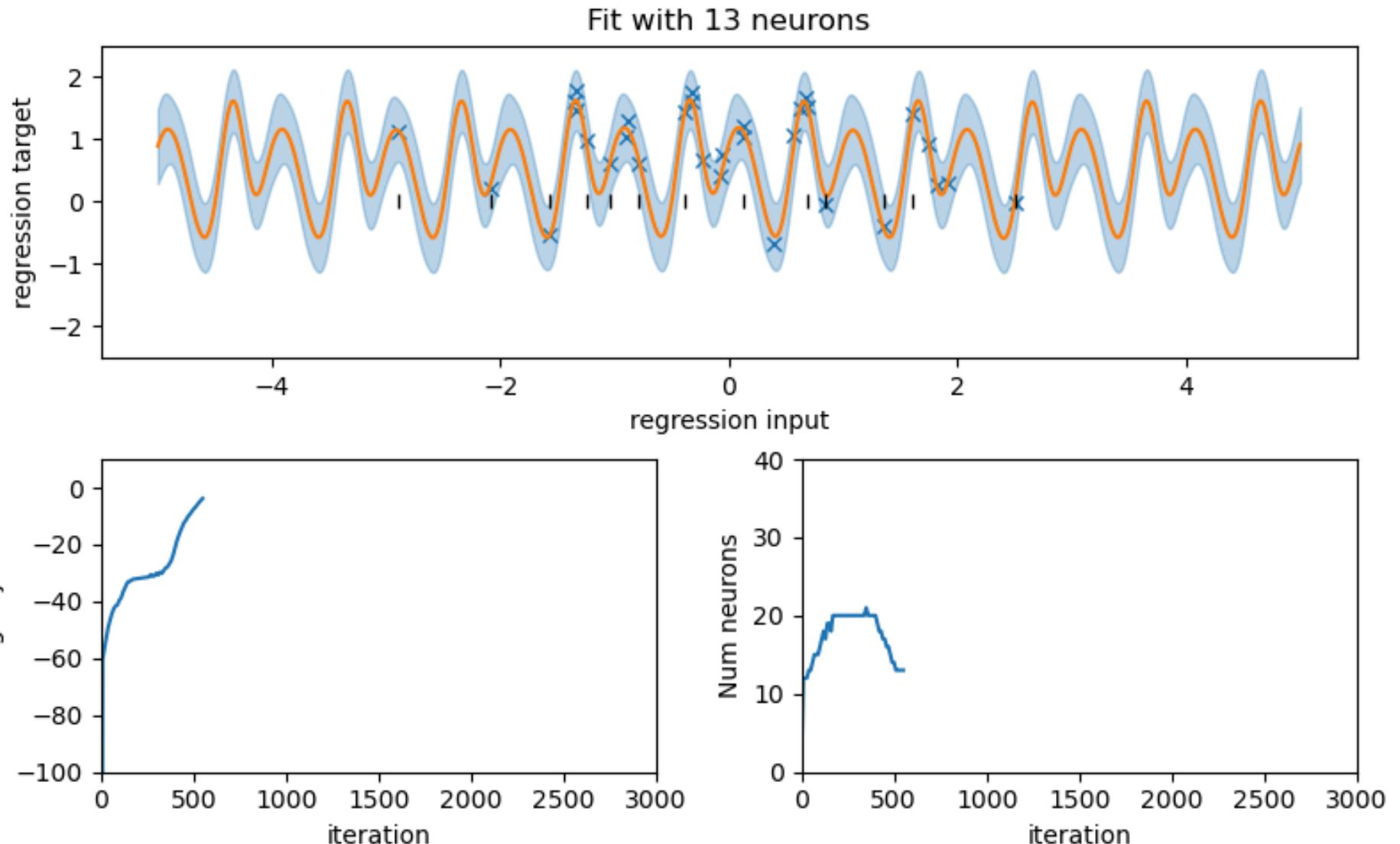
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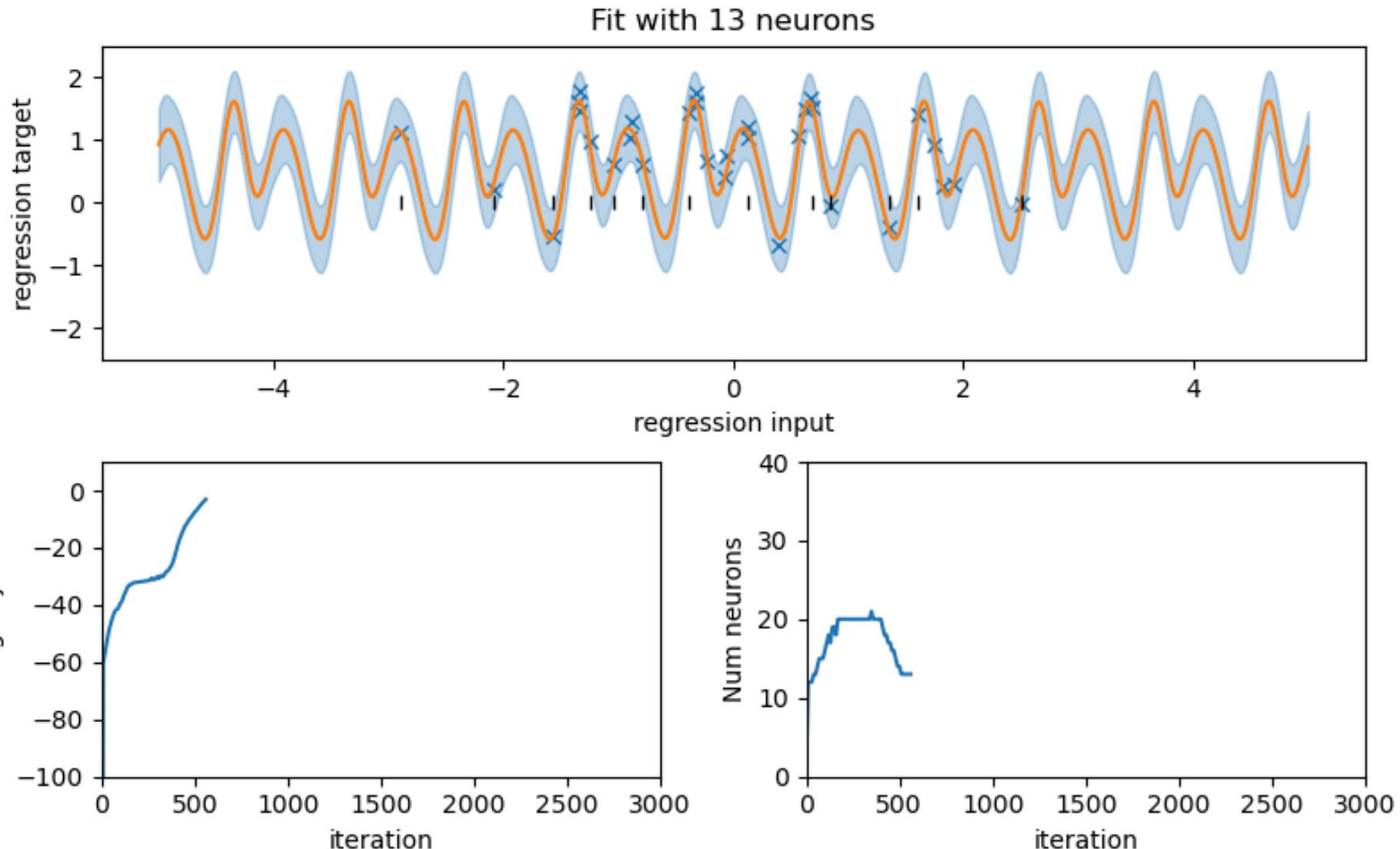
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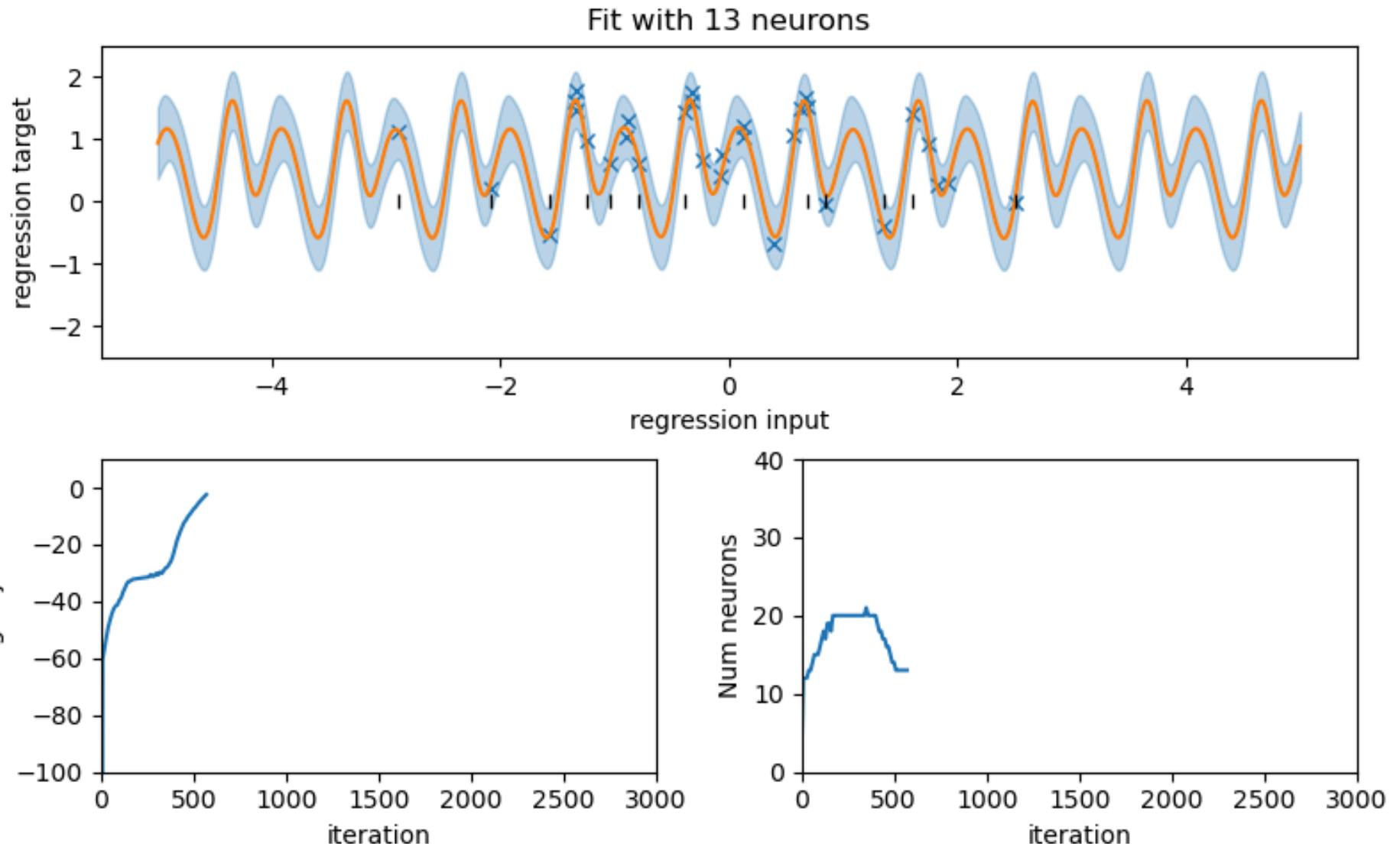
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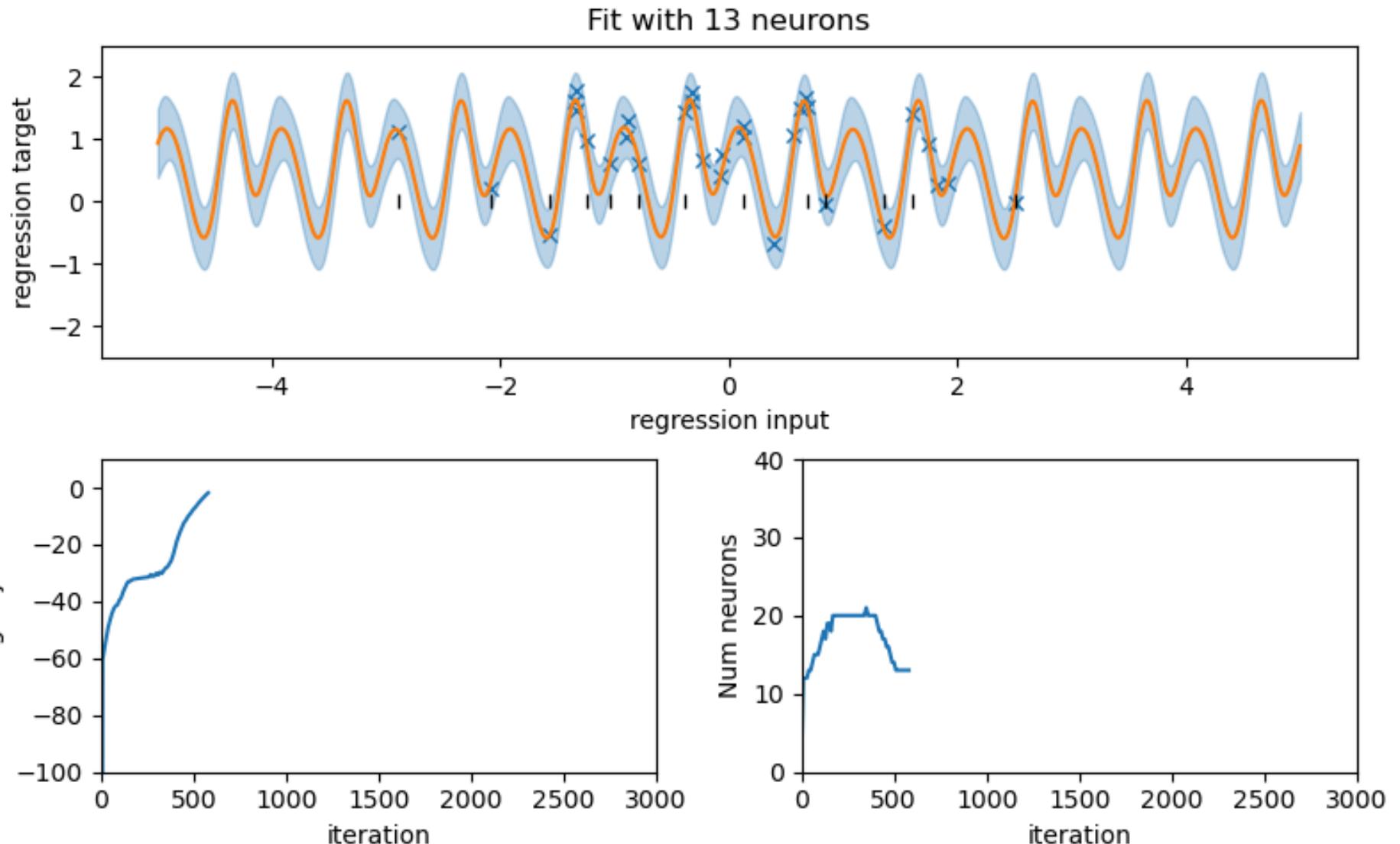
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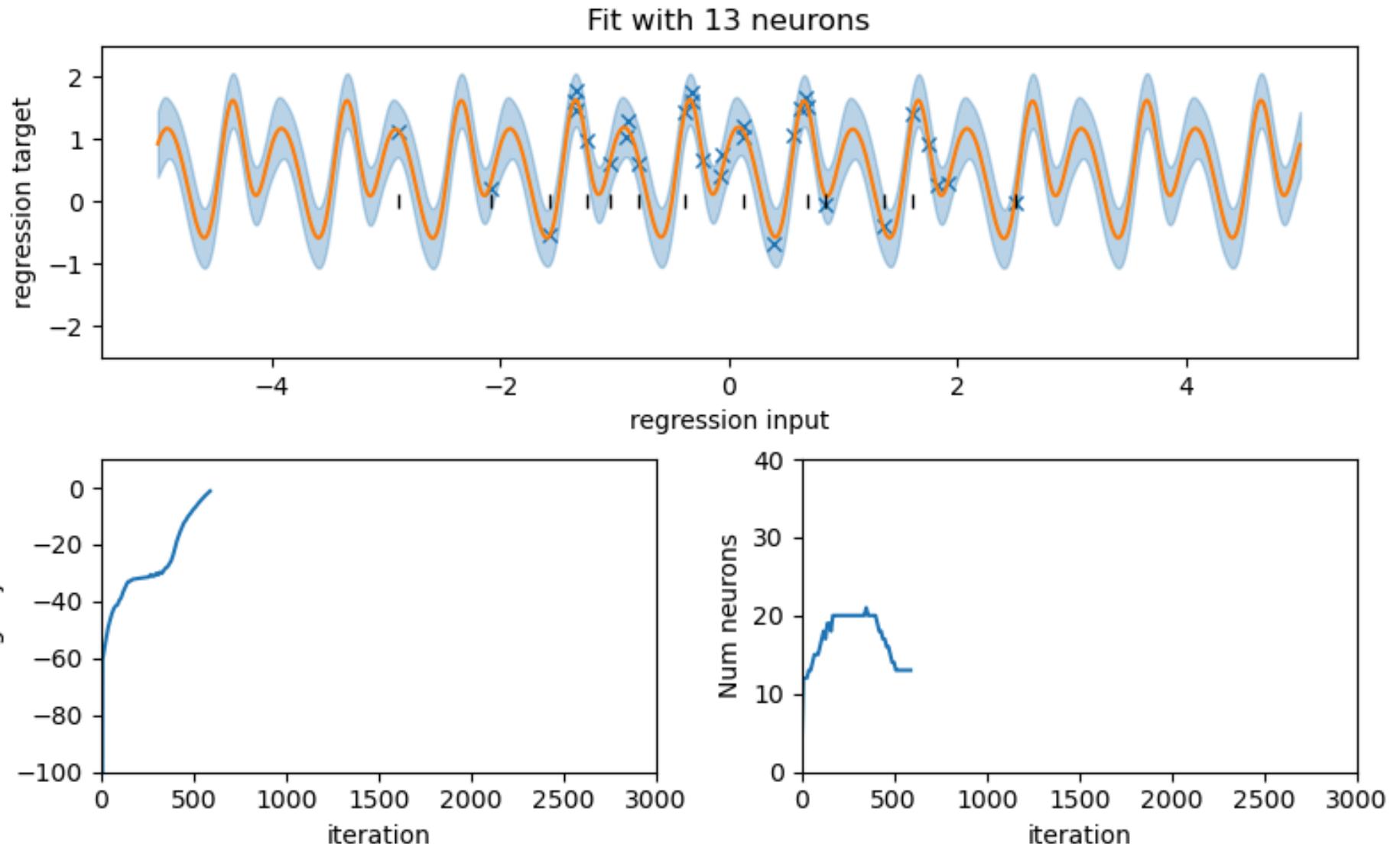
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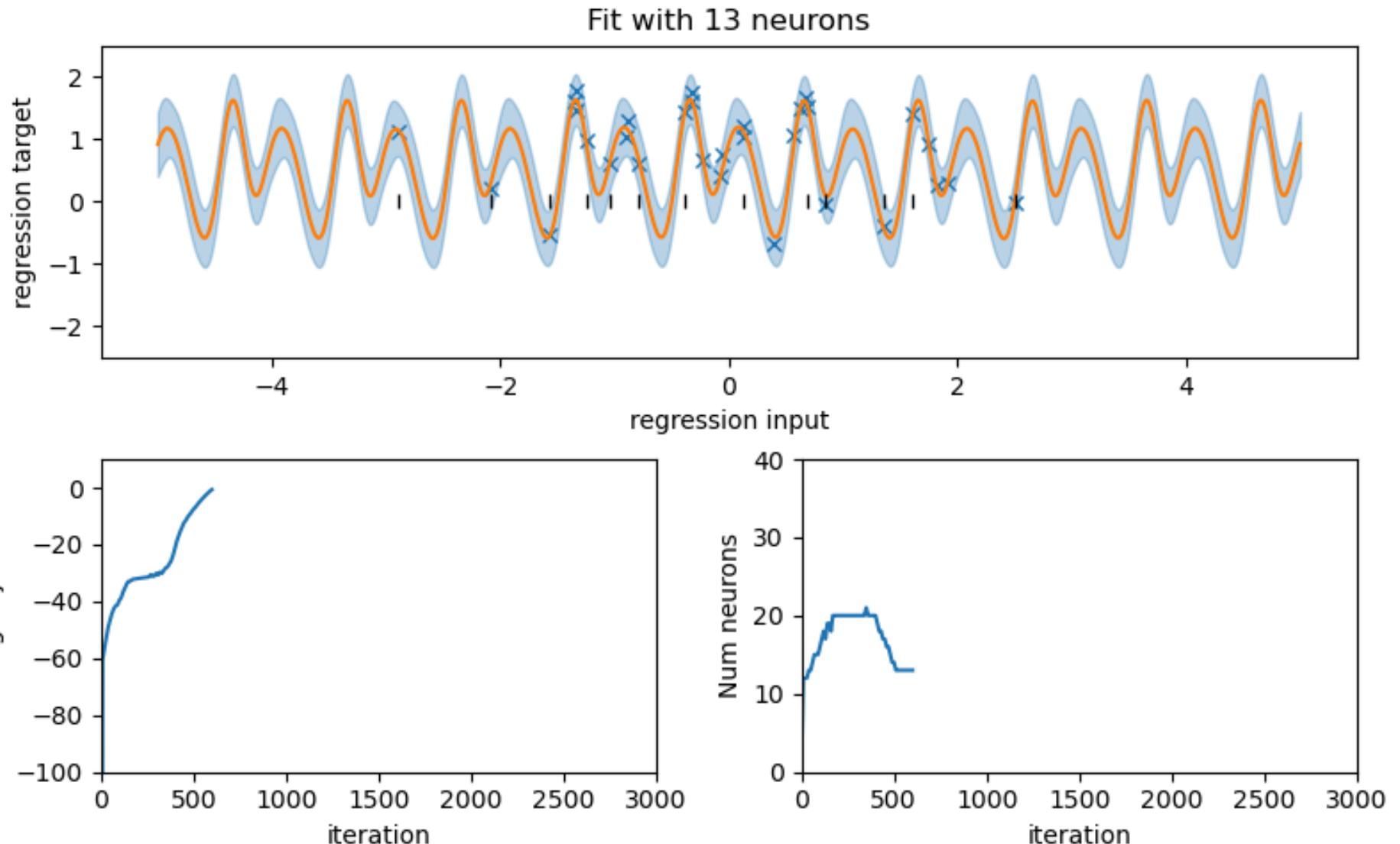
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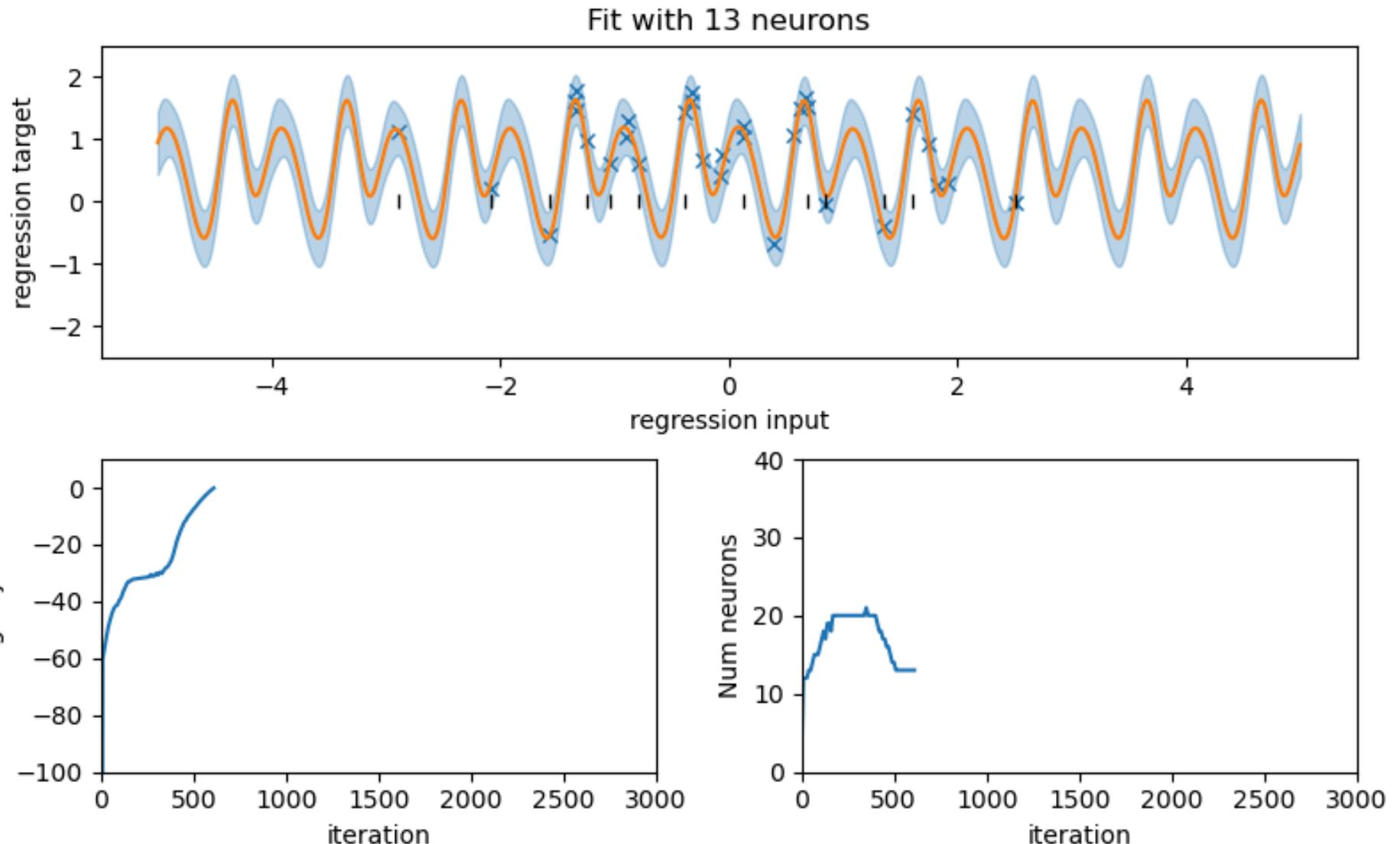
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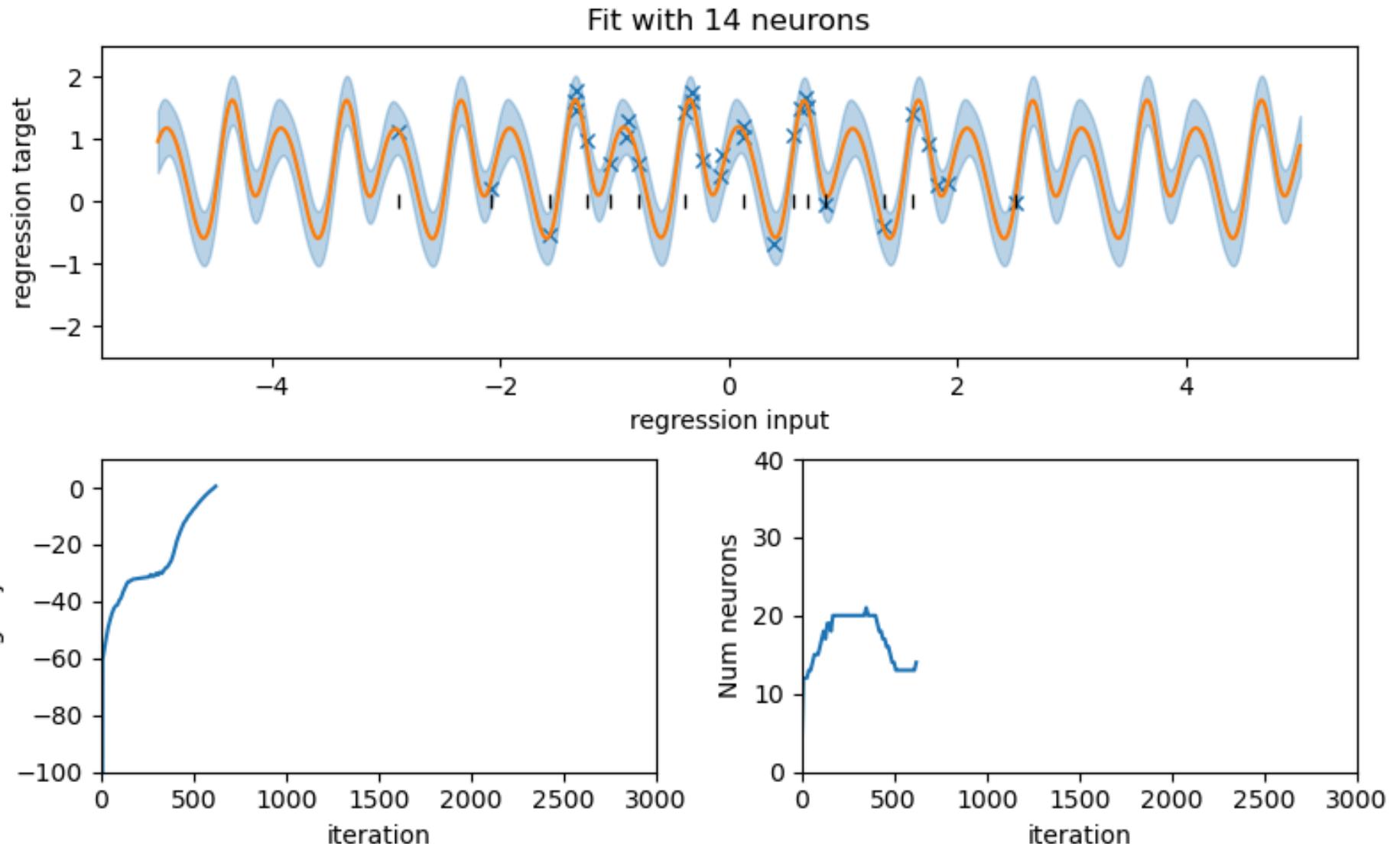
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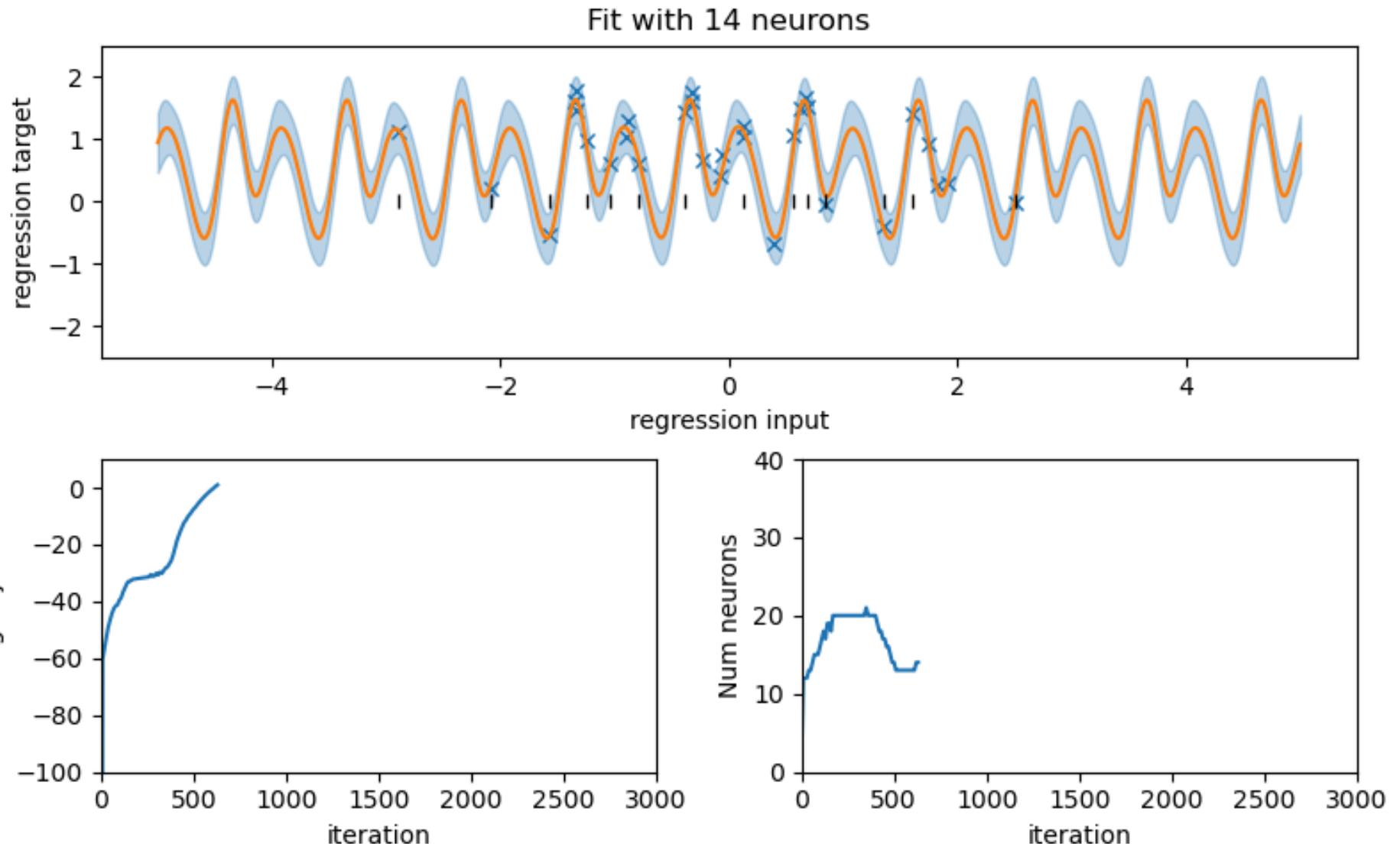
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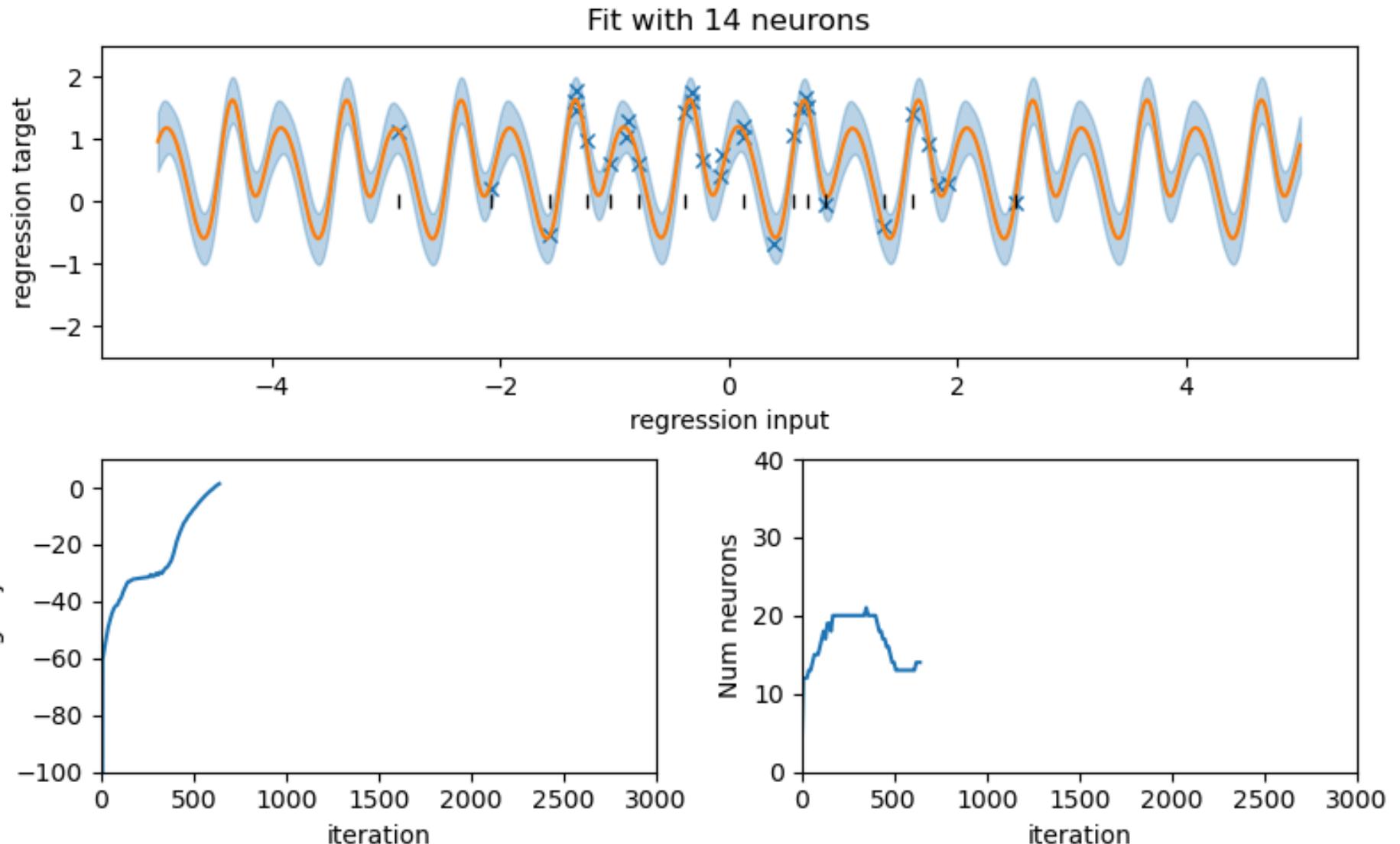
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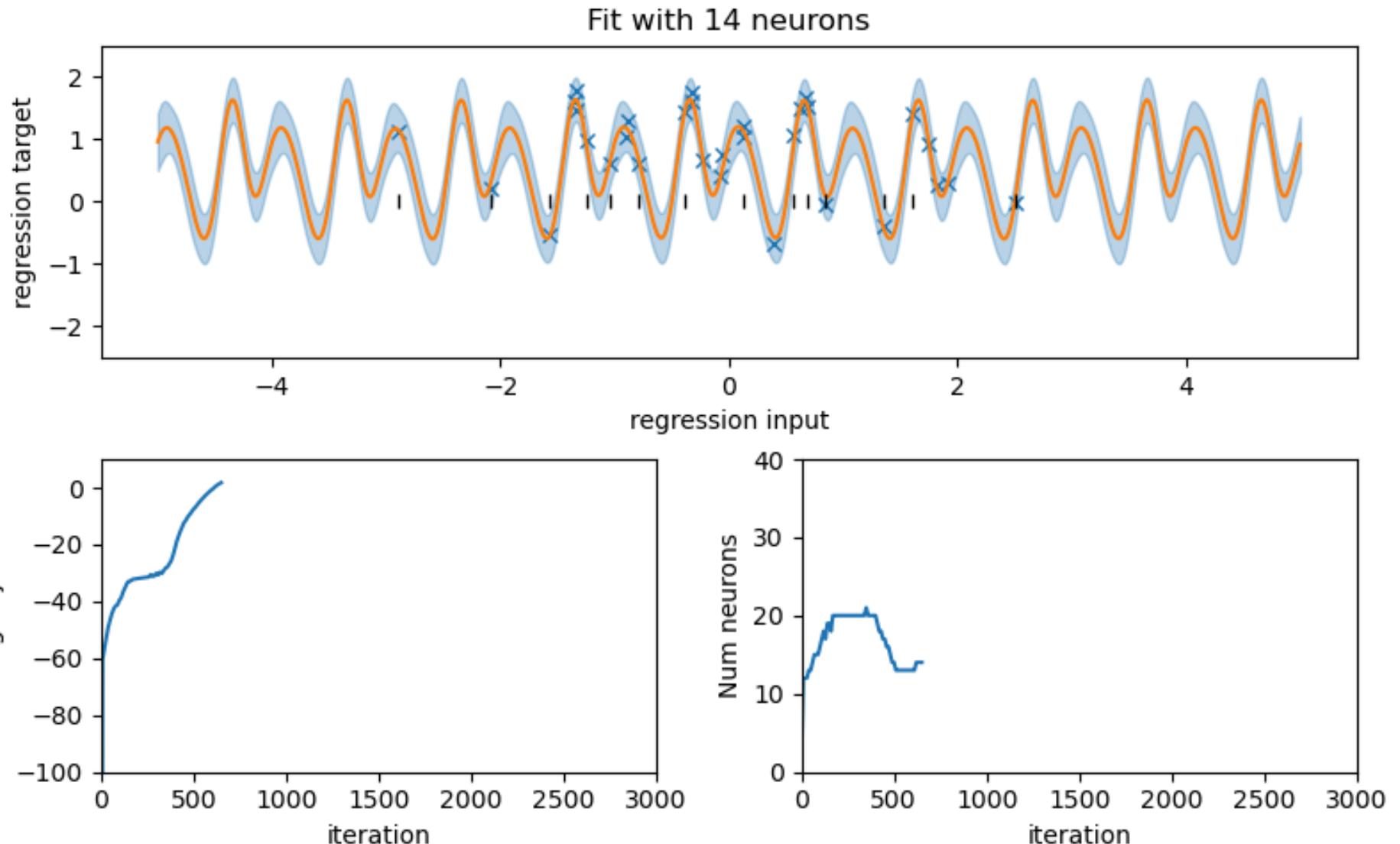
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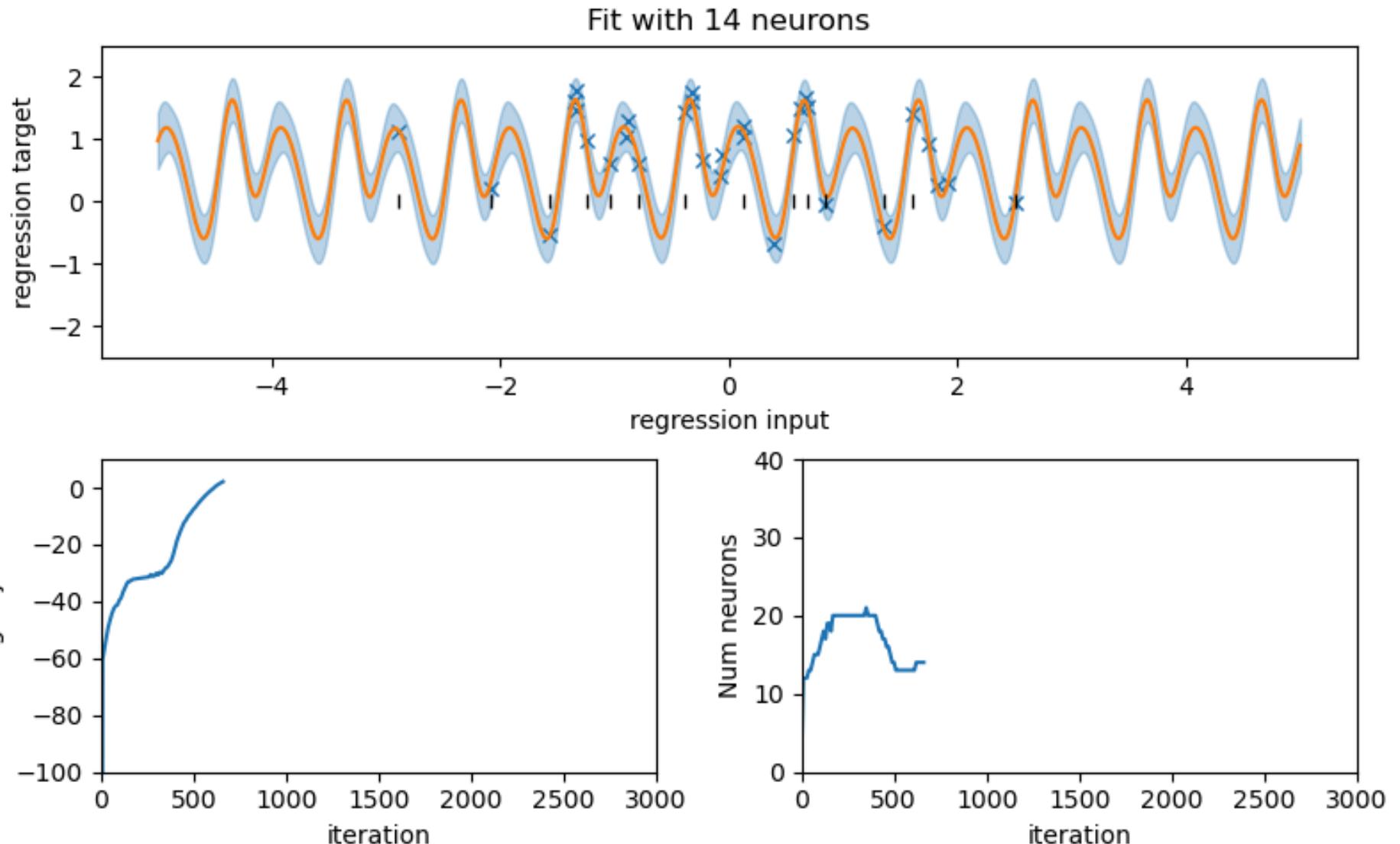
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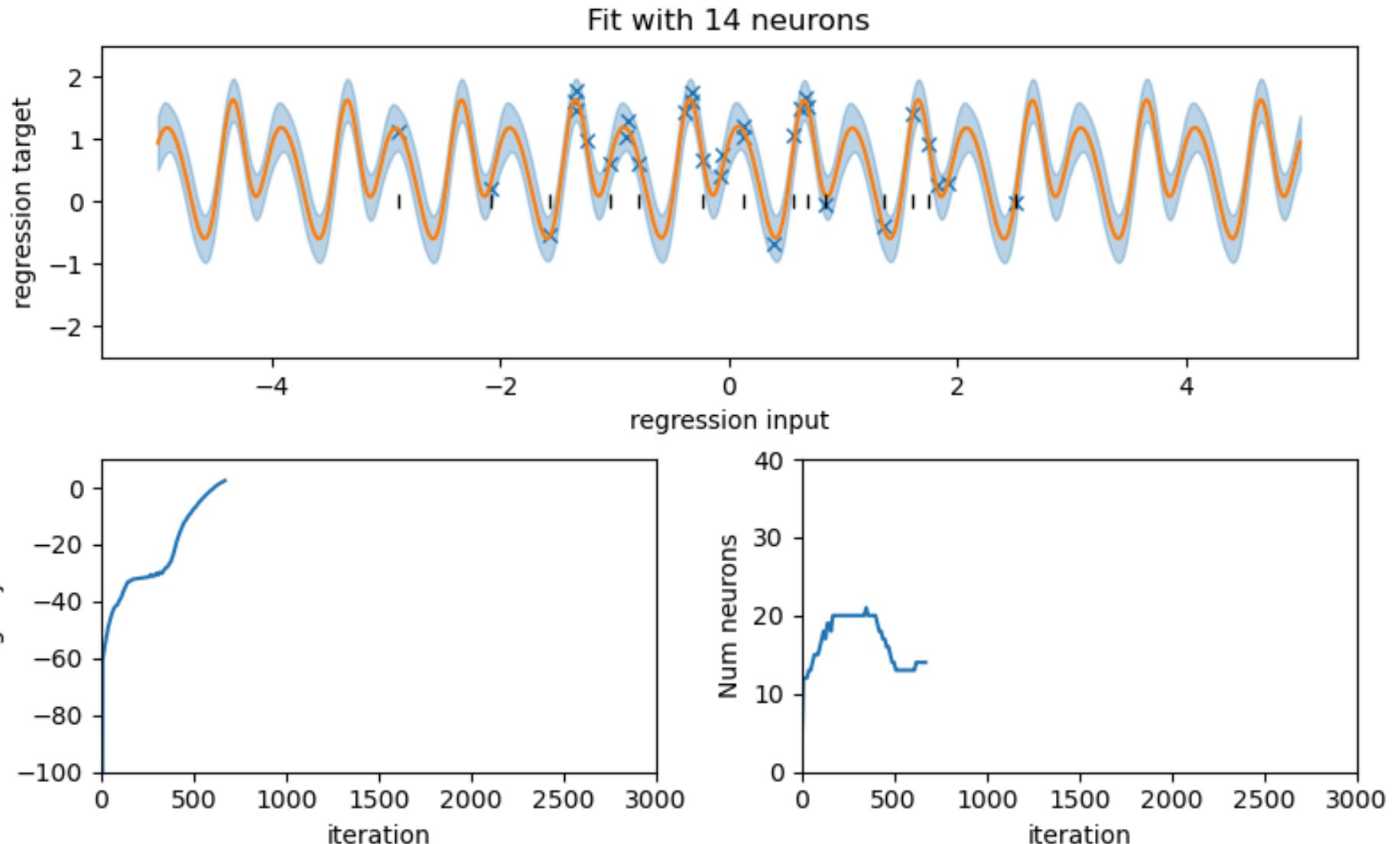
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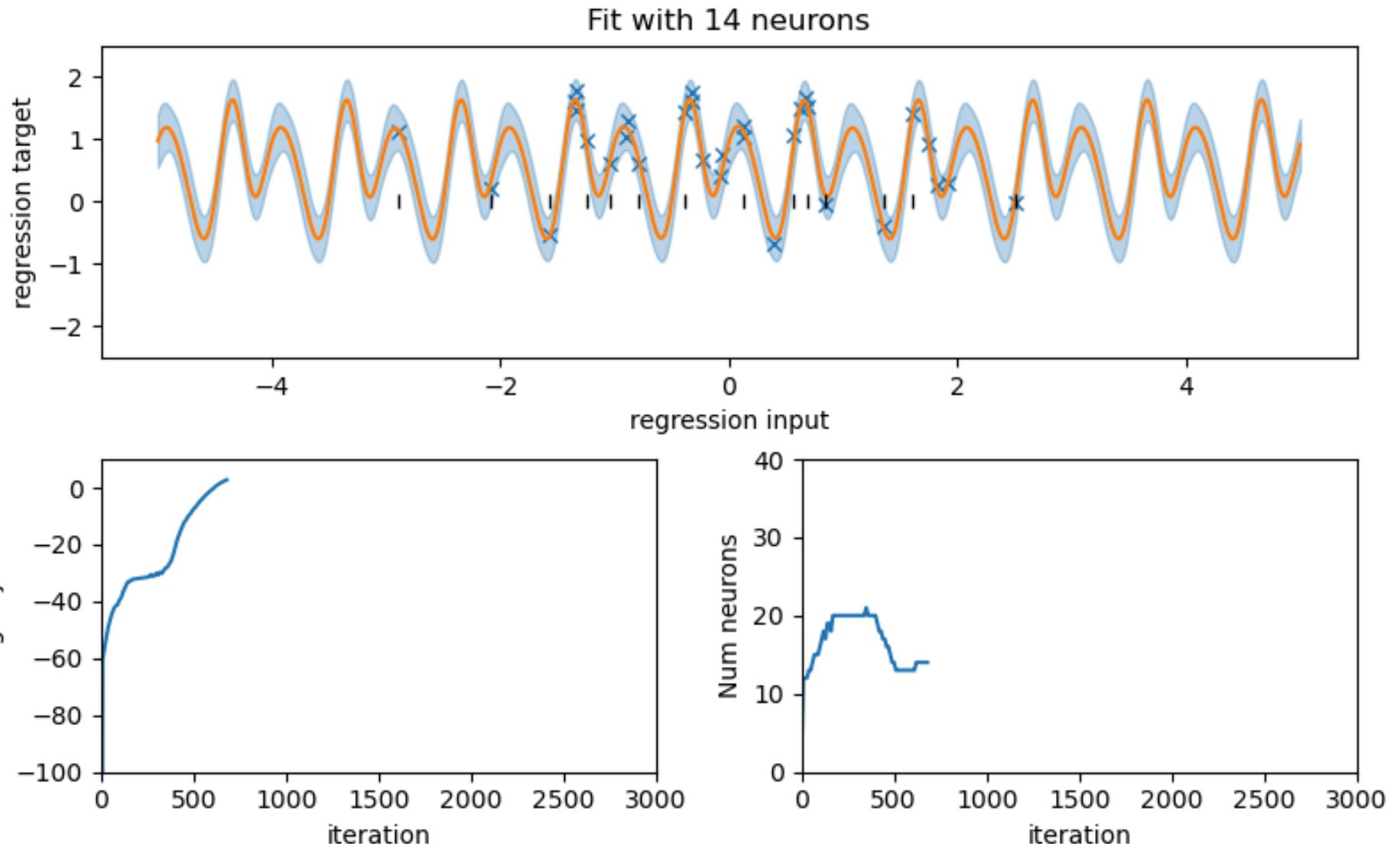
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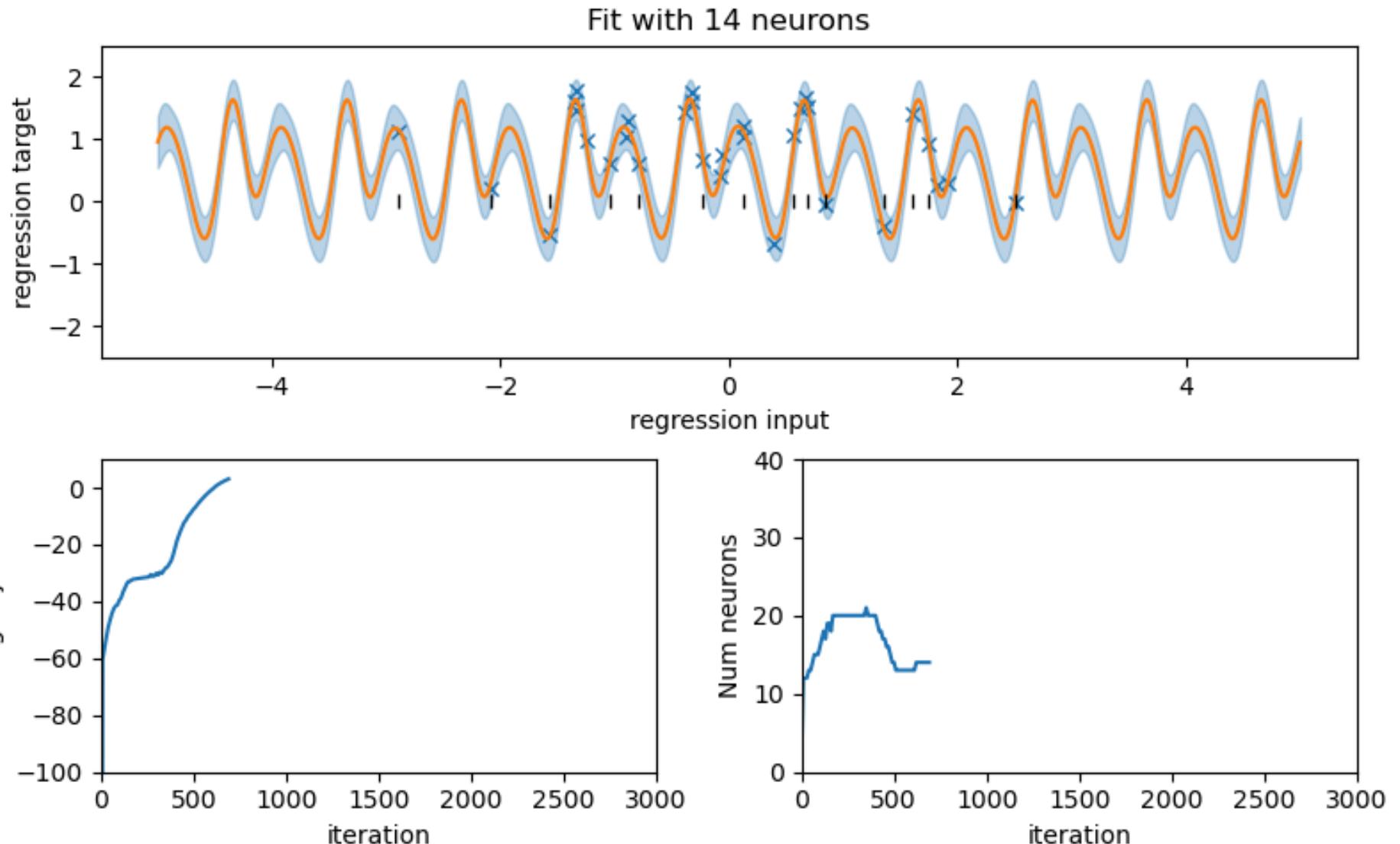
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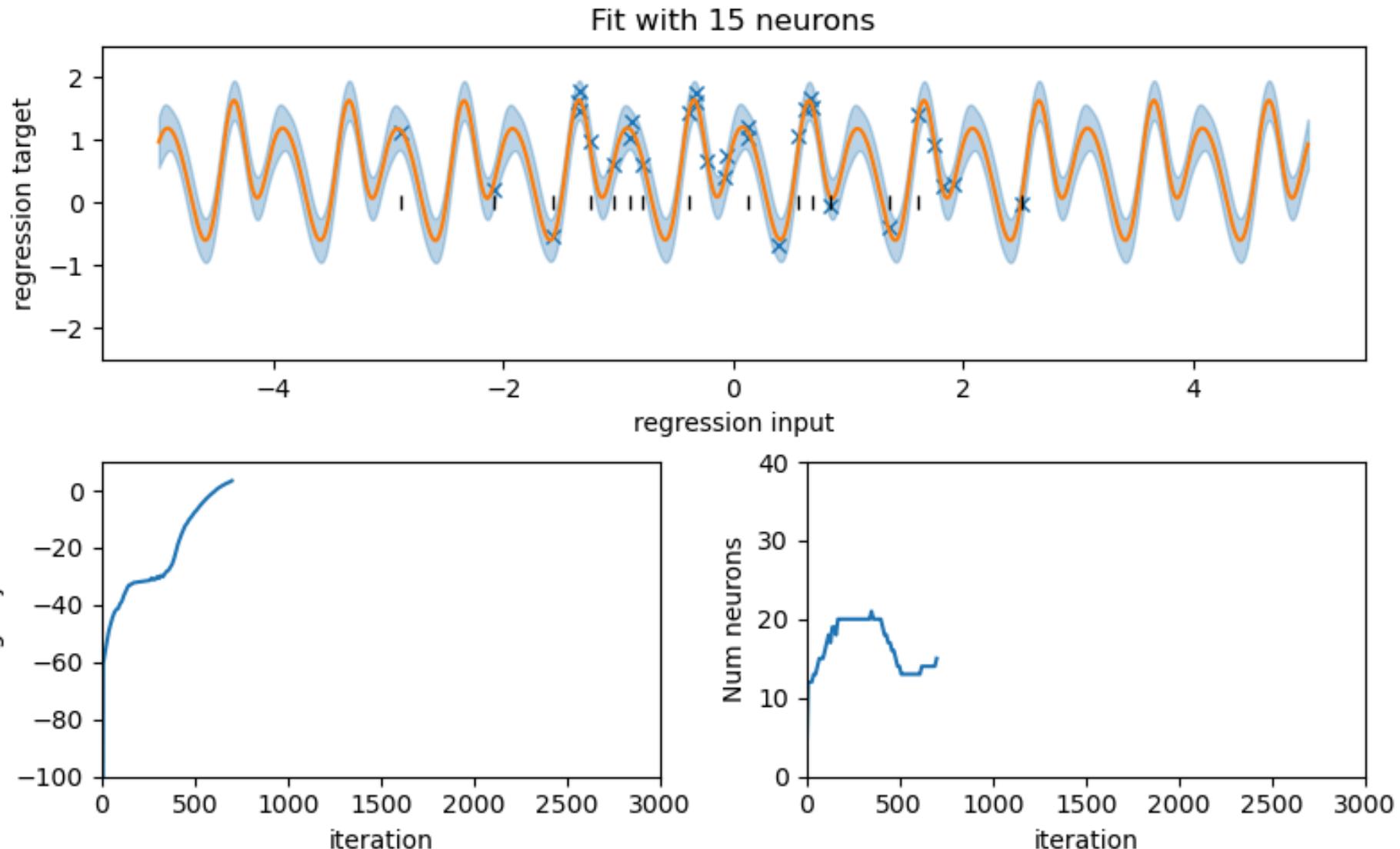
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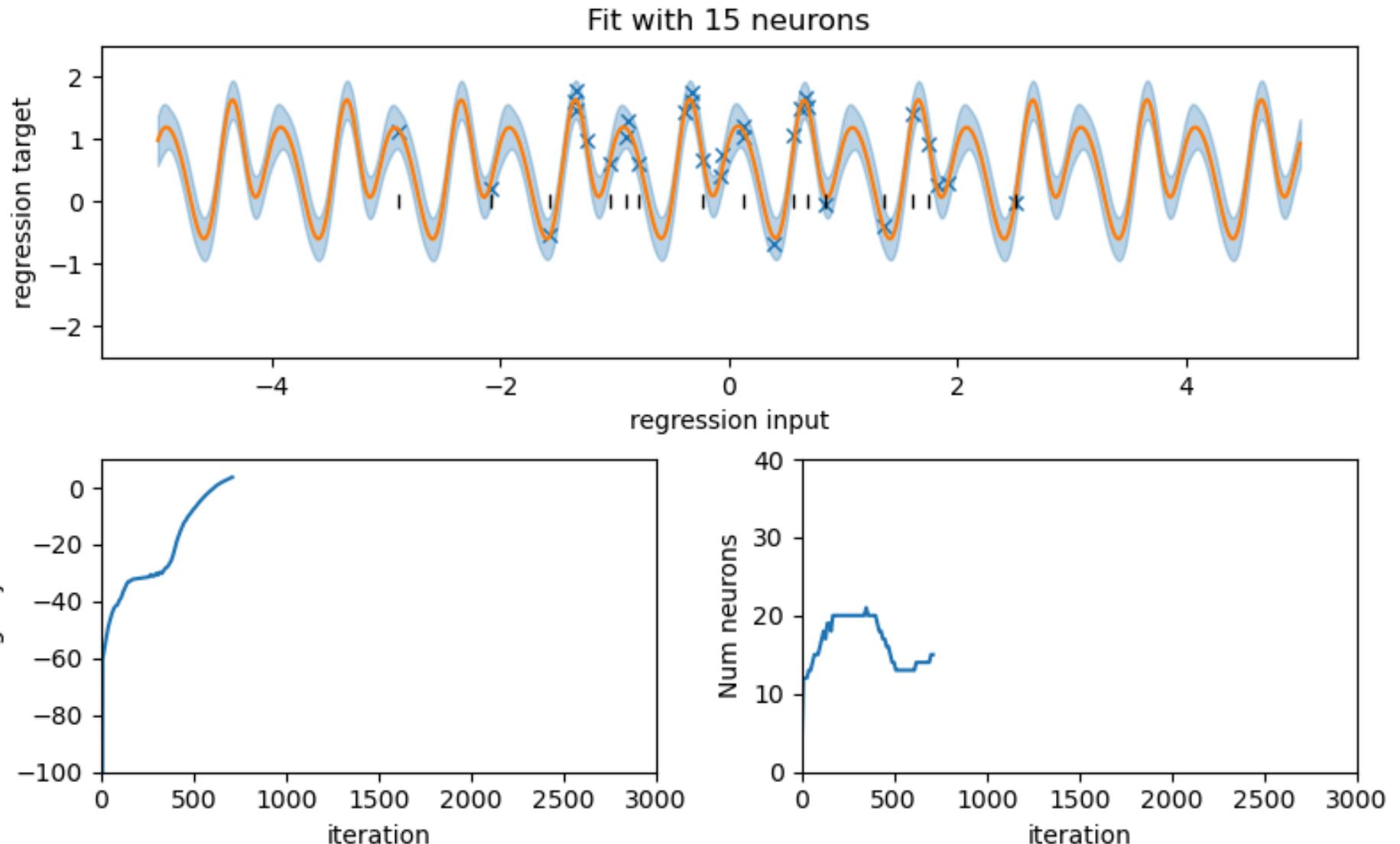
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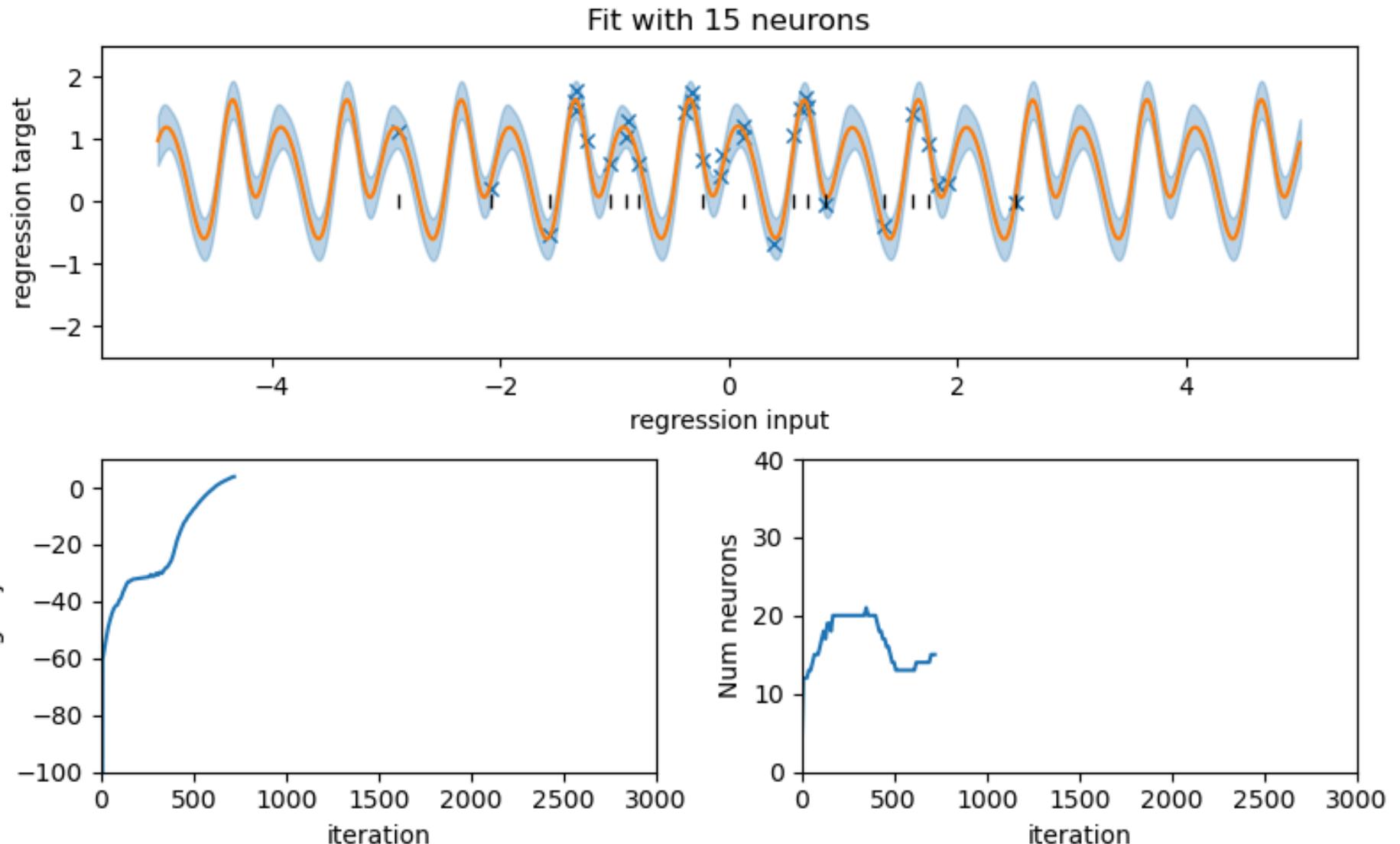
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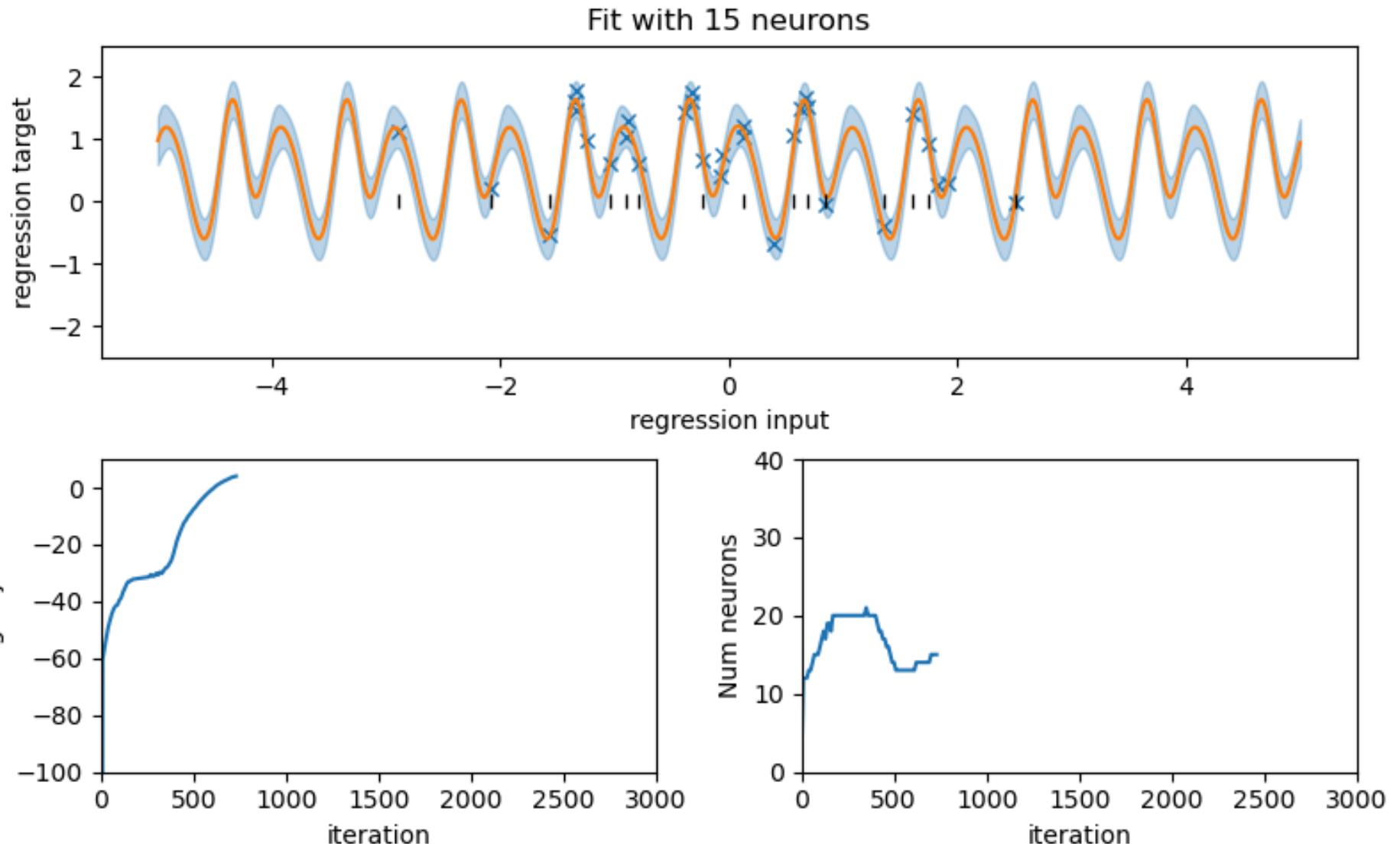
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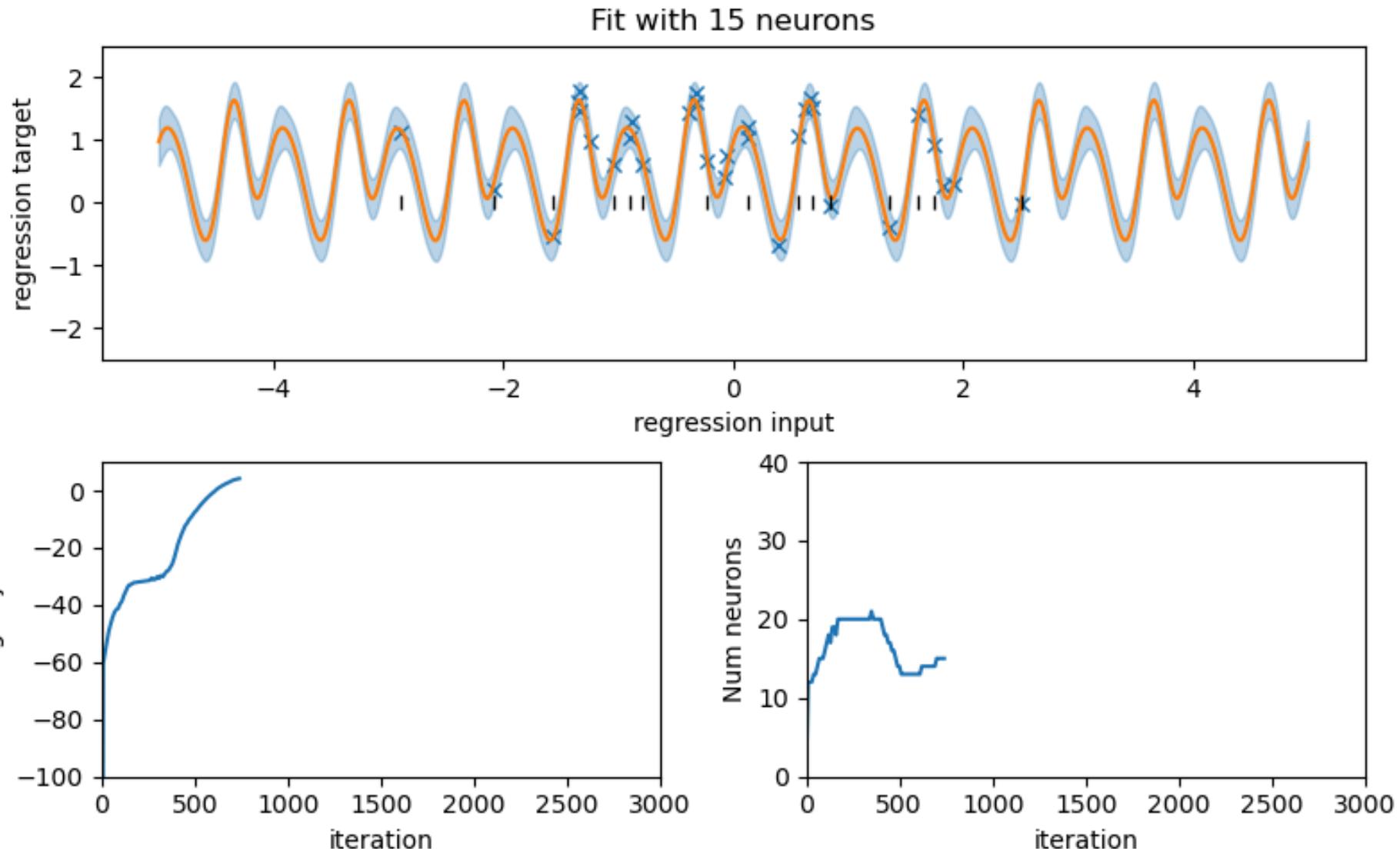
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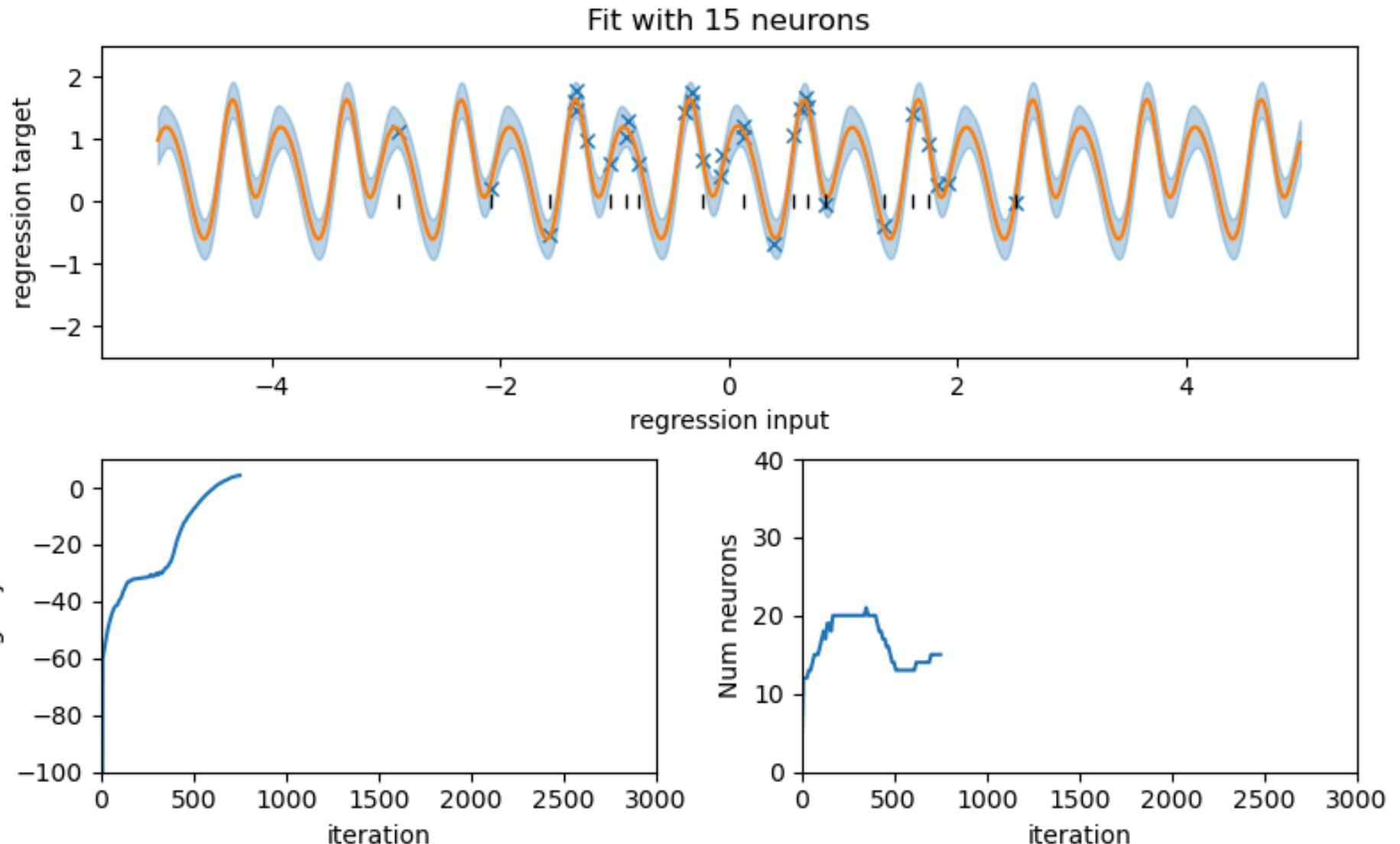
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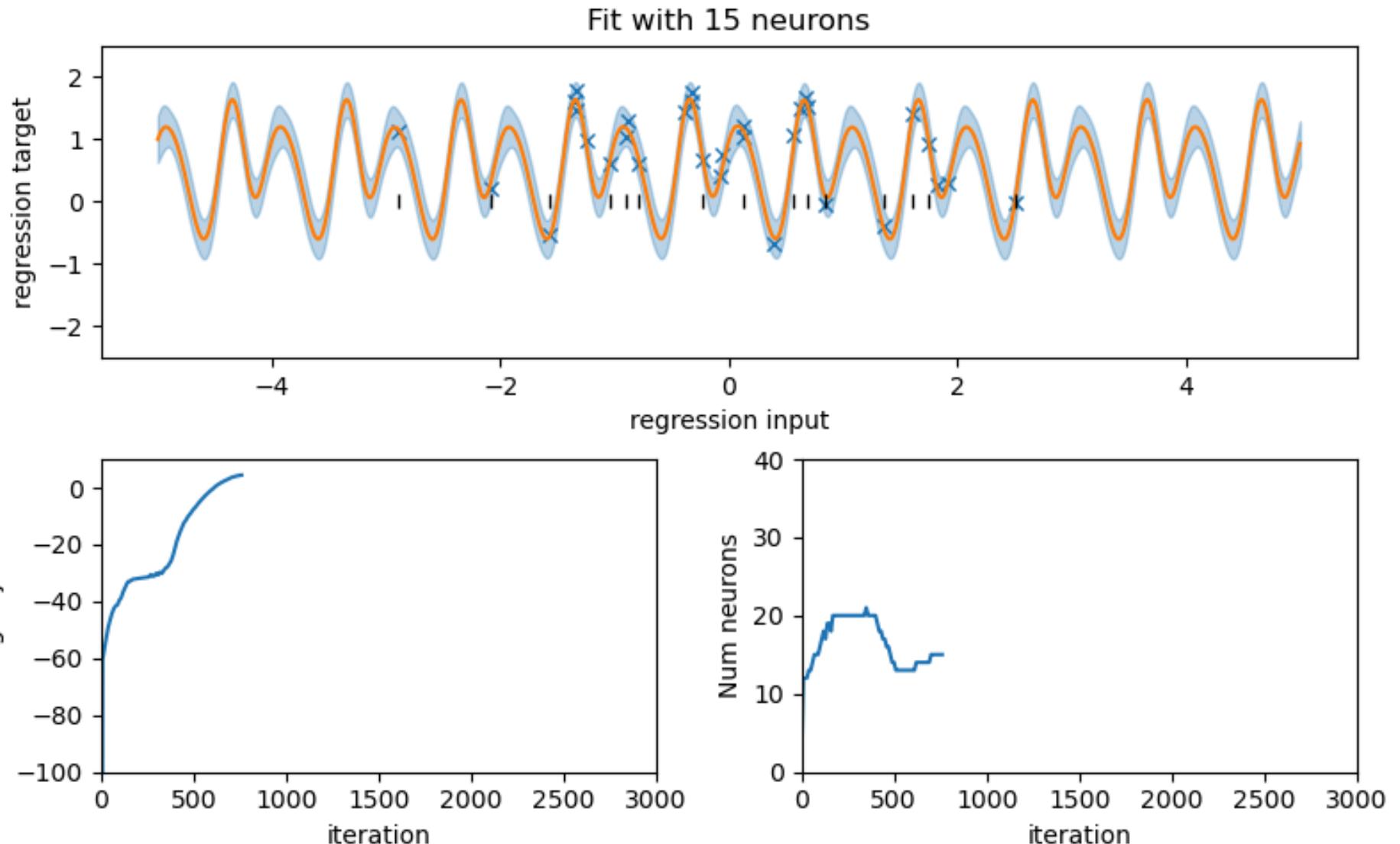
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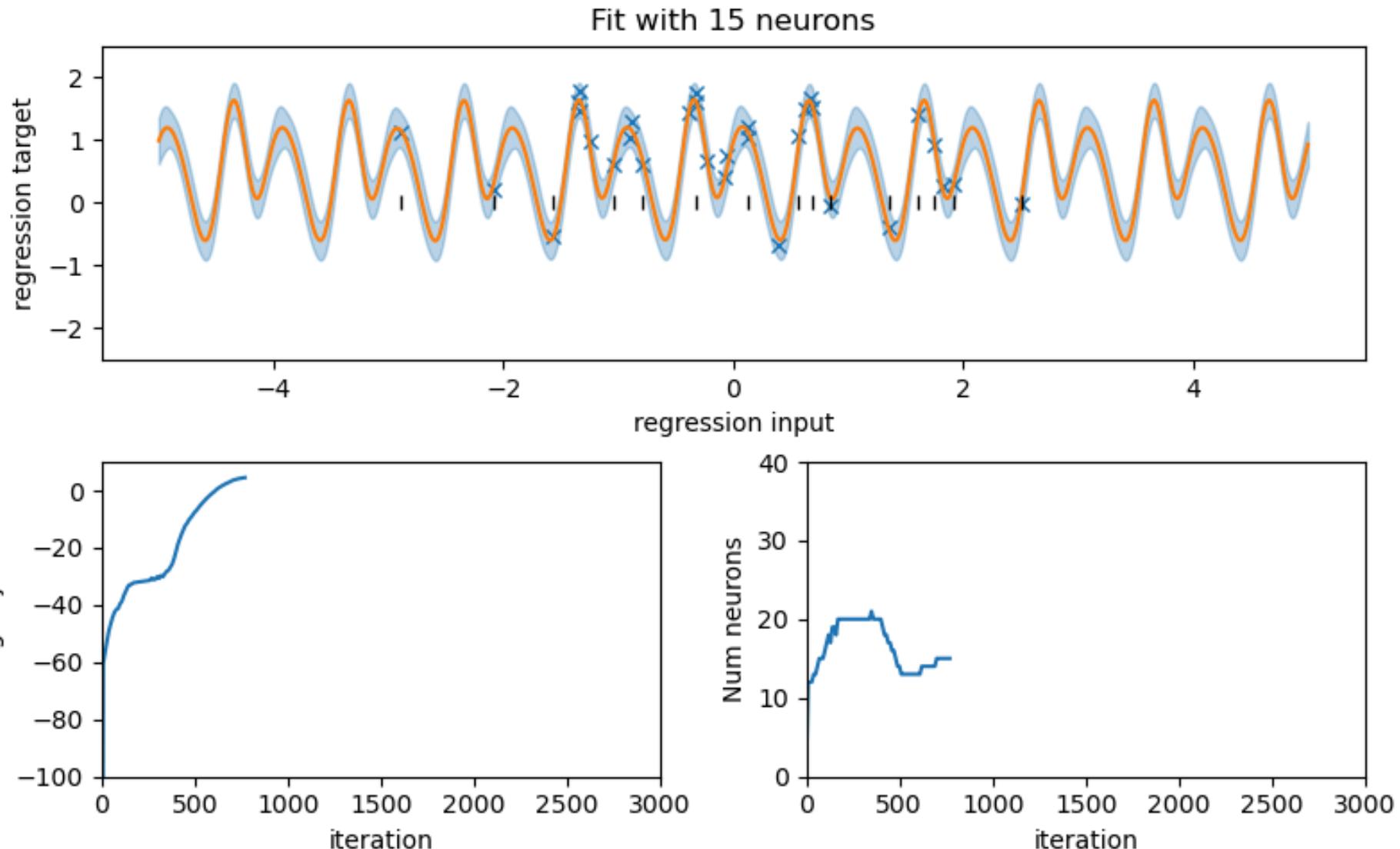
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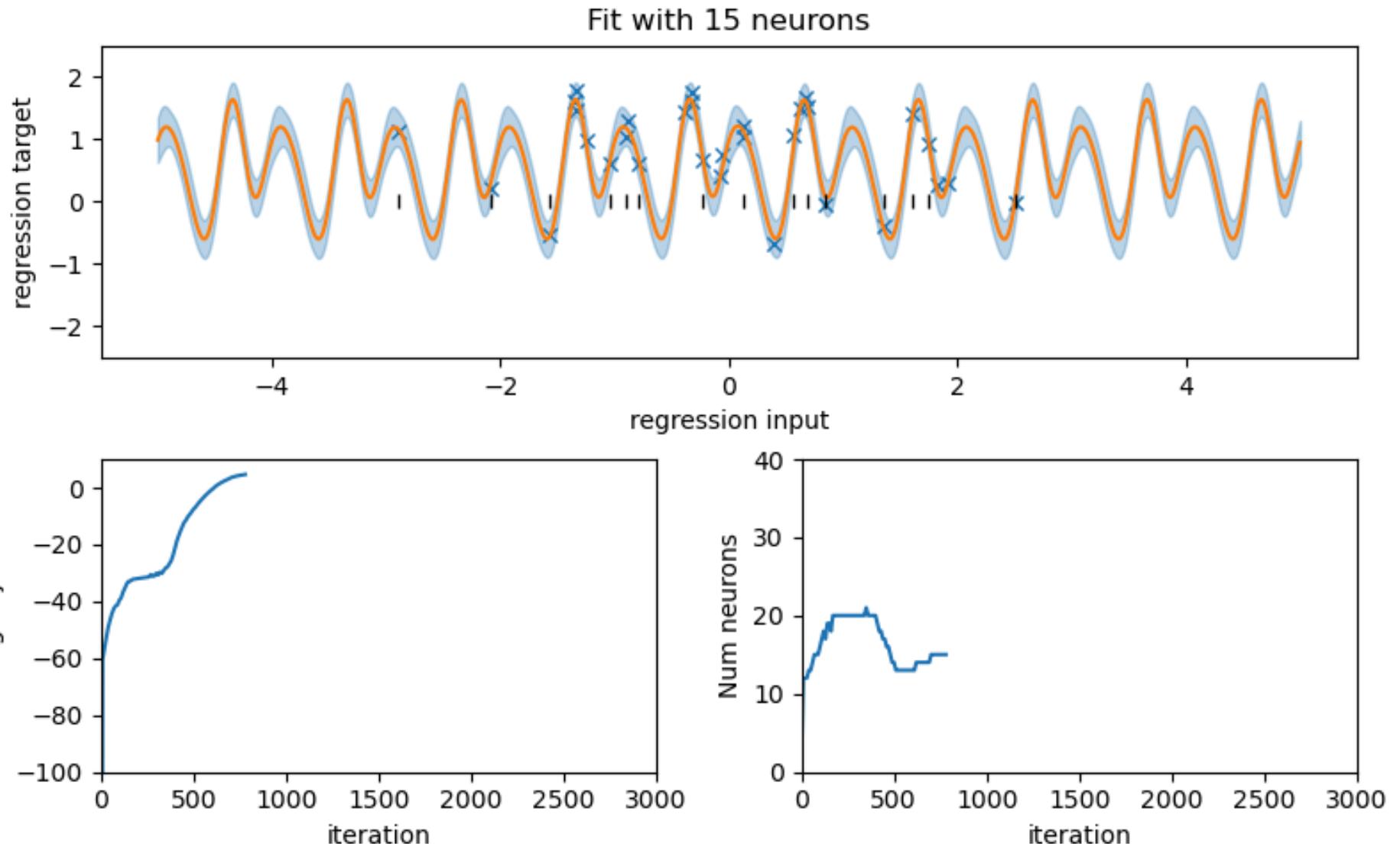
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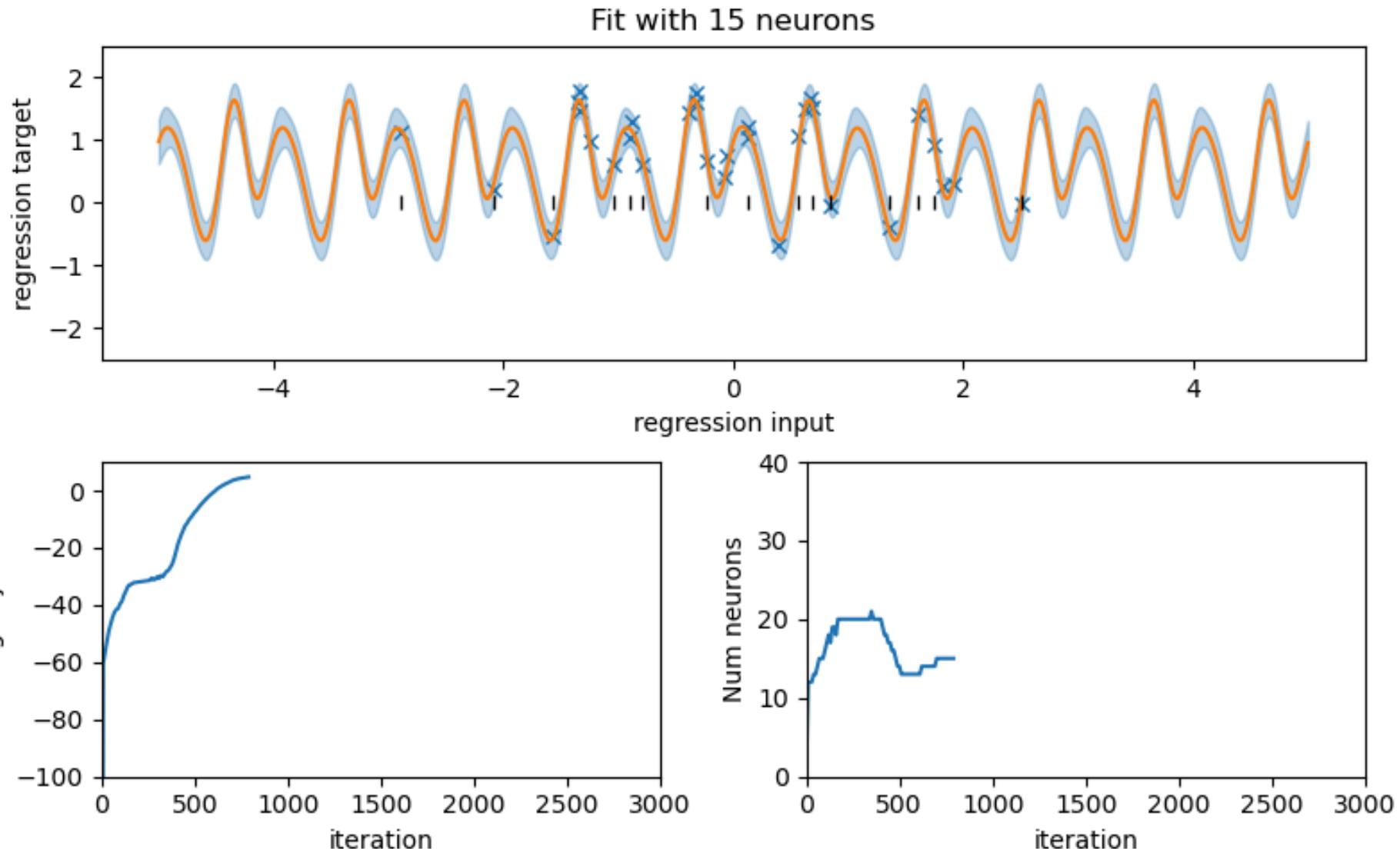
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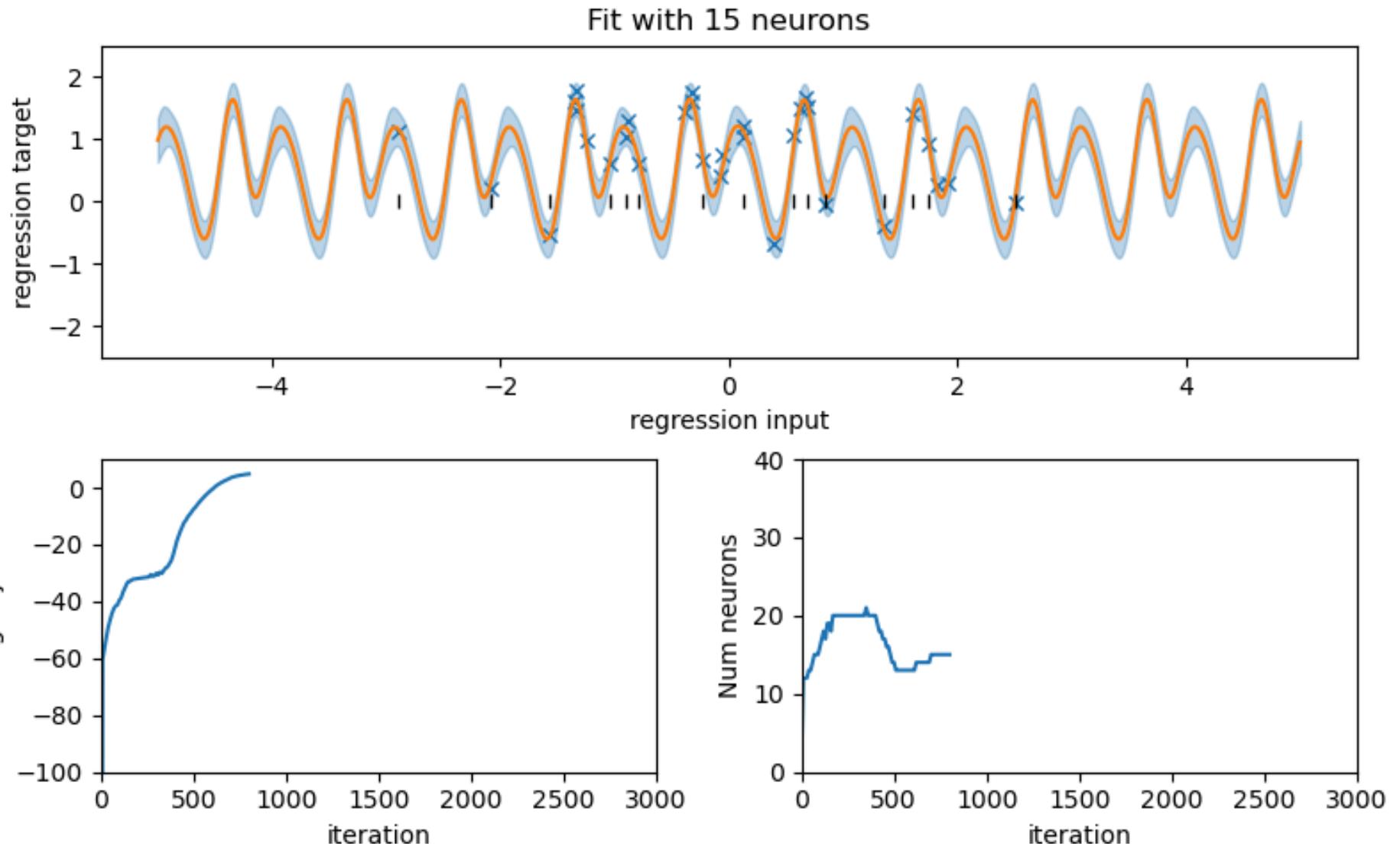
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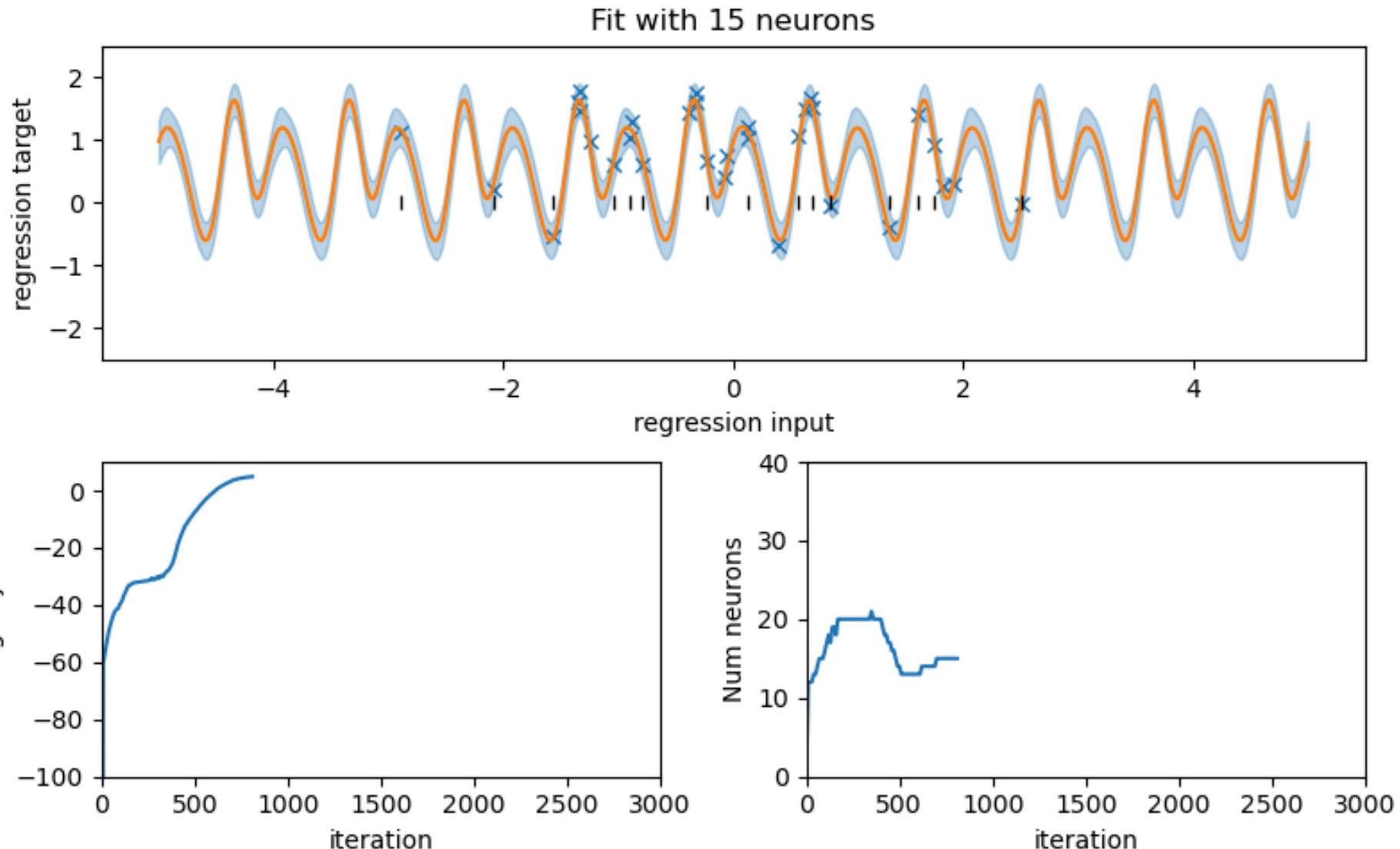
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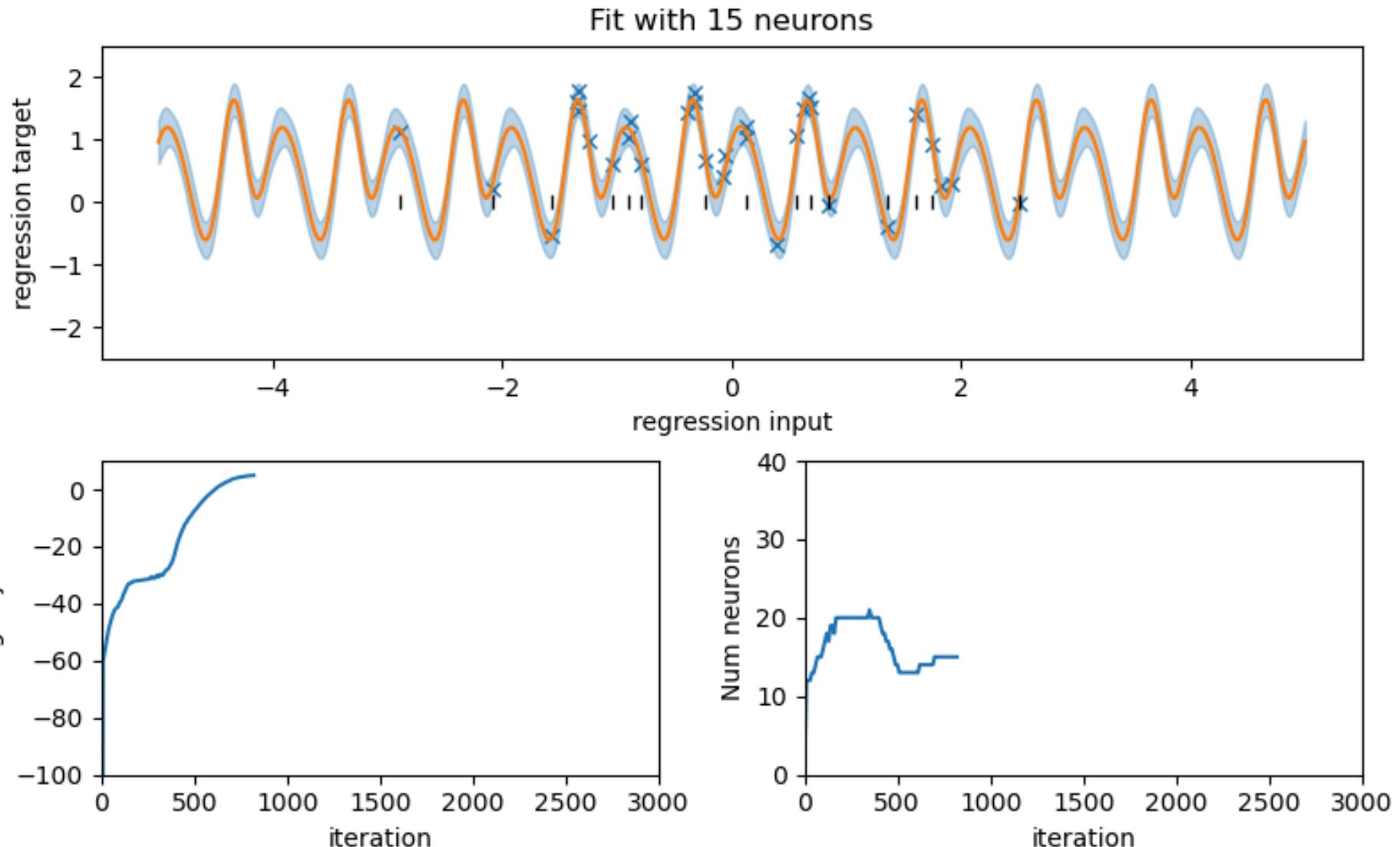
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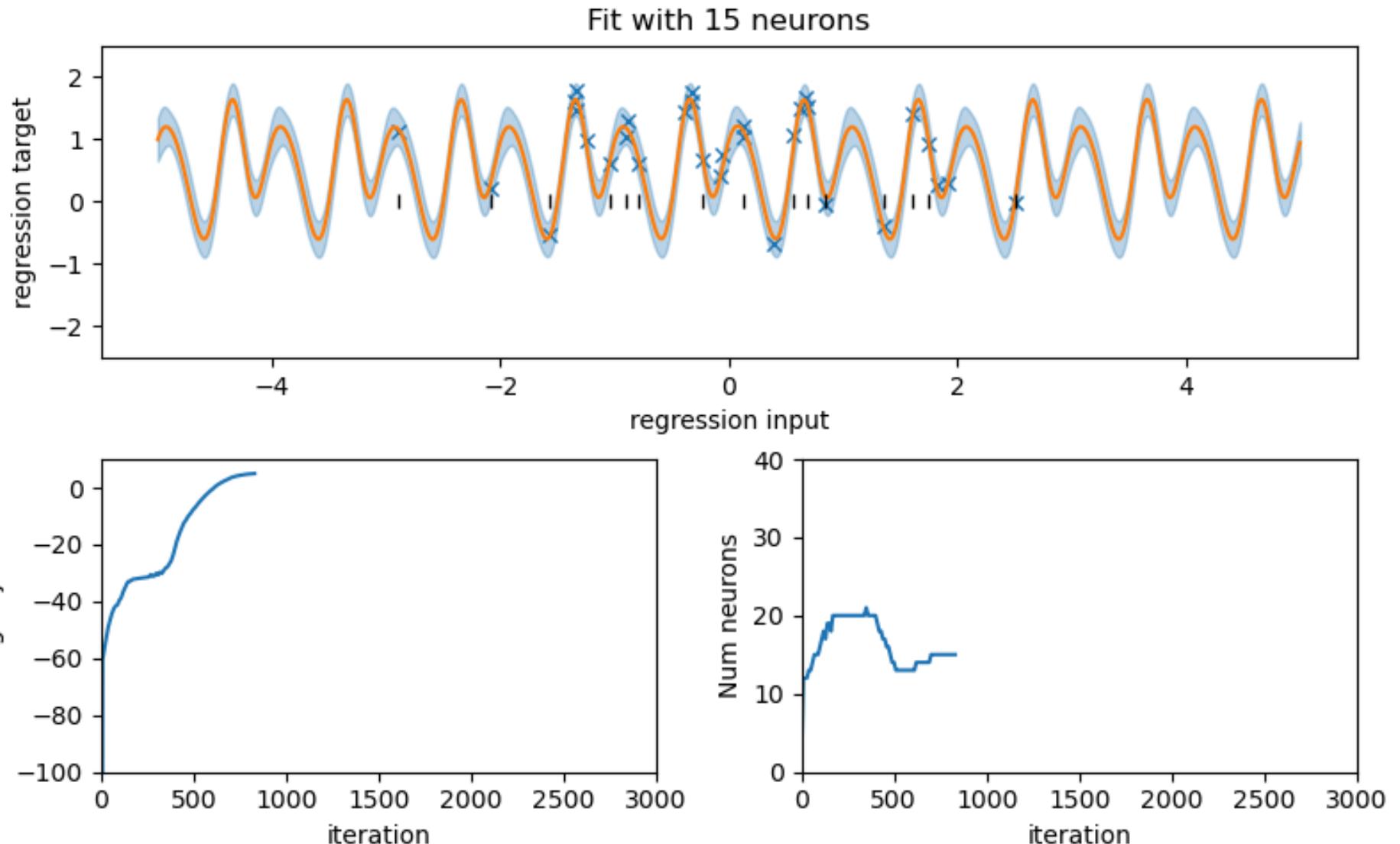
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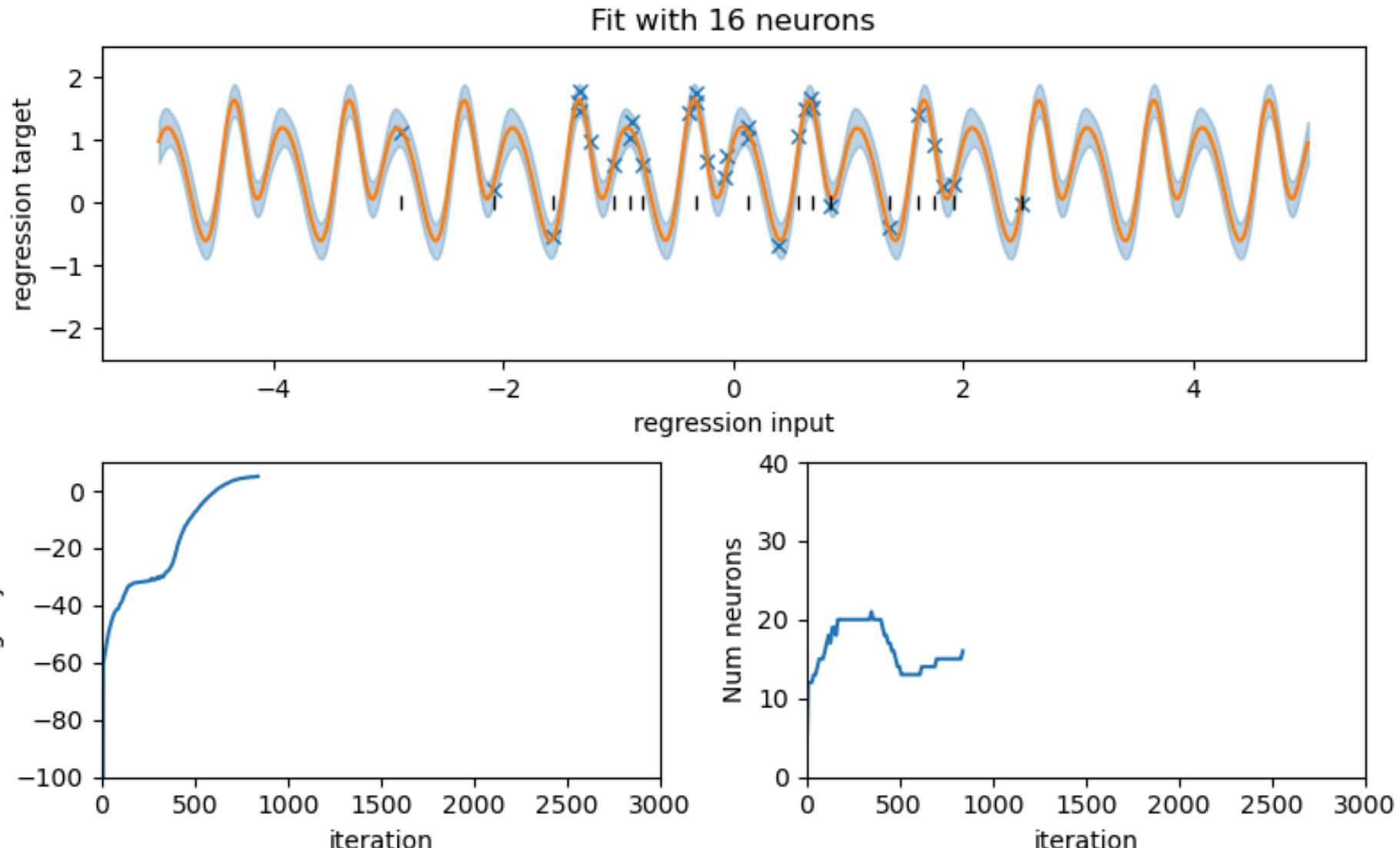
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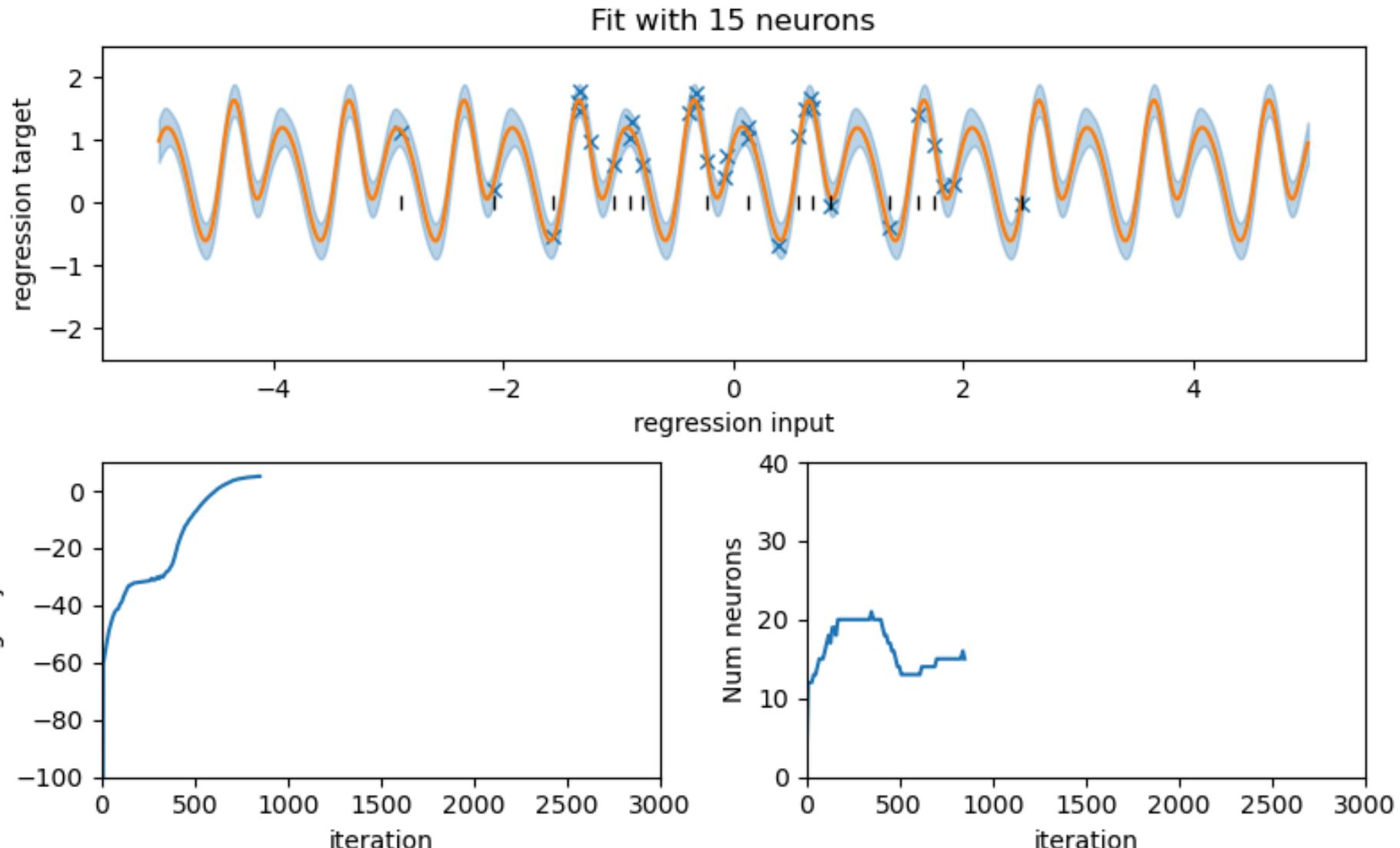
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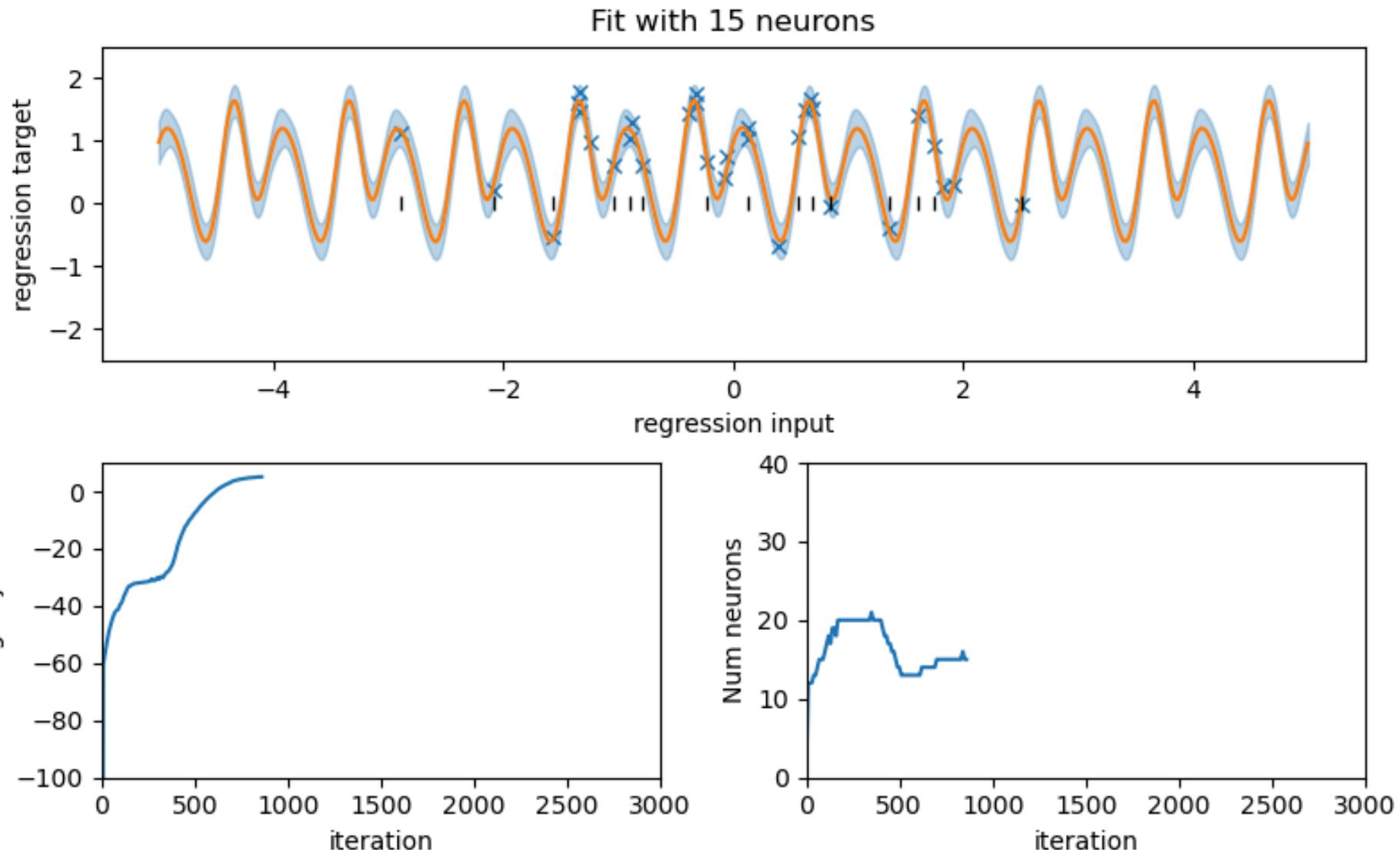
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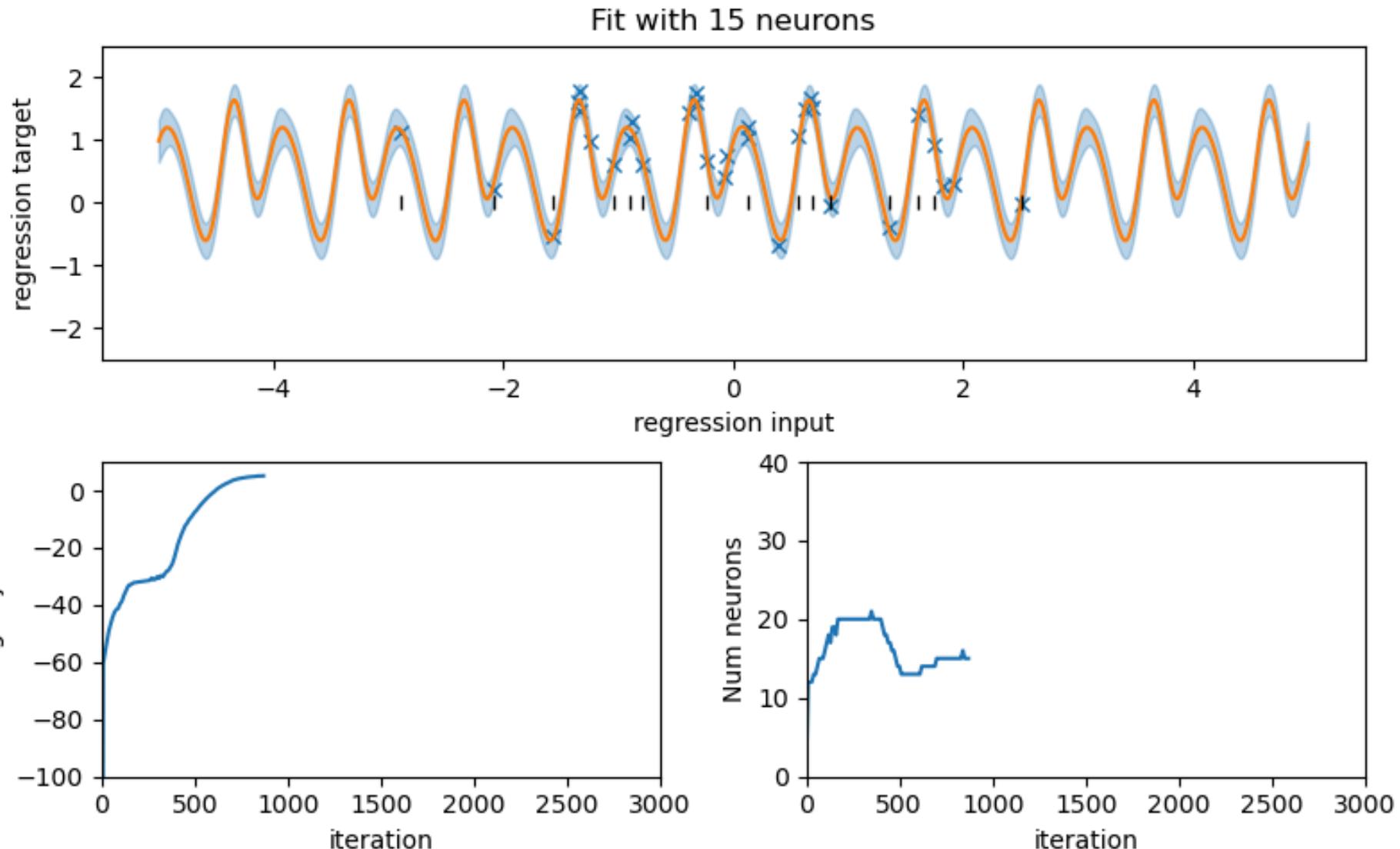
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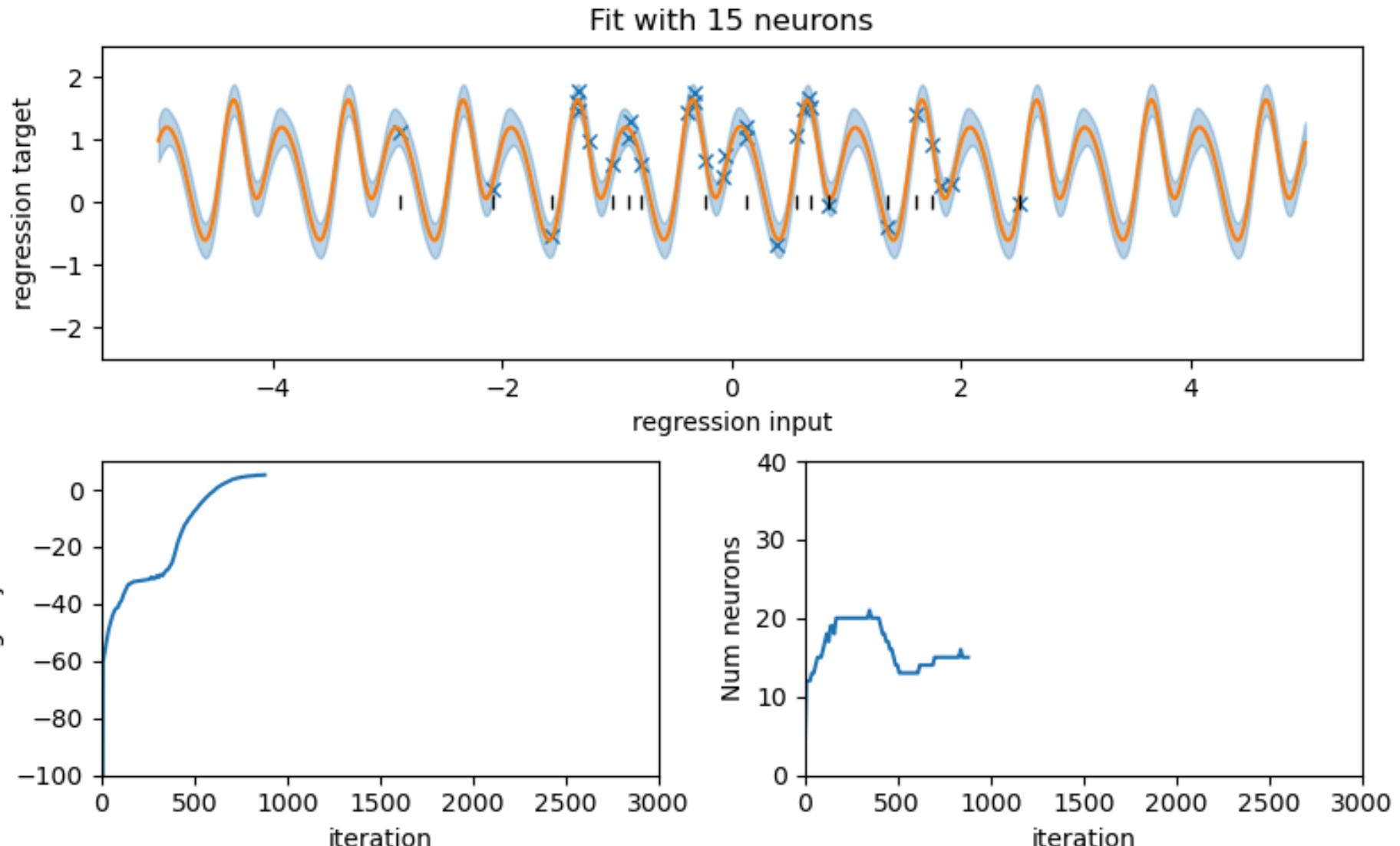
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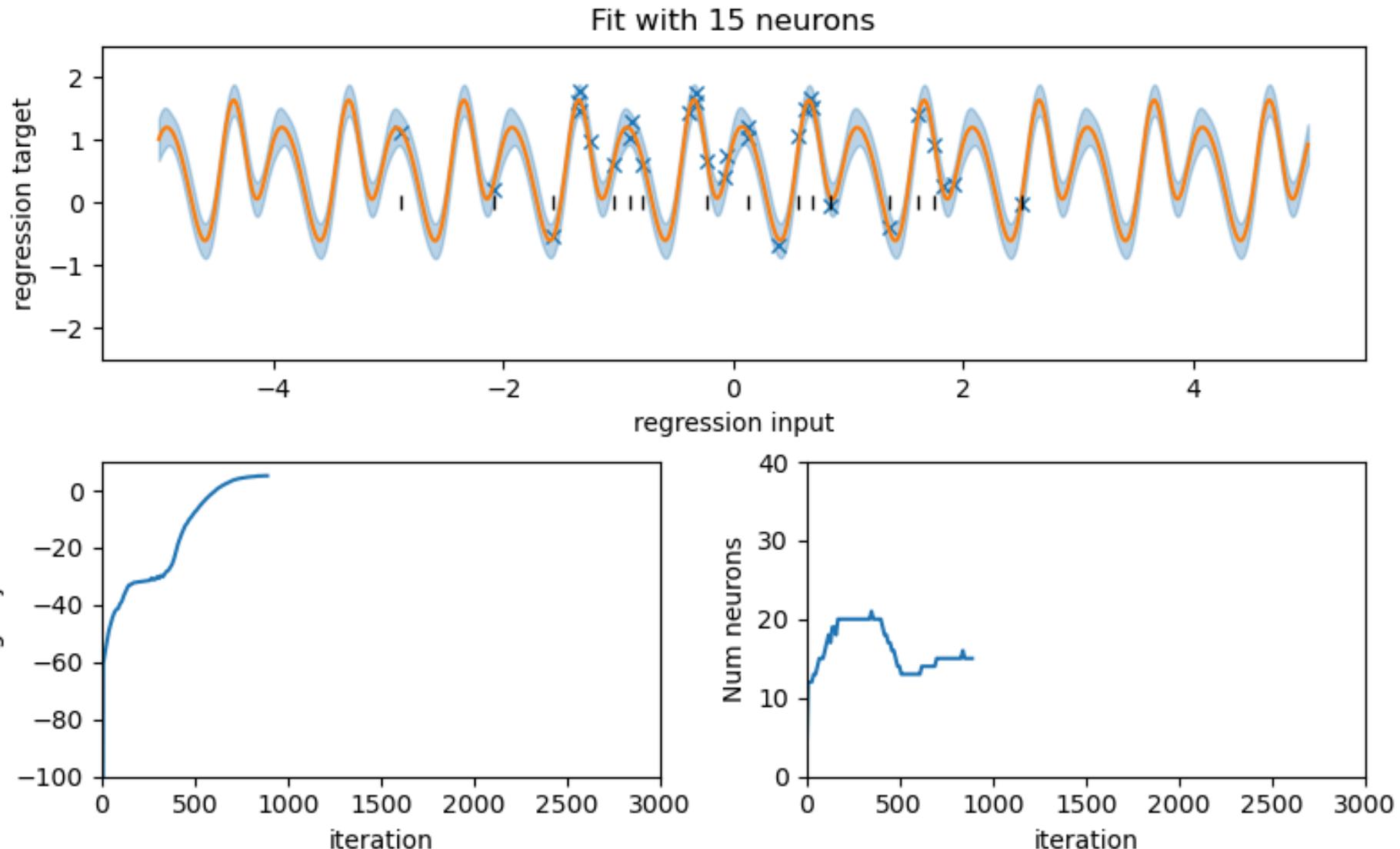
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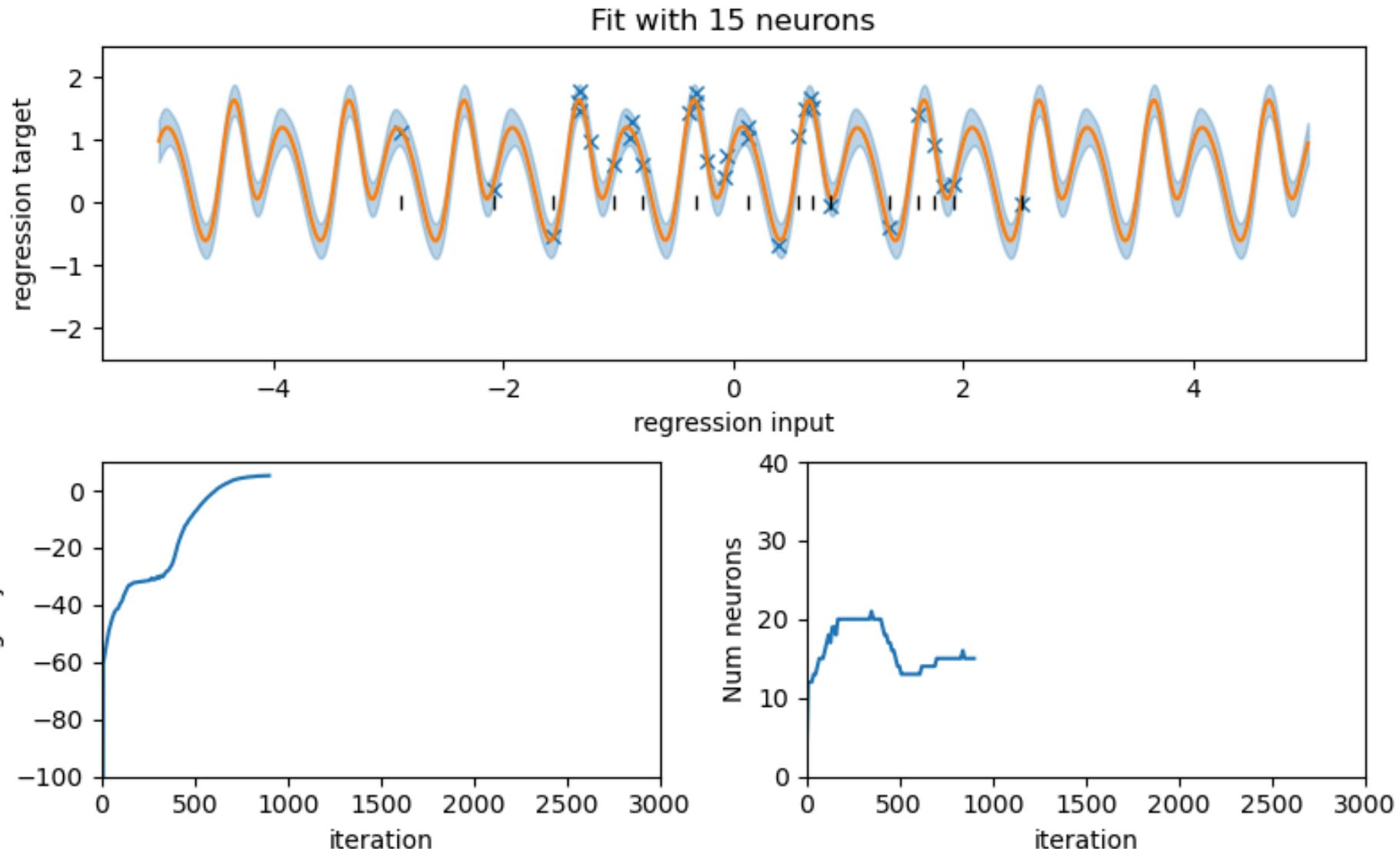
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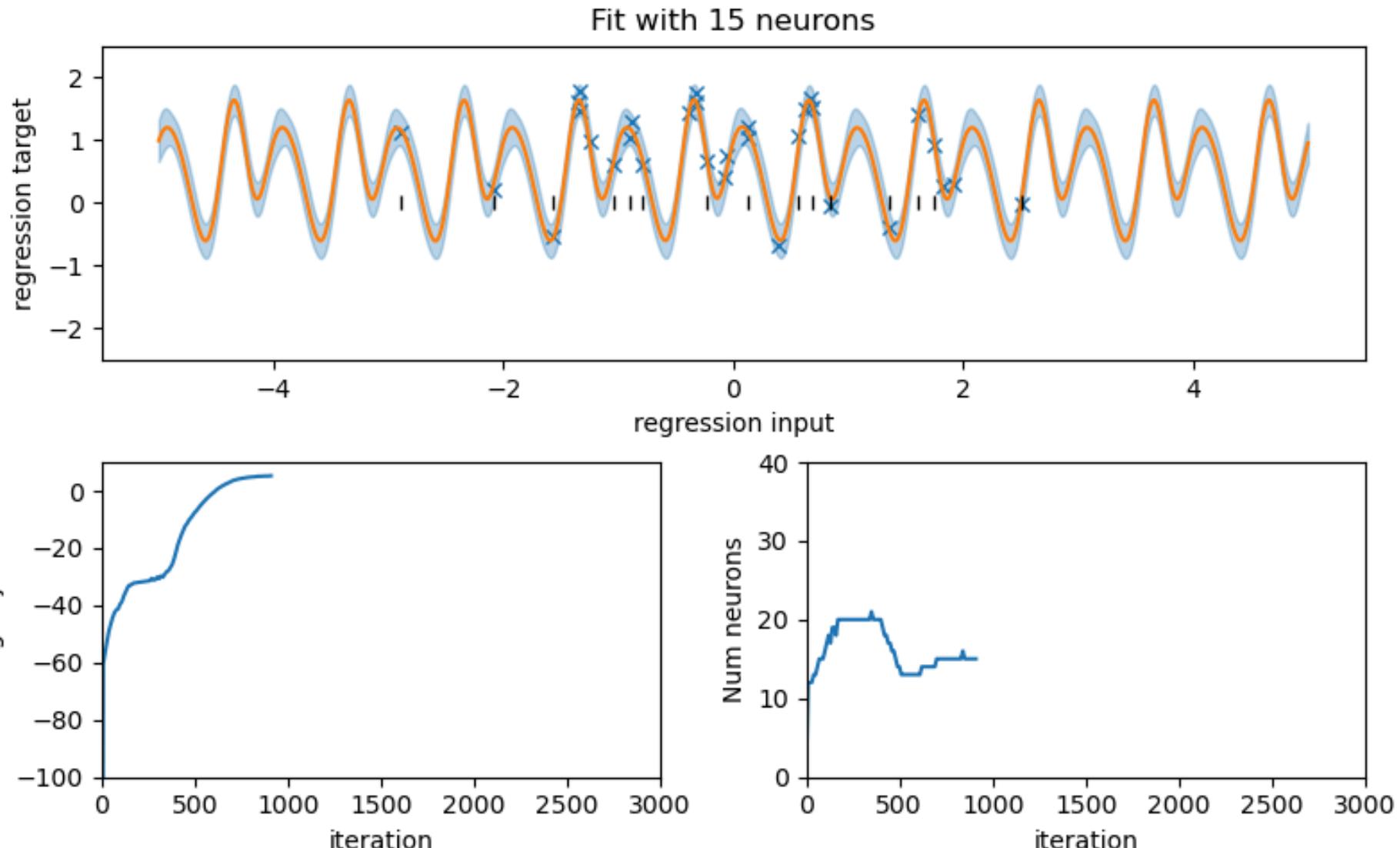
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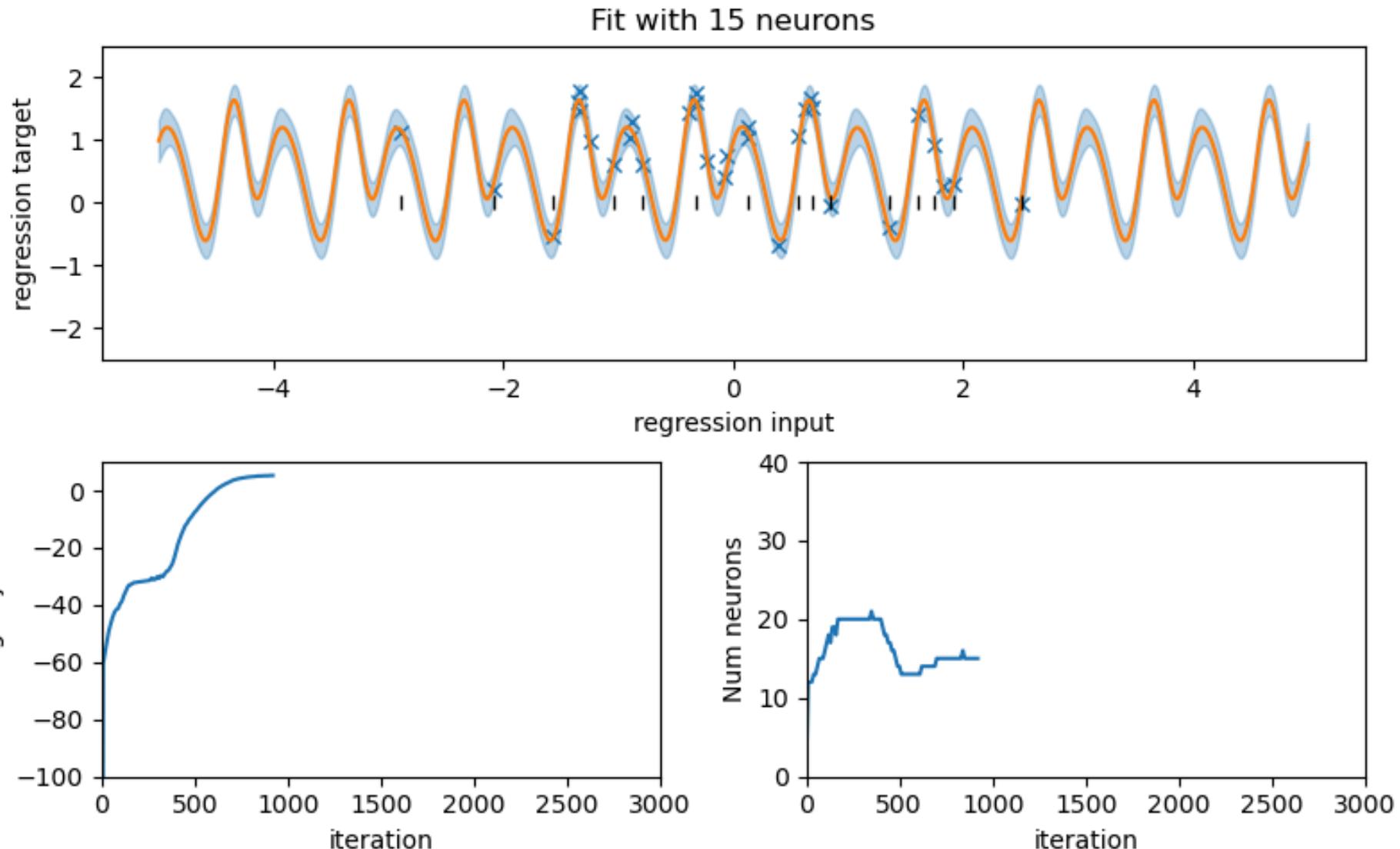
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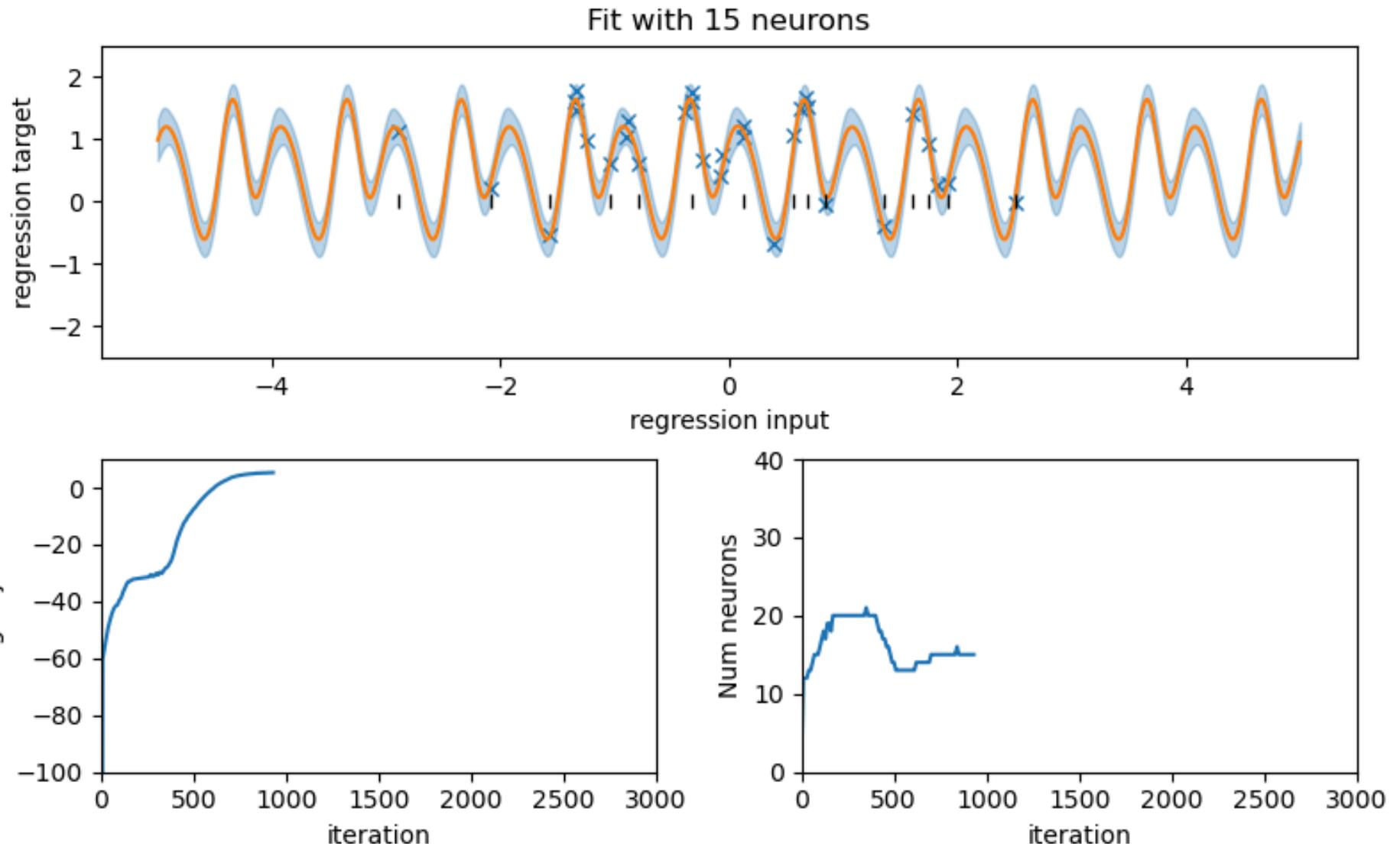
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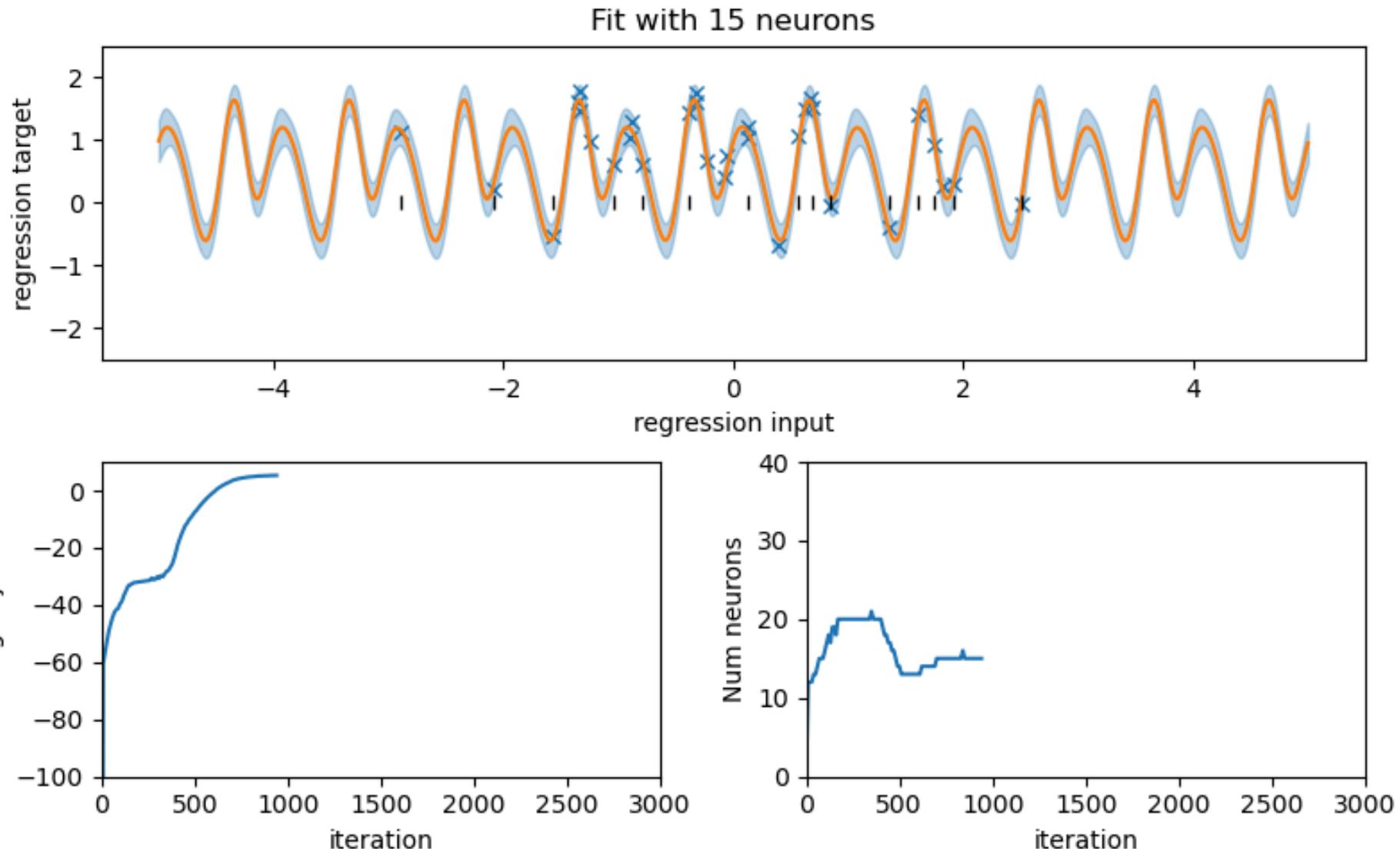
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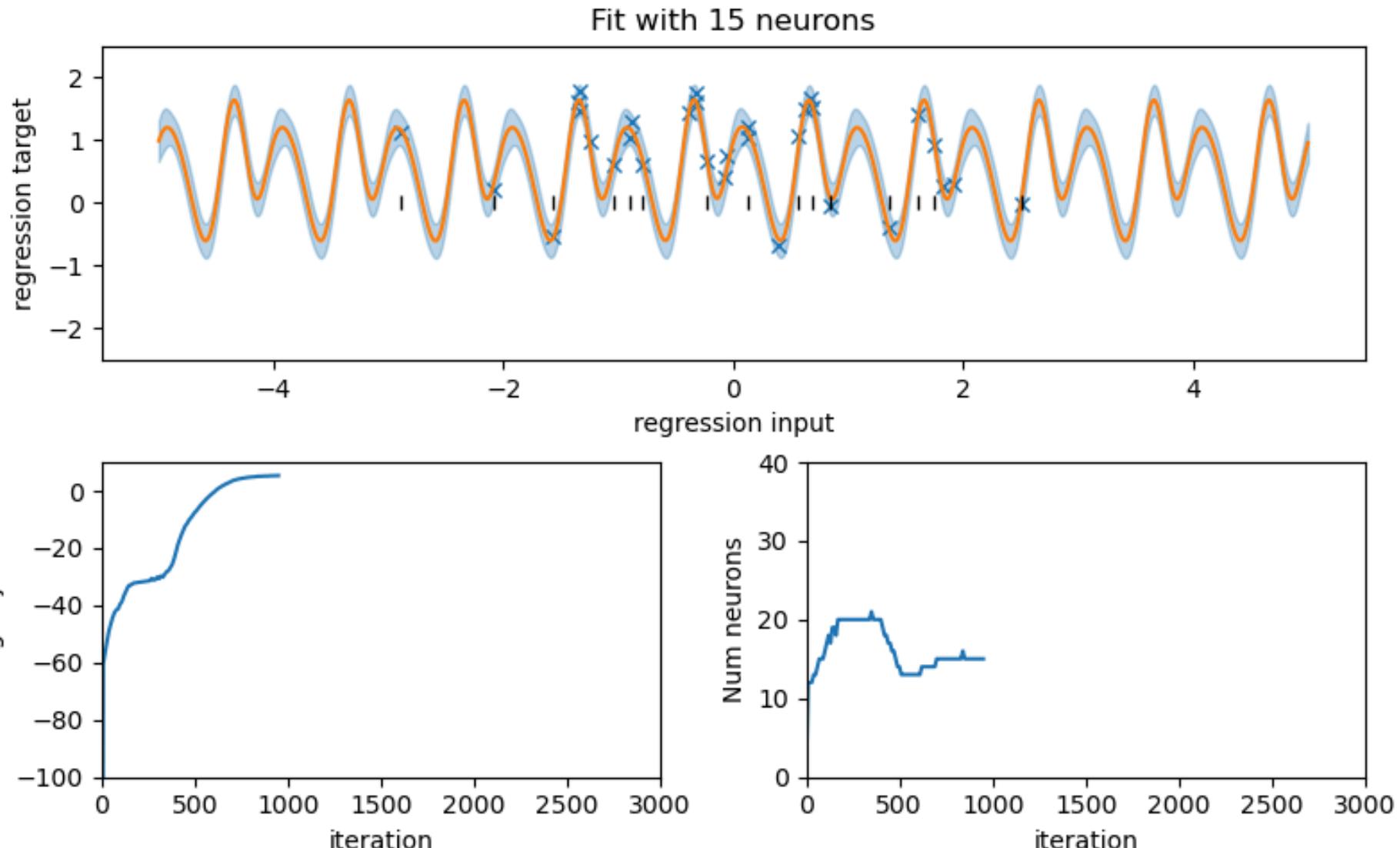
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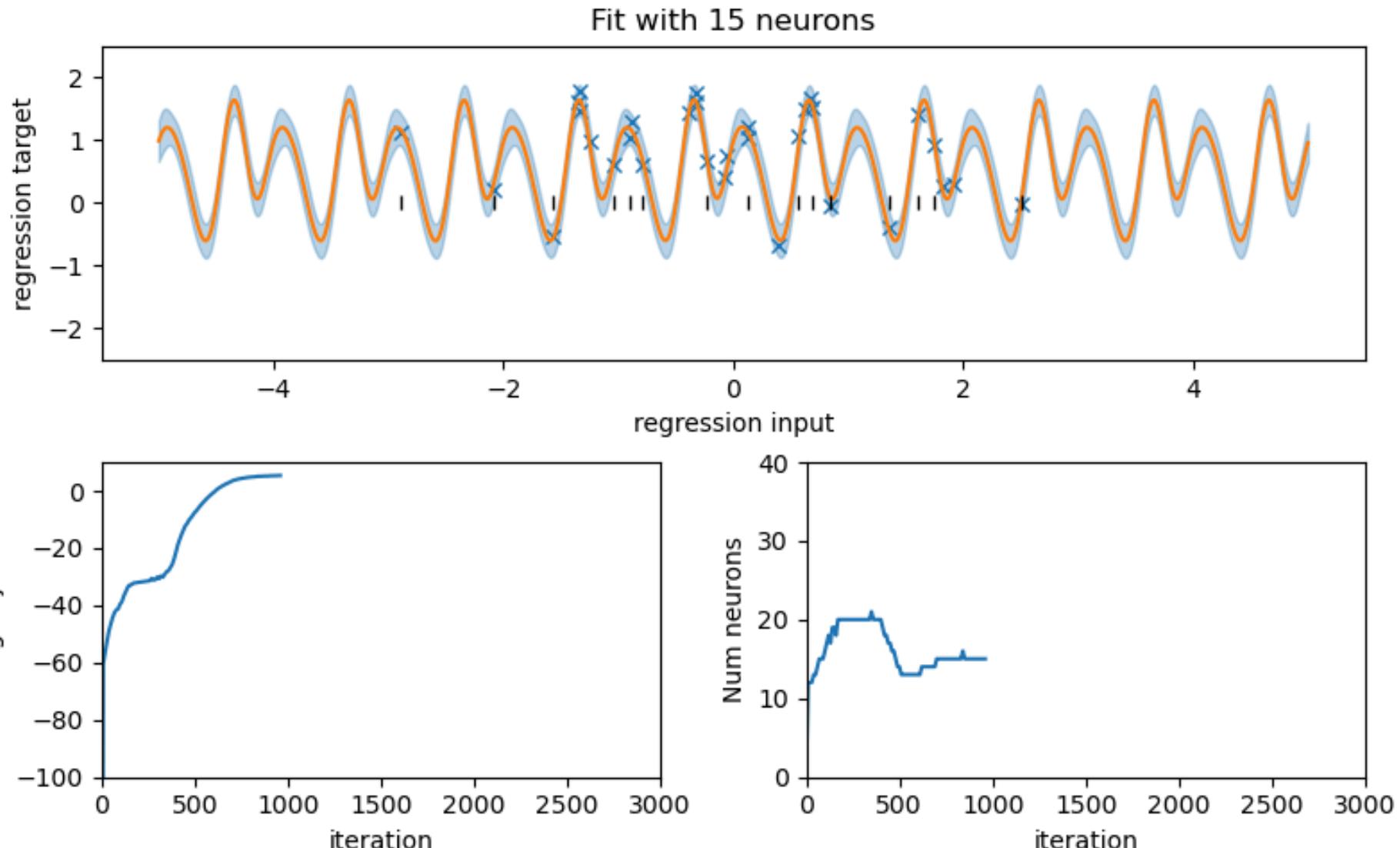
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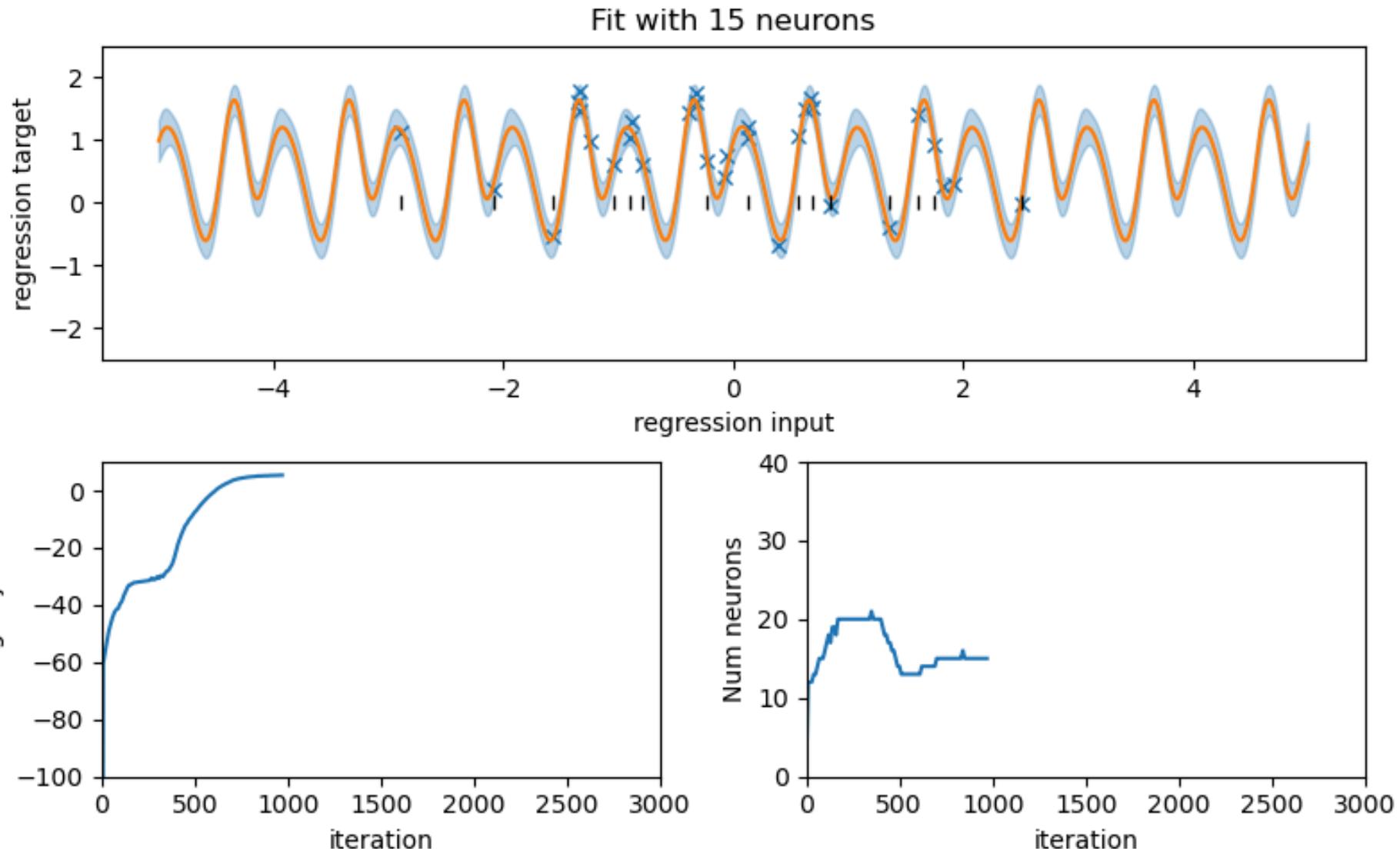
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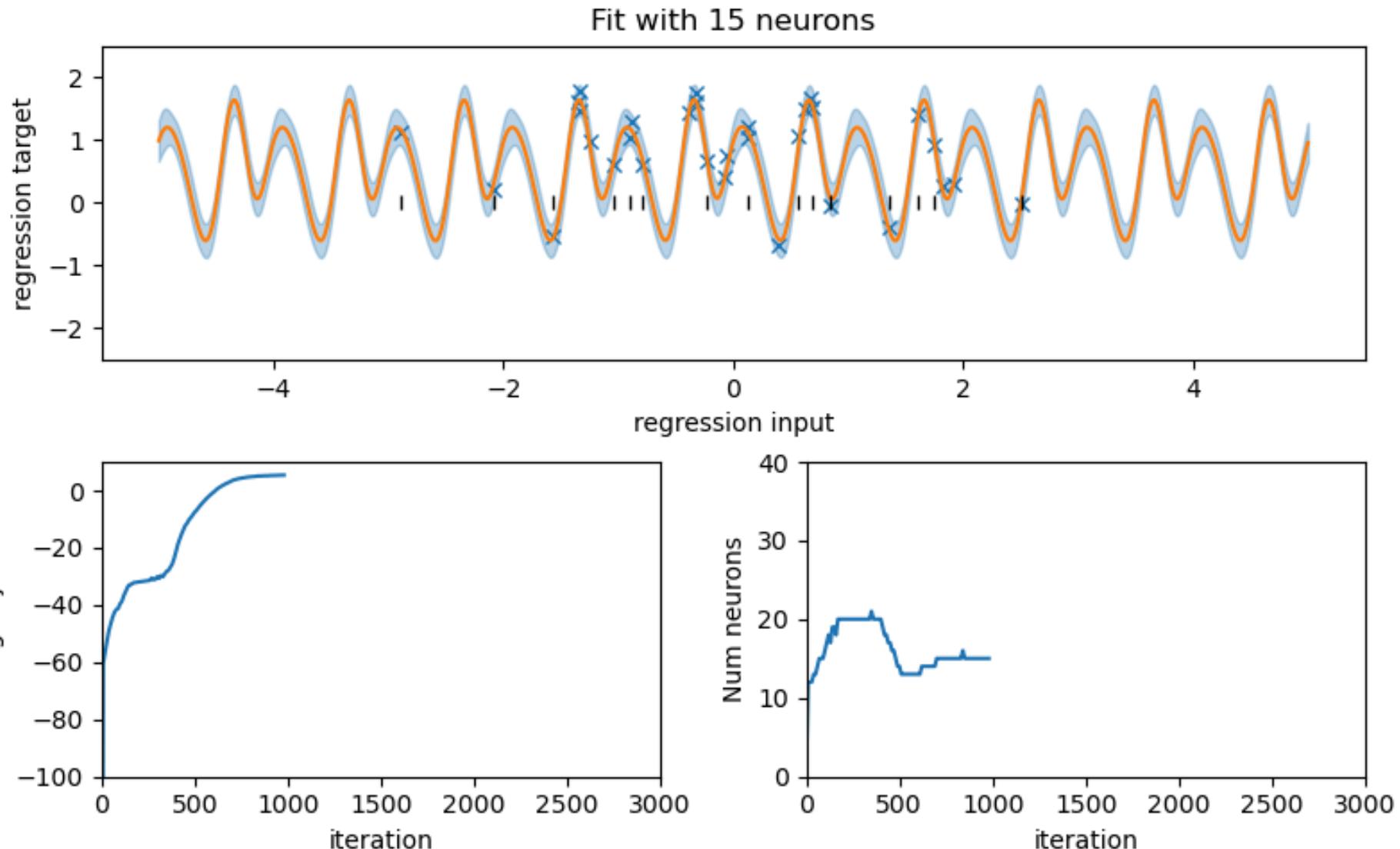
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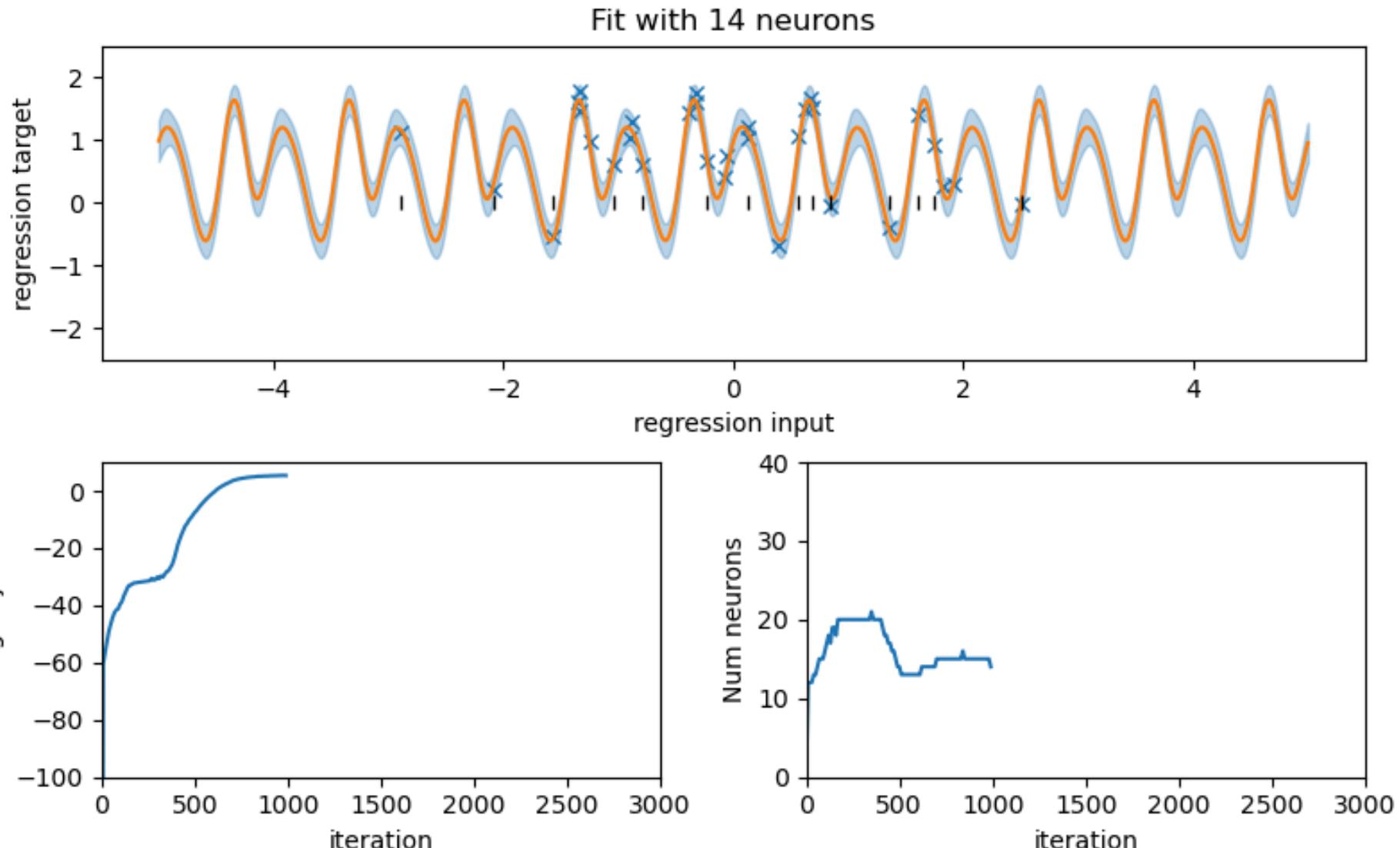
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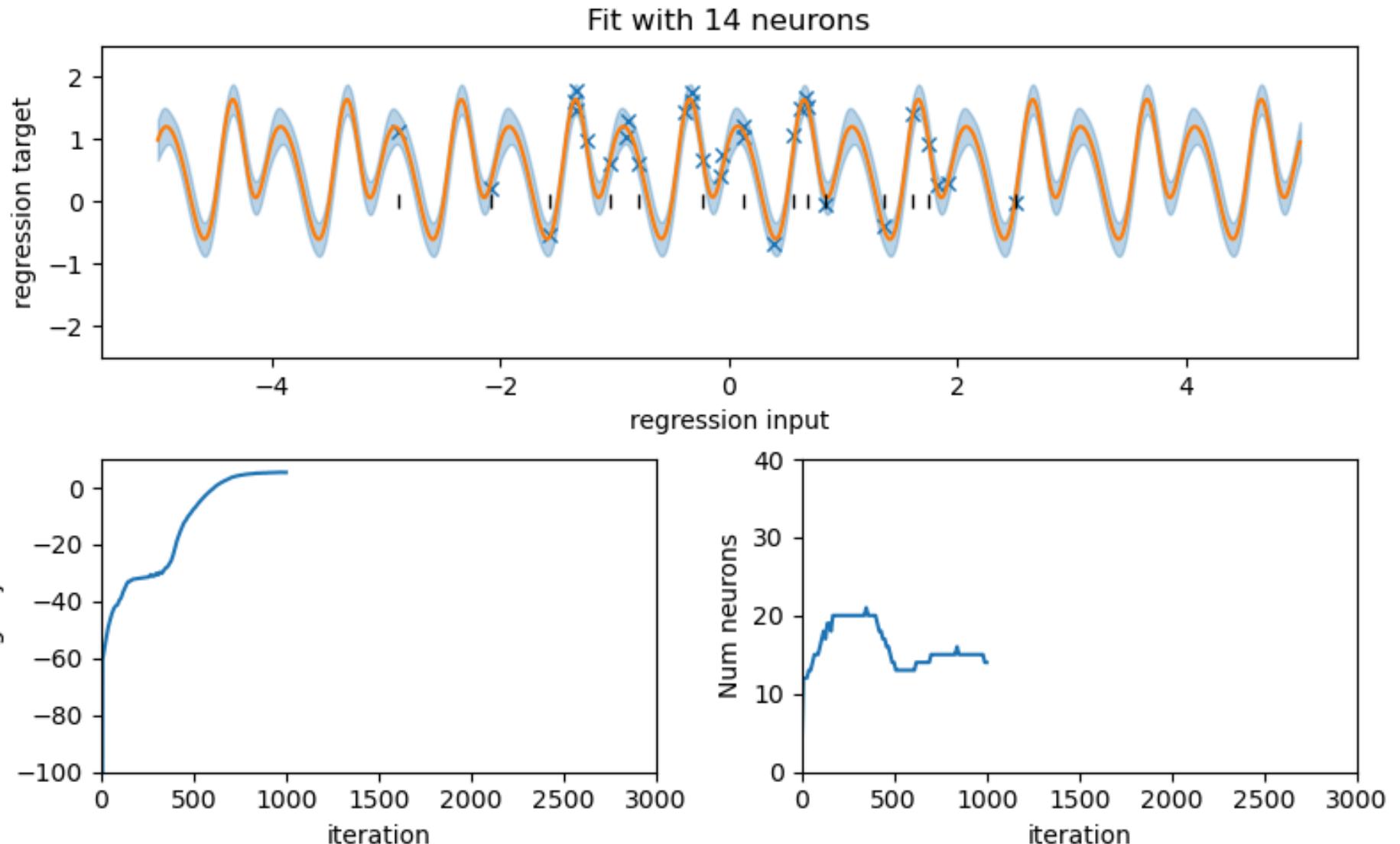
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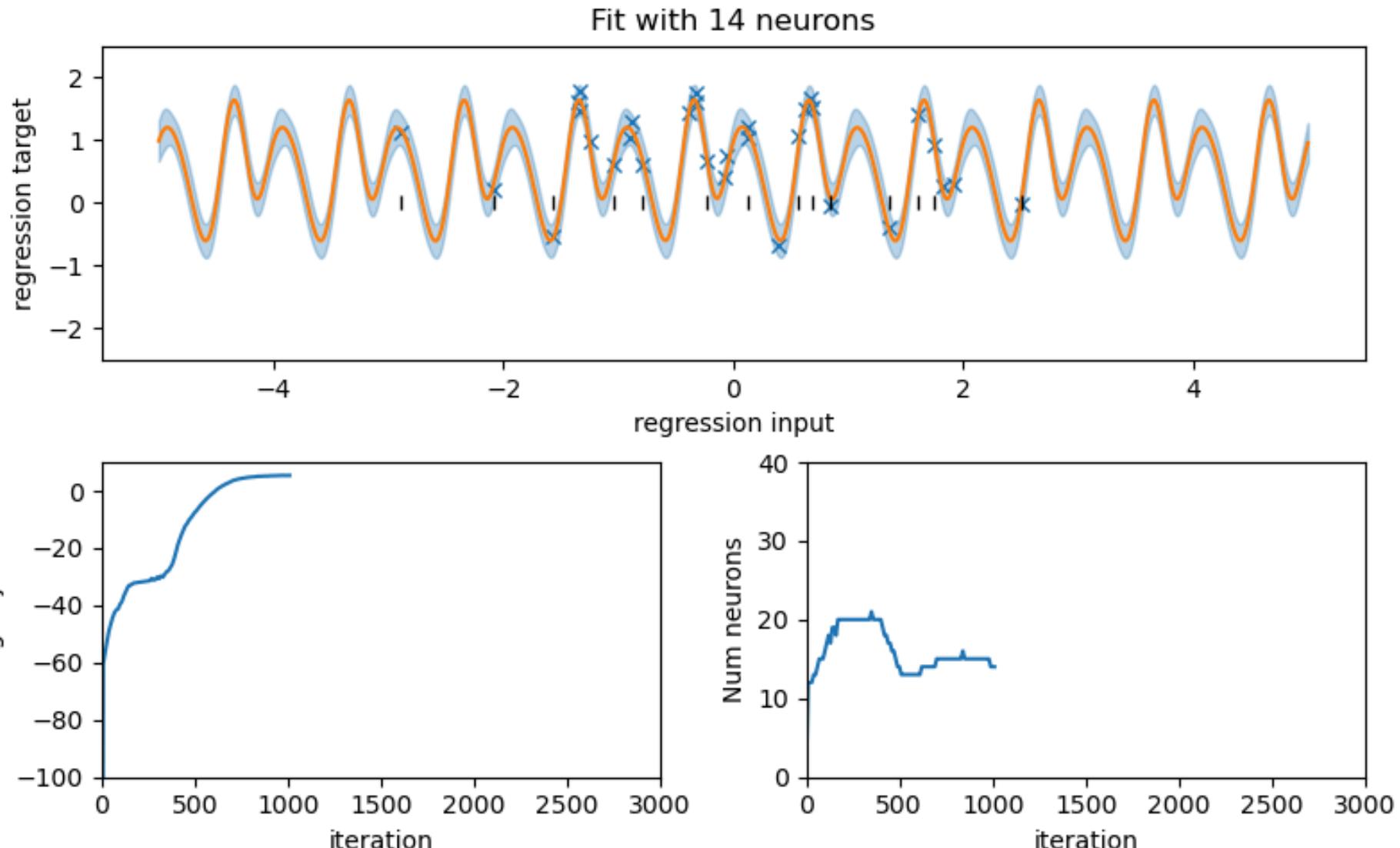
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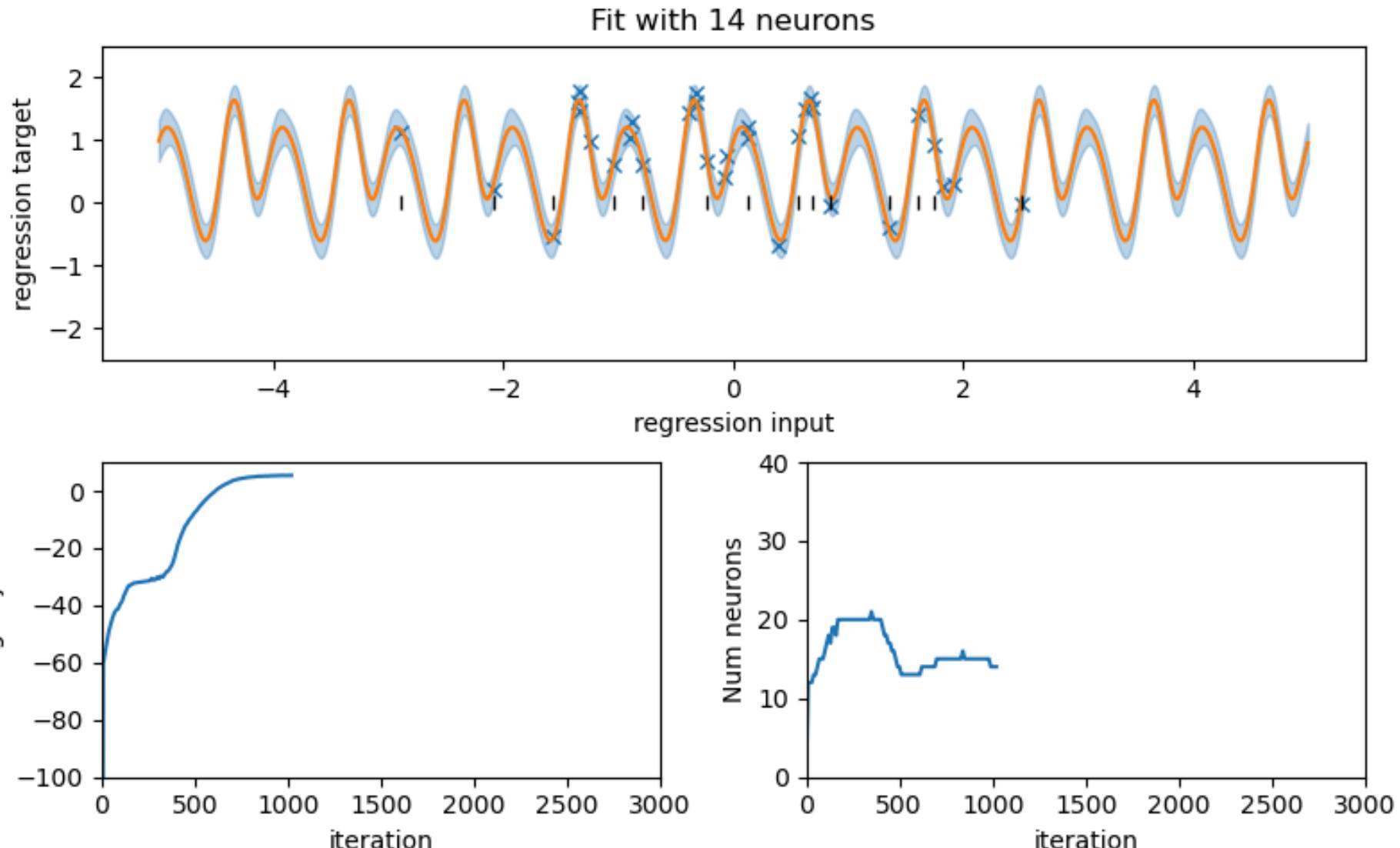
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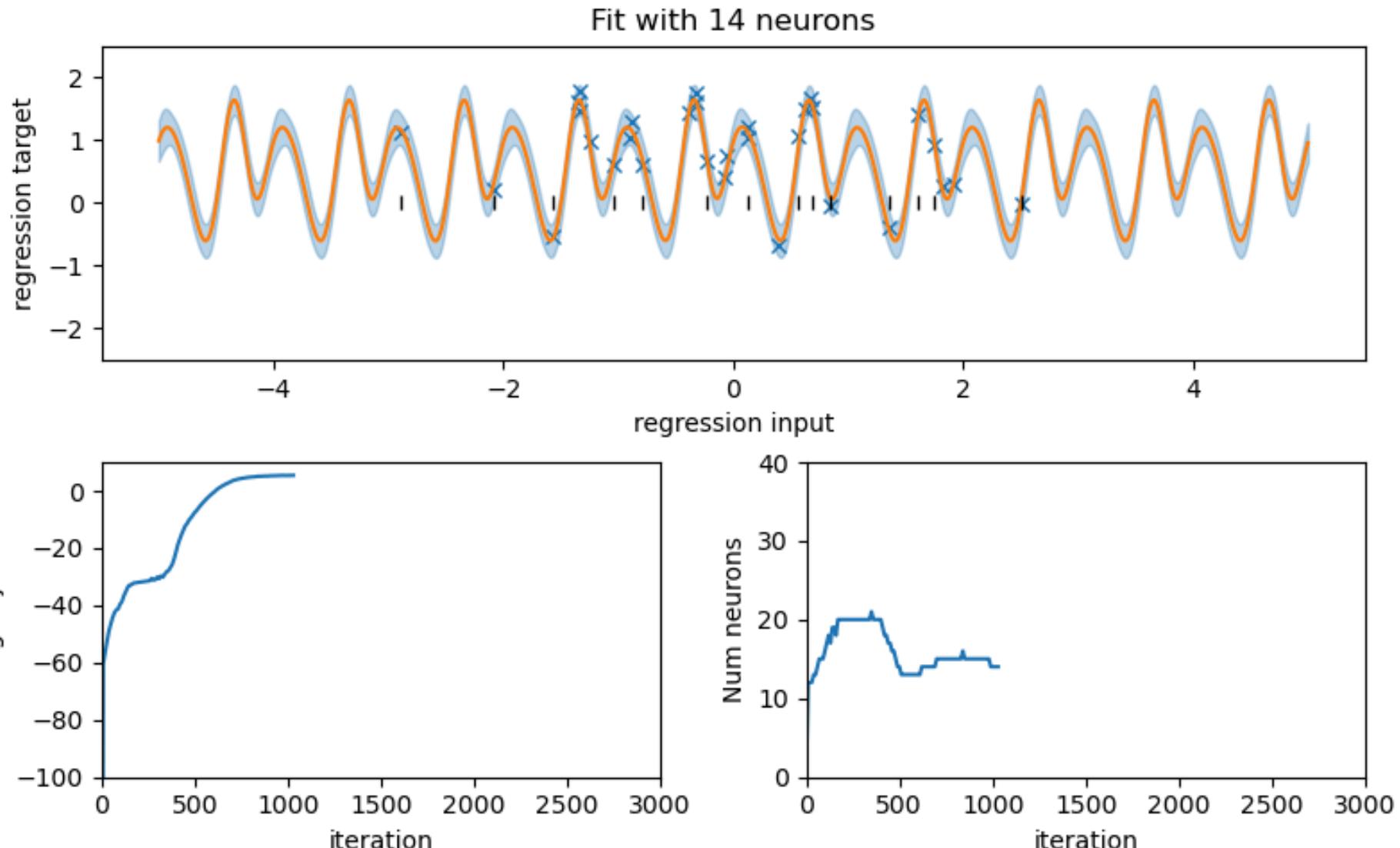
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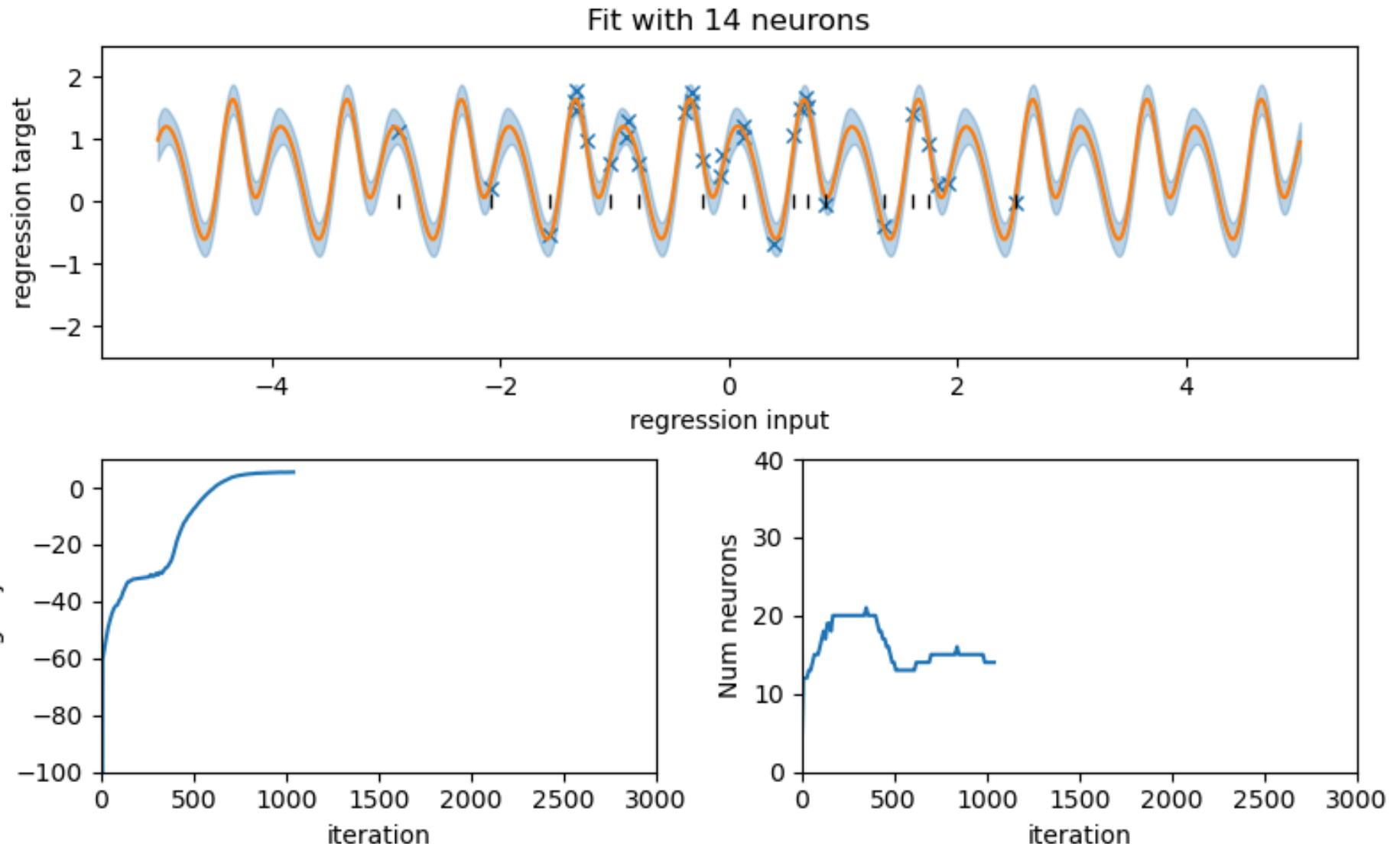
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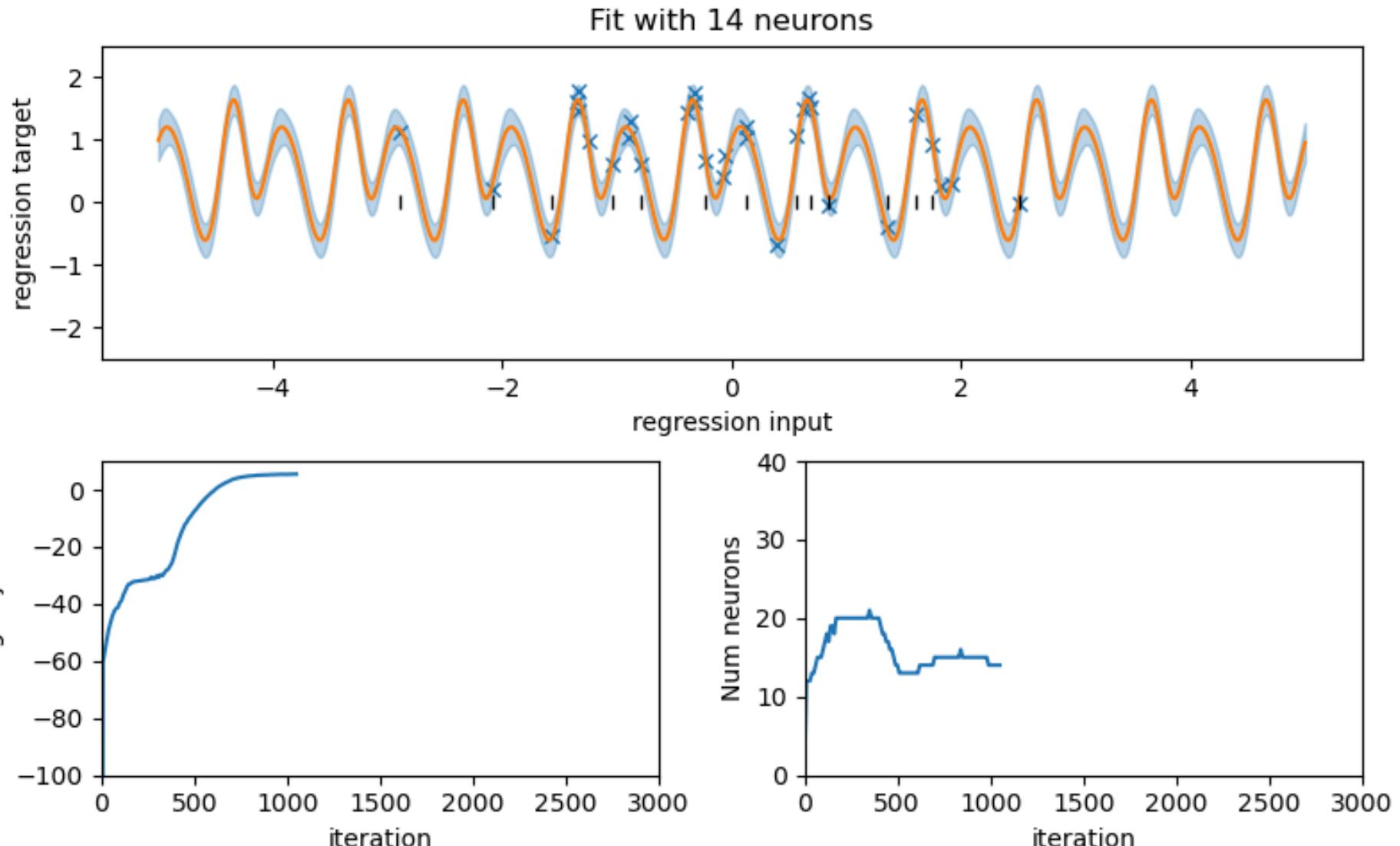
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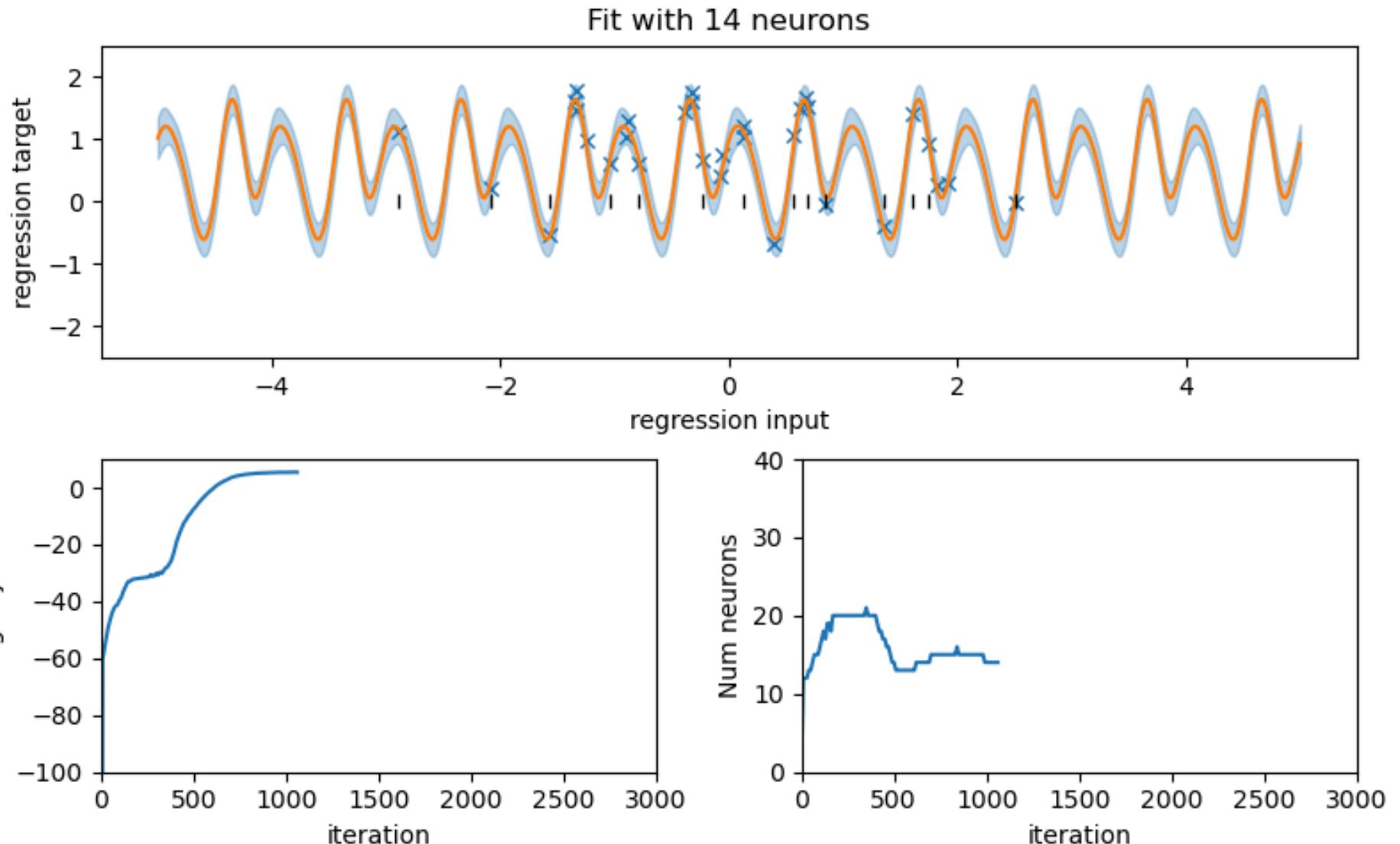
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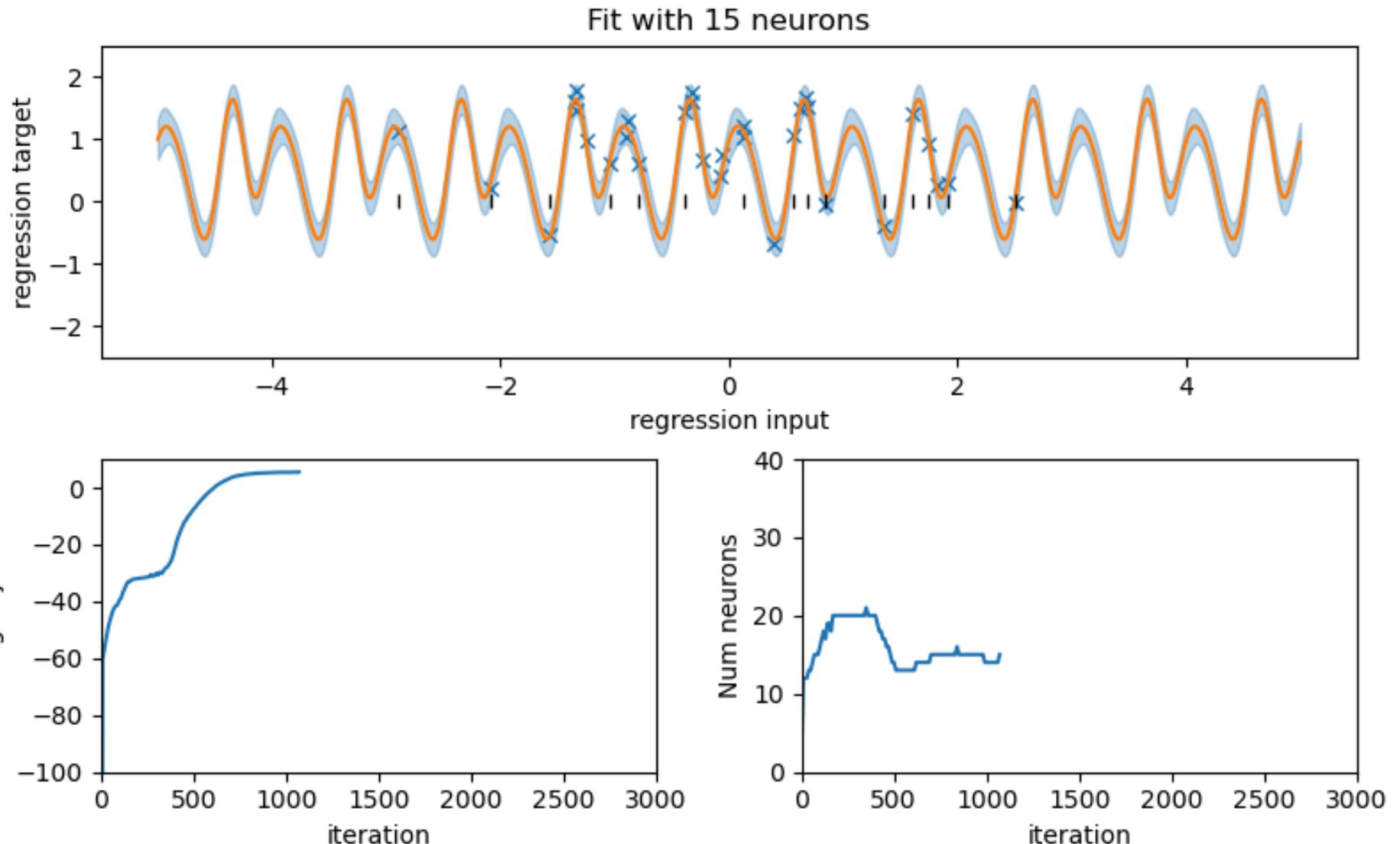
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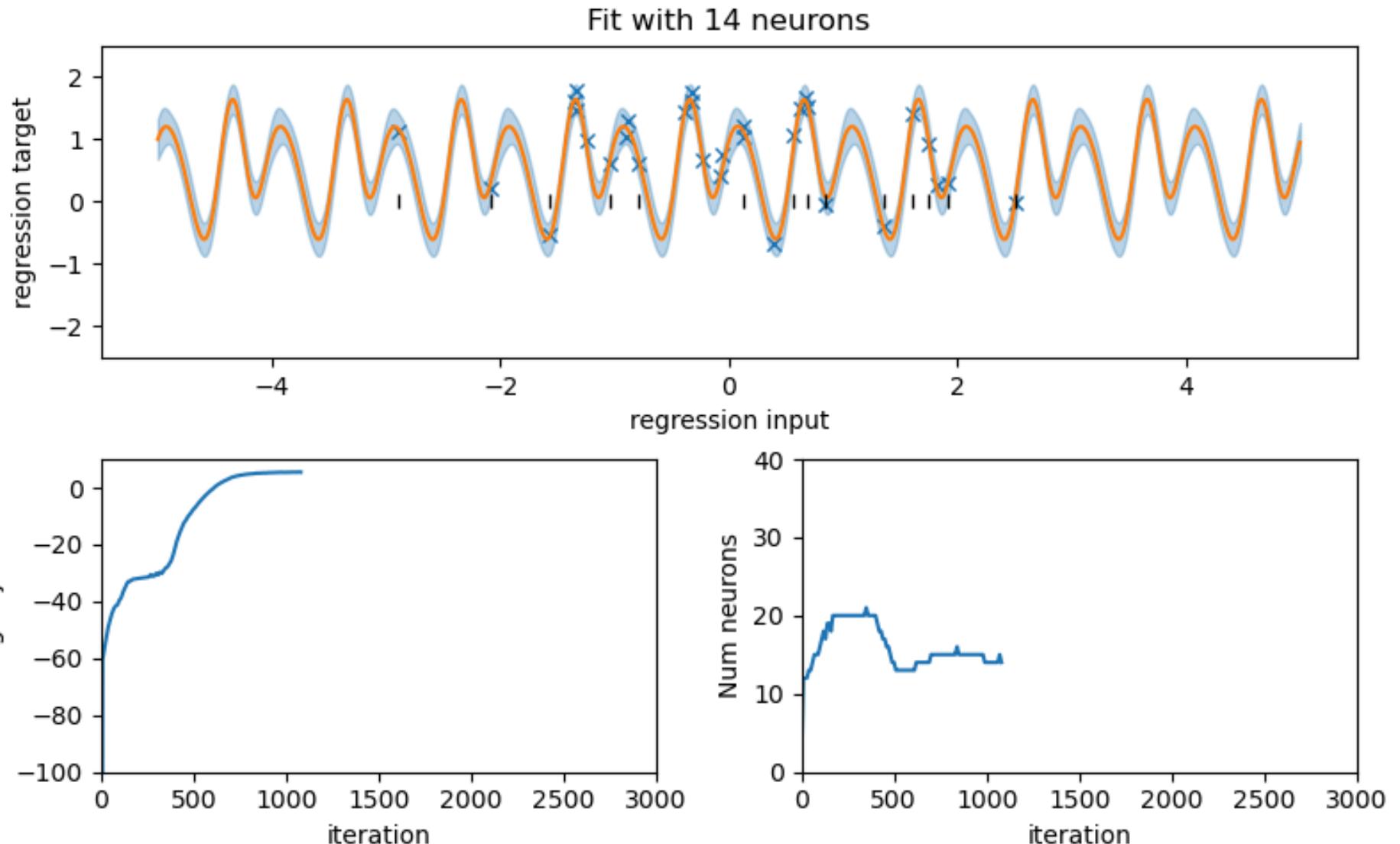
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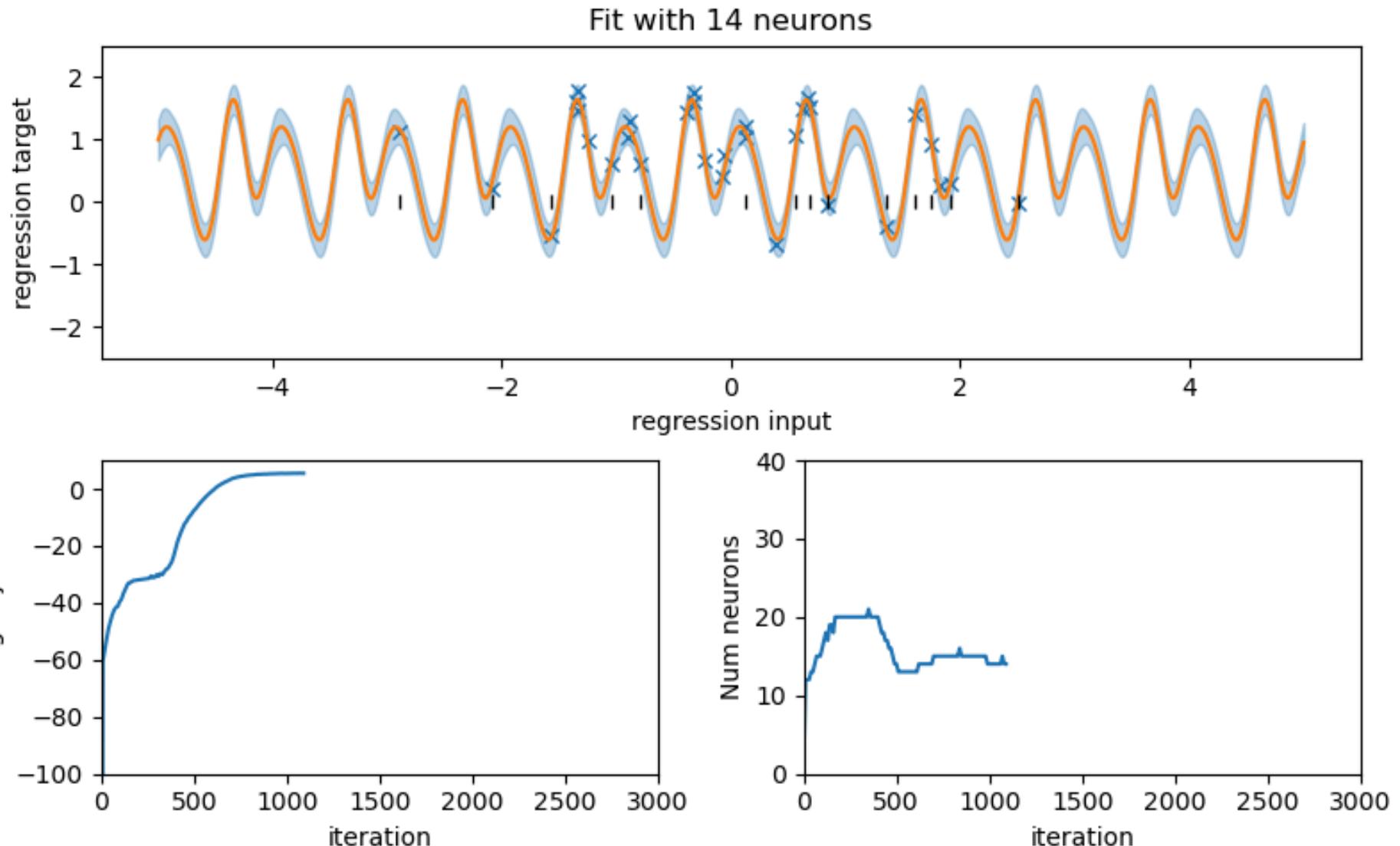
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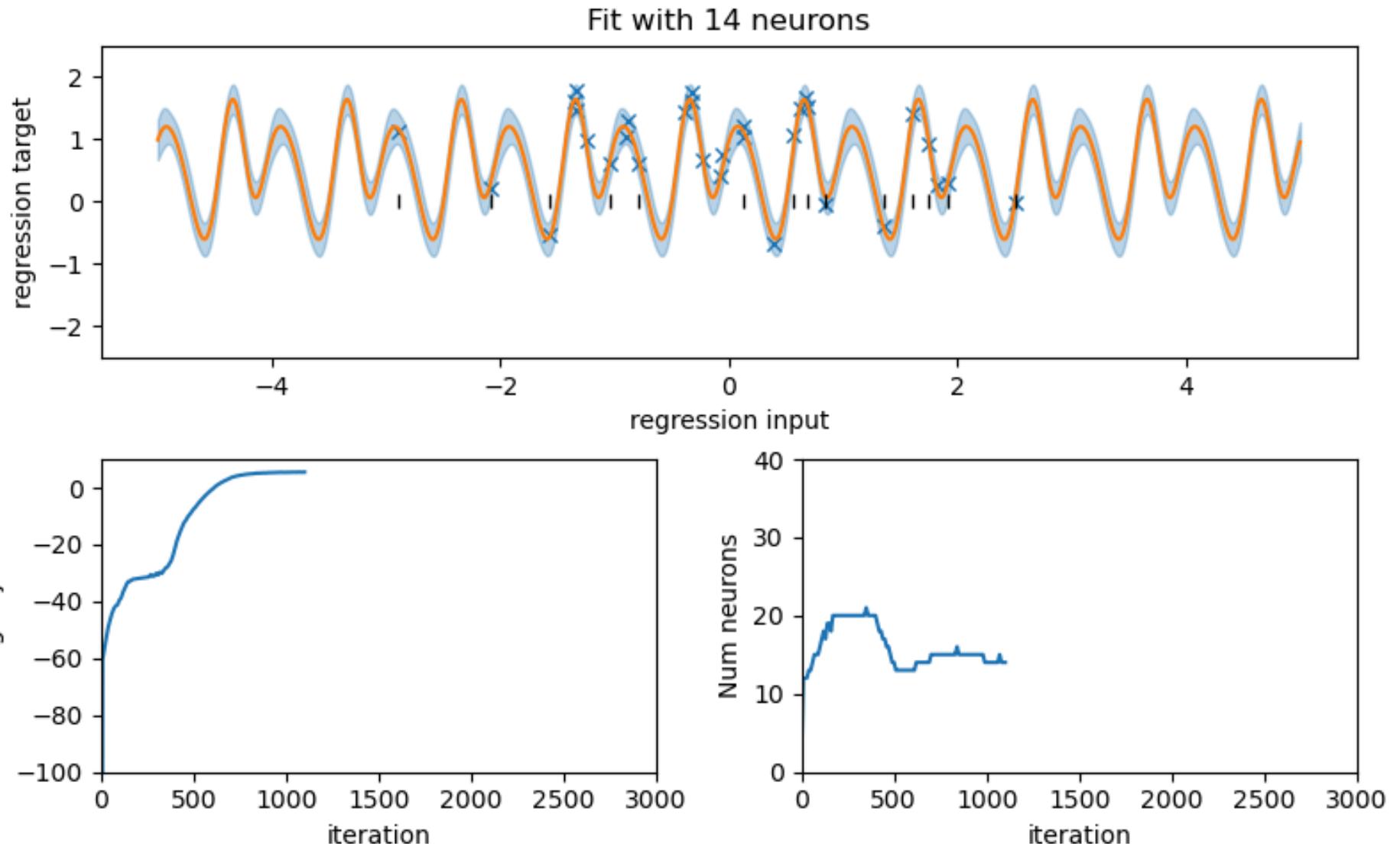
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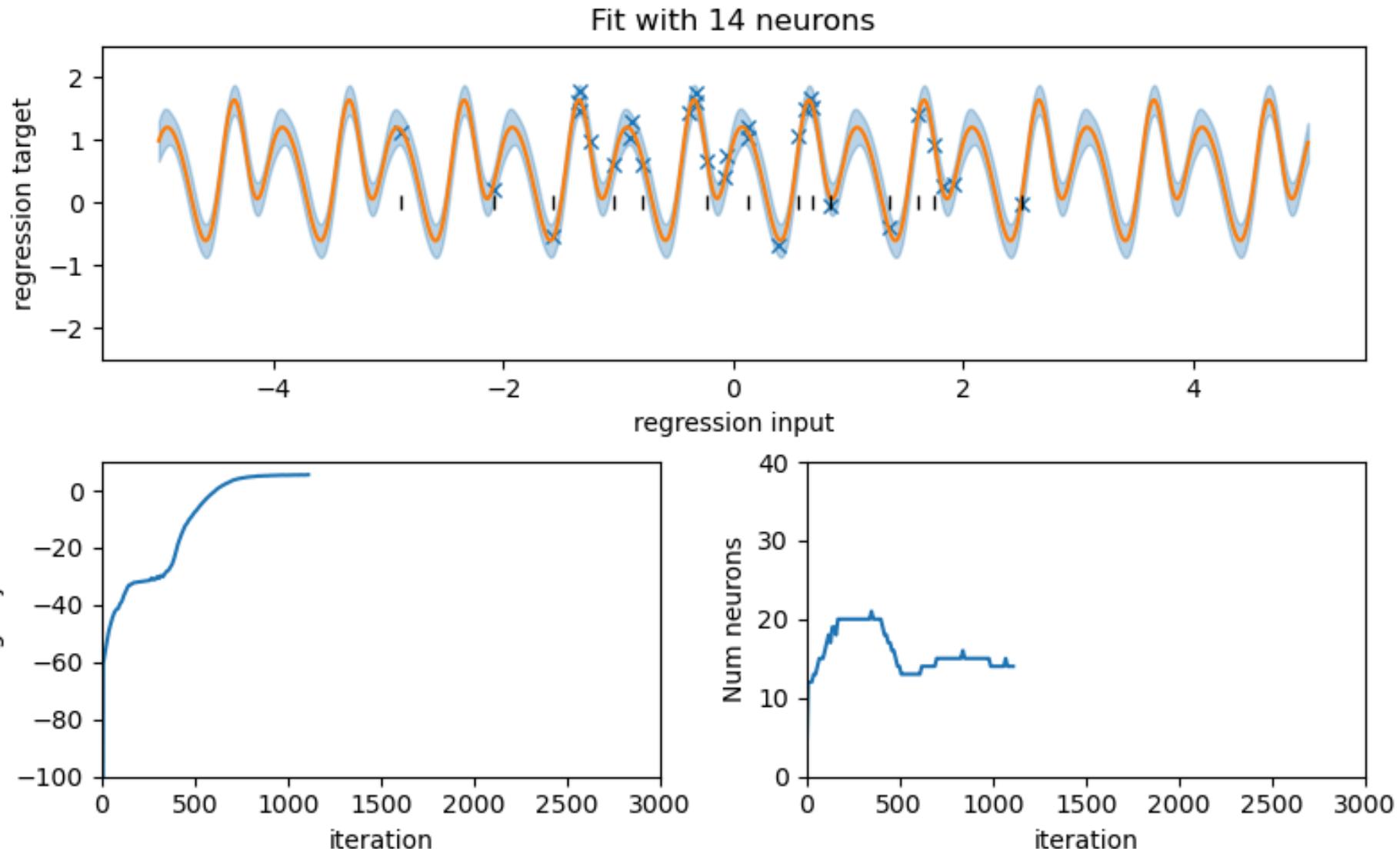
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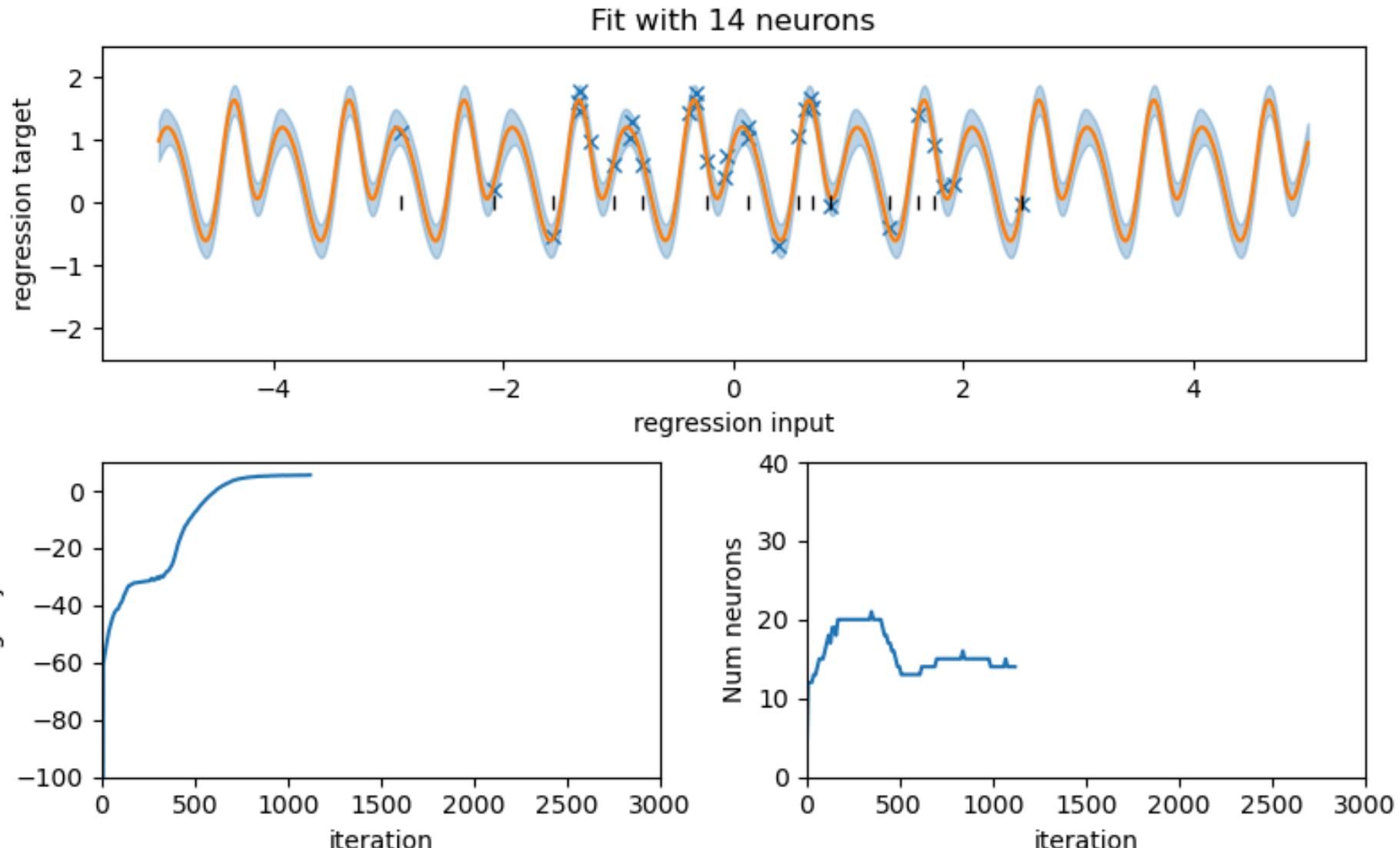
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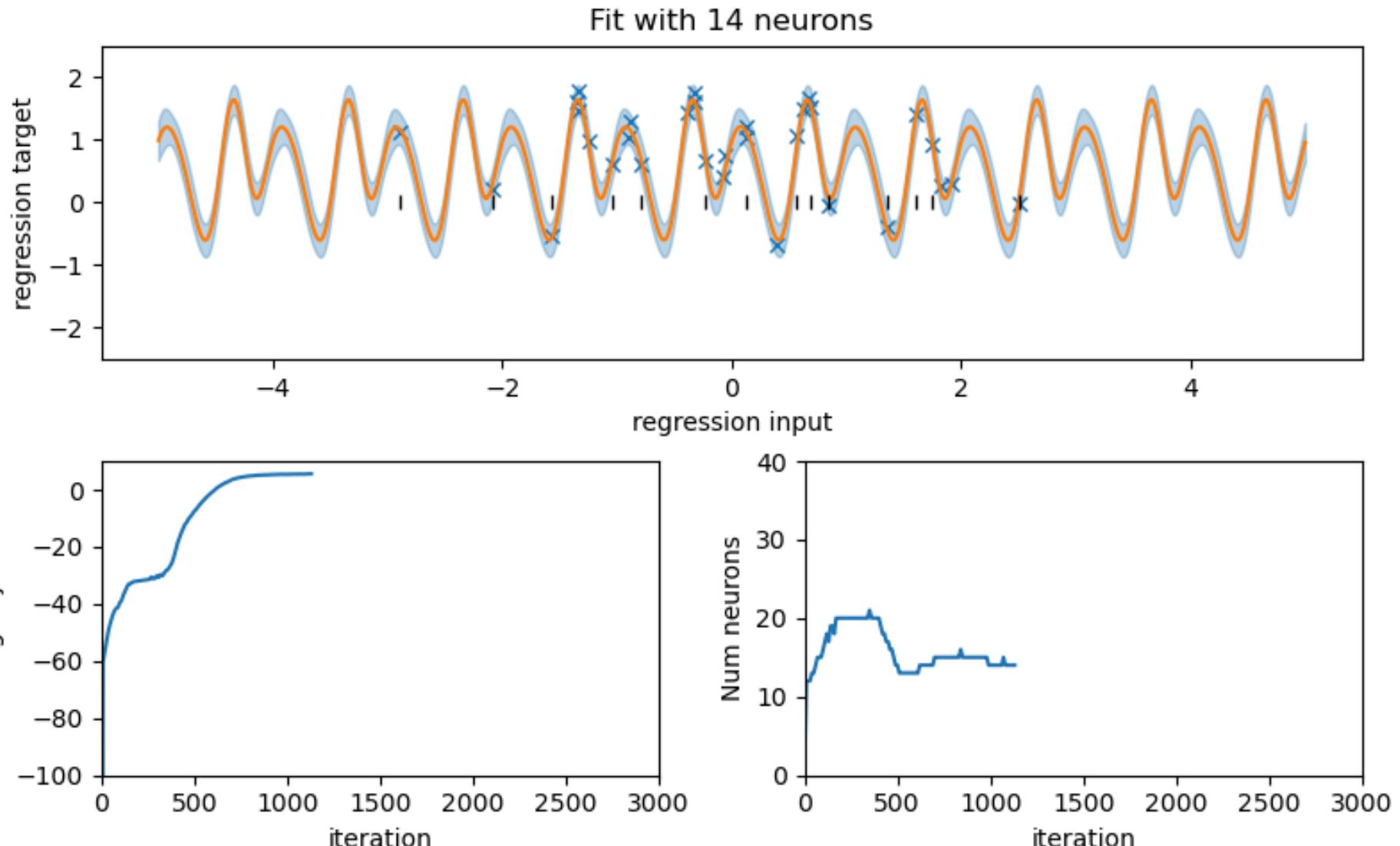
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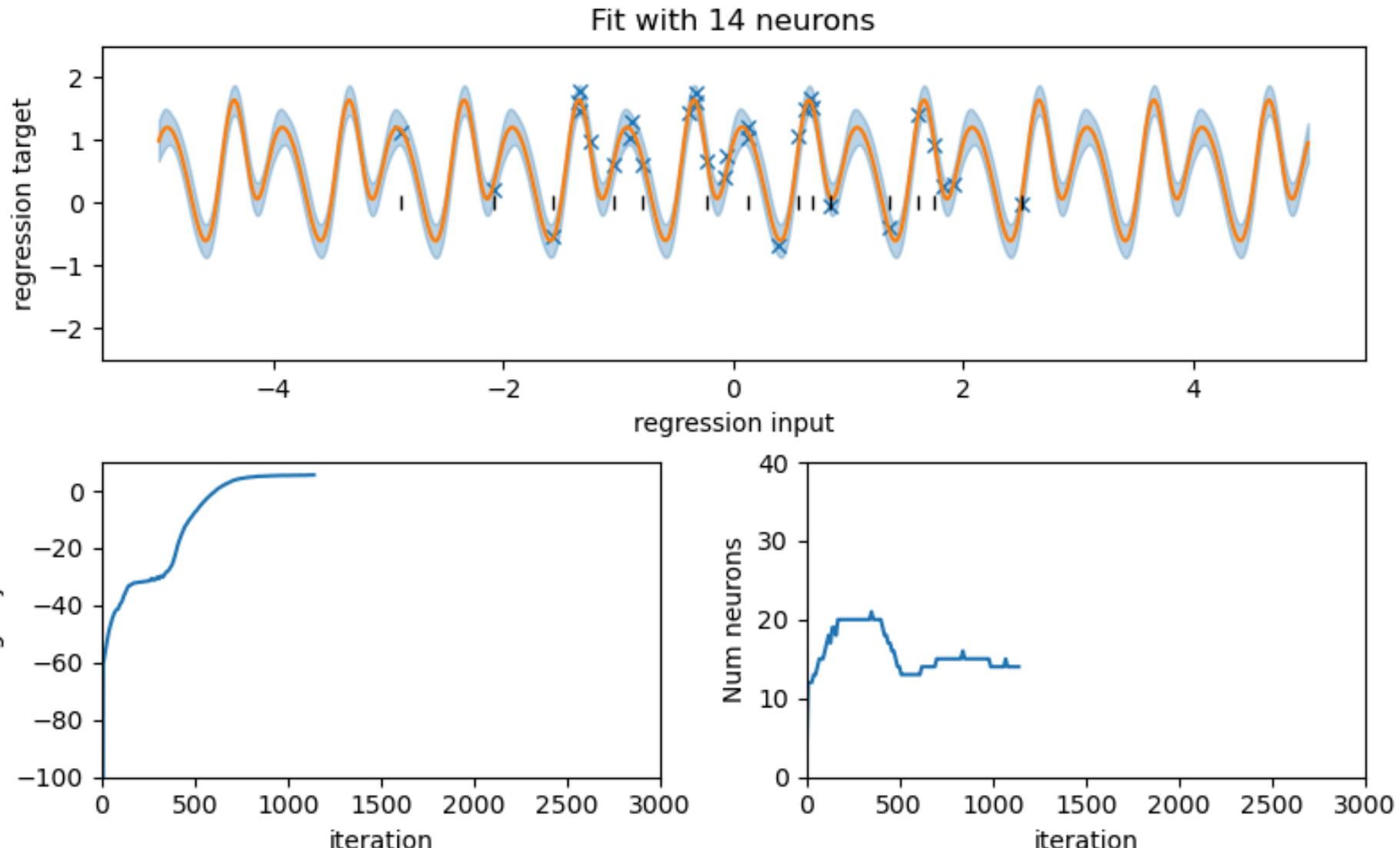
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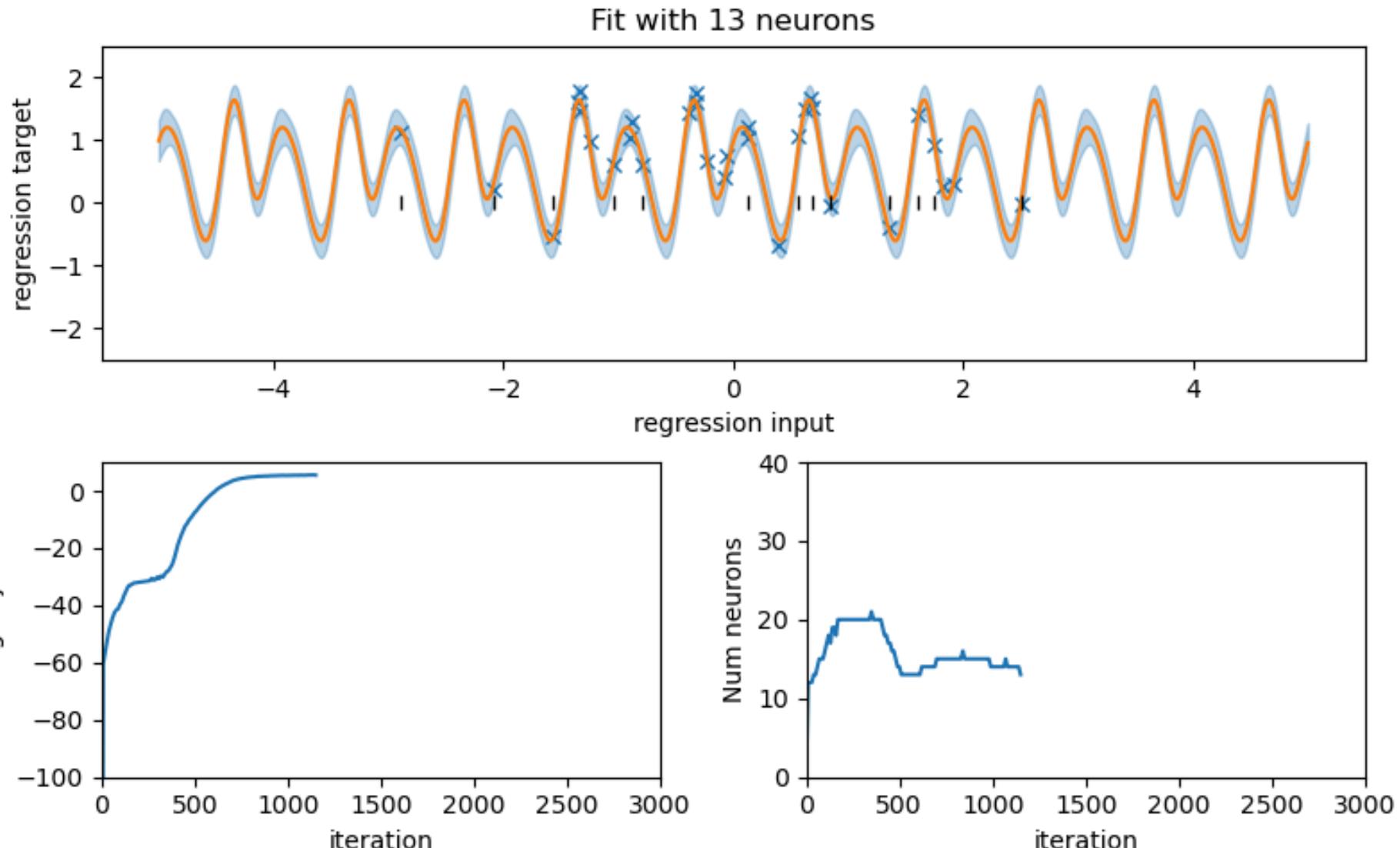
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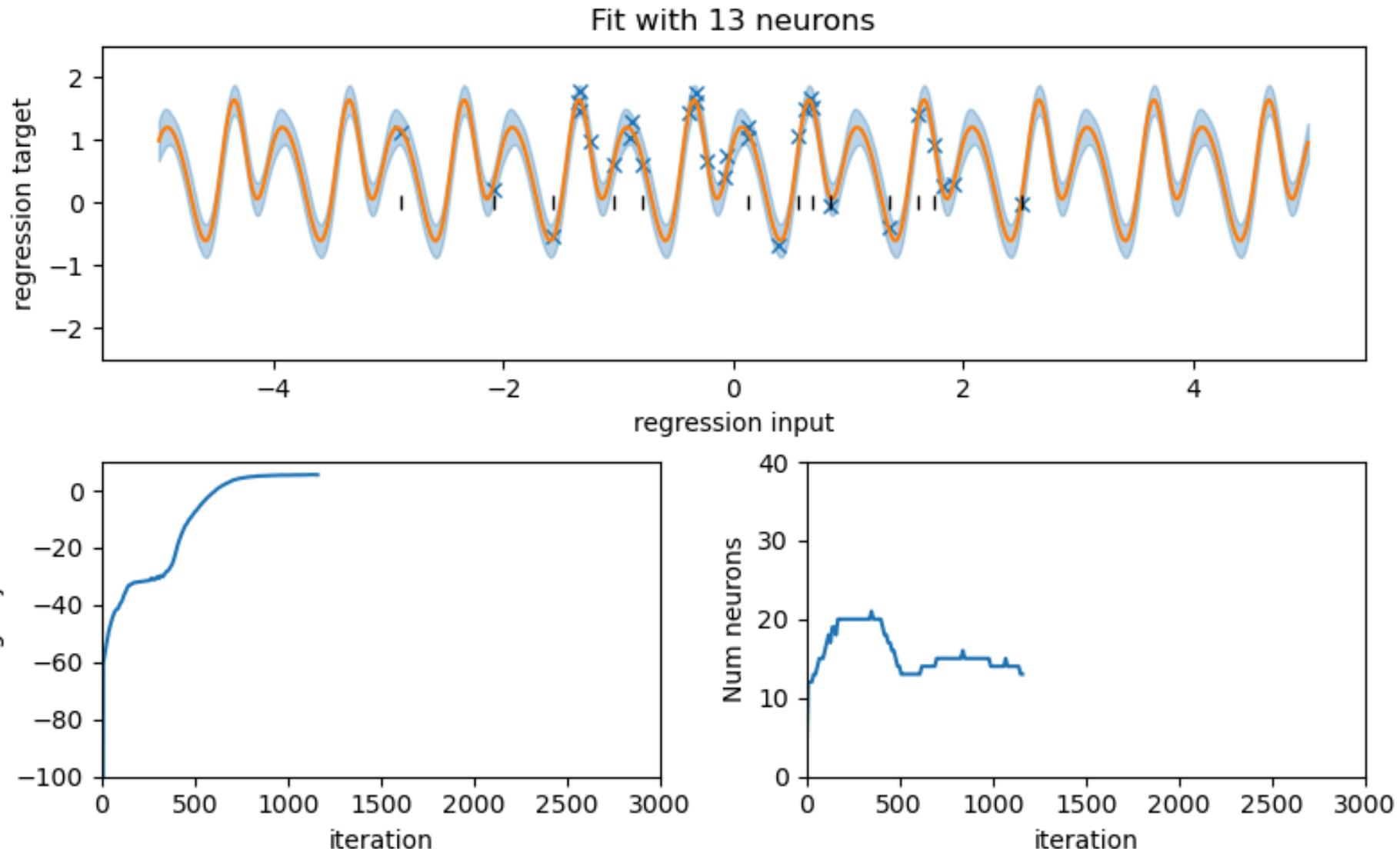
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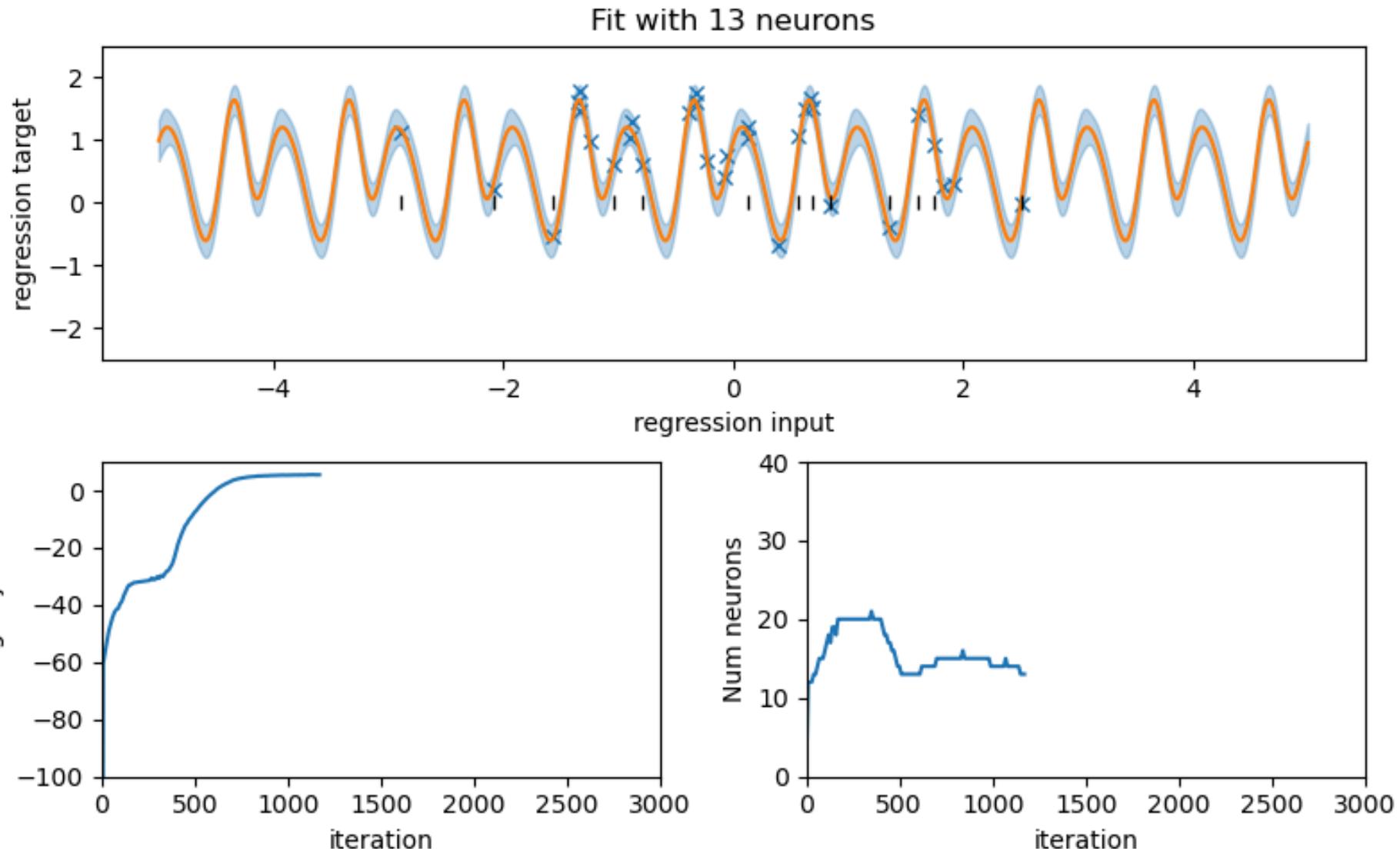
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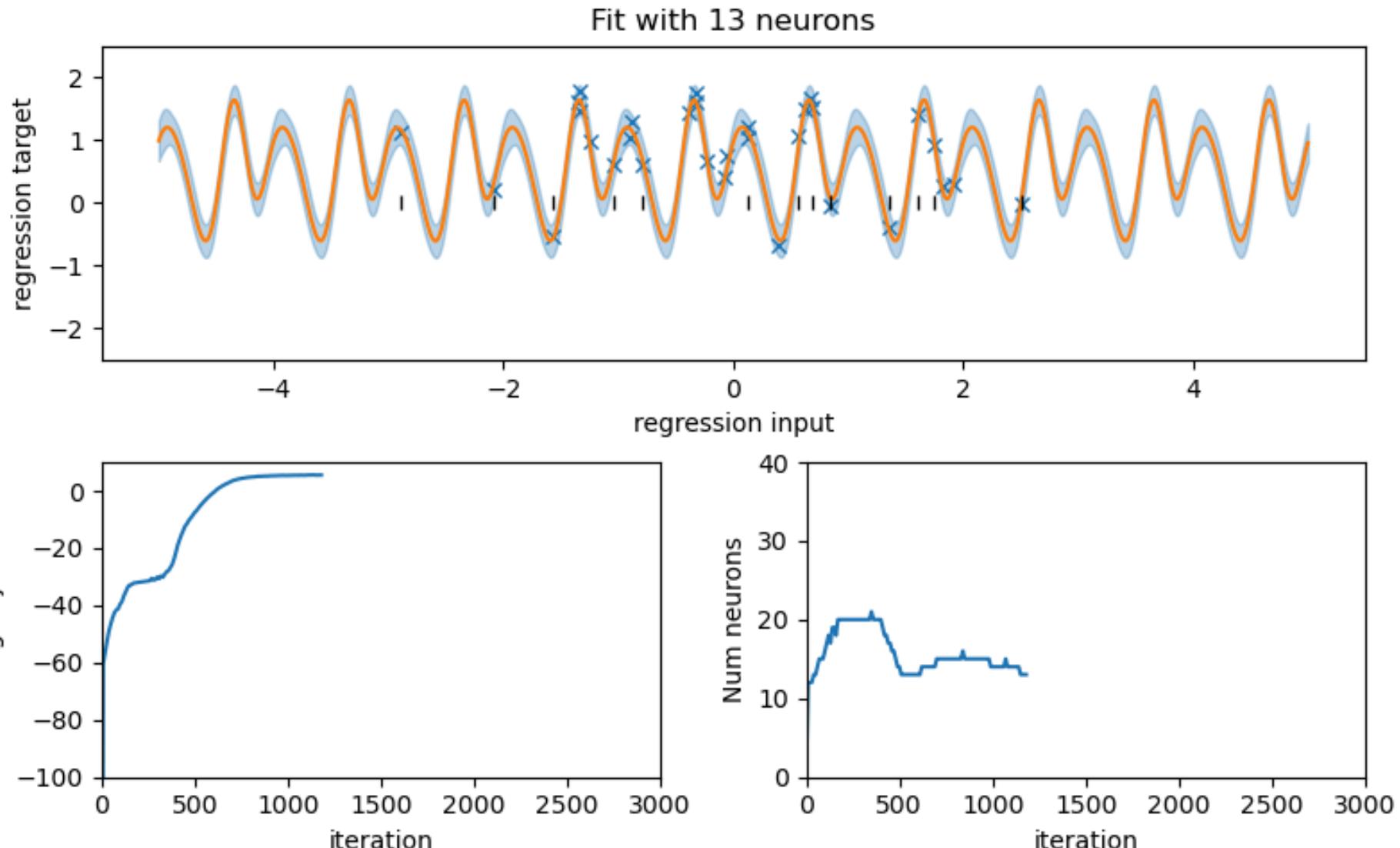
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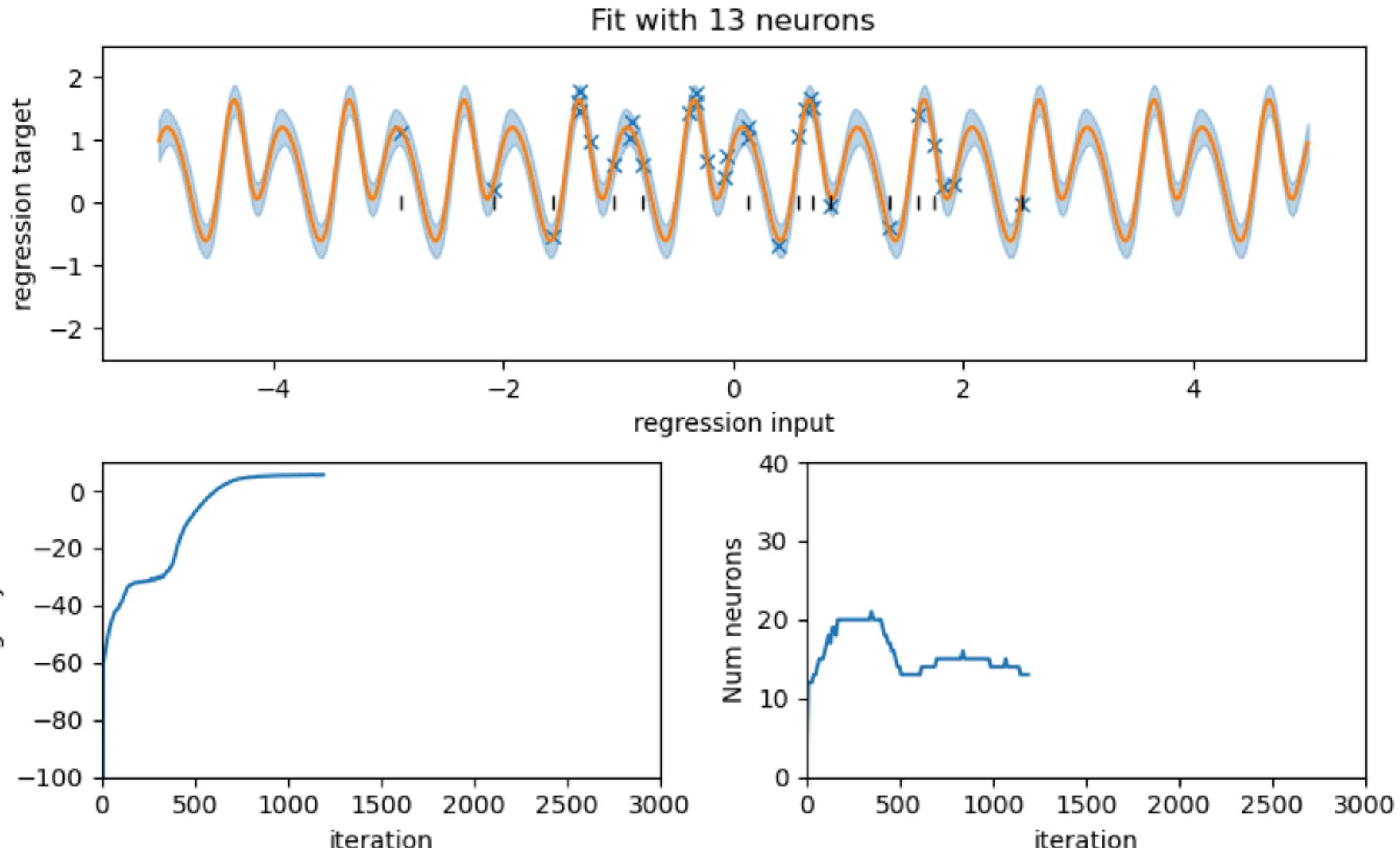
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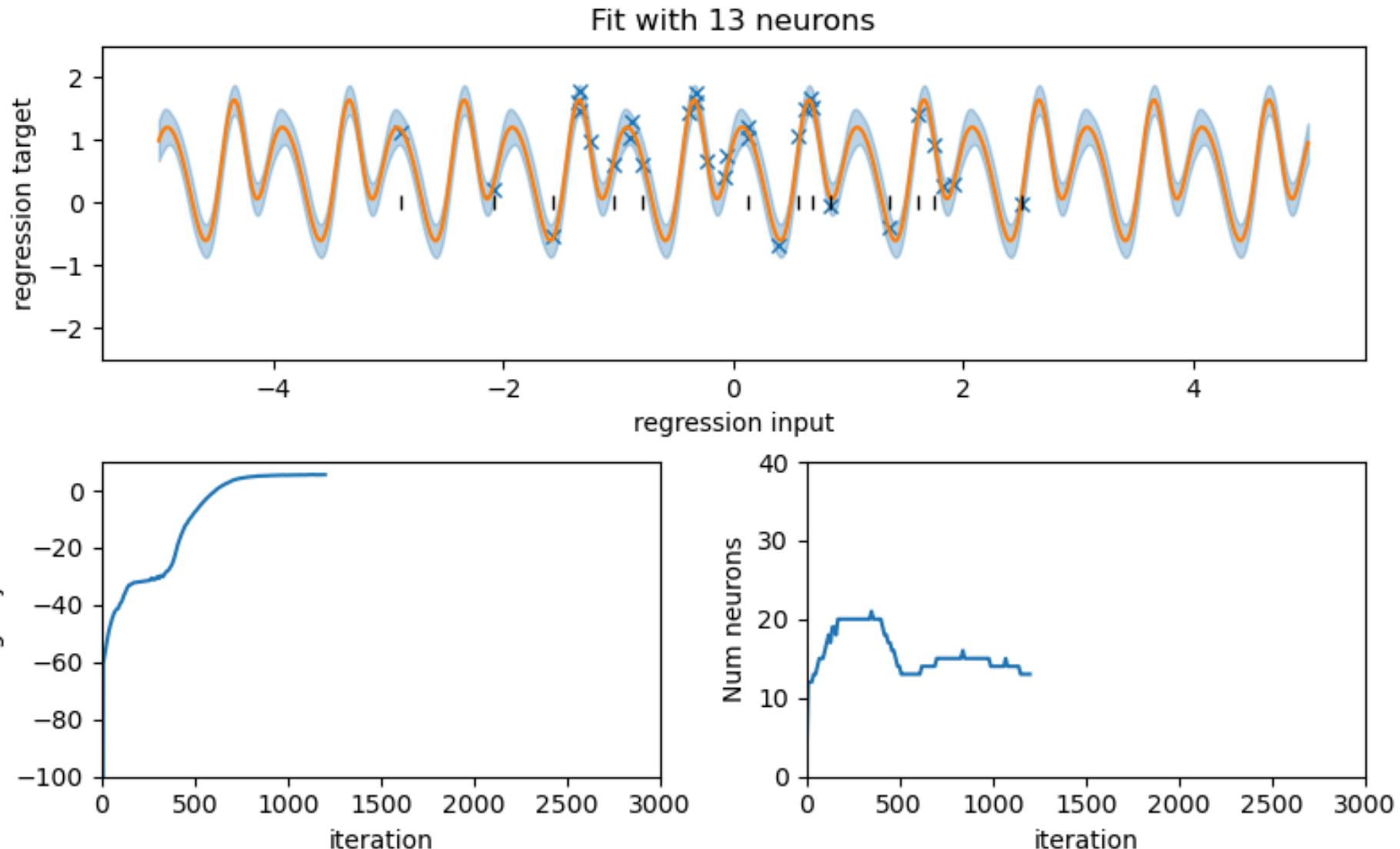
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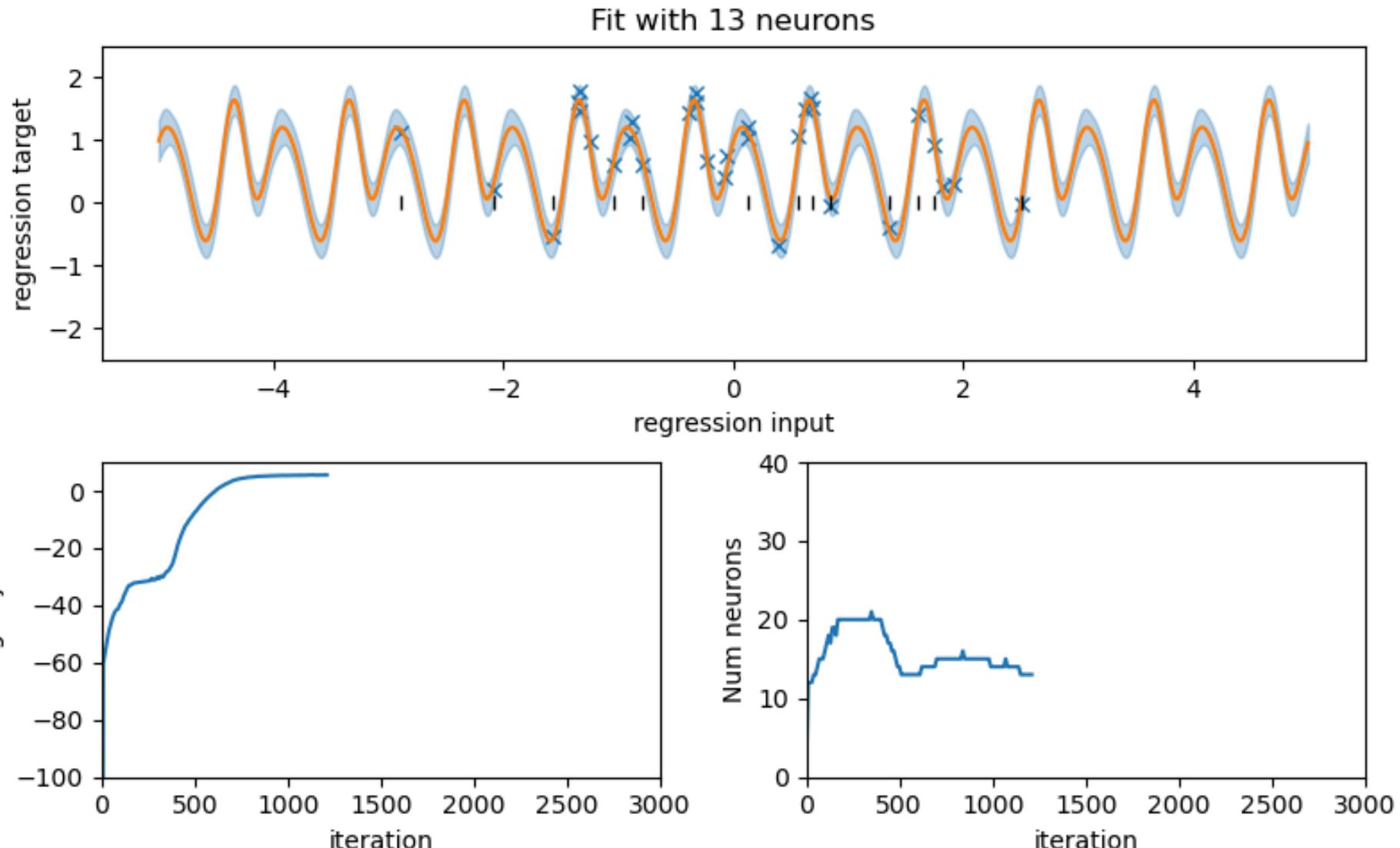
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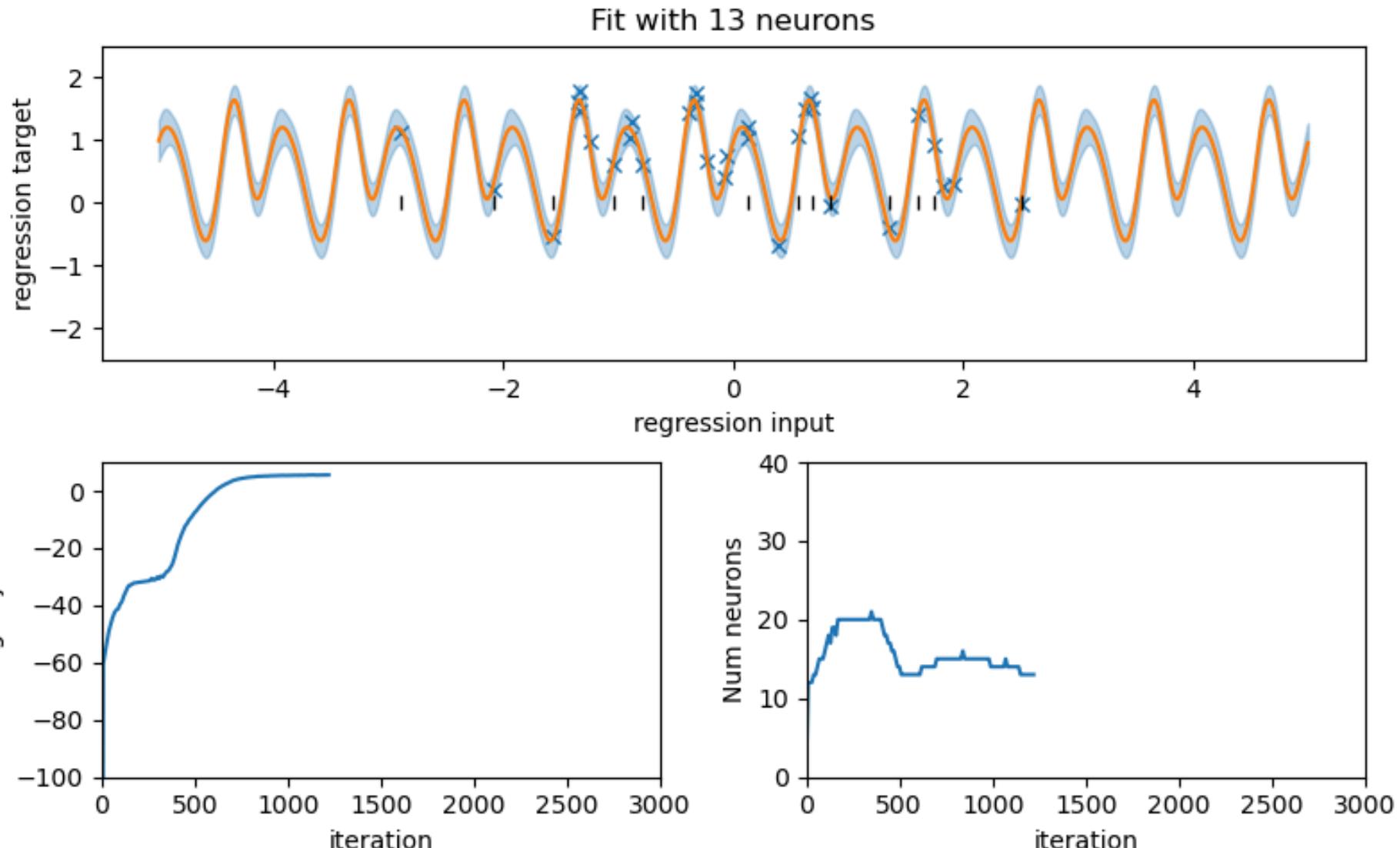
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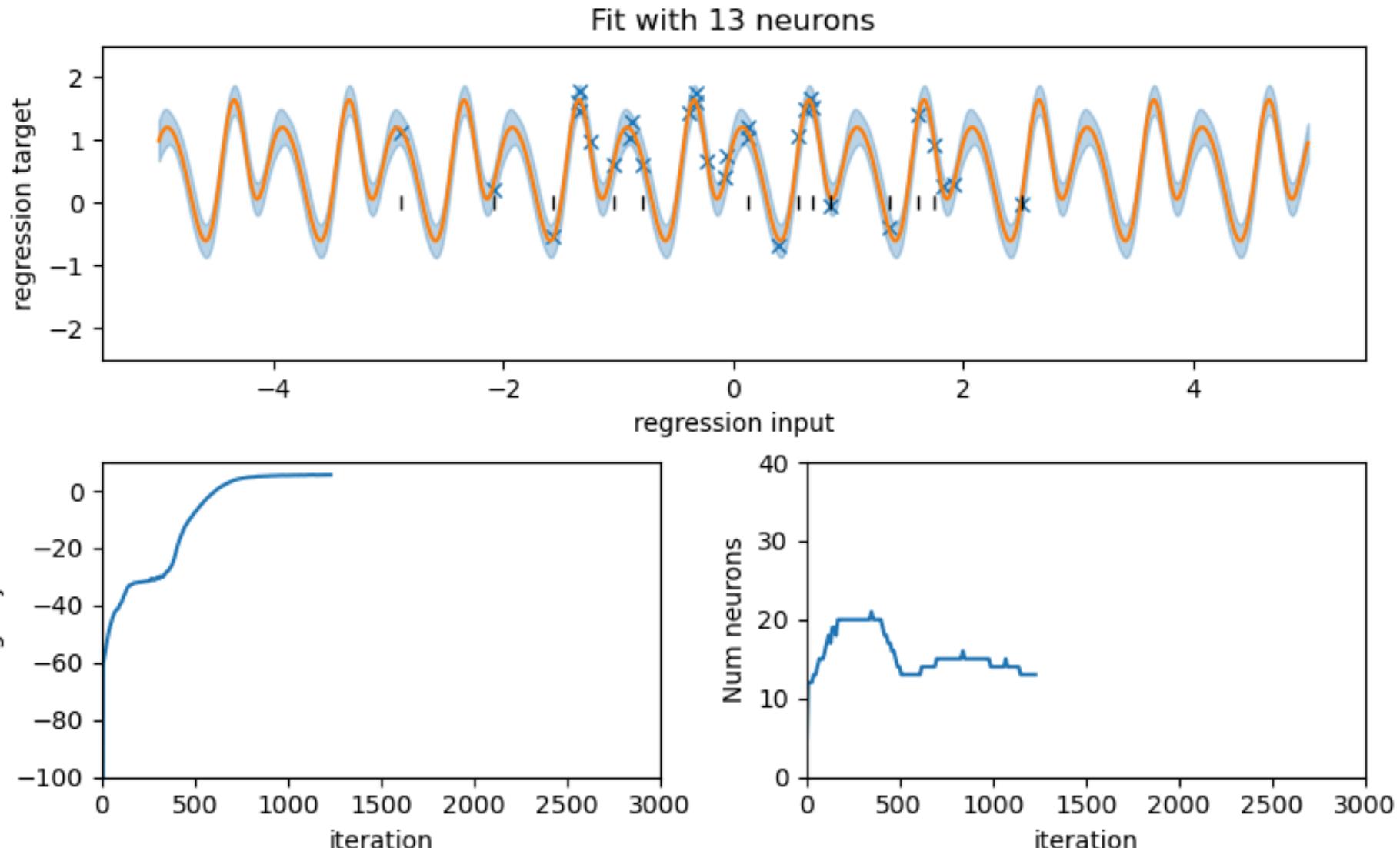
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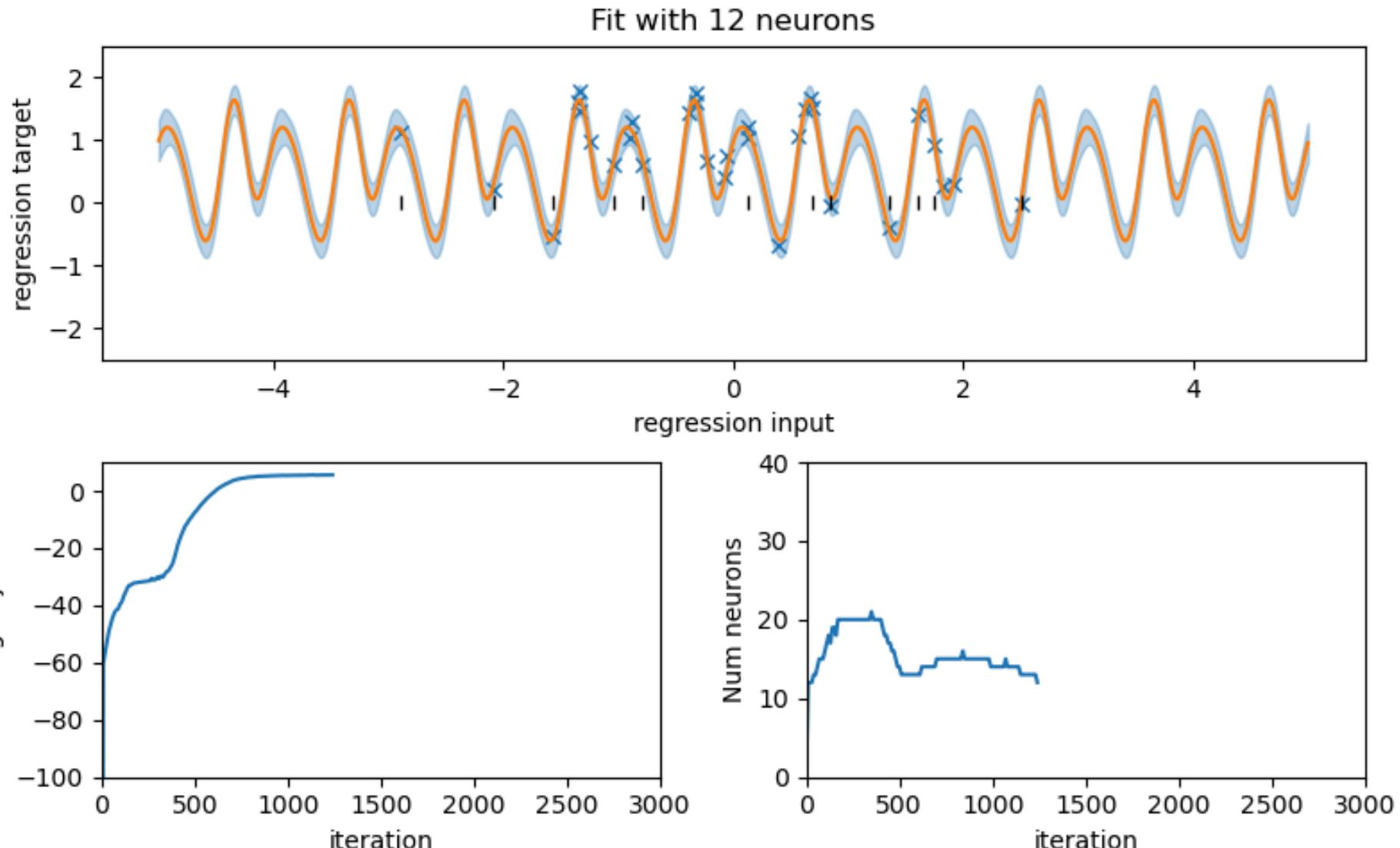
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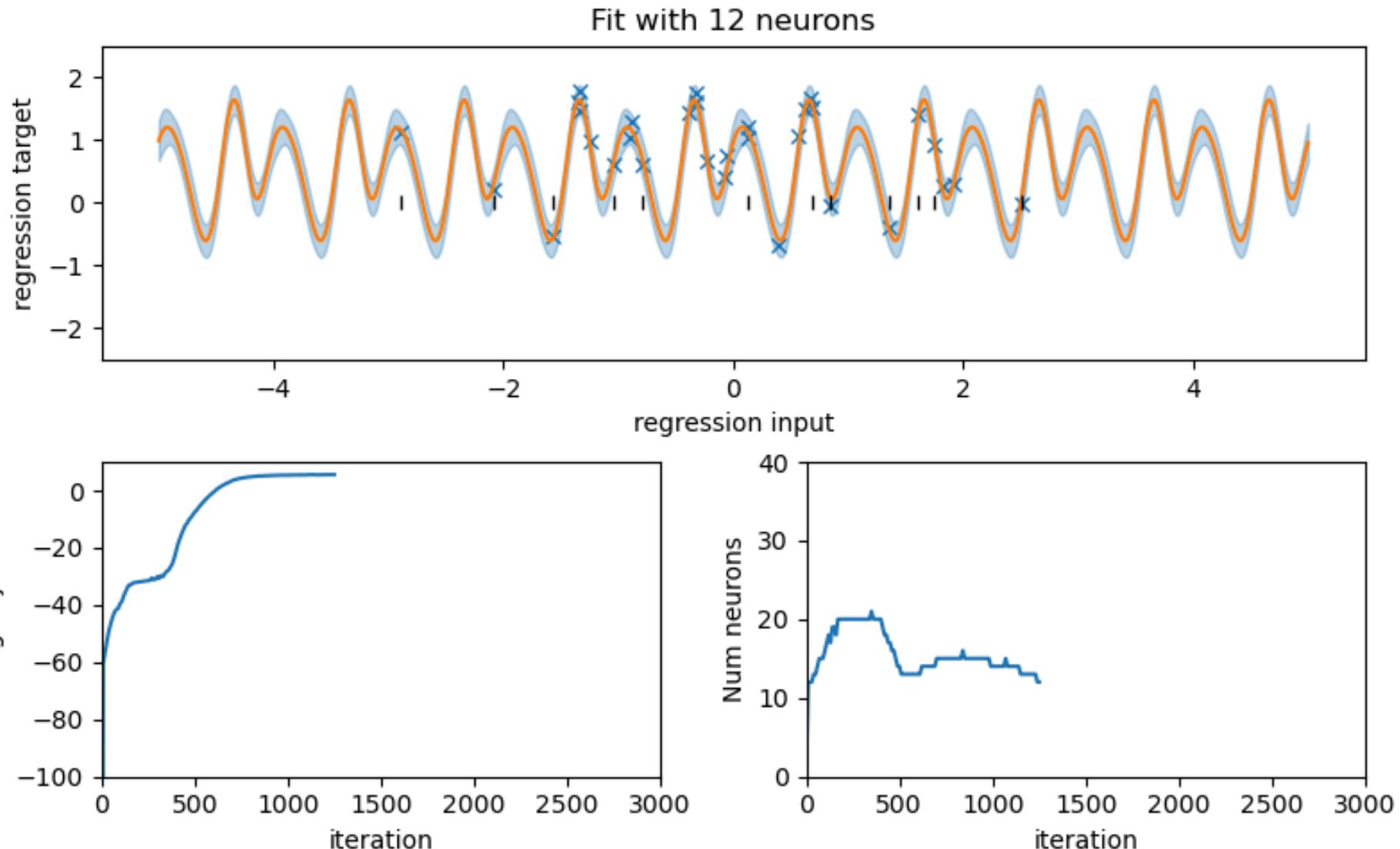
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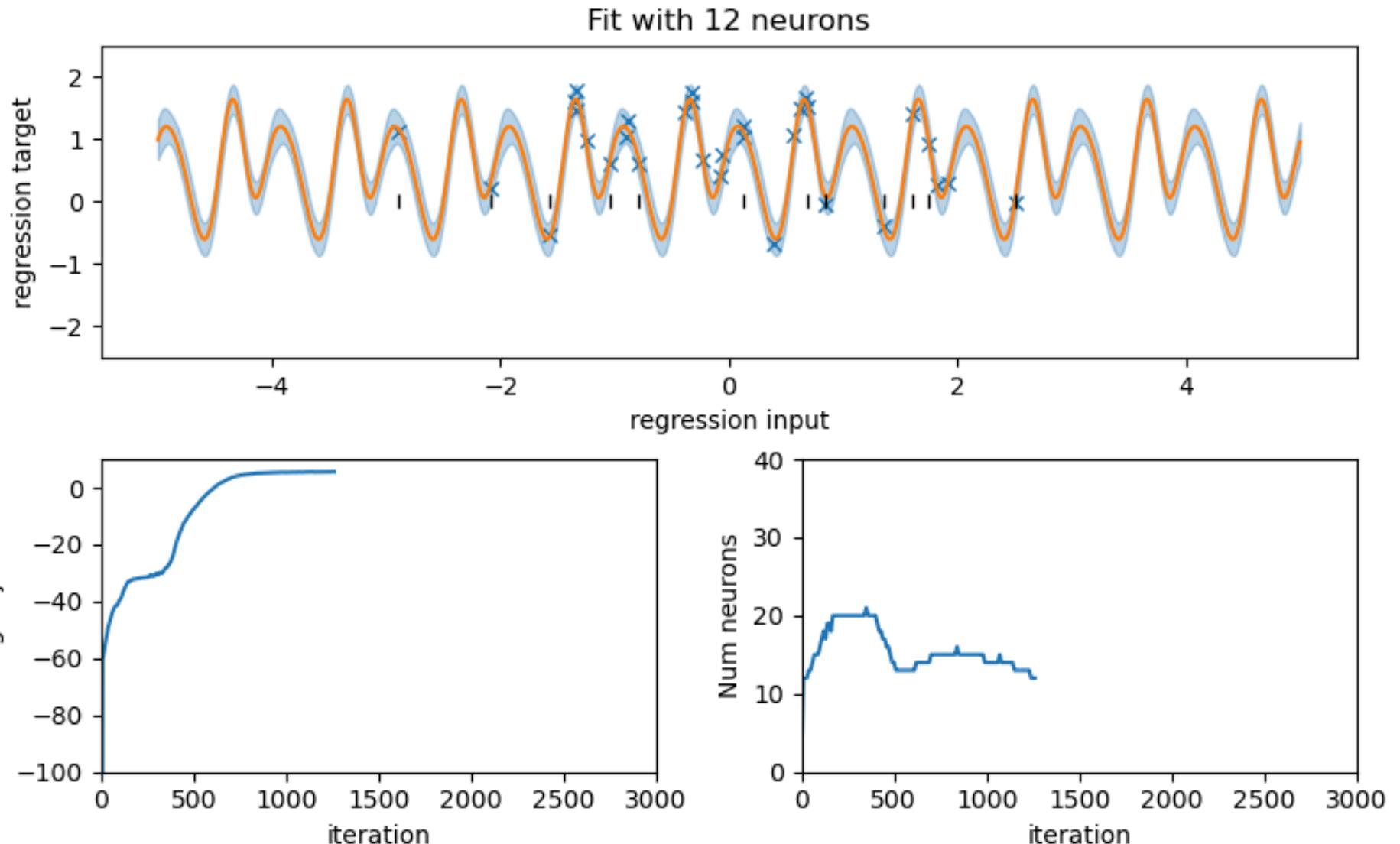
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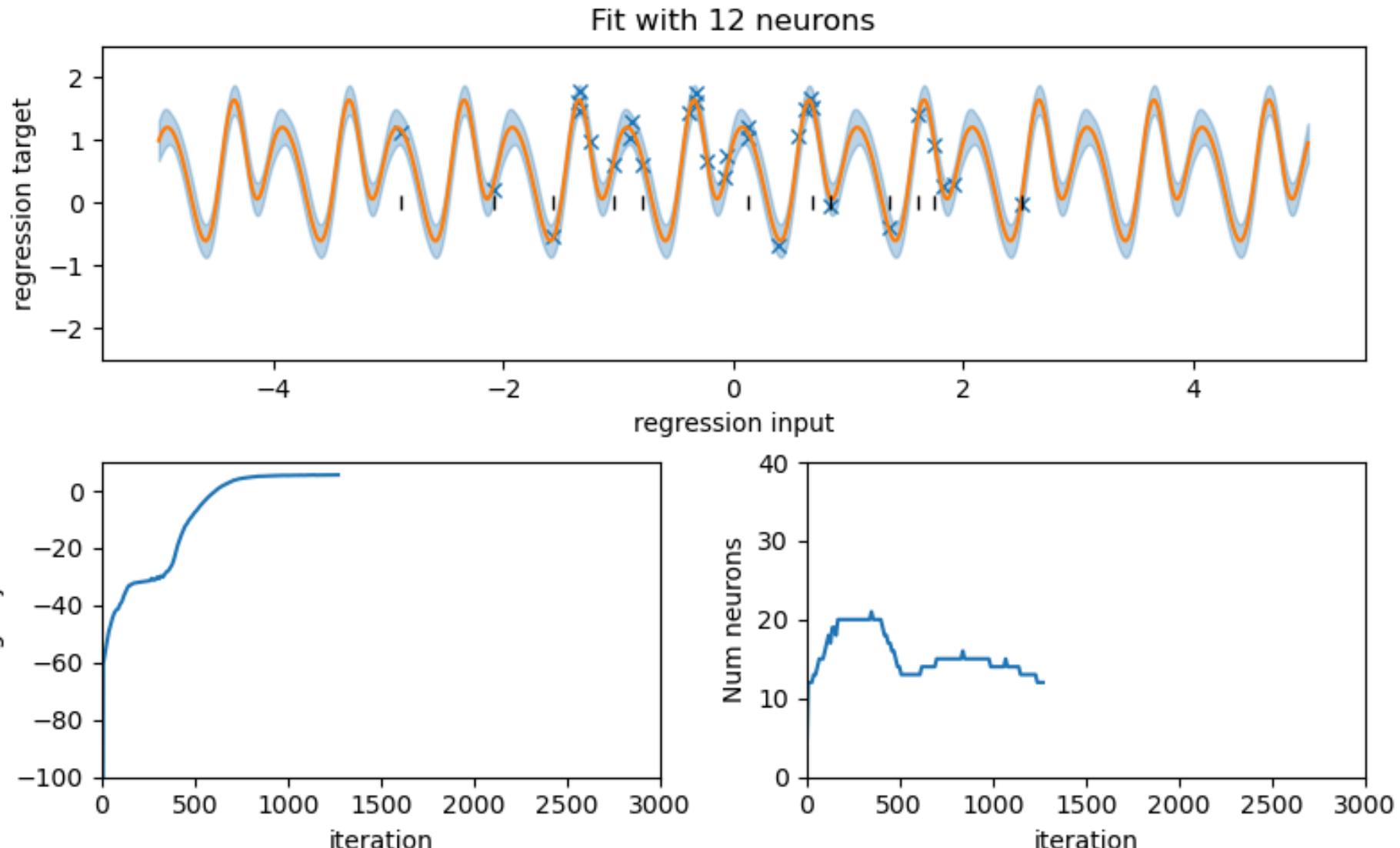
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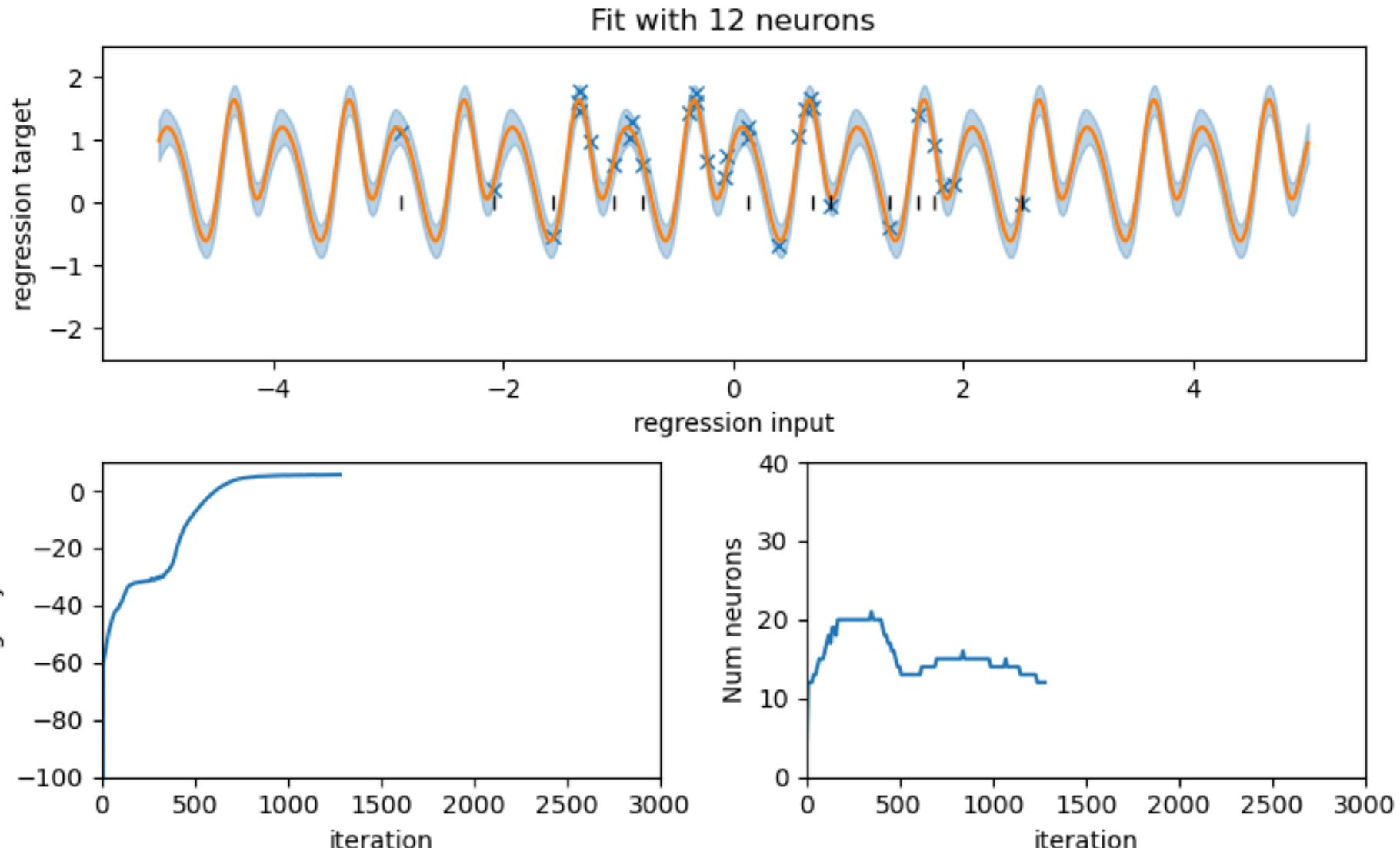
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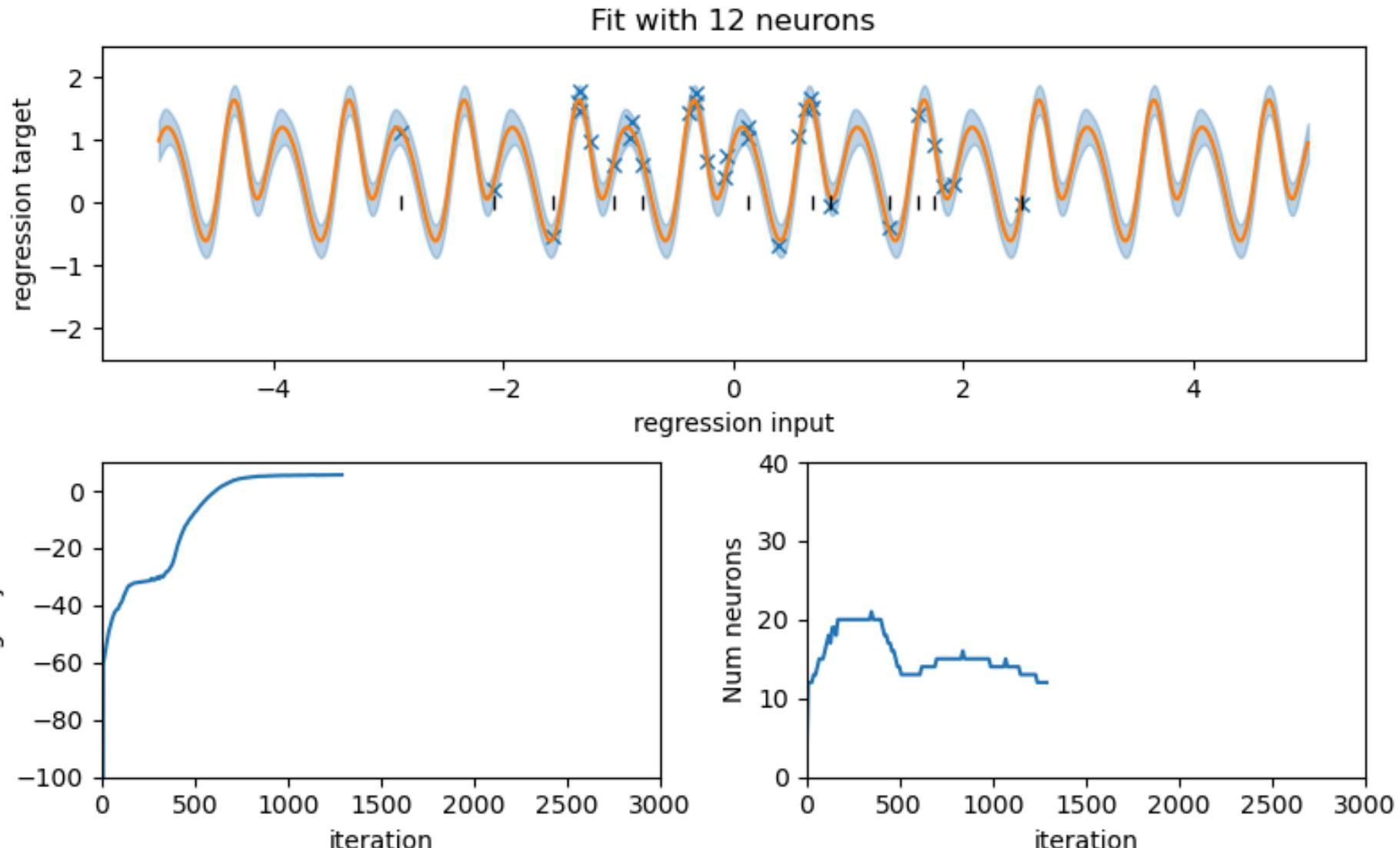
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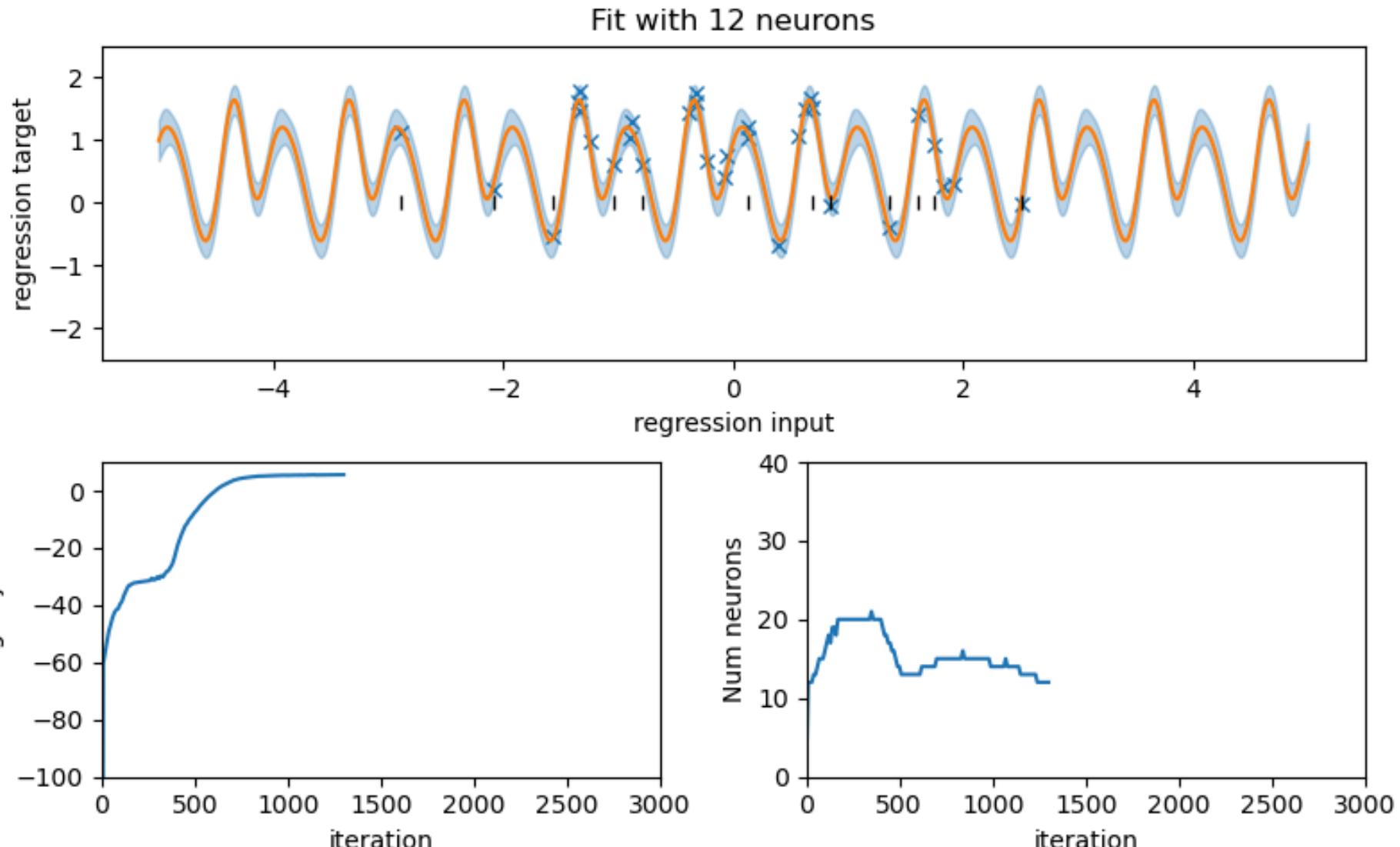
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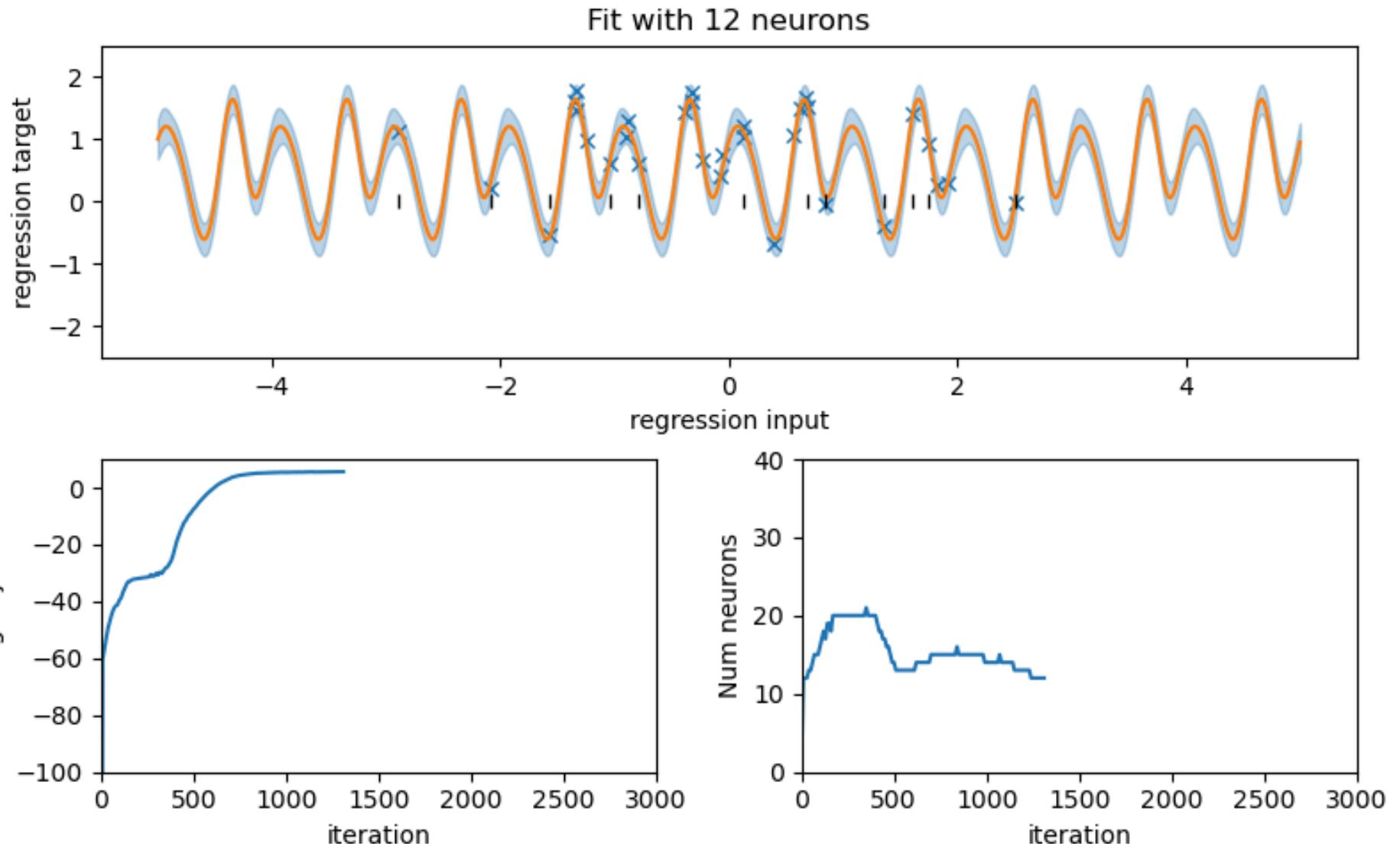
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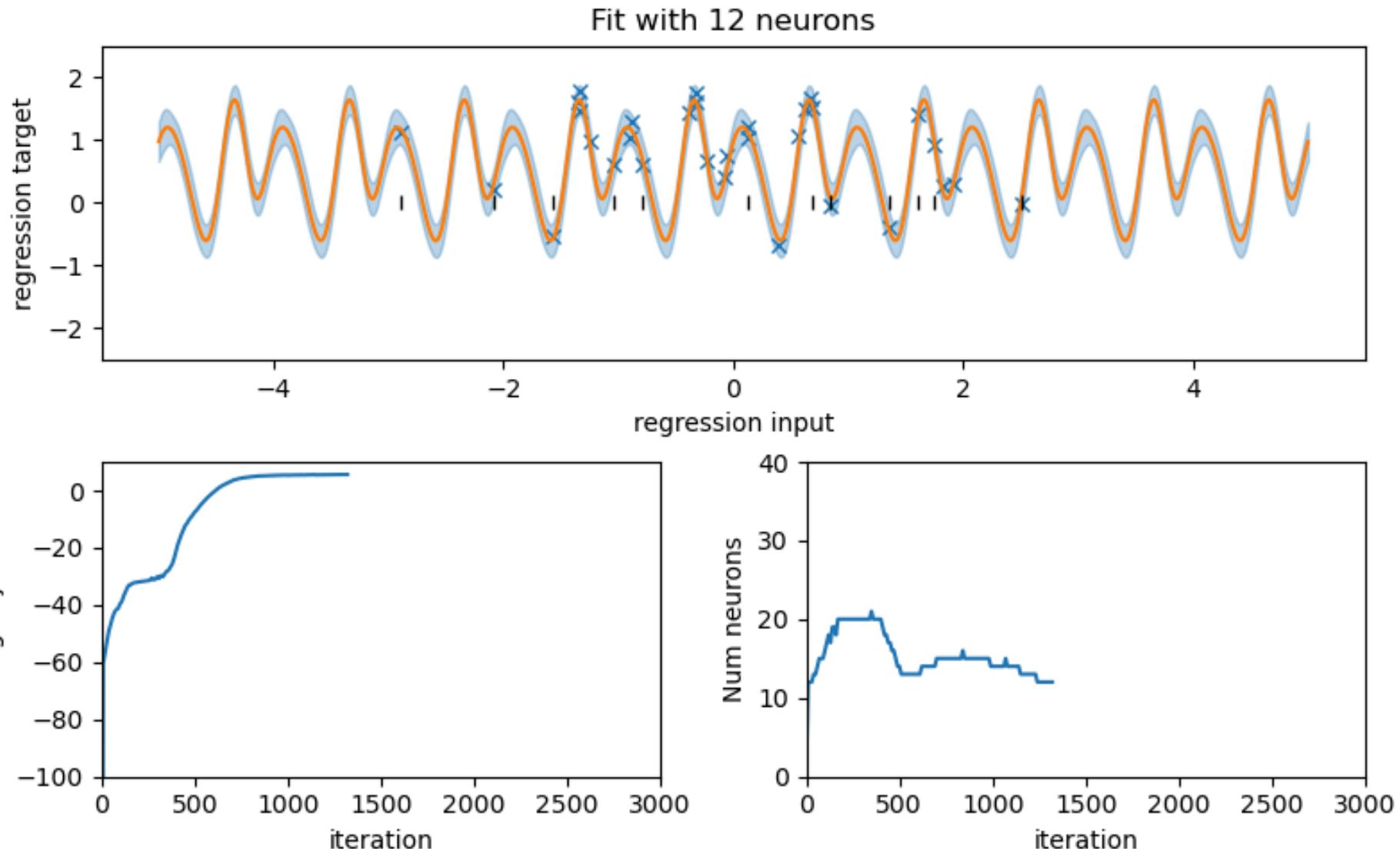
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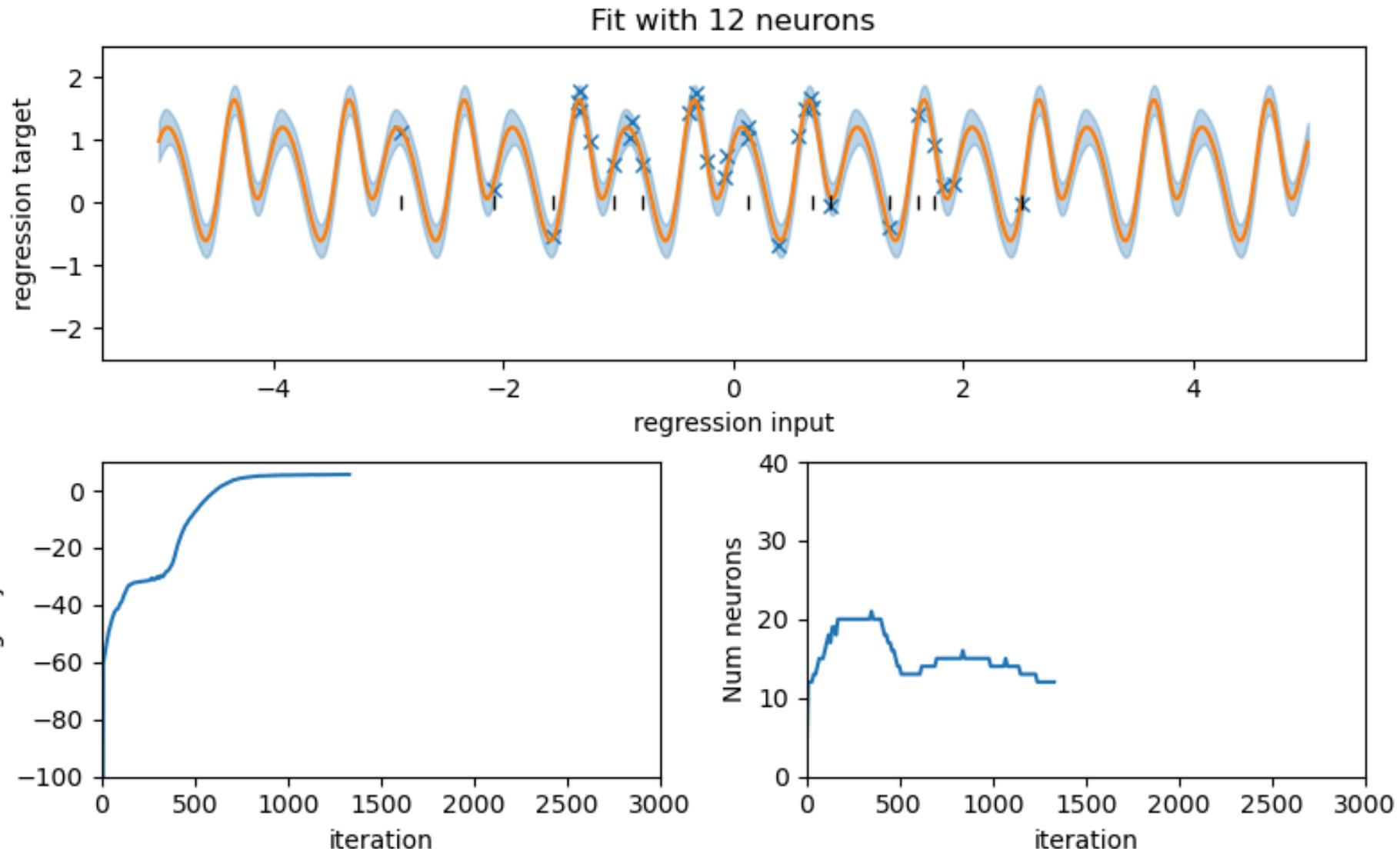
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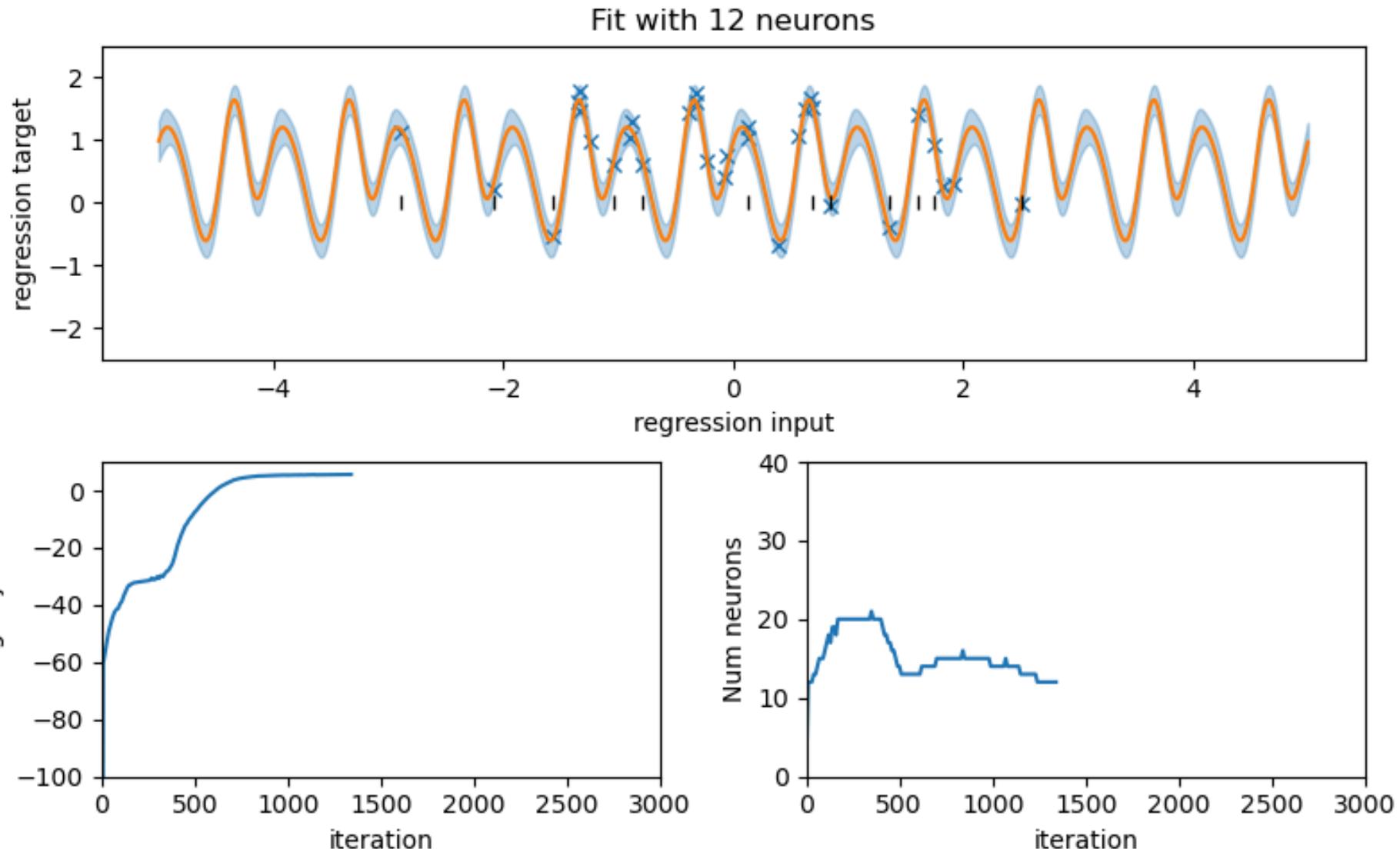
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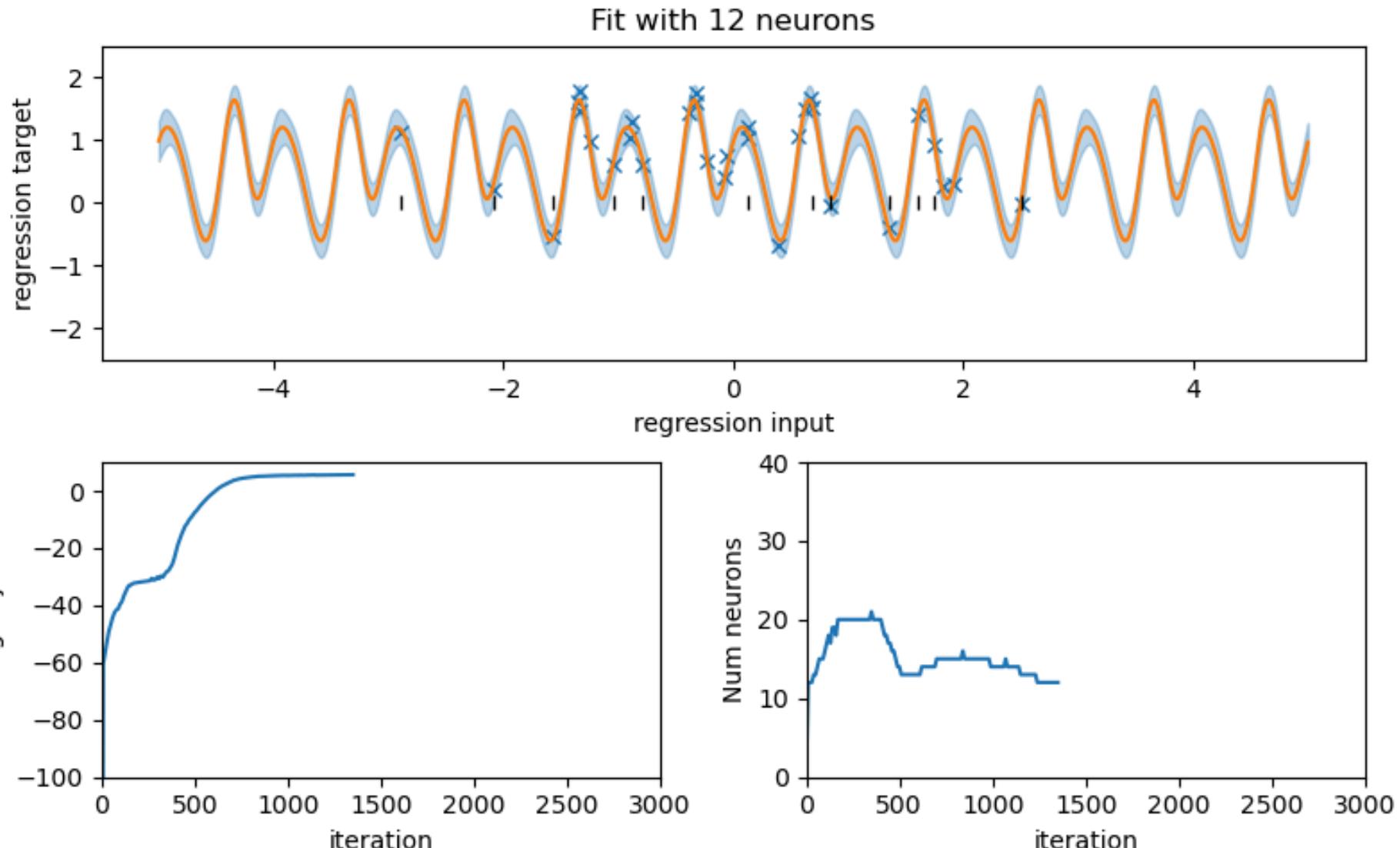
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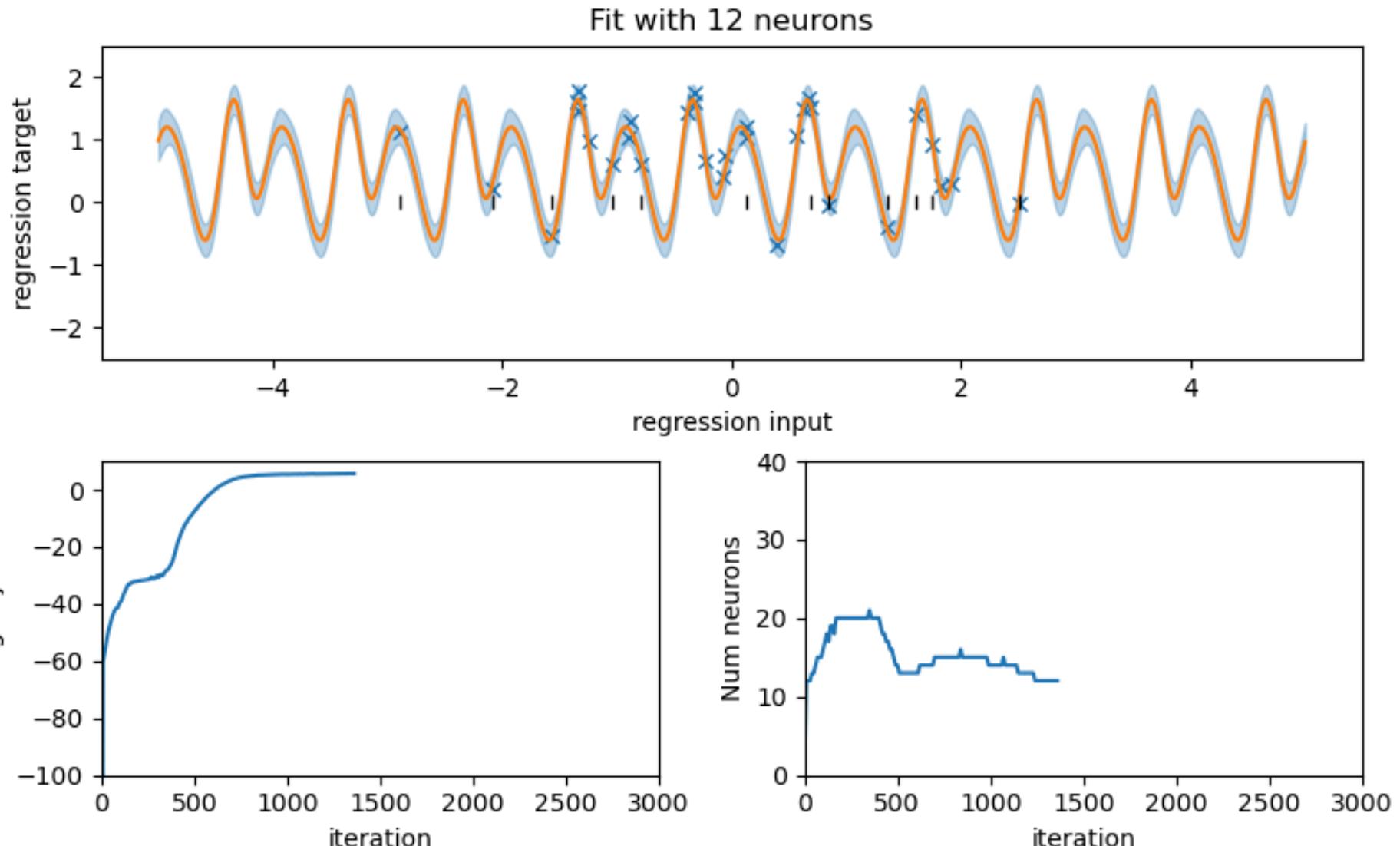
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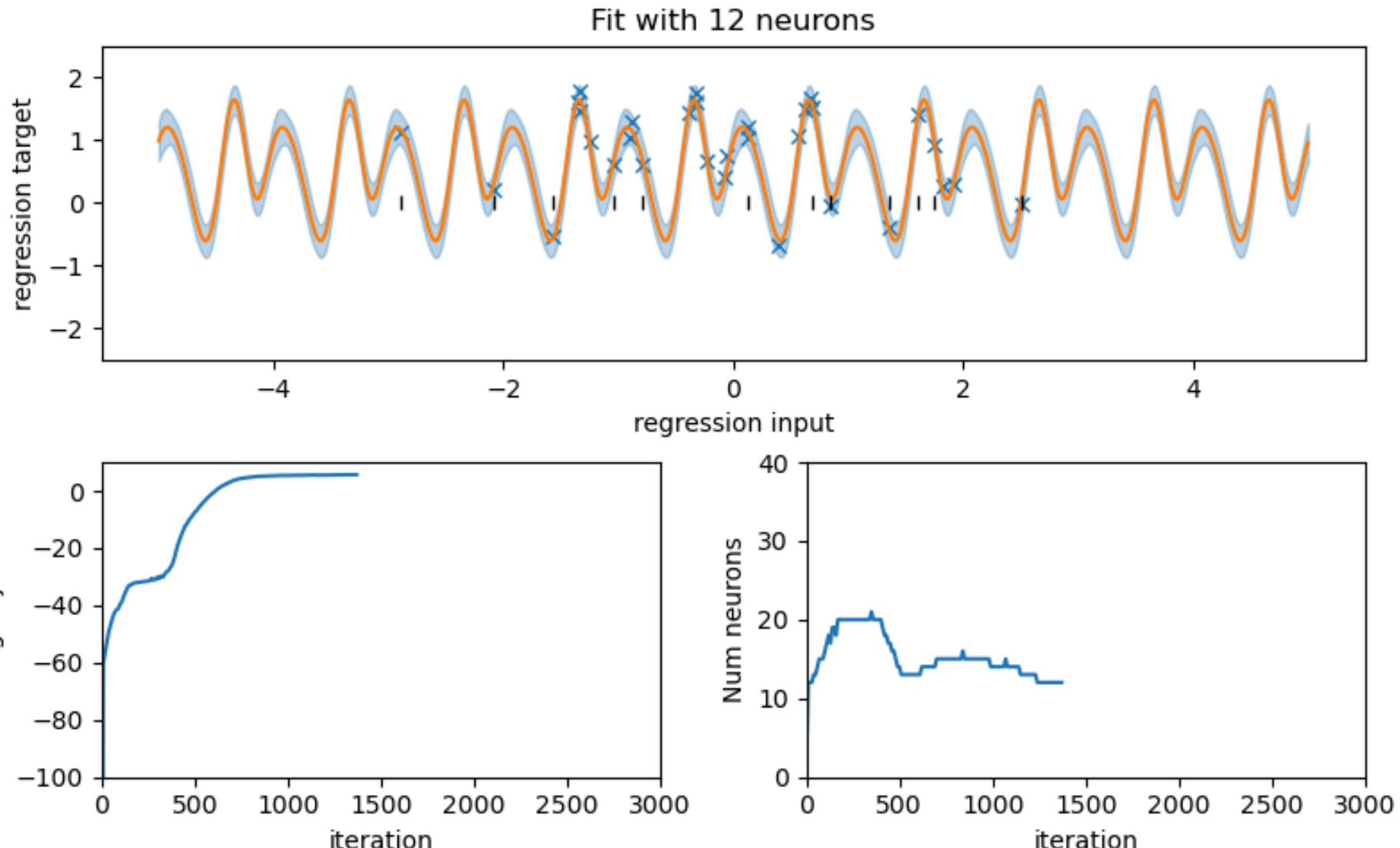
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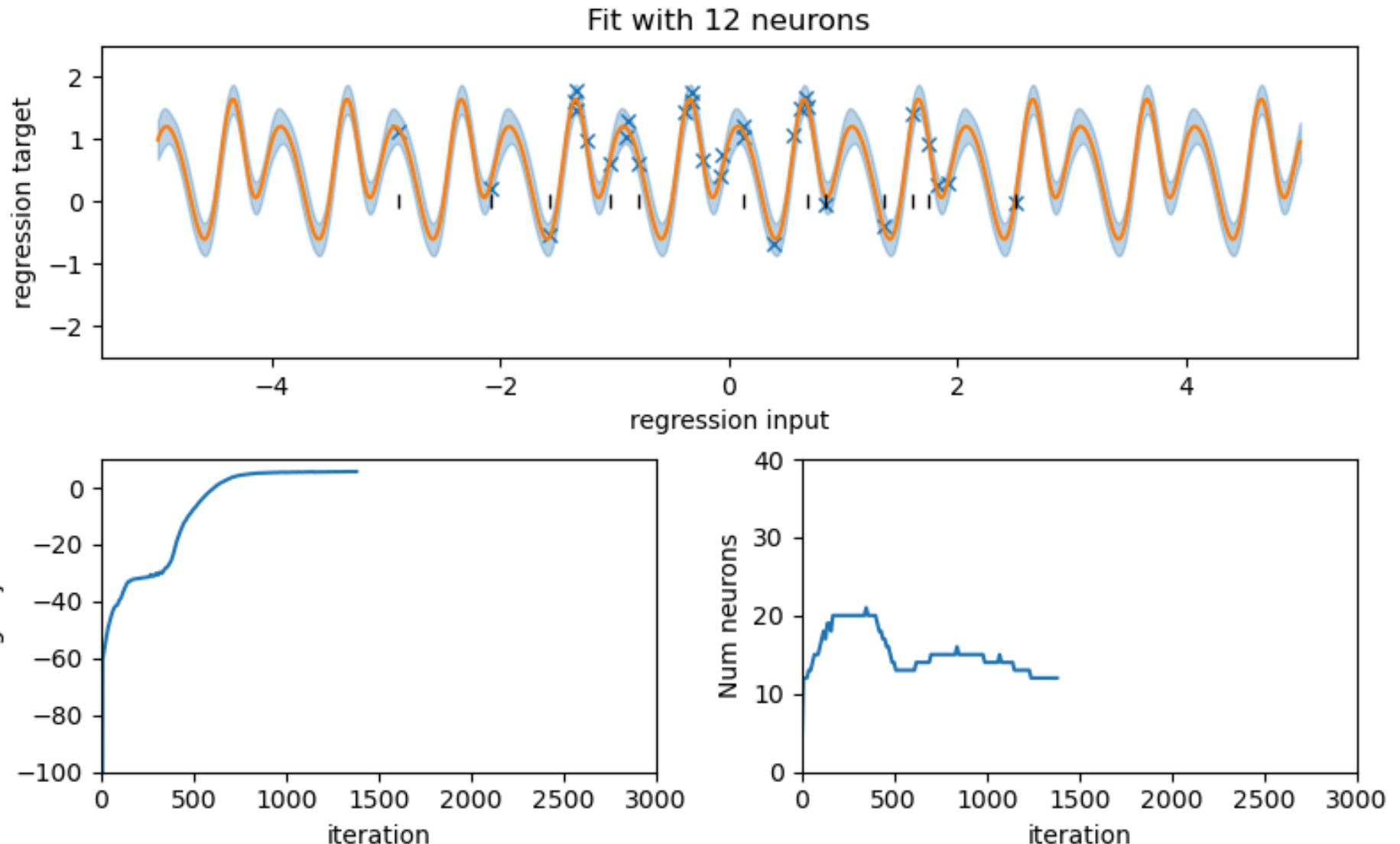
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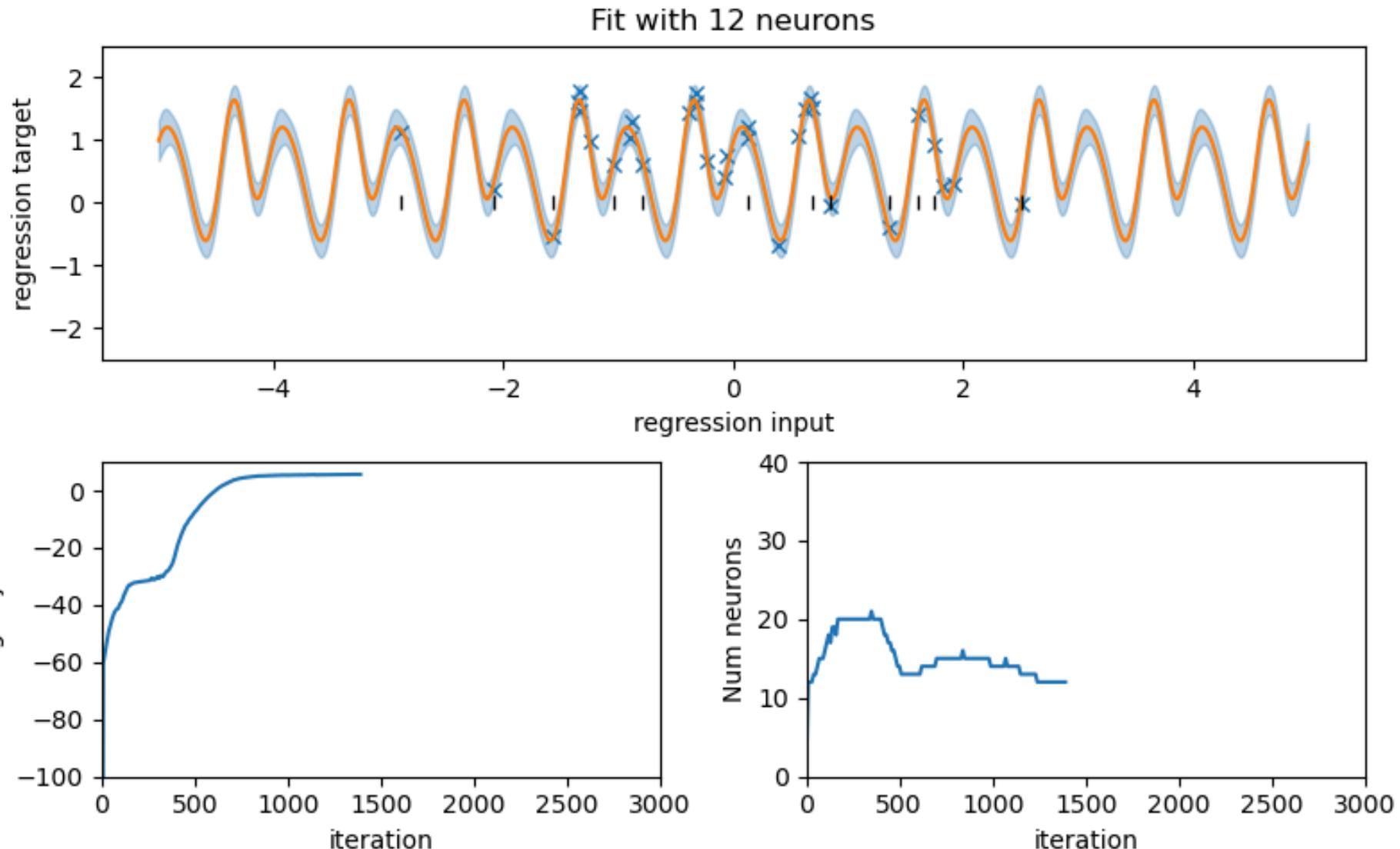
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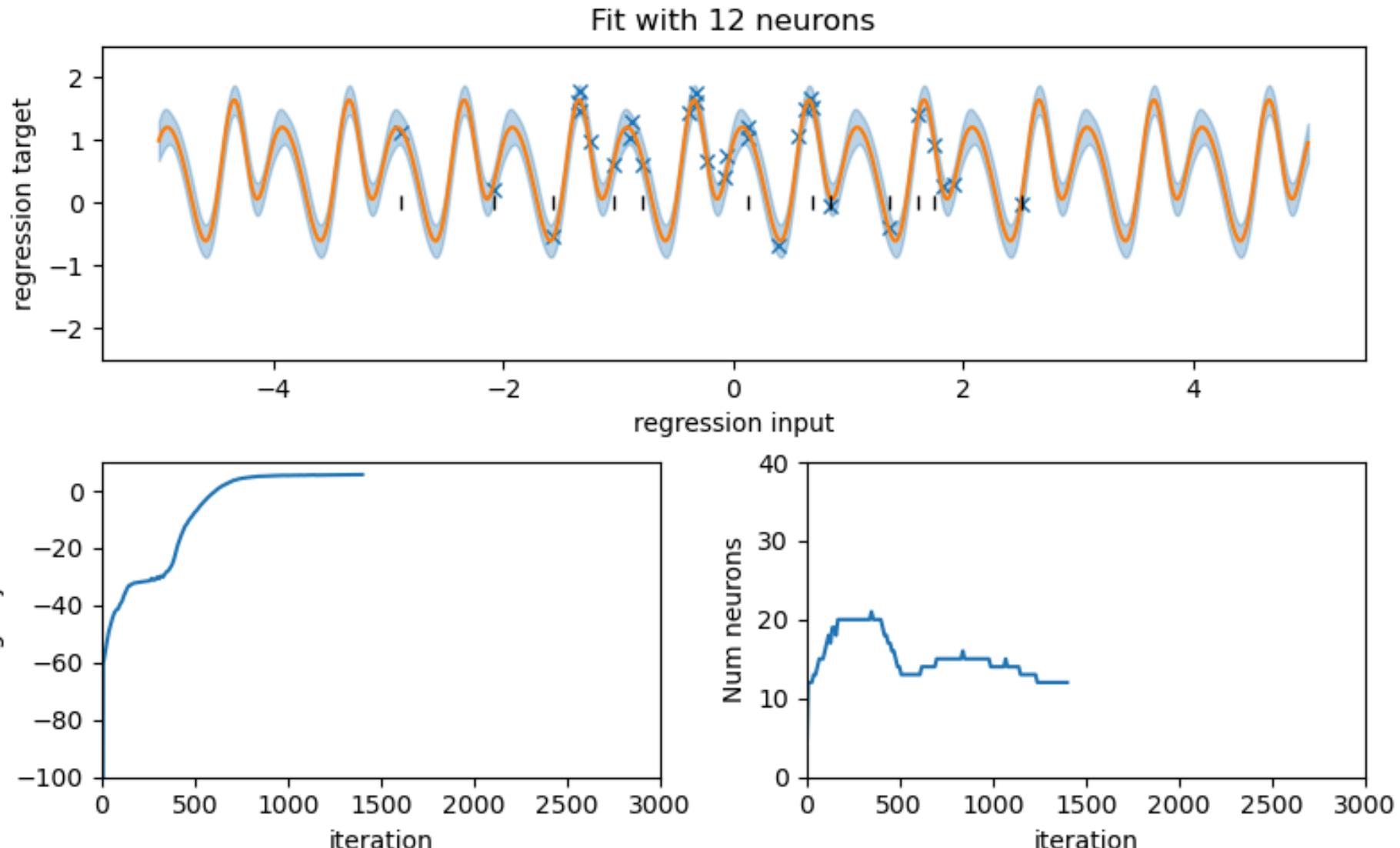
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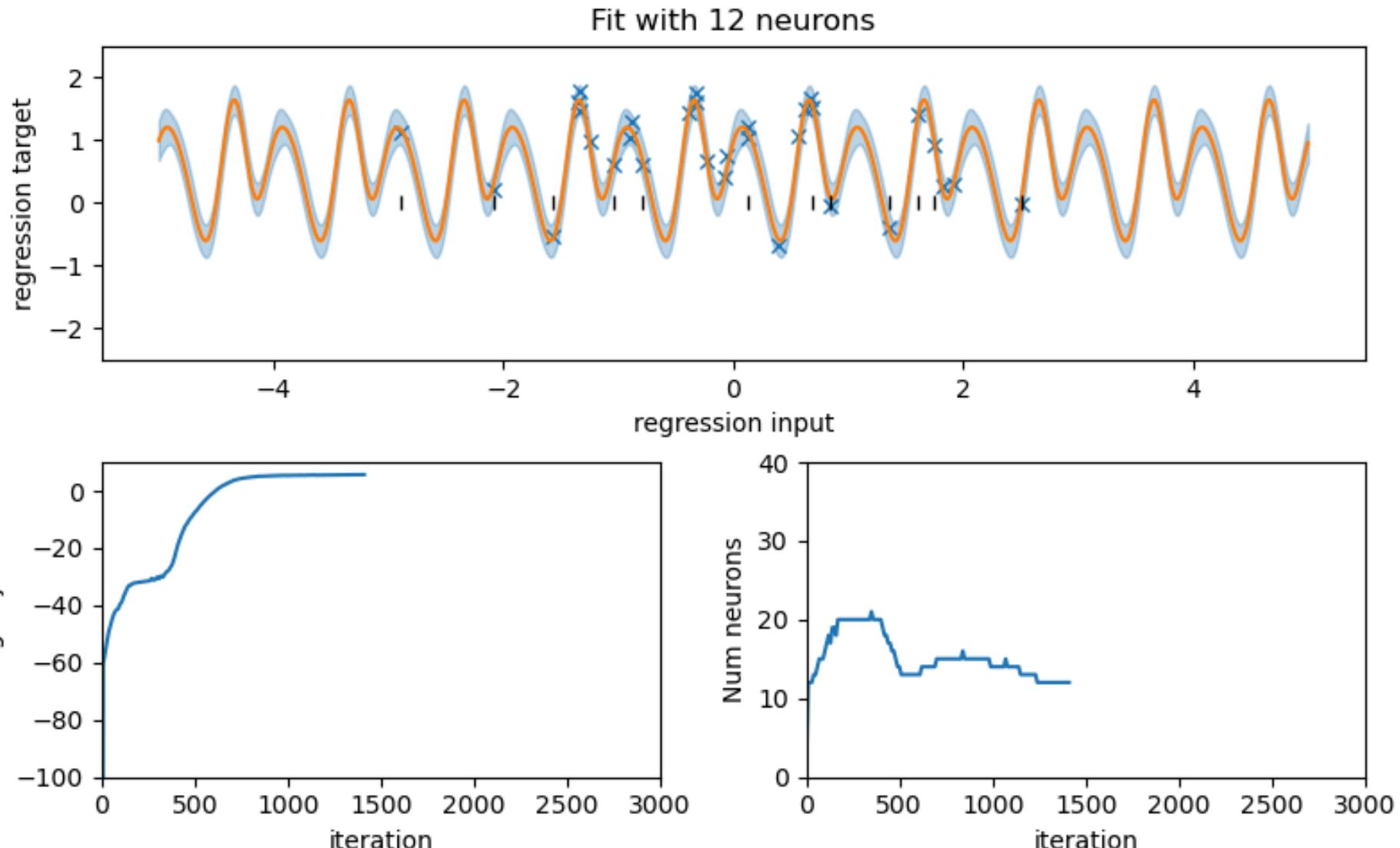
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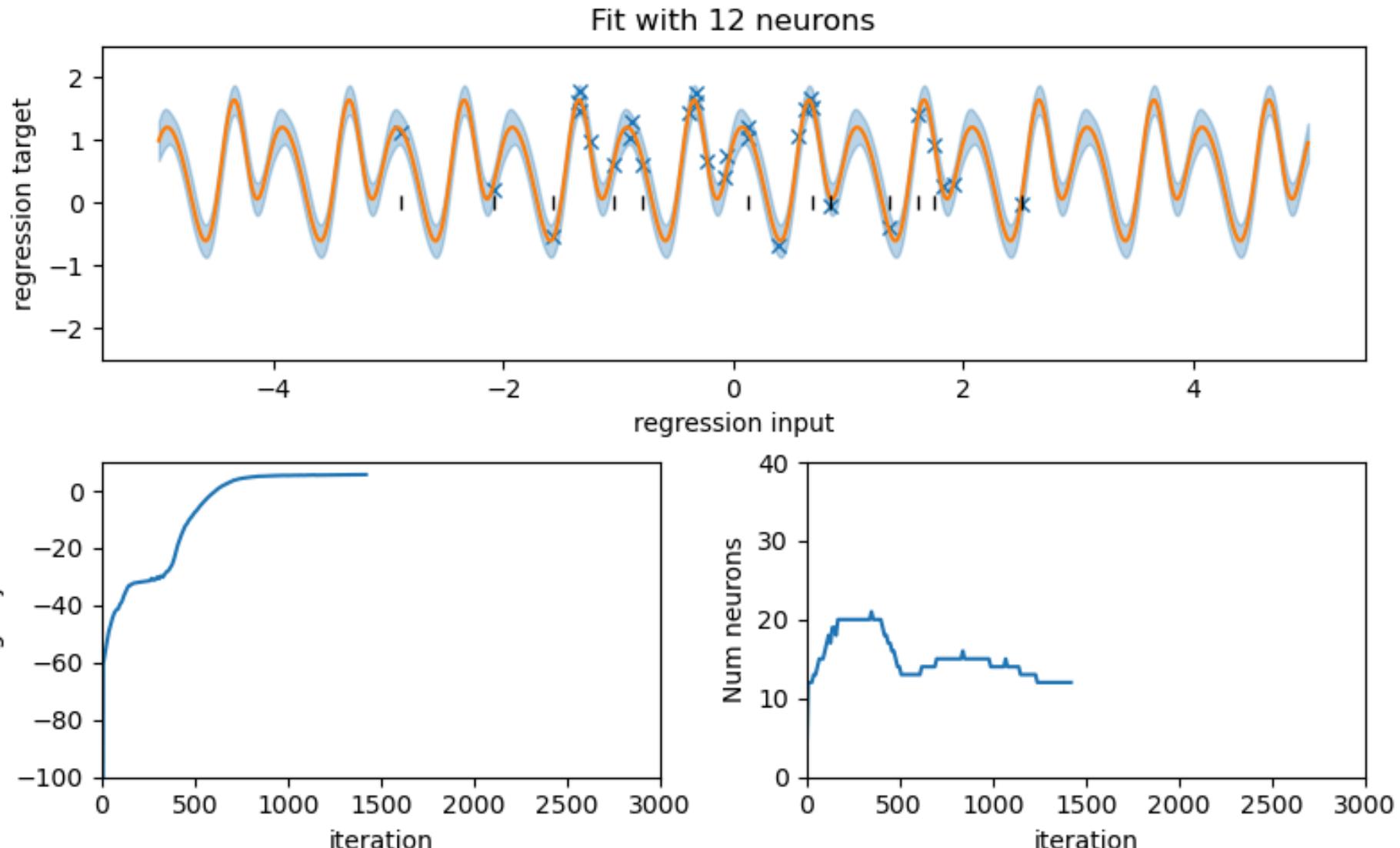
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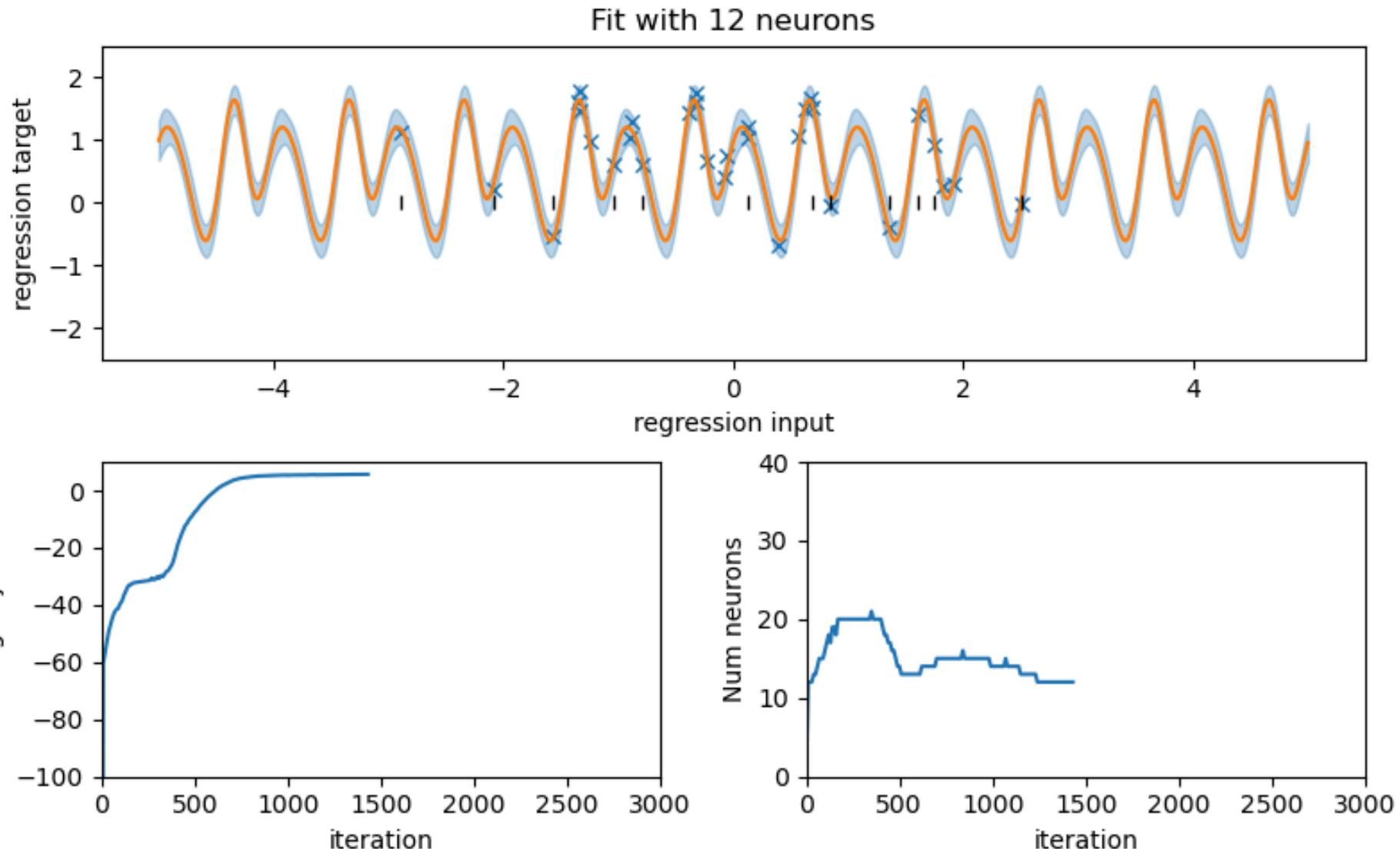
Growing Neurons, Grokking, Pruning

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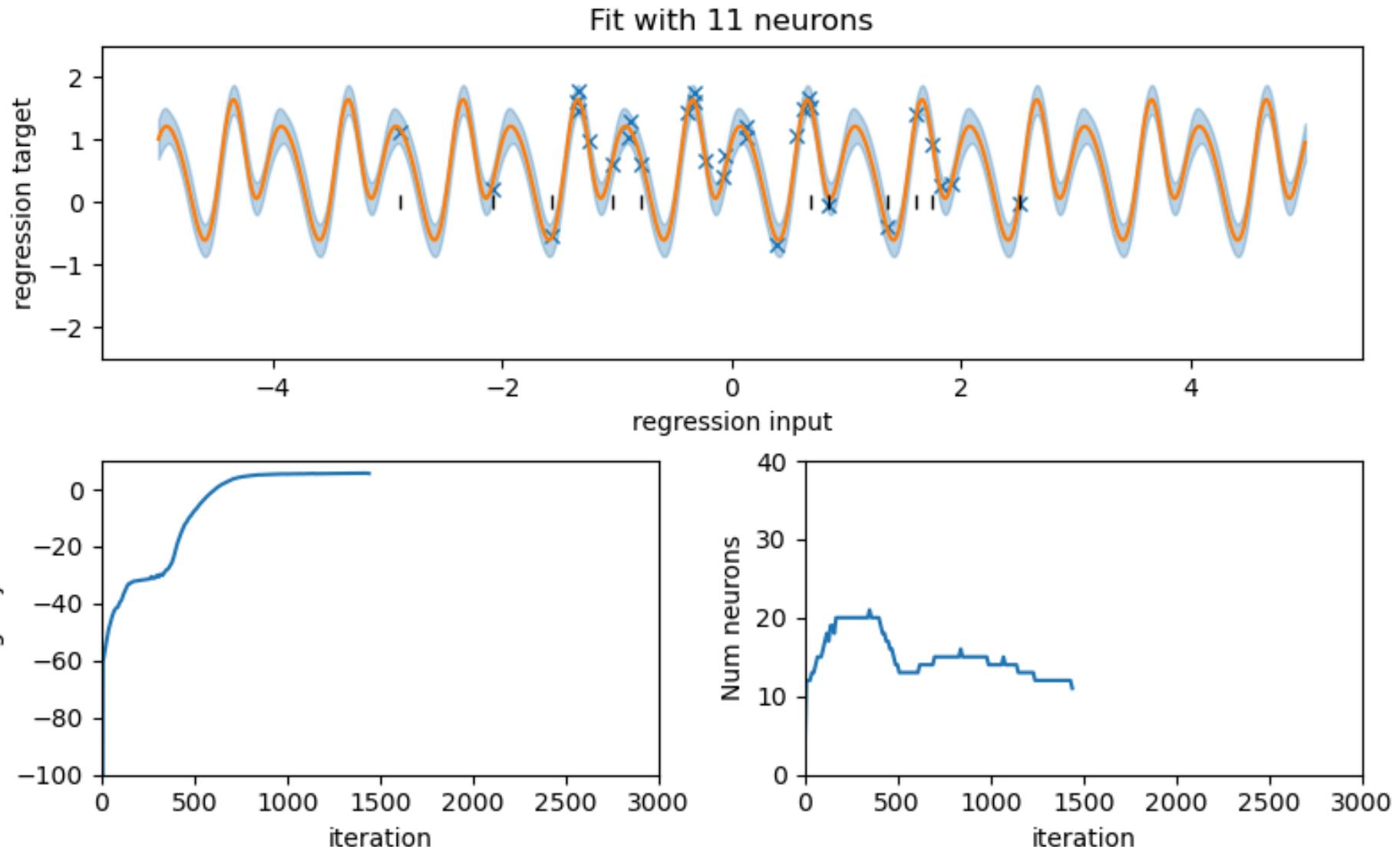
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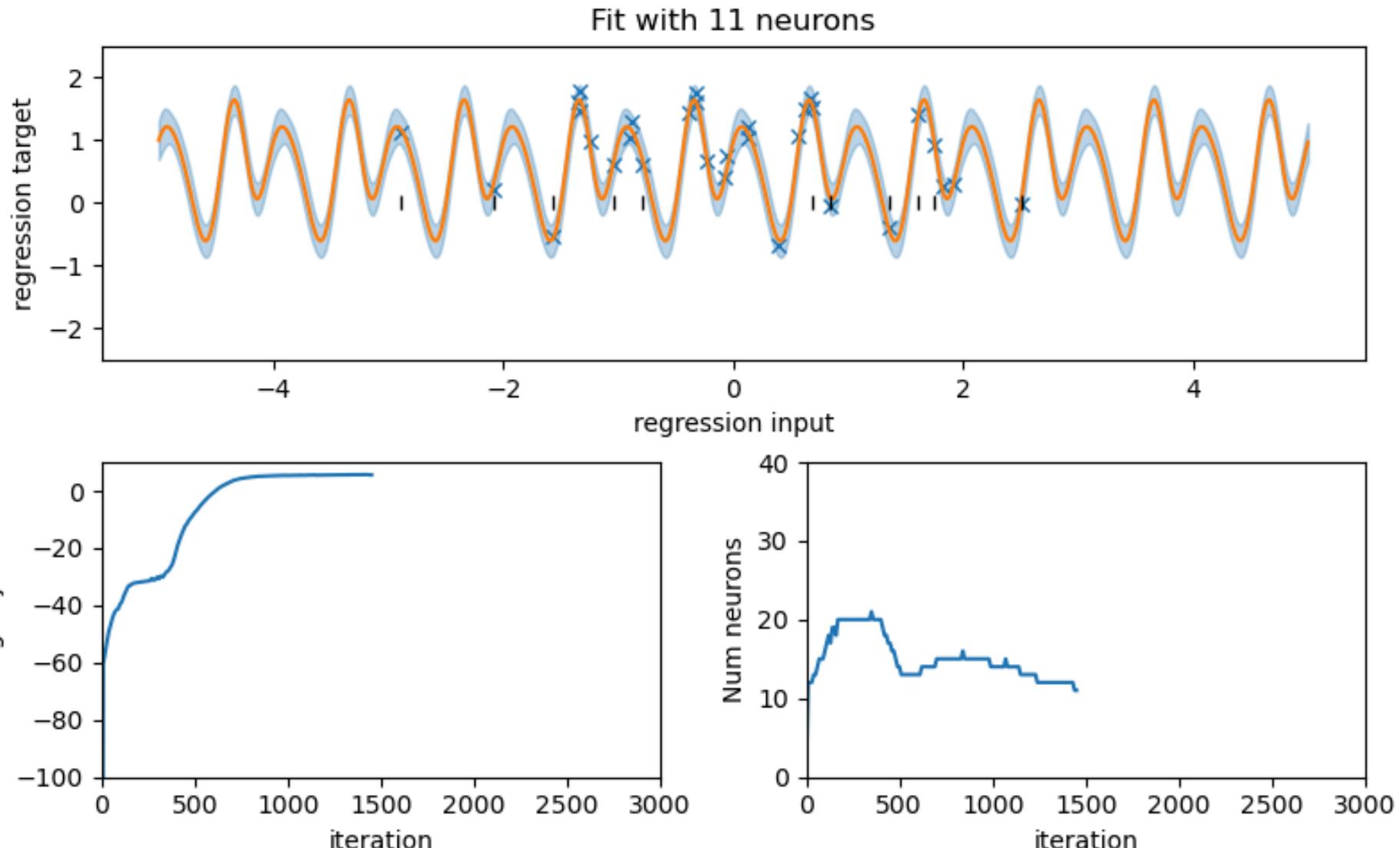
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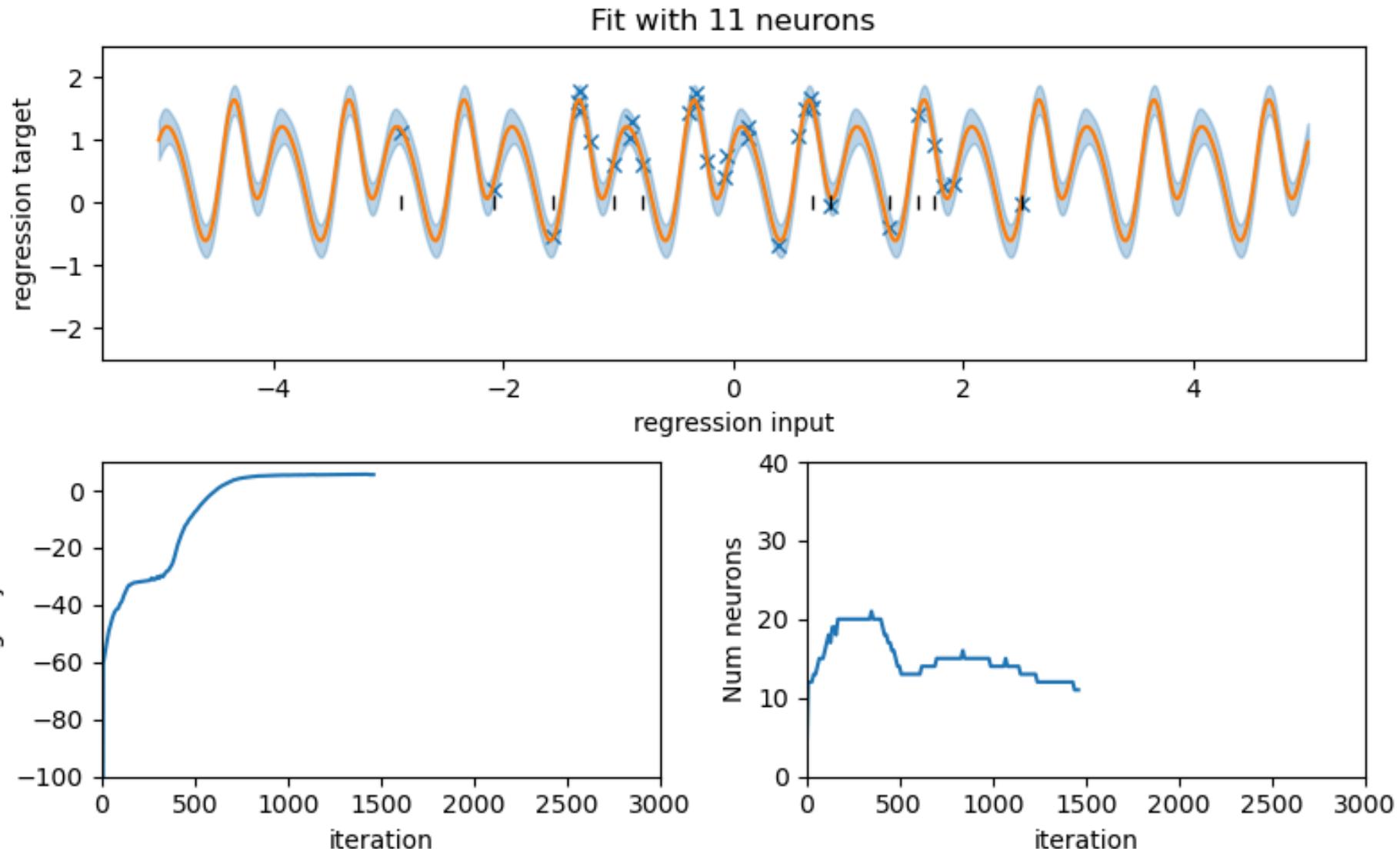
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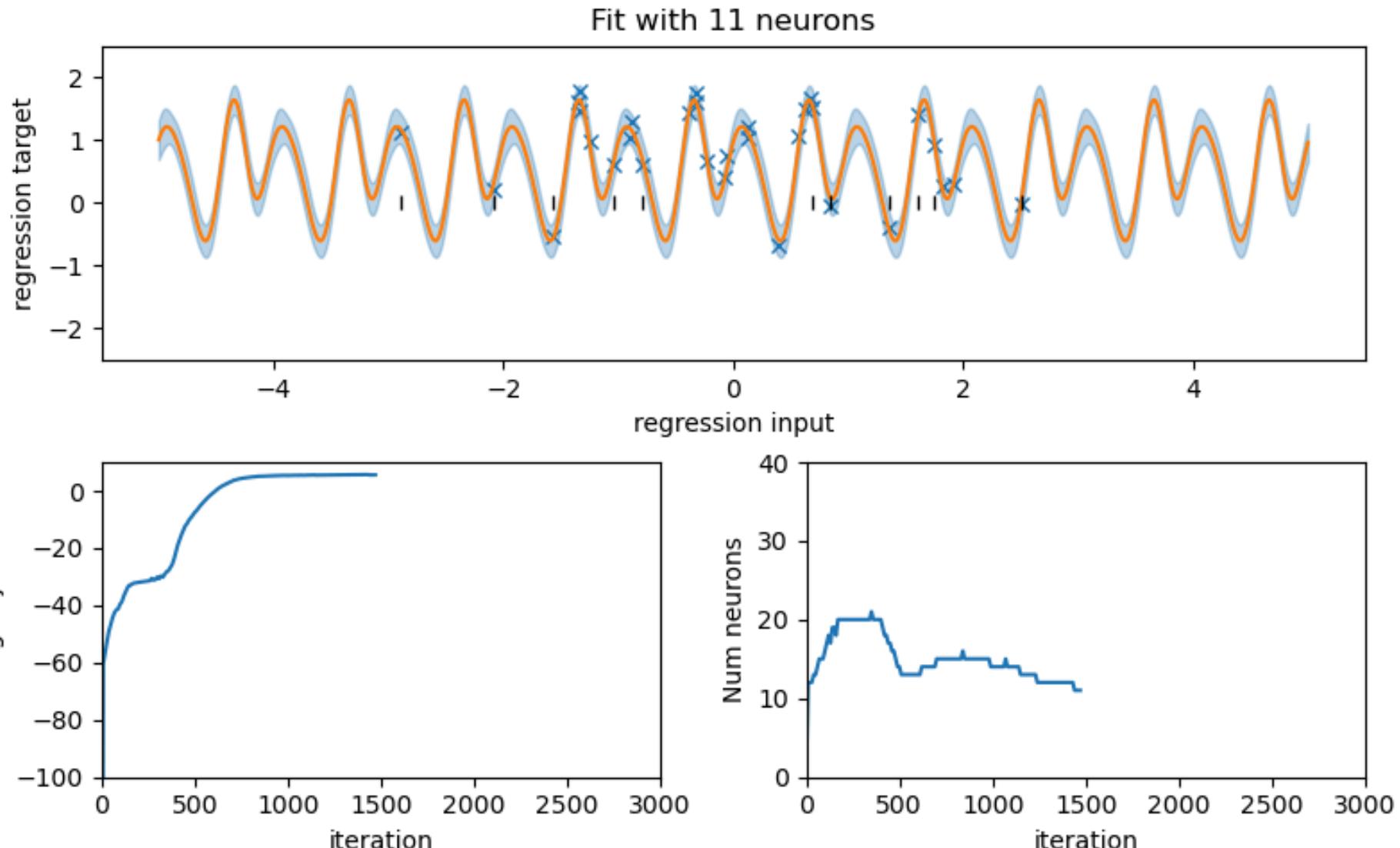
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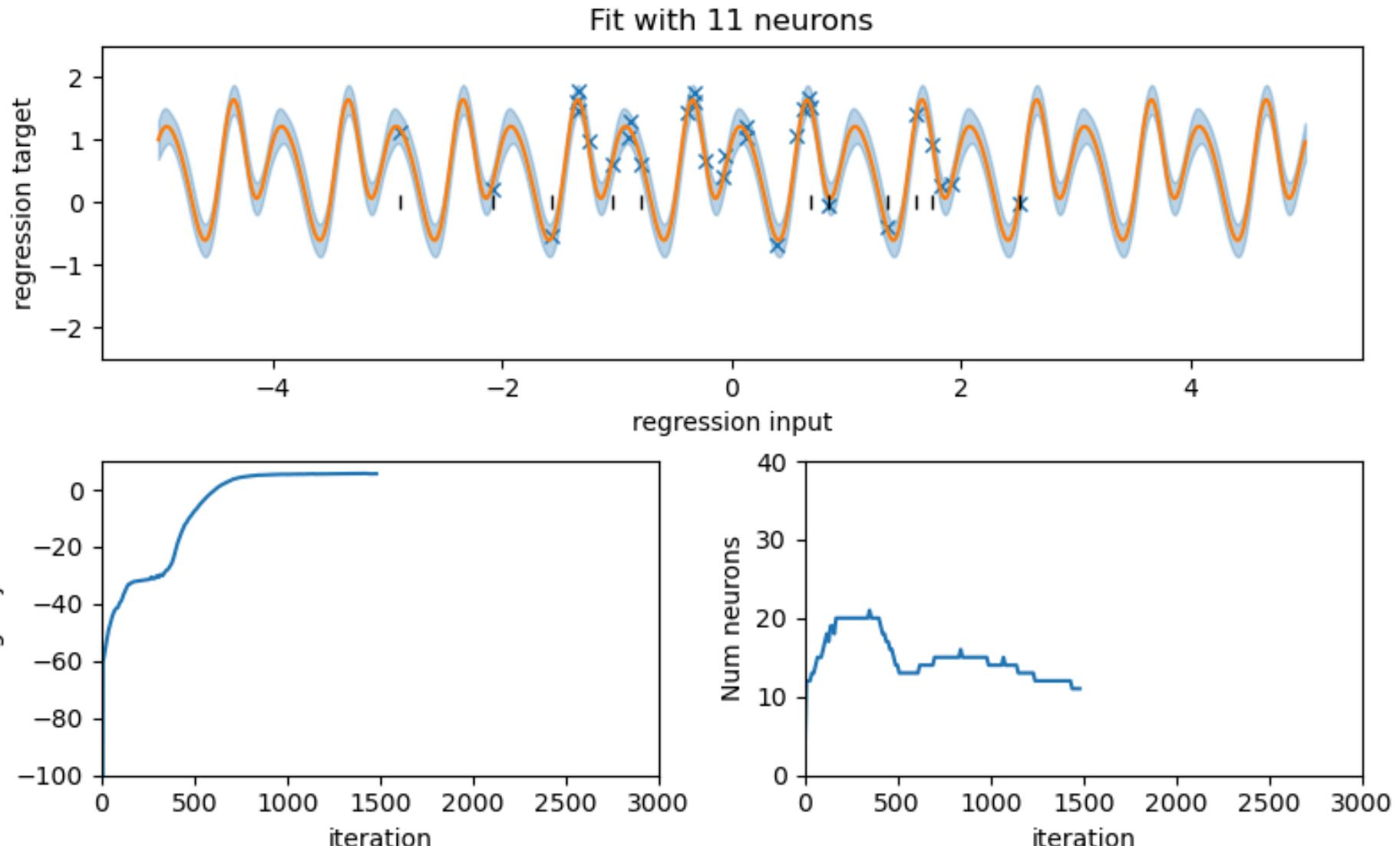
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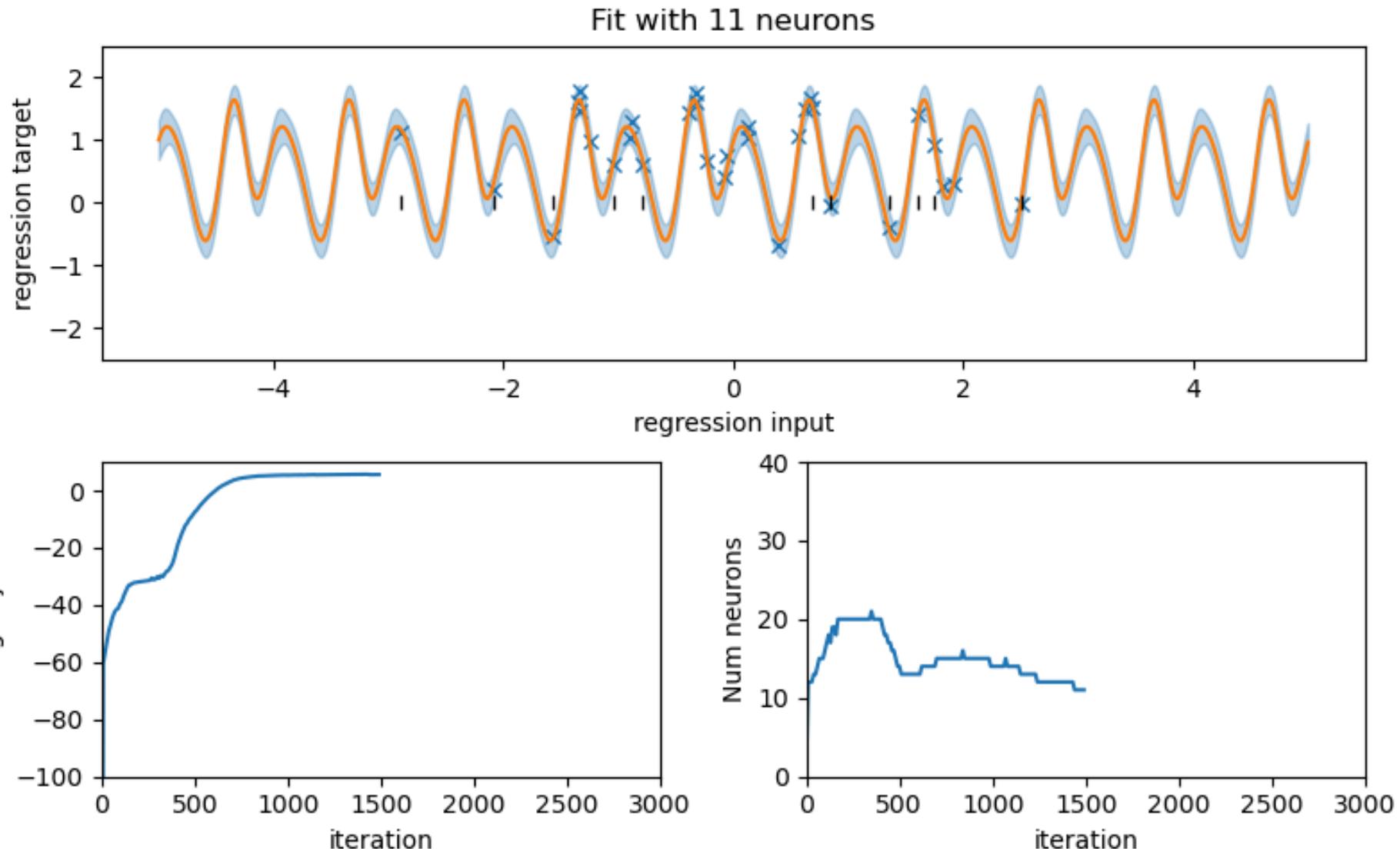
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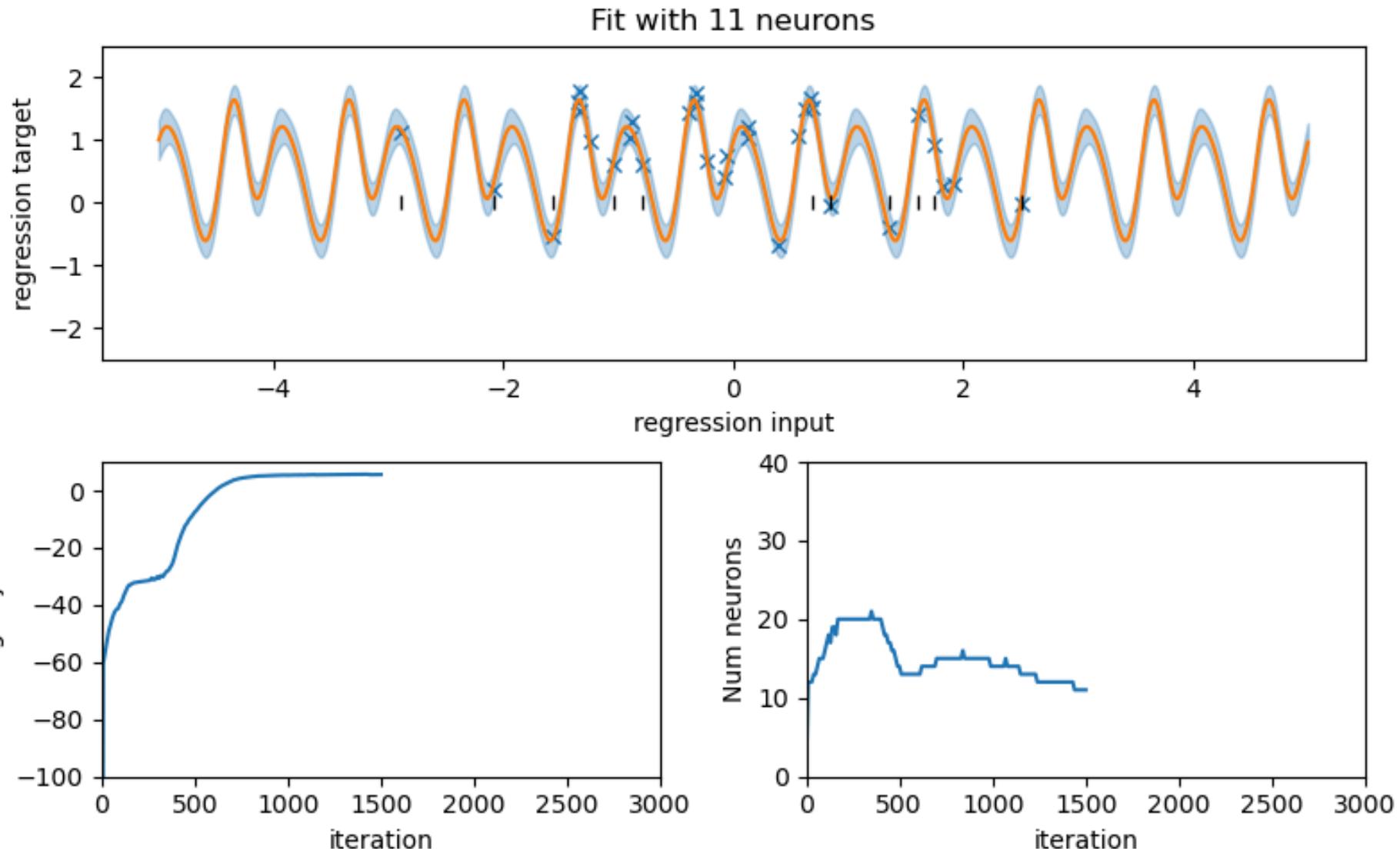
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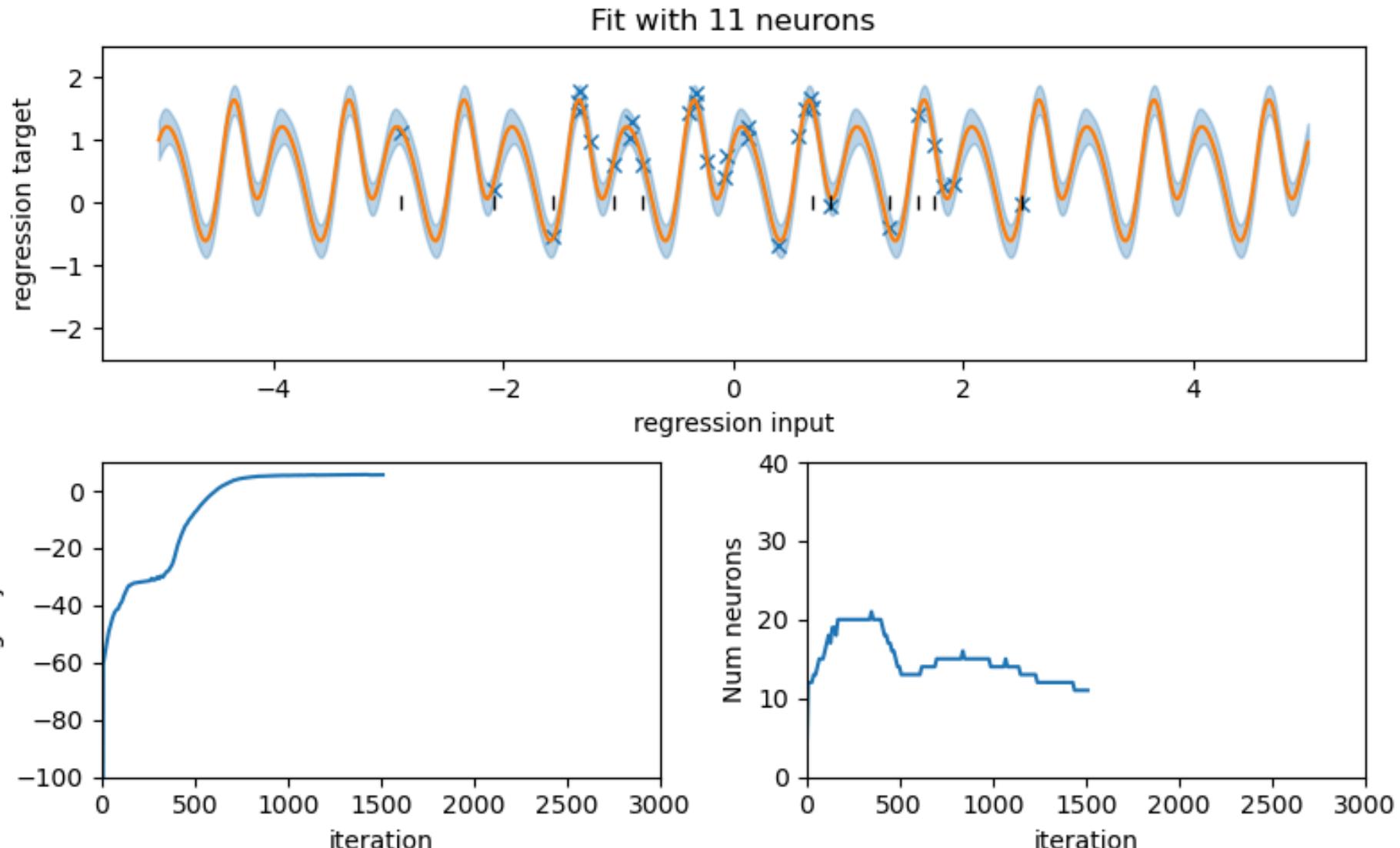
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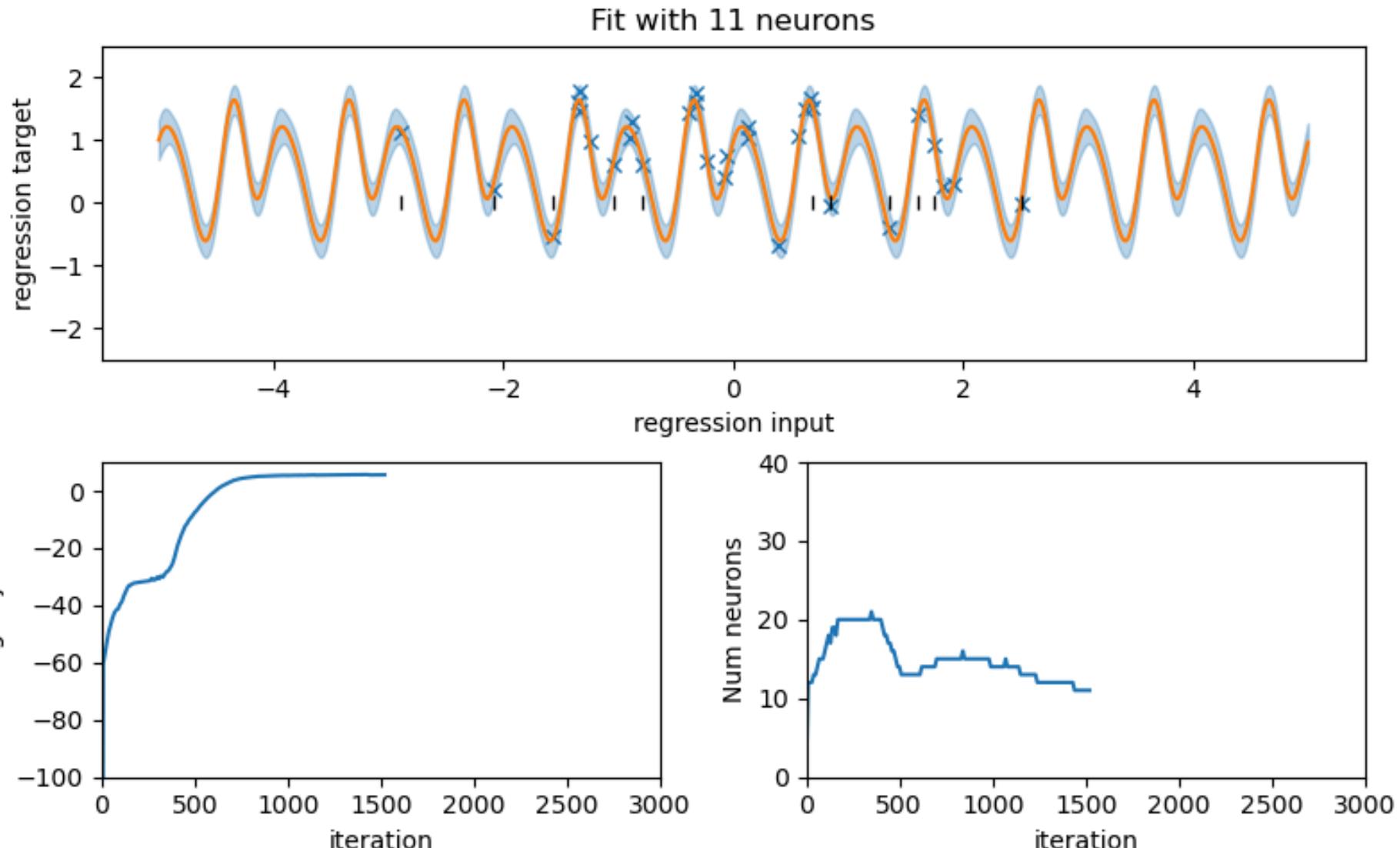
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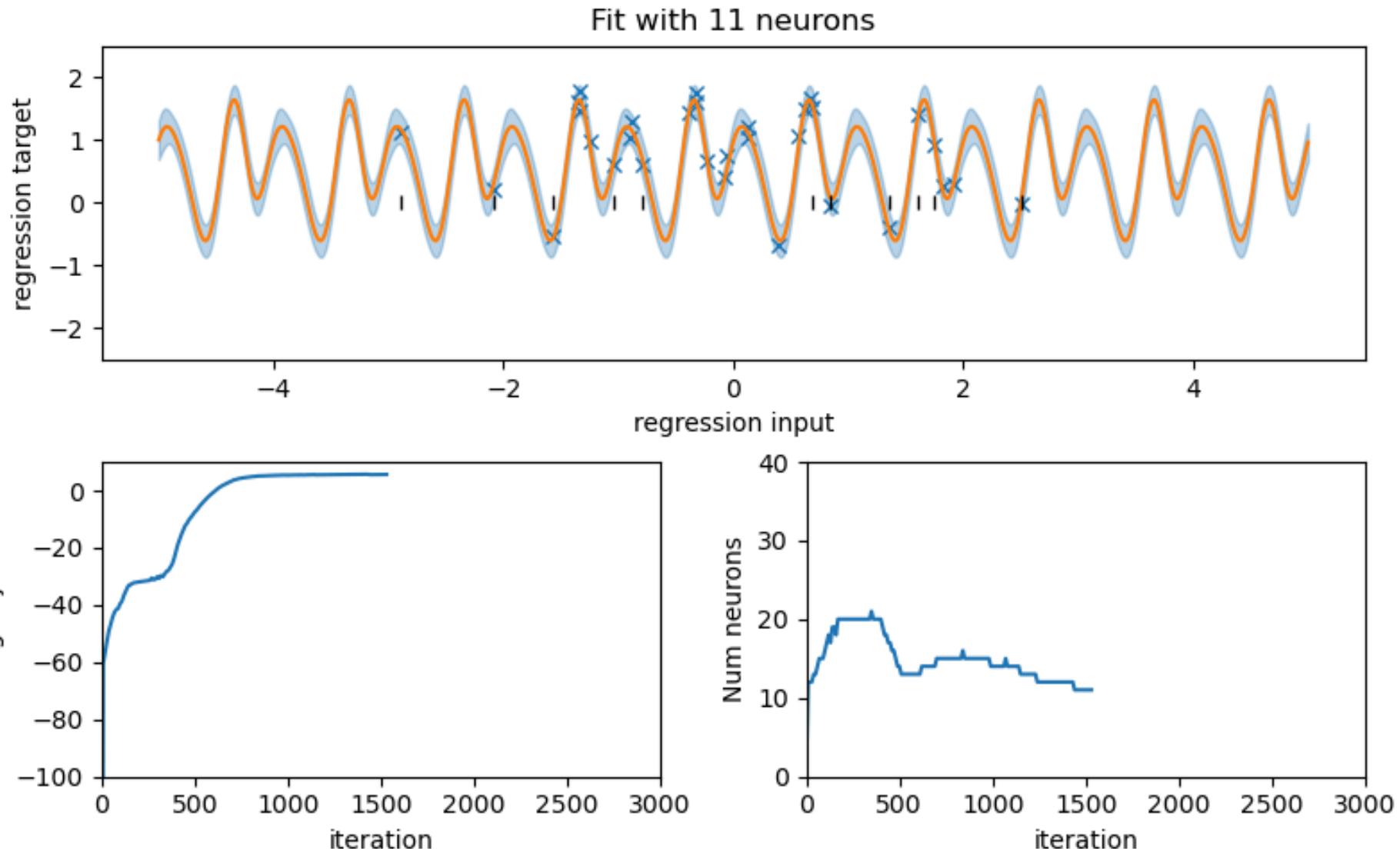
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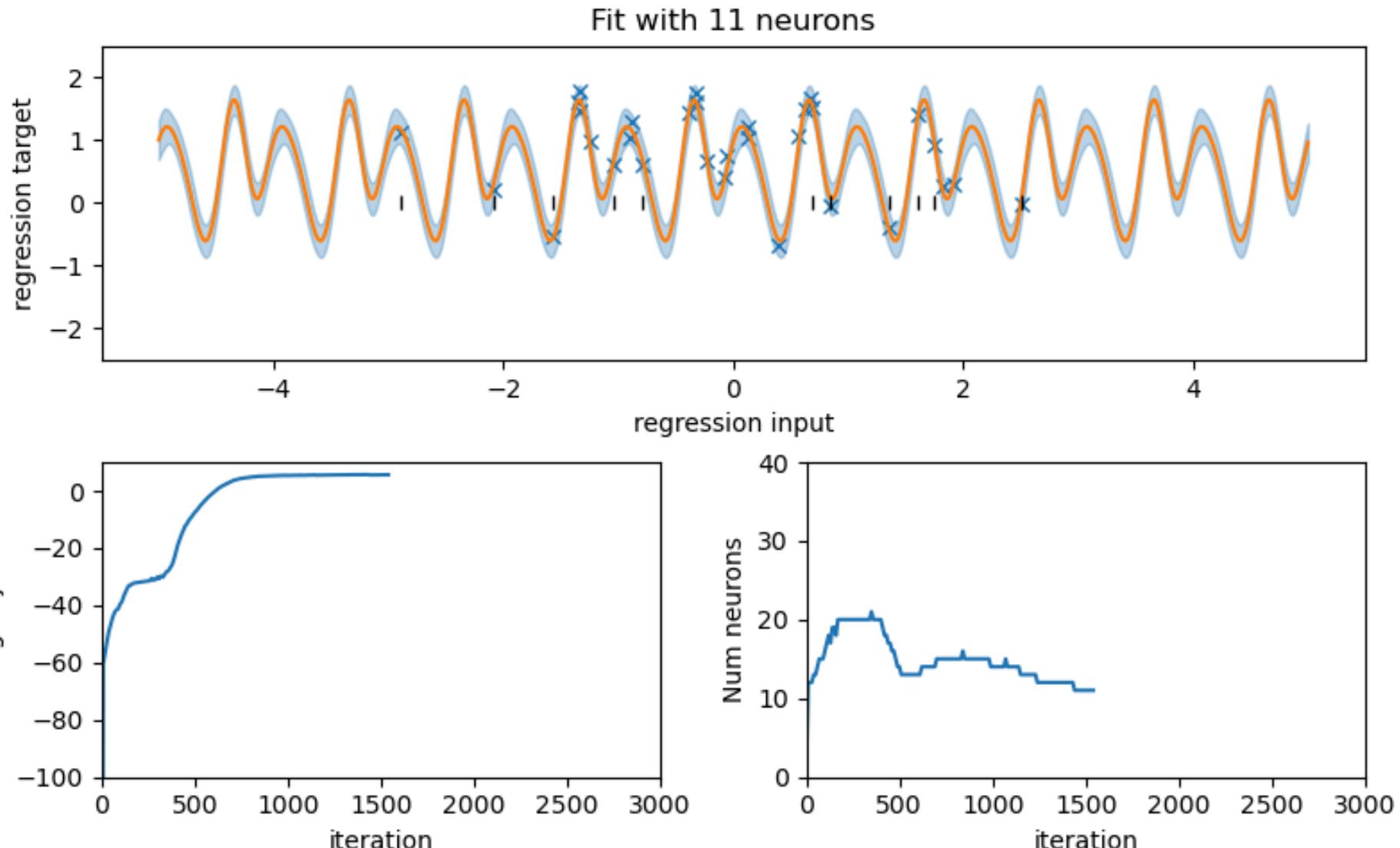
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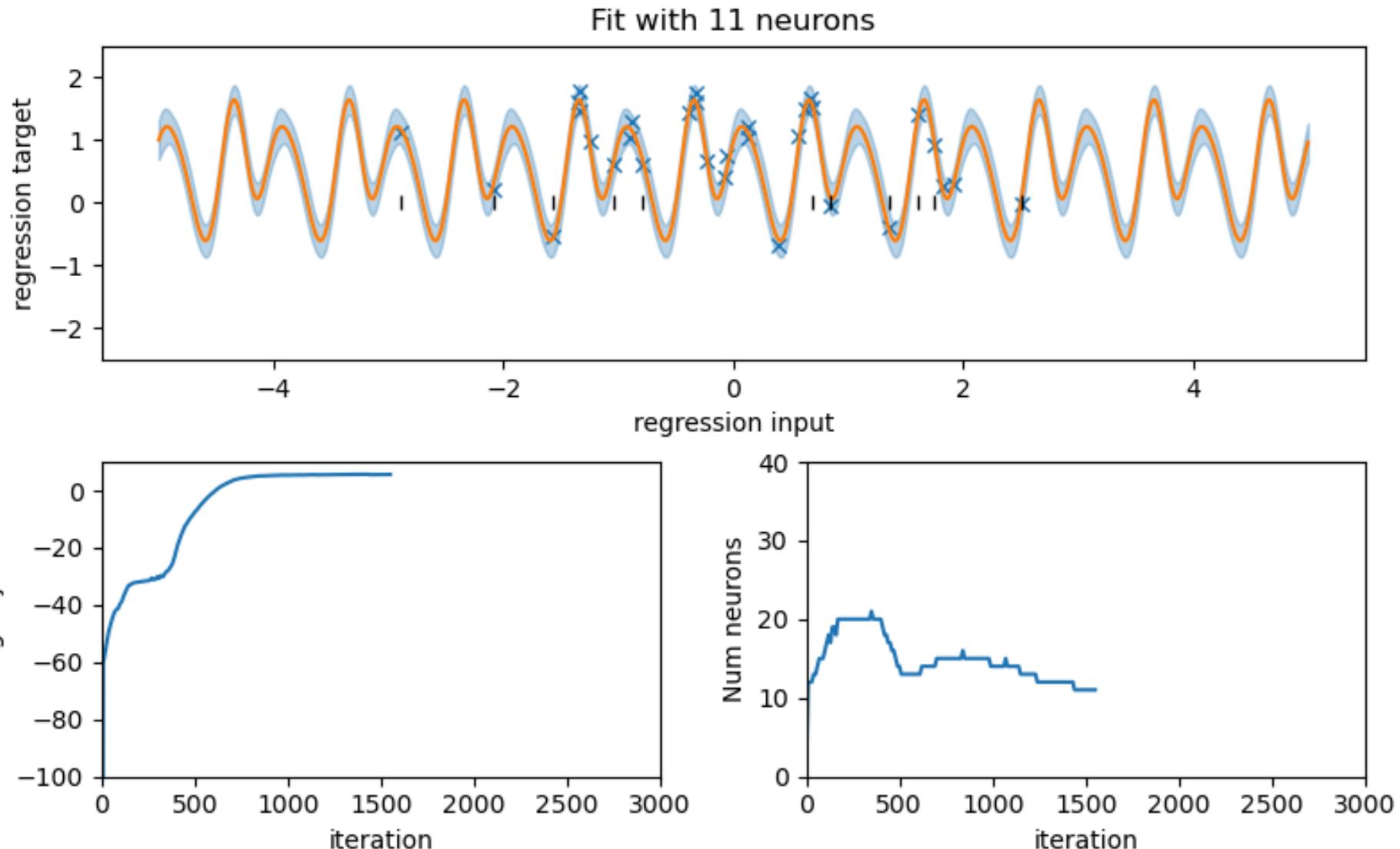
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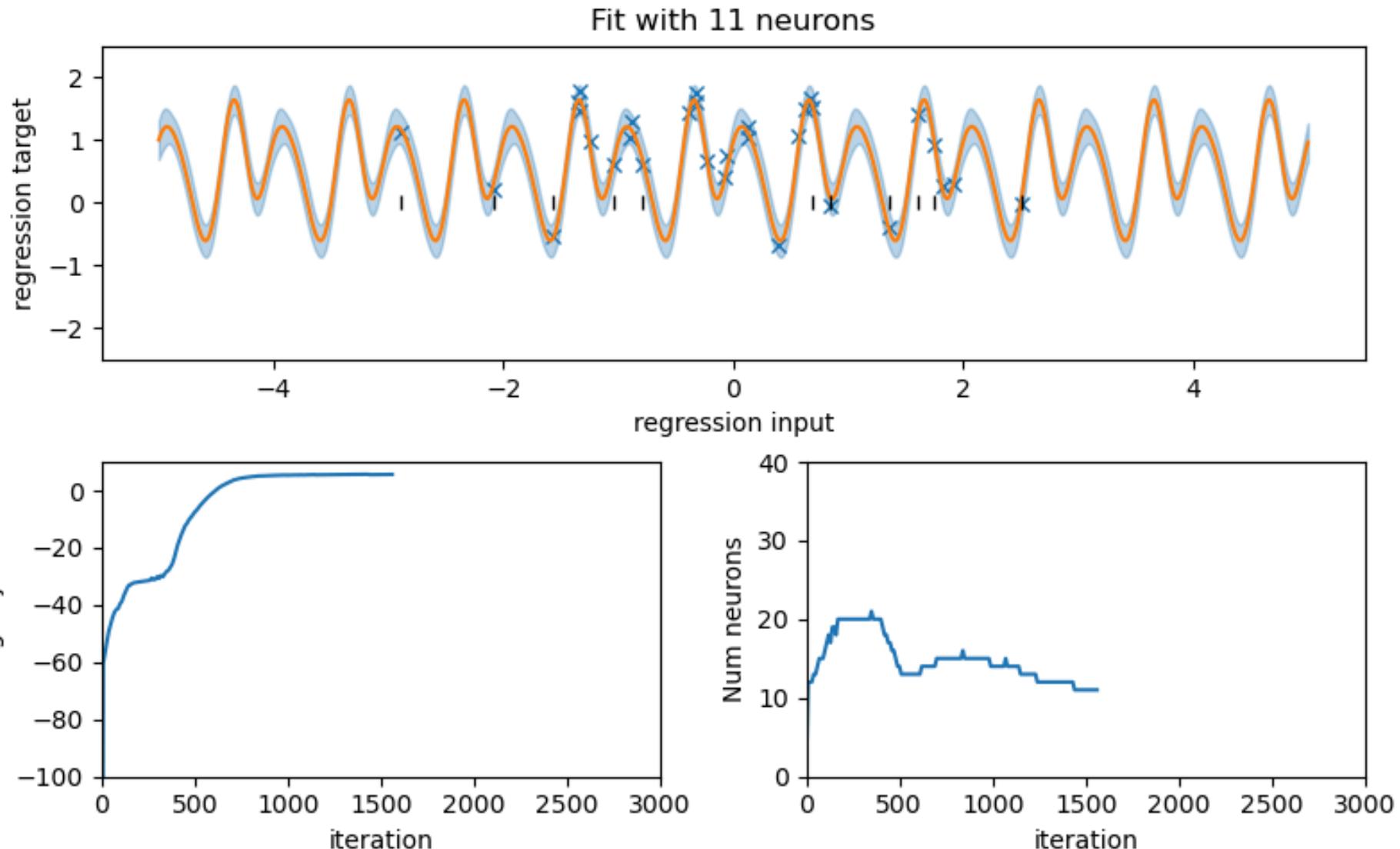
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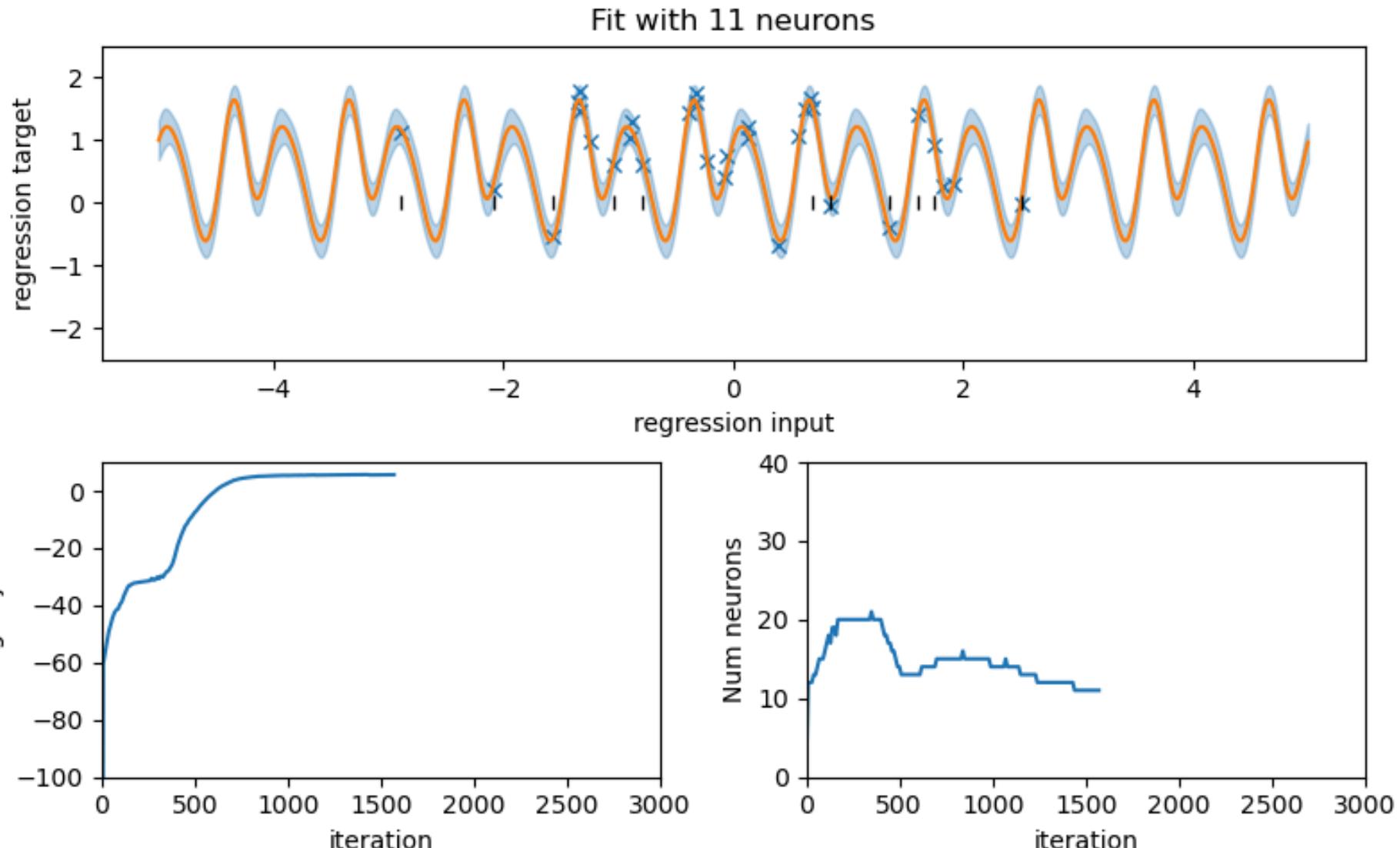
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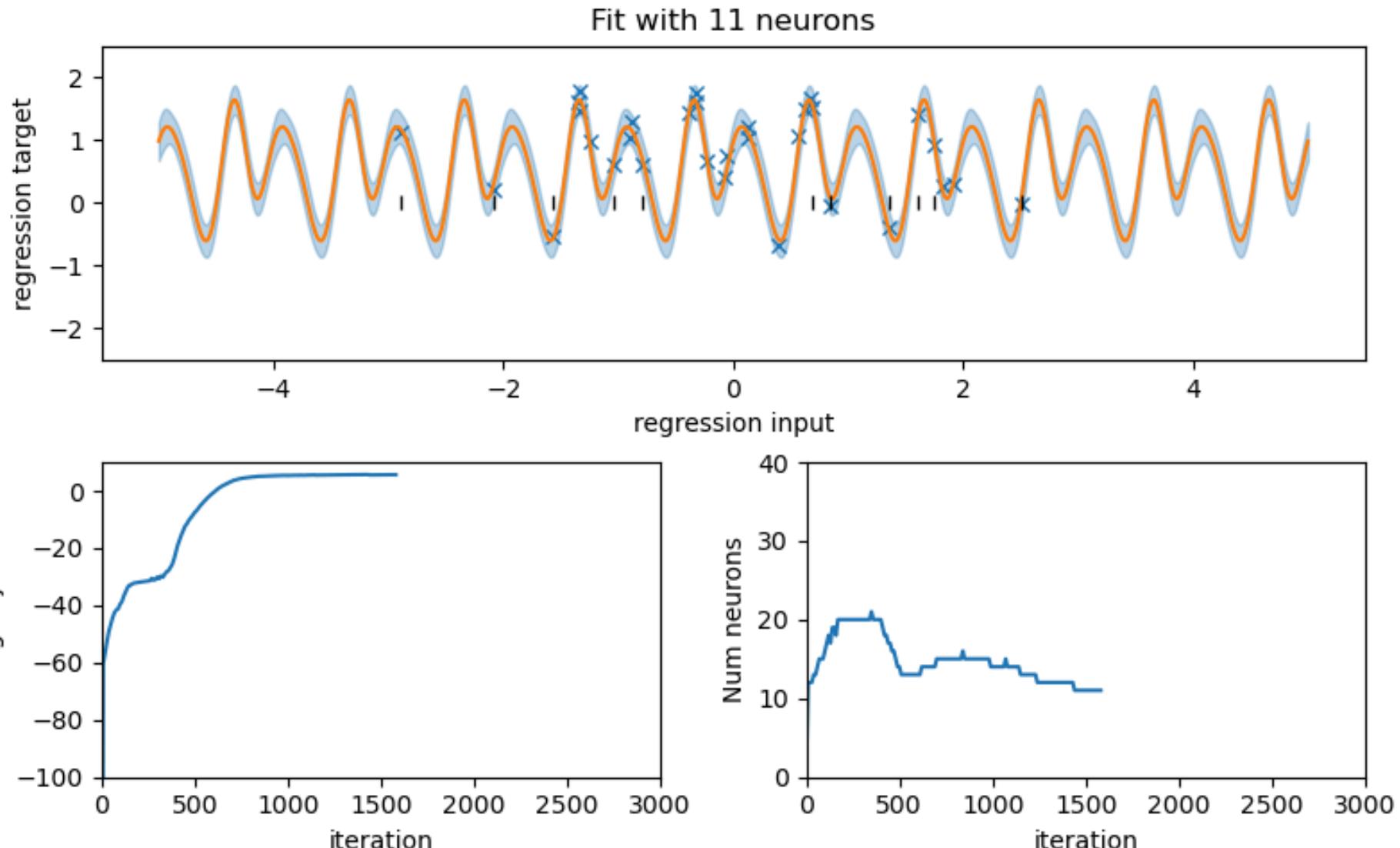
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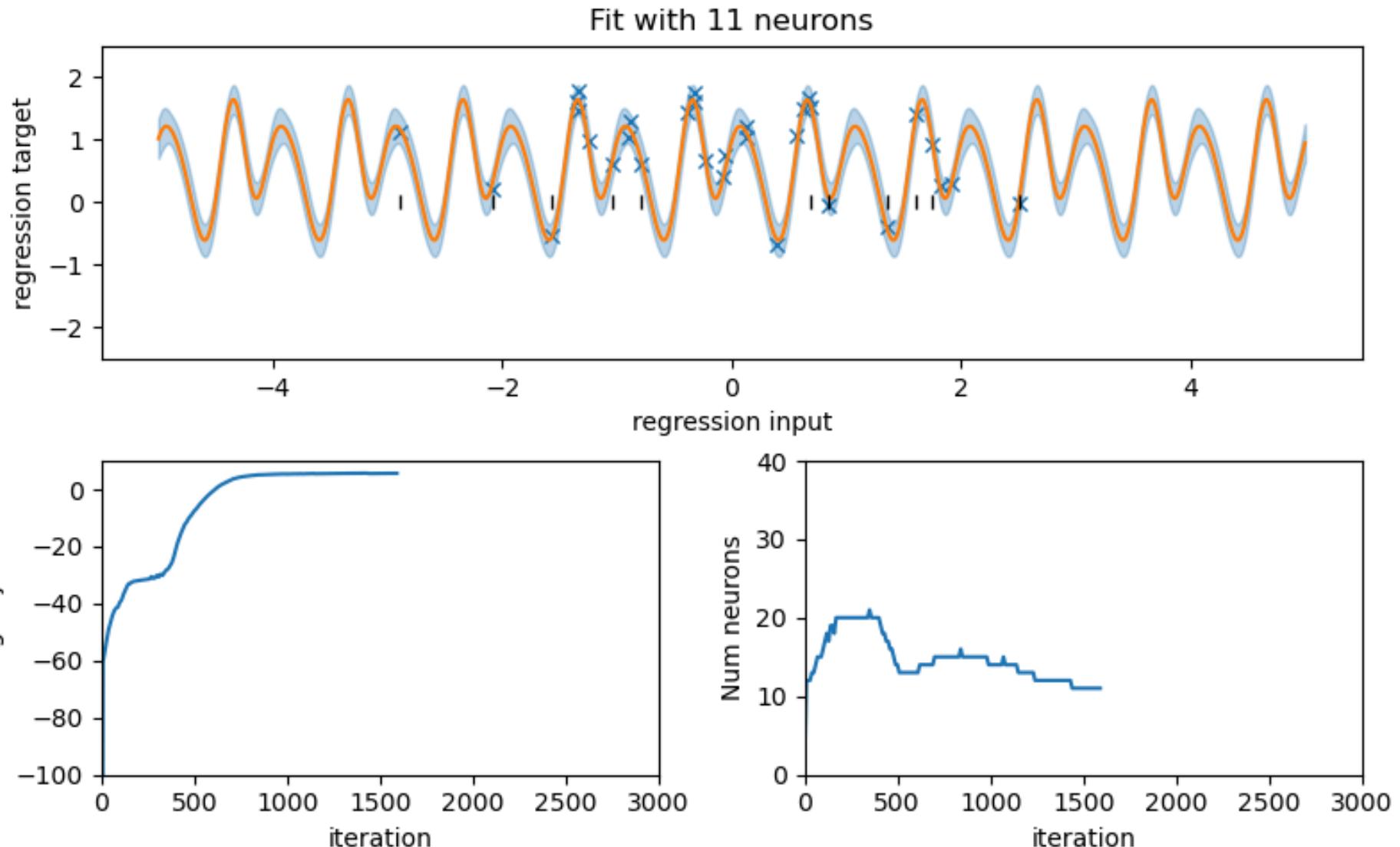
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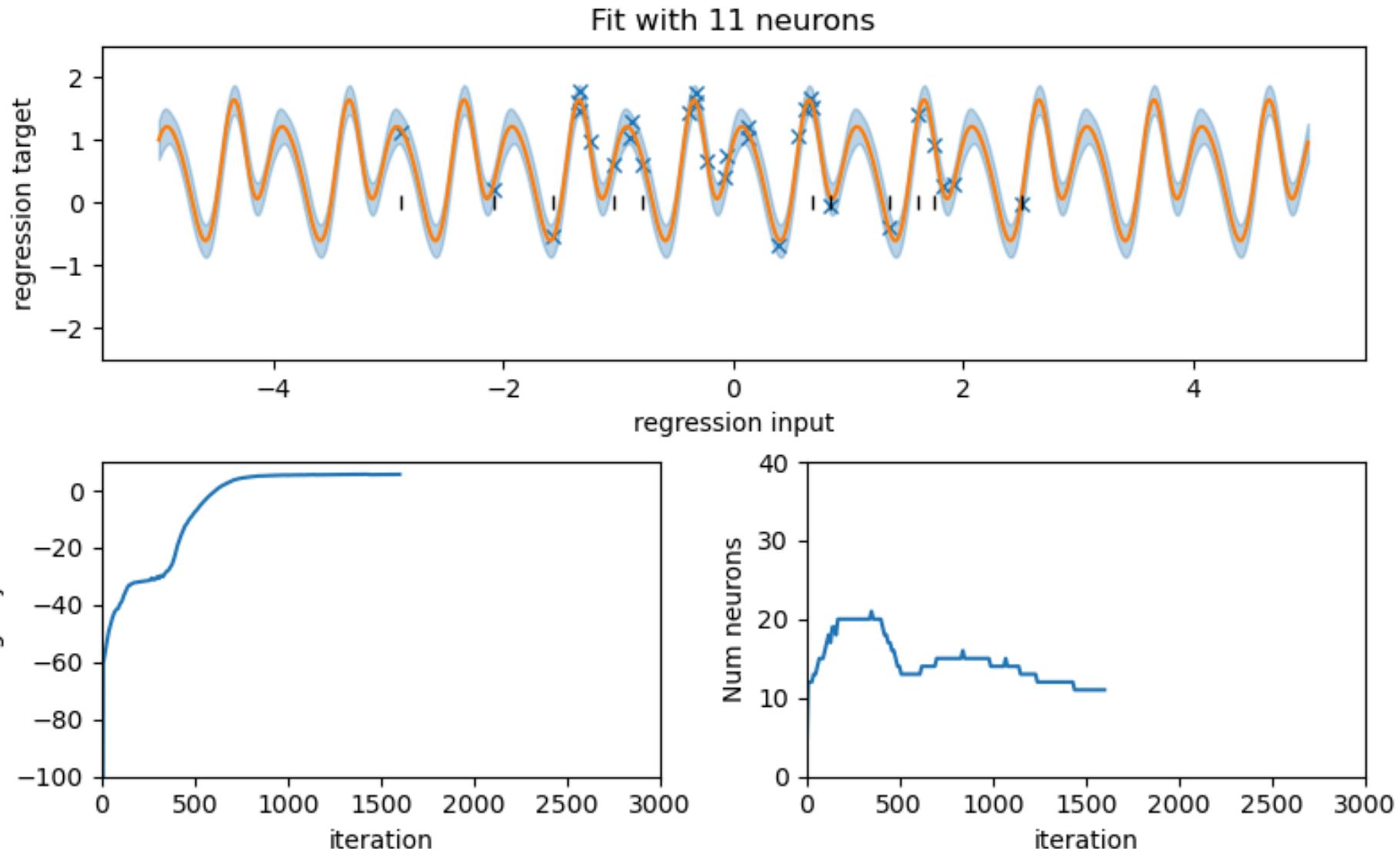
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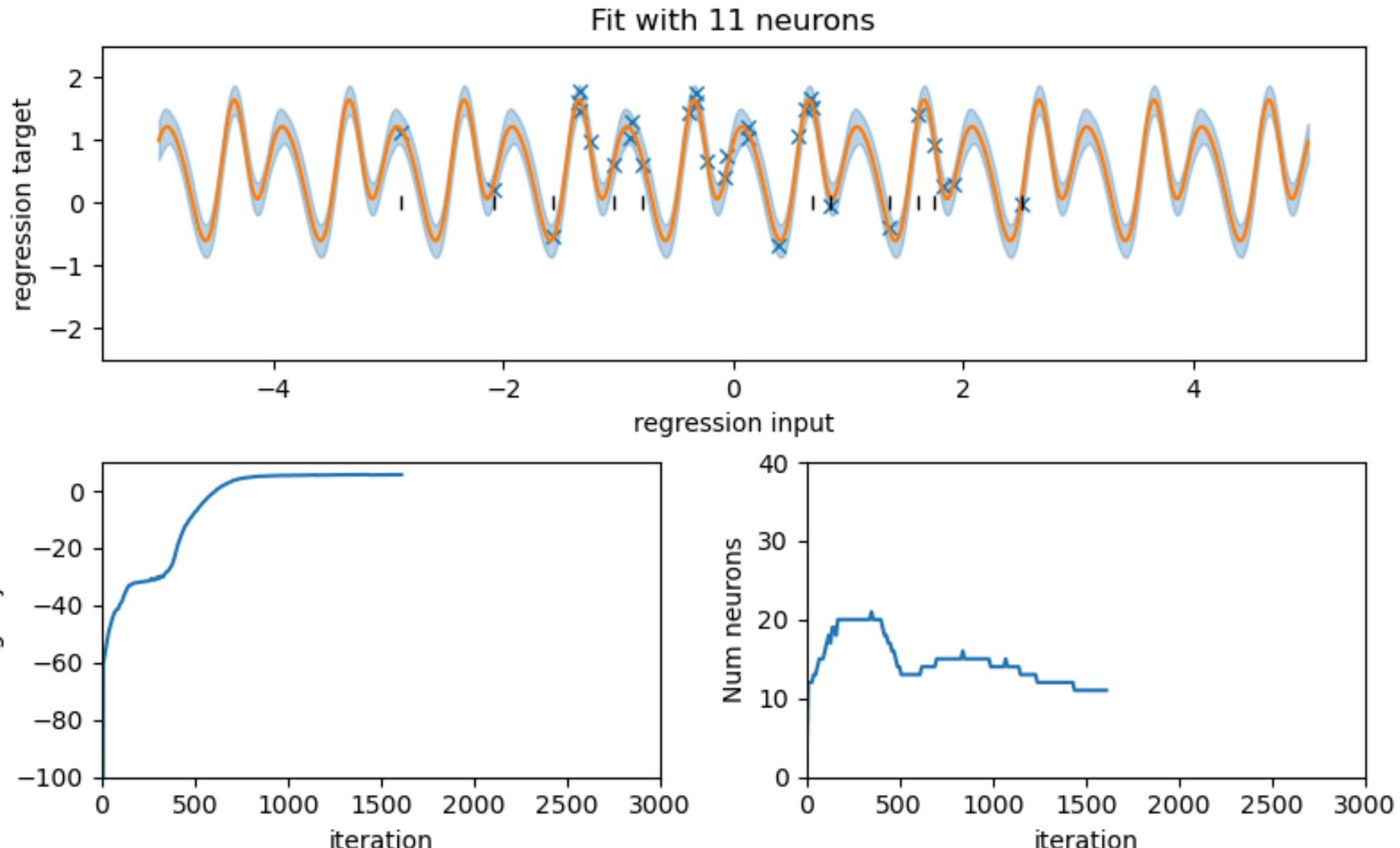
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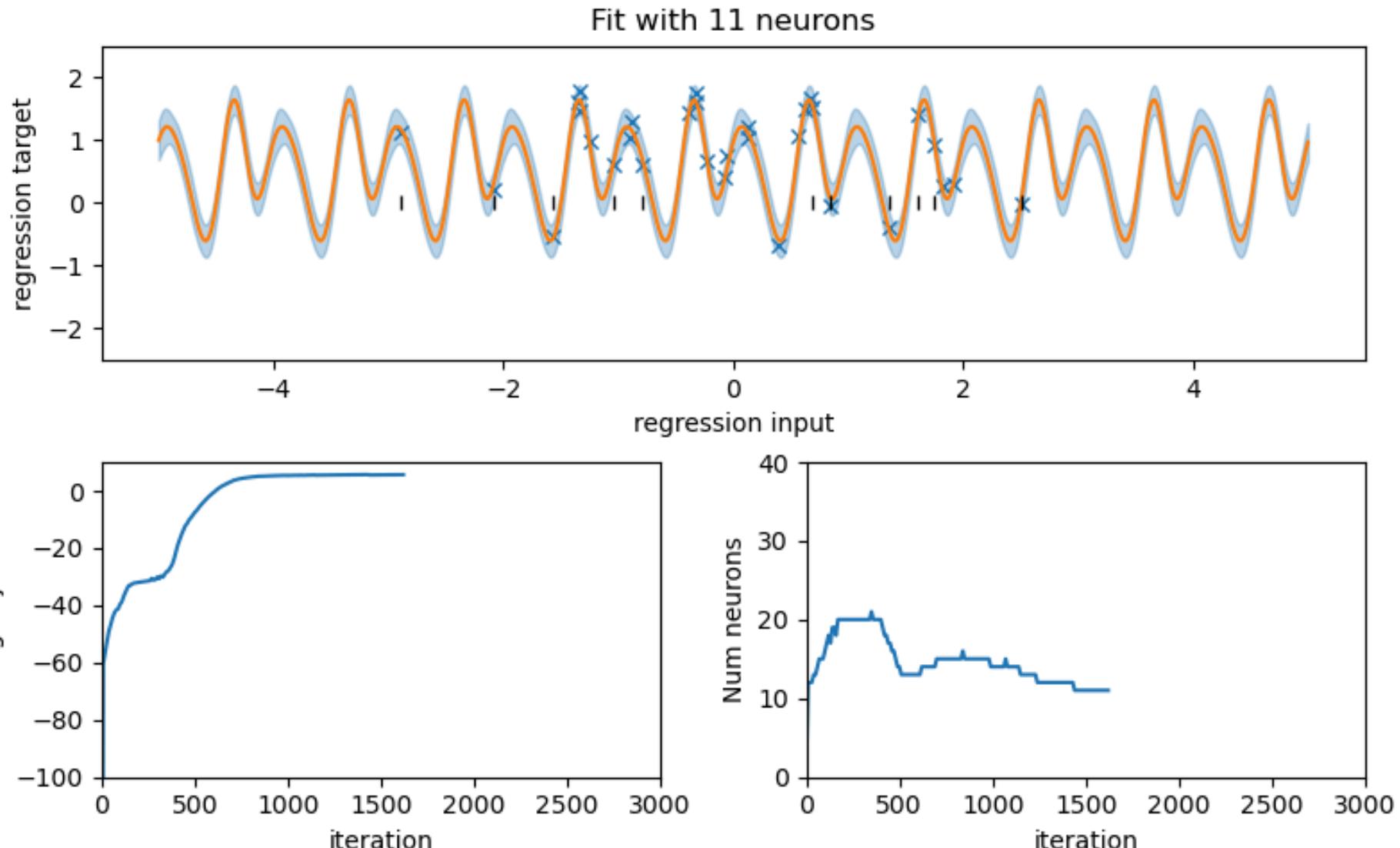
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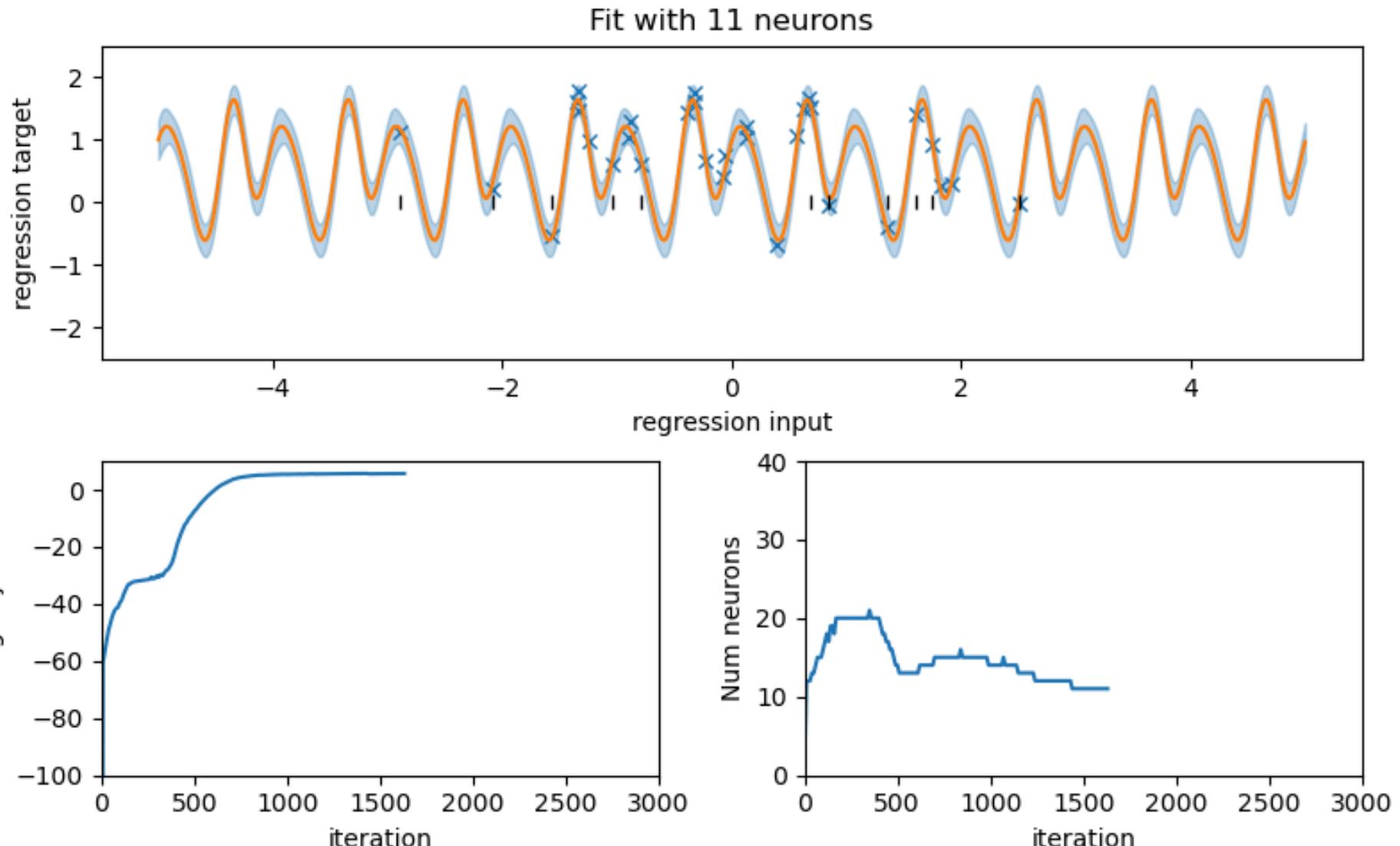
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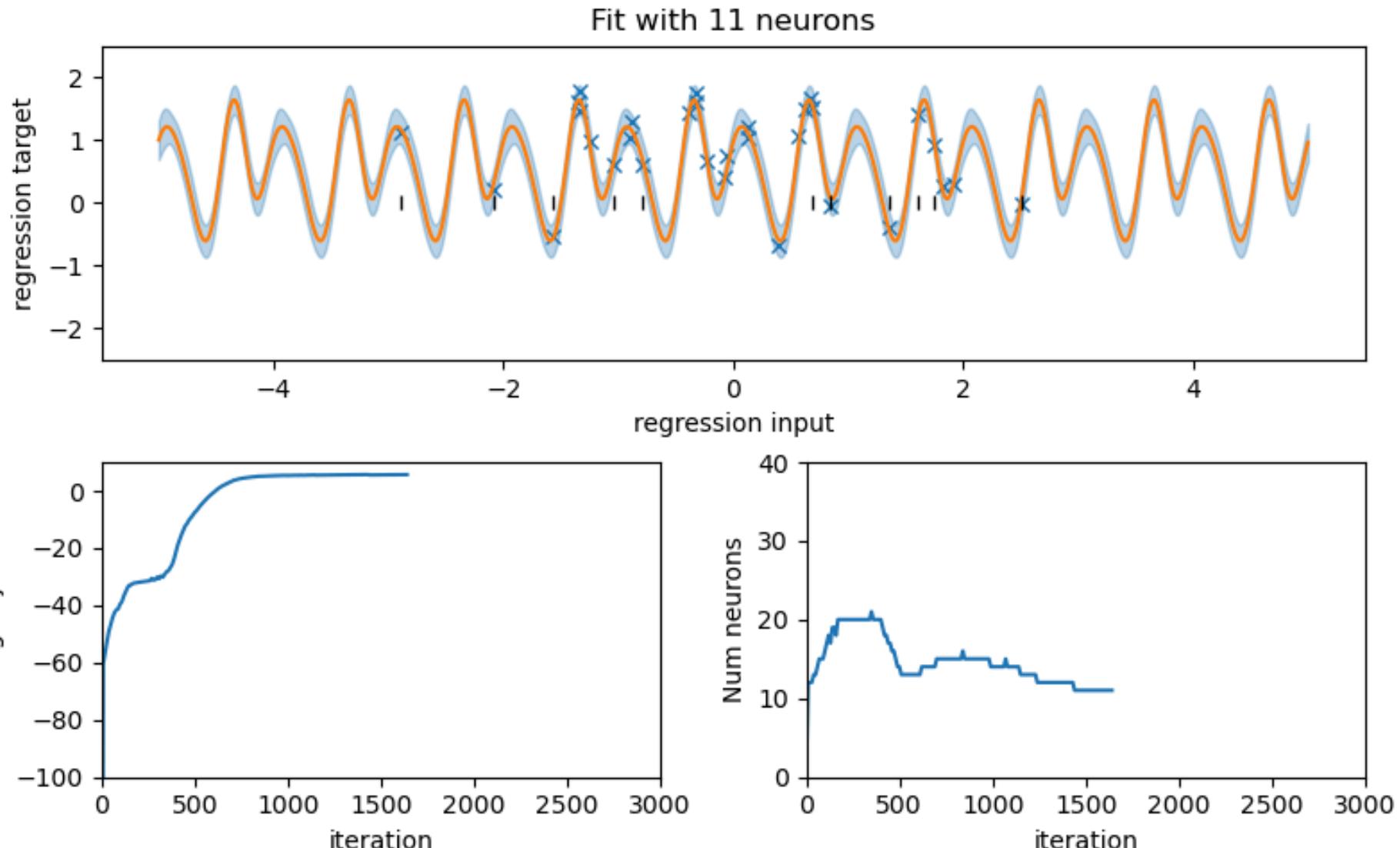
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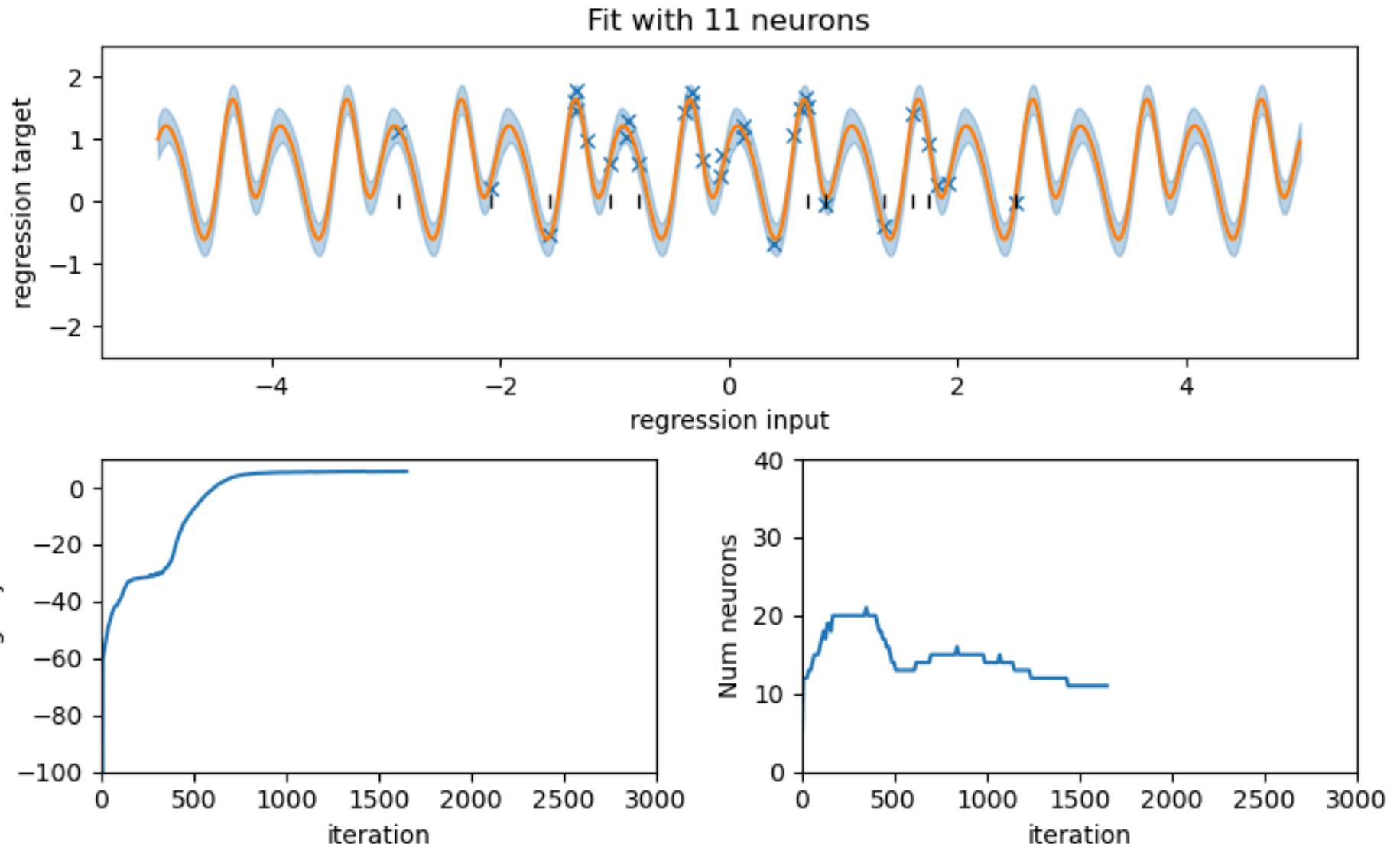
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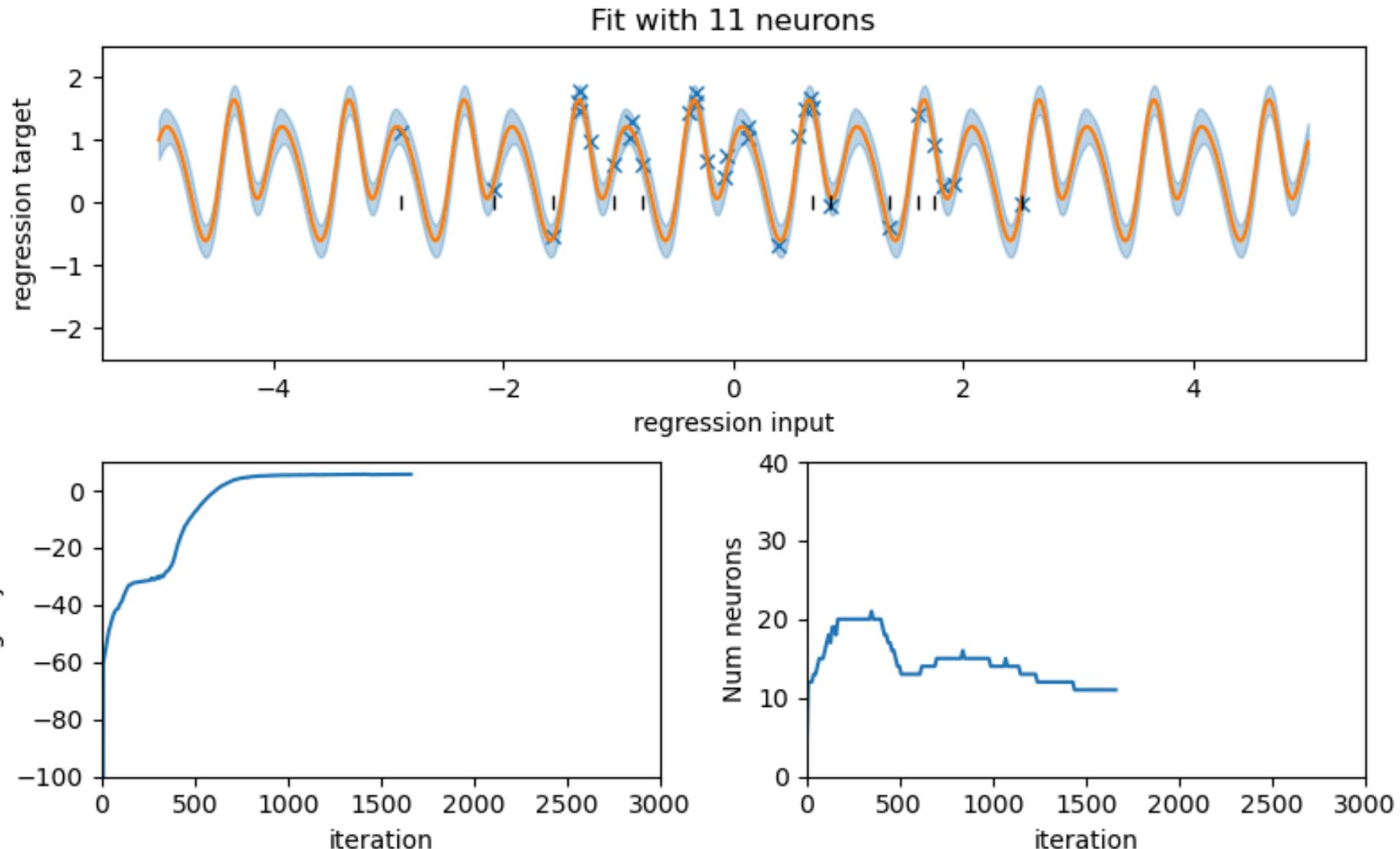
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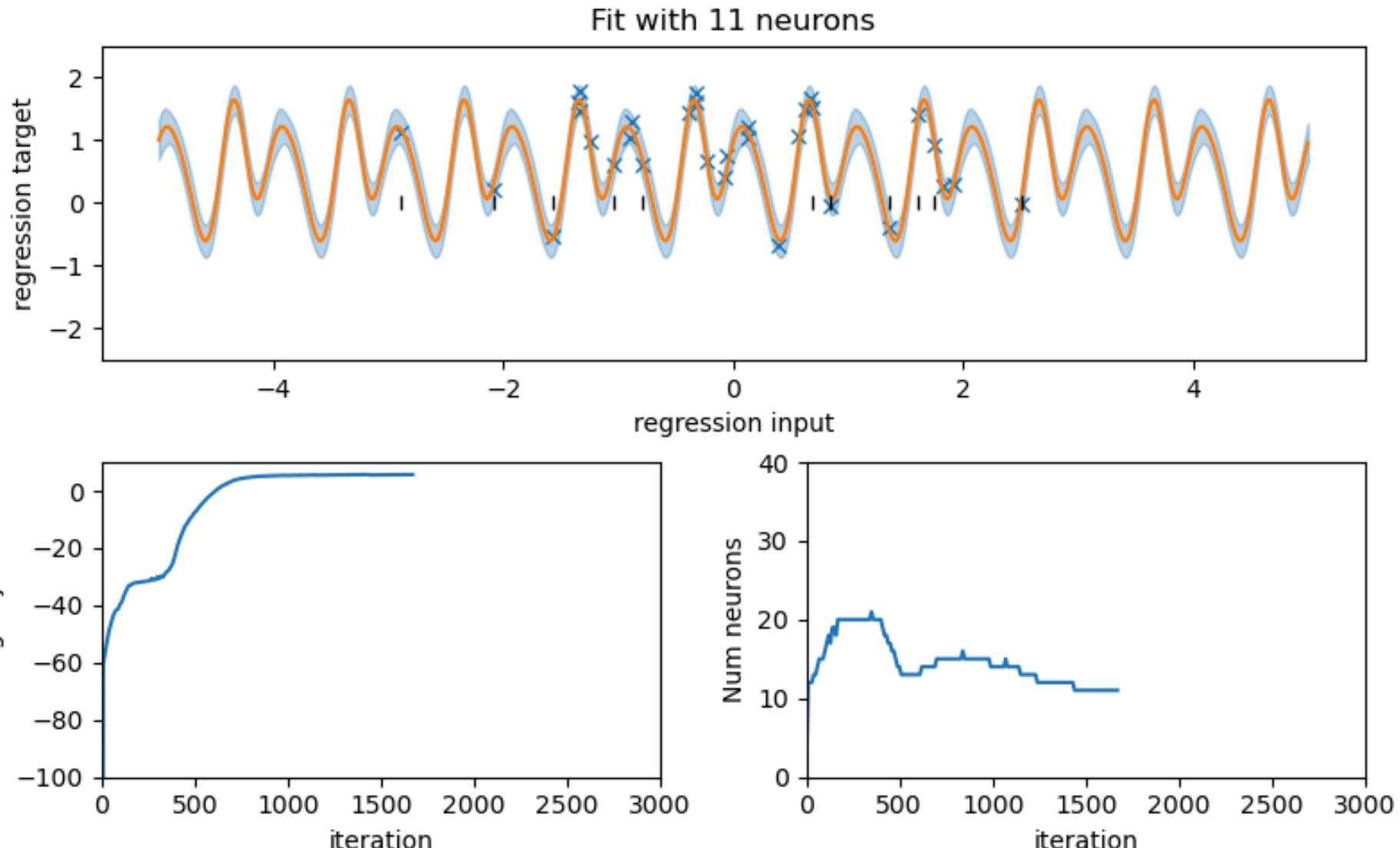
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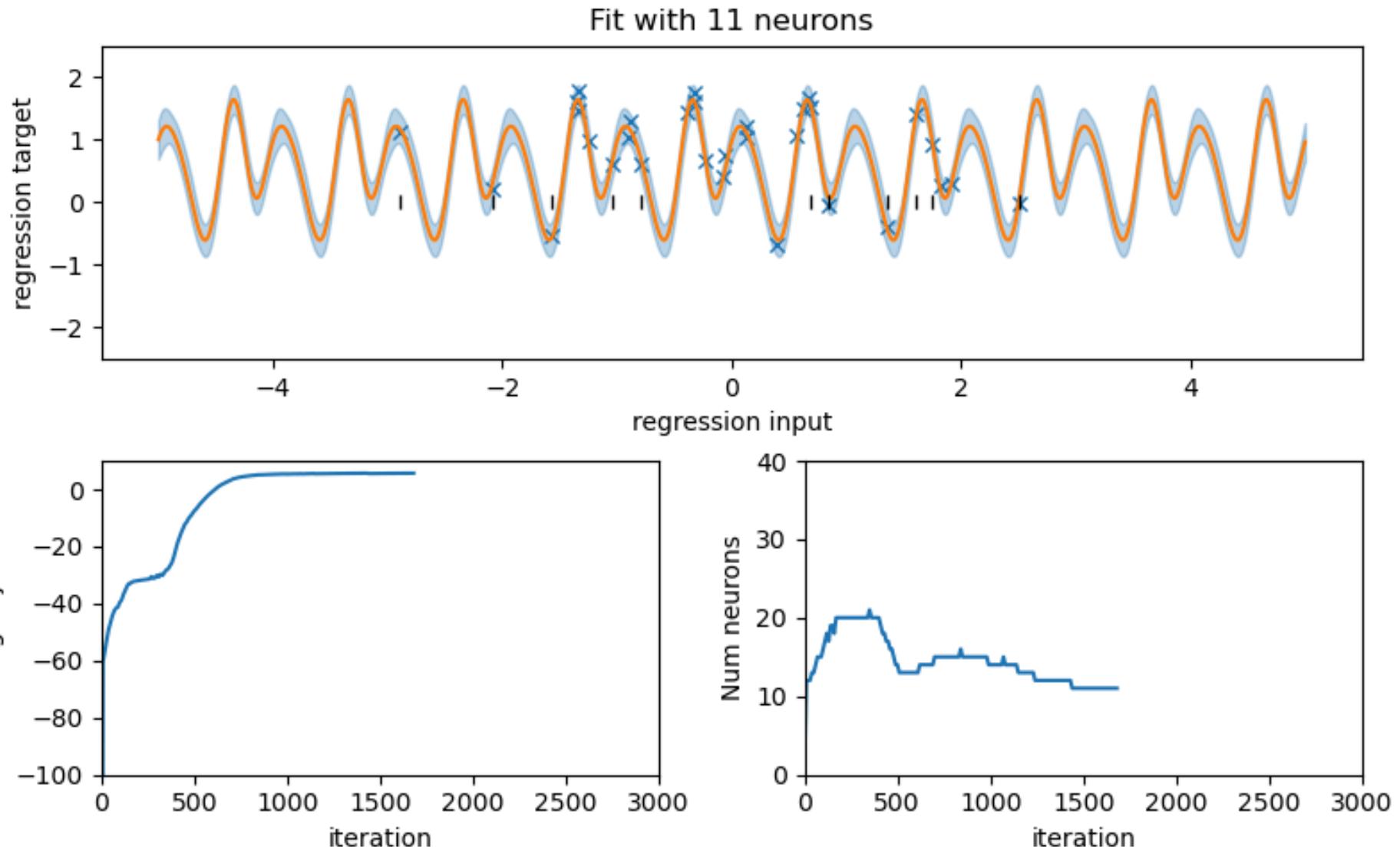
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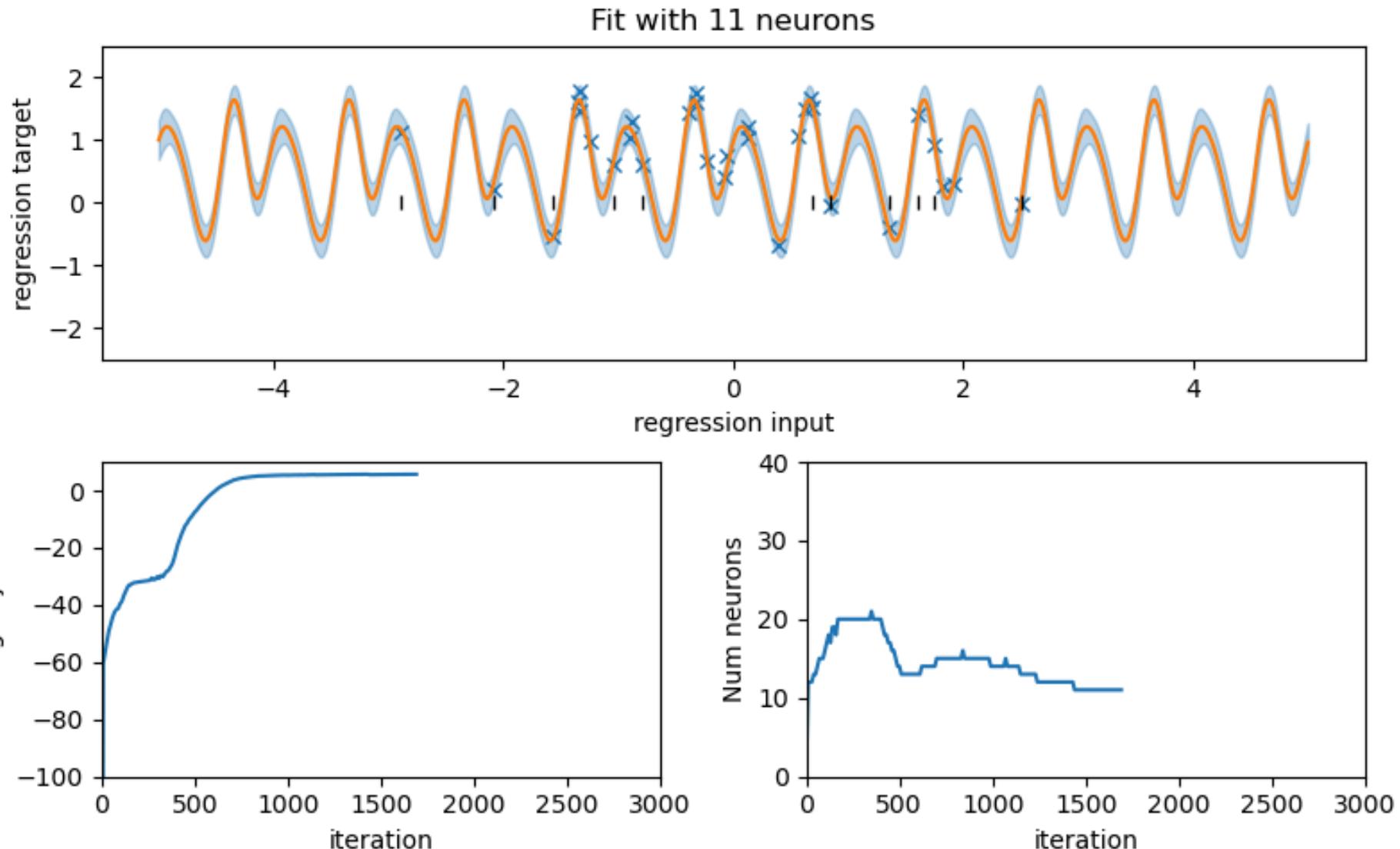
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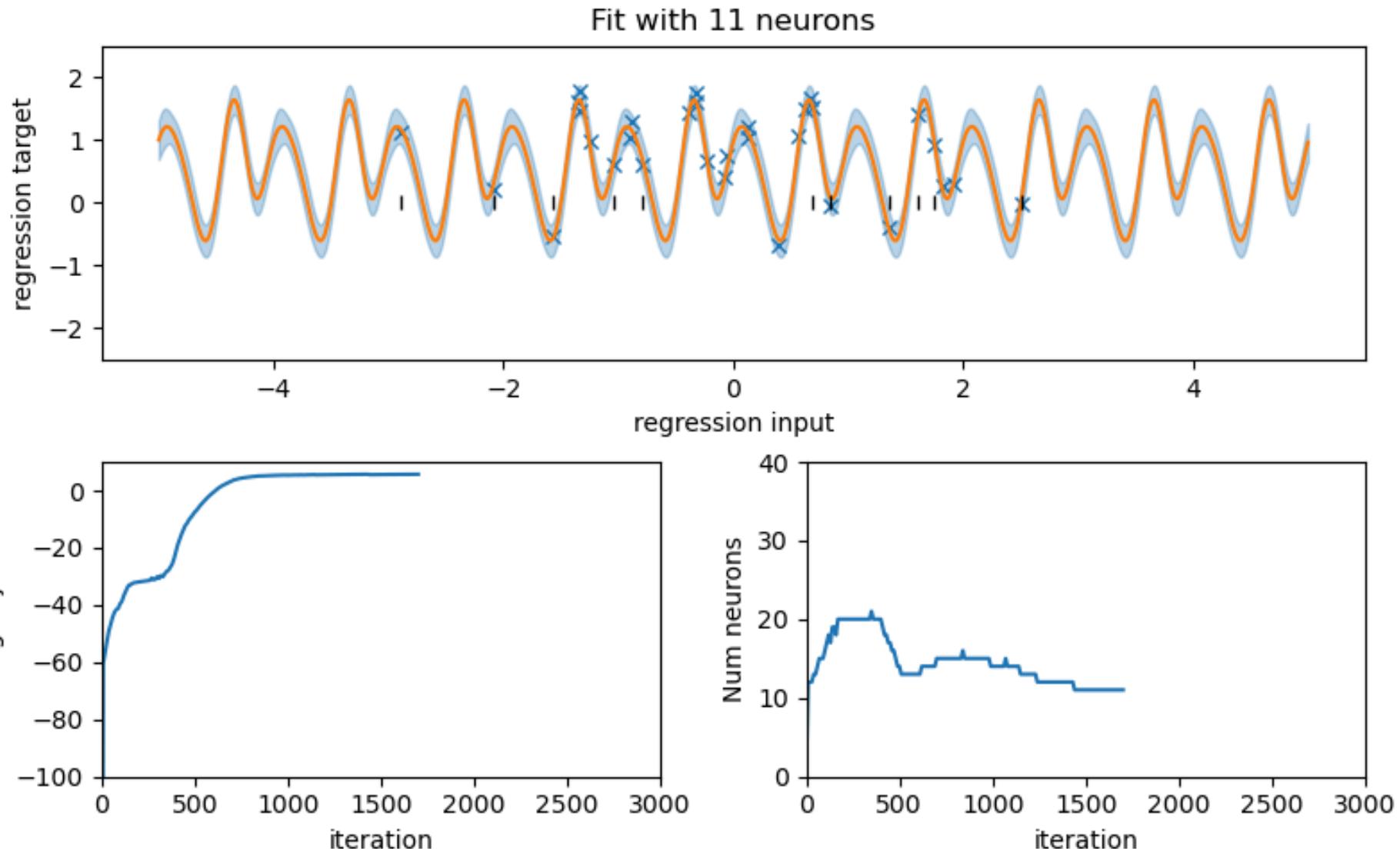
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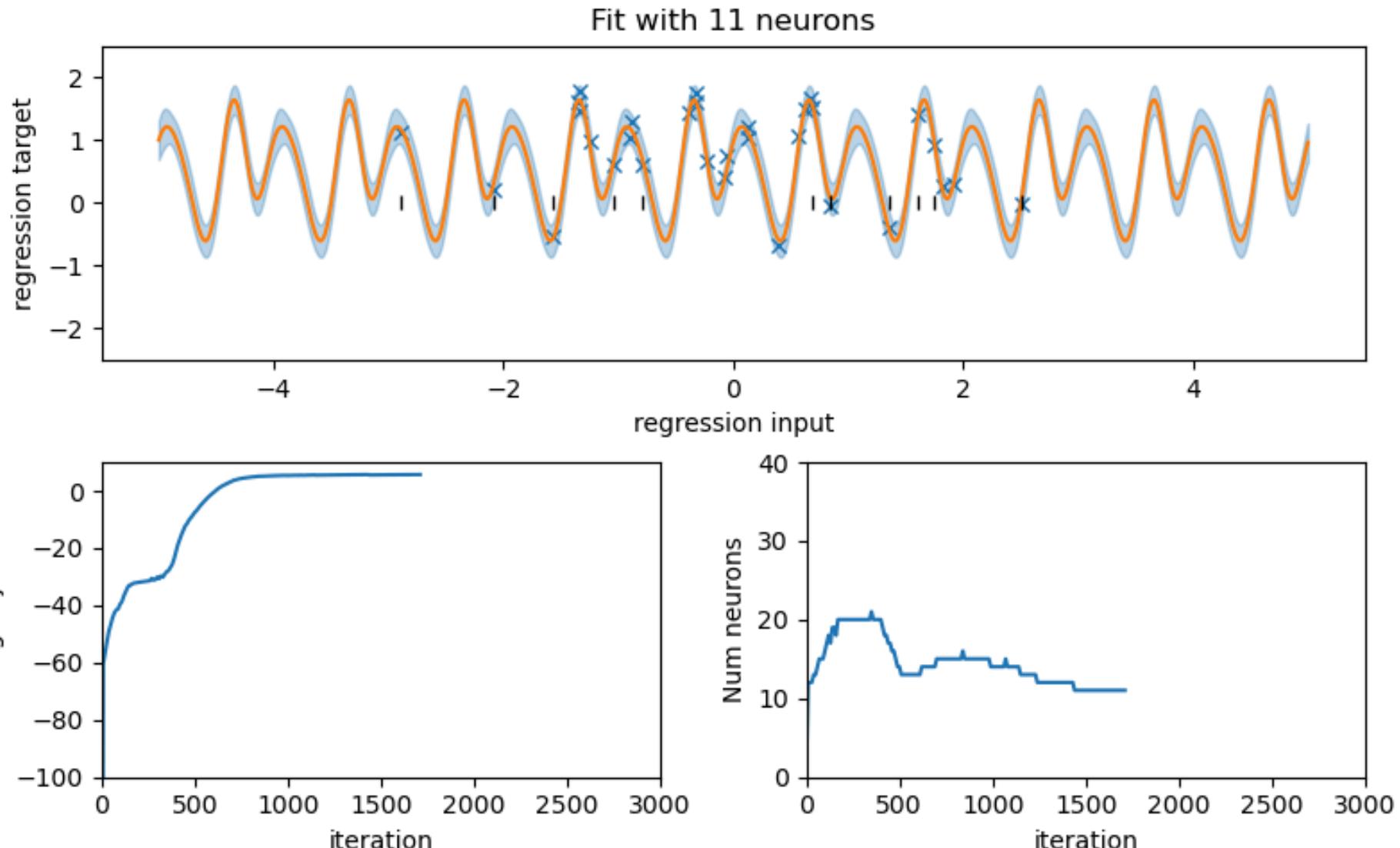
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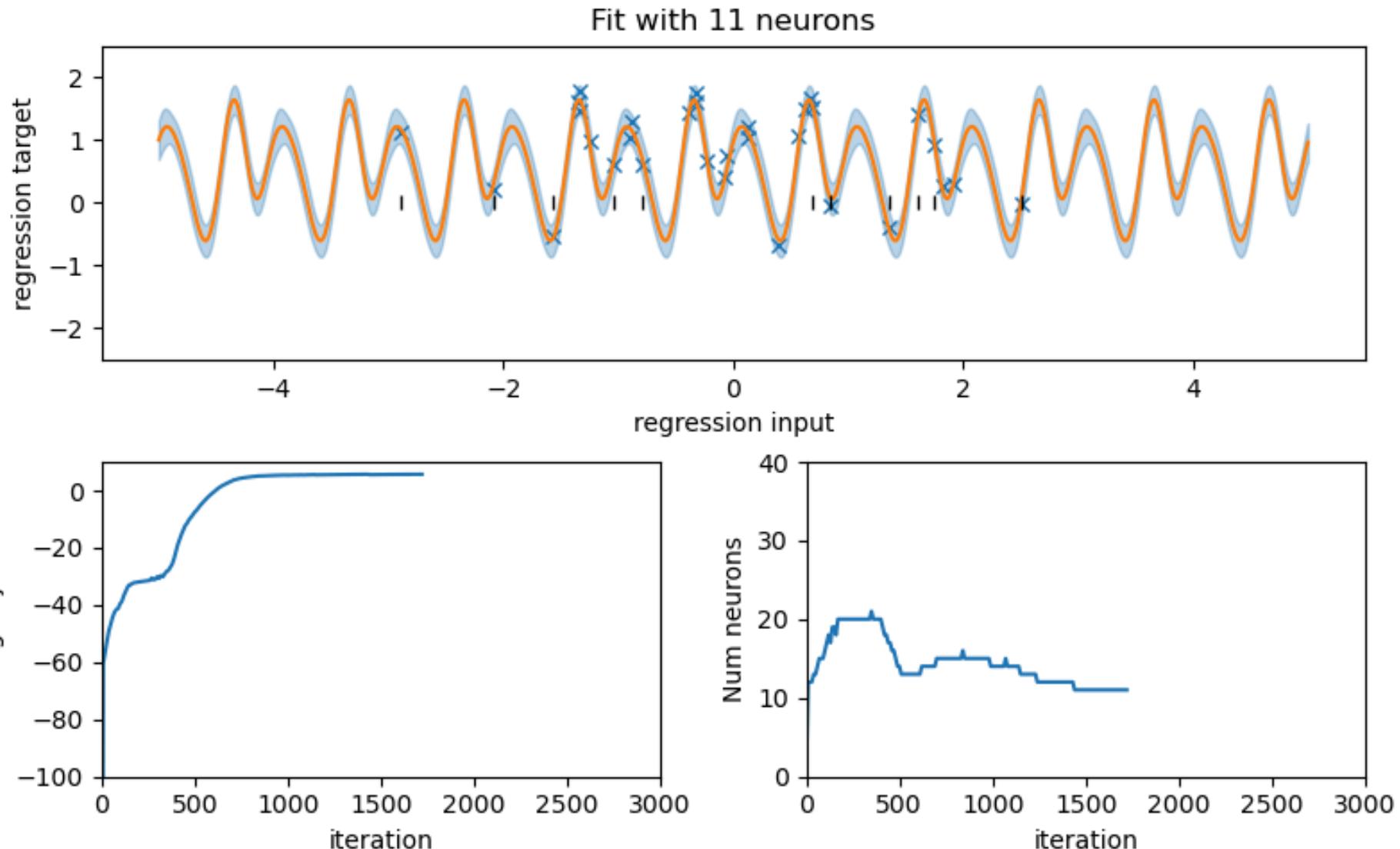
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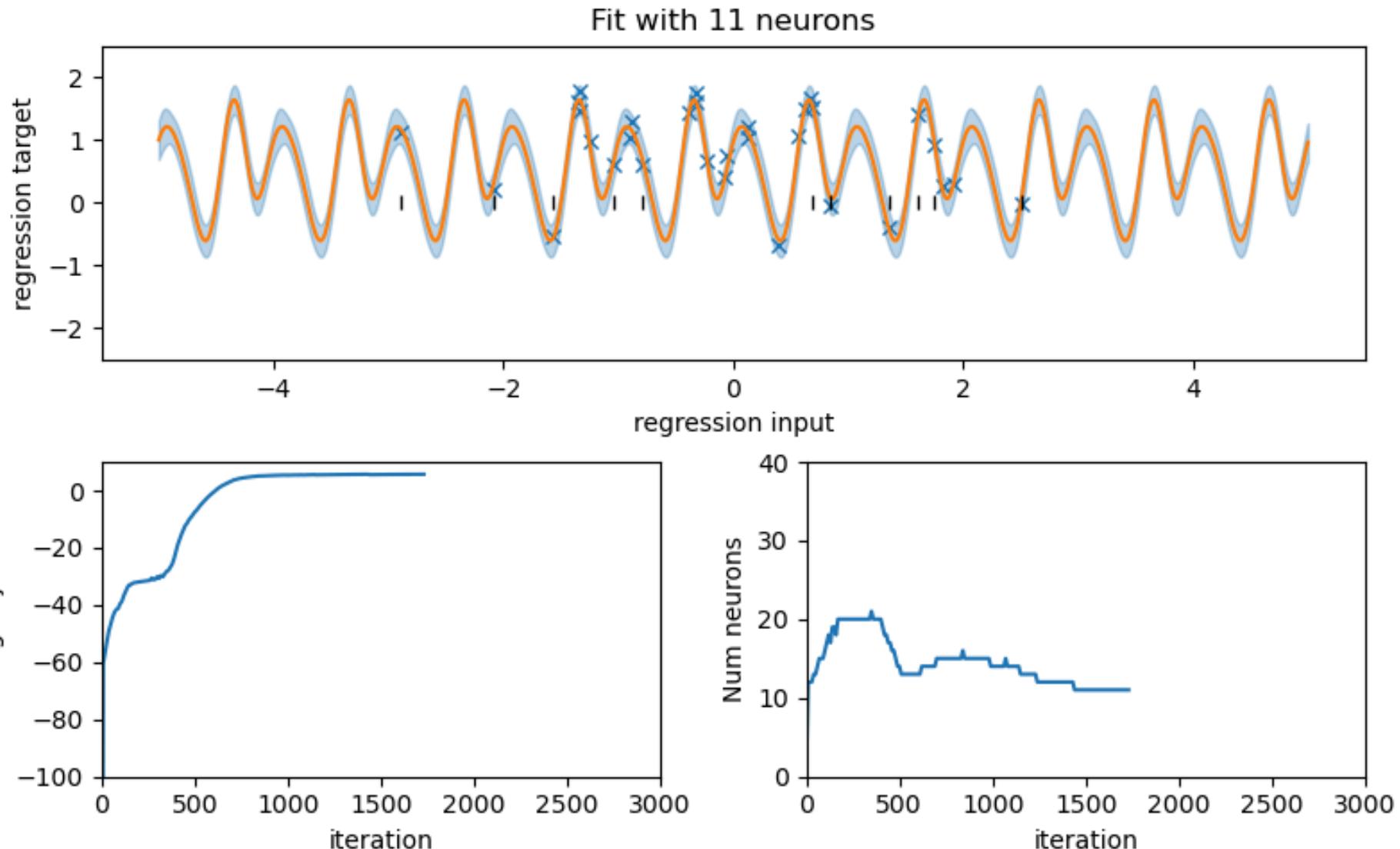
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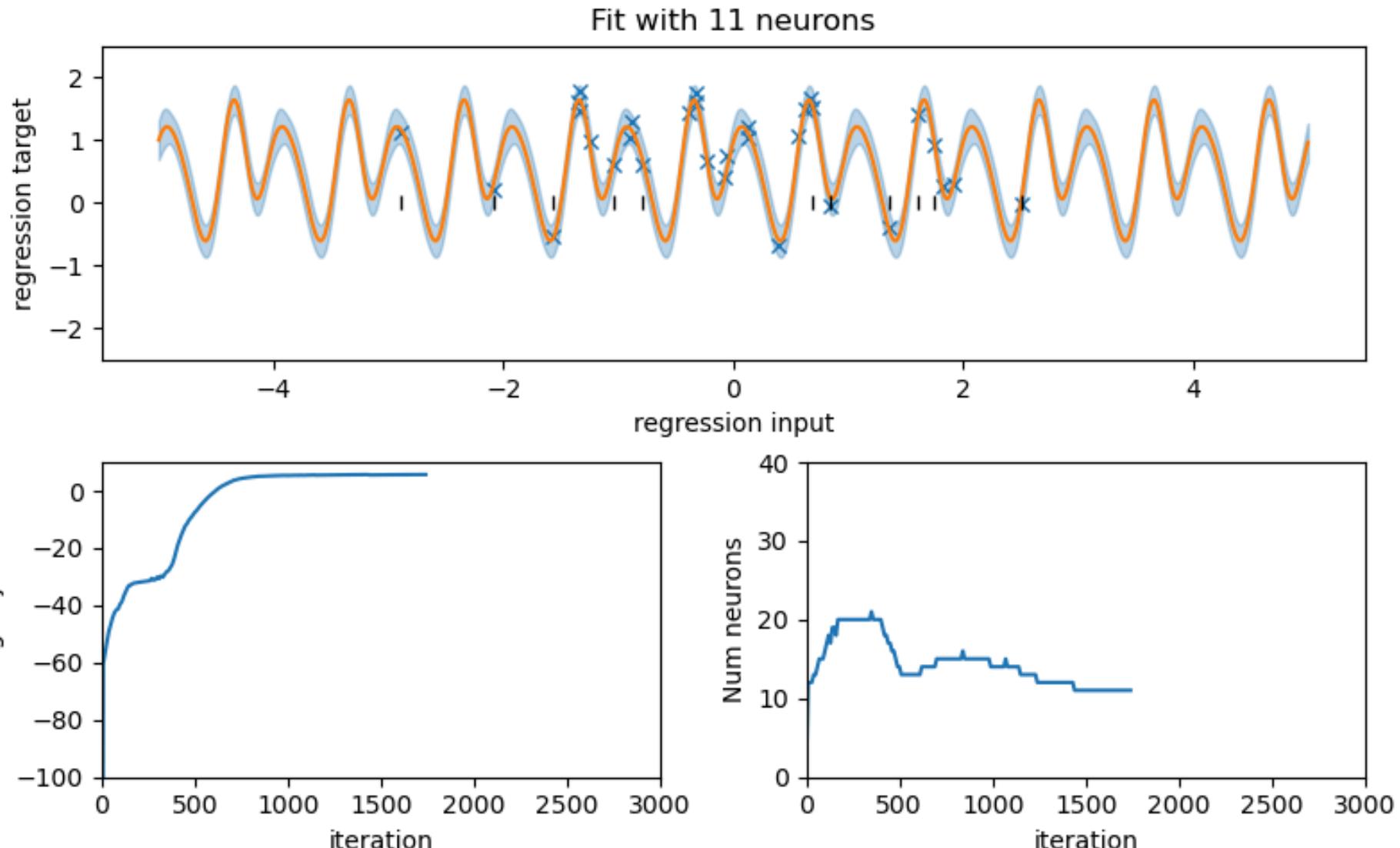
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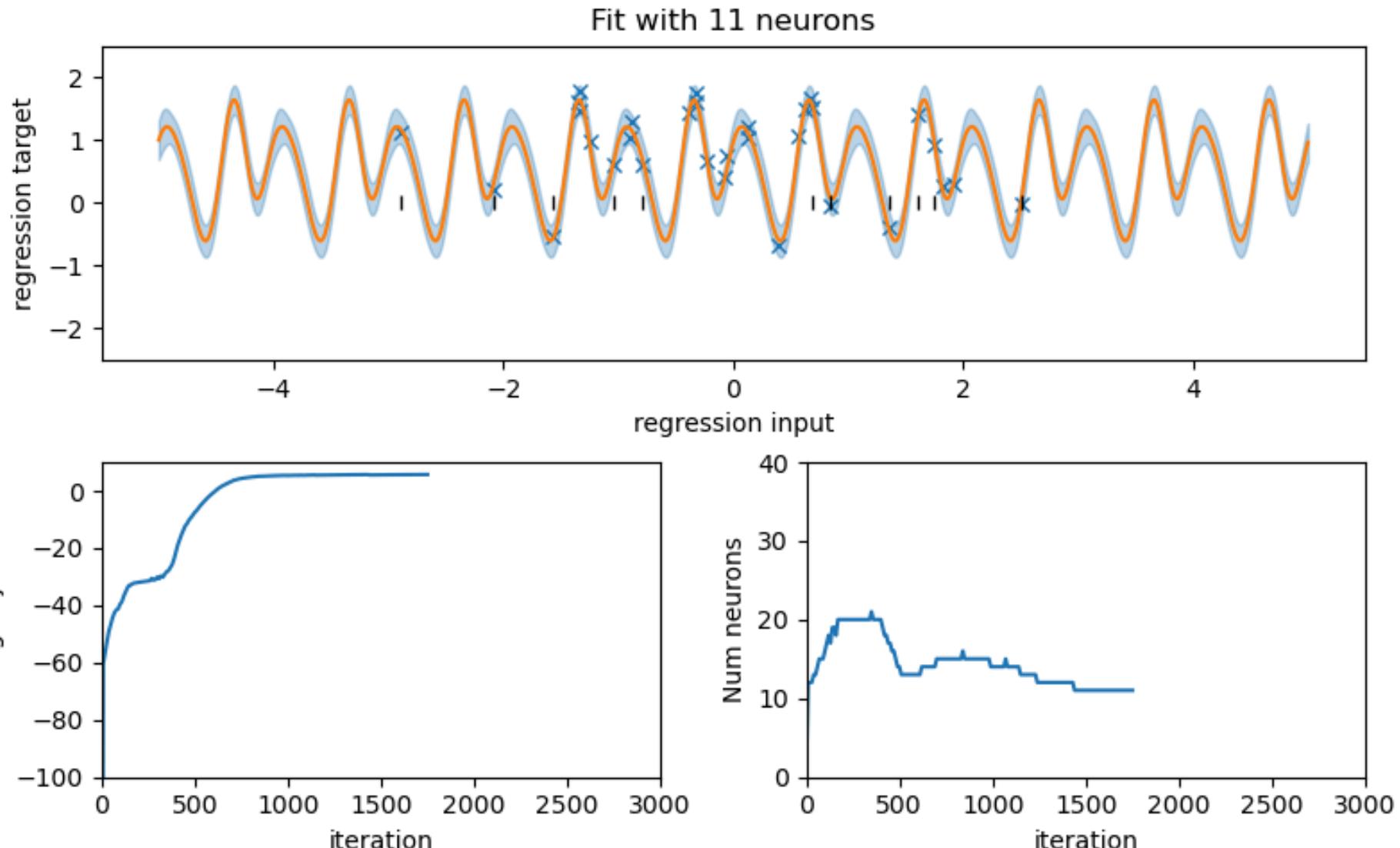
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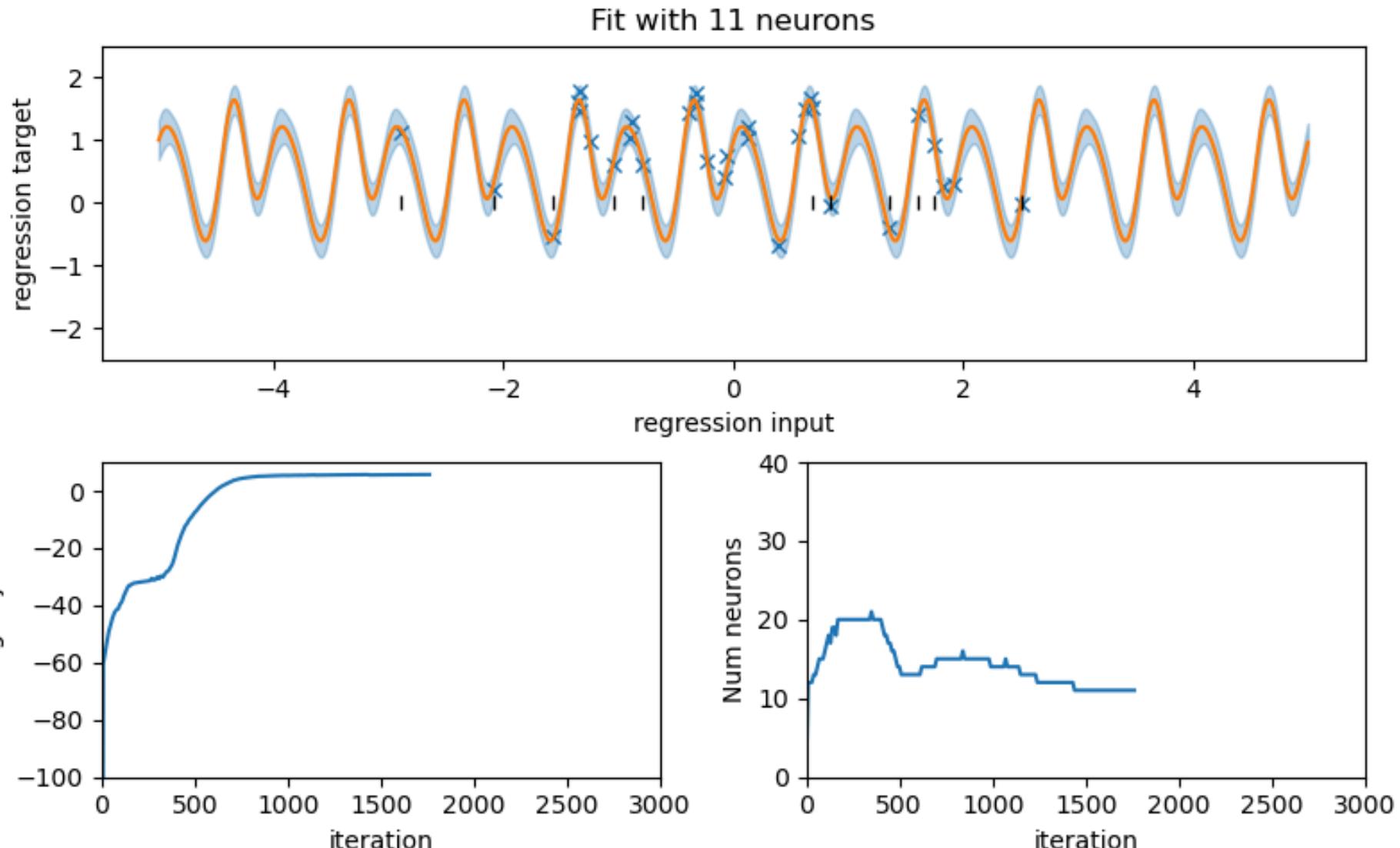
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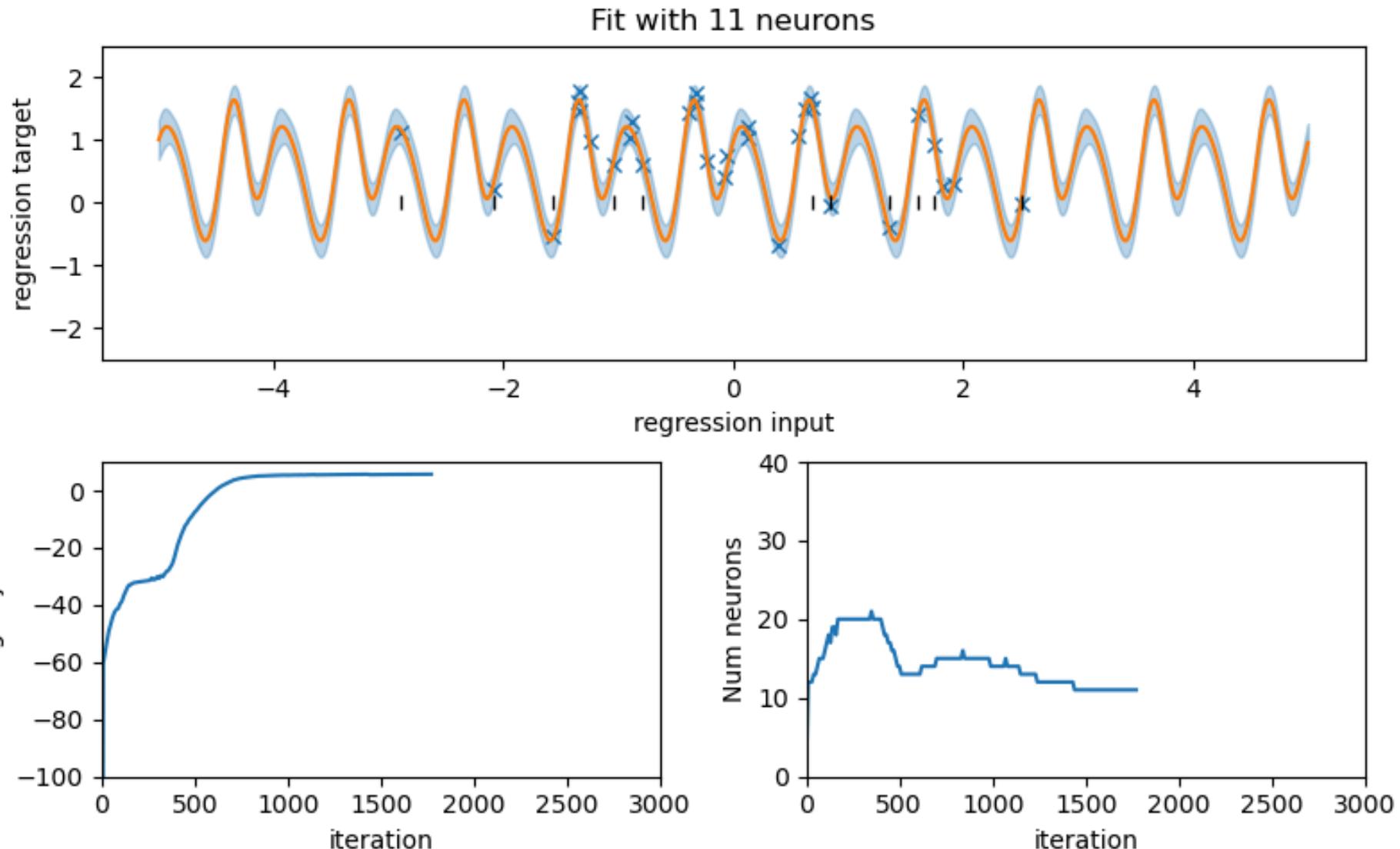
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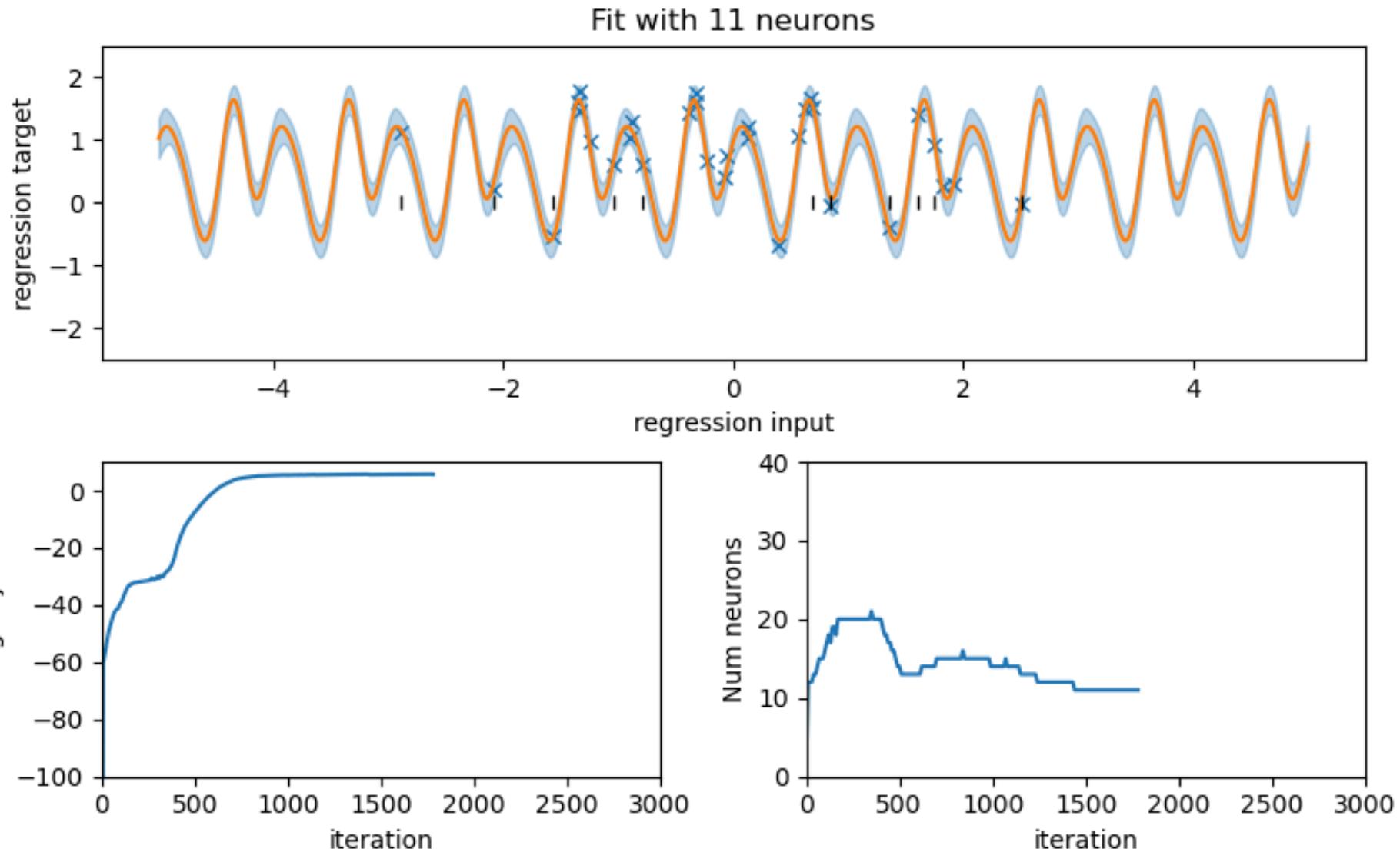
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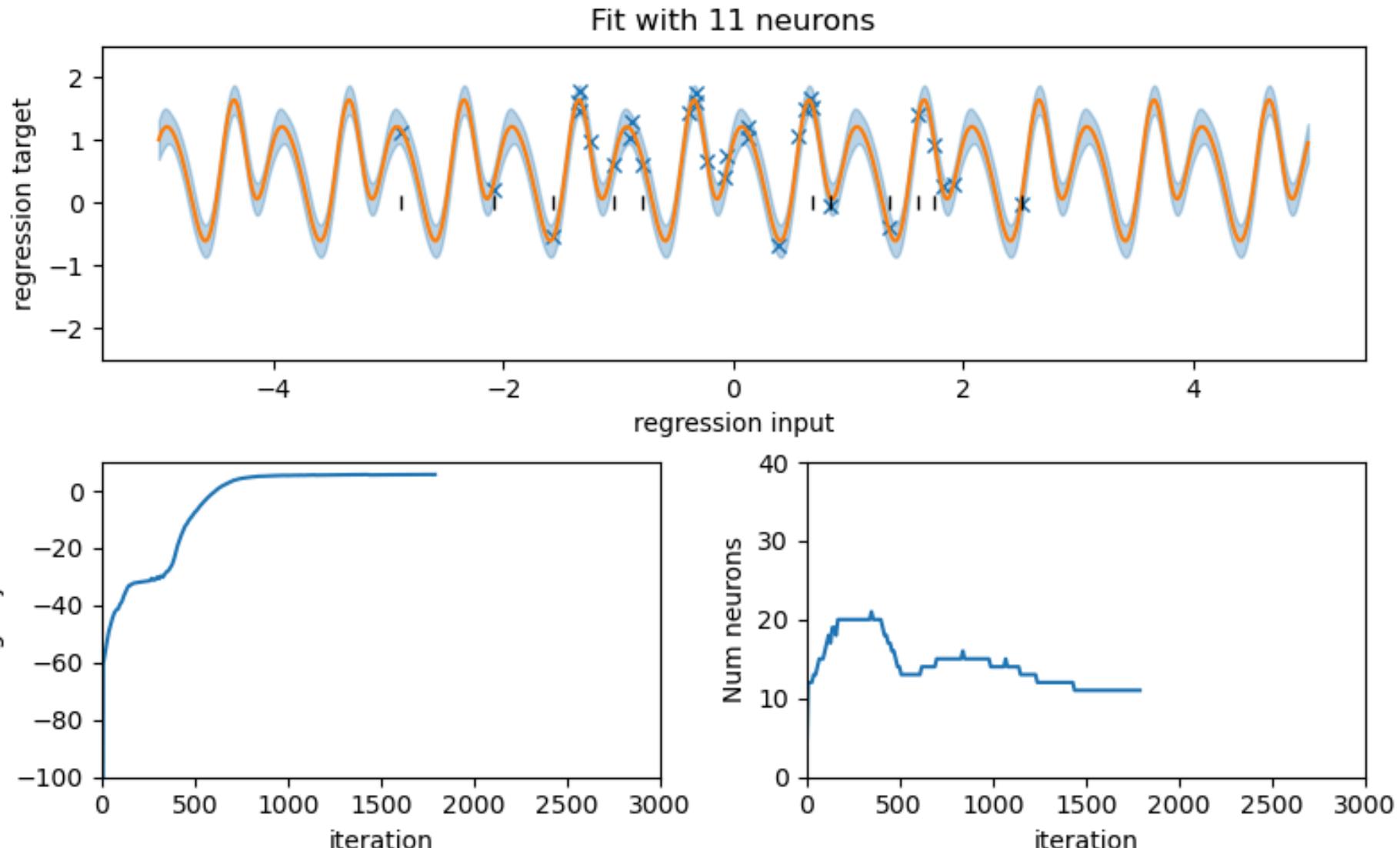
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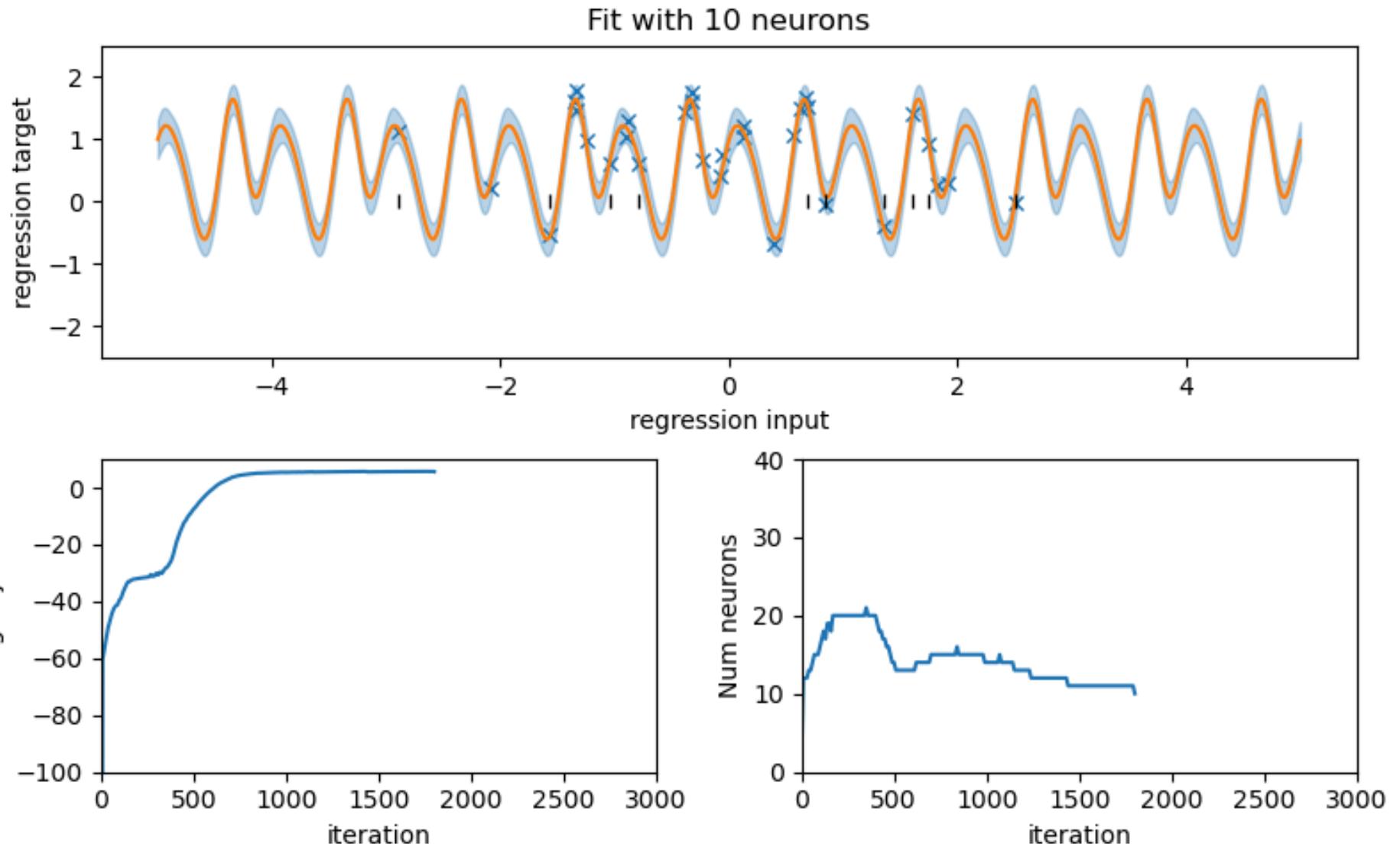
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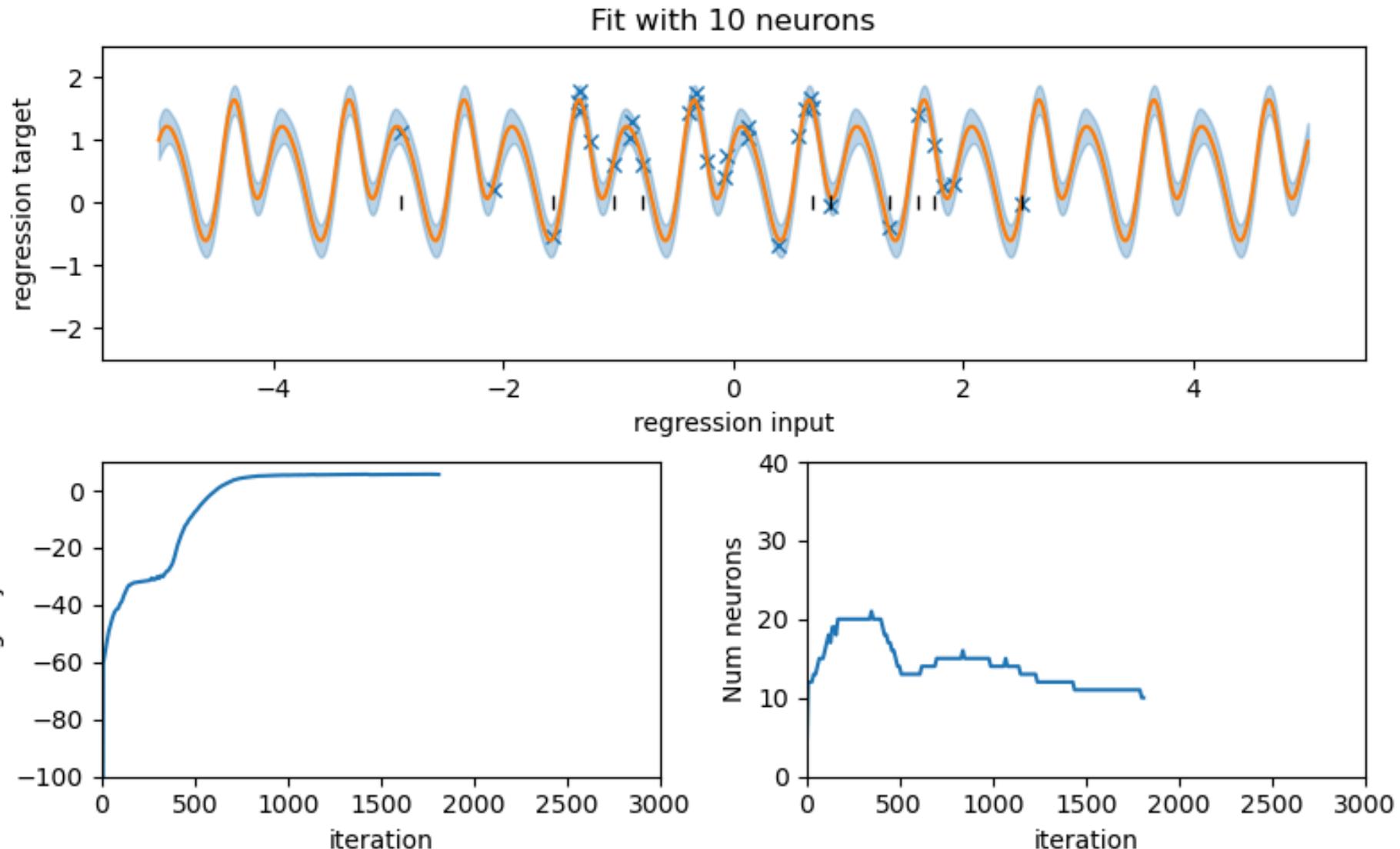
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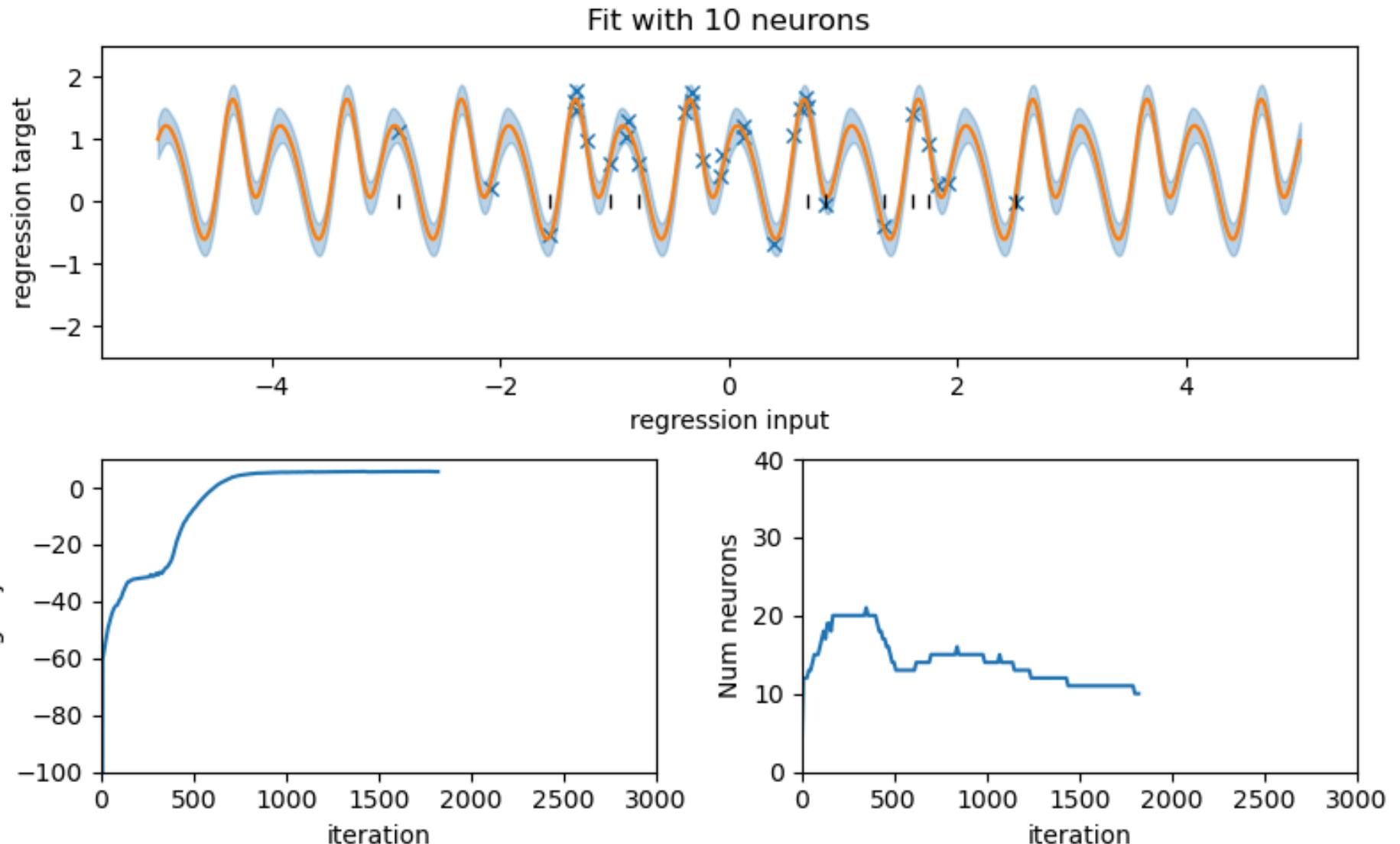
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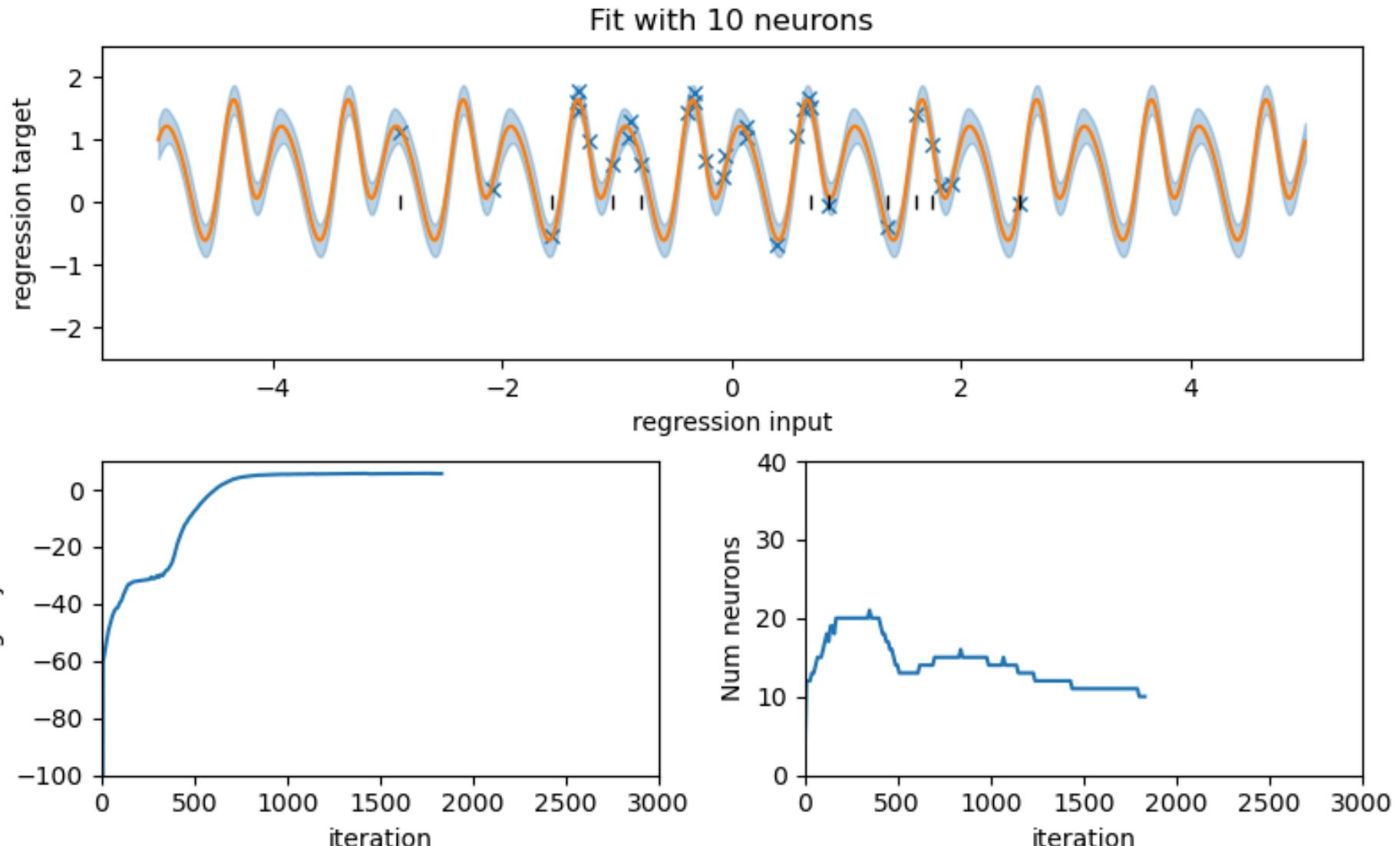
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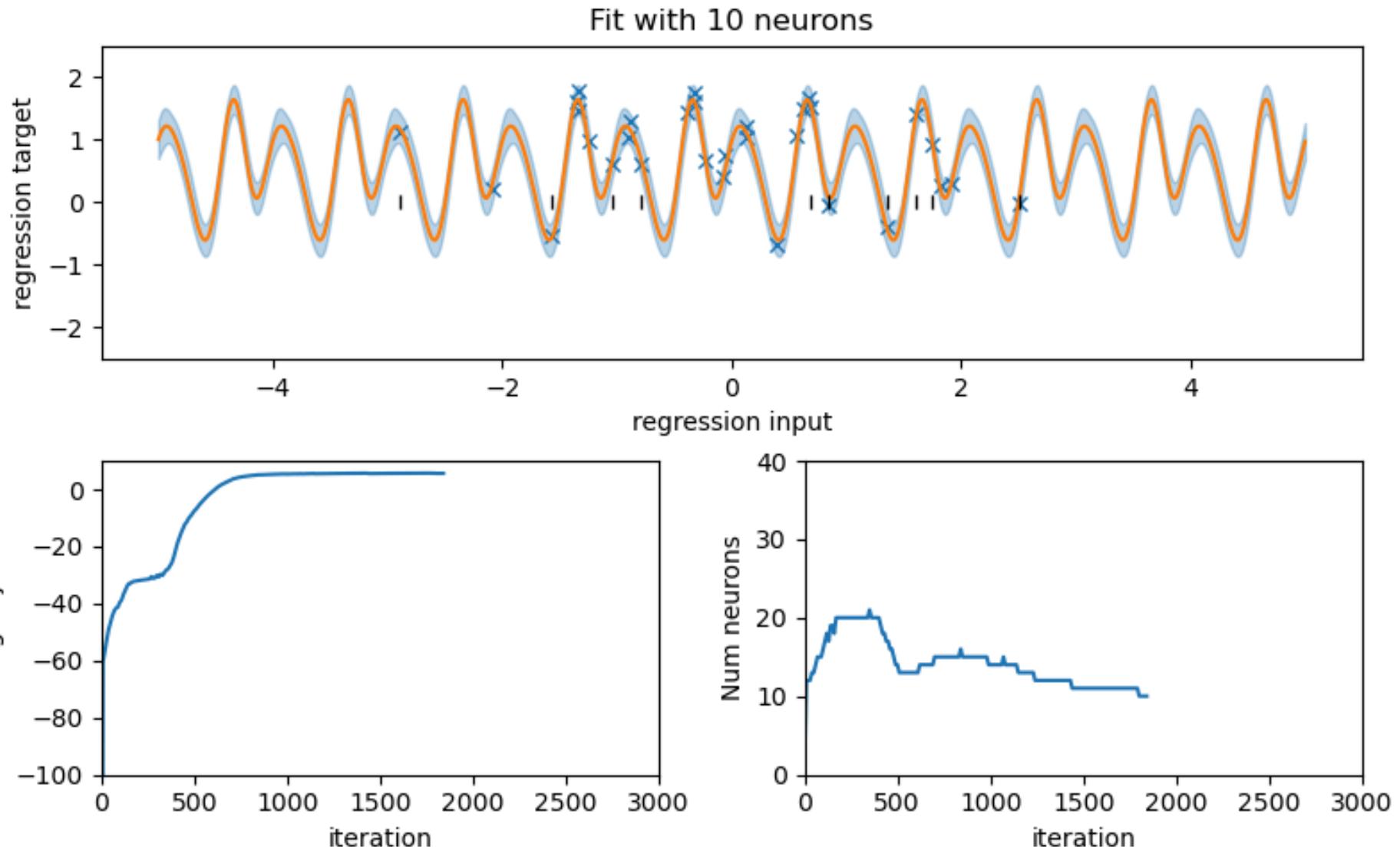
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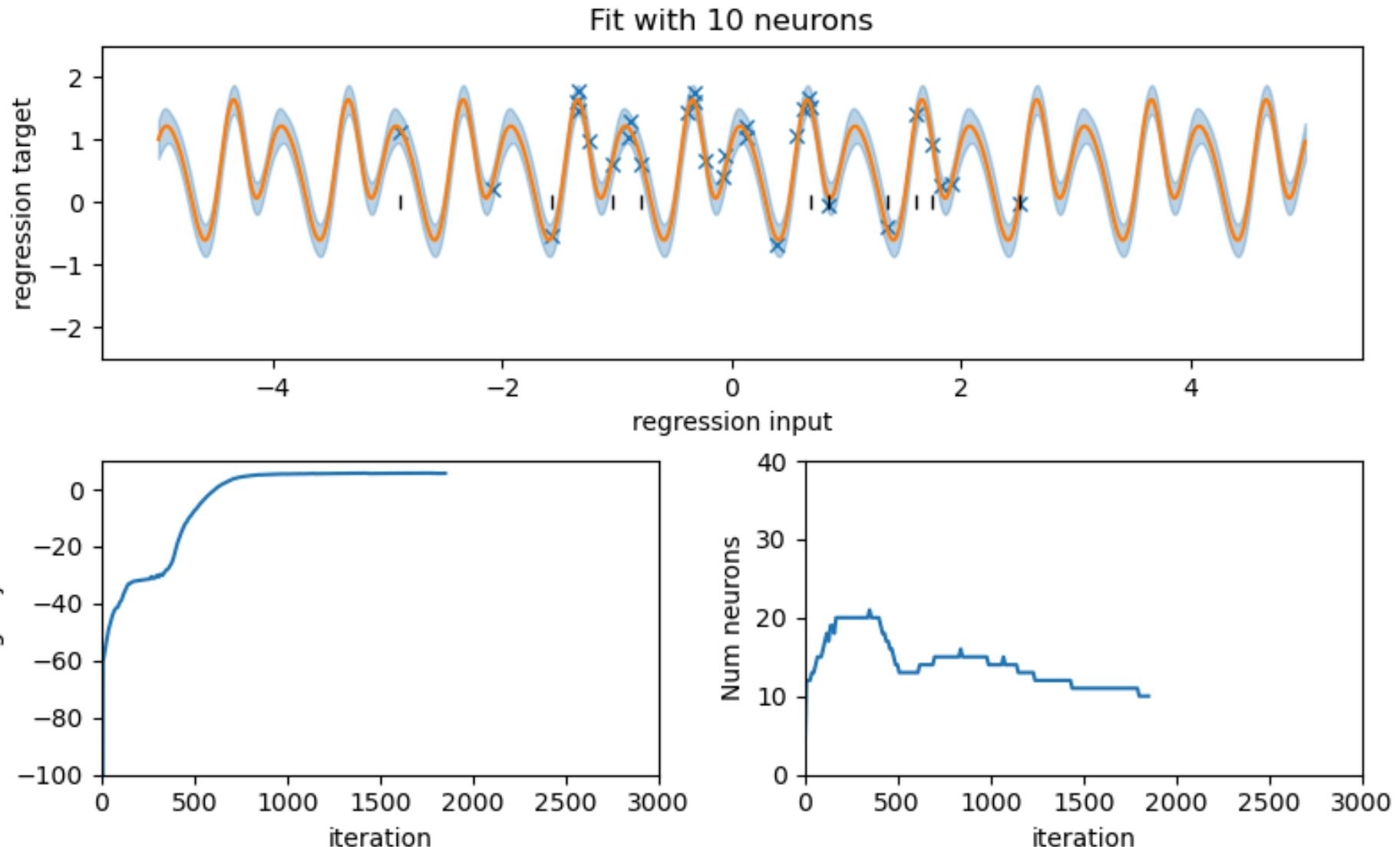
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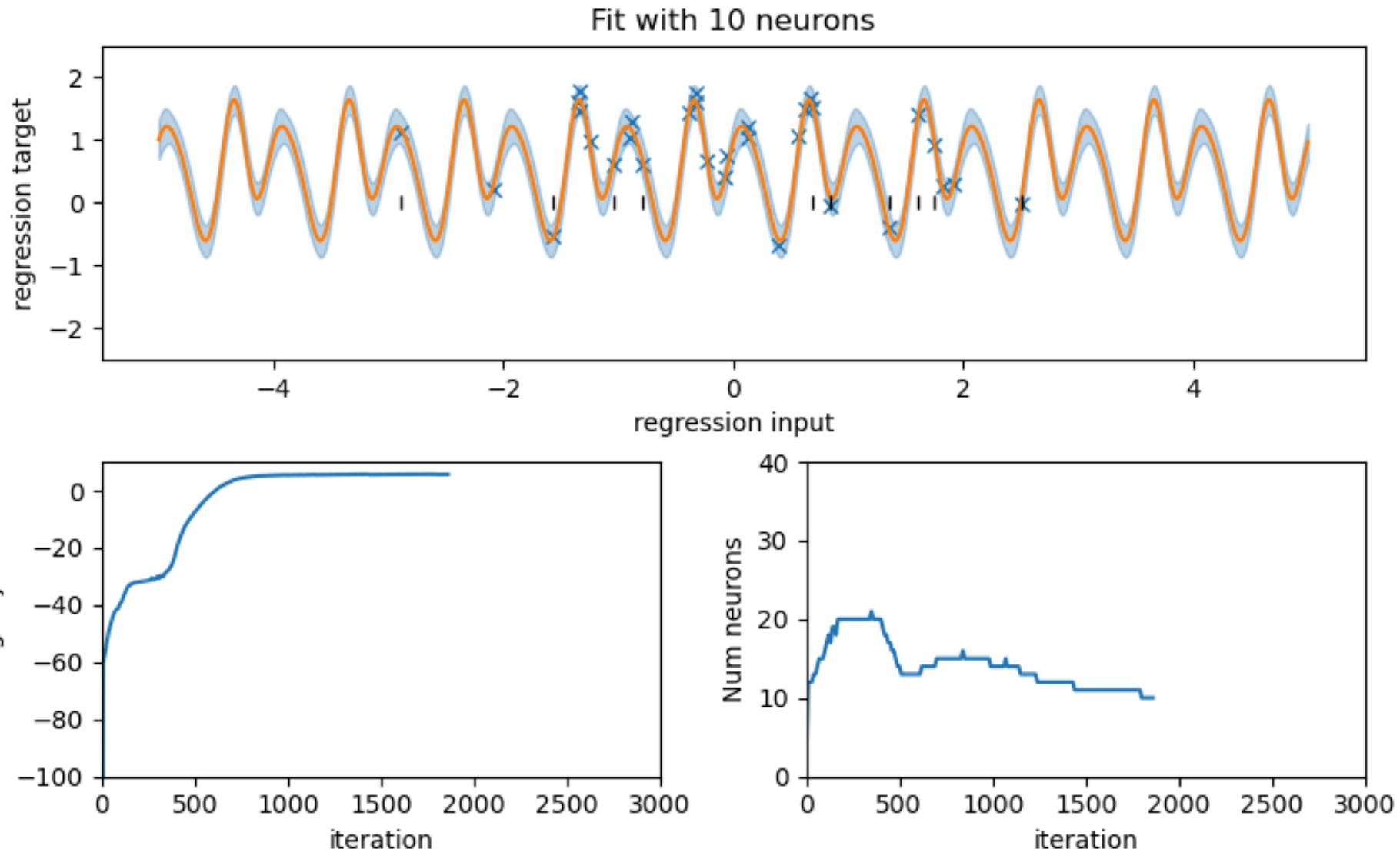
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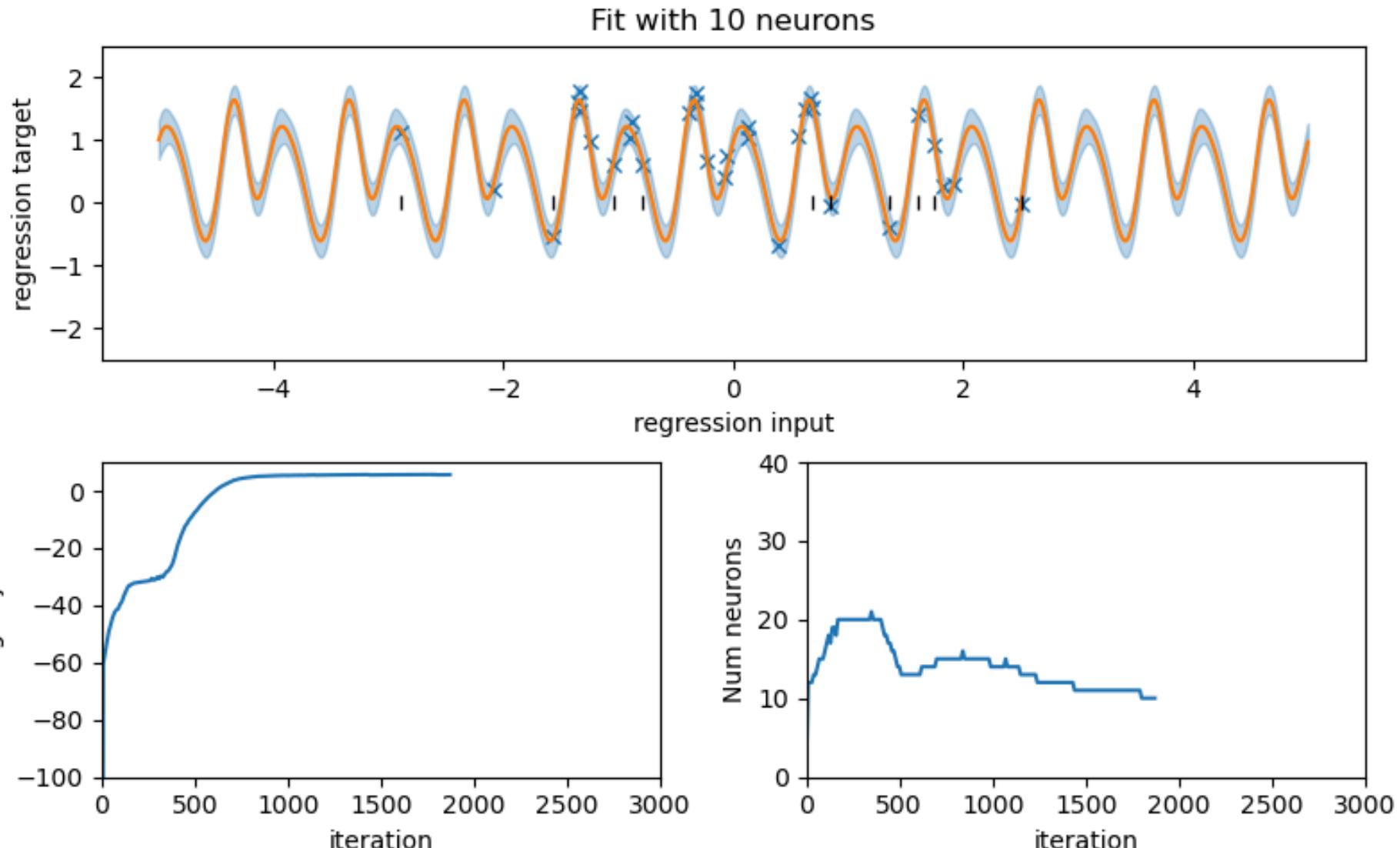
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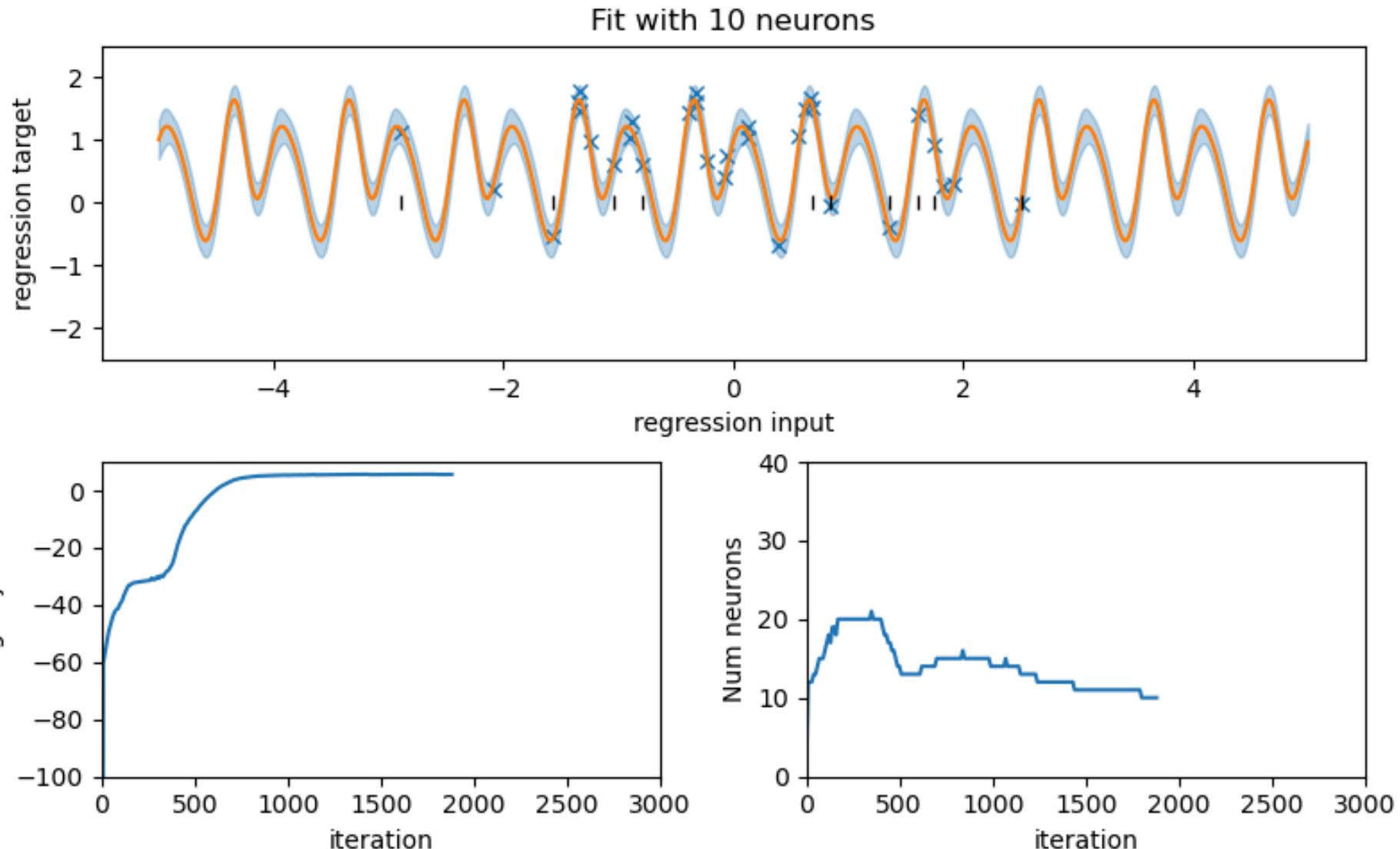
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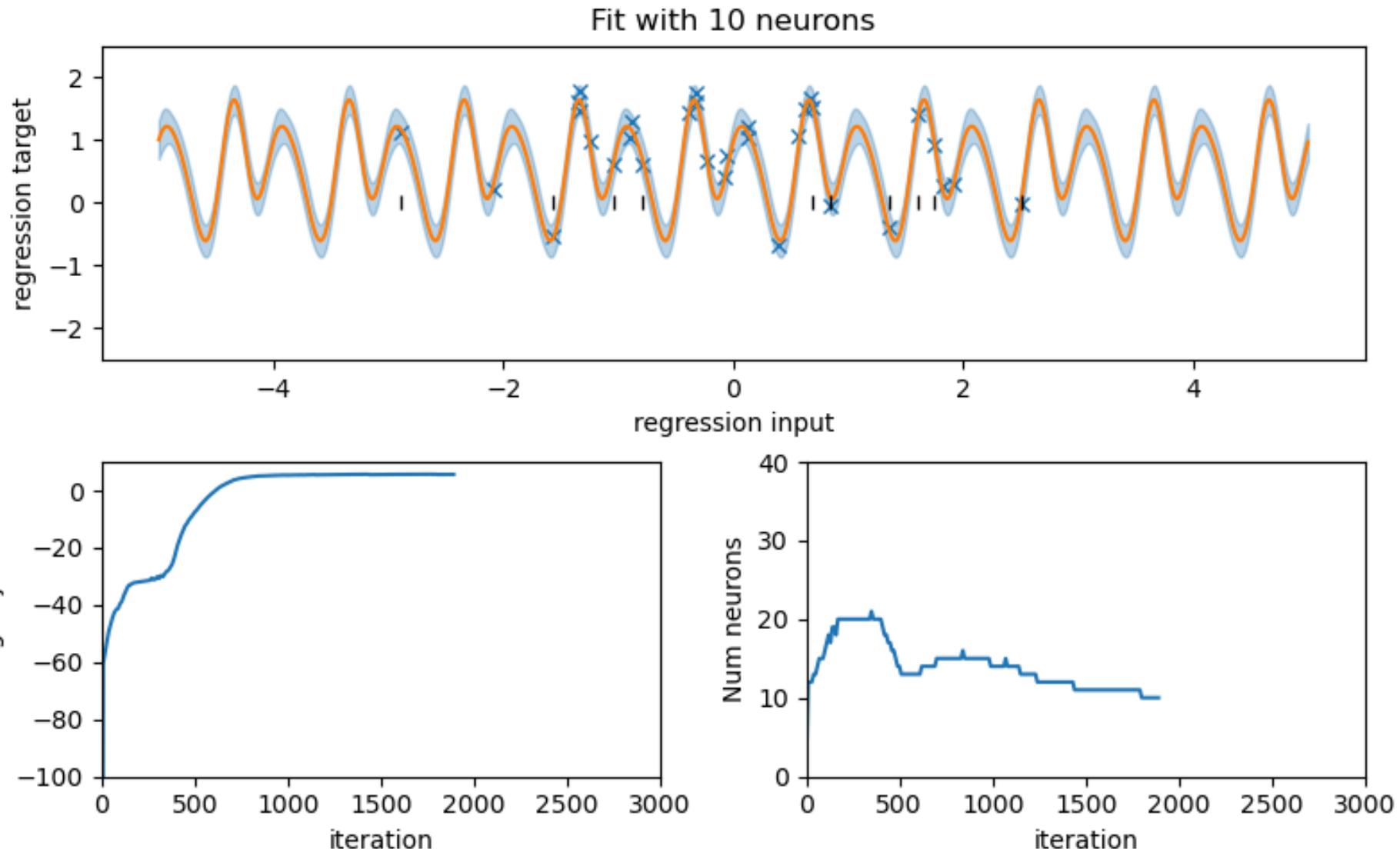
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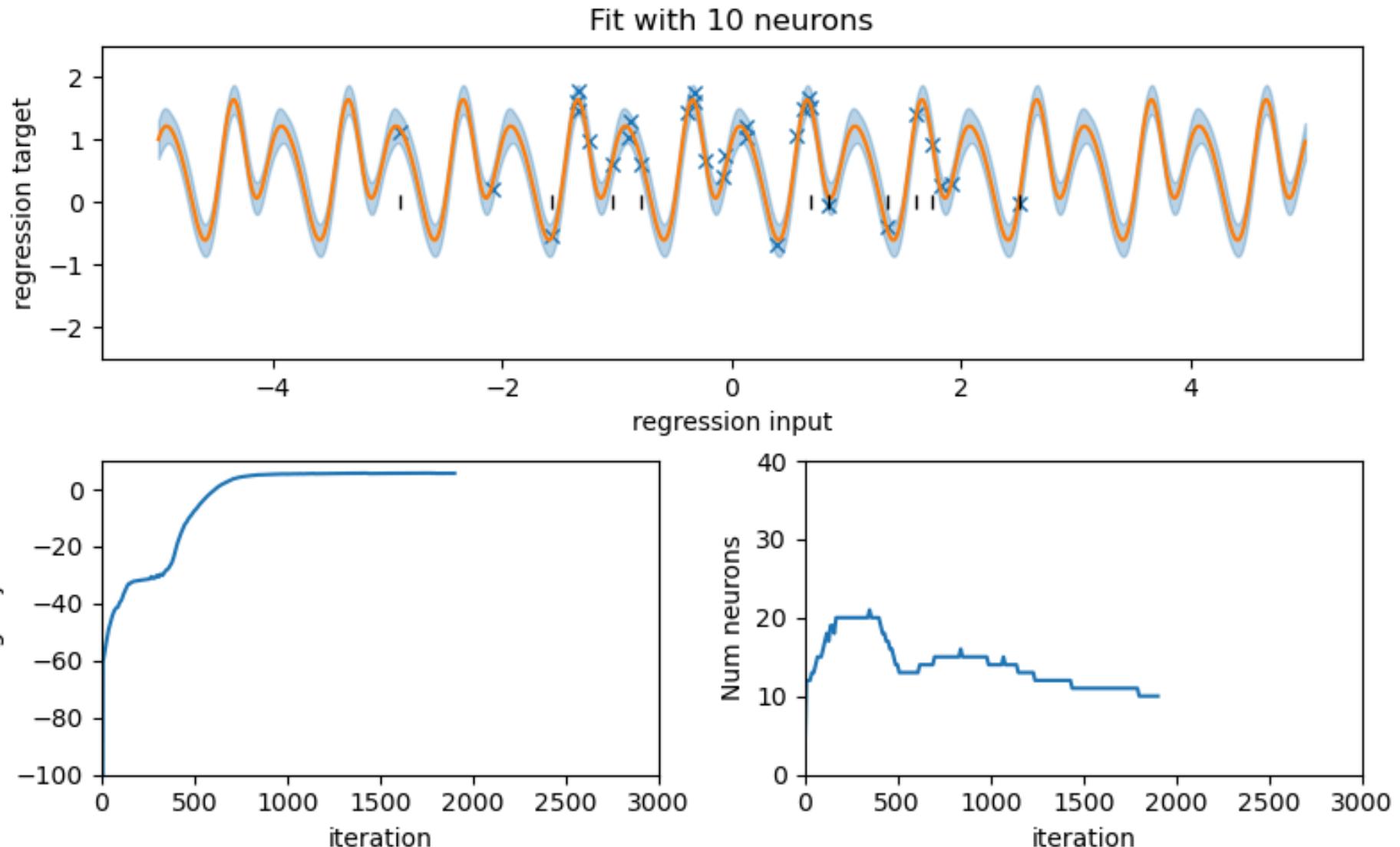
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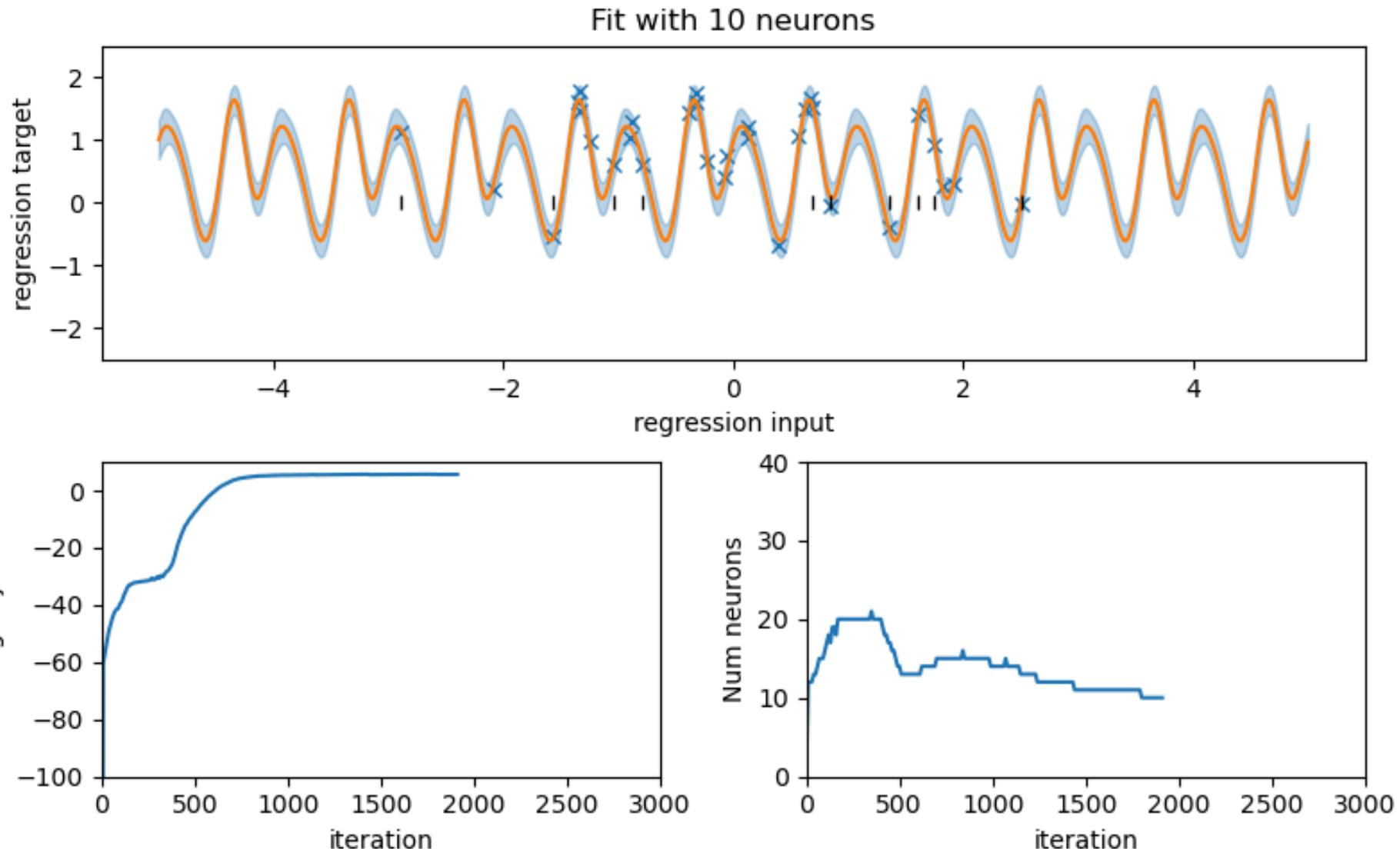
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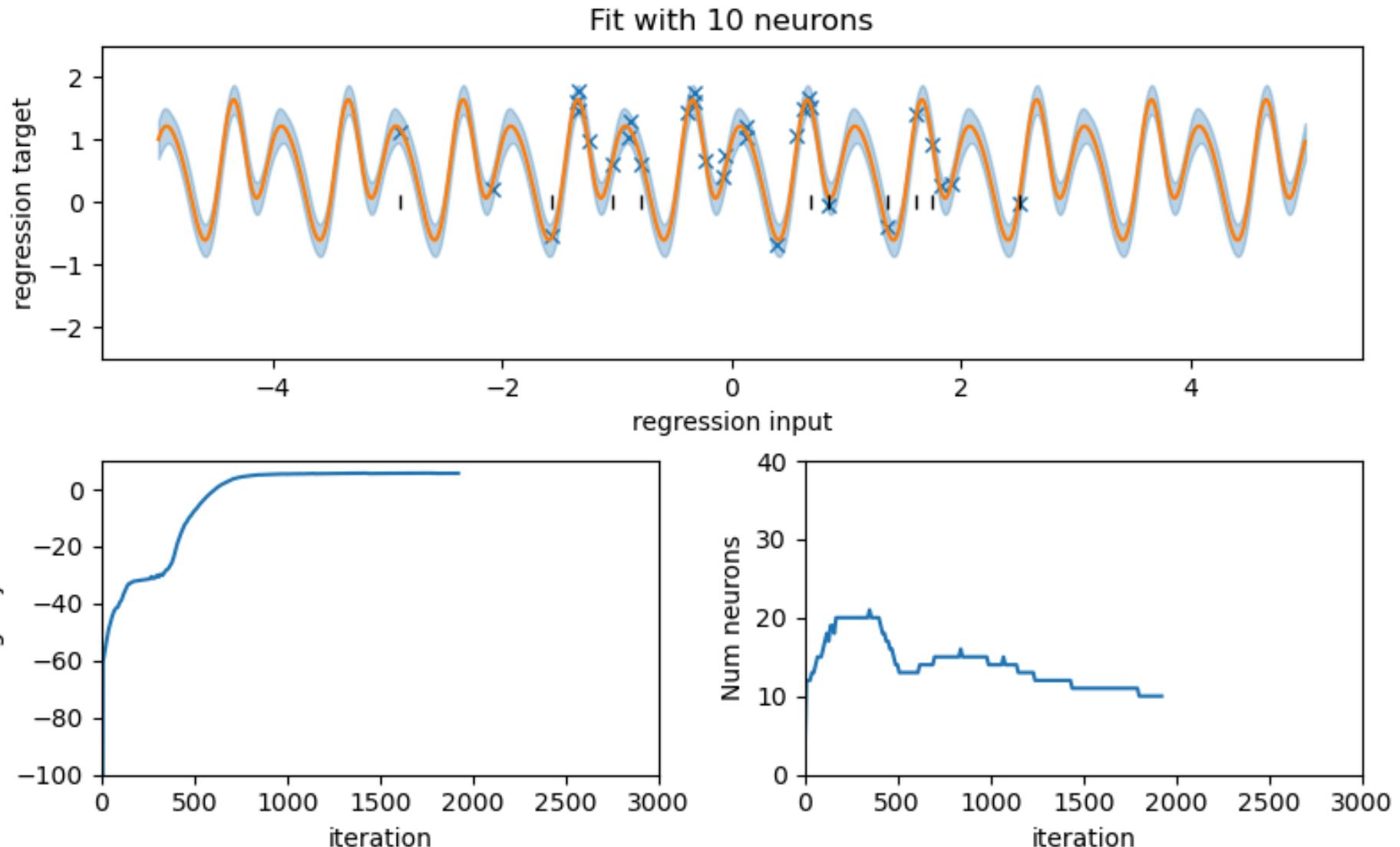
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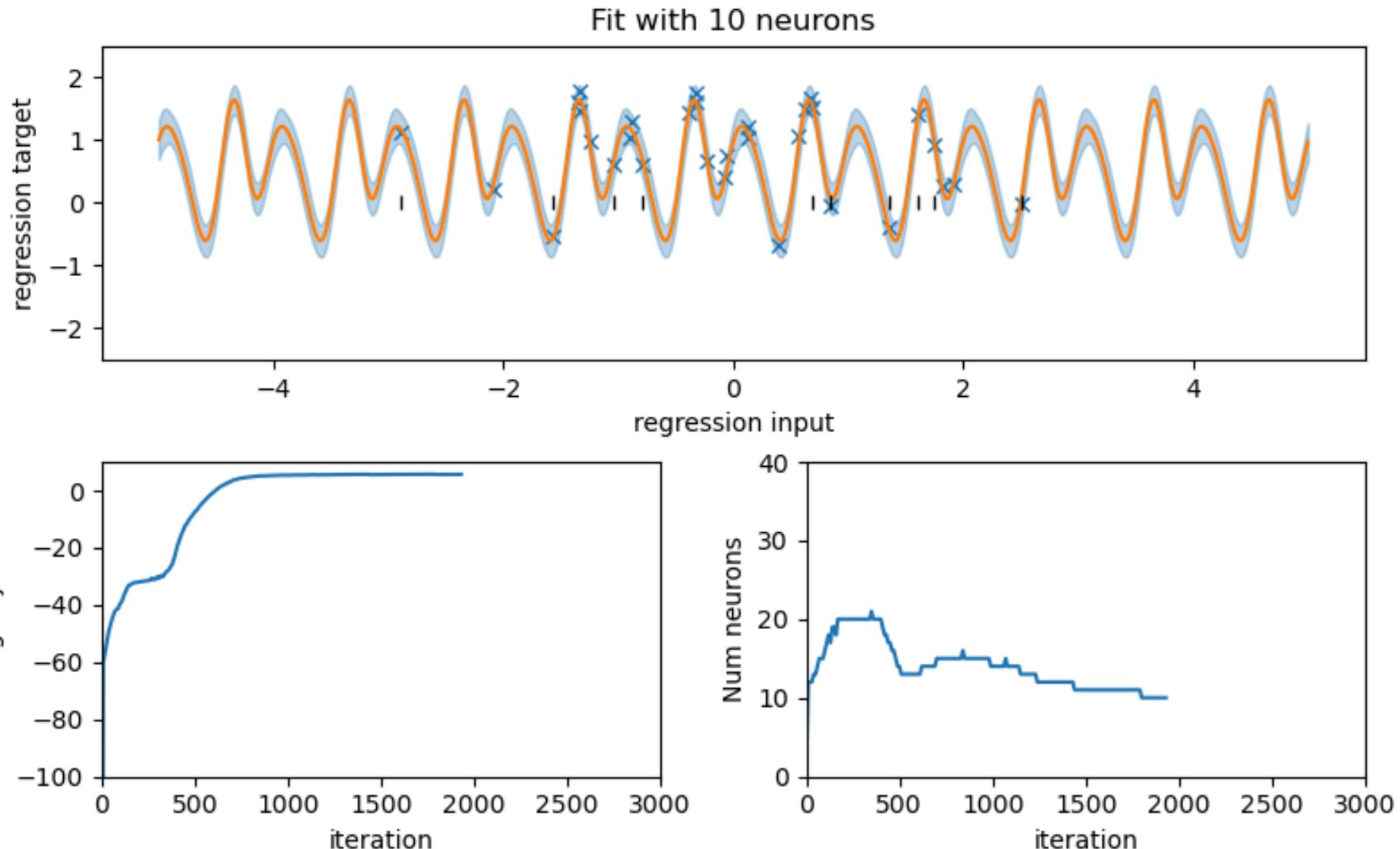
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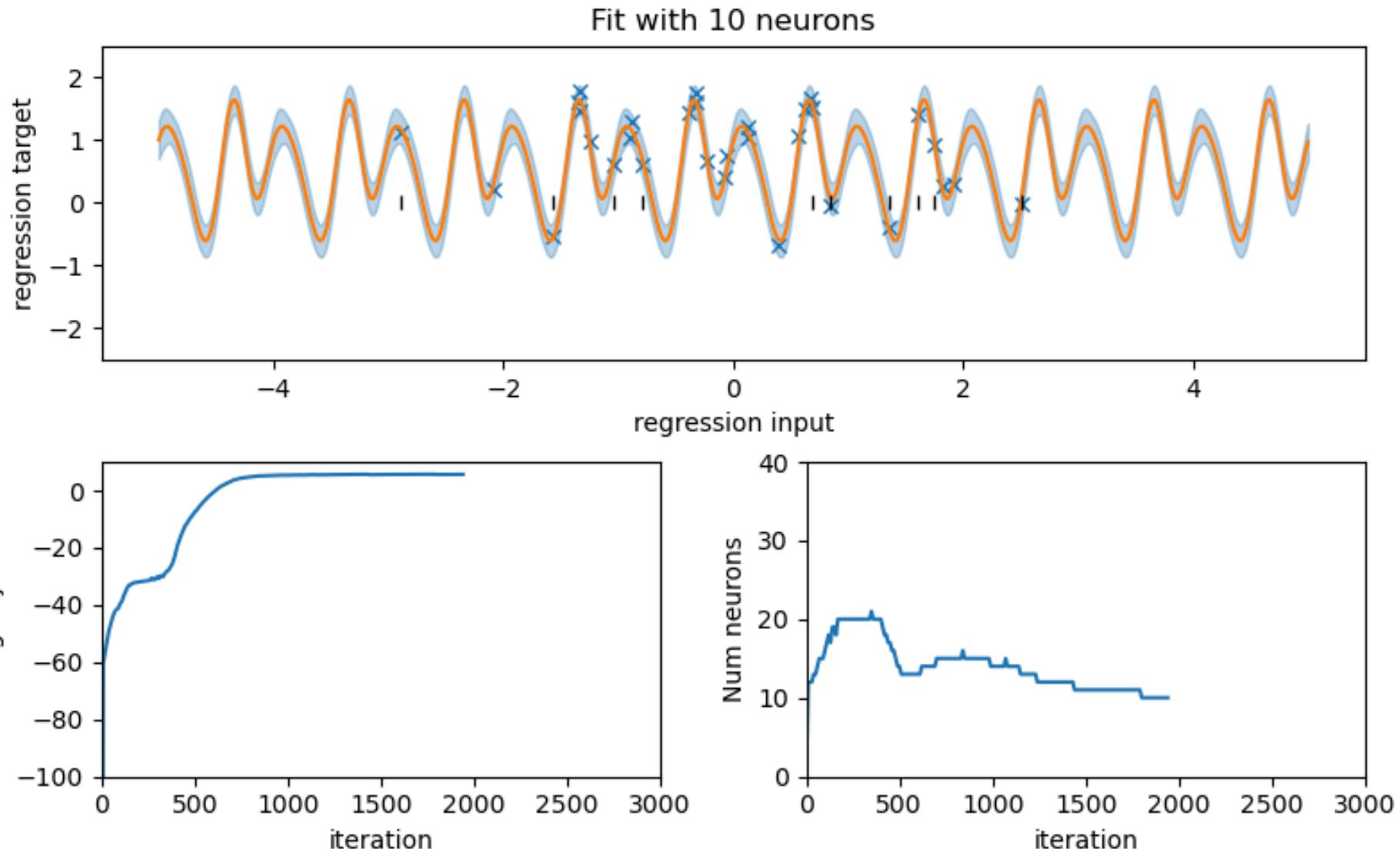
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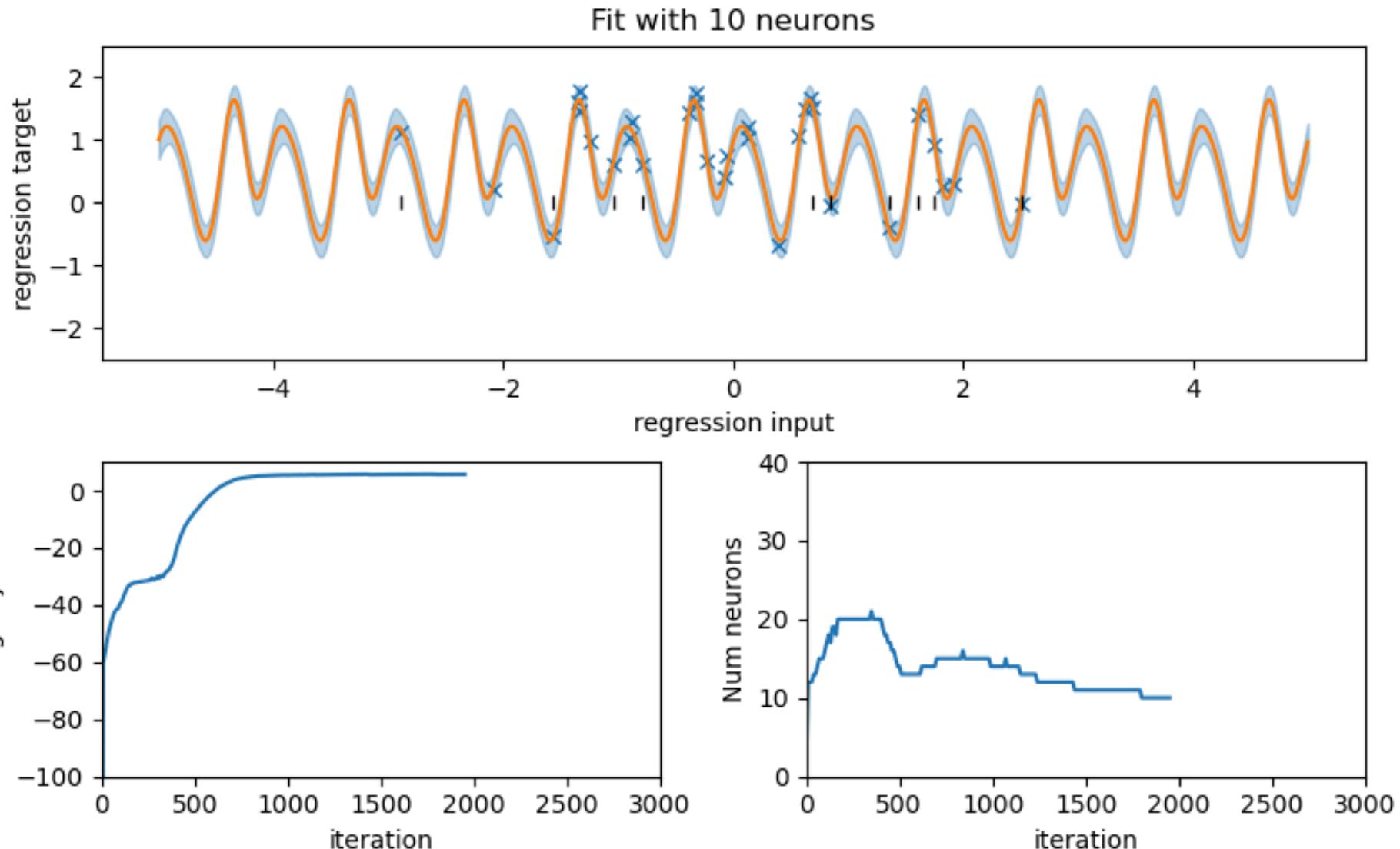
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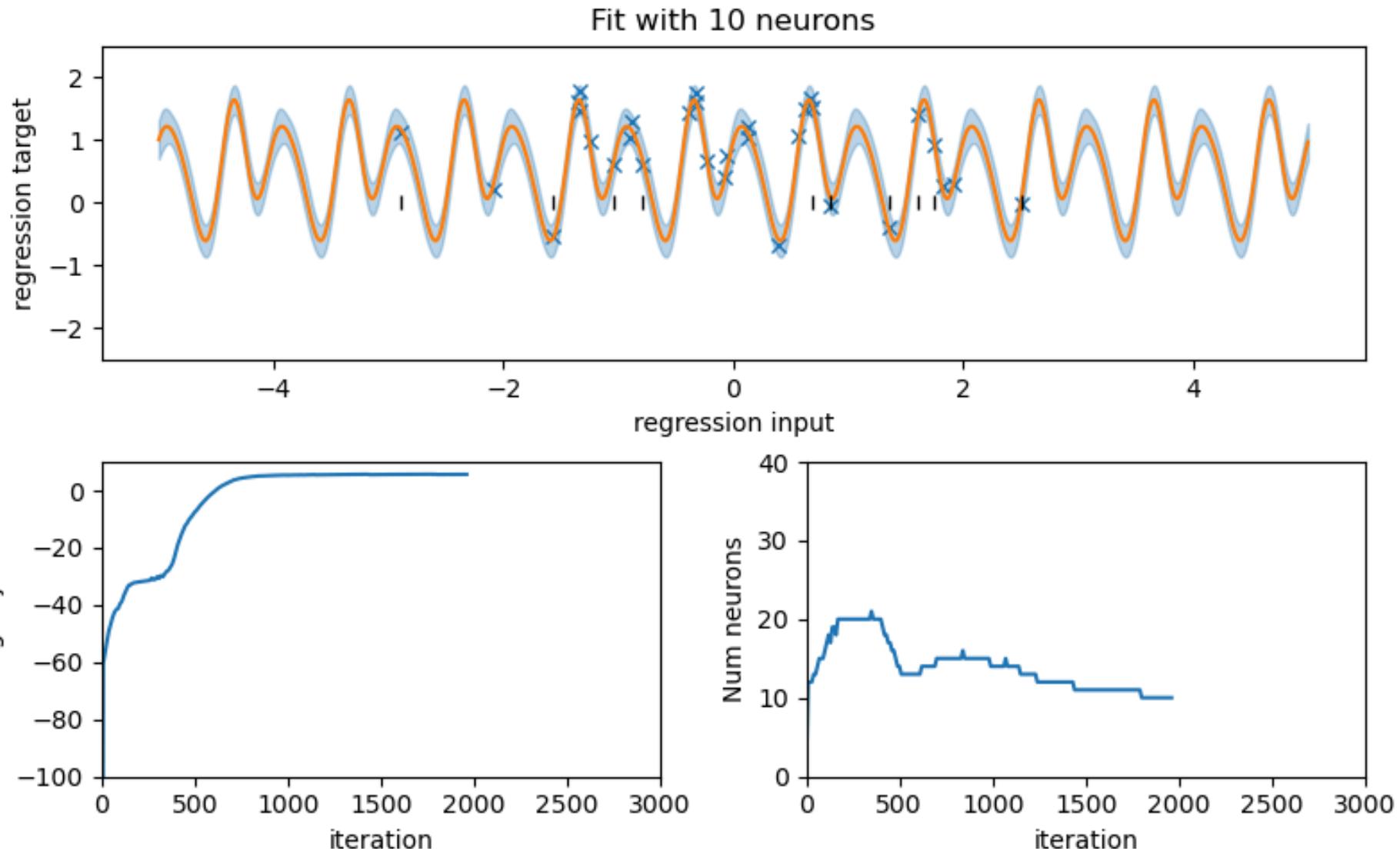
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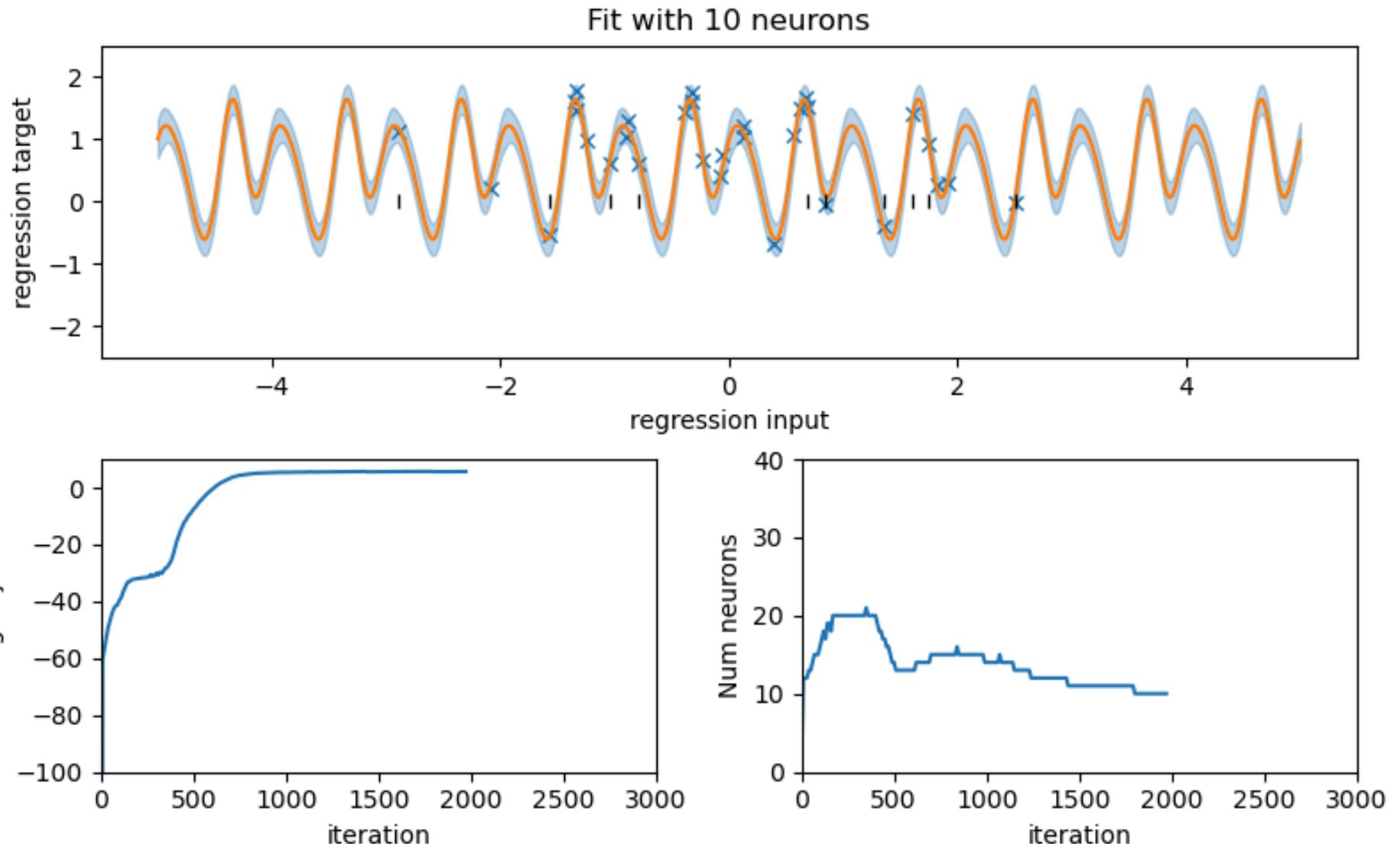
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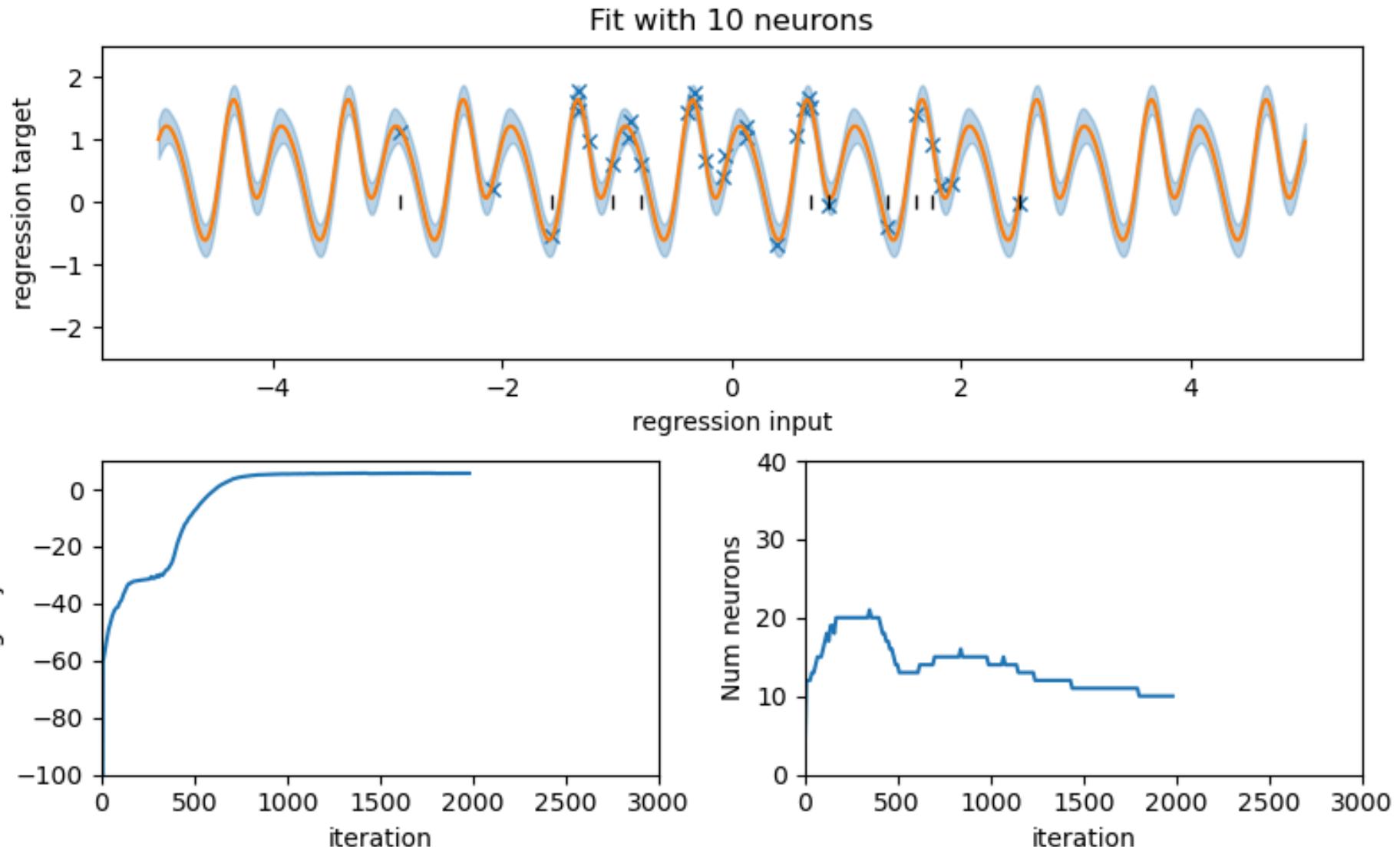
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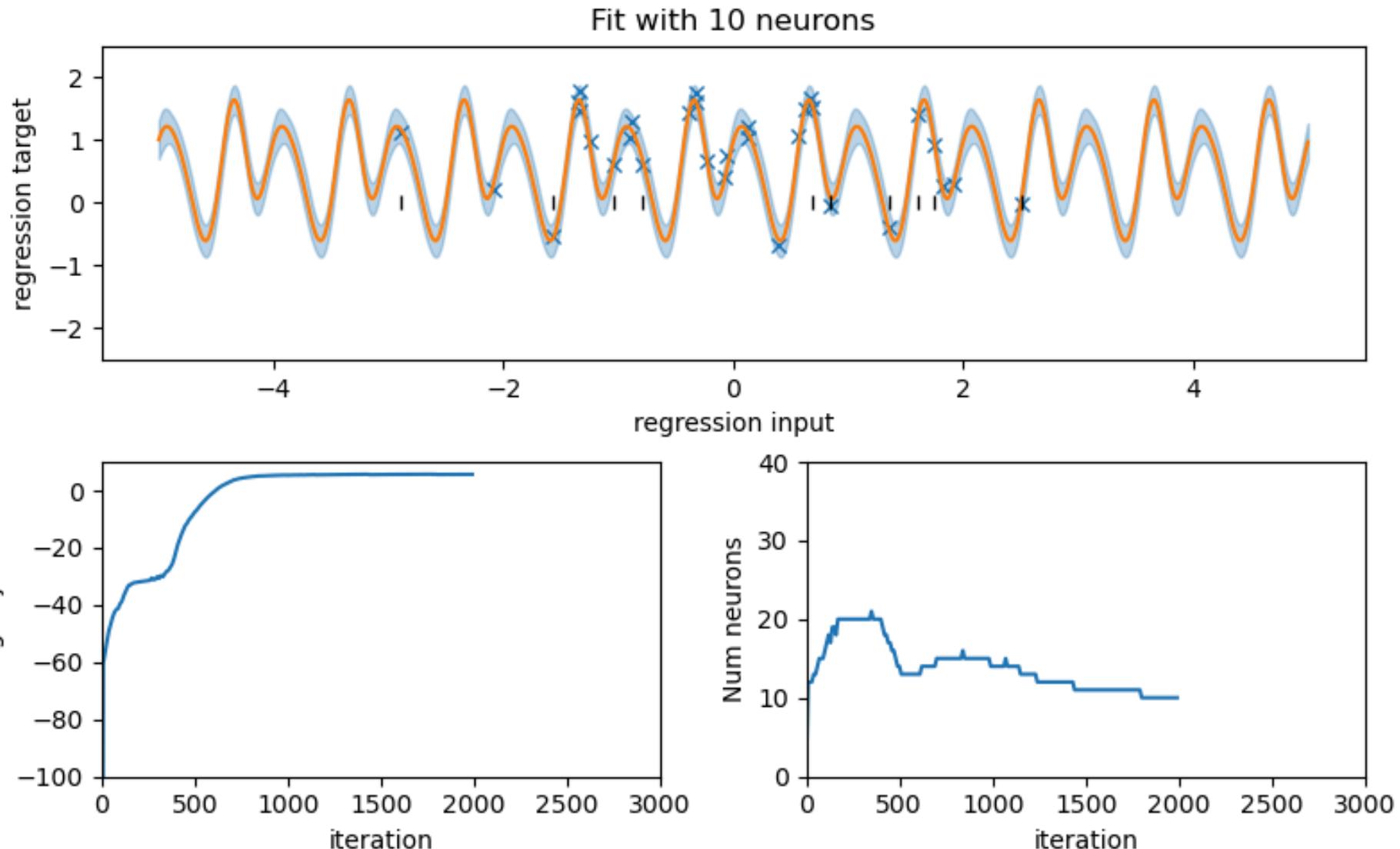
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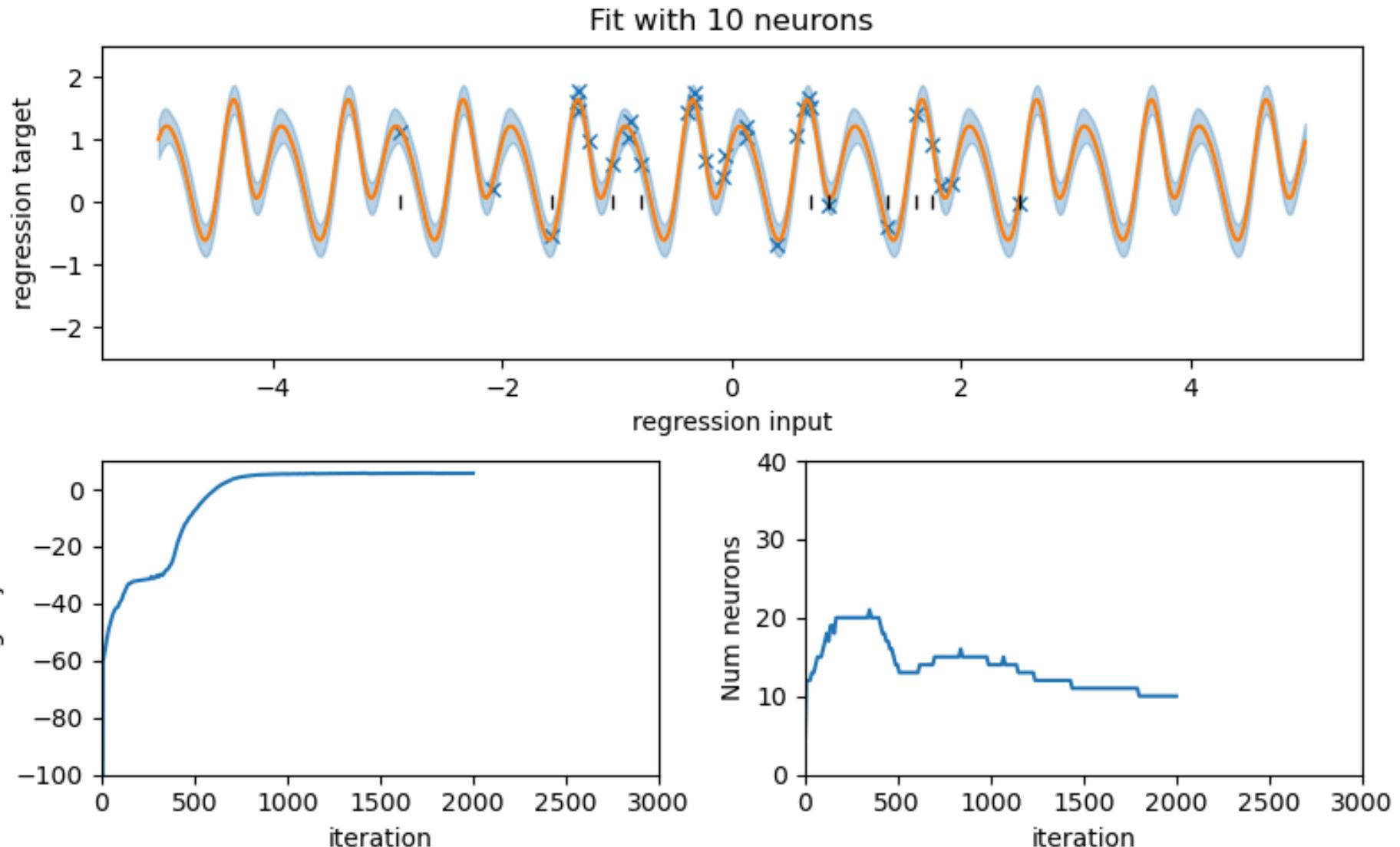
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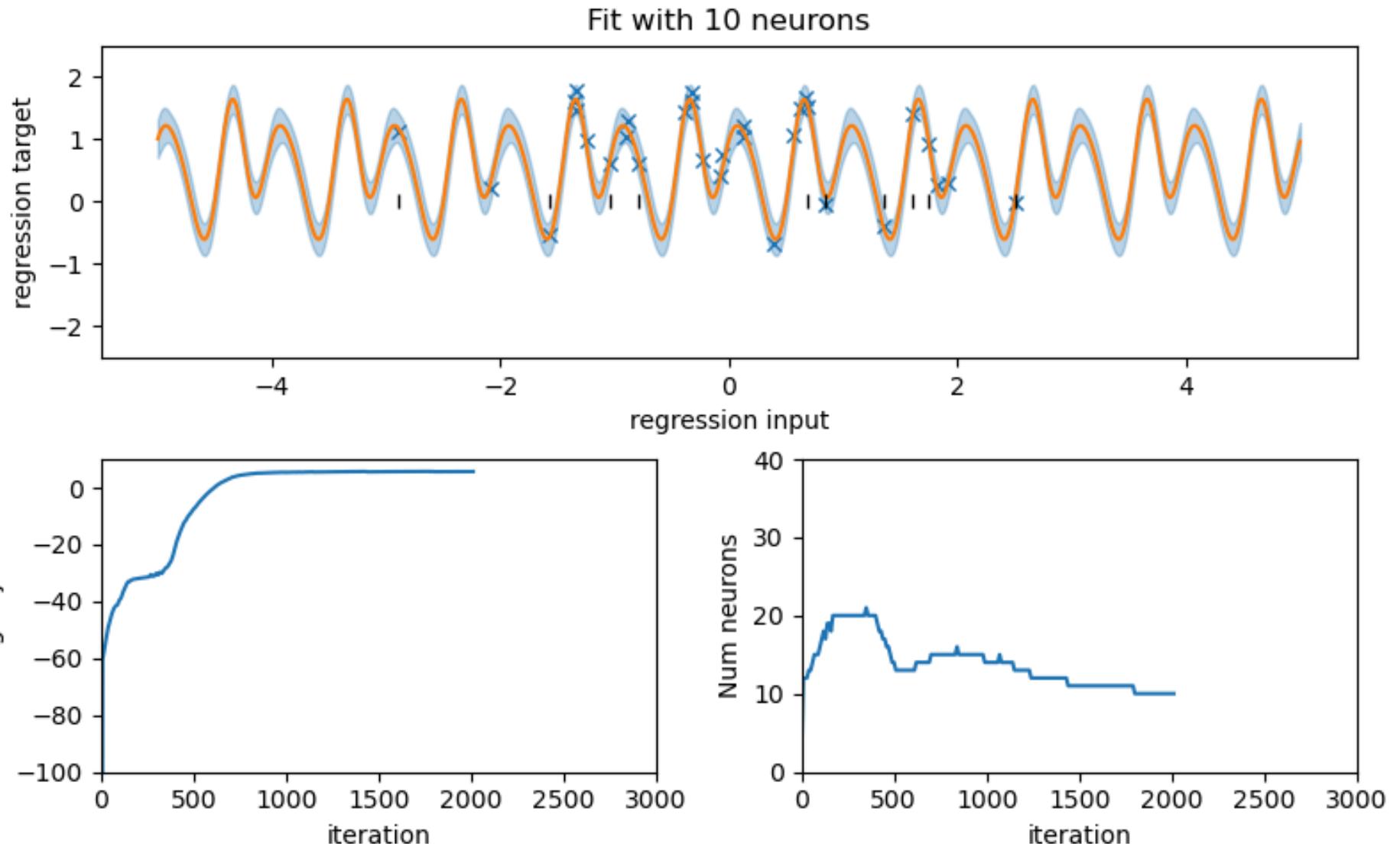
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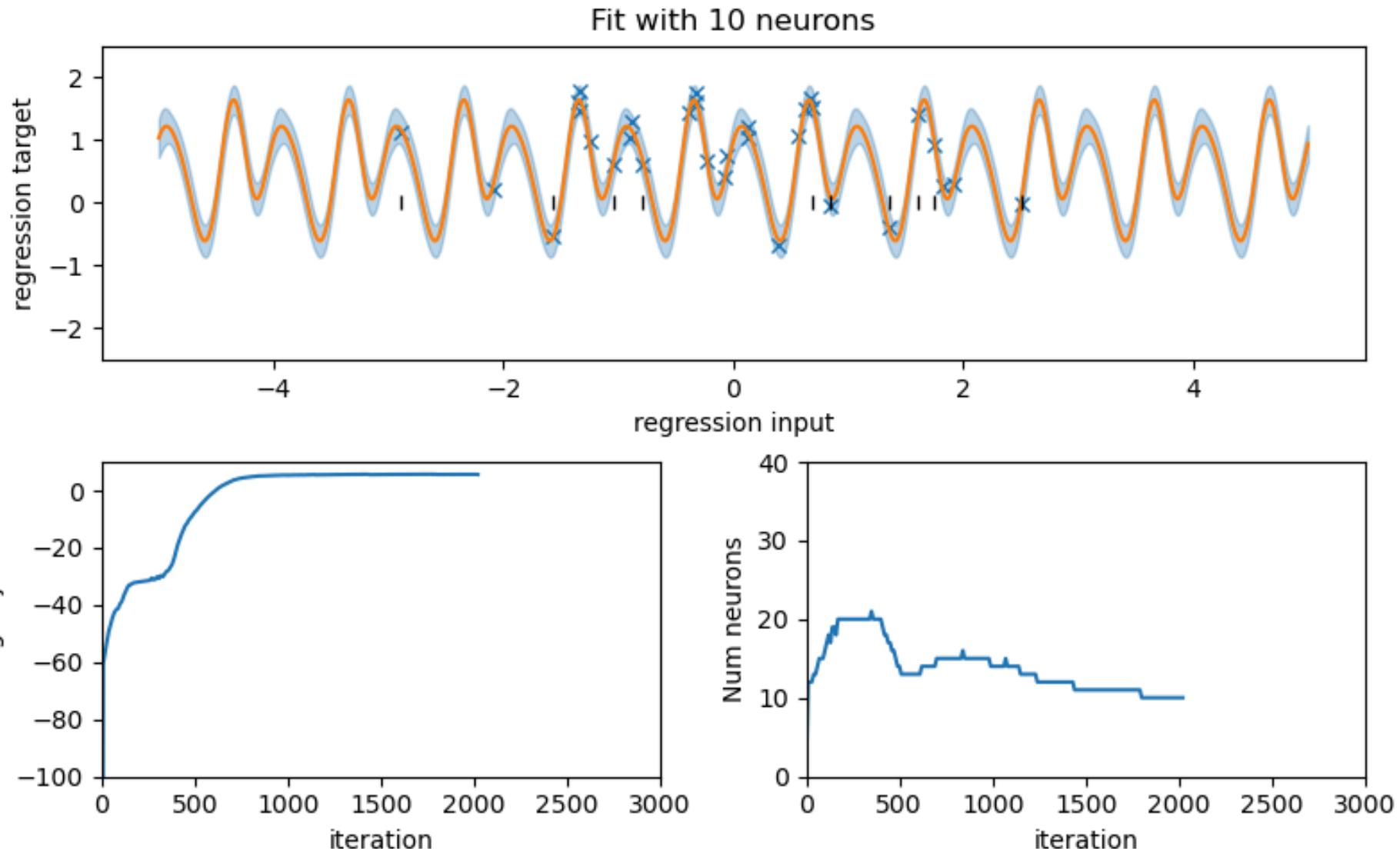
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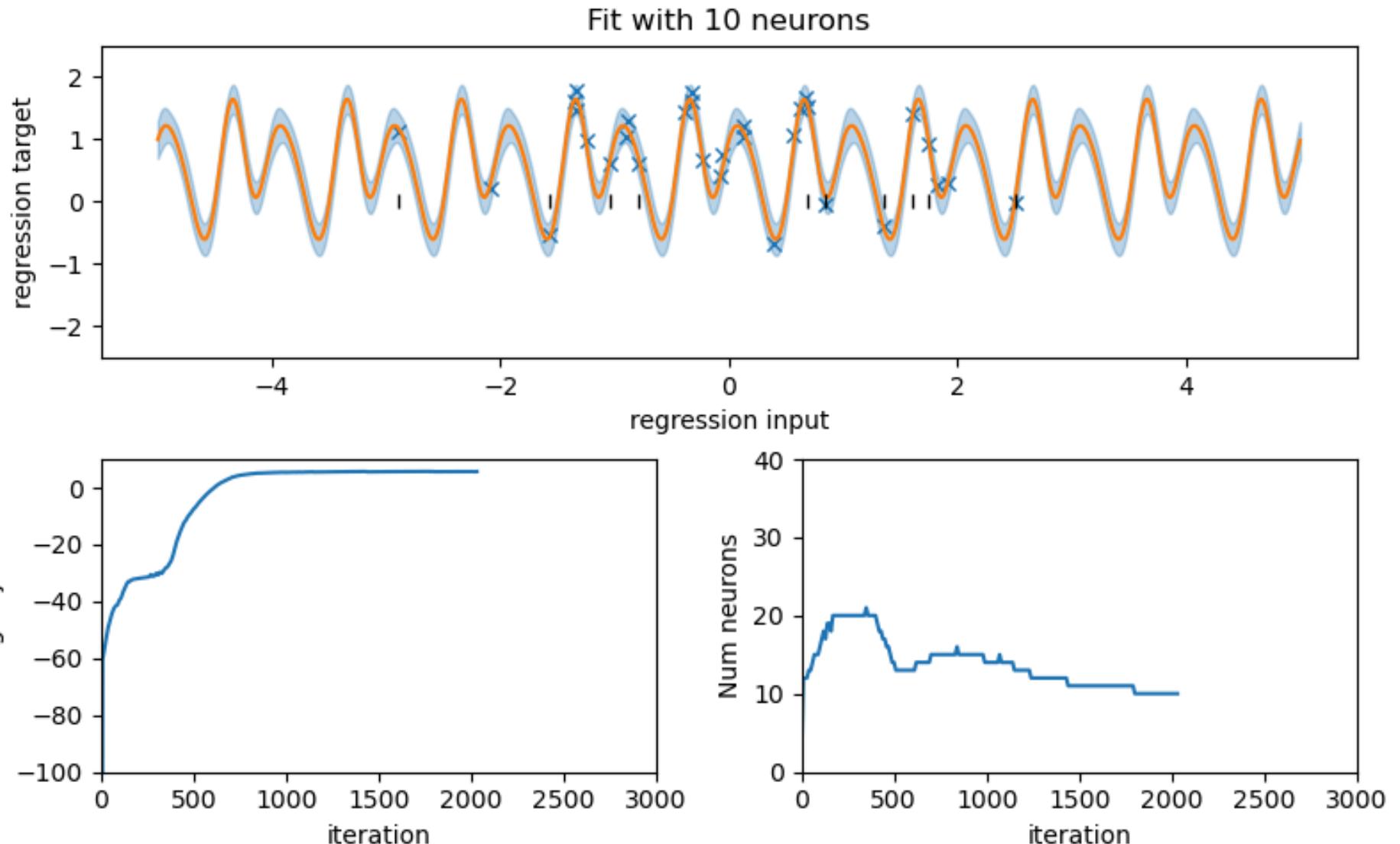
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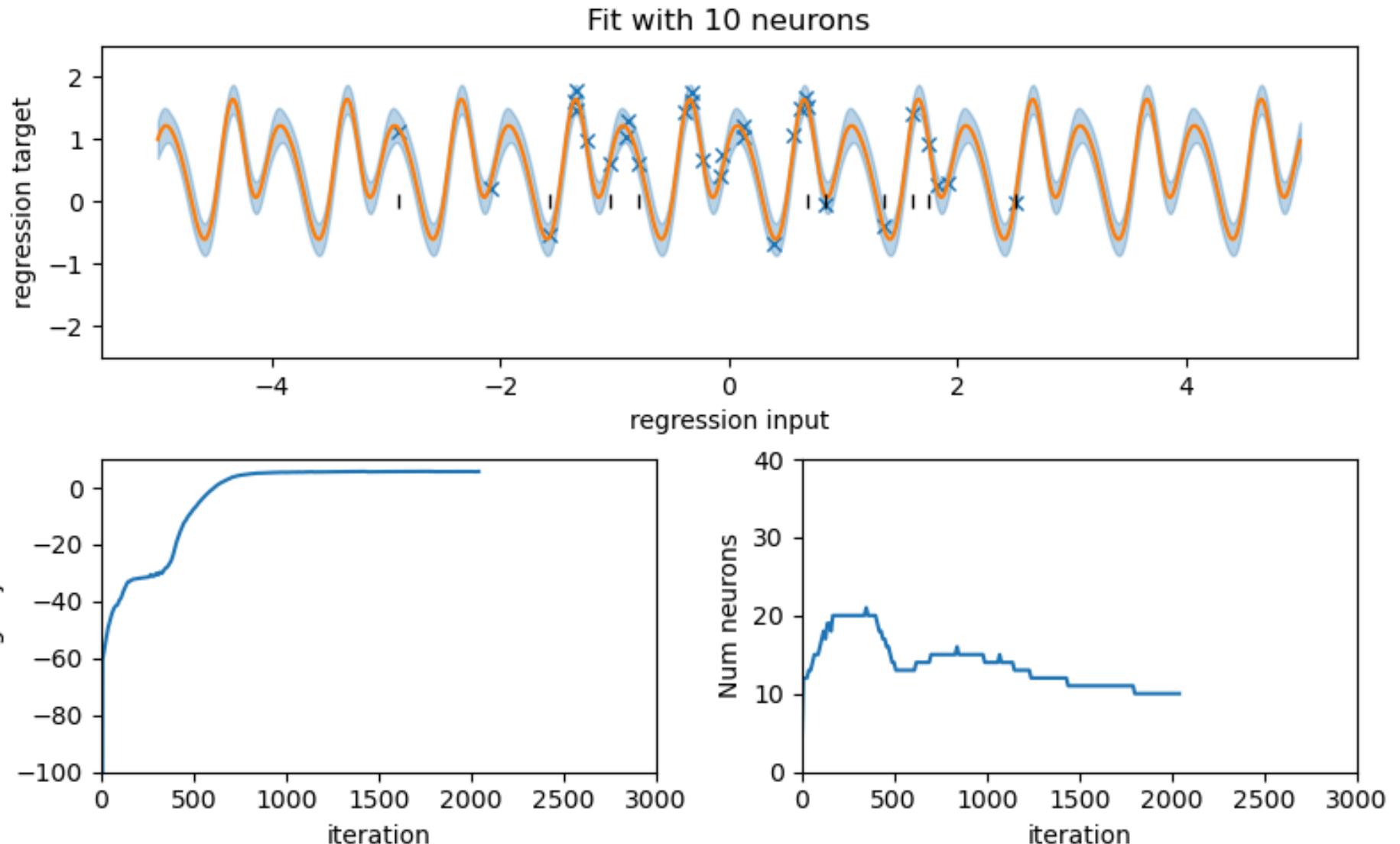
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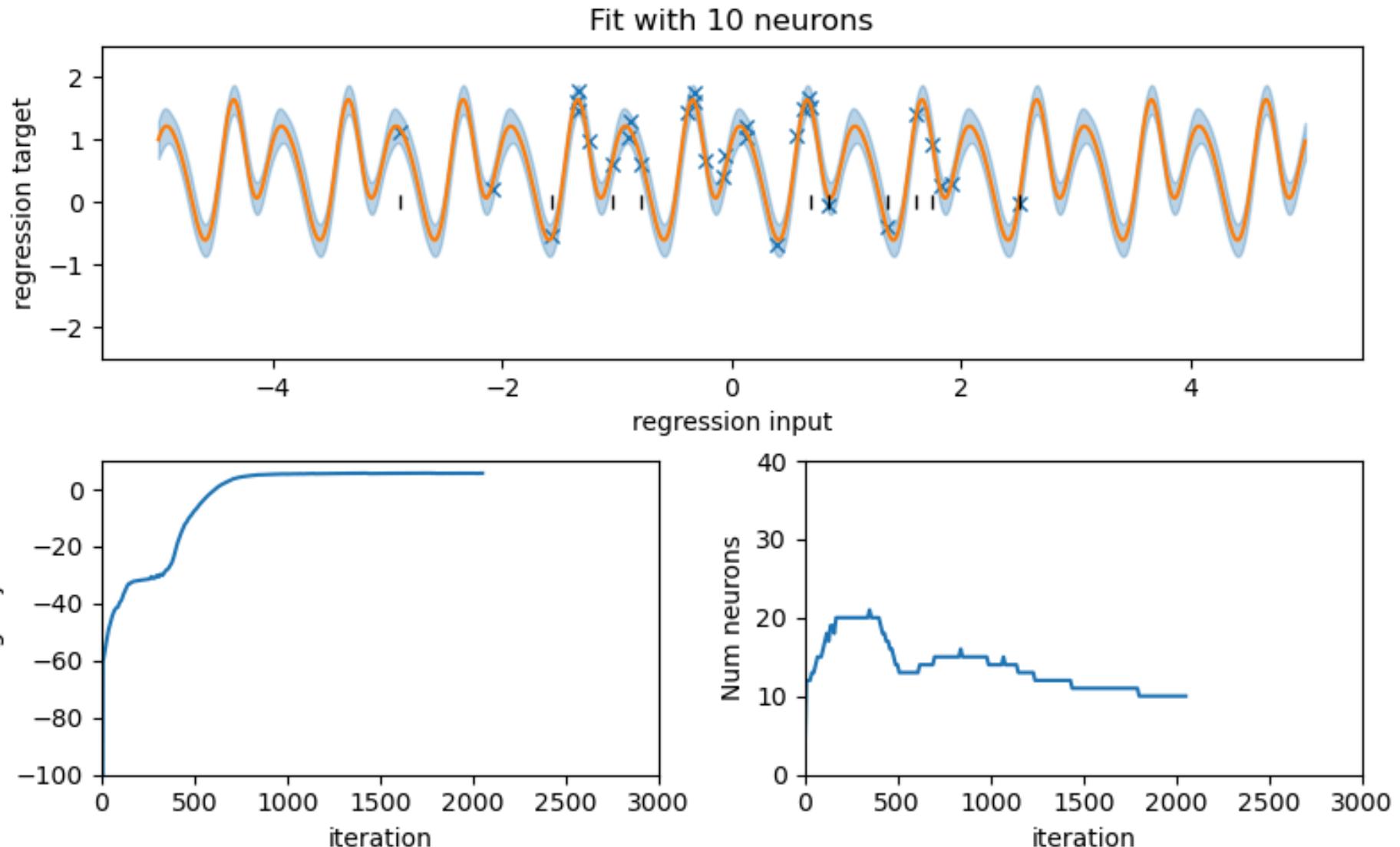
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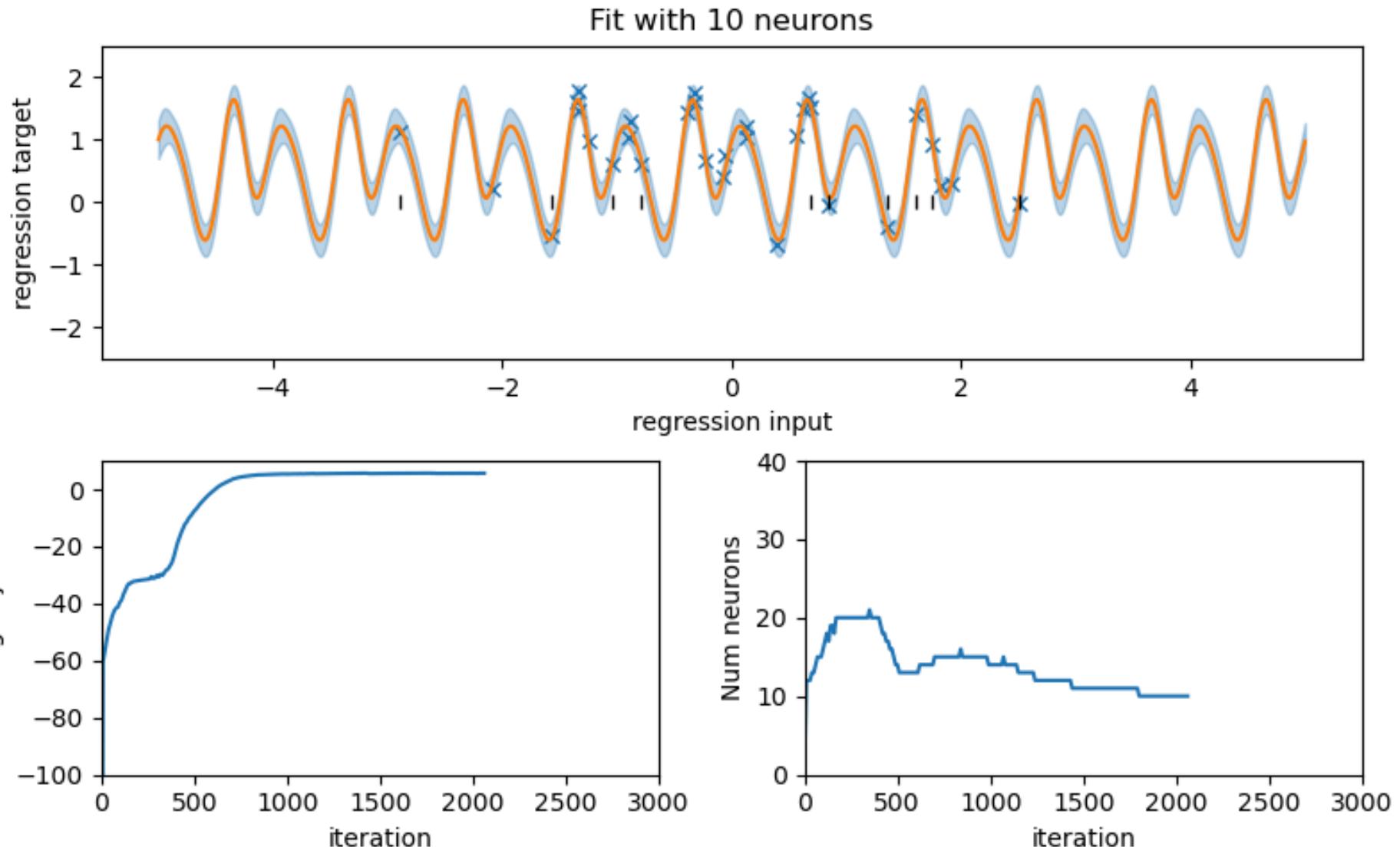
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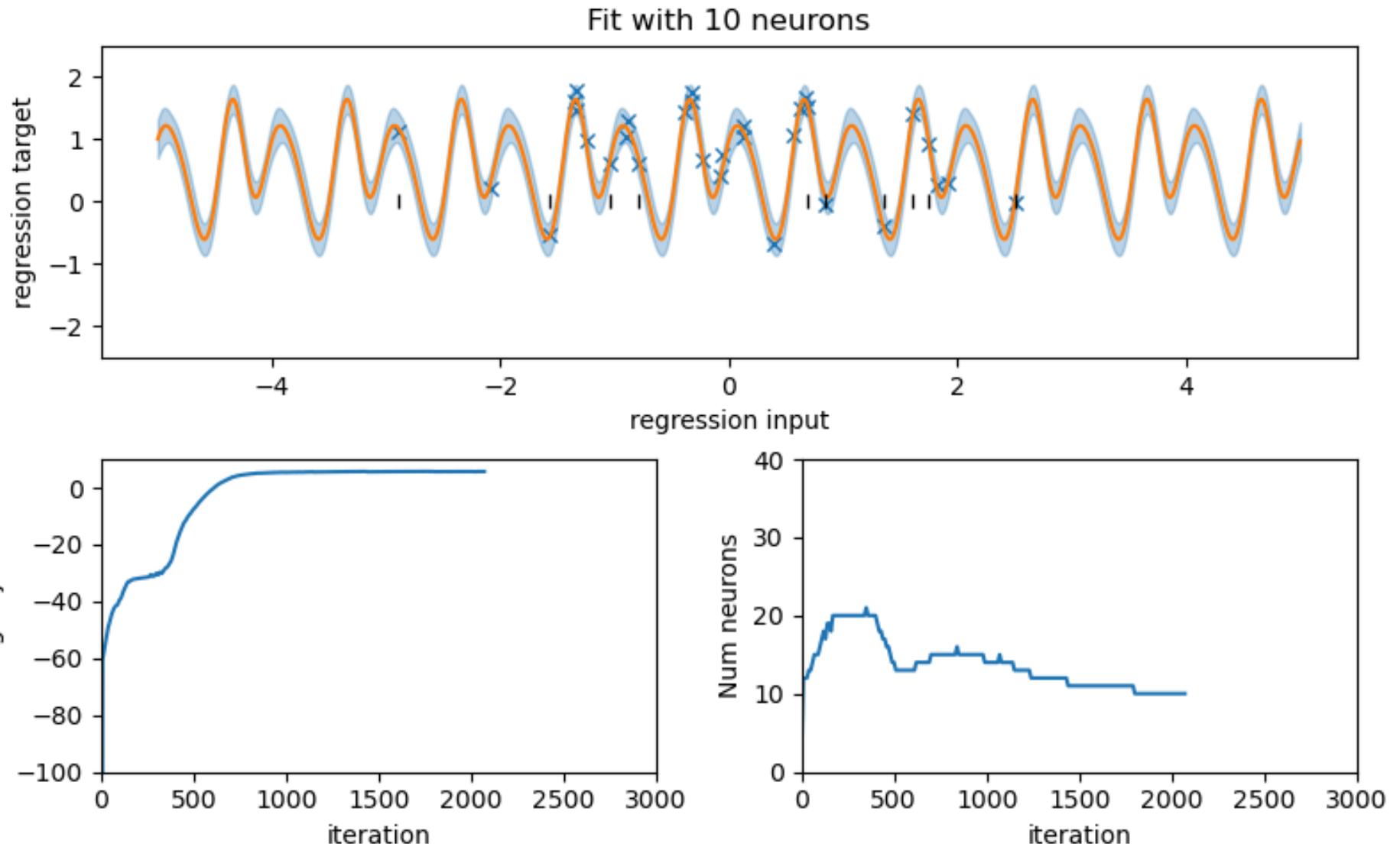
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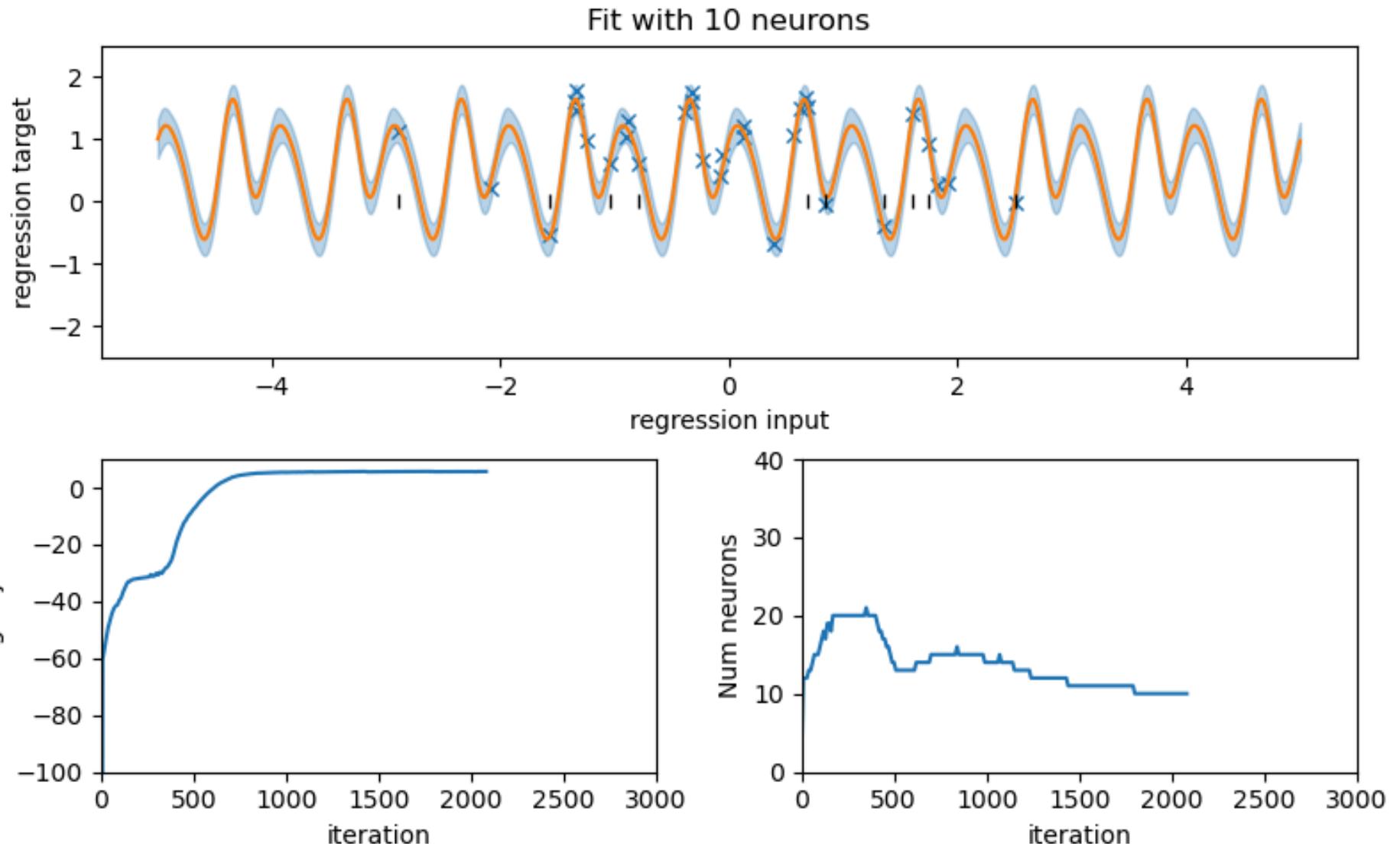
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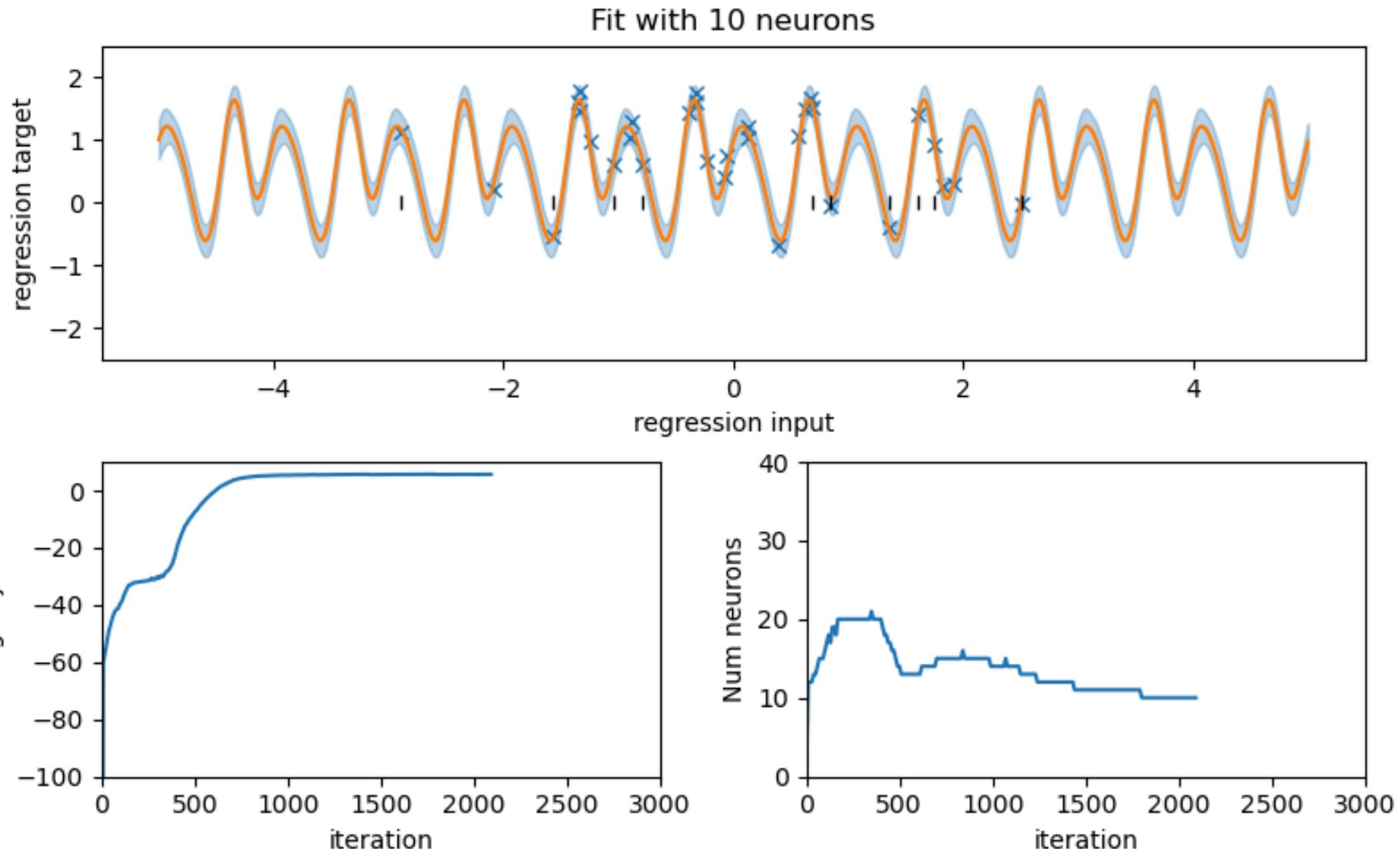
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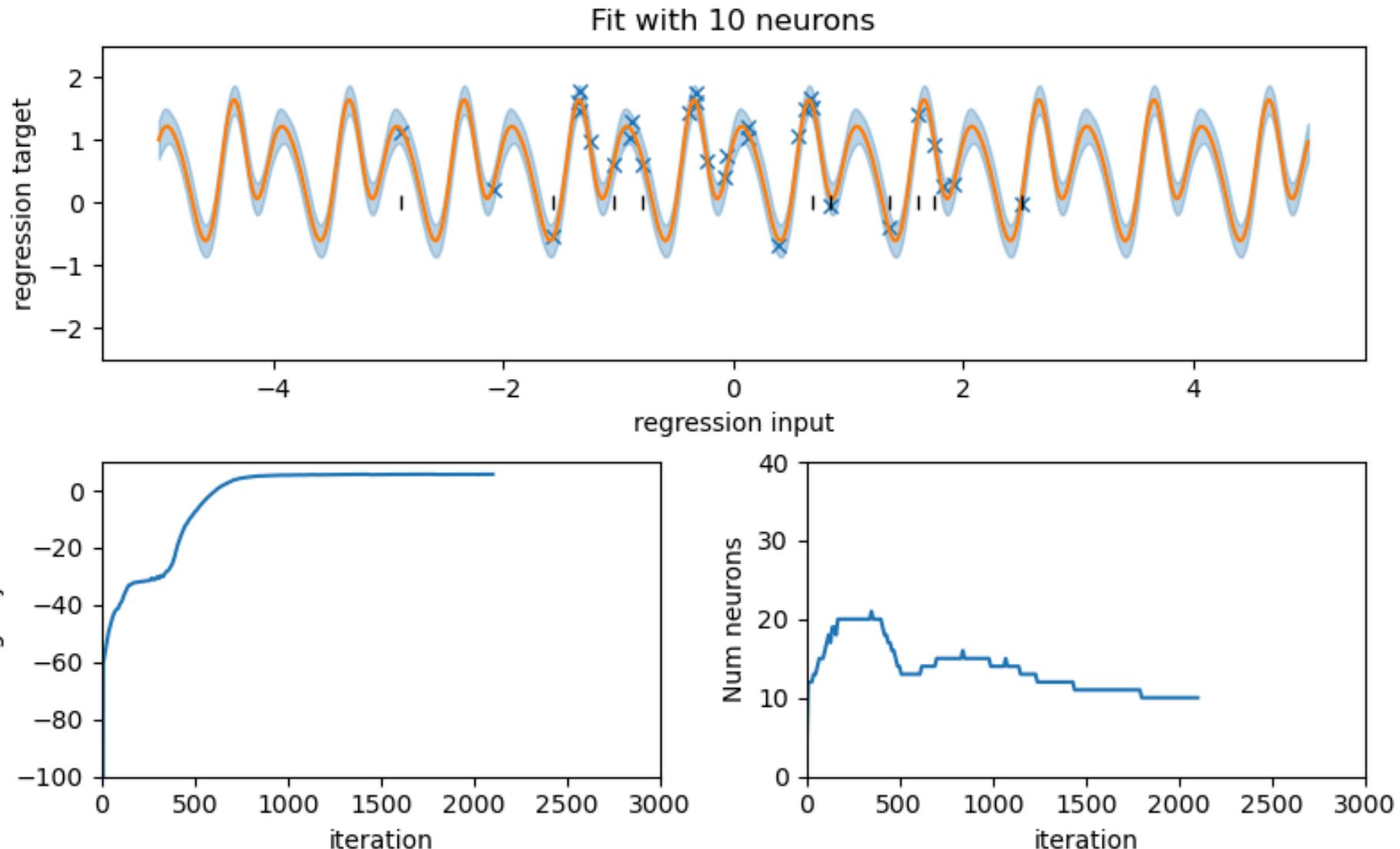
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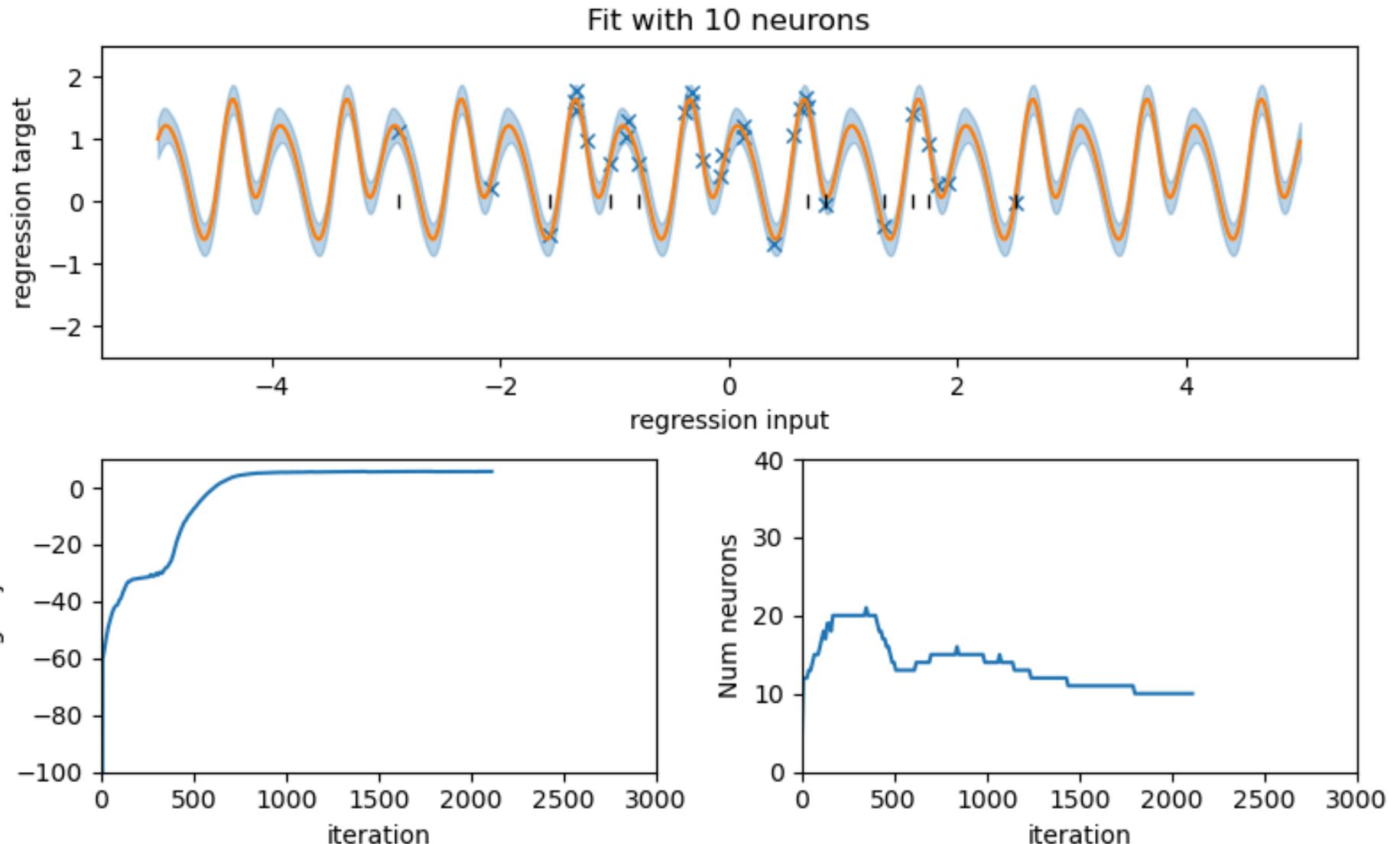
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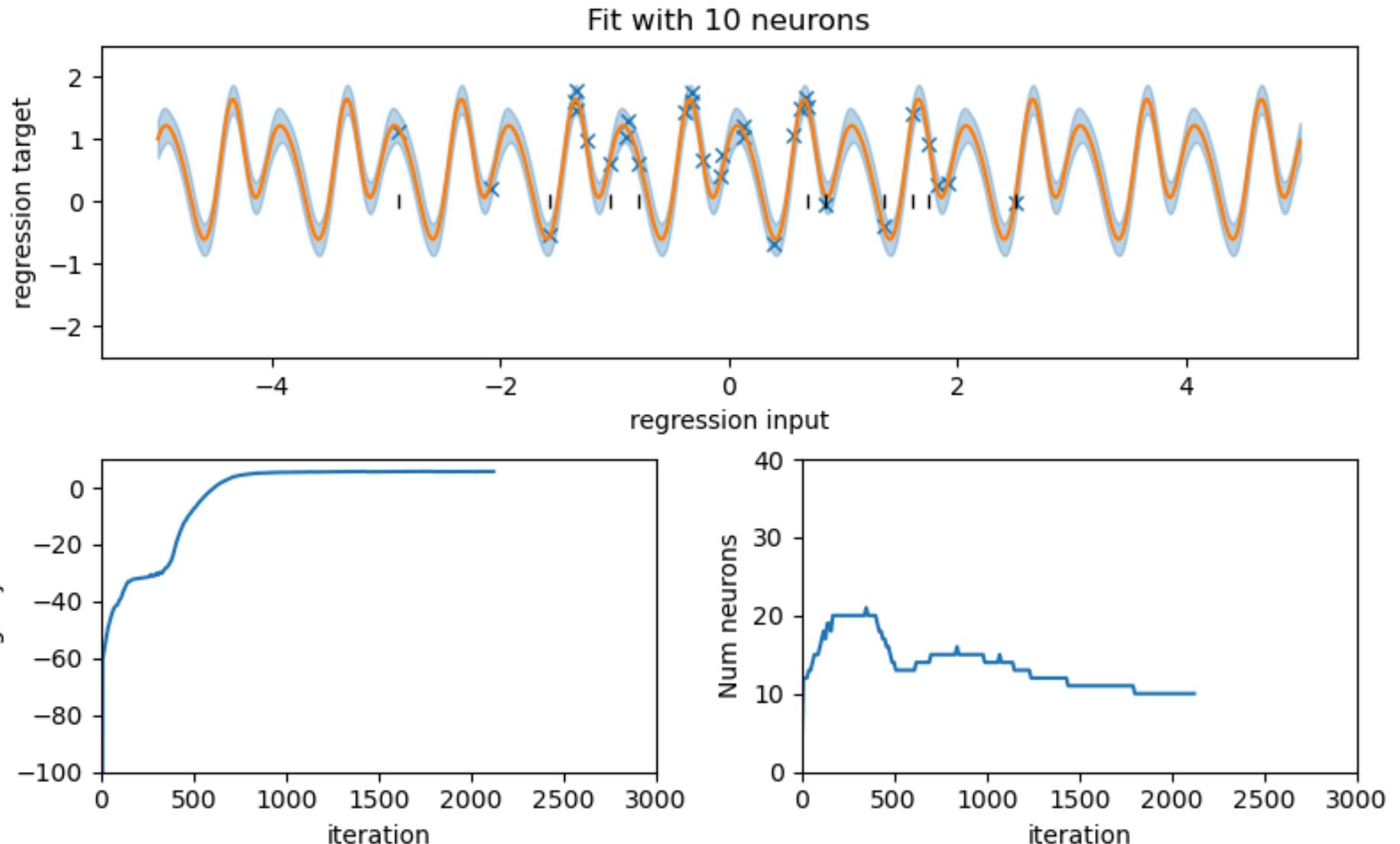
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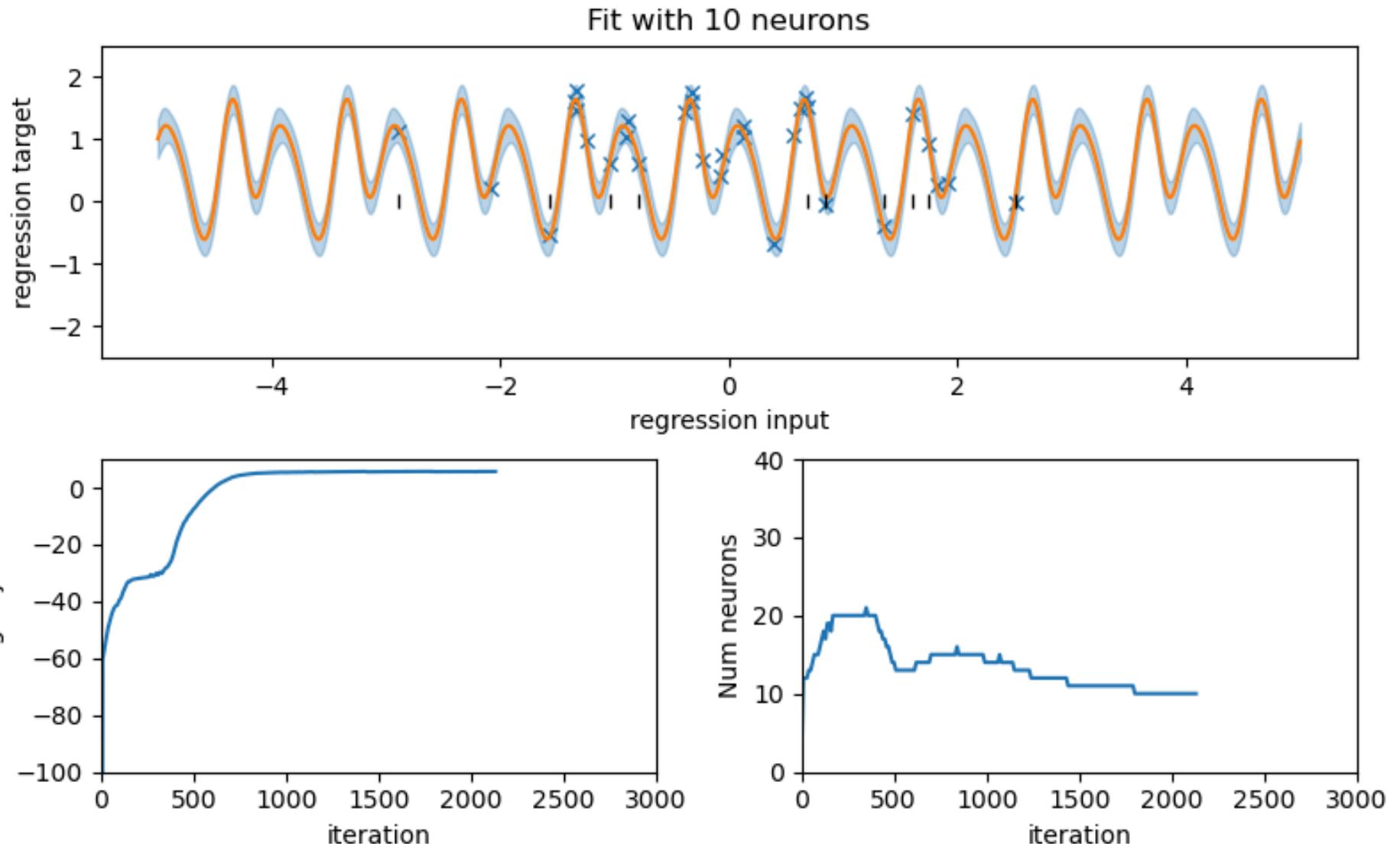
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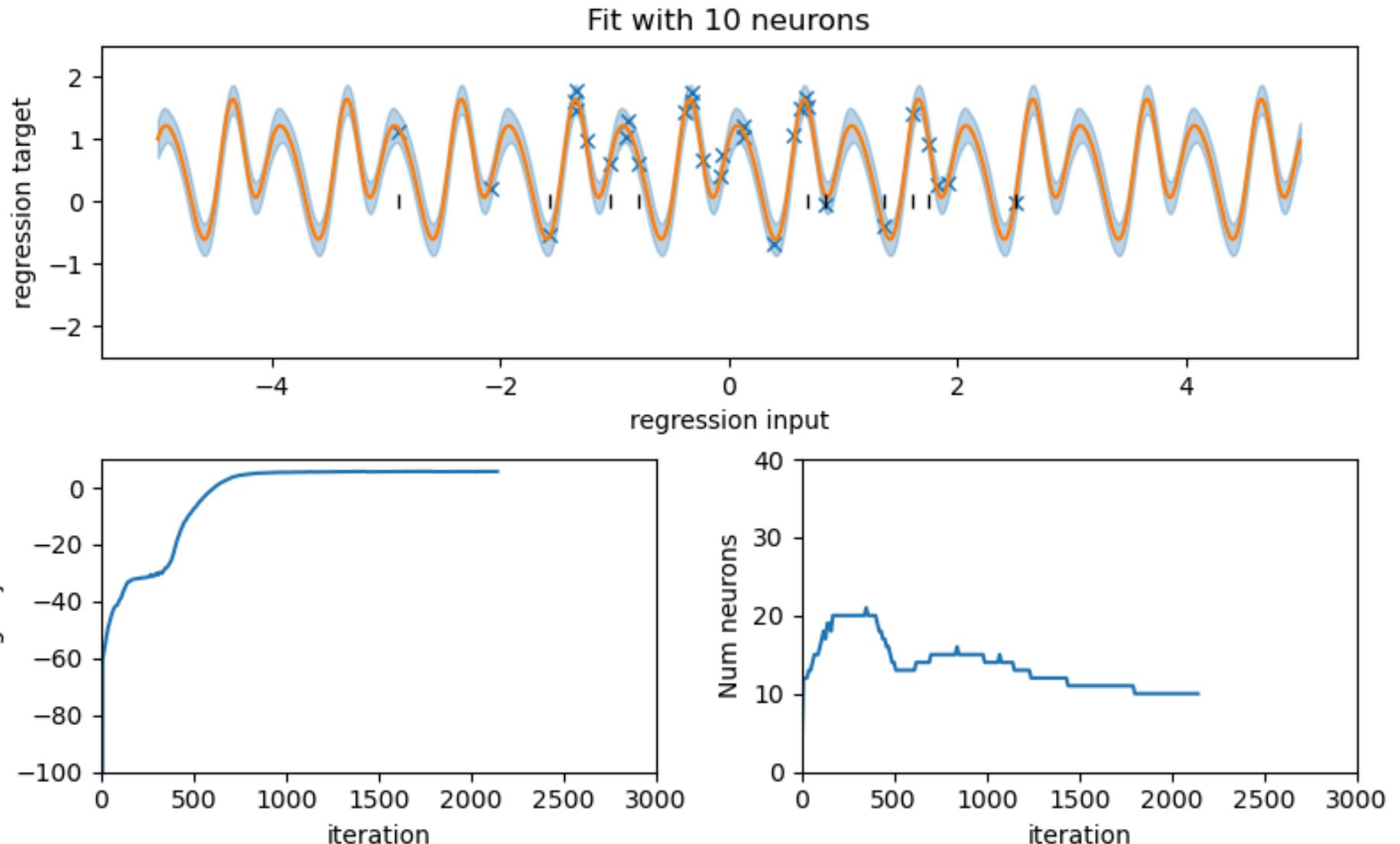
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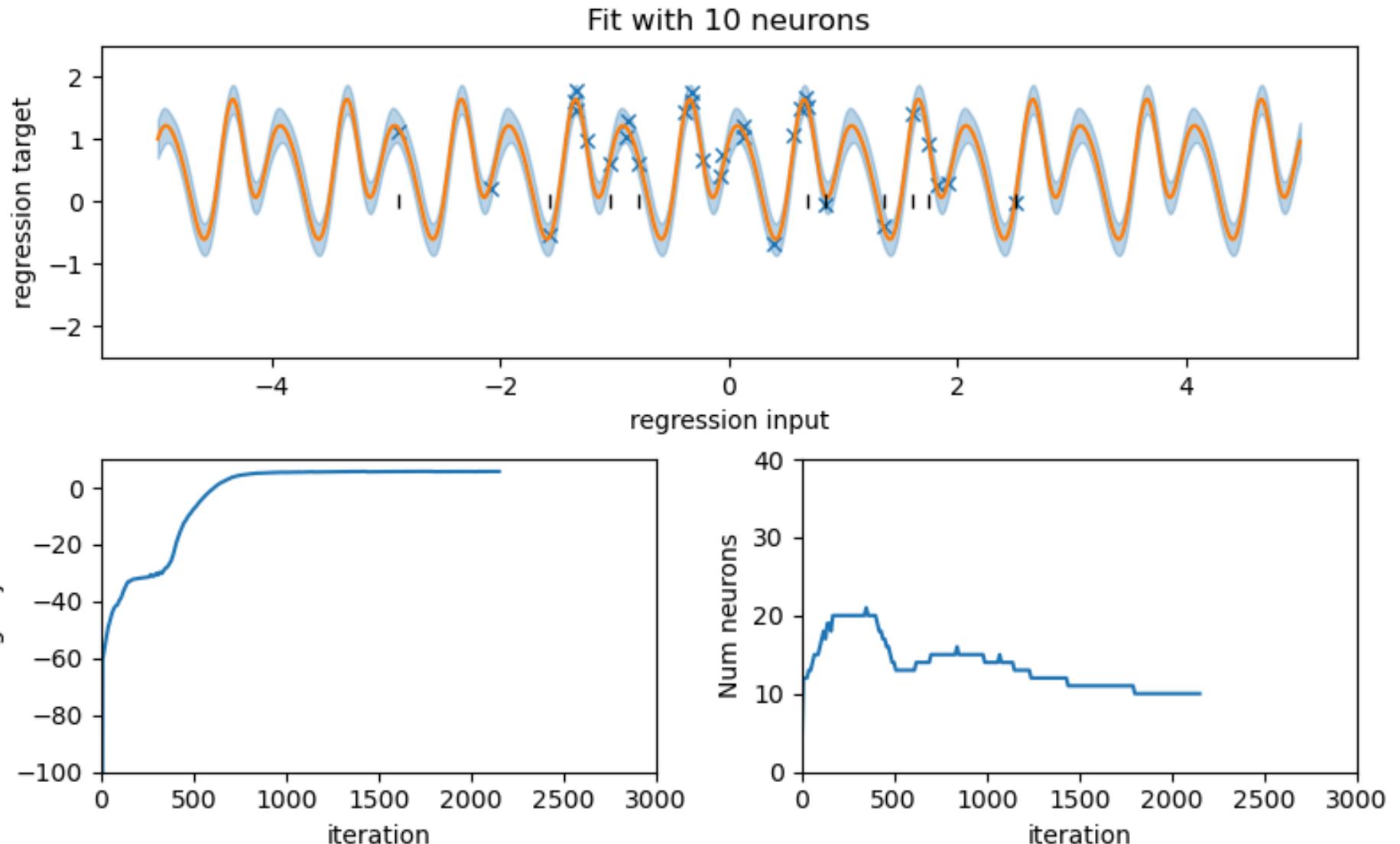
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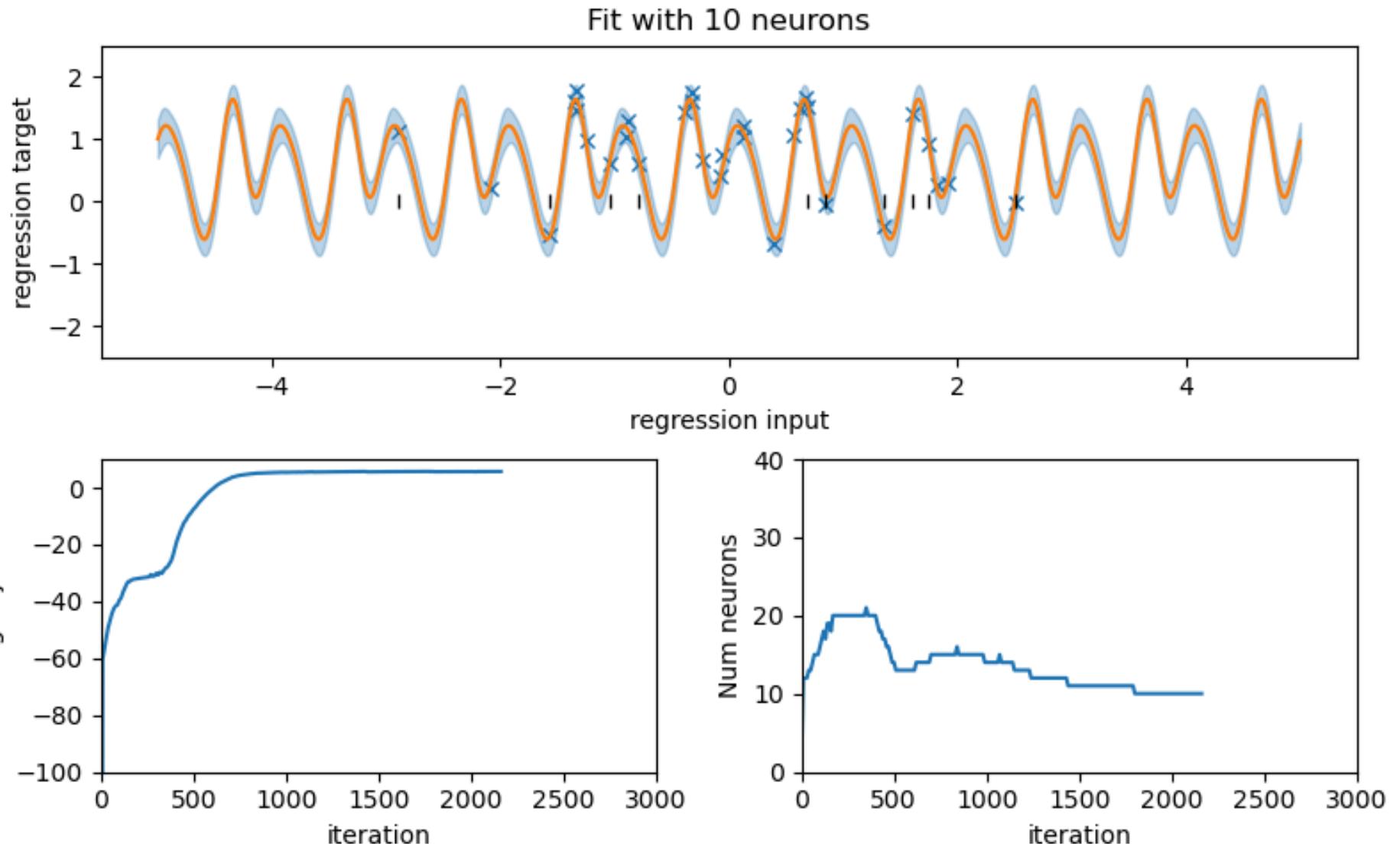
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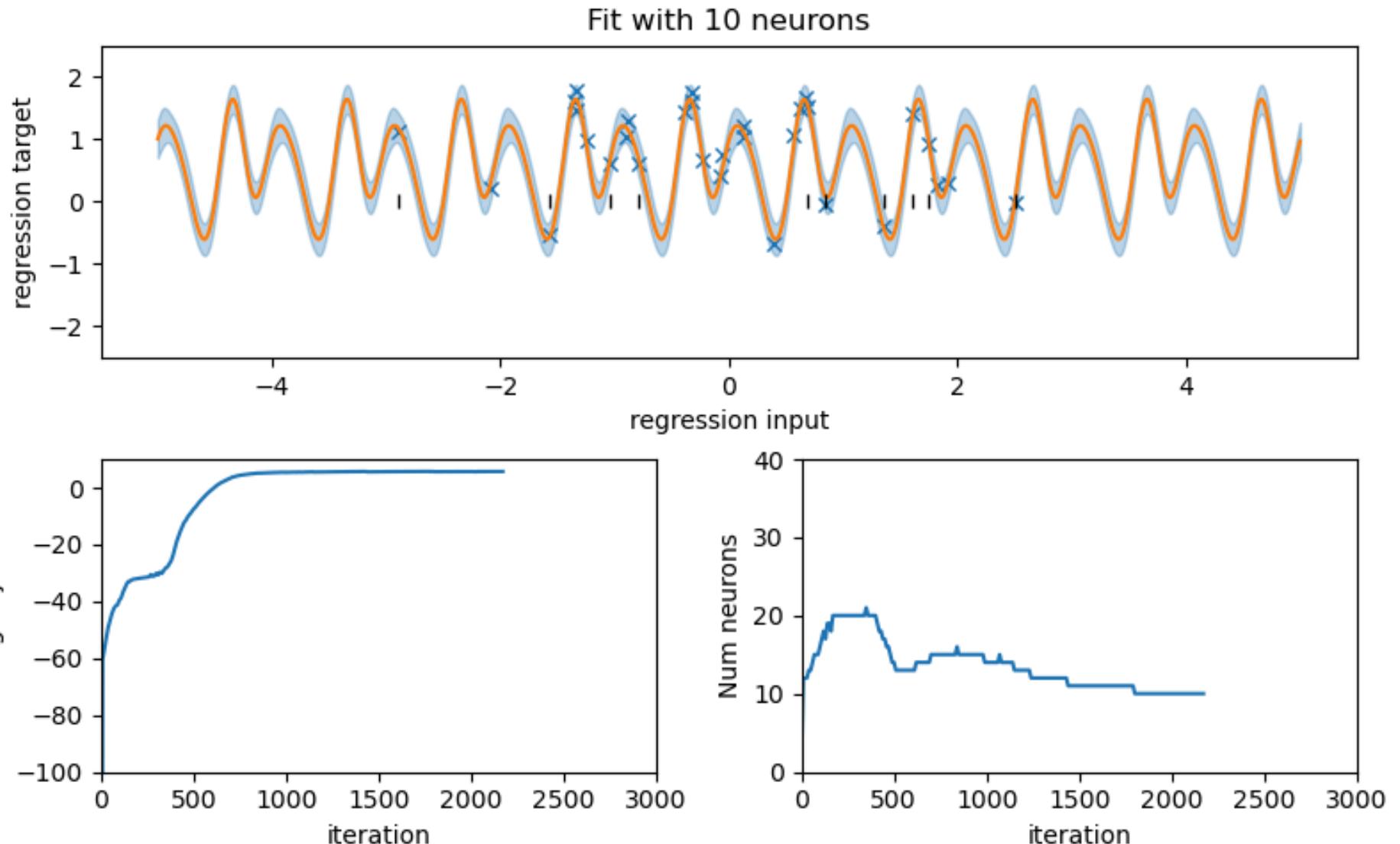
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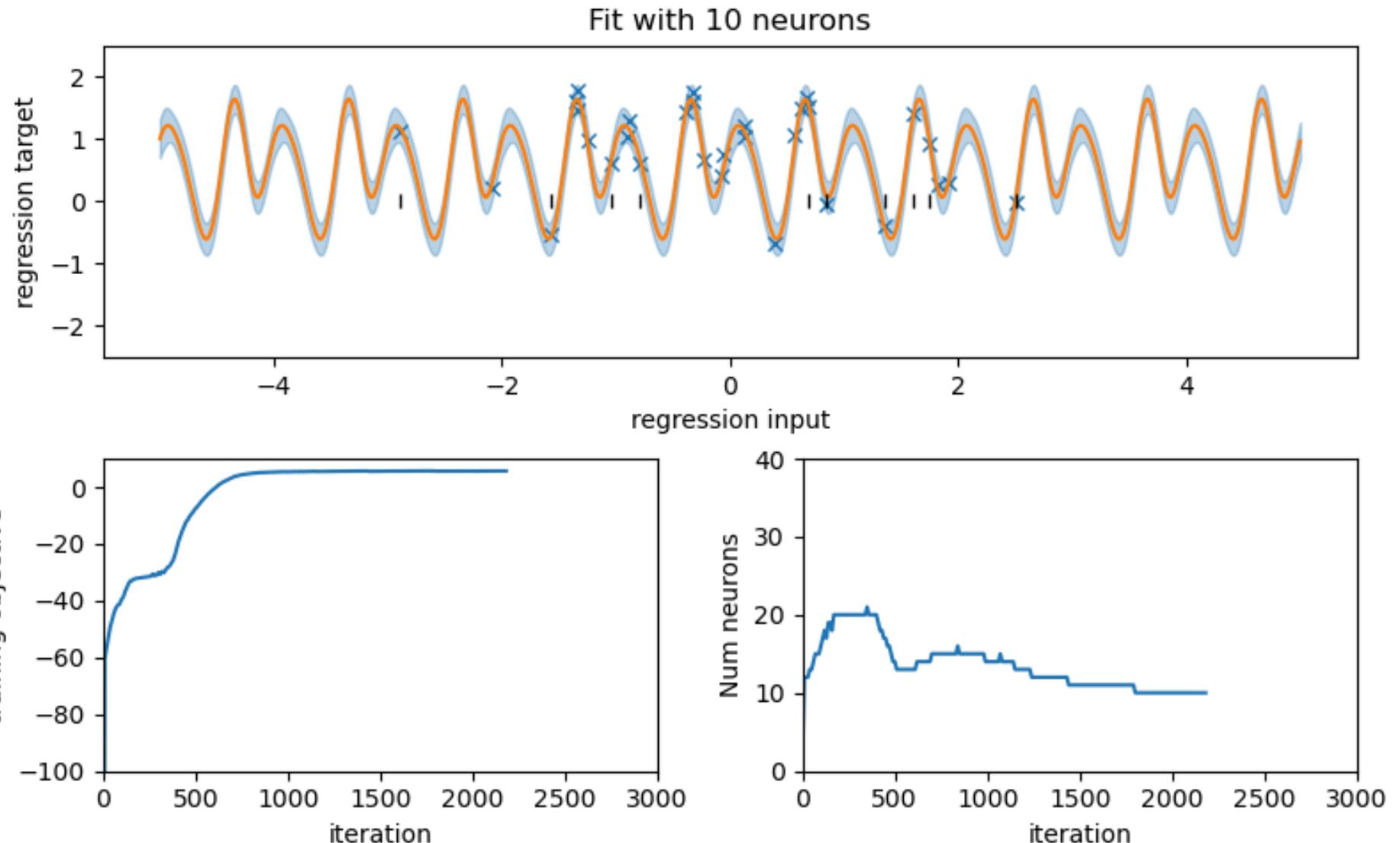
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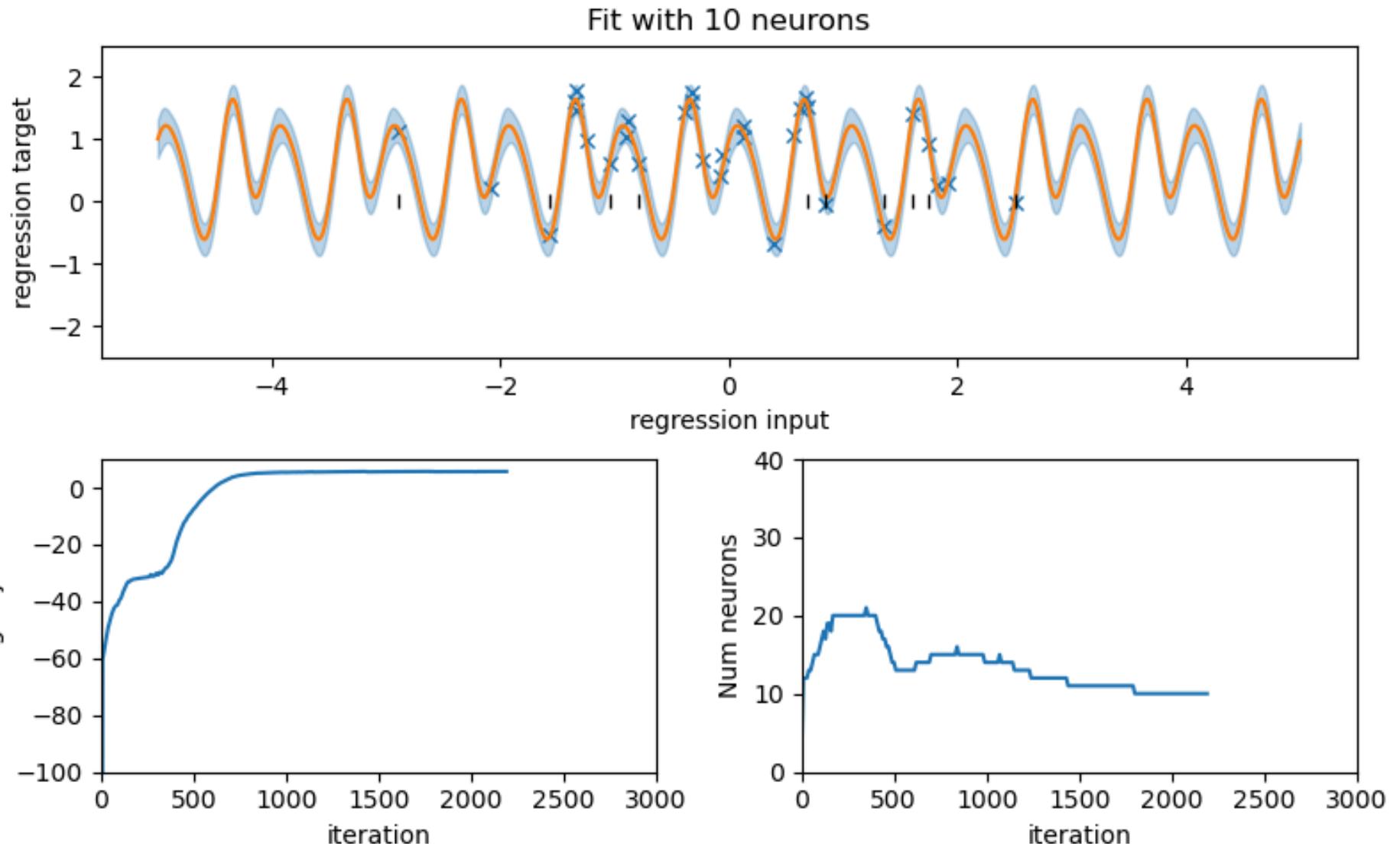
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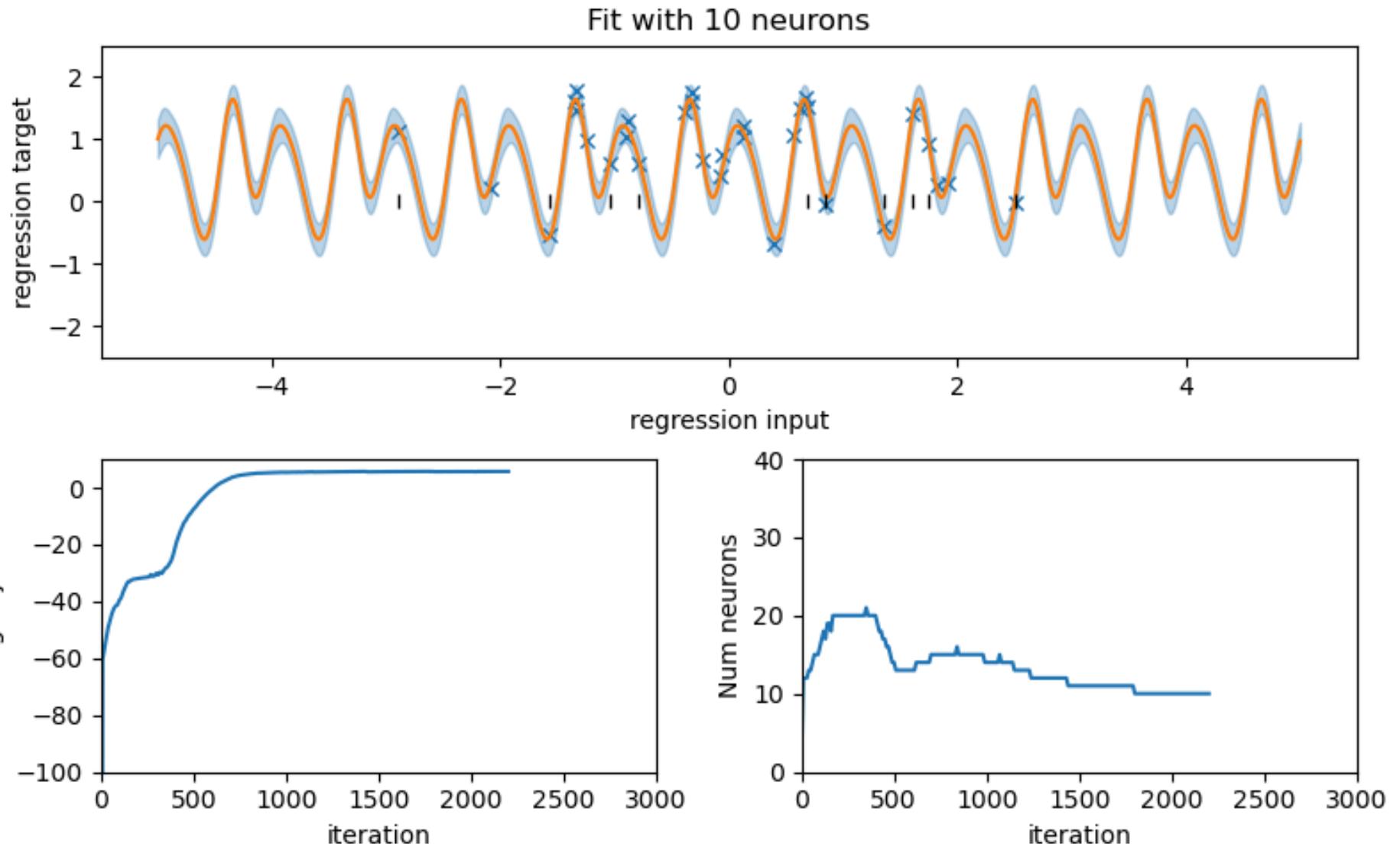
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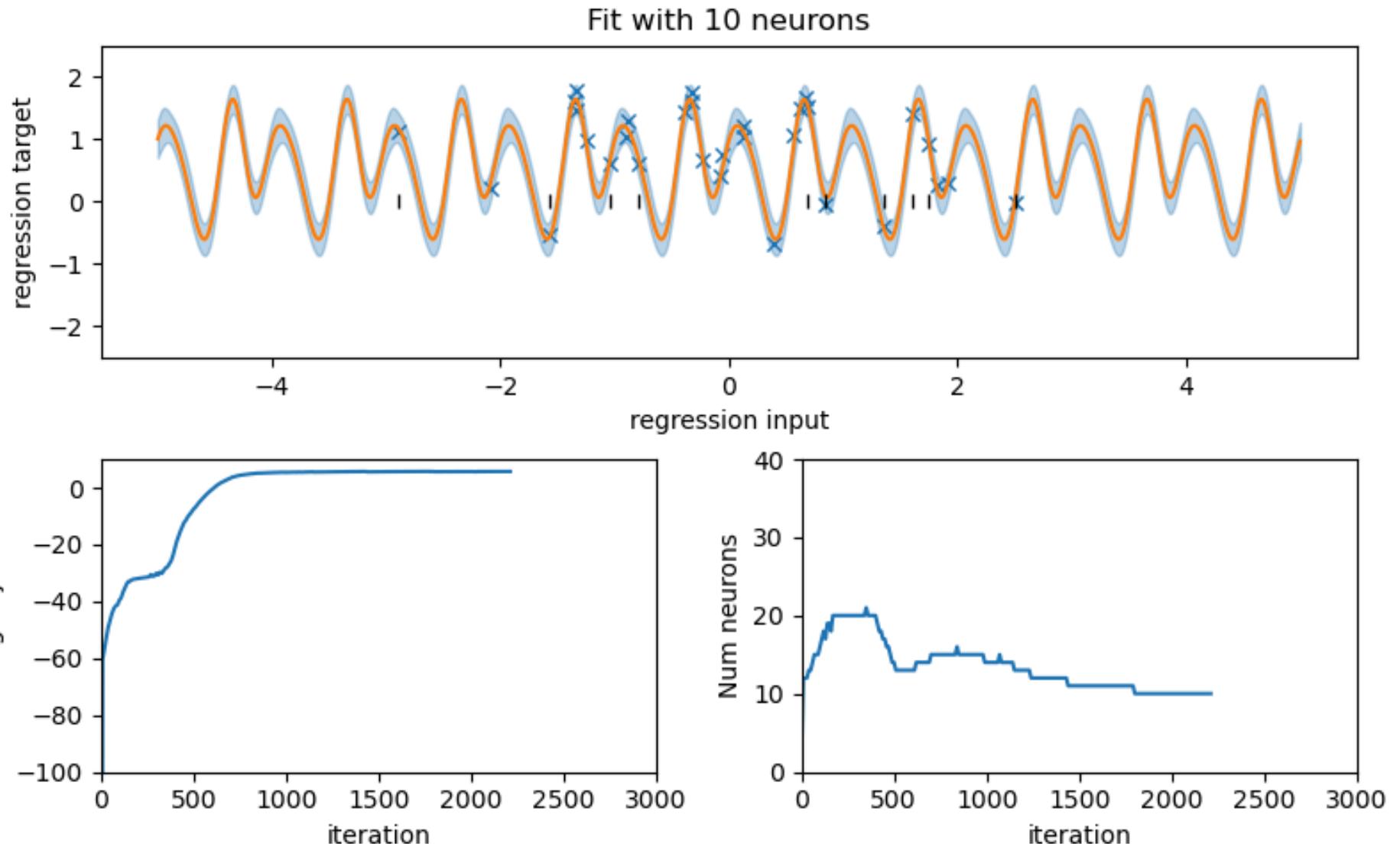
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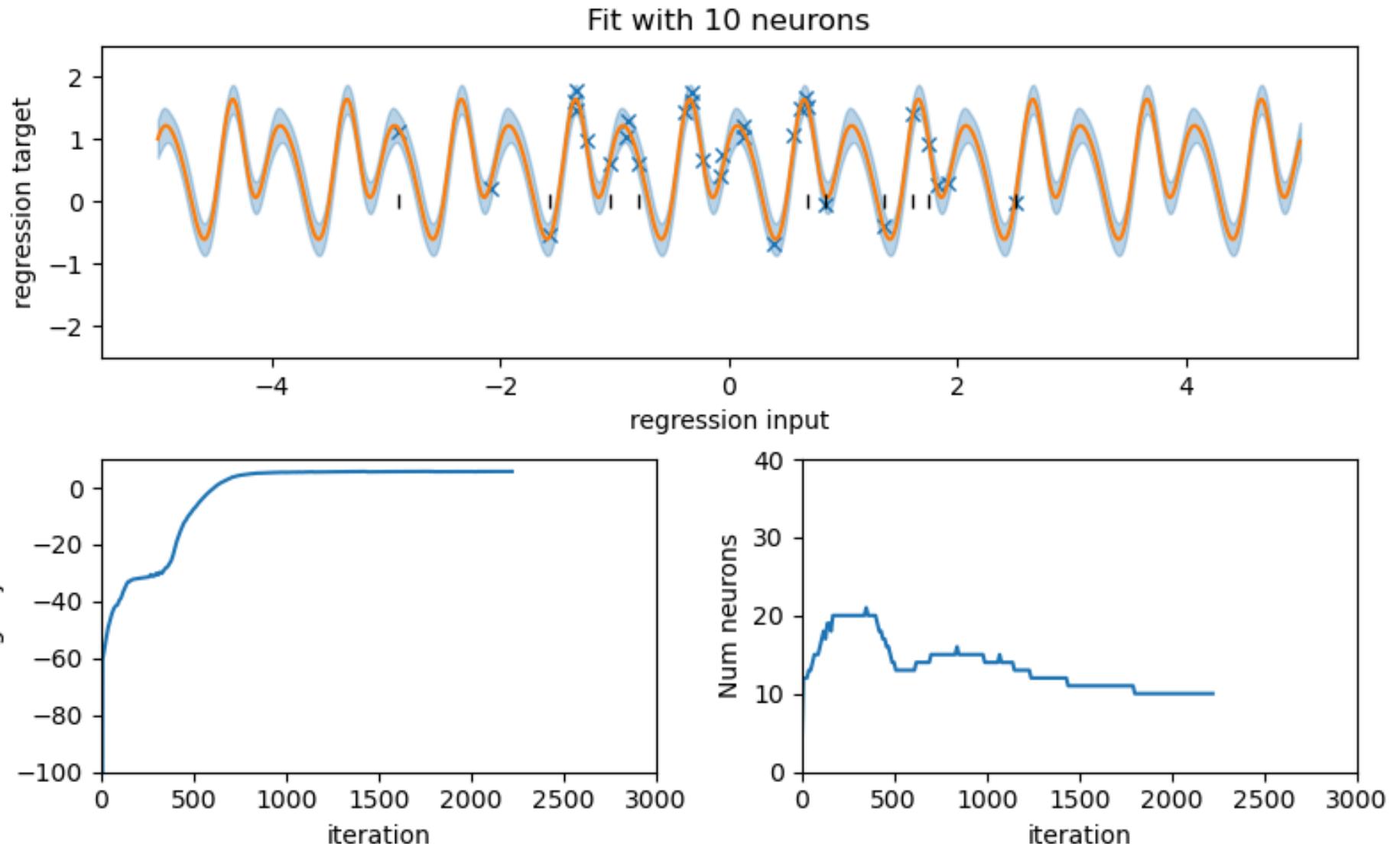
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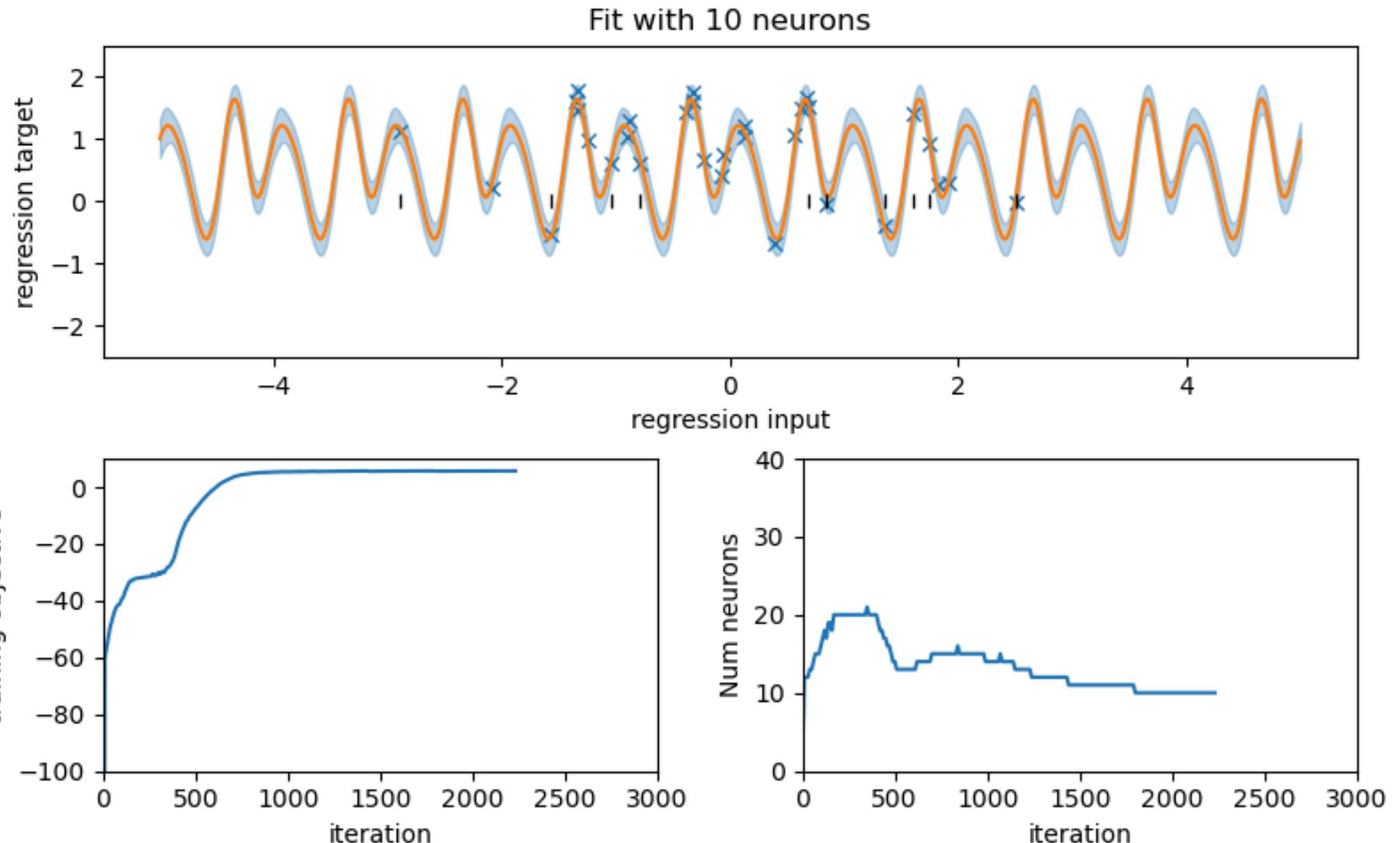
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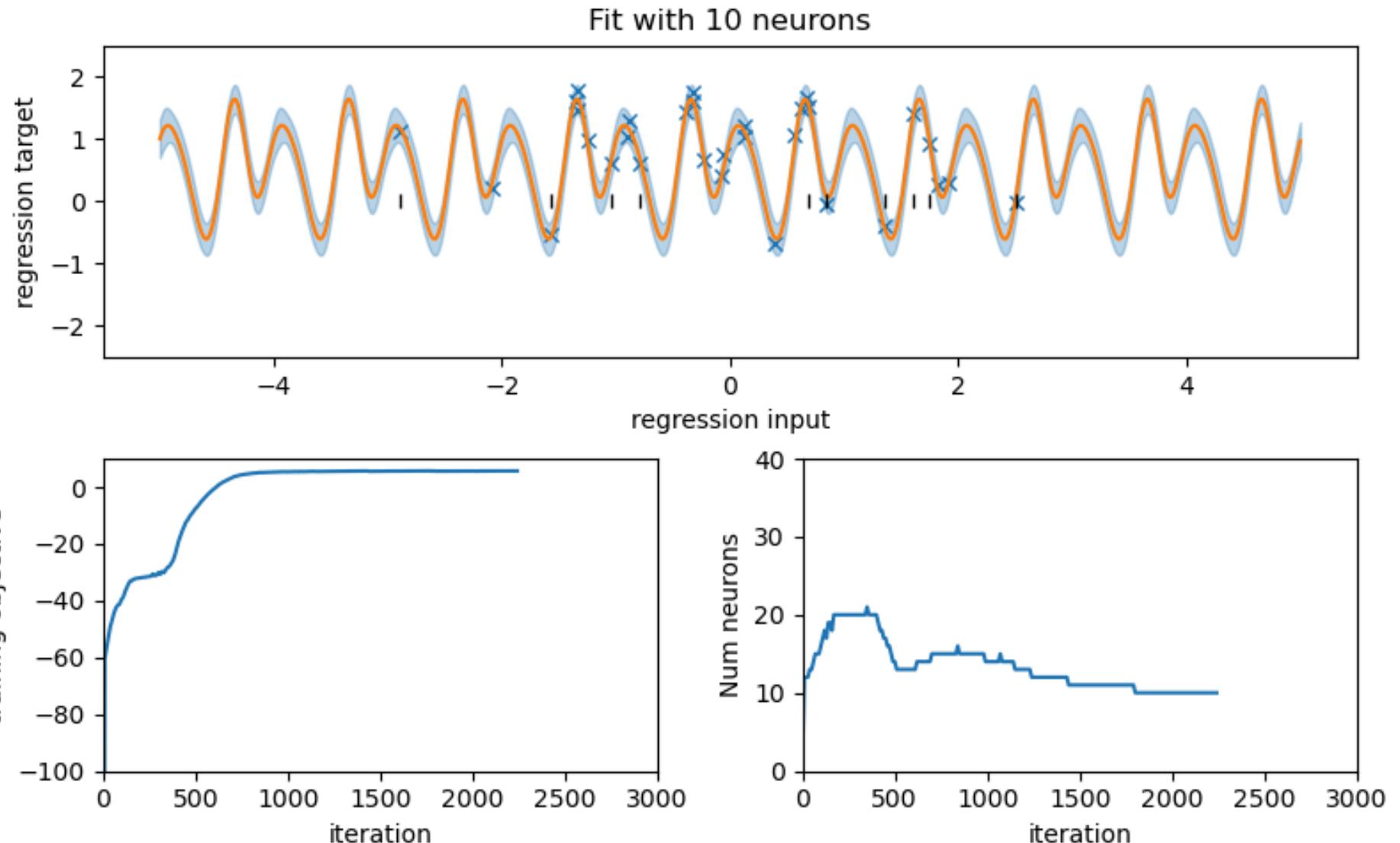
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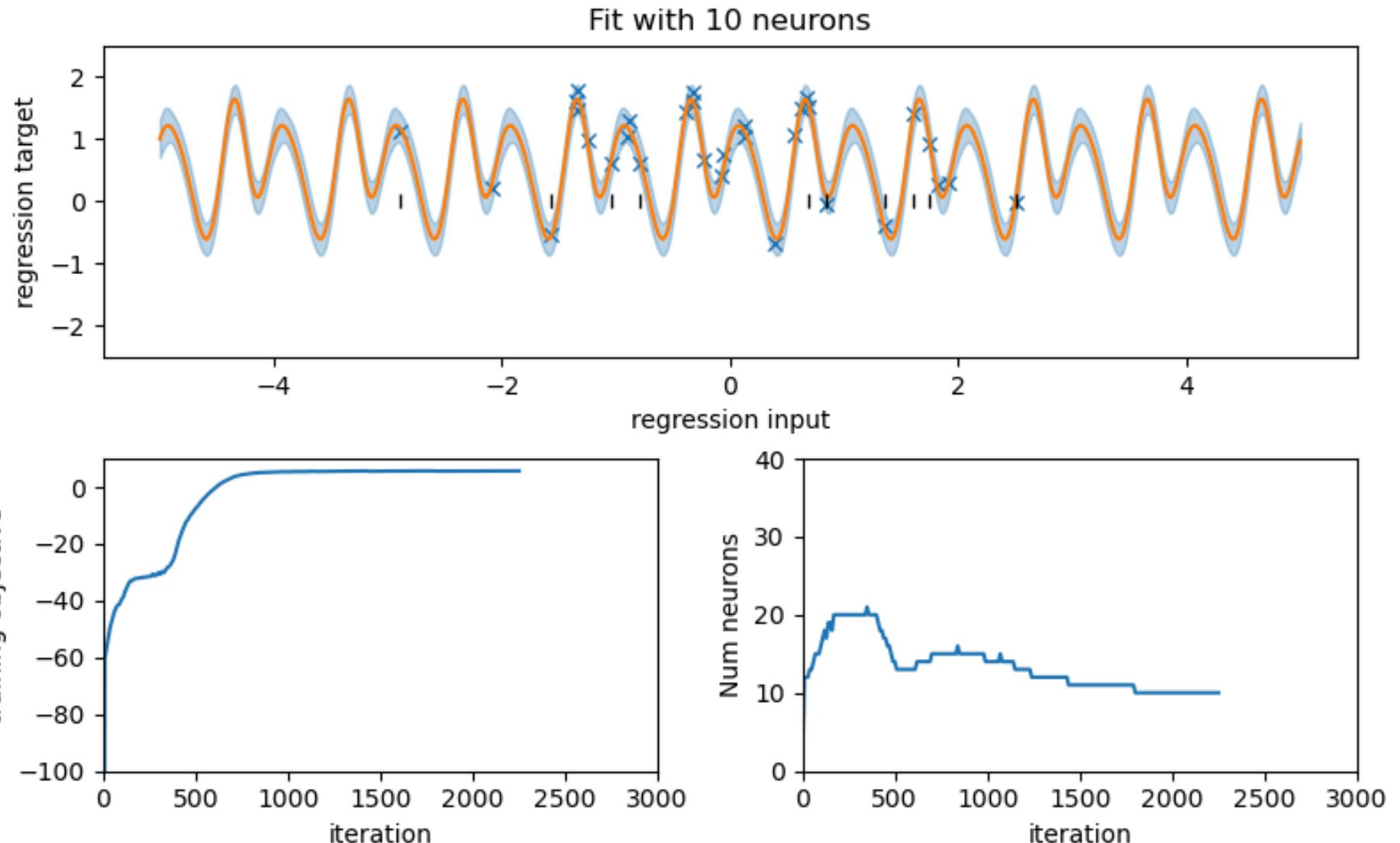
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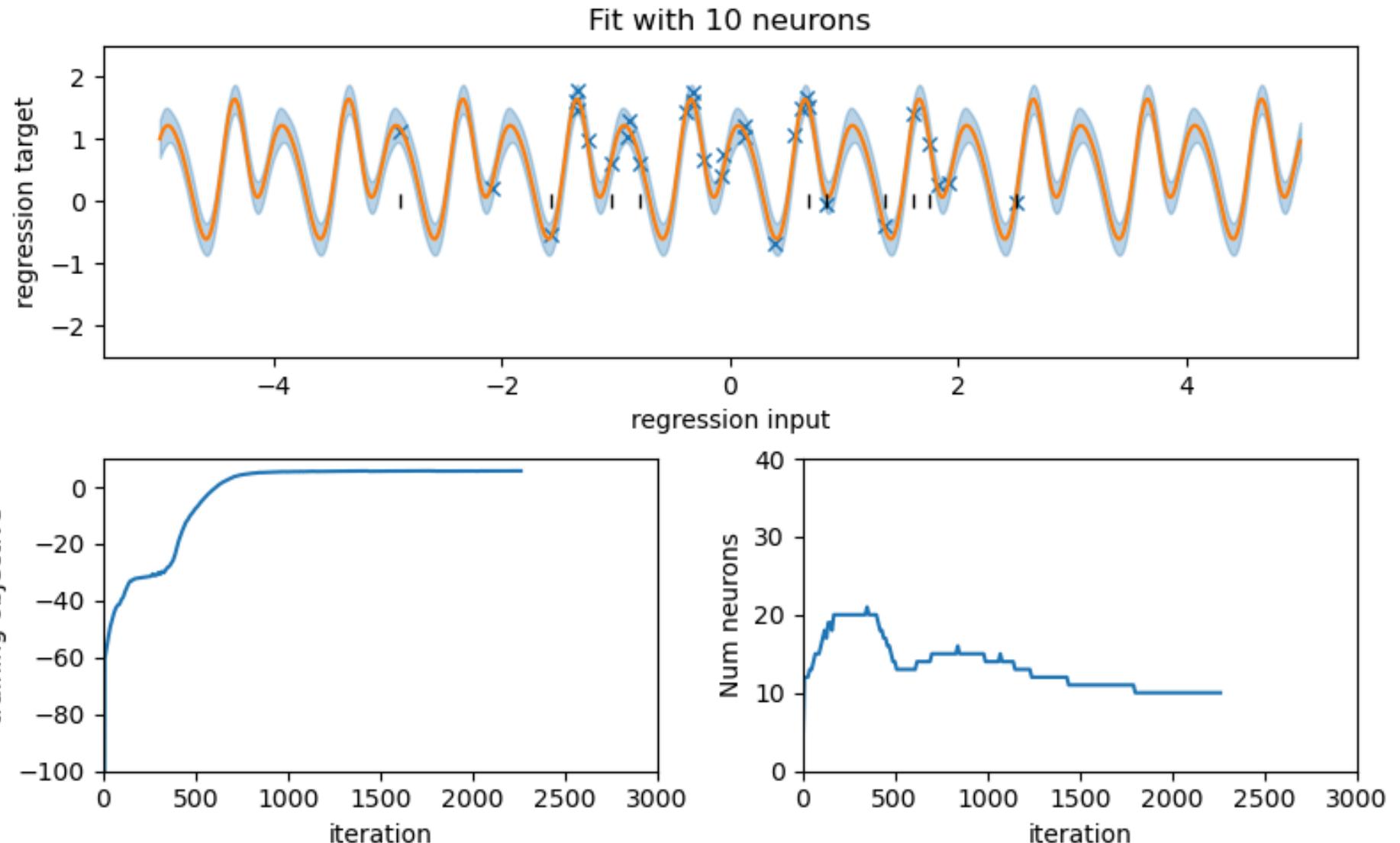
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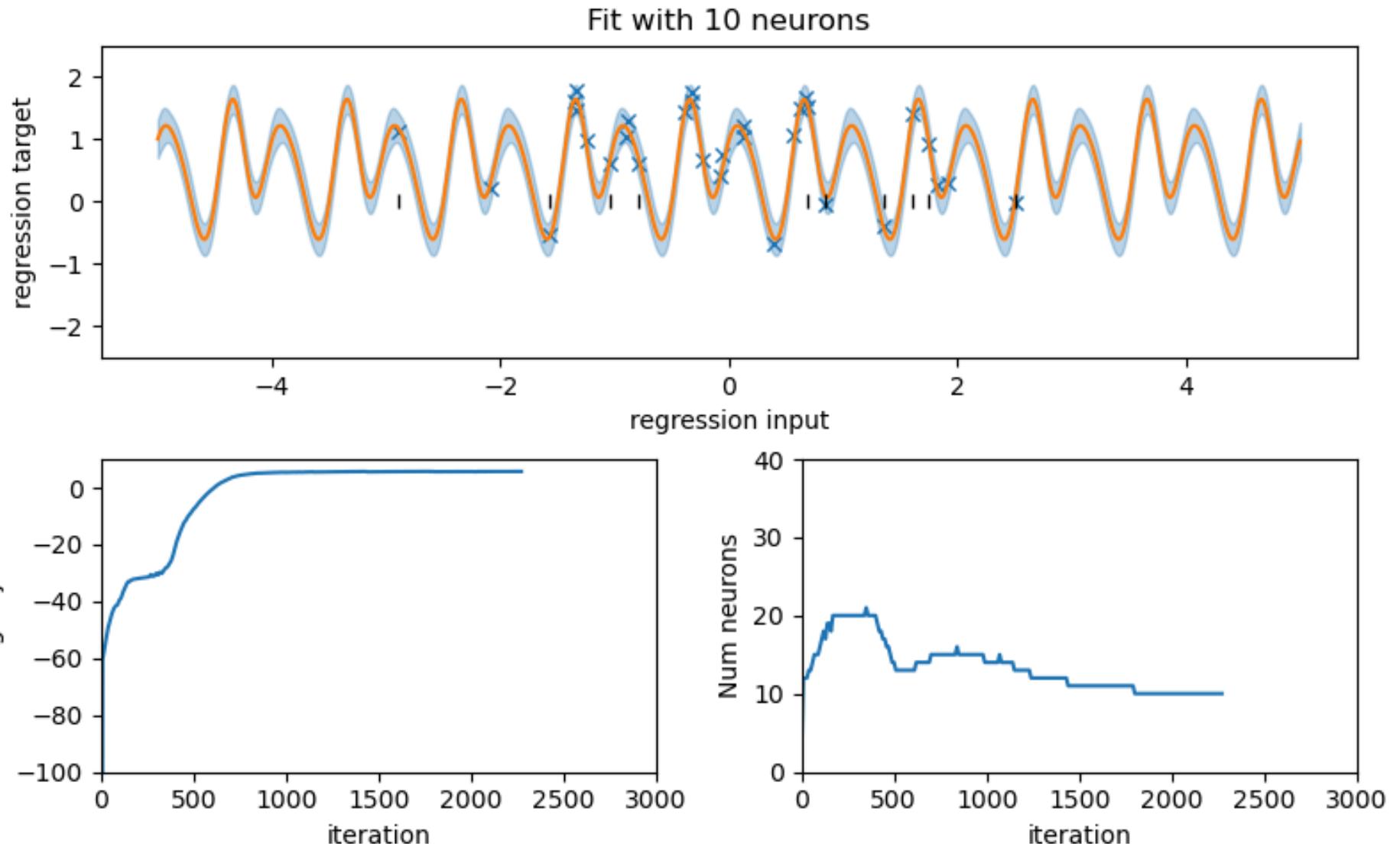
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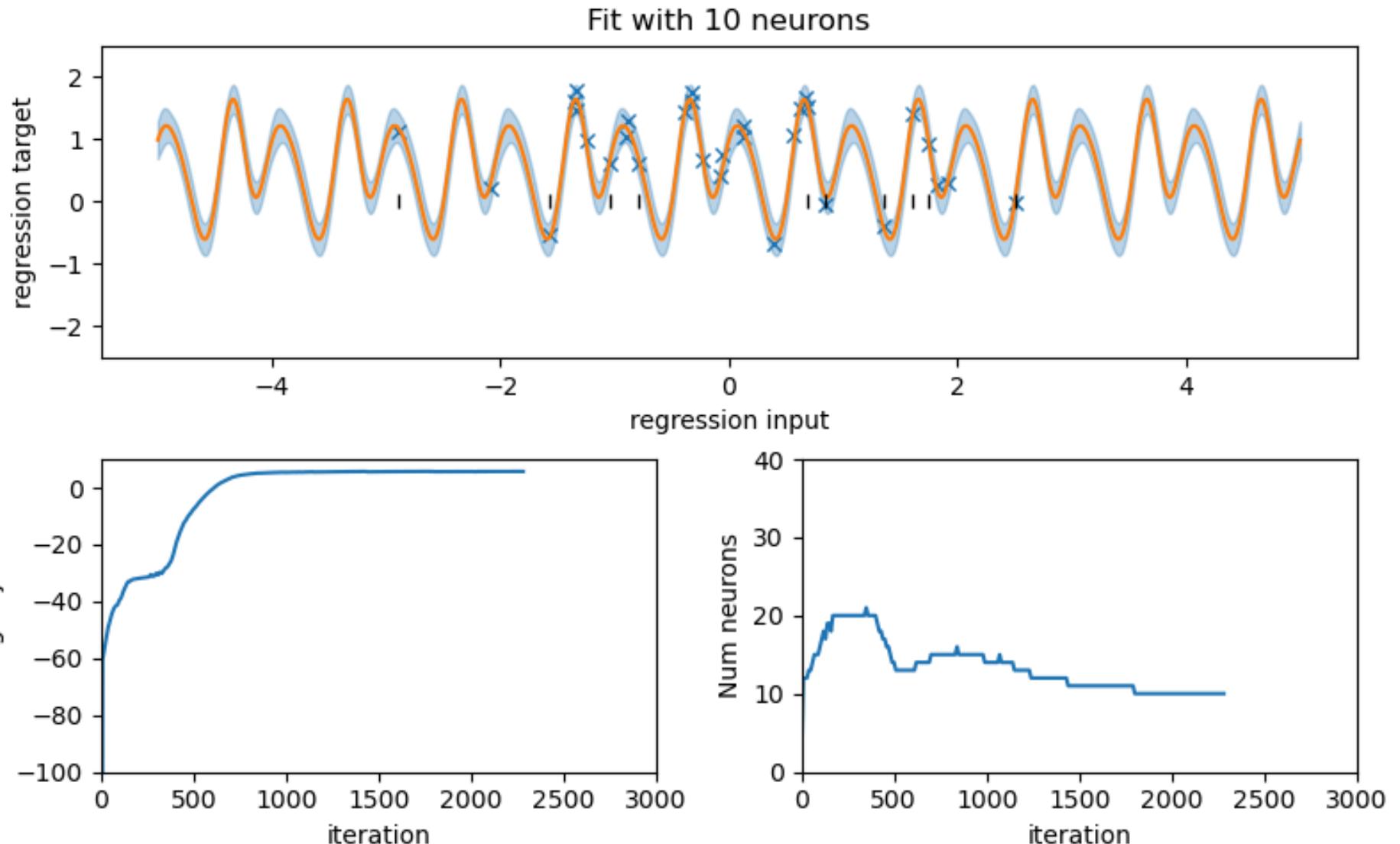
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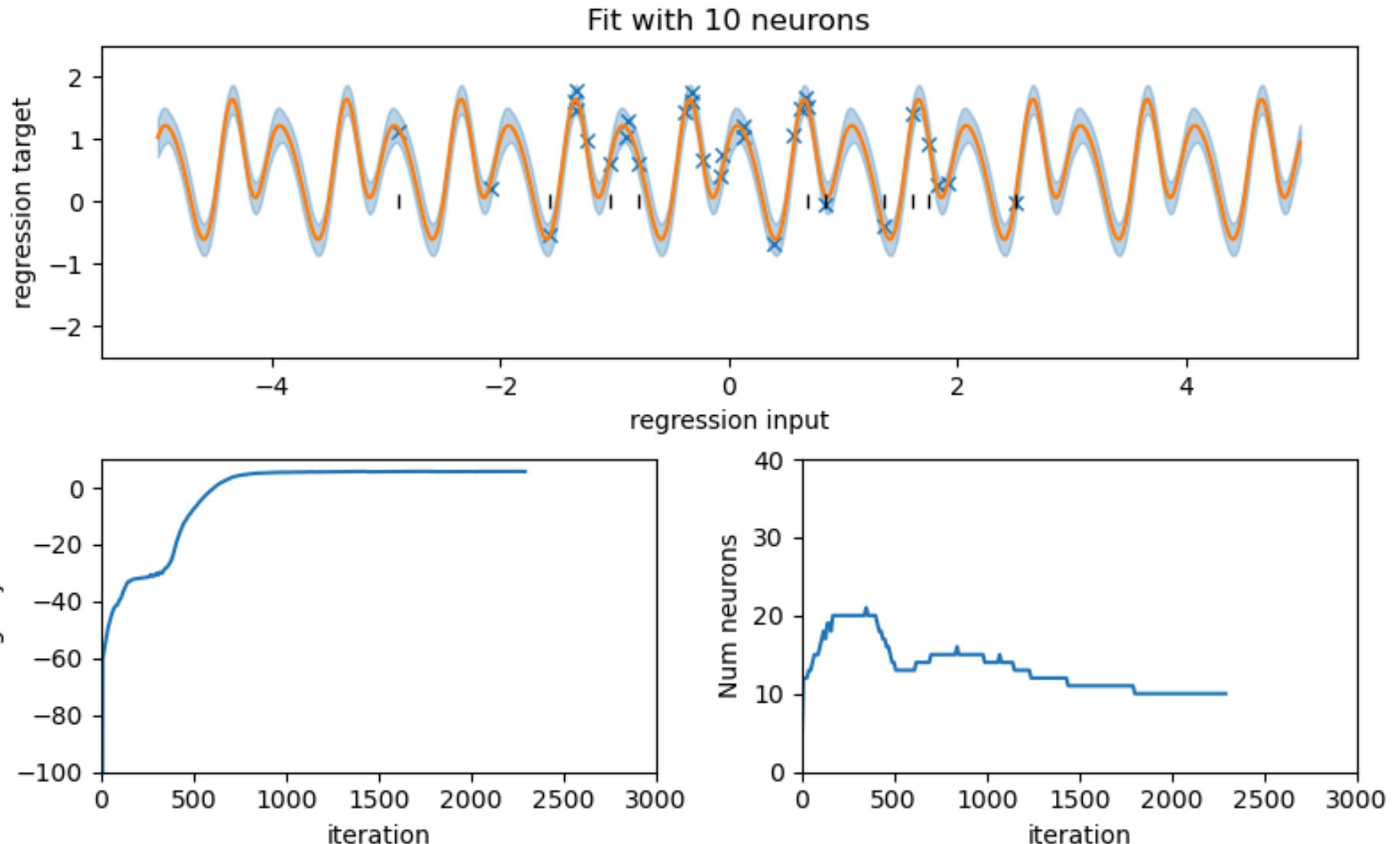
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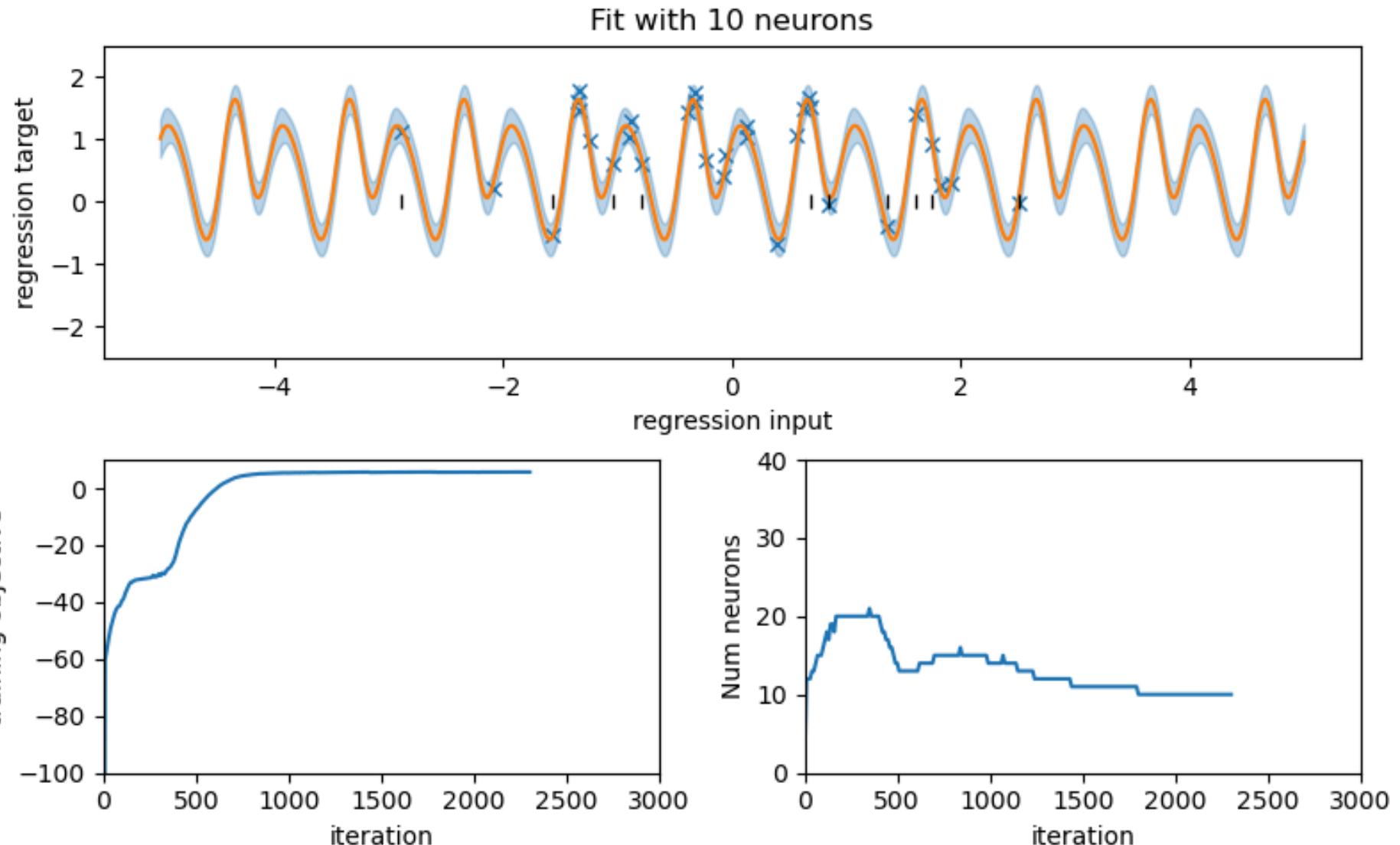
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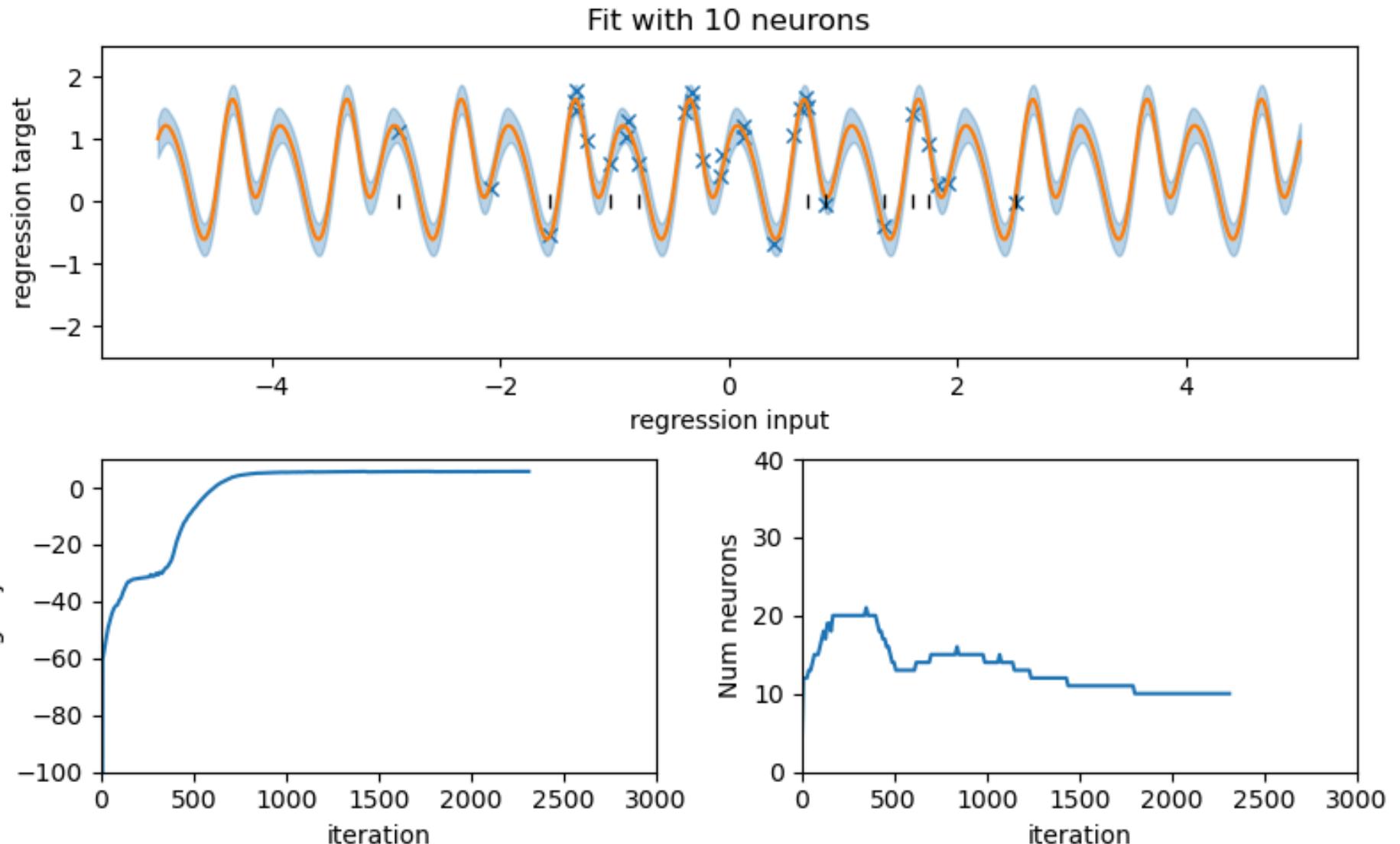
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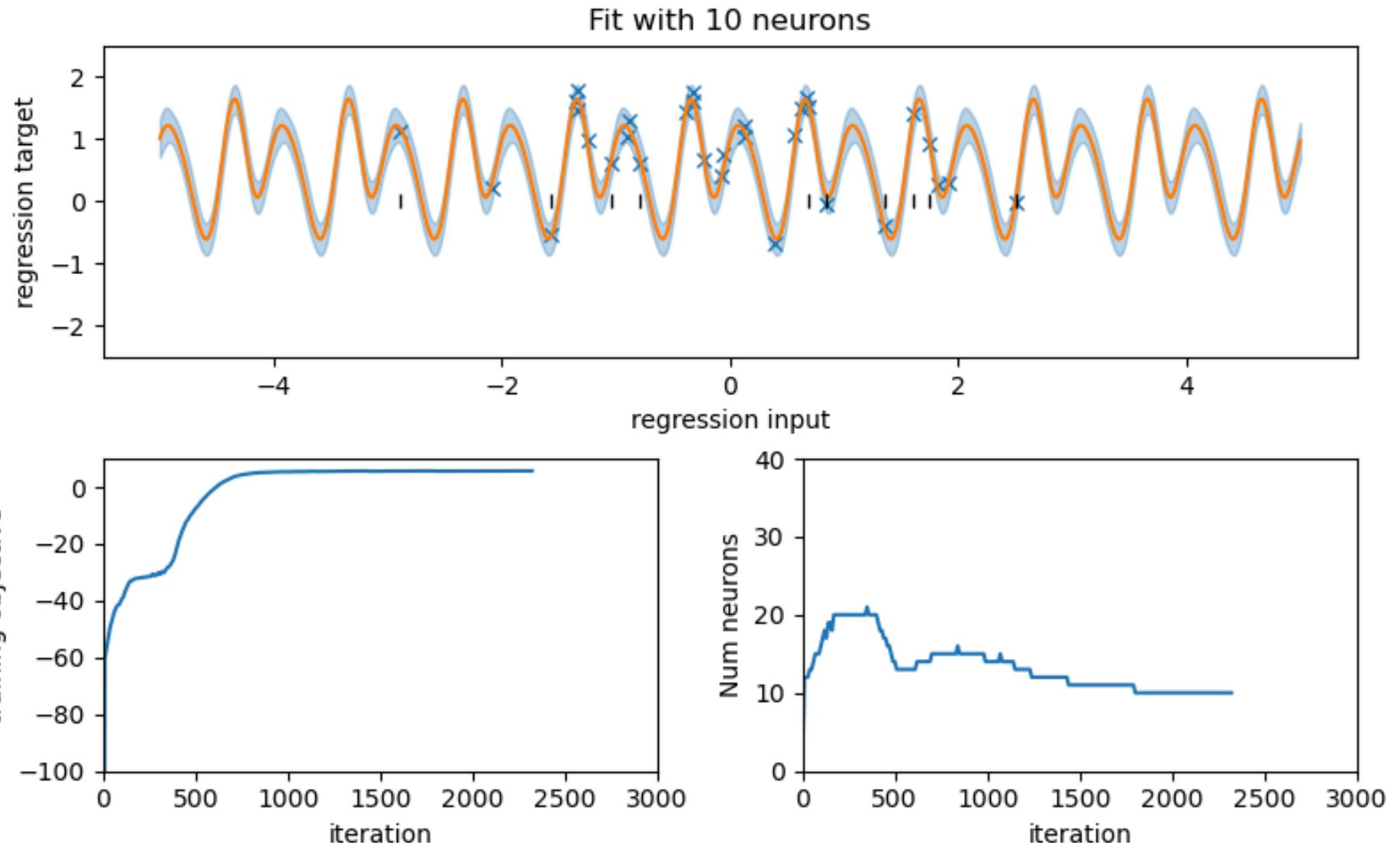
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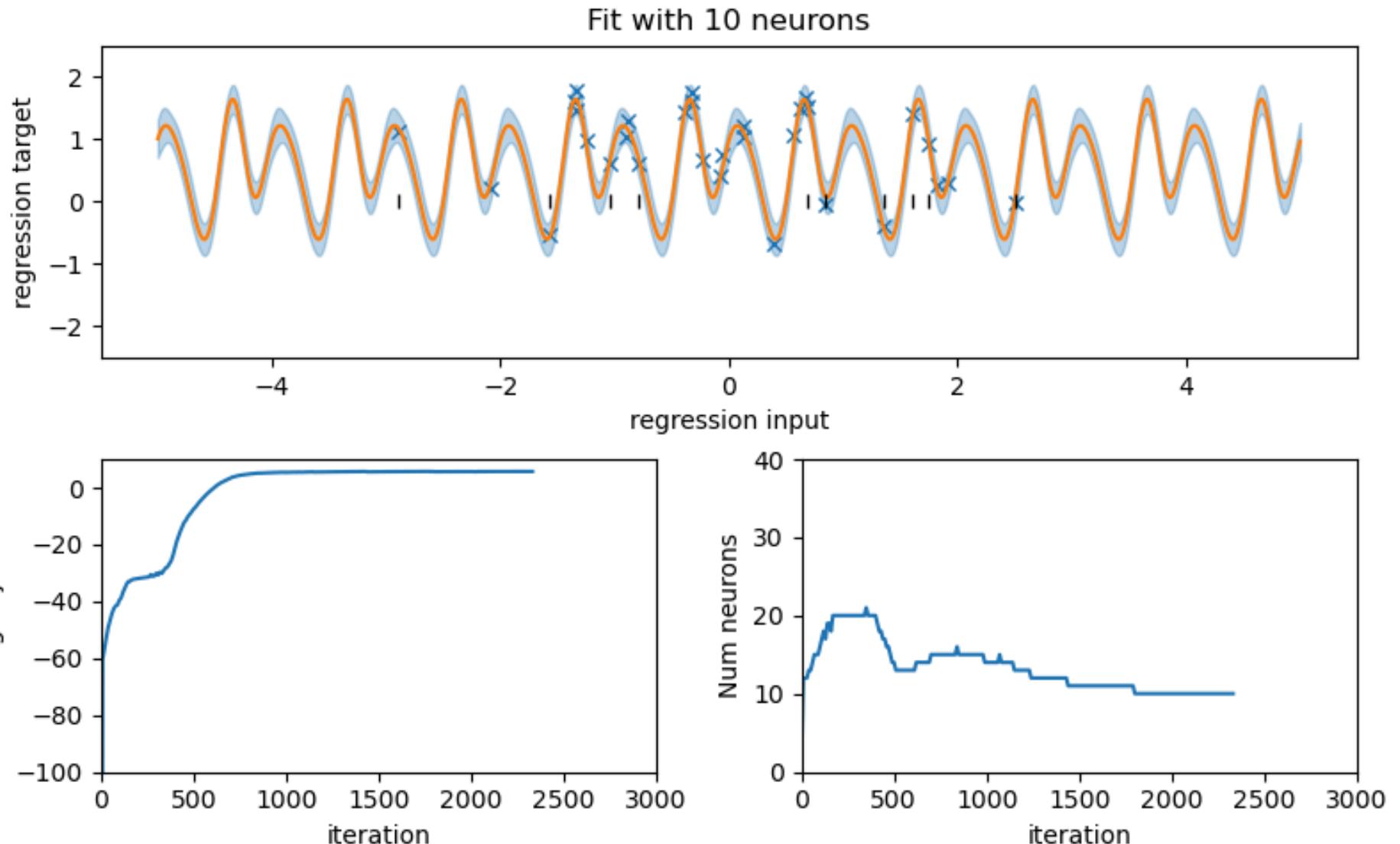
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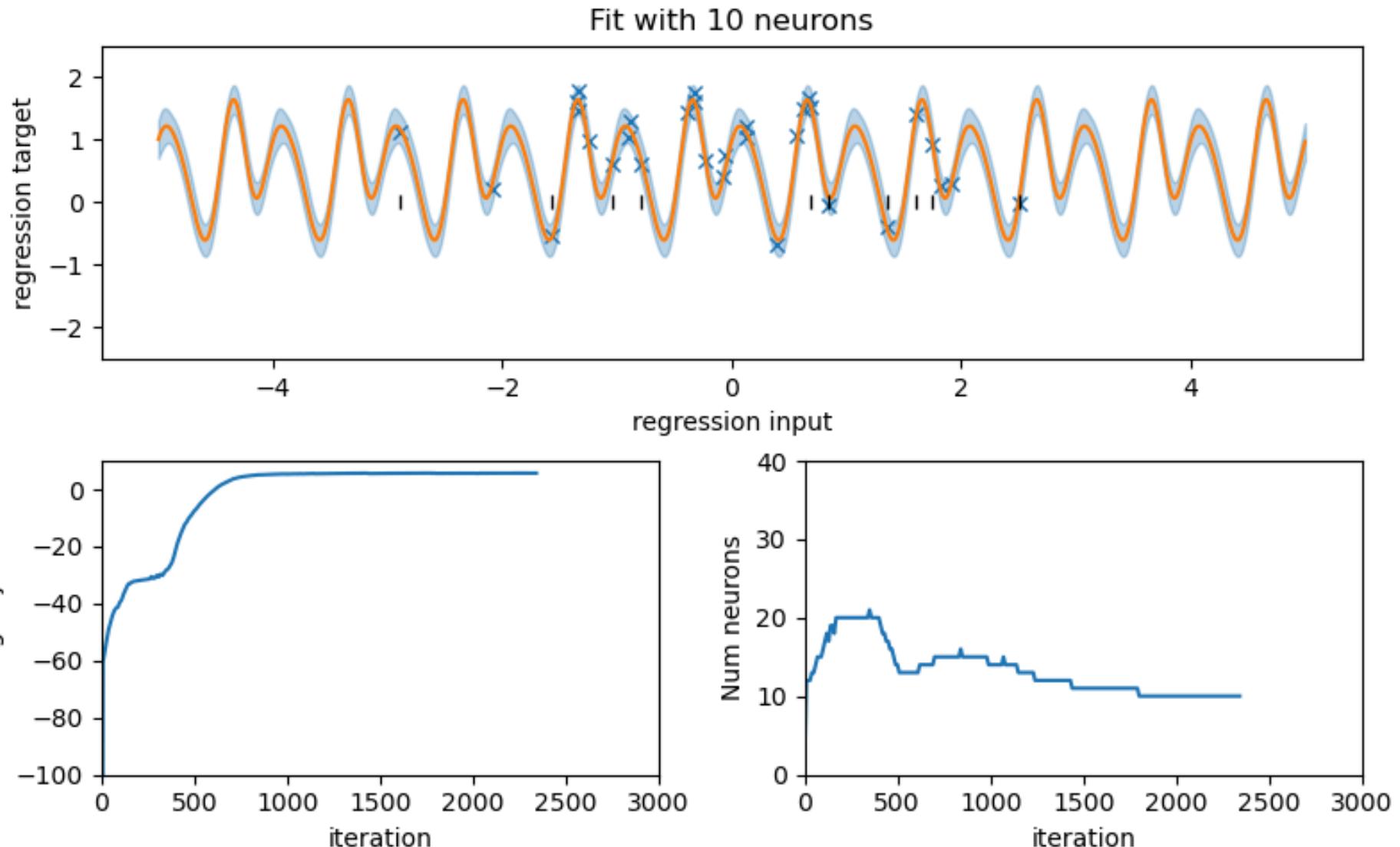
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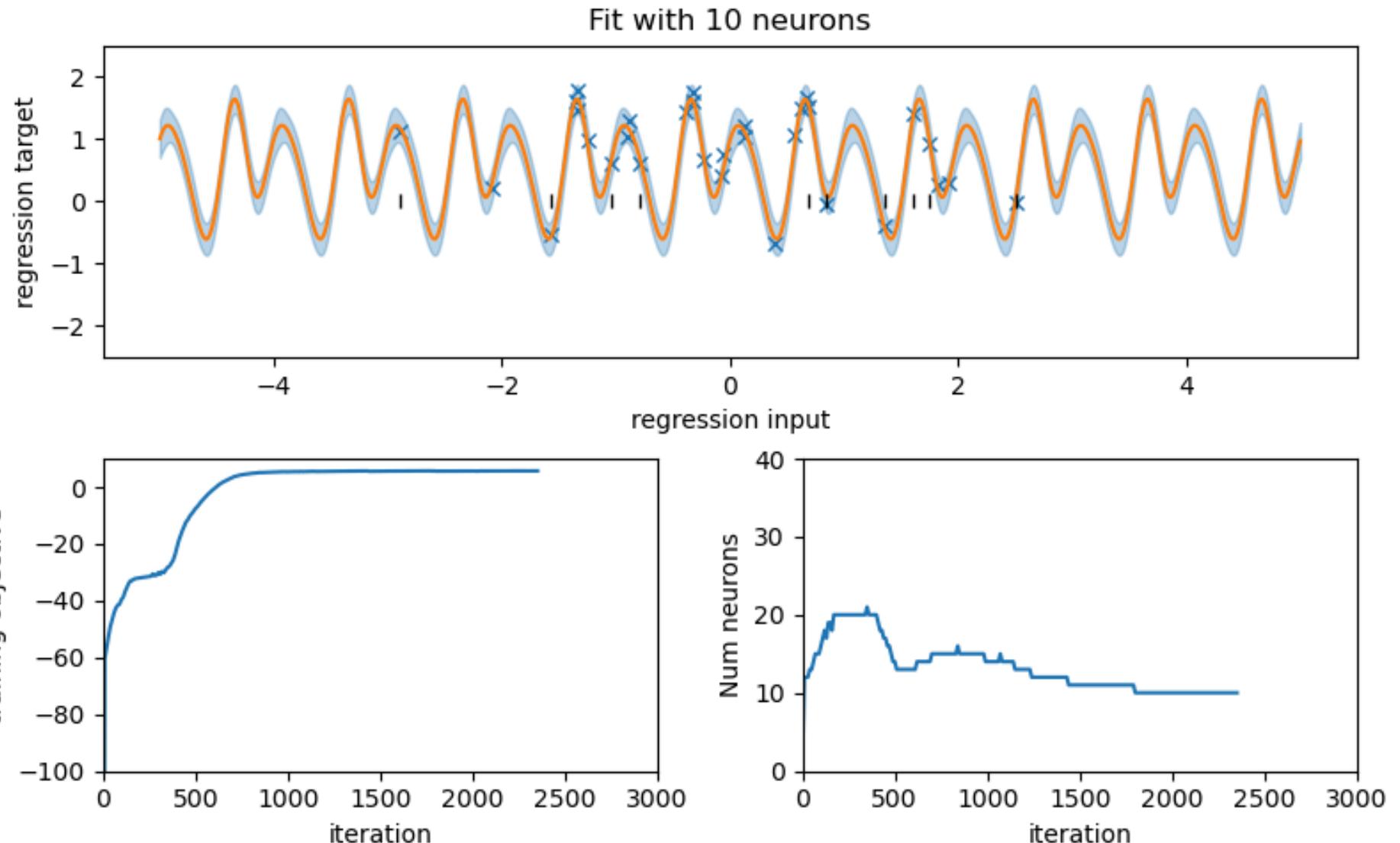
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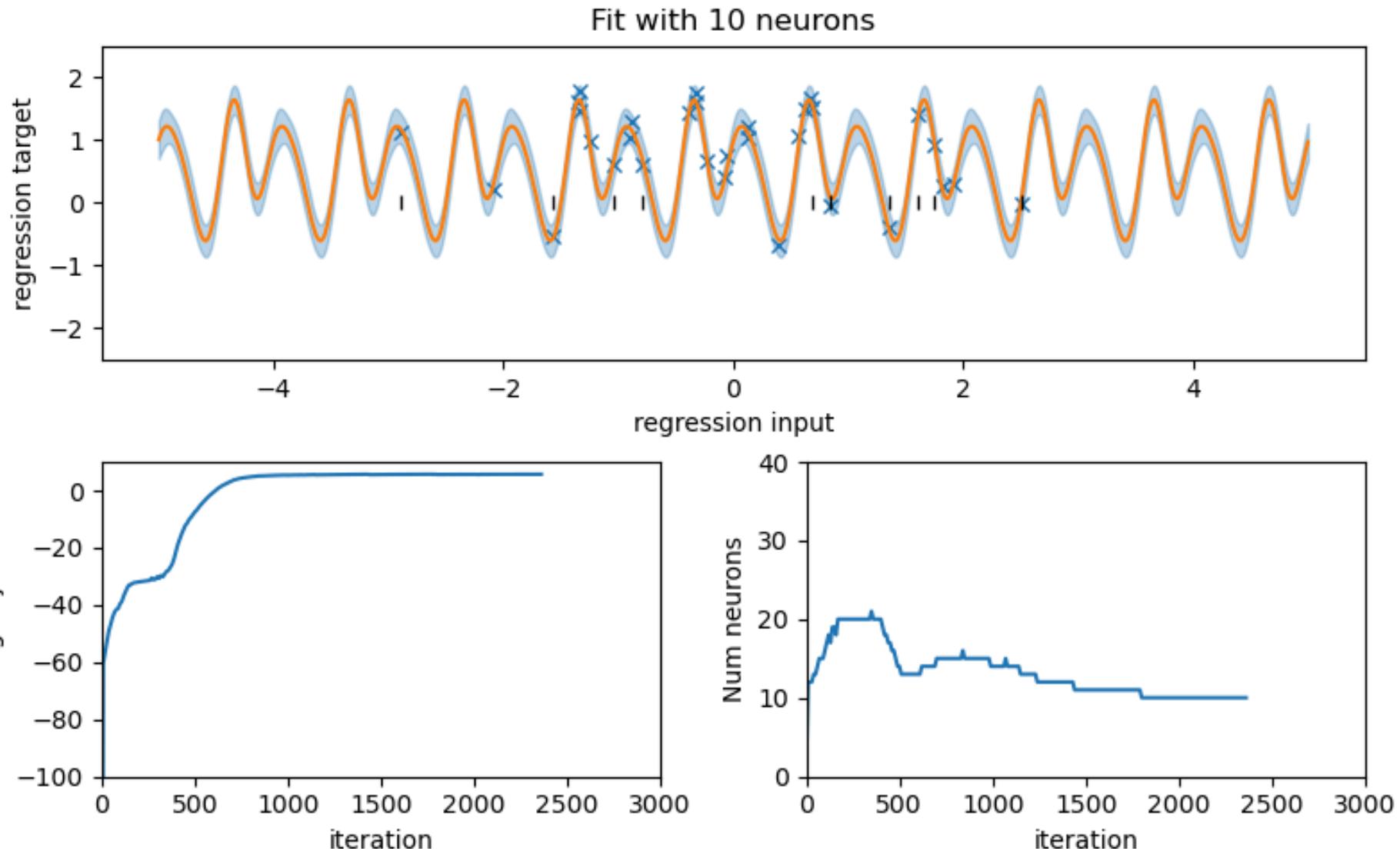
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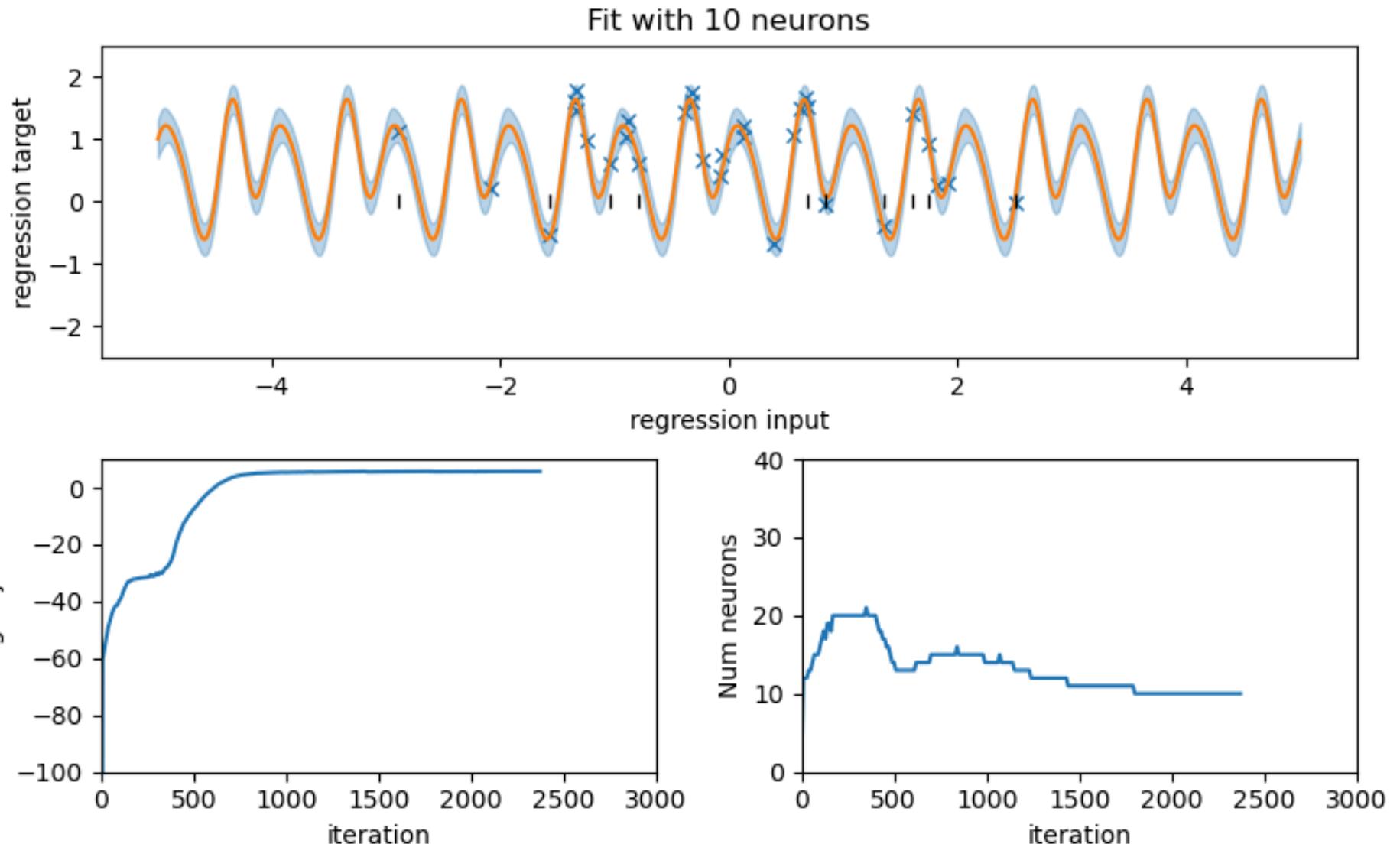
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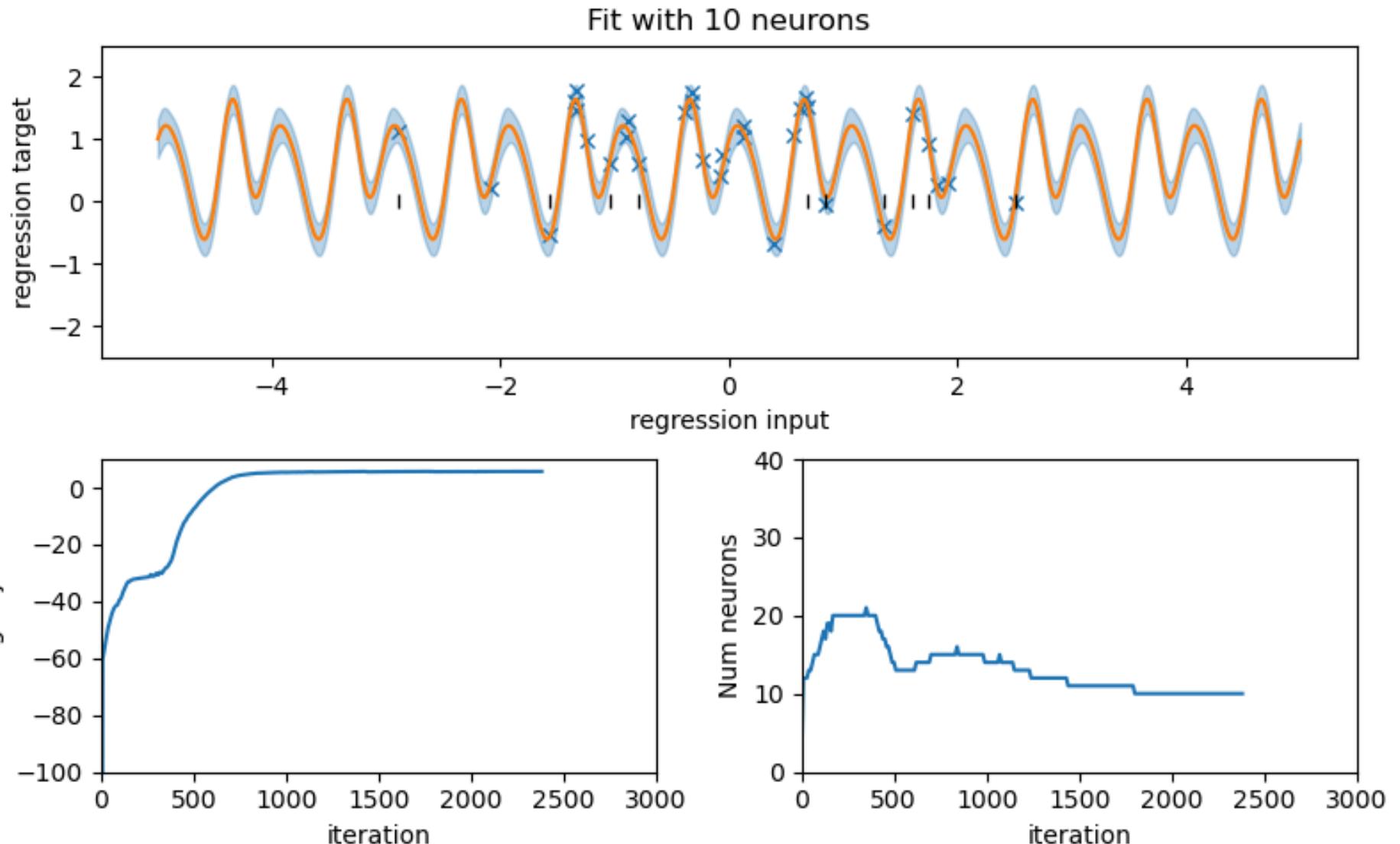
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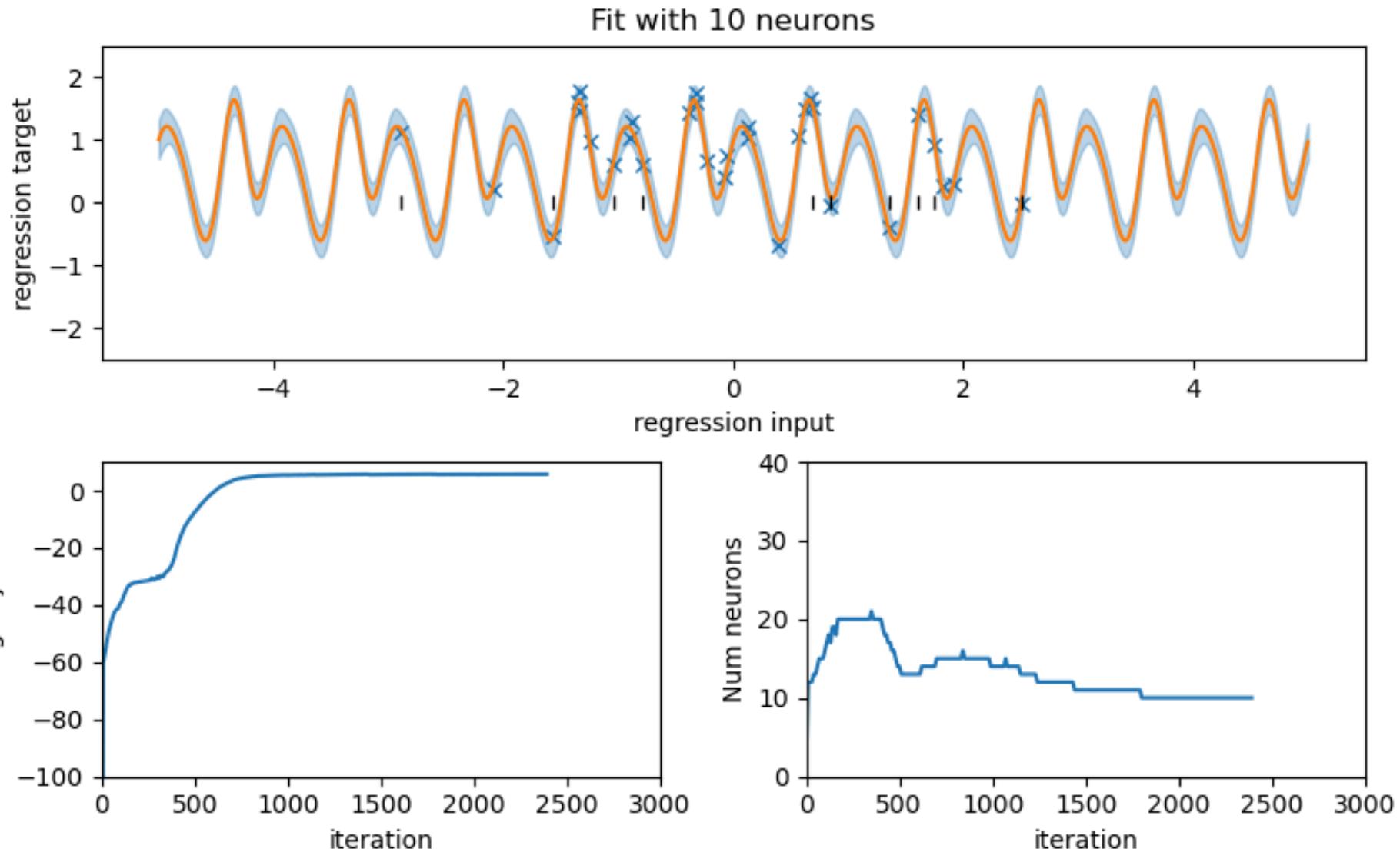
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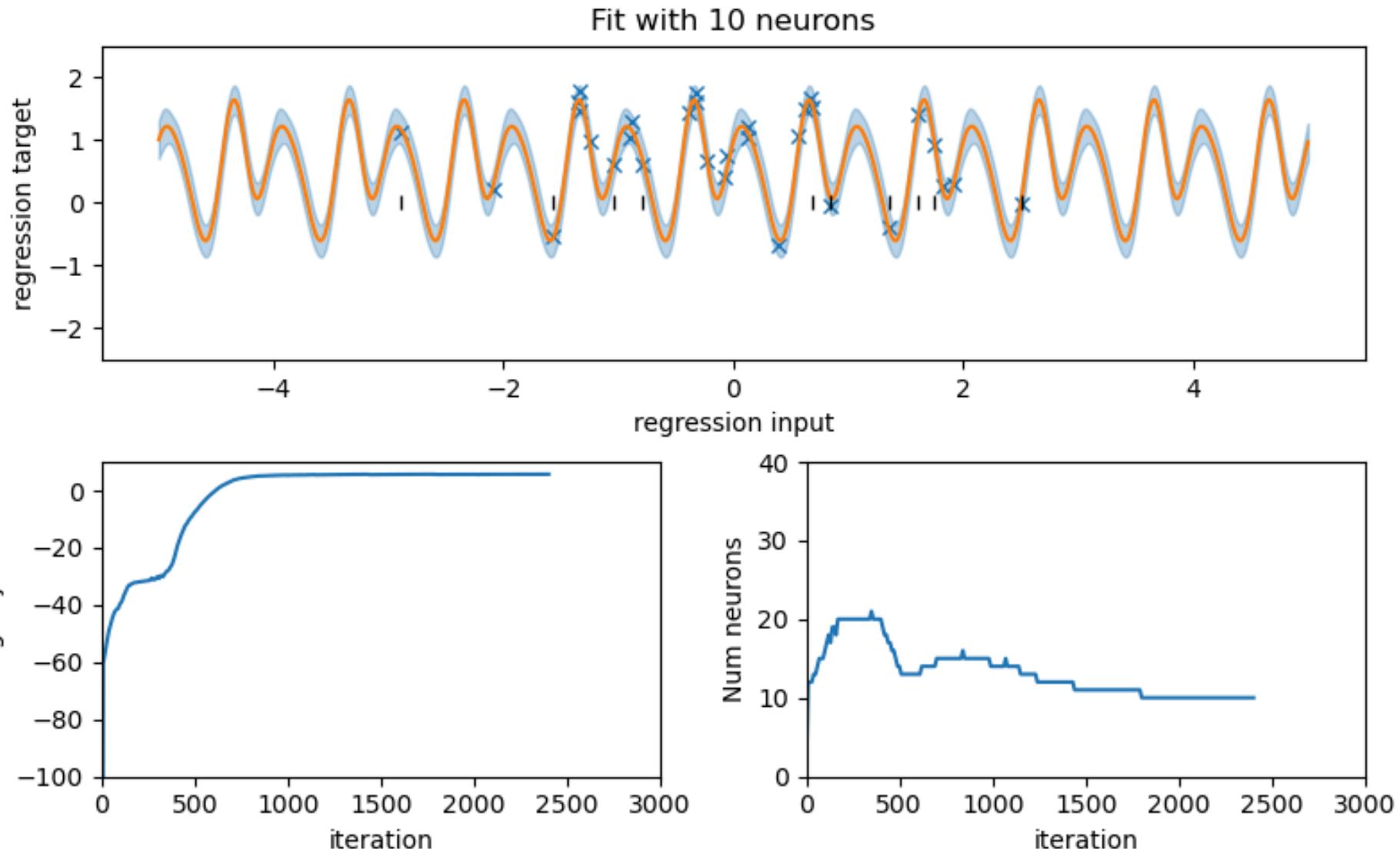
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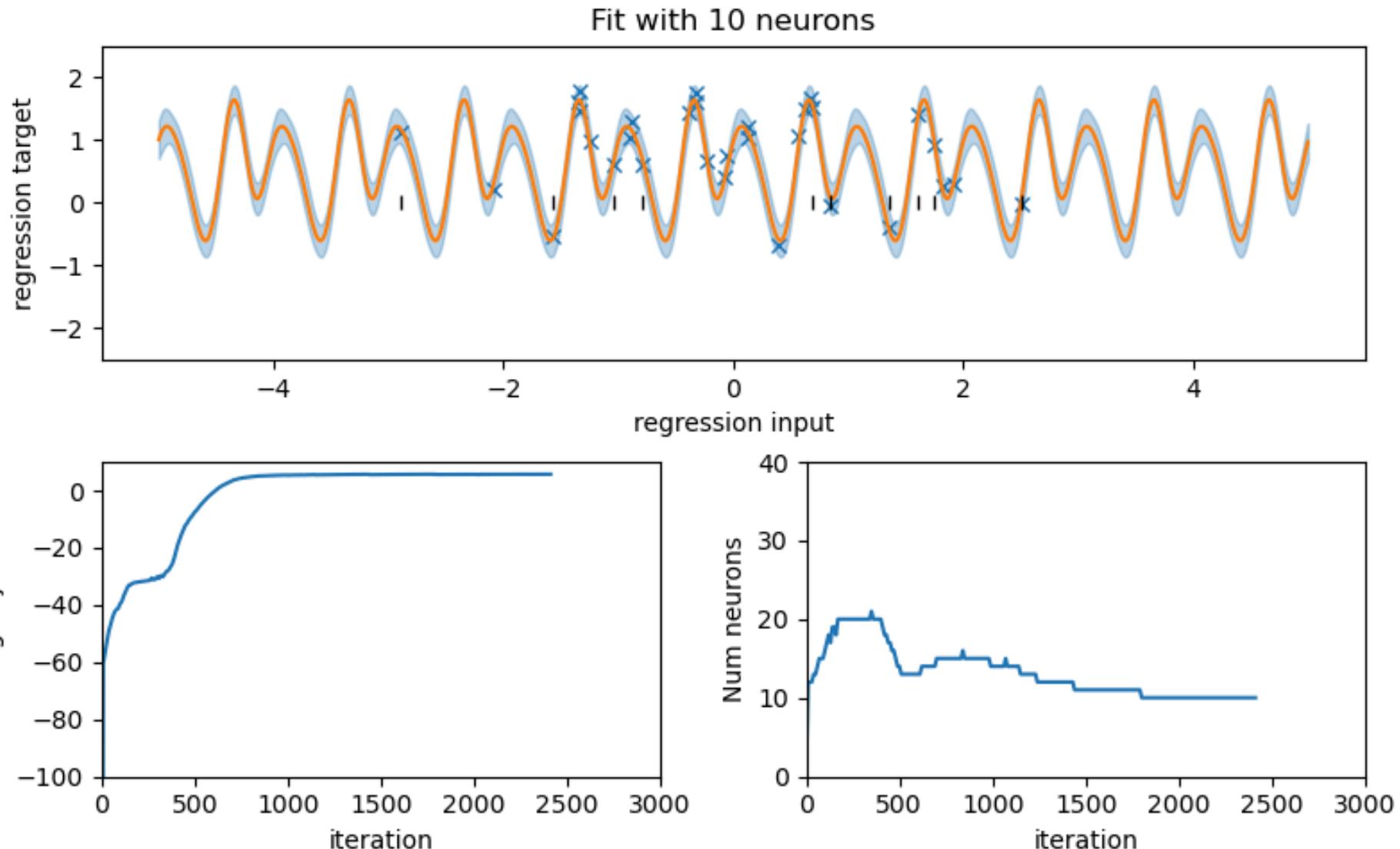
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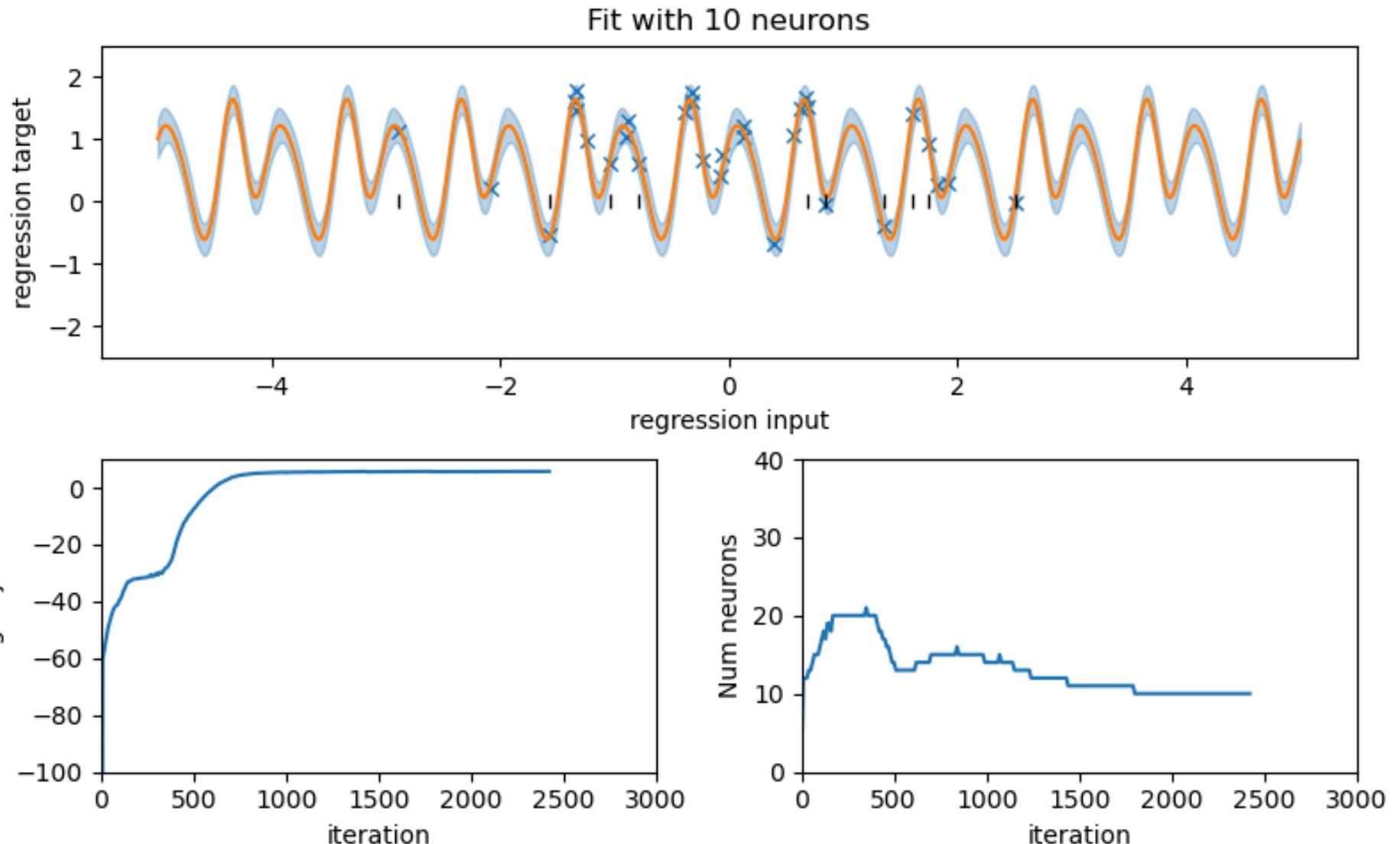
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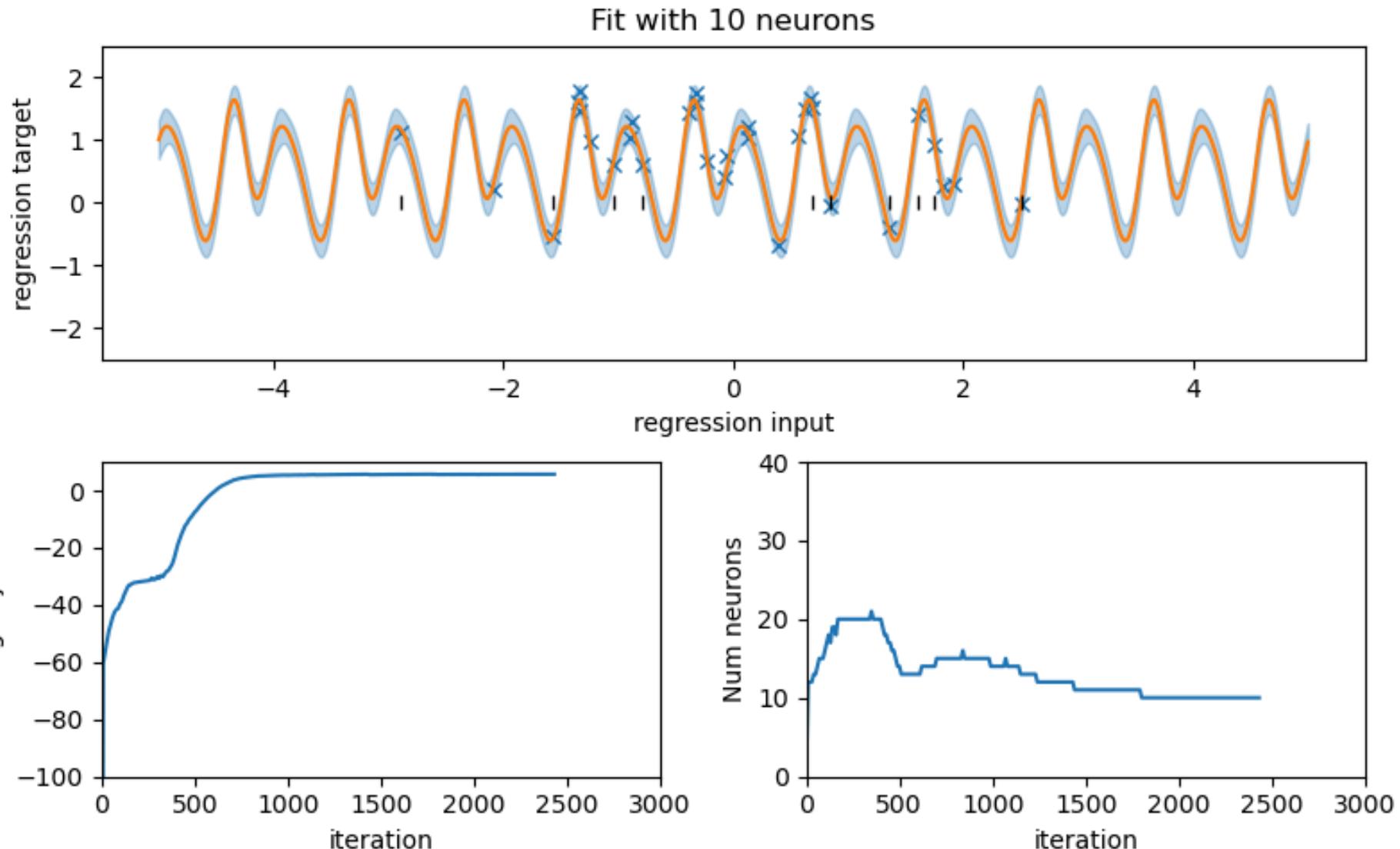
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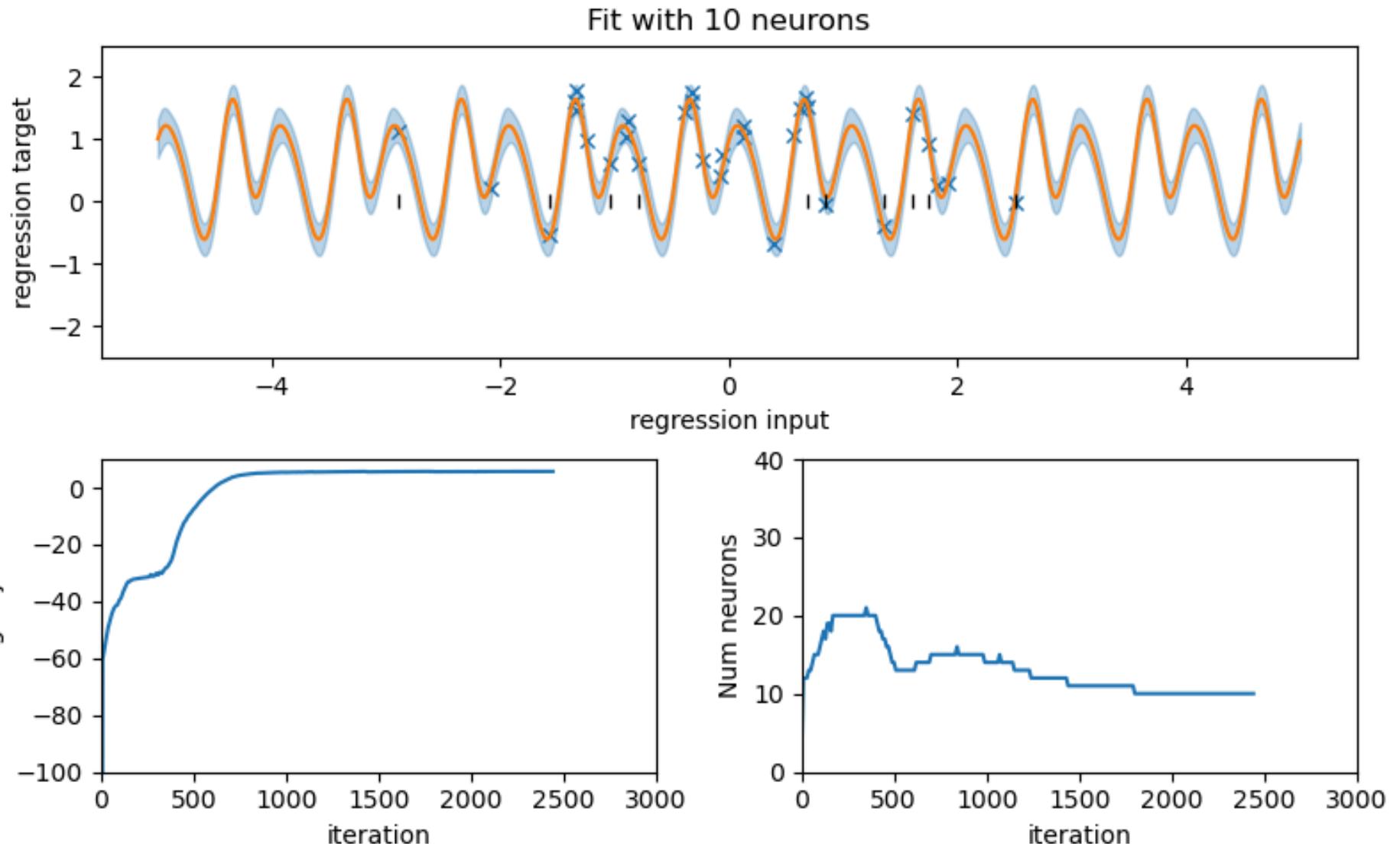
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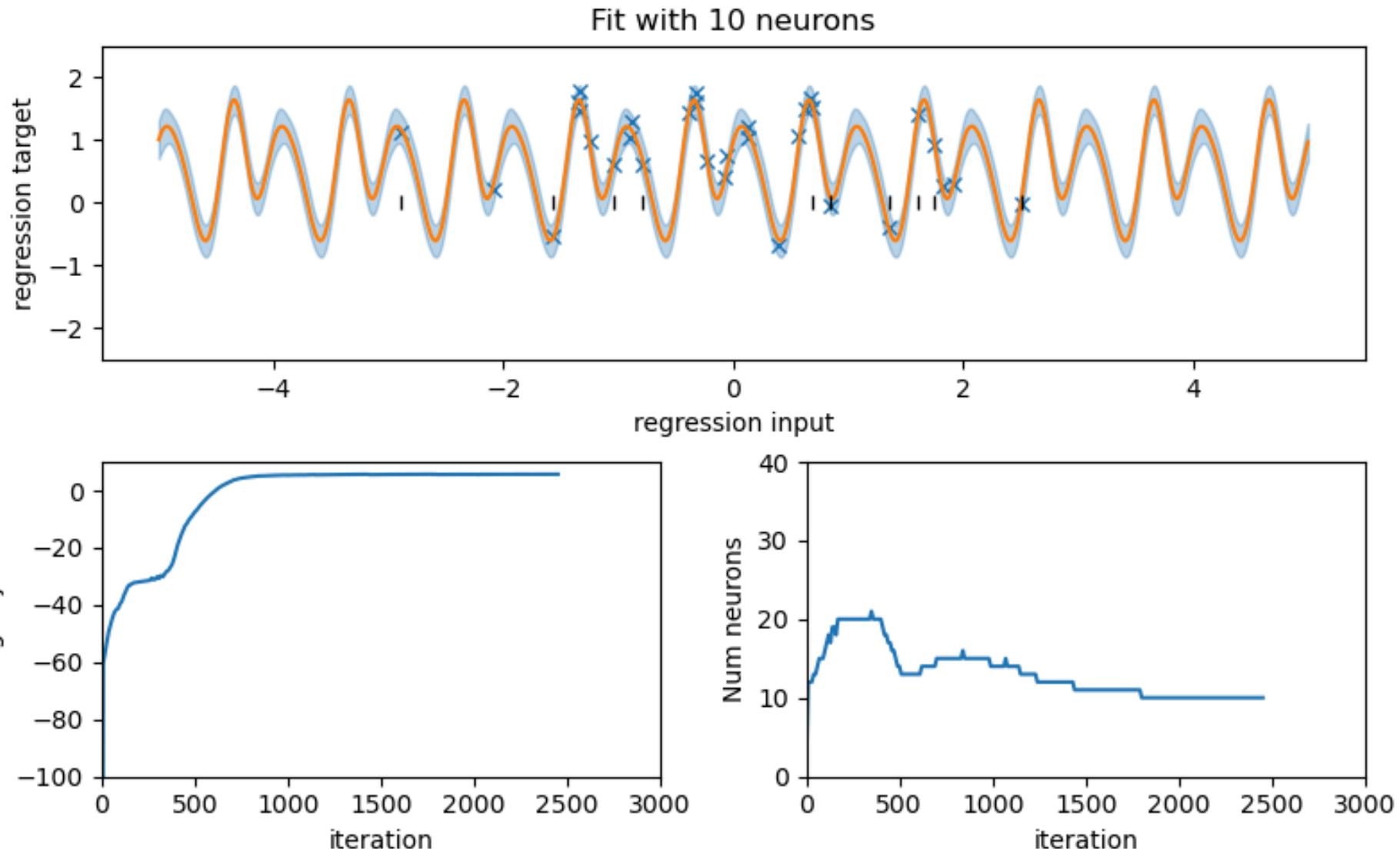
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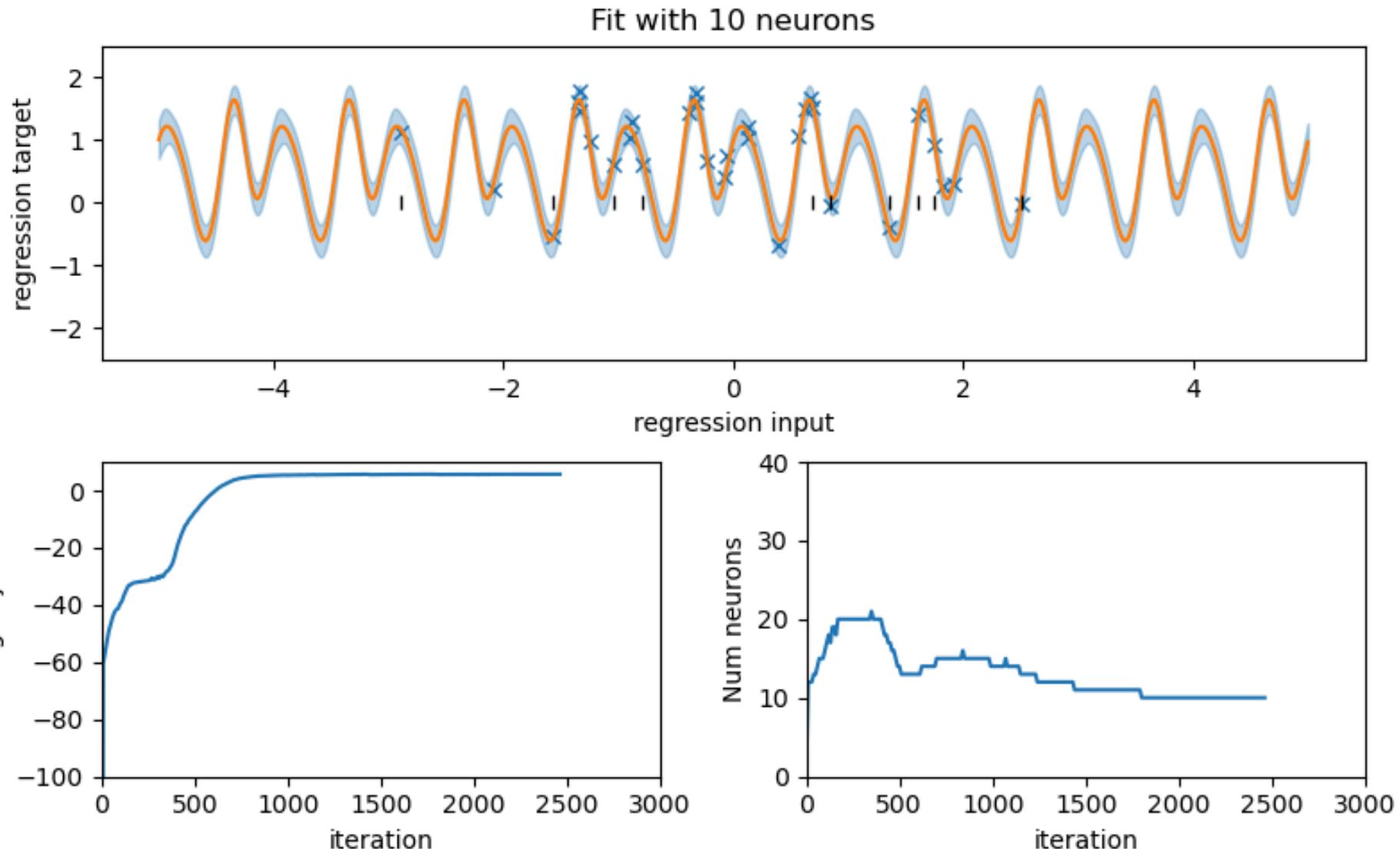
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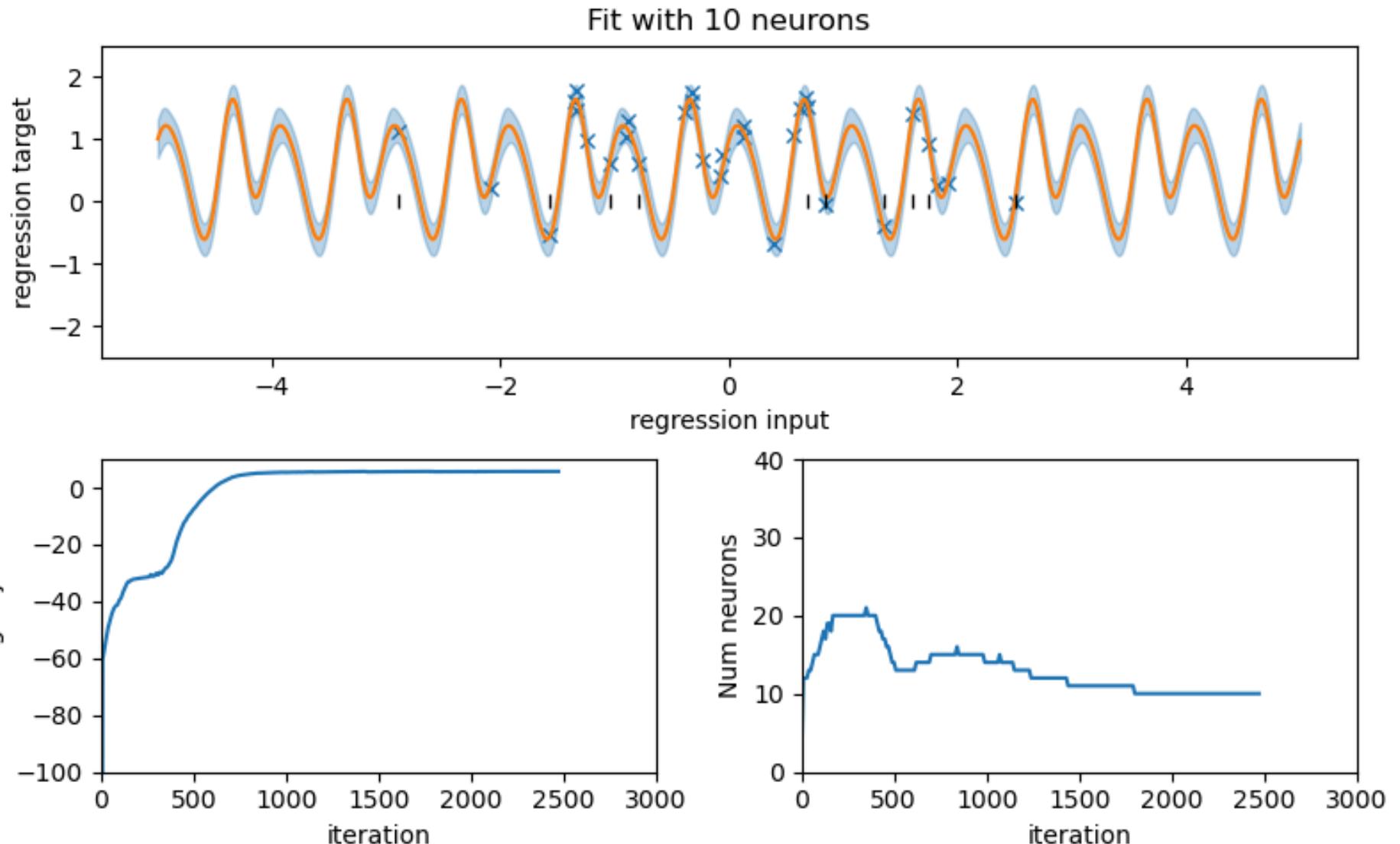
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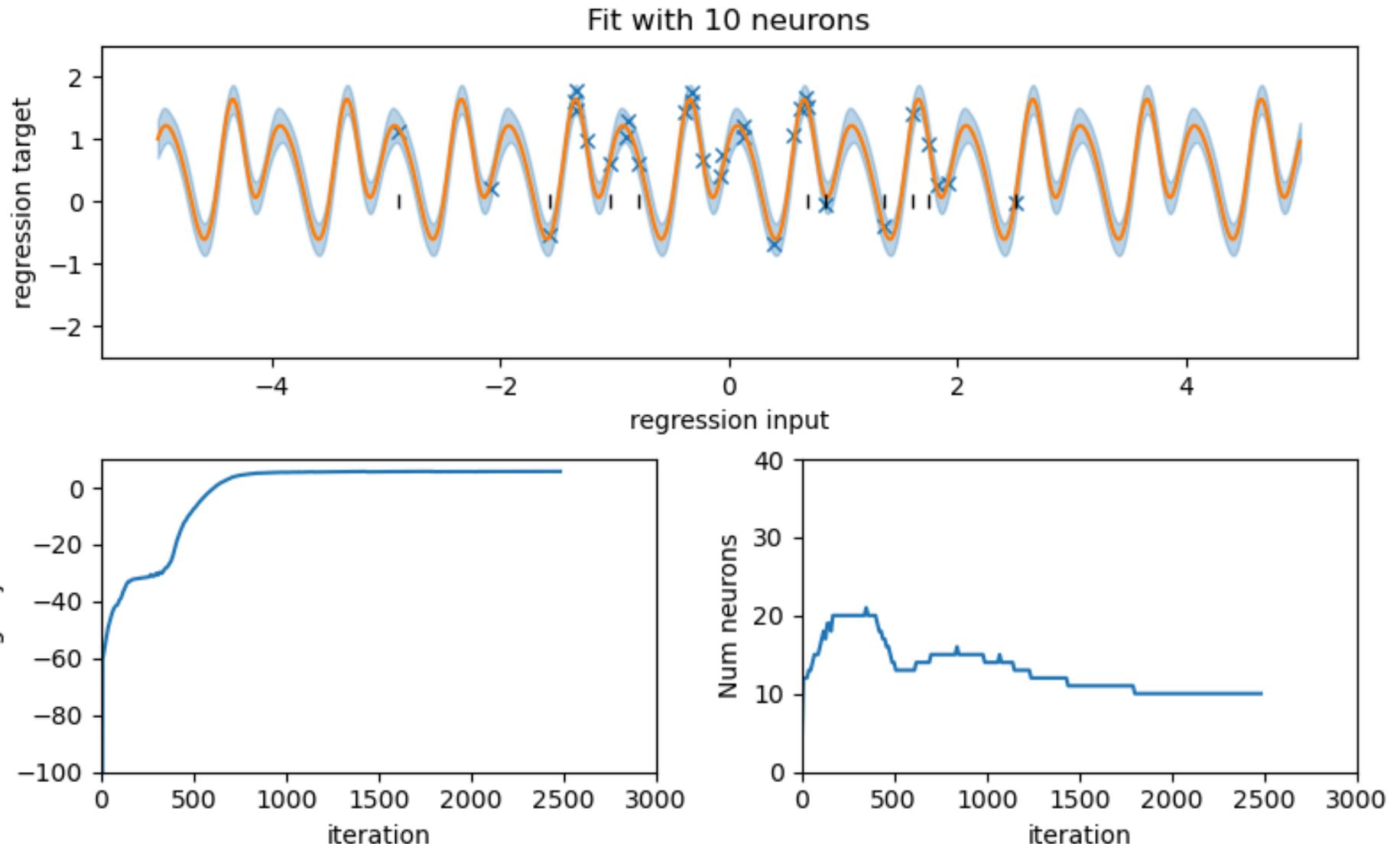
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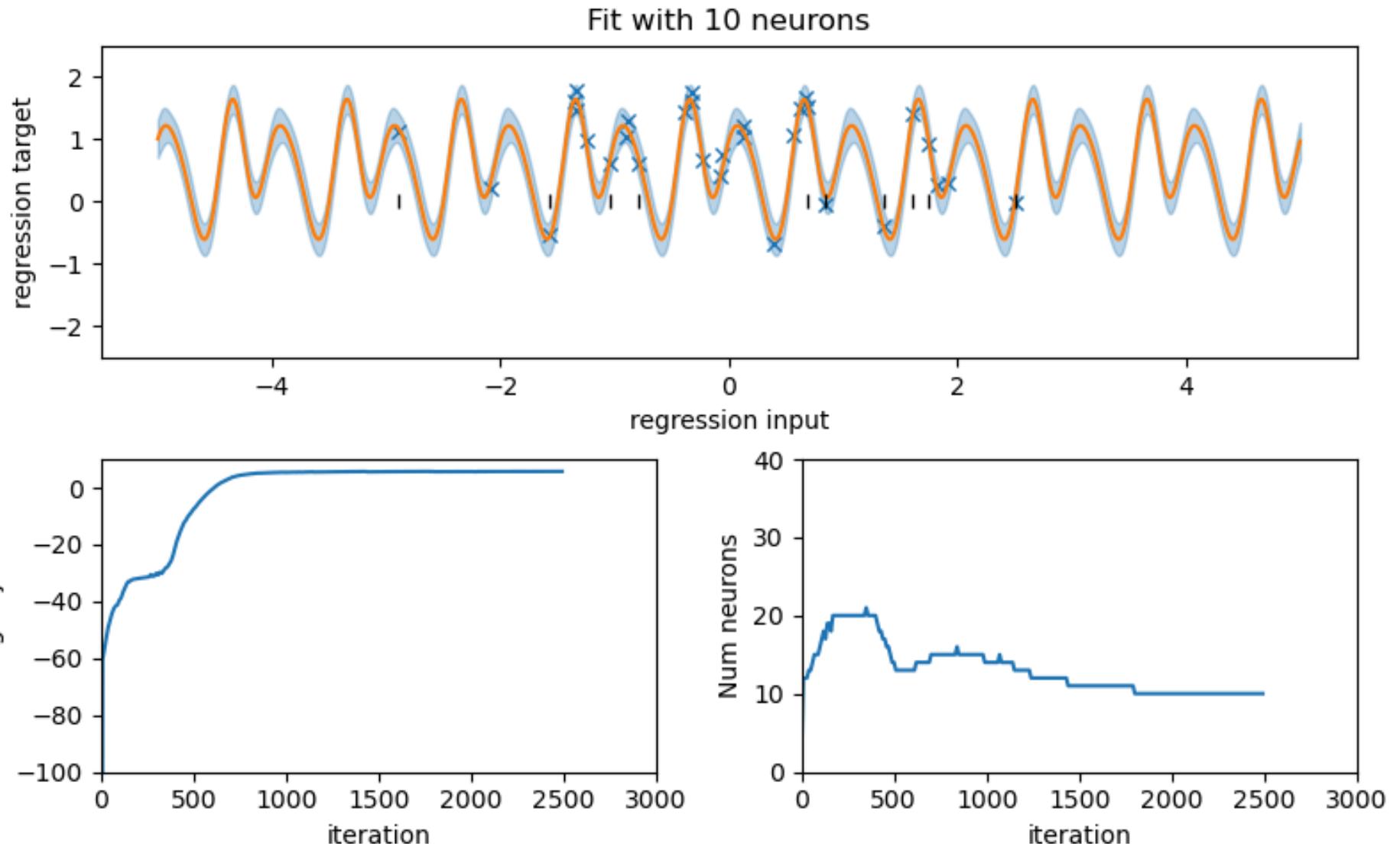
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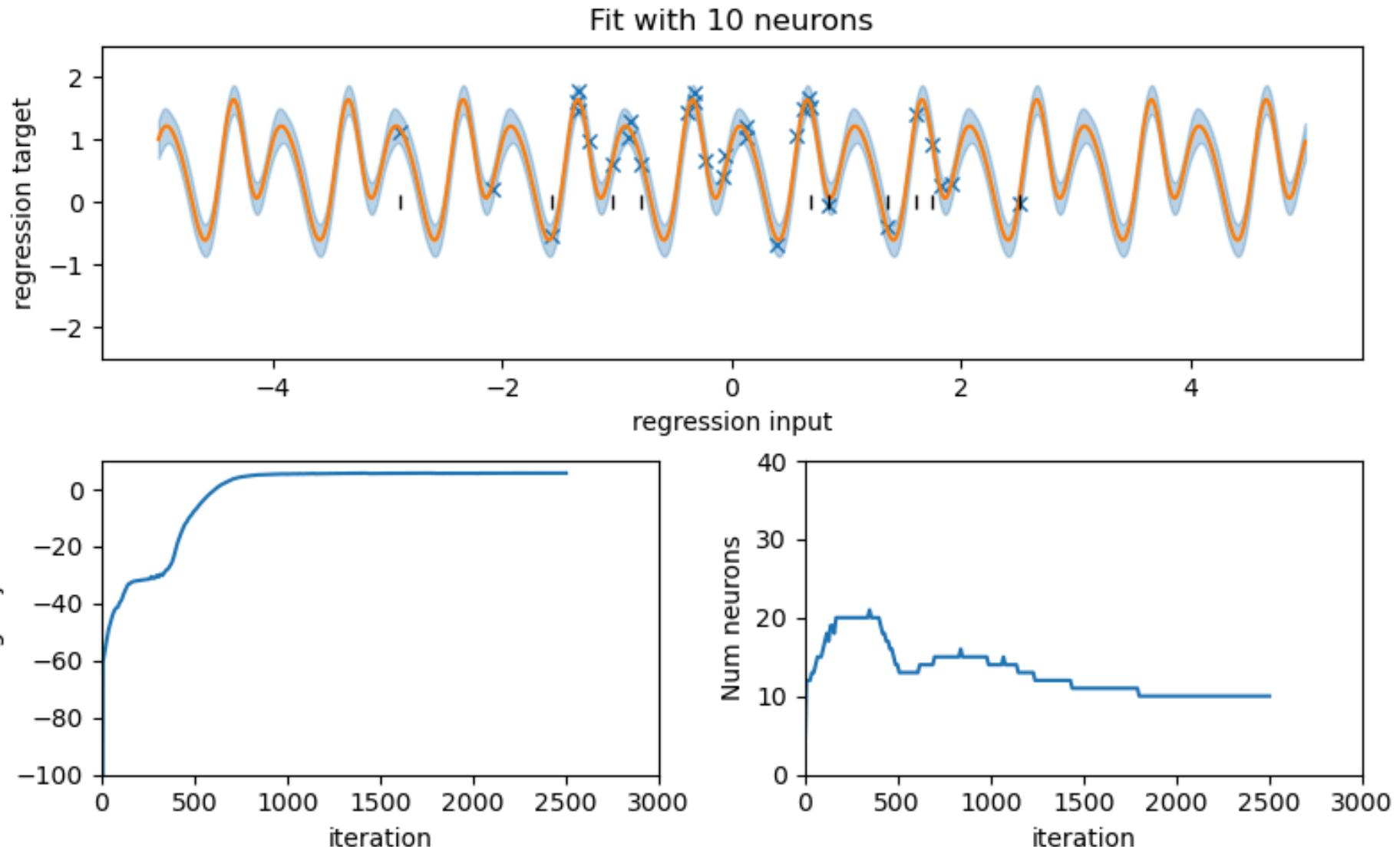
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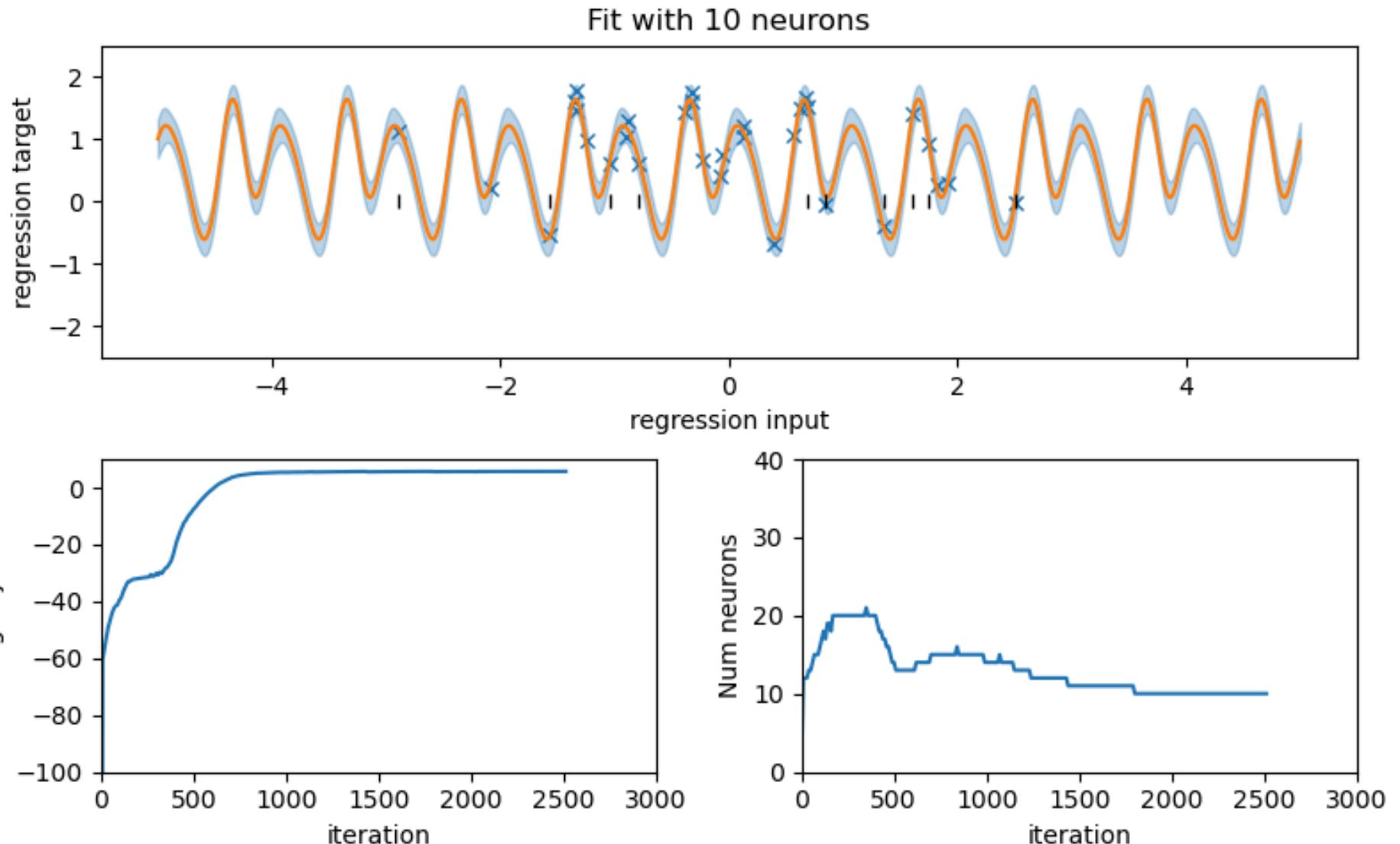
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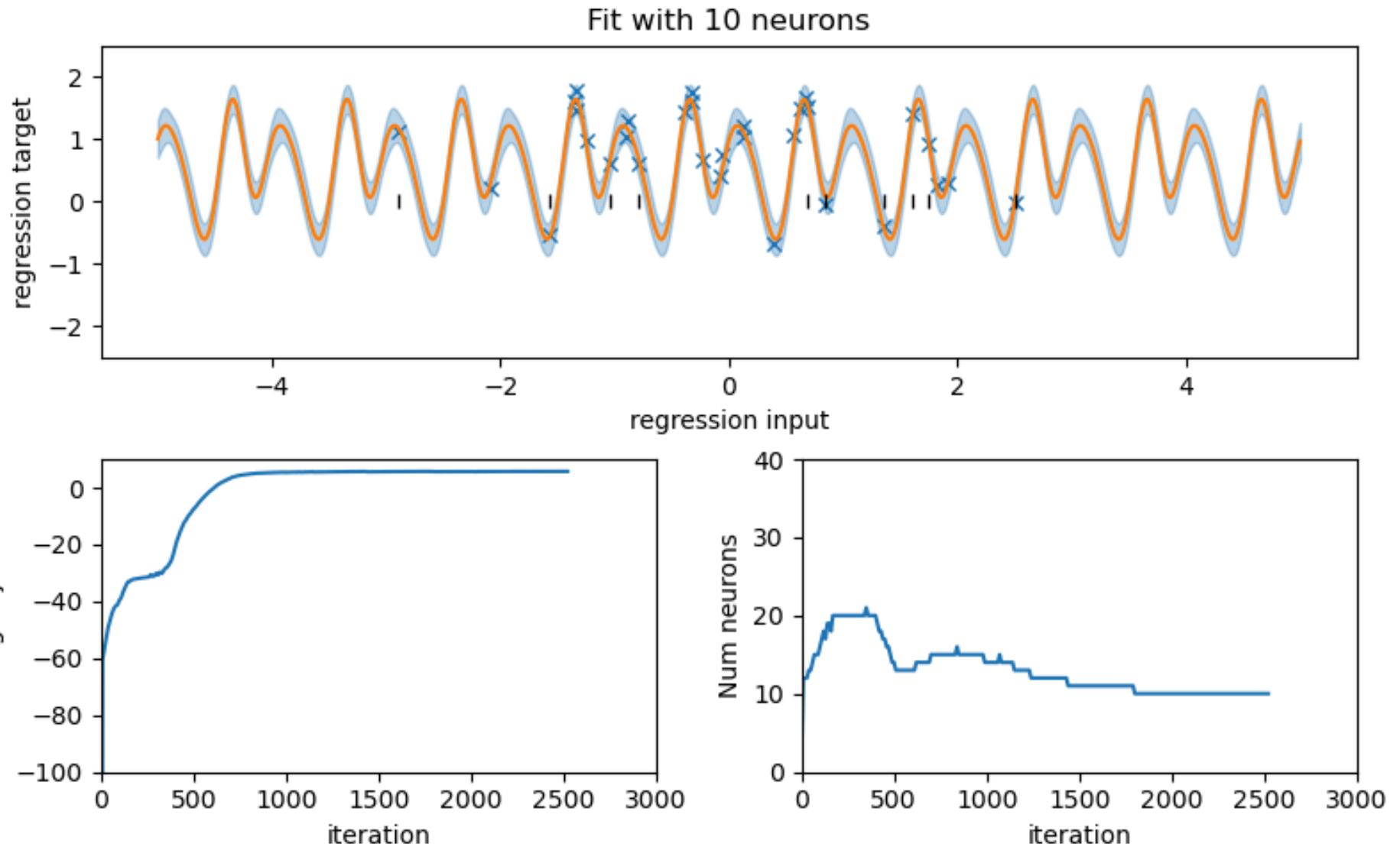
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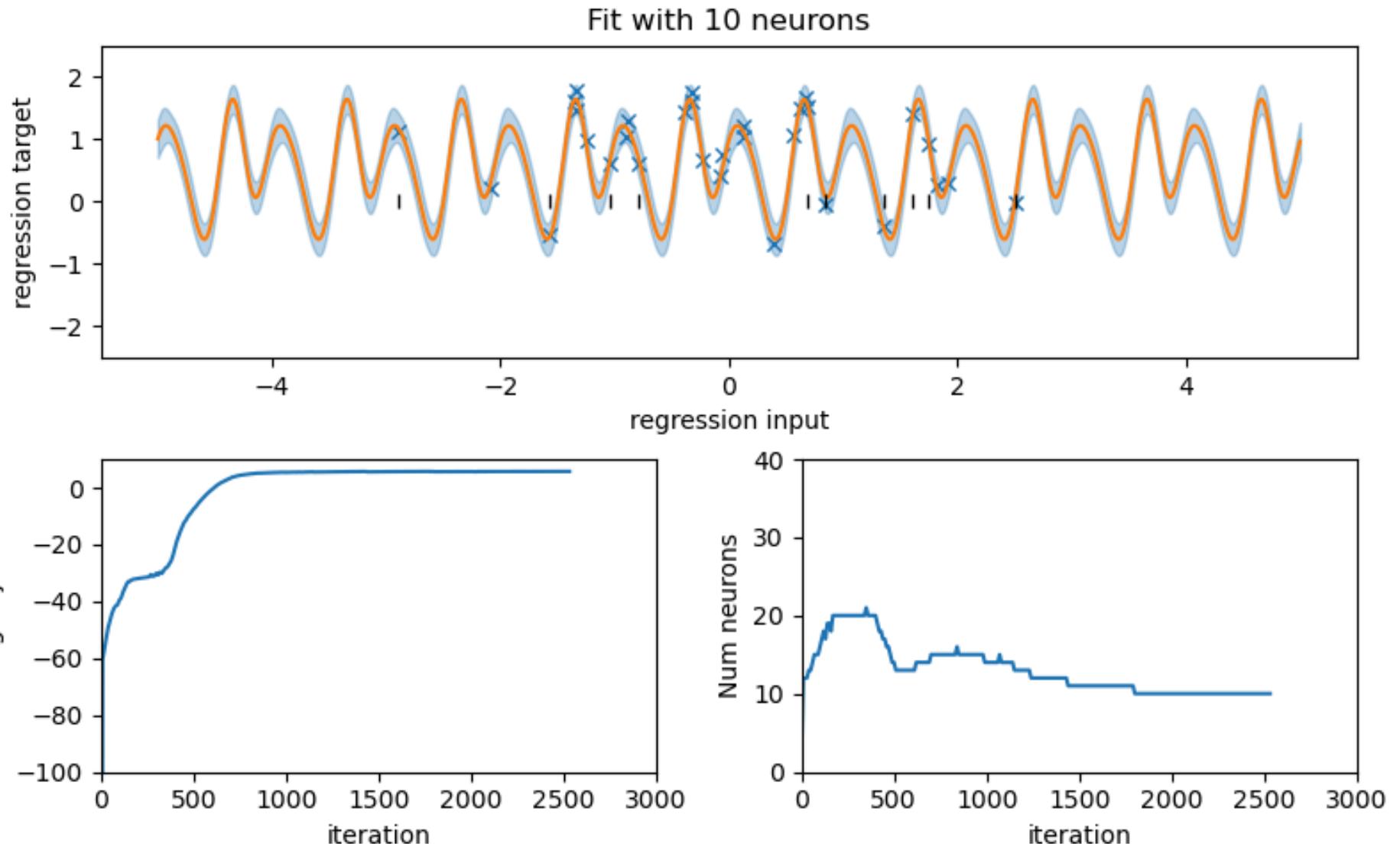
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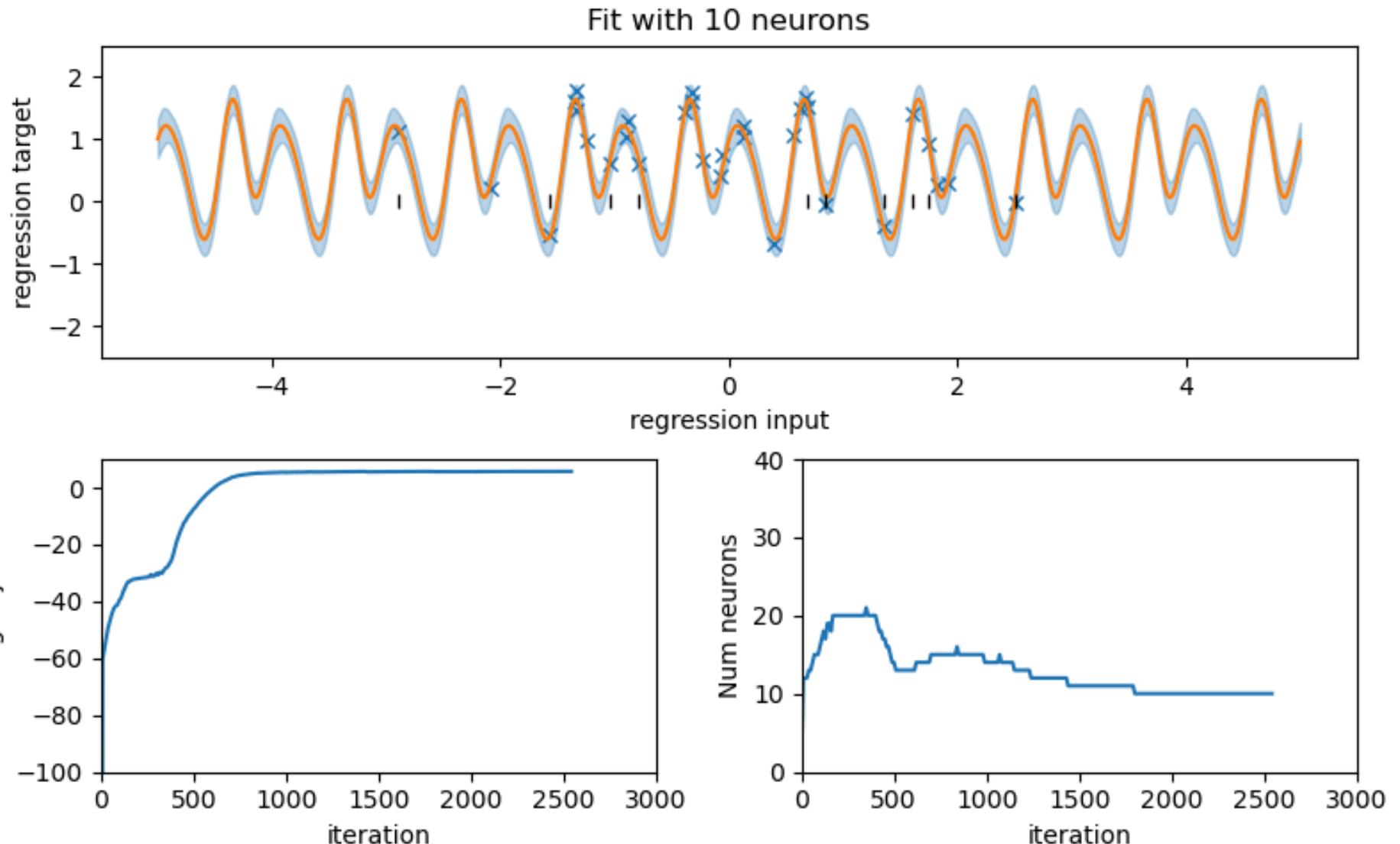
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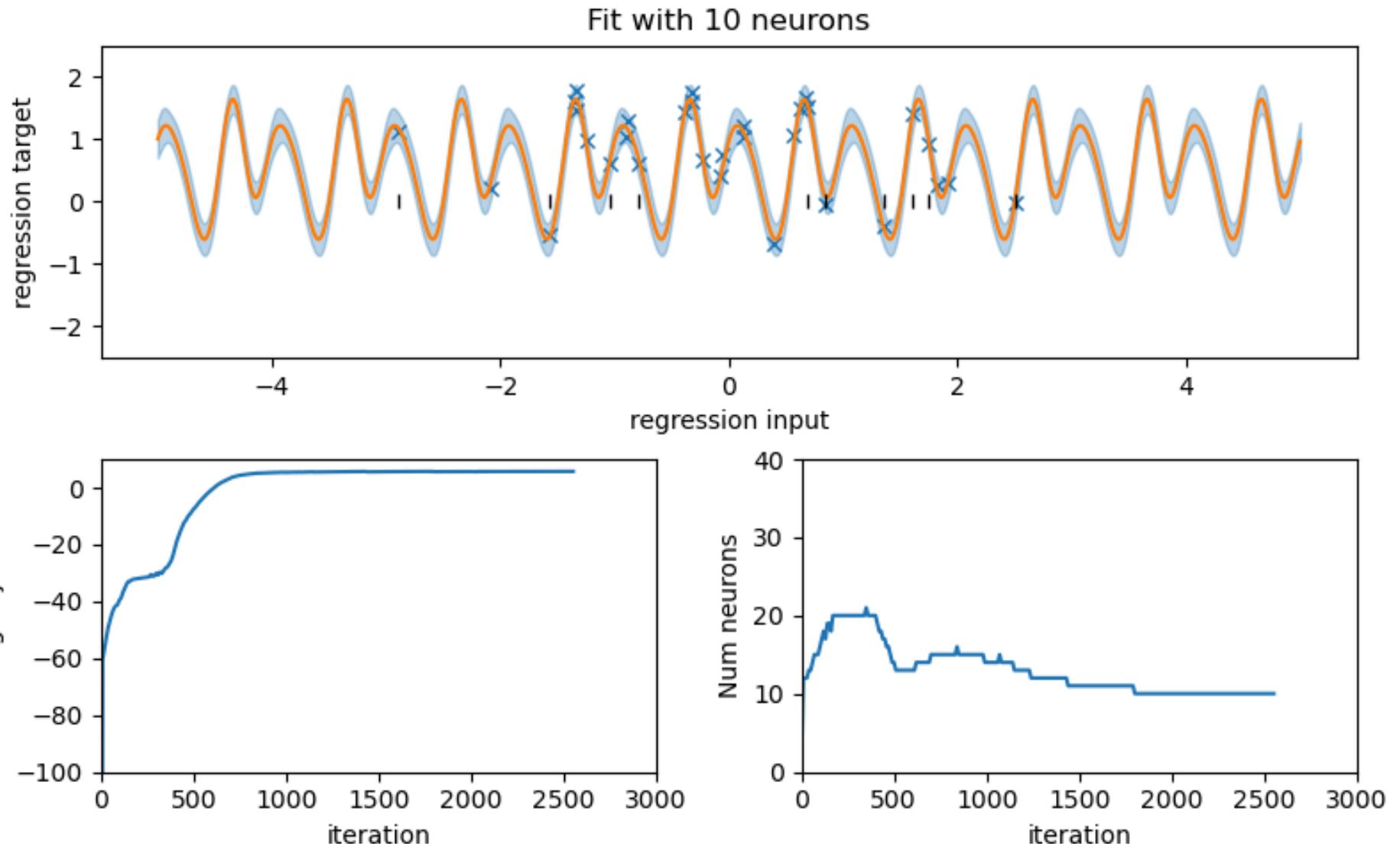
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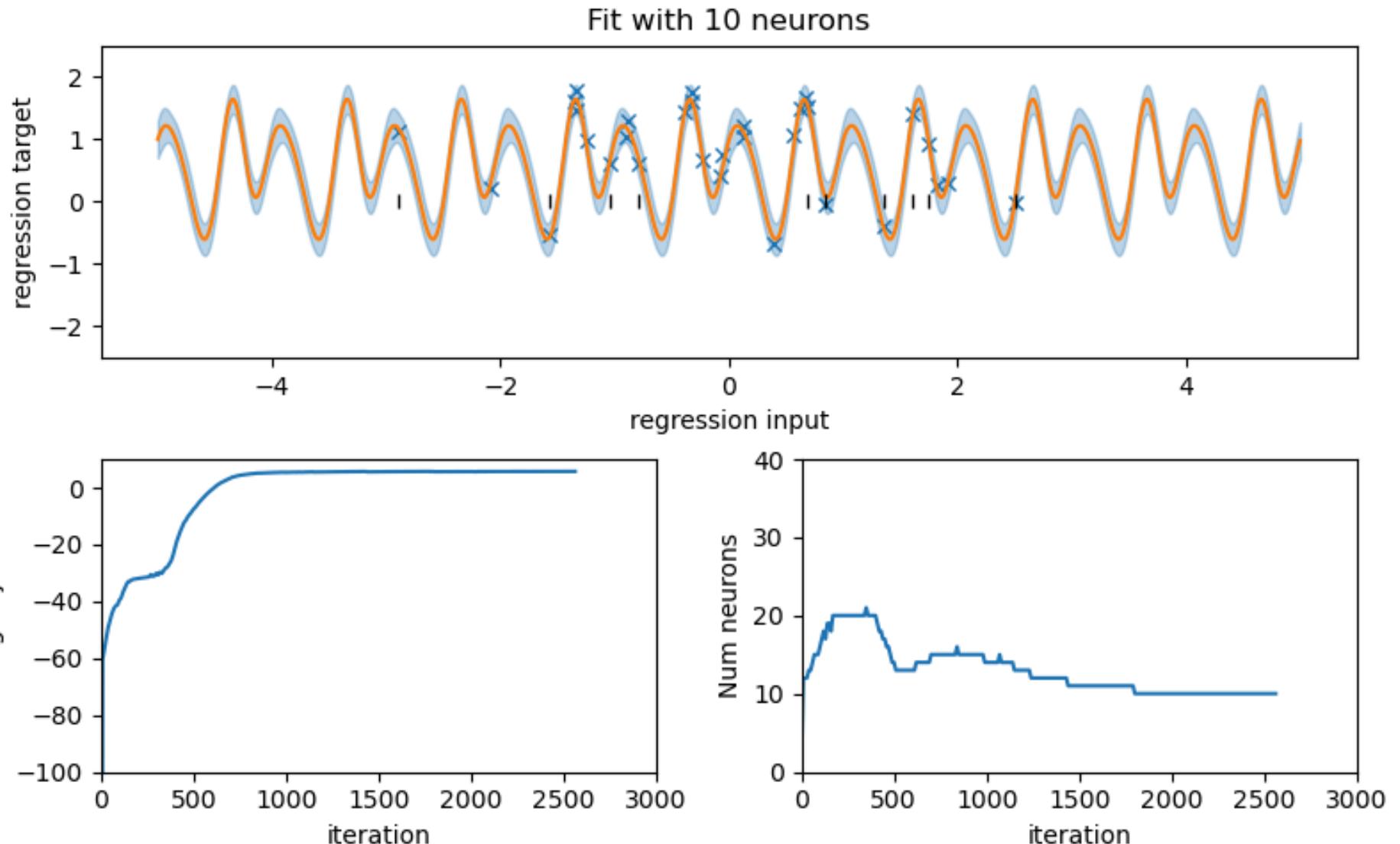
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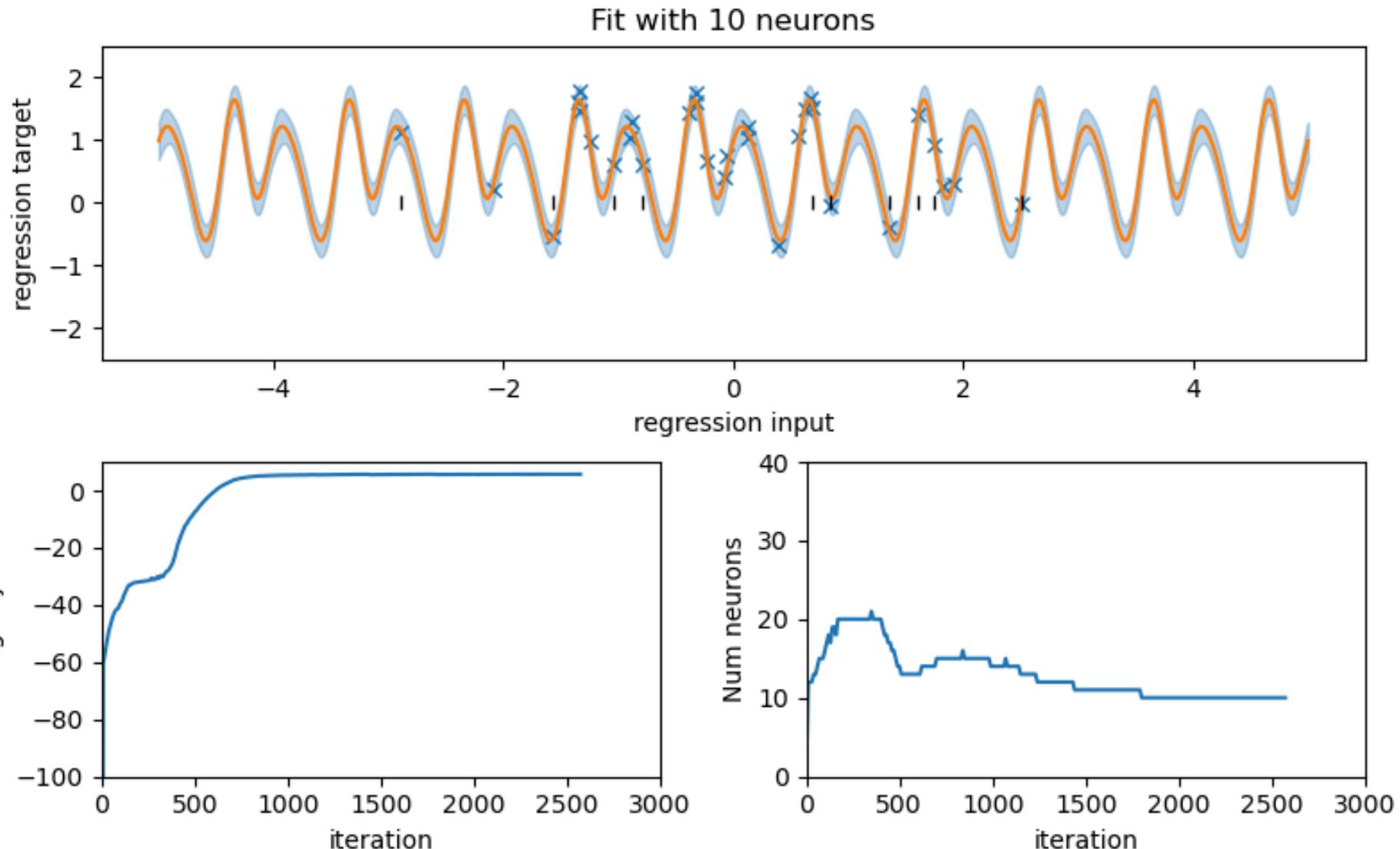
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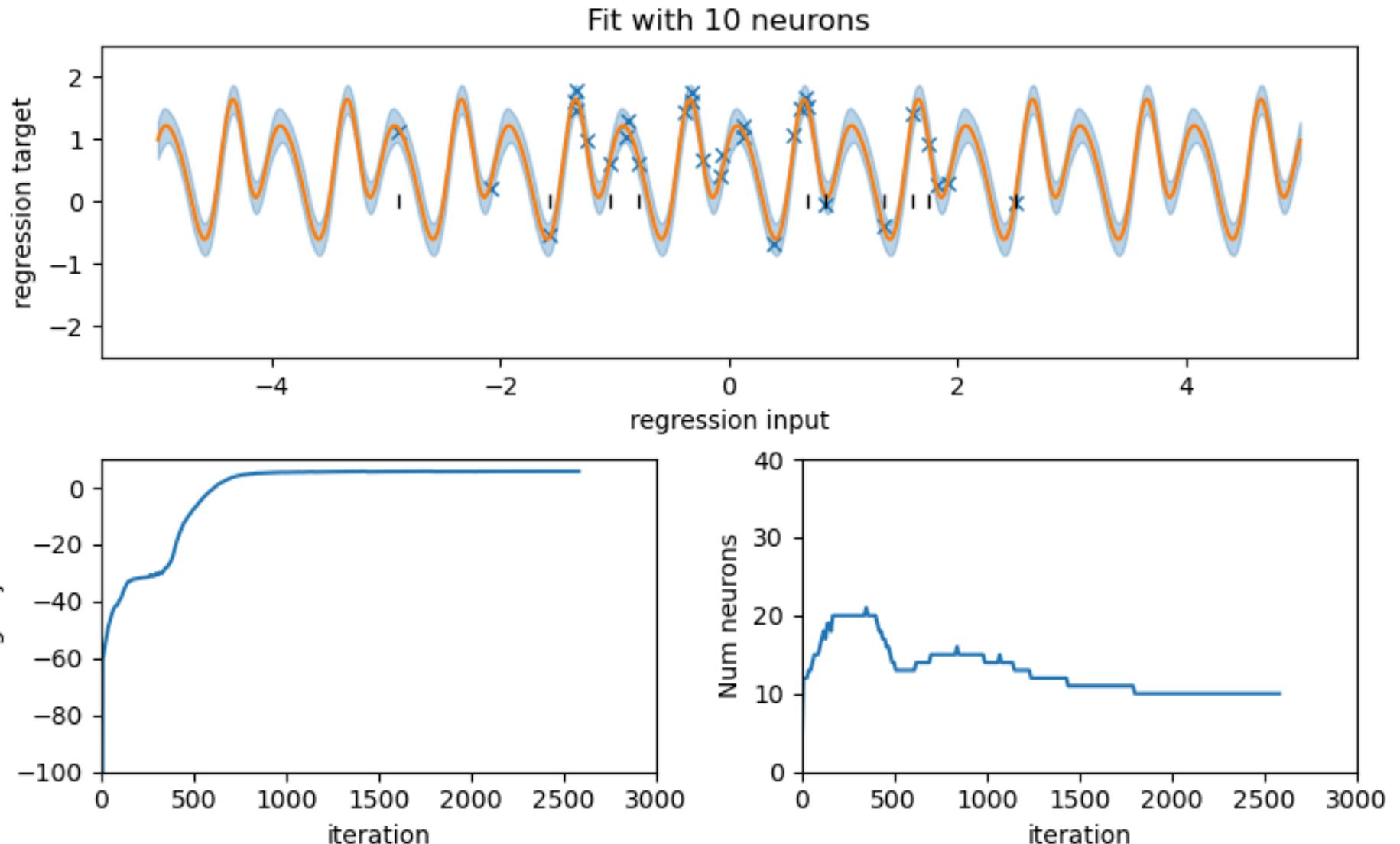
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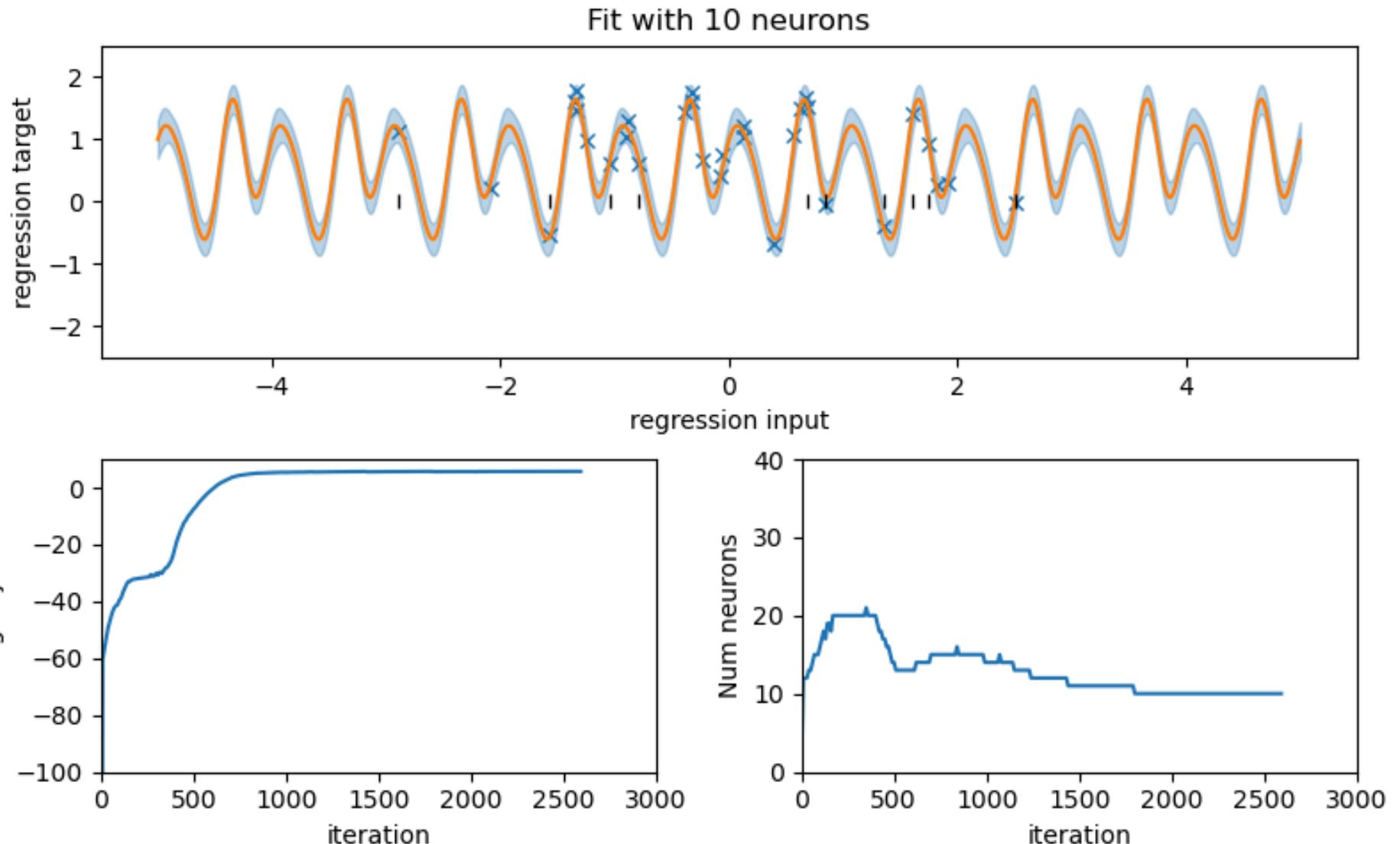
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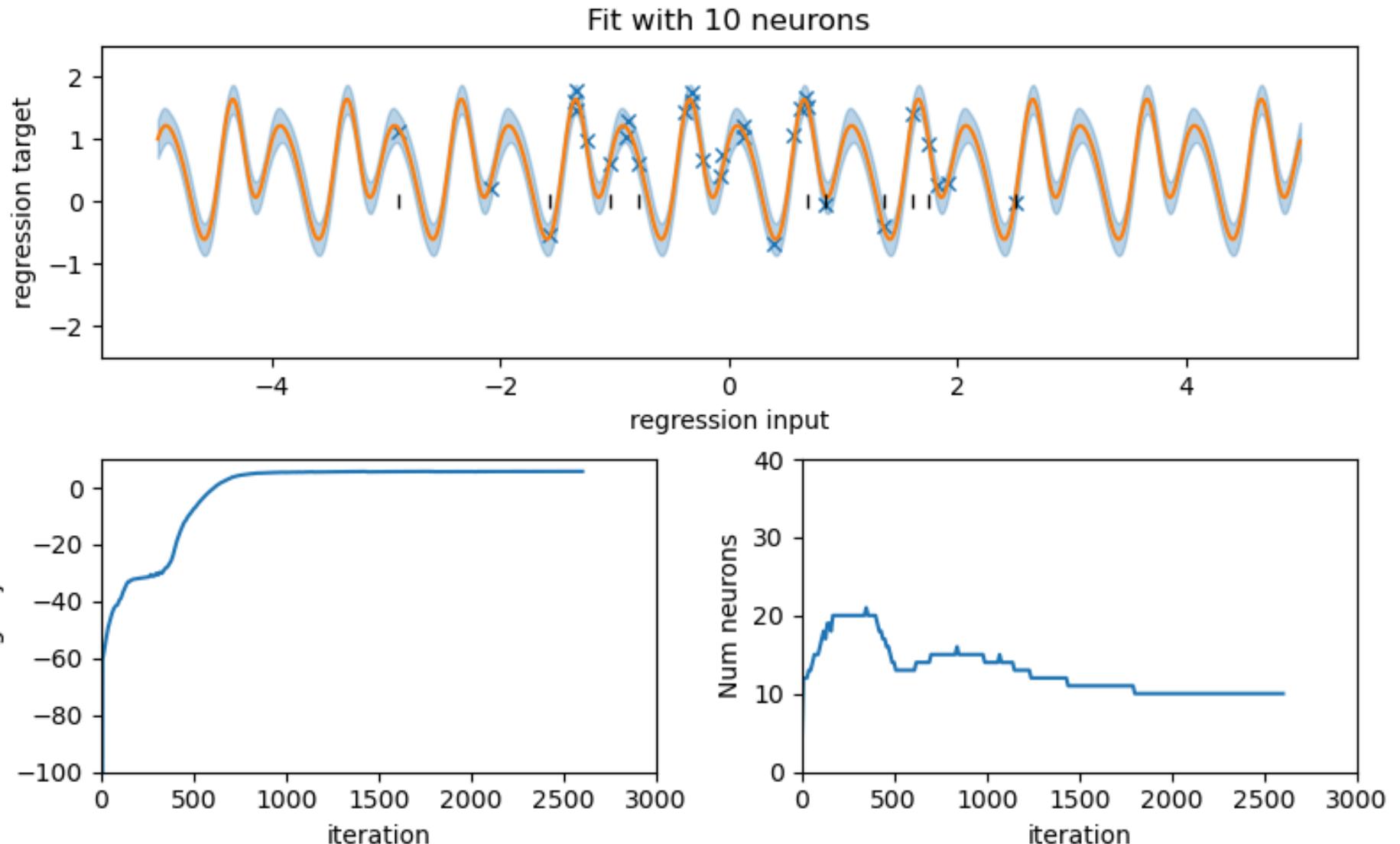
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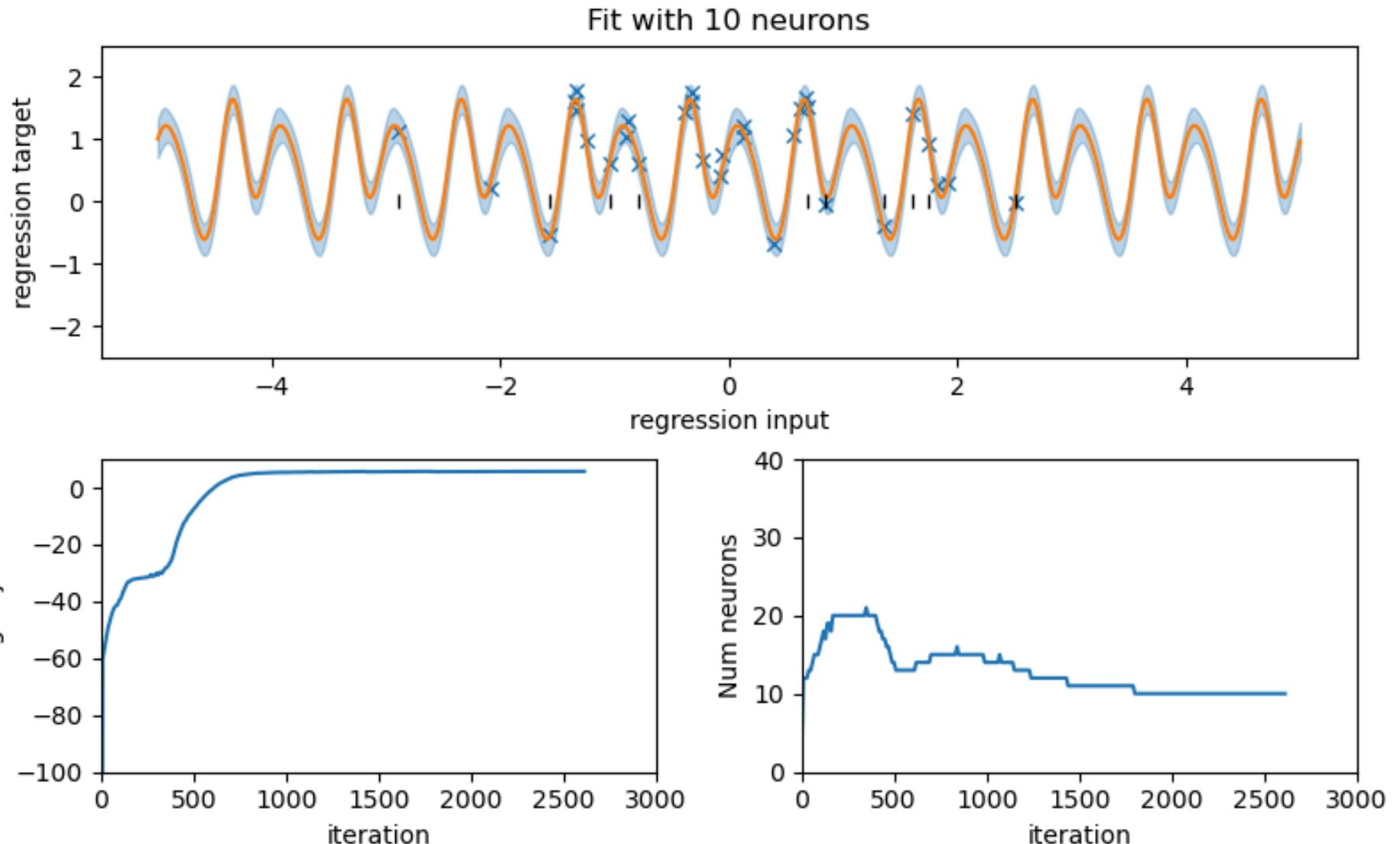
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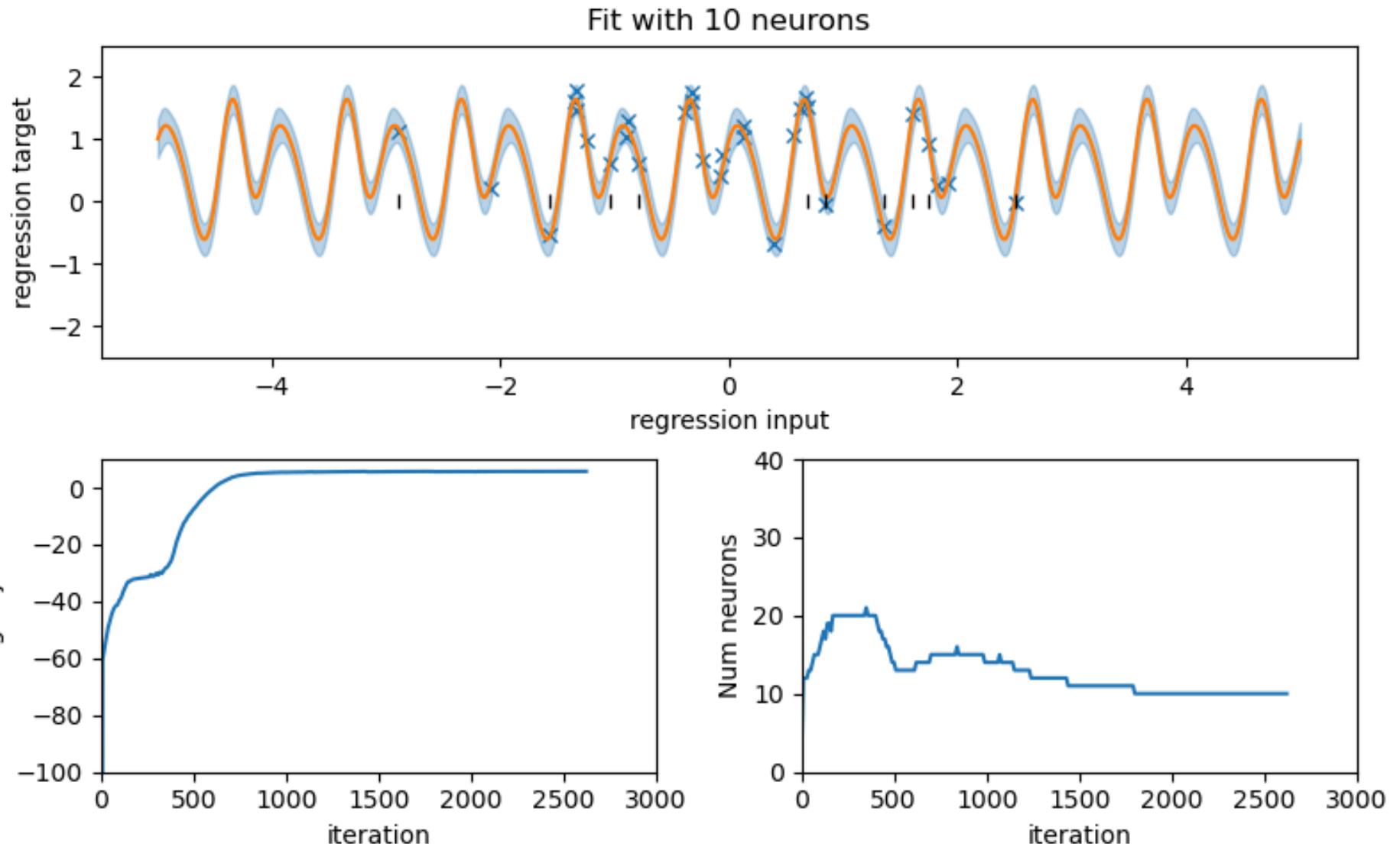
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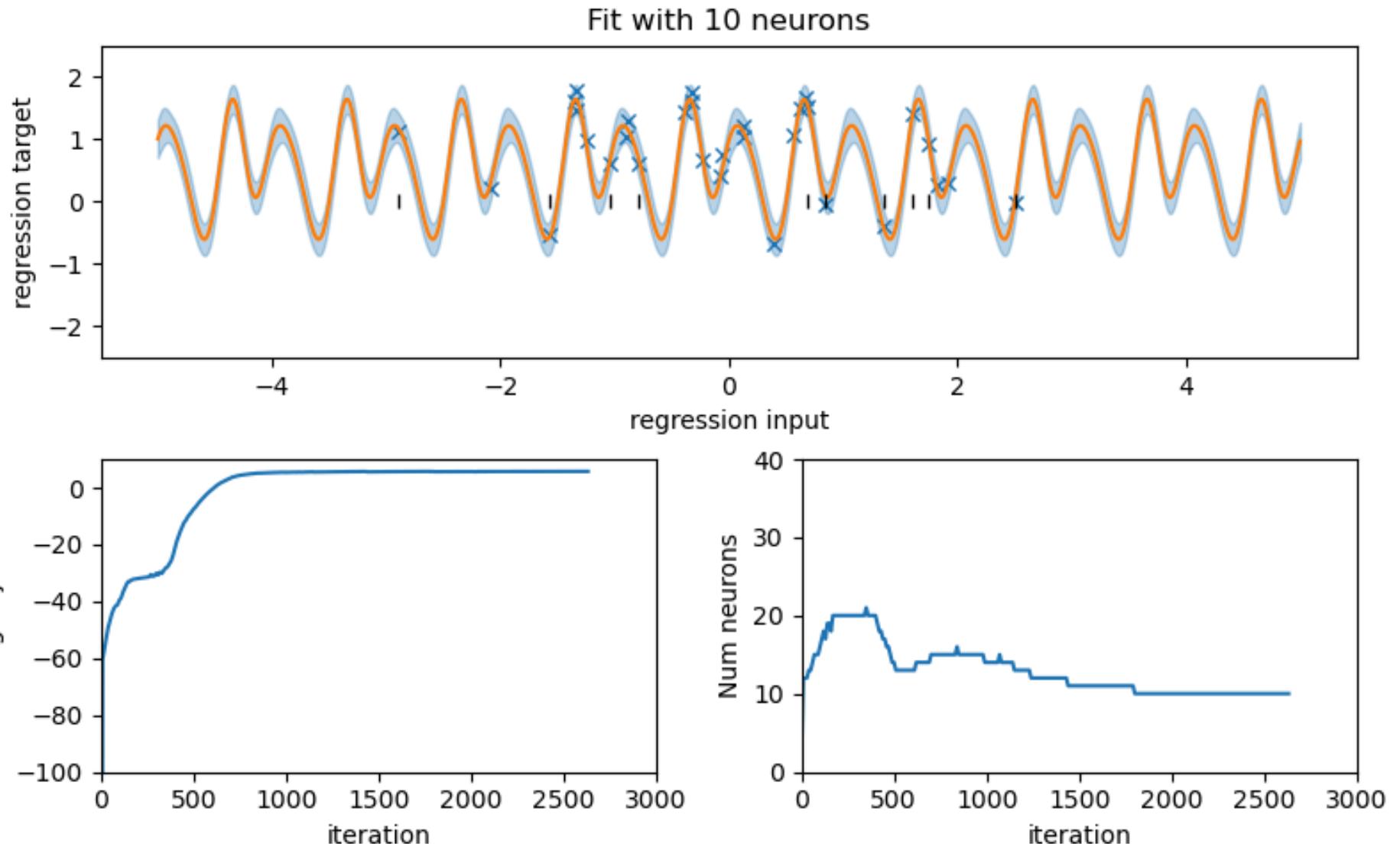
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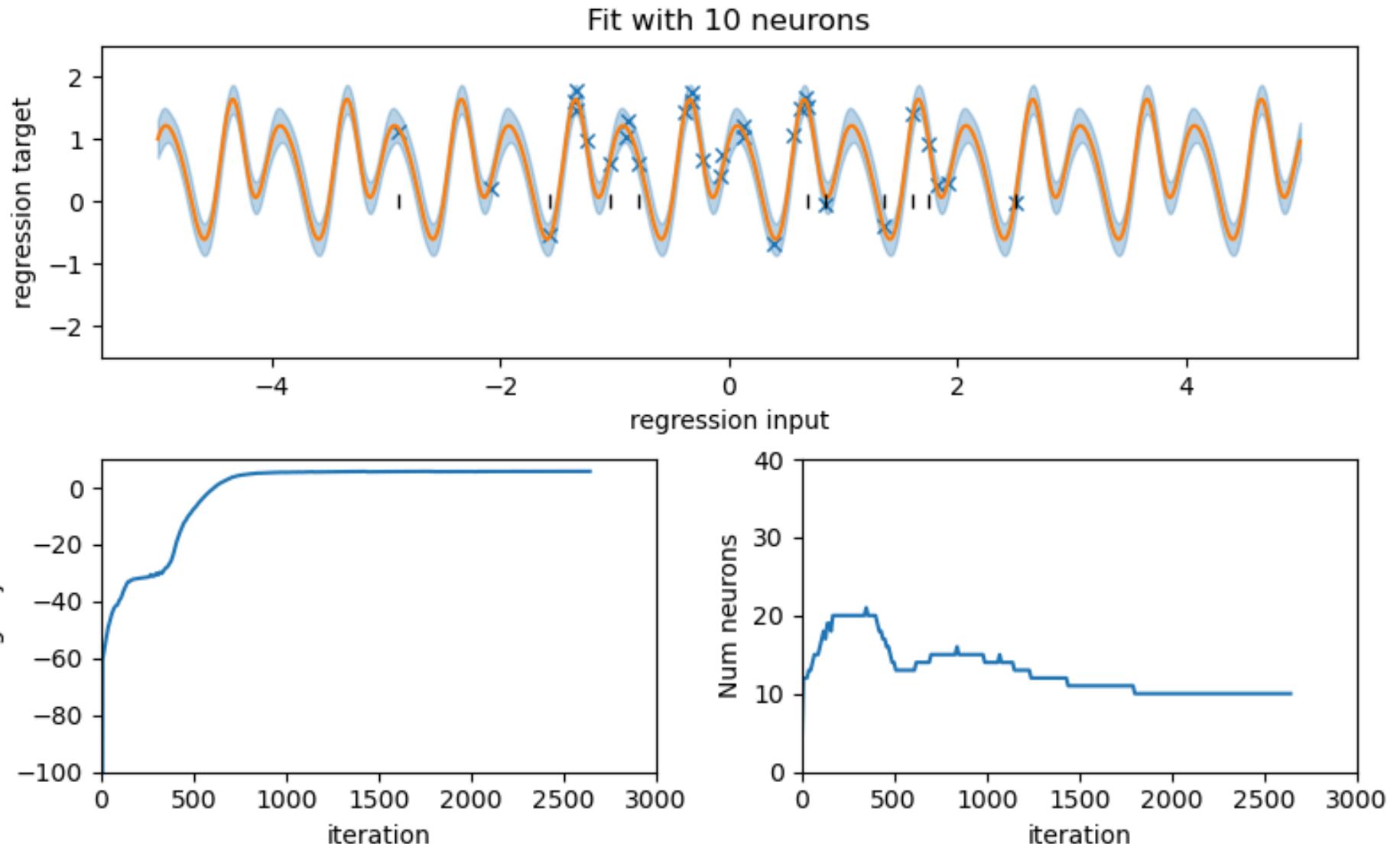
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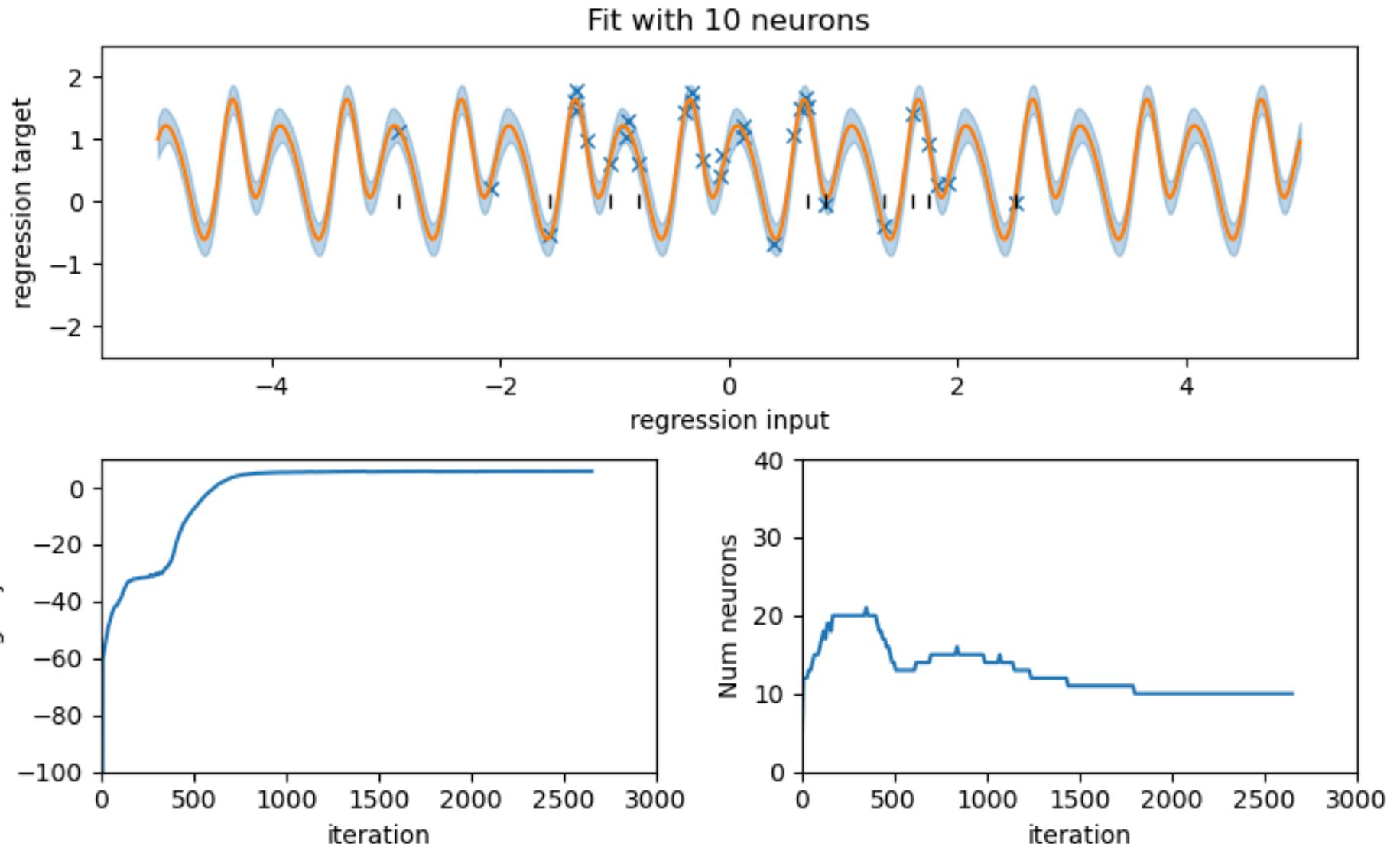
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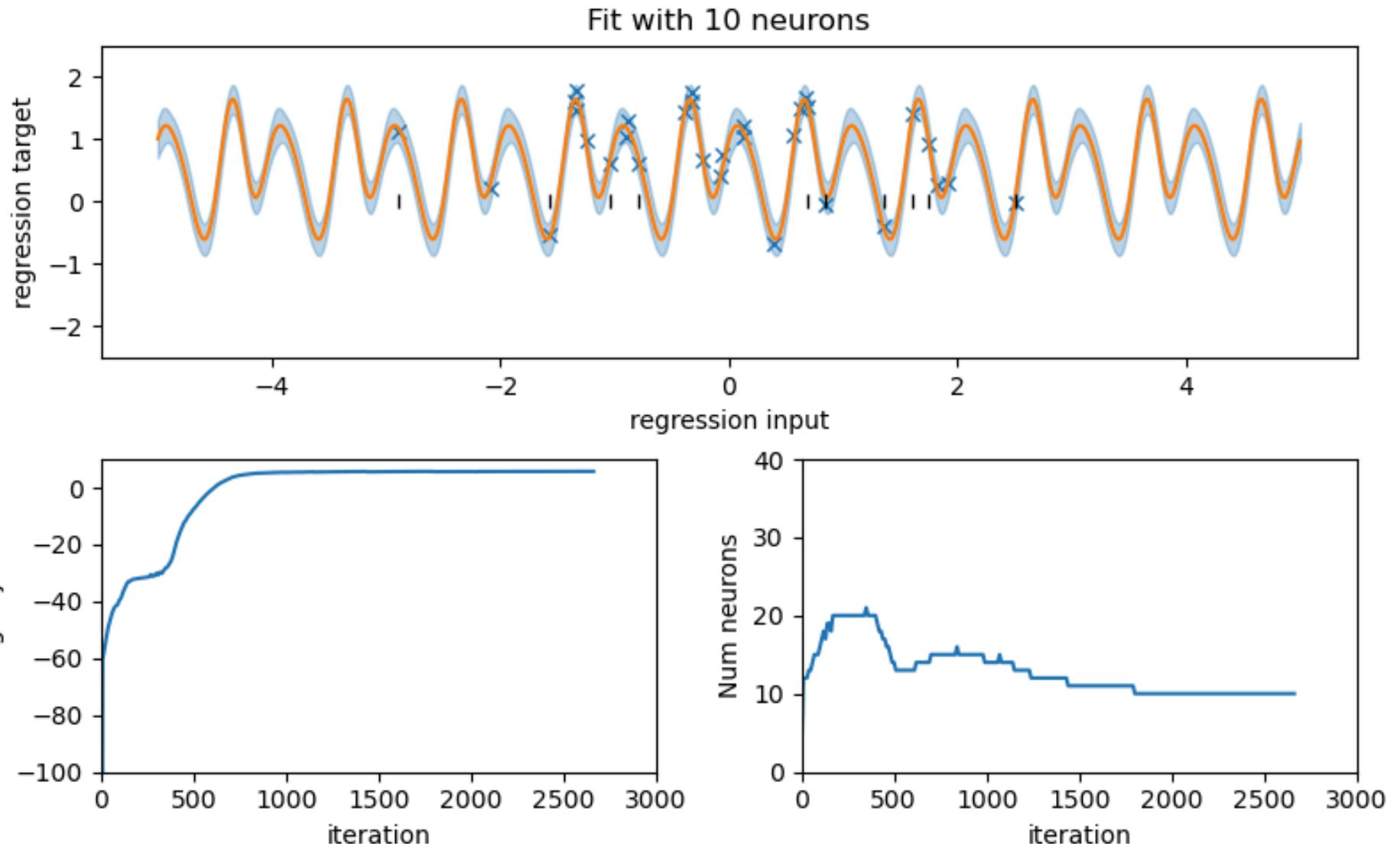
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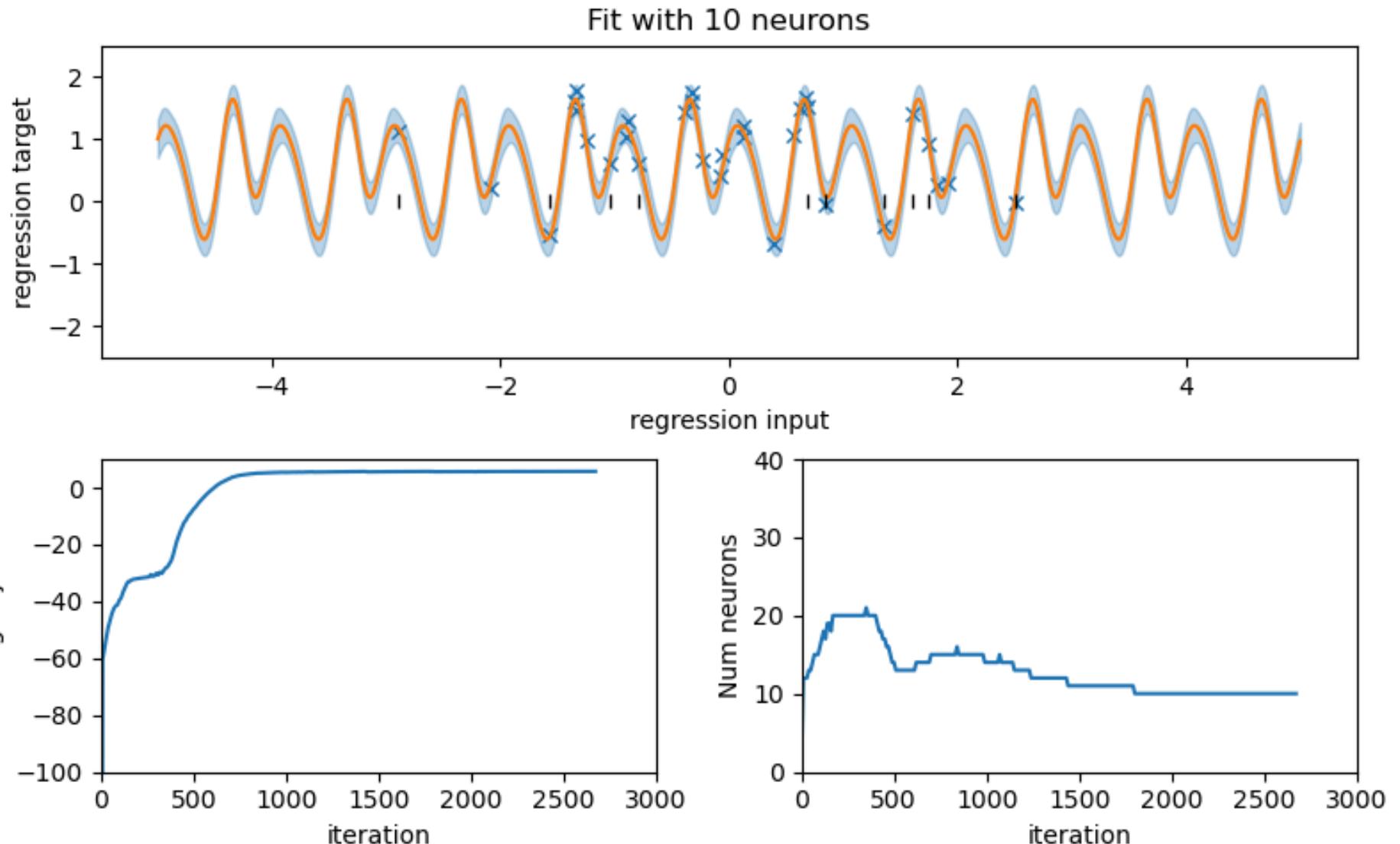
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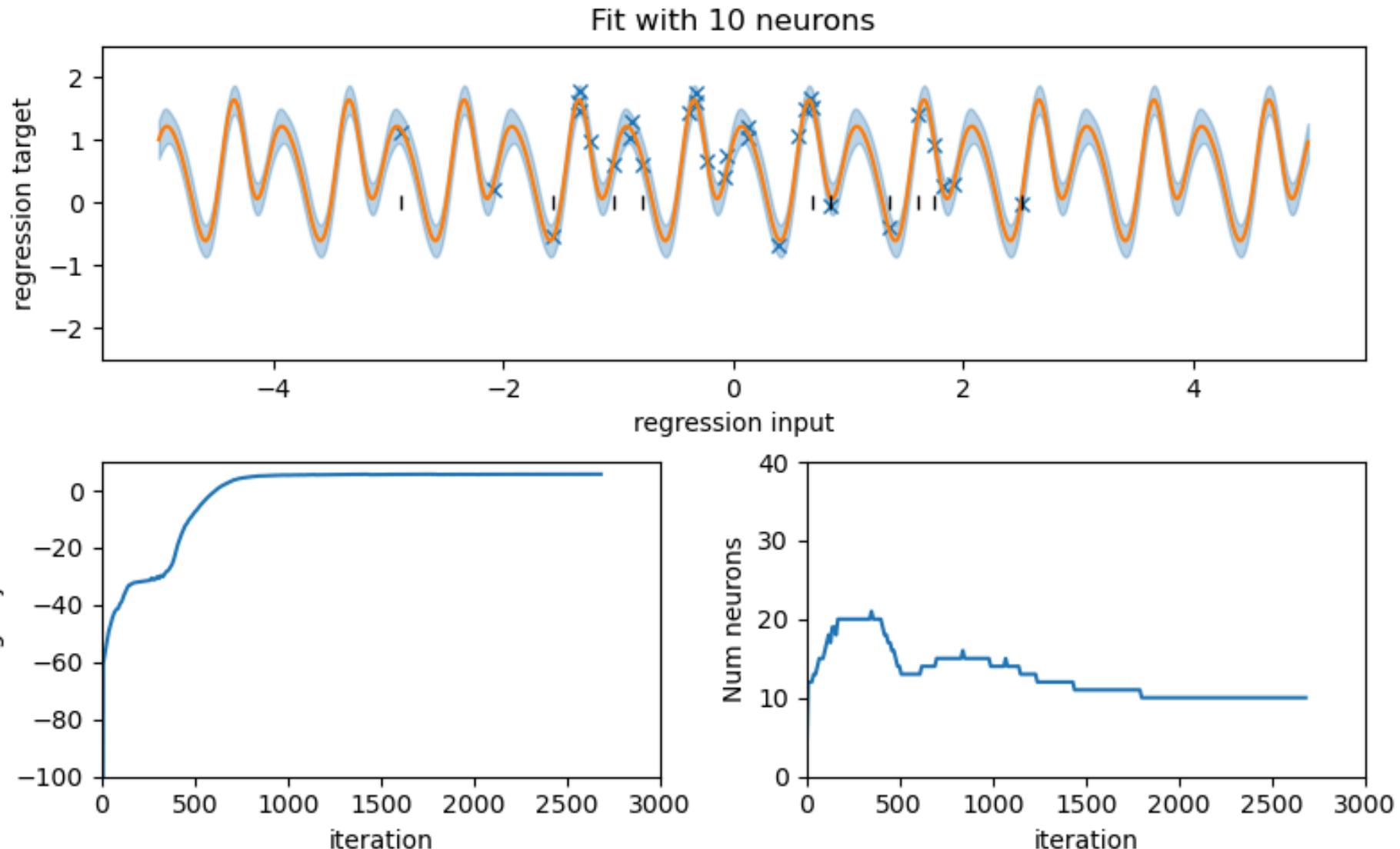
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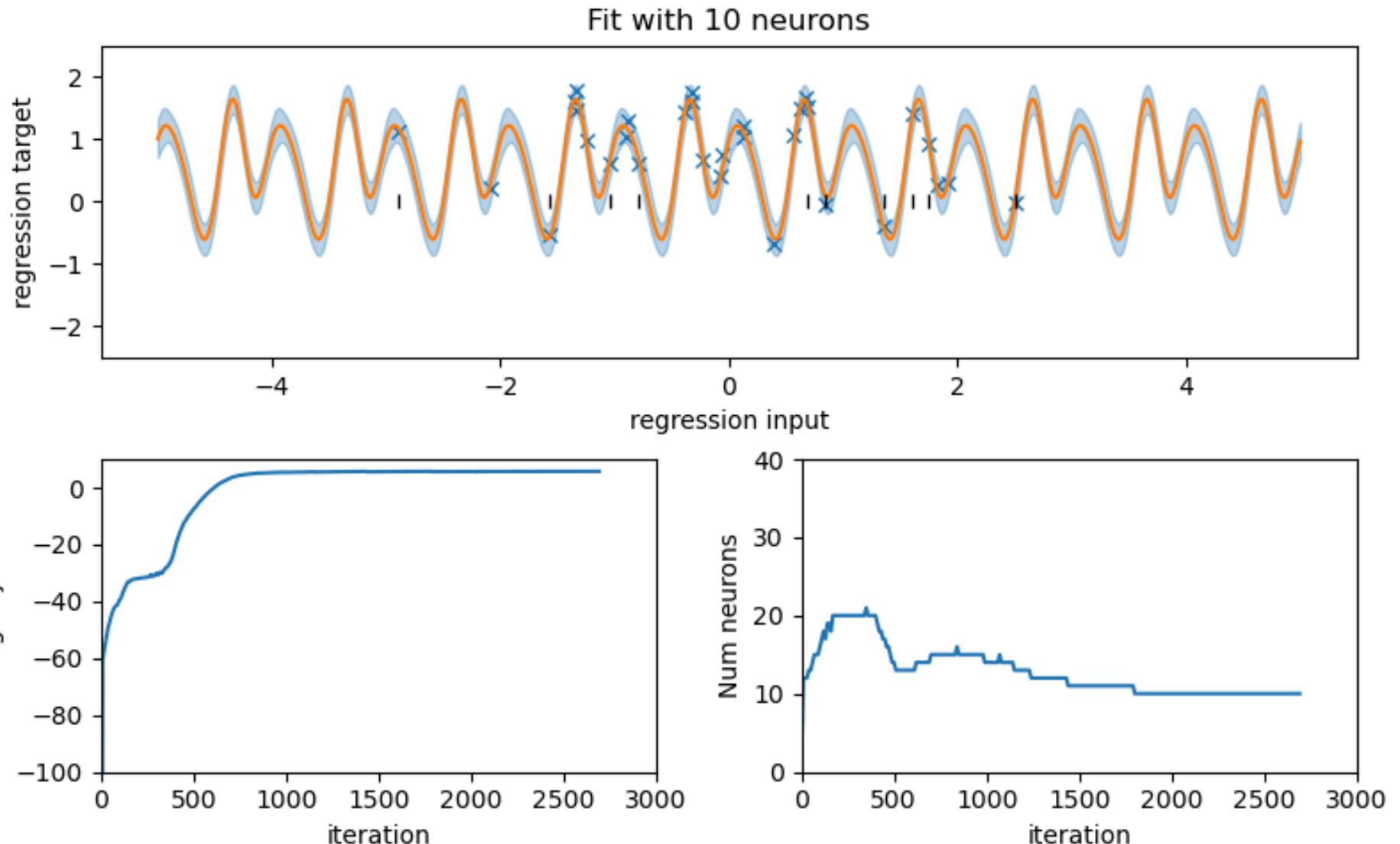
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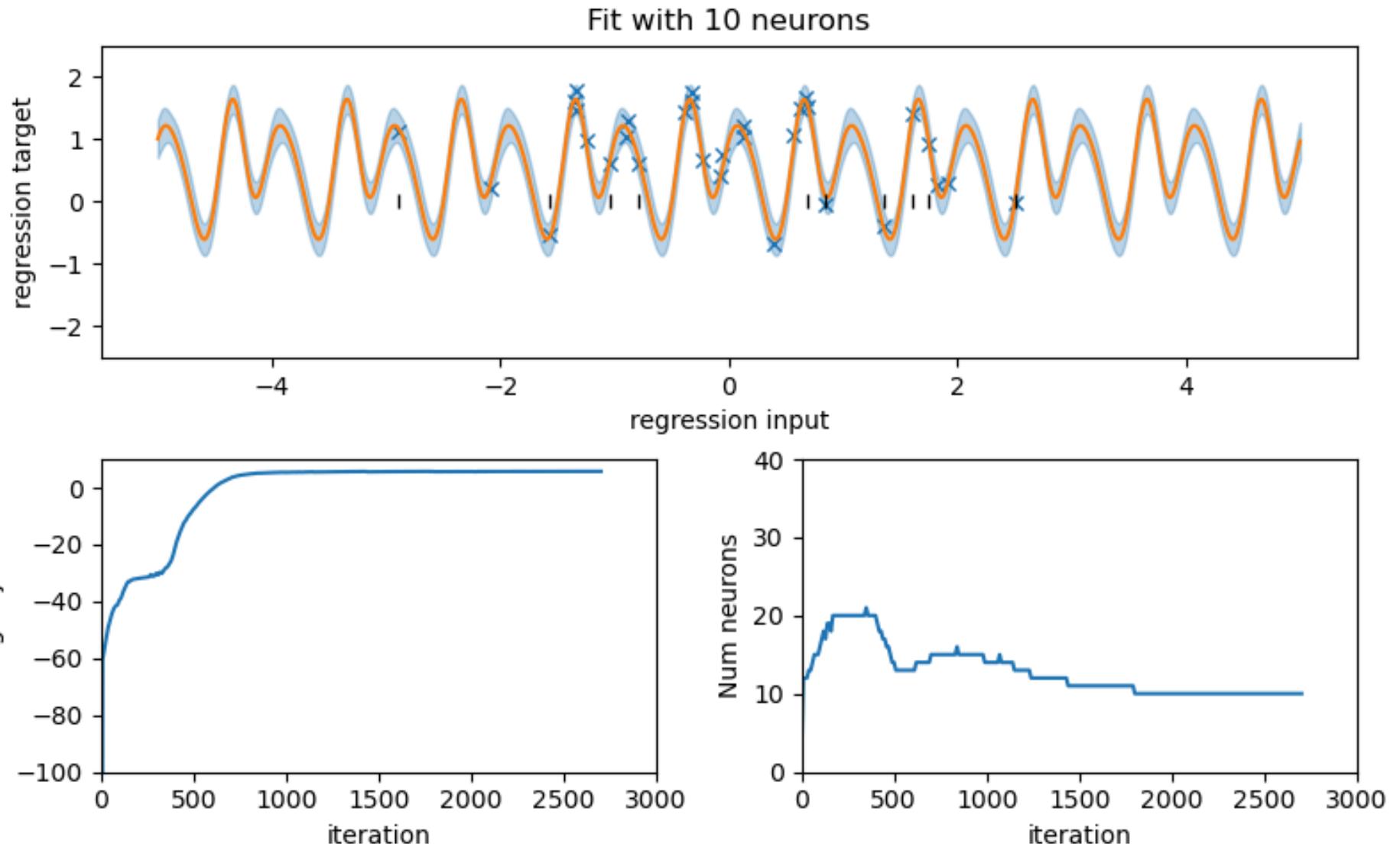
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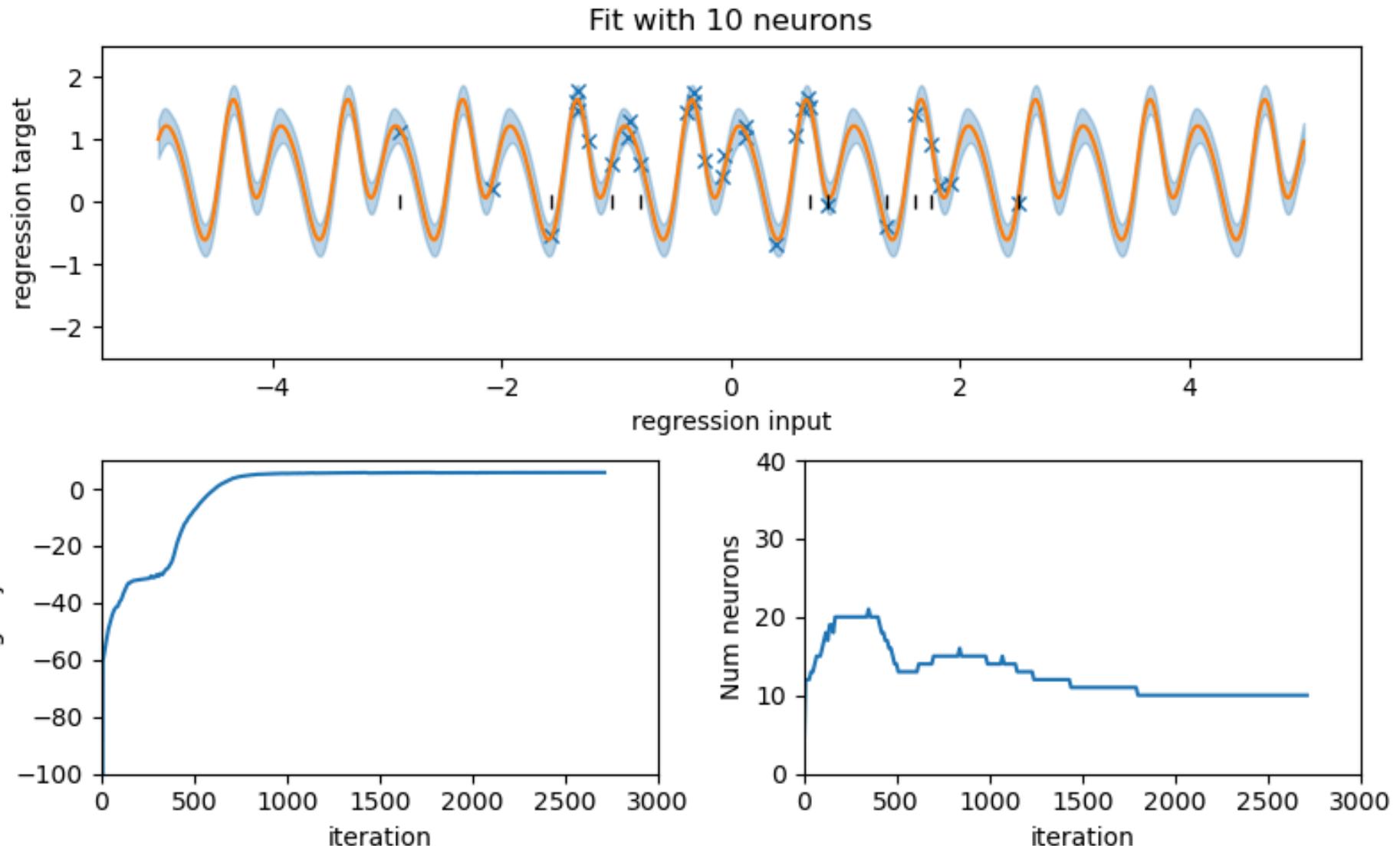
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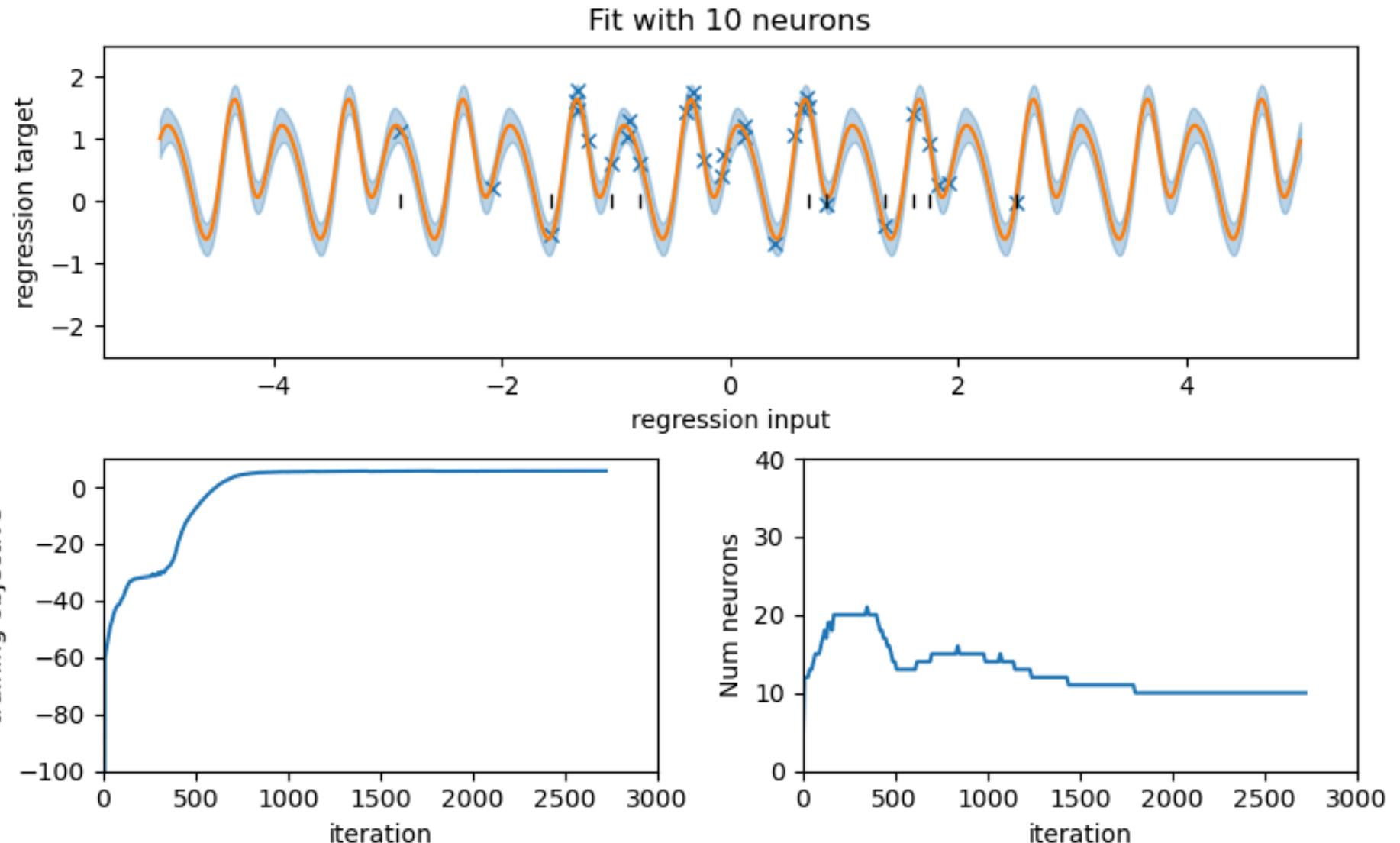
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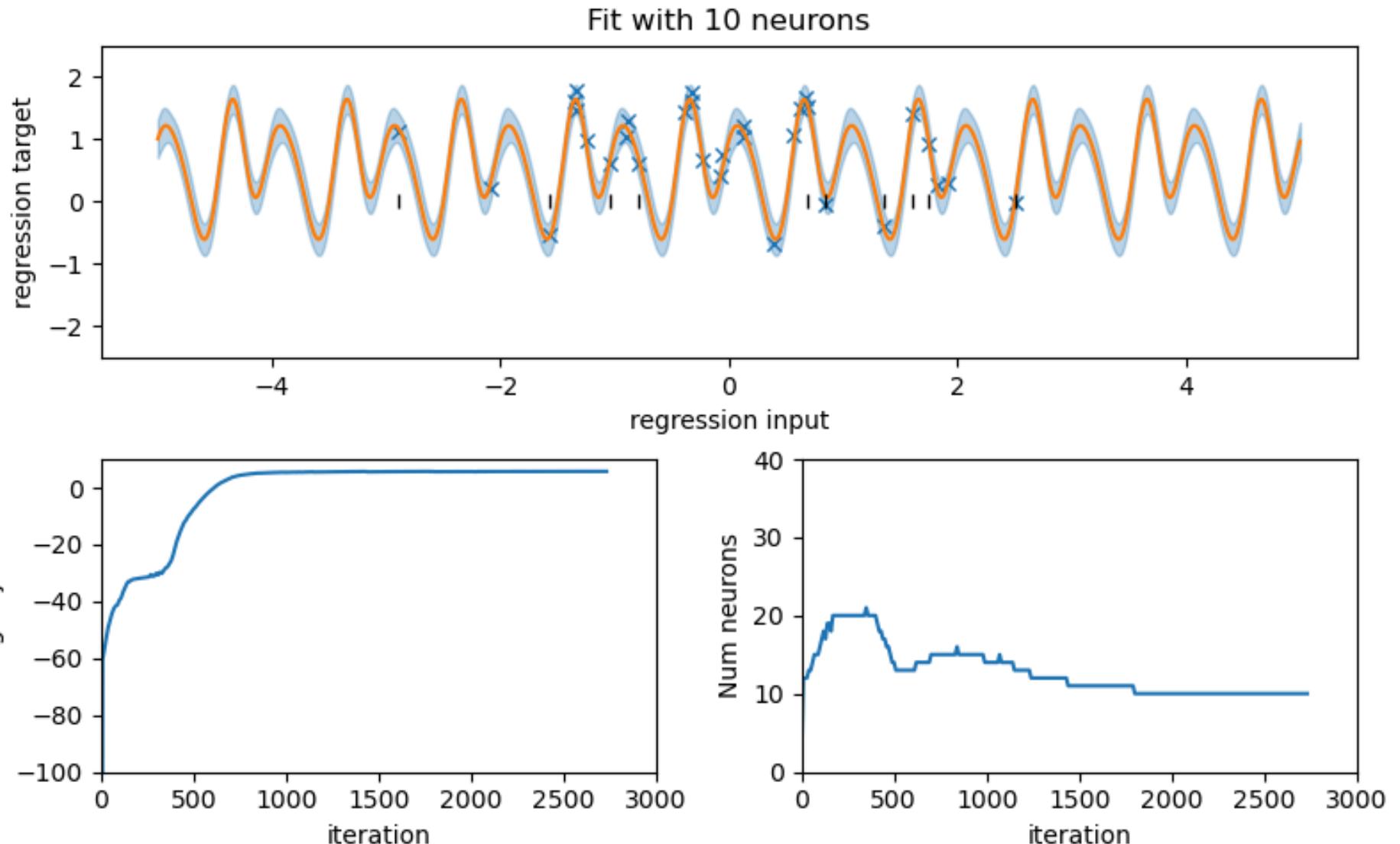
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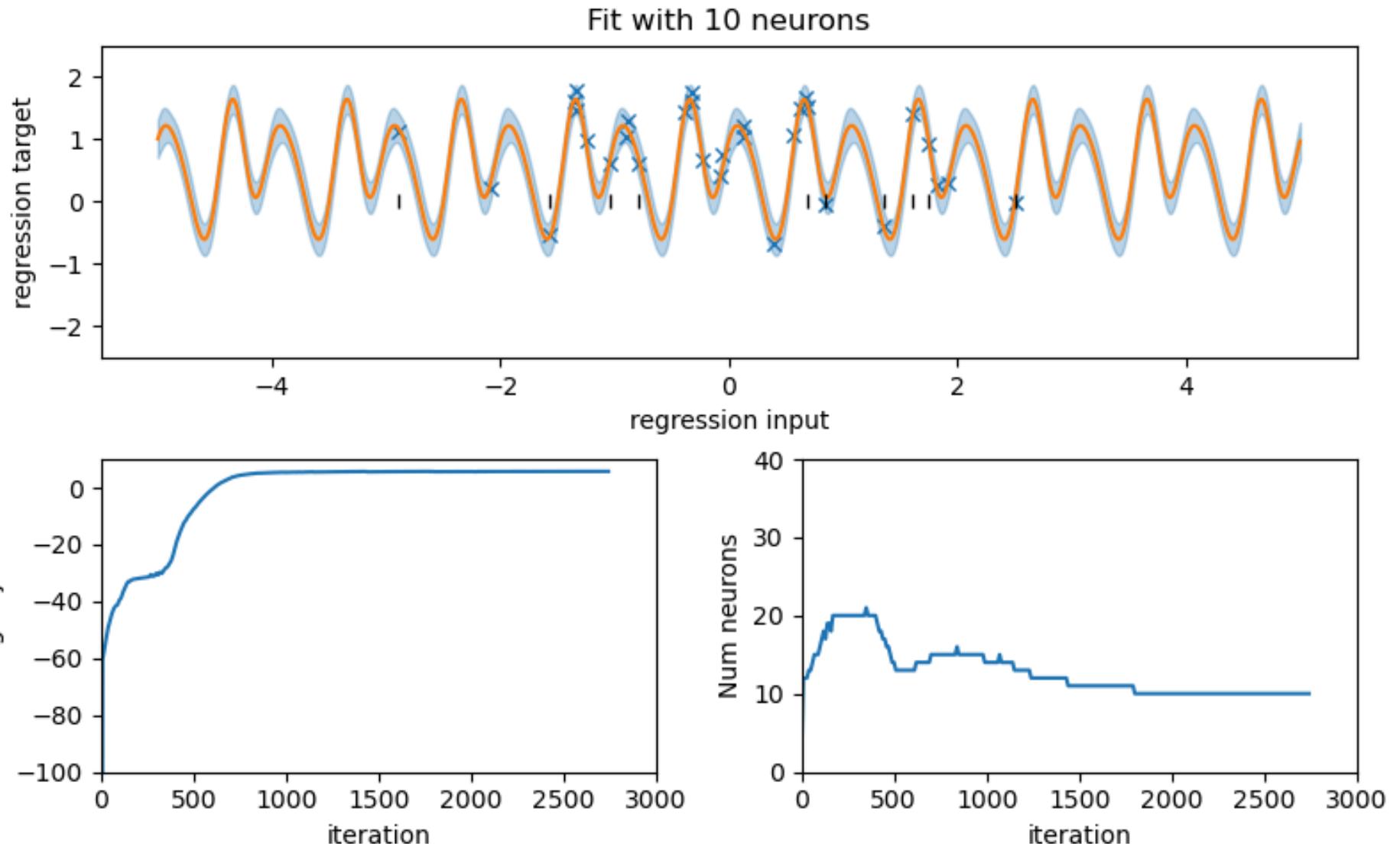
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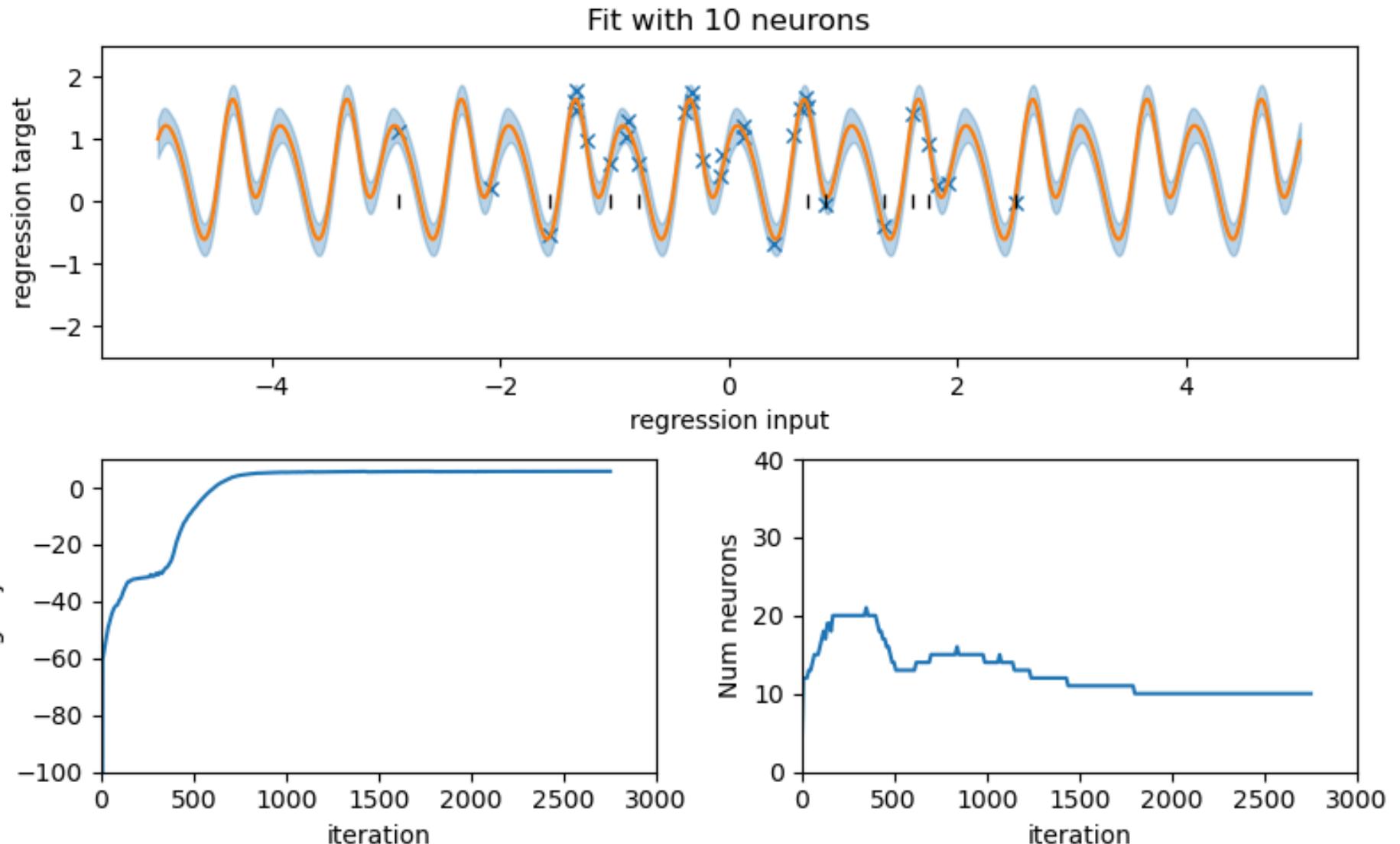
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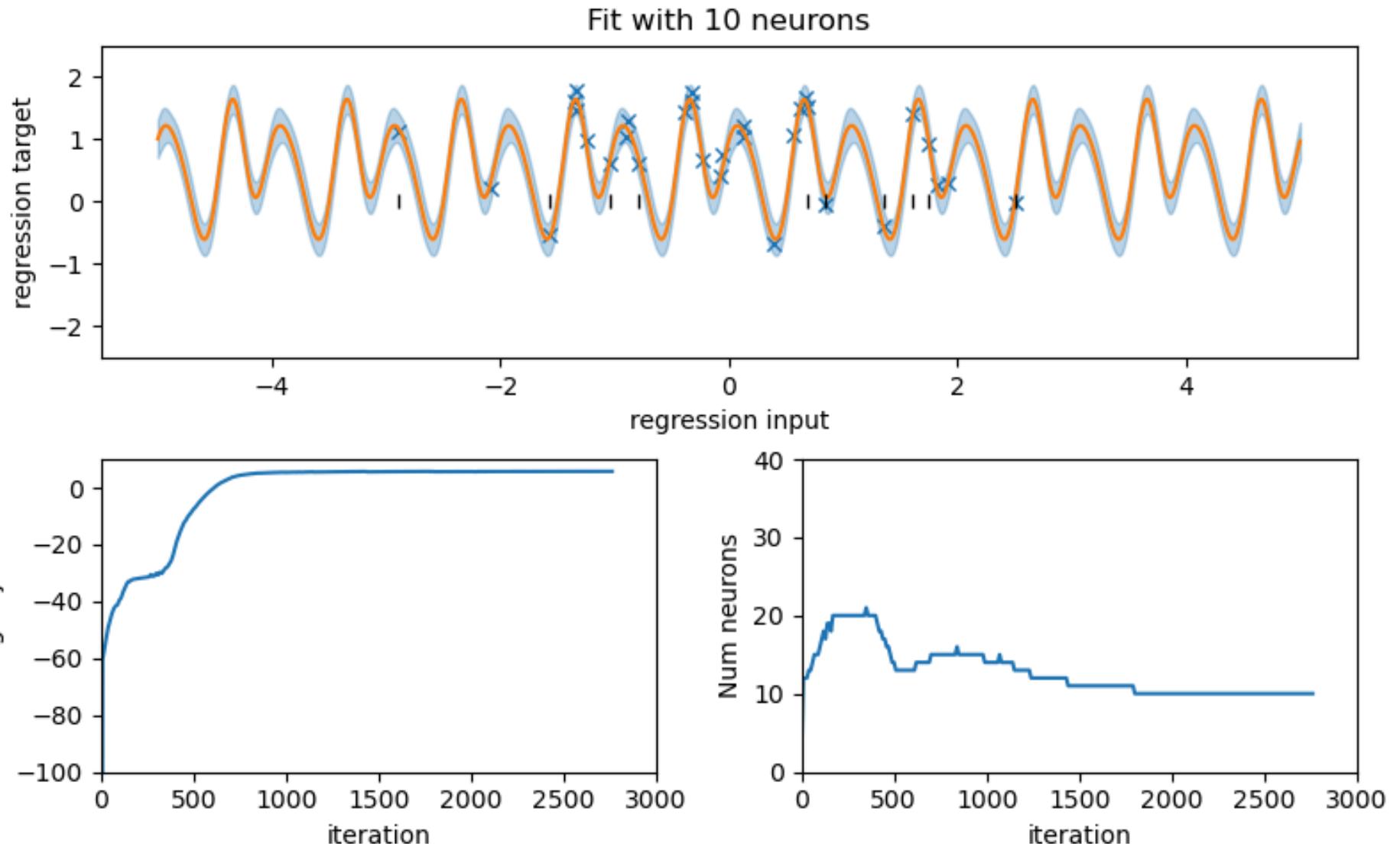
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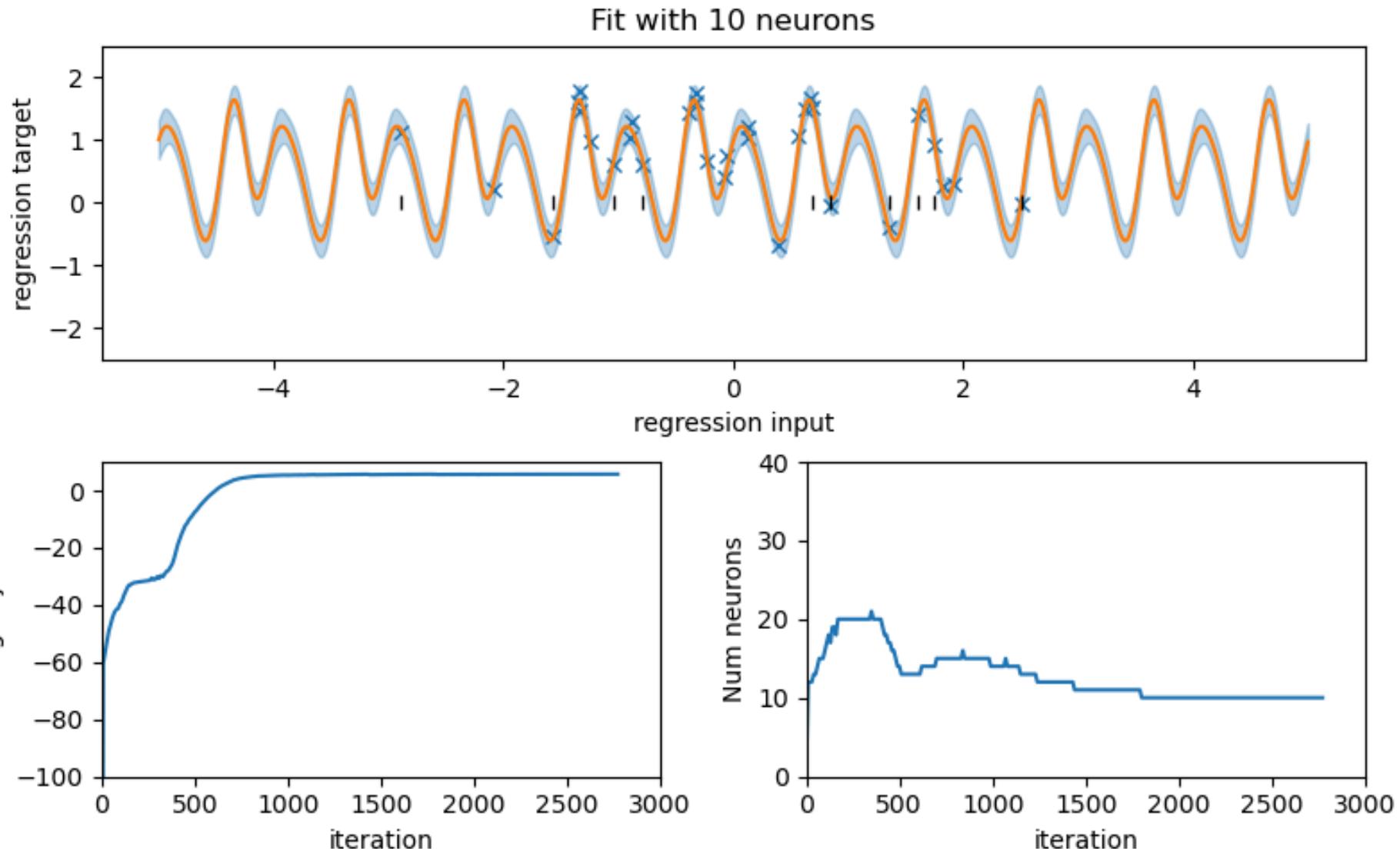
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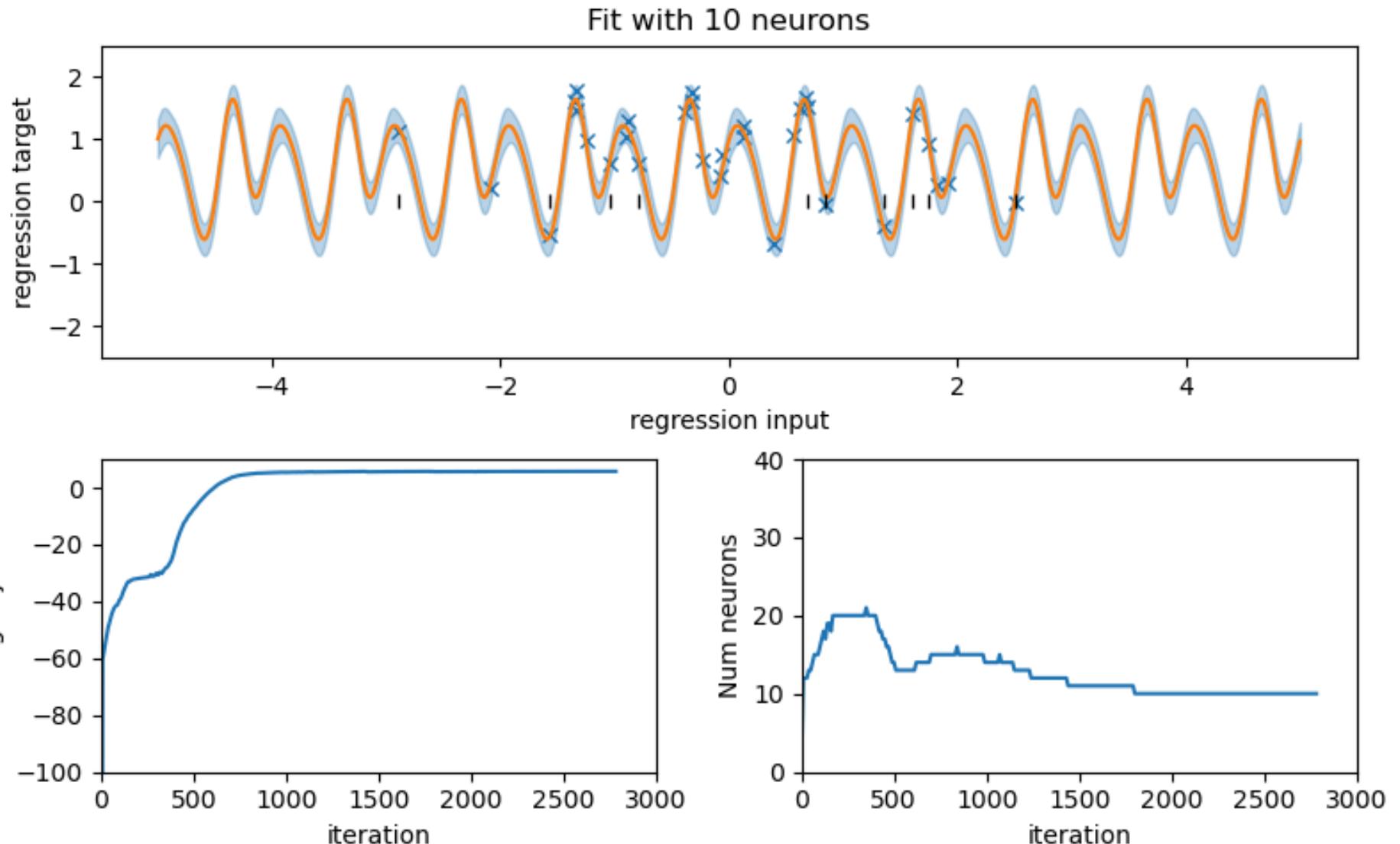
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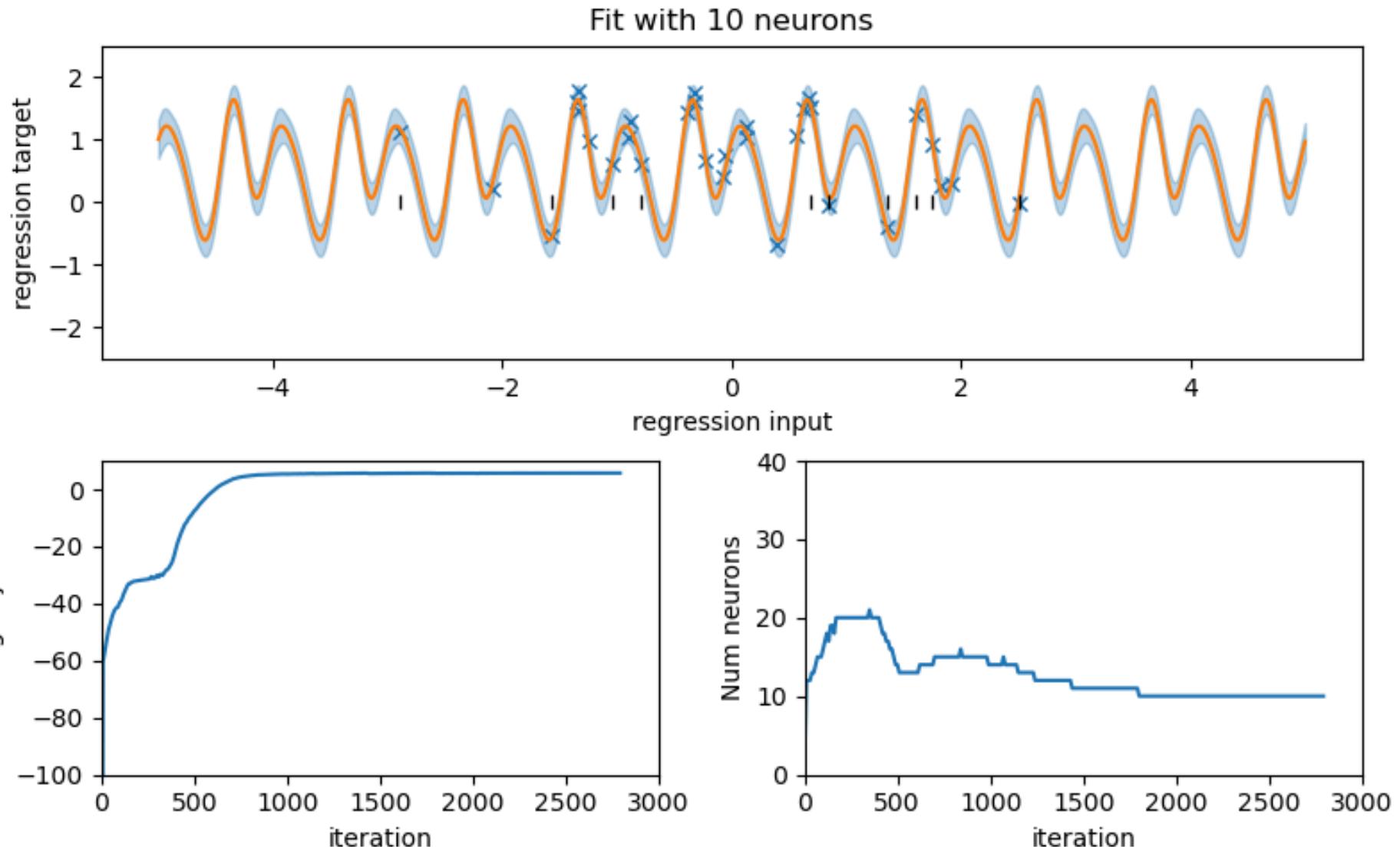
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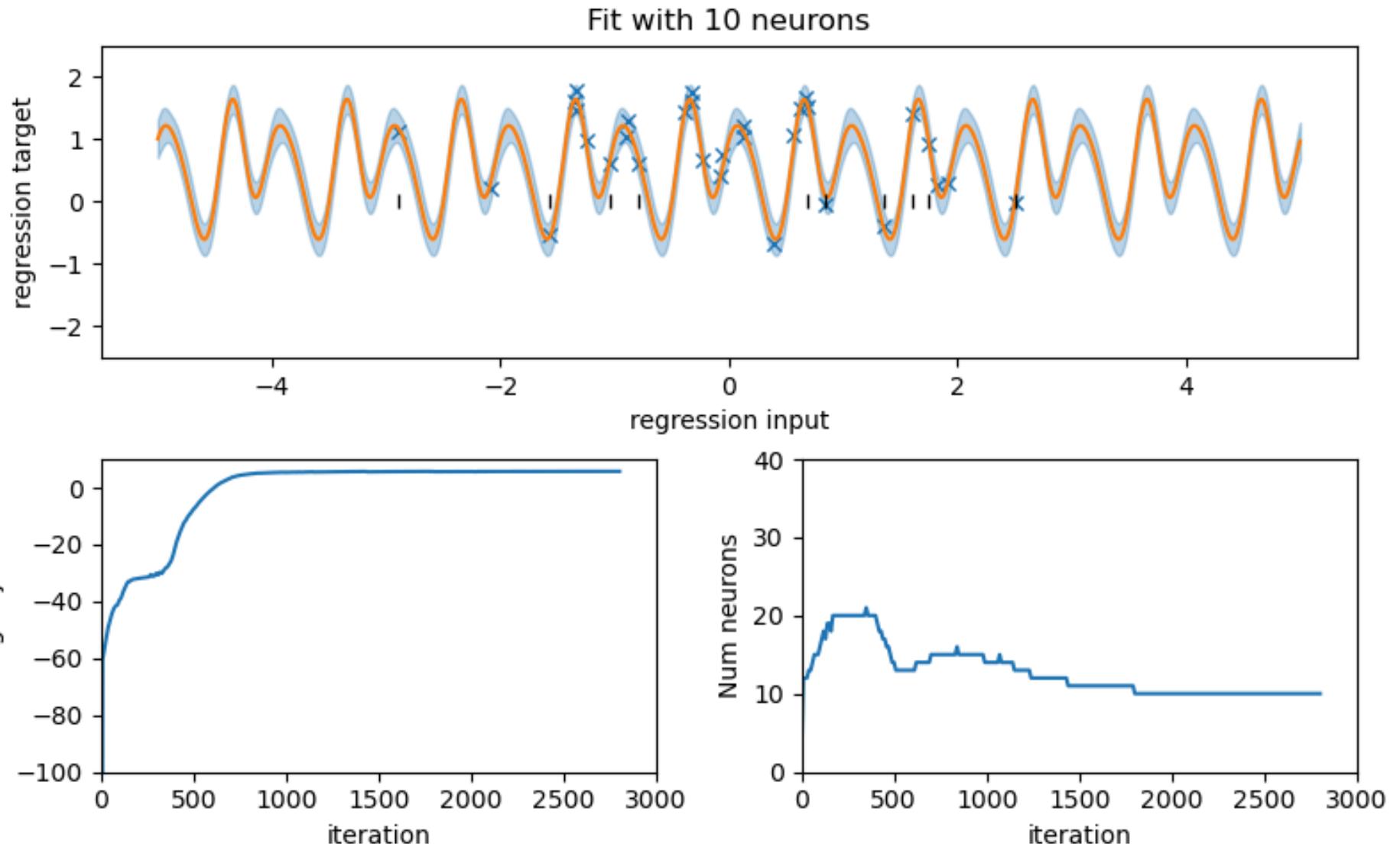
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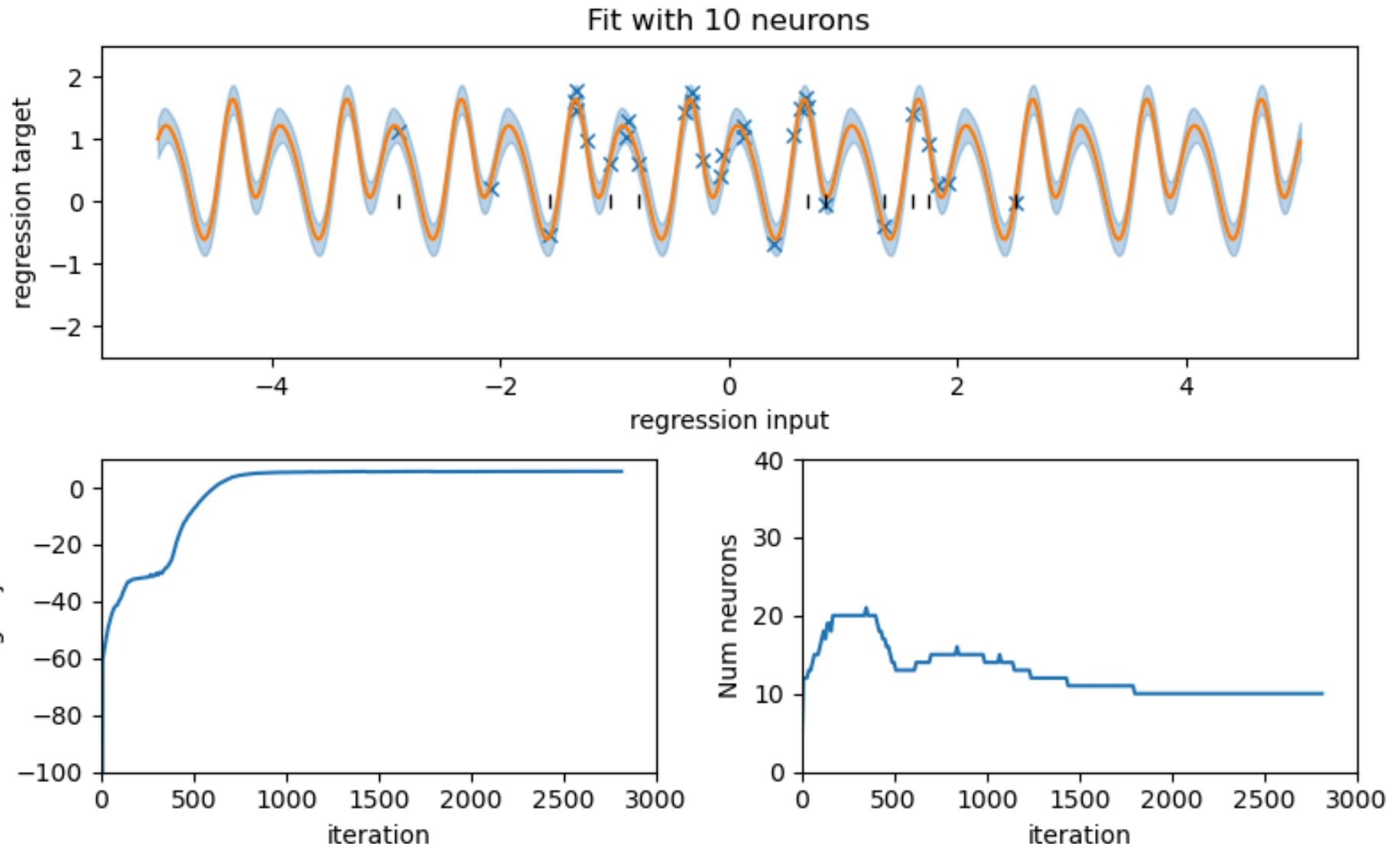
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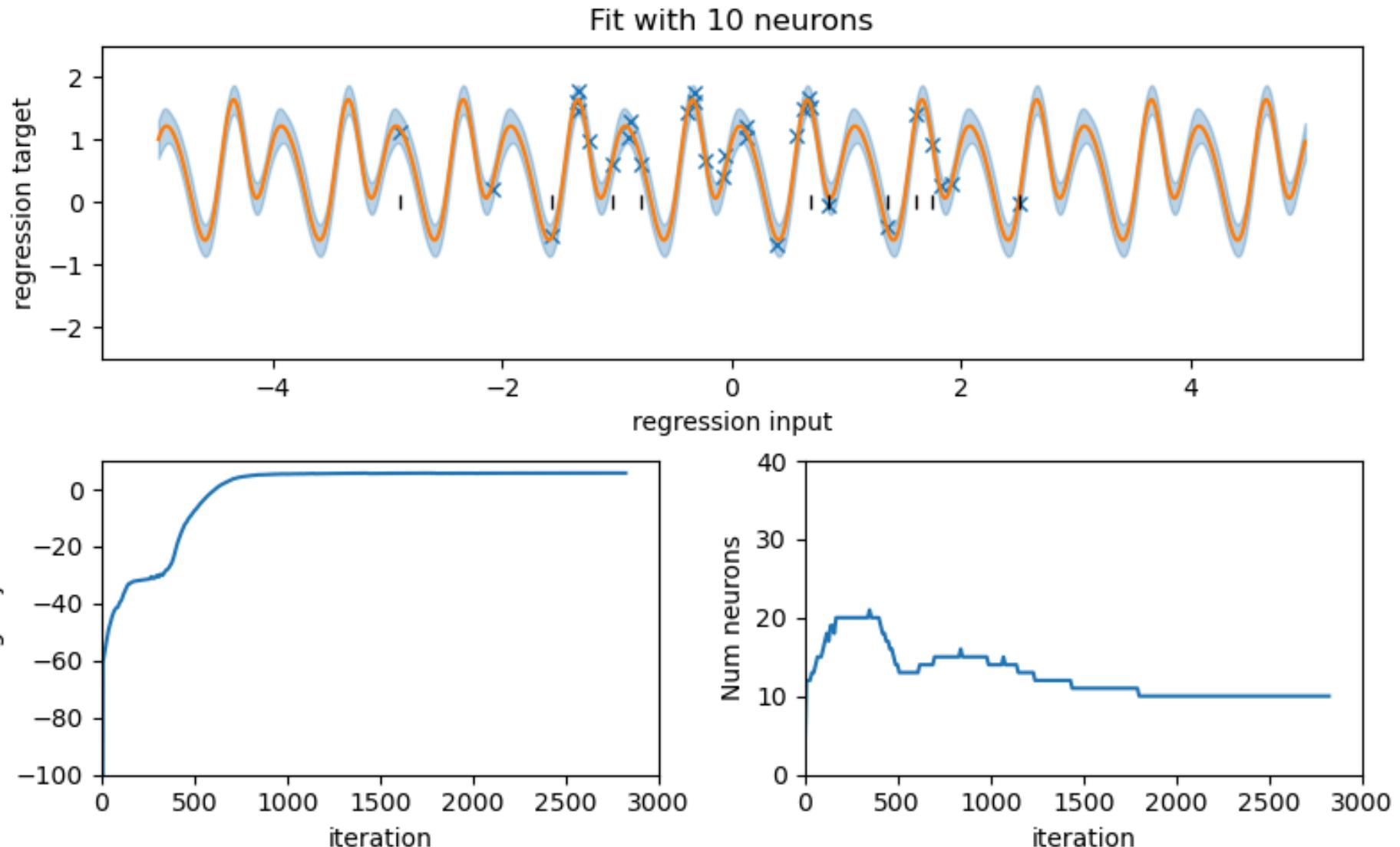
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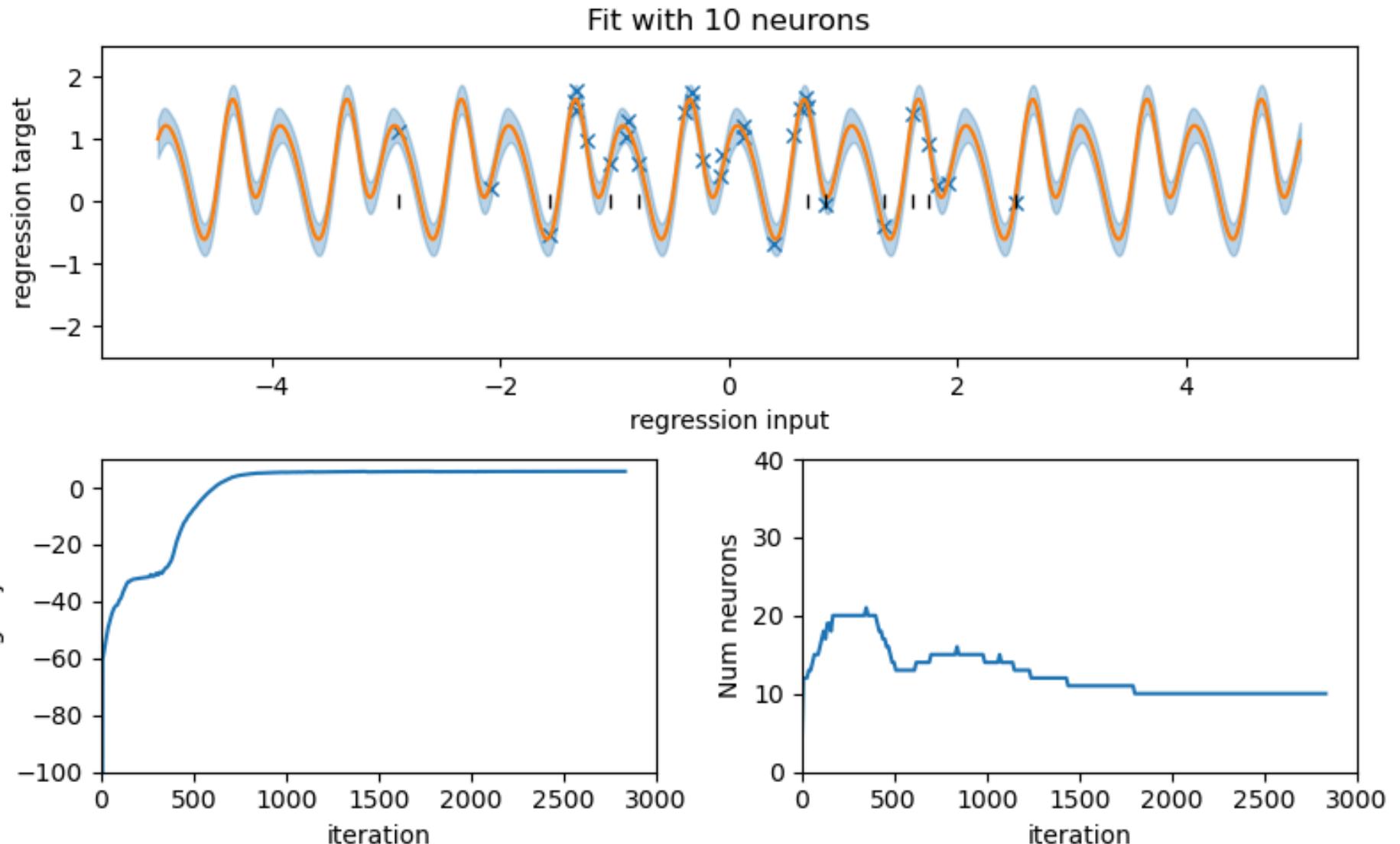
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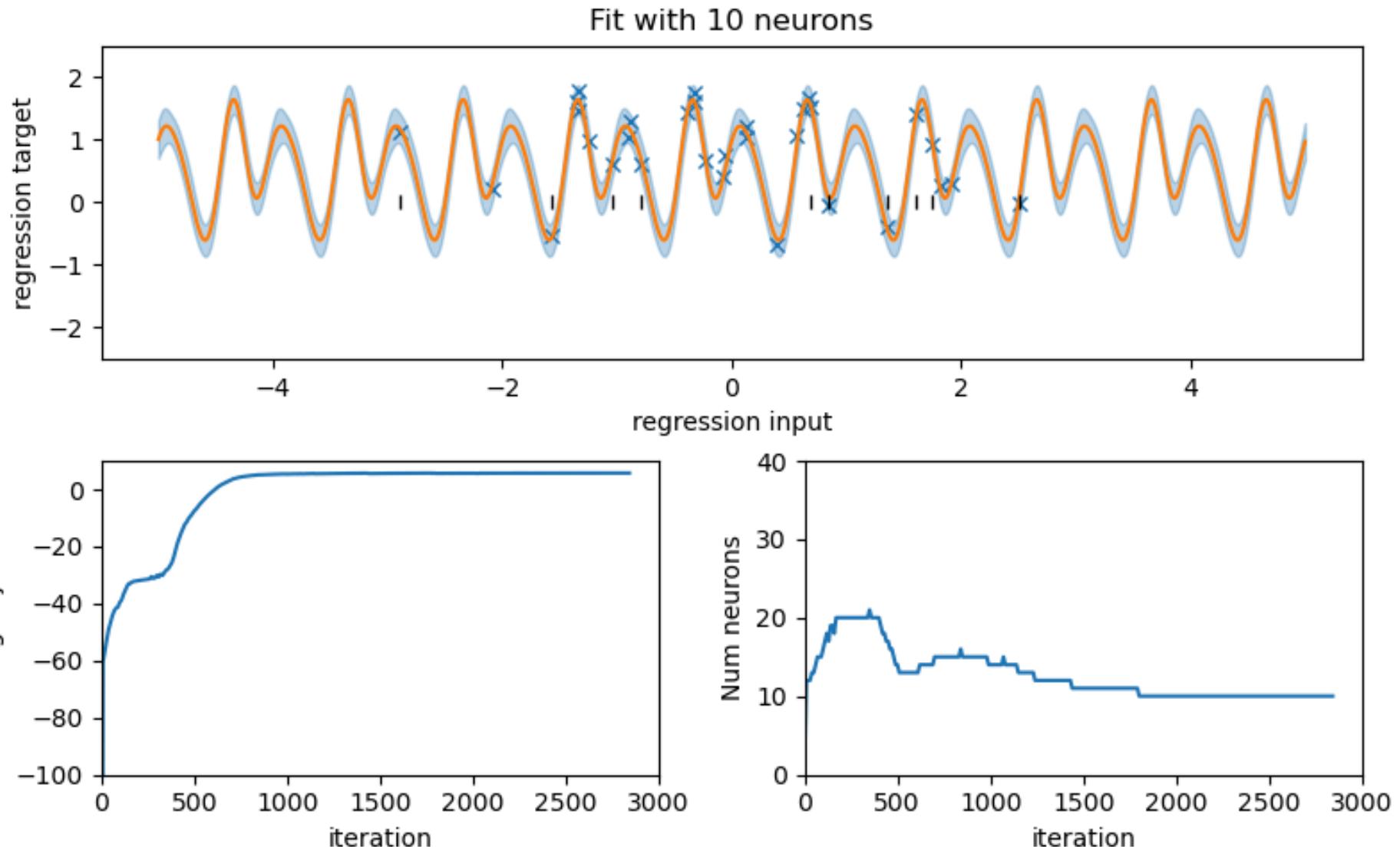
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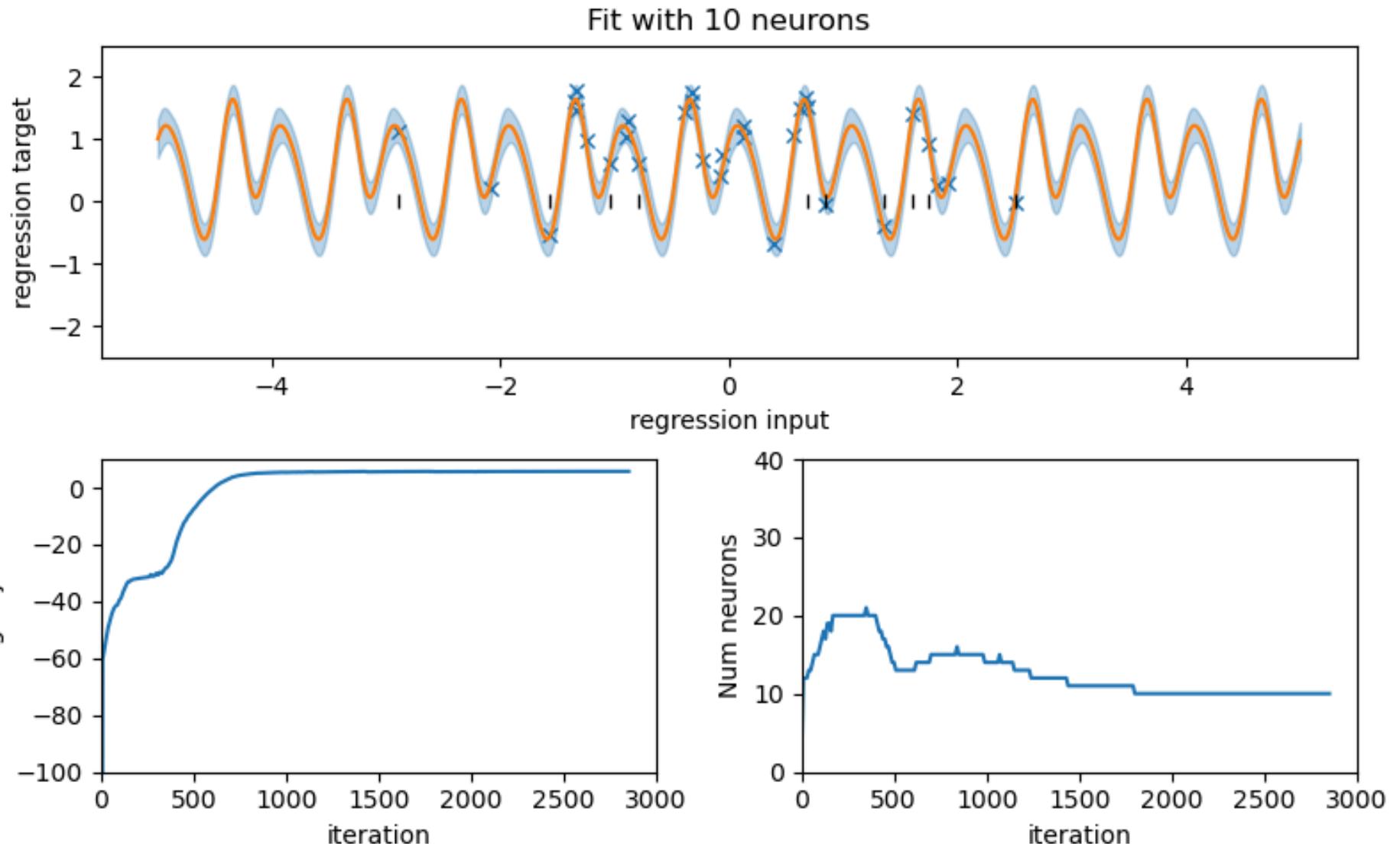
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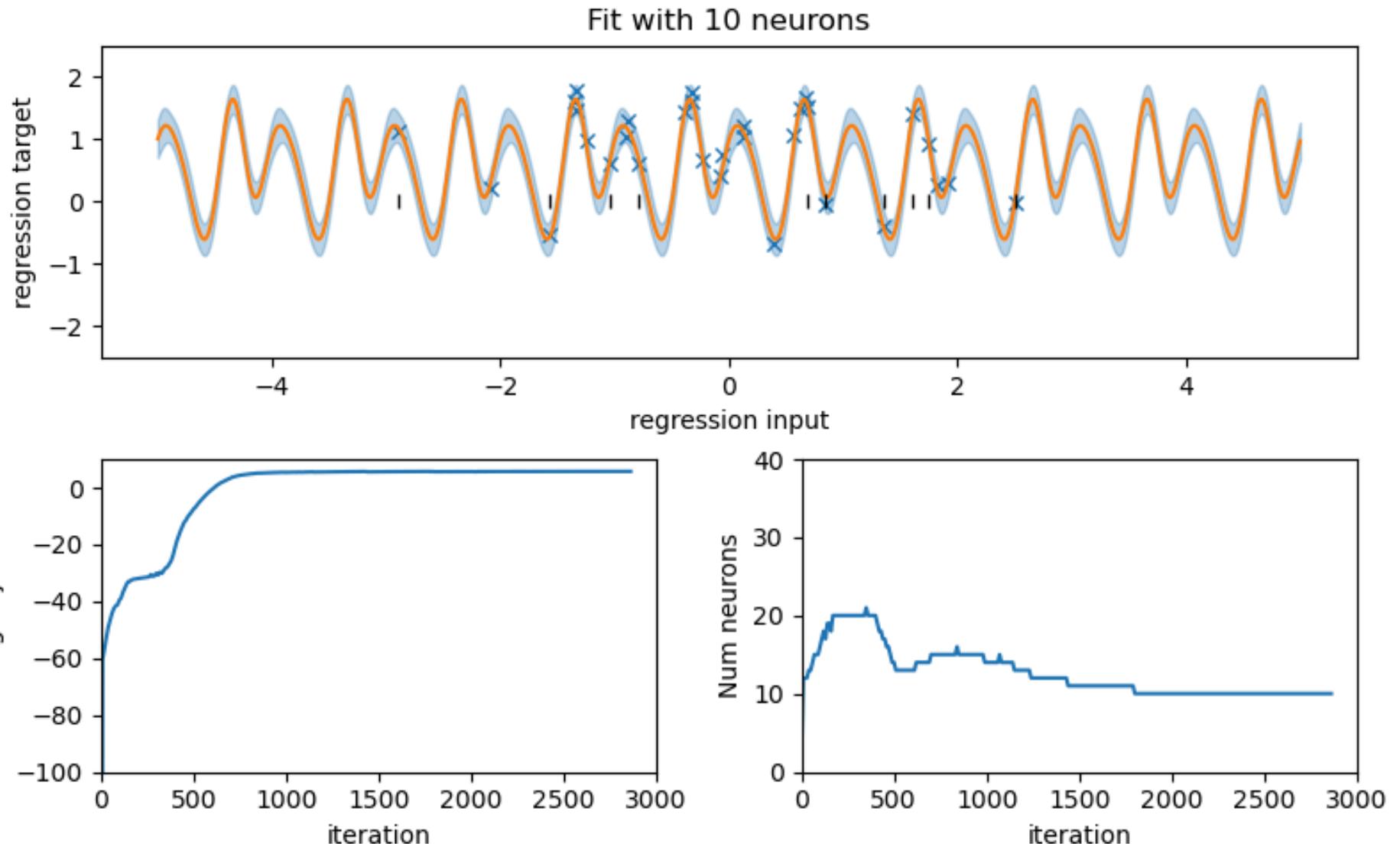
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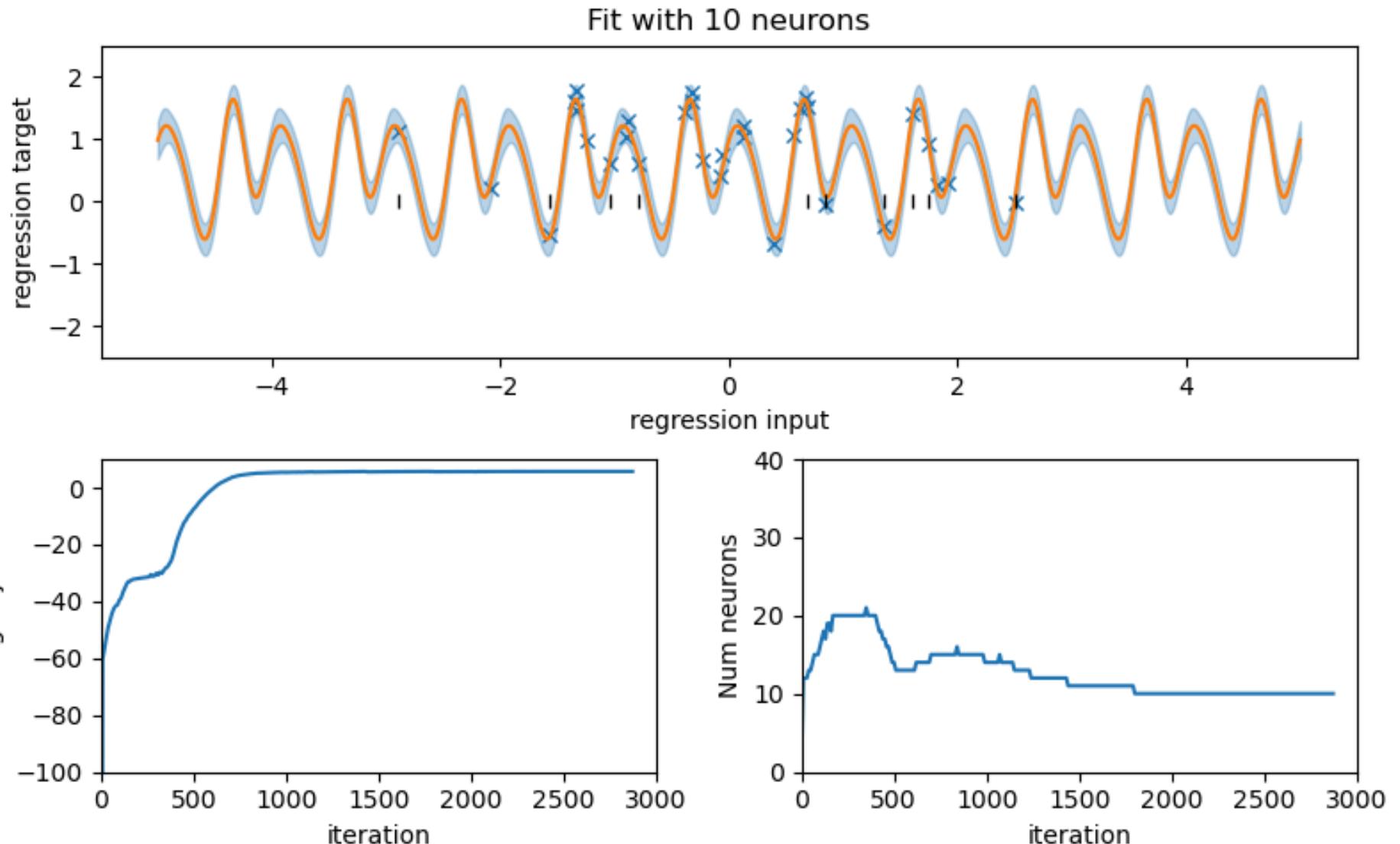
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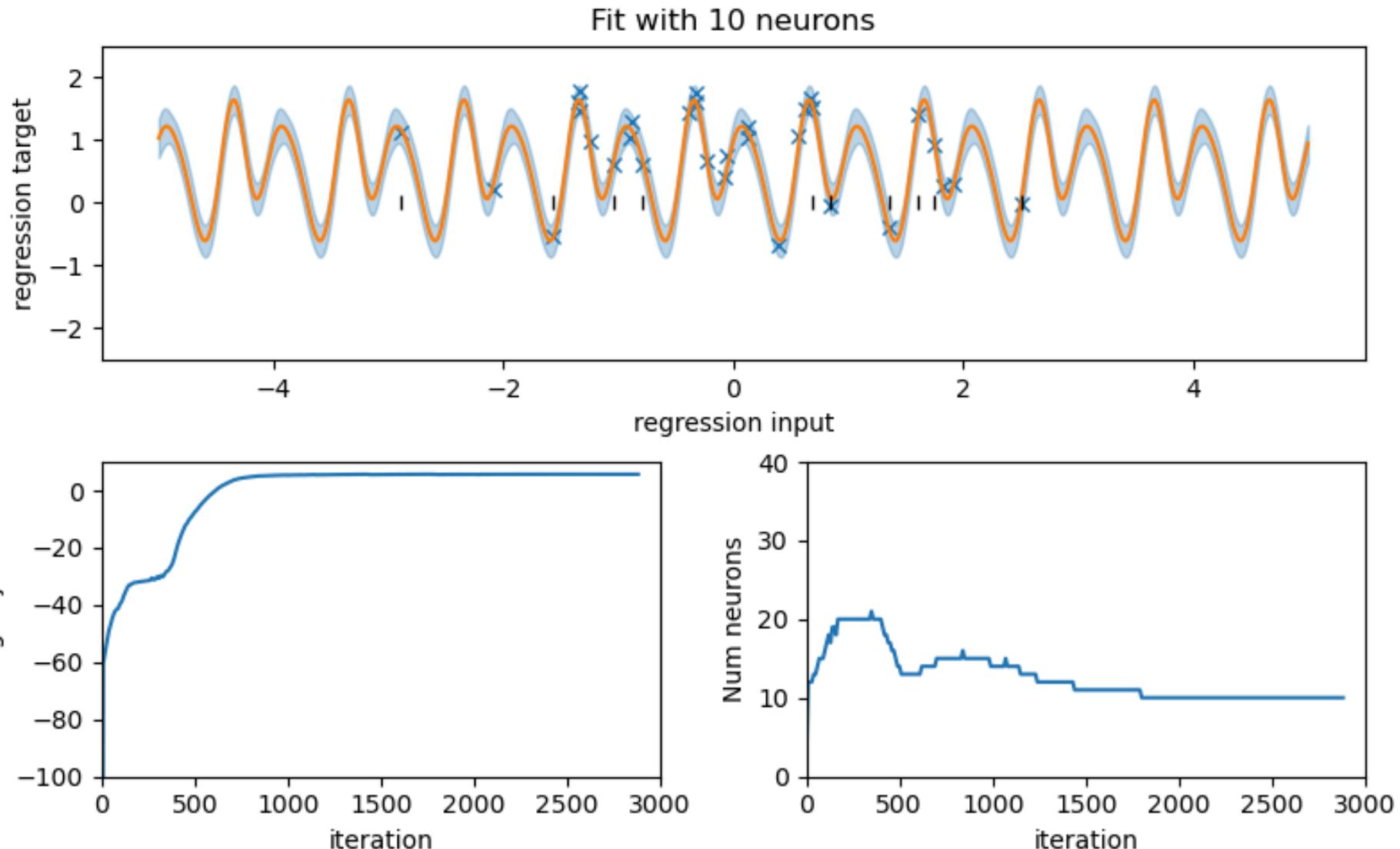
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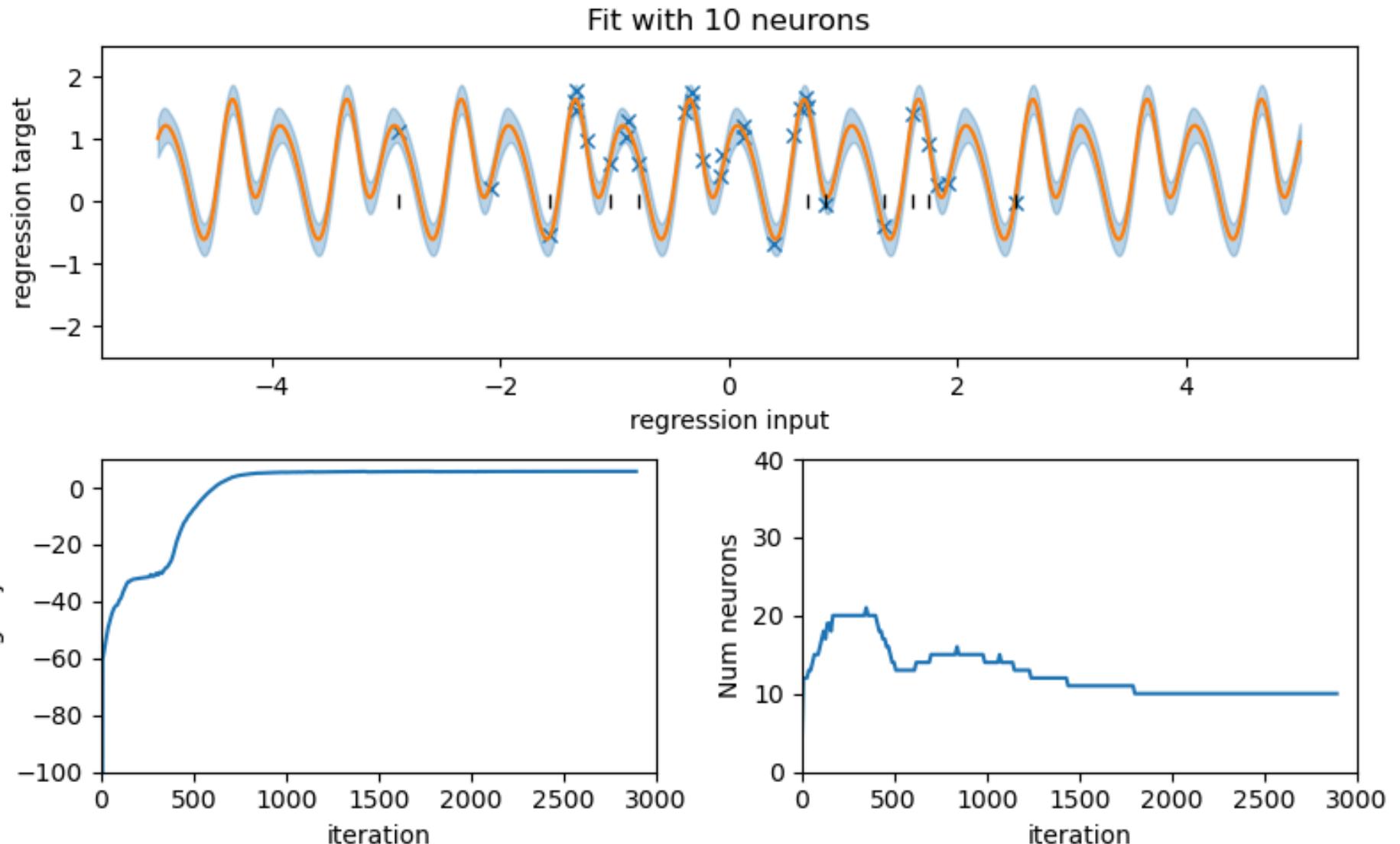
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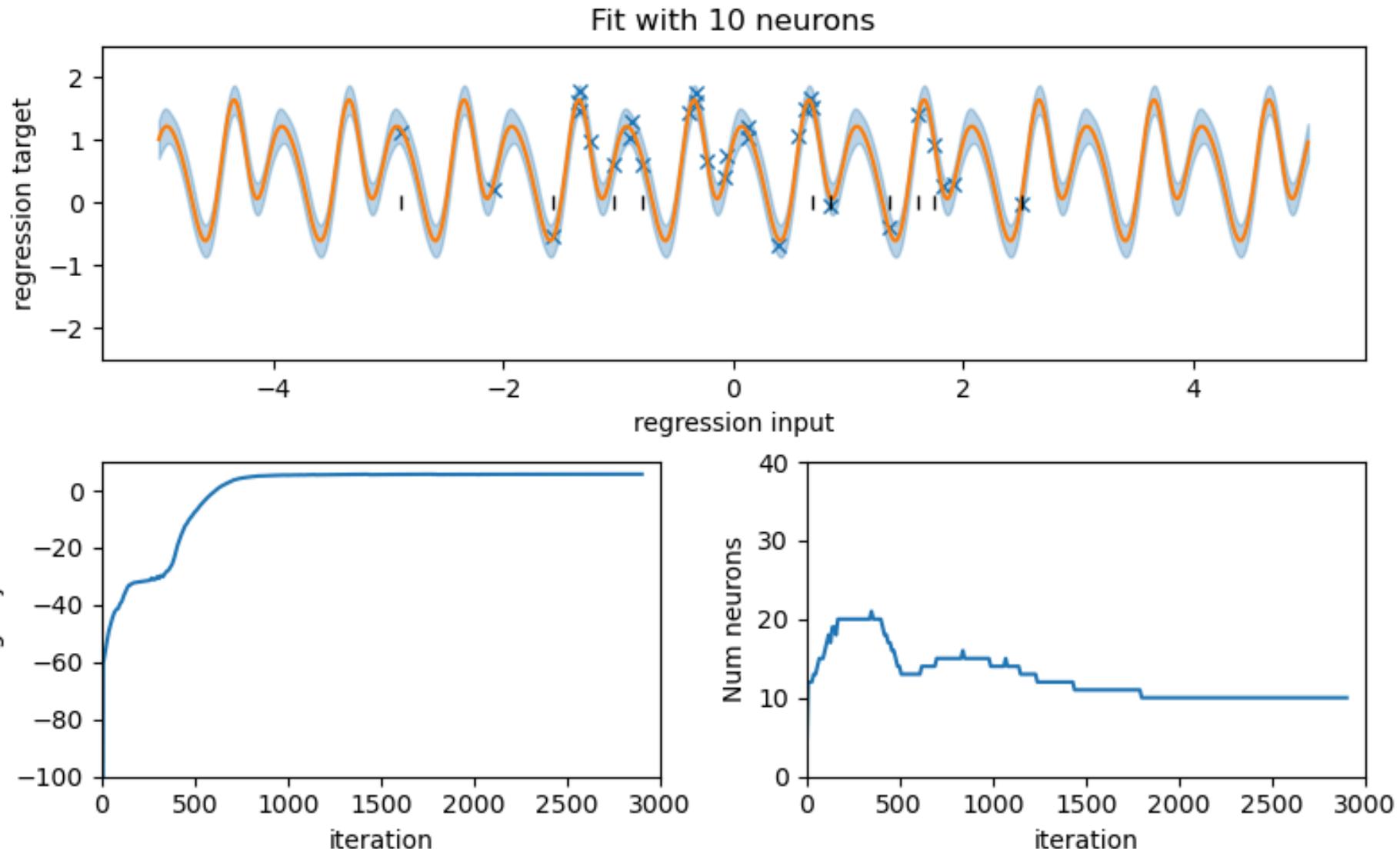
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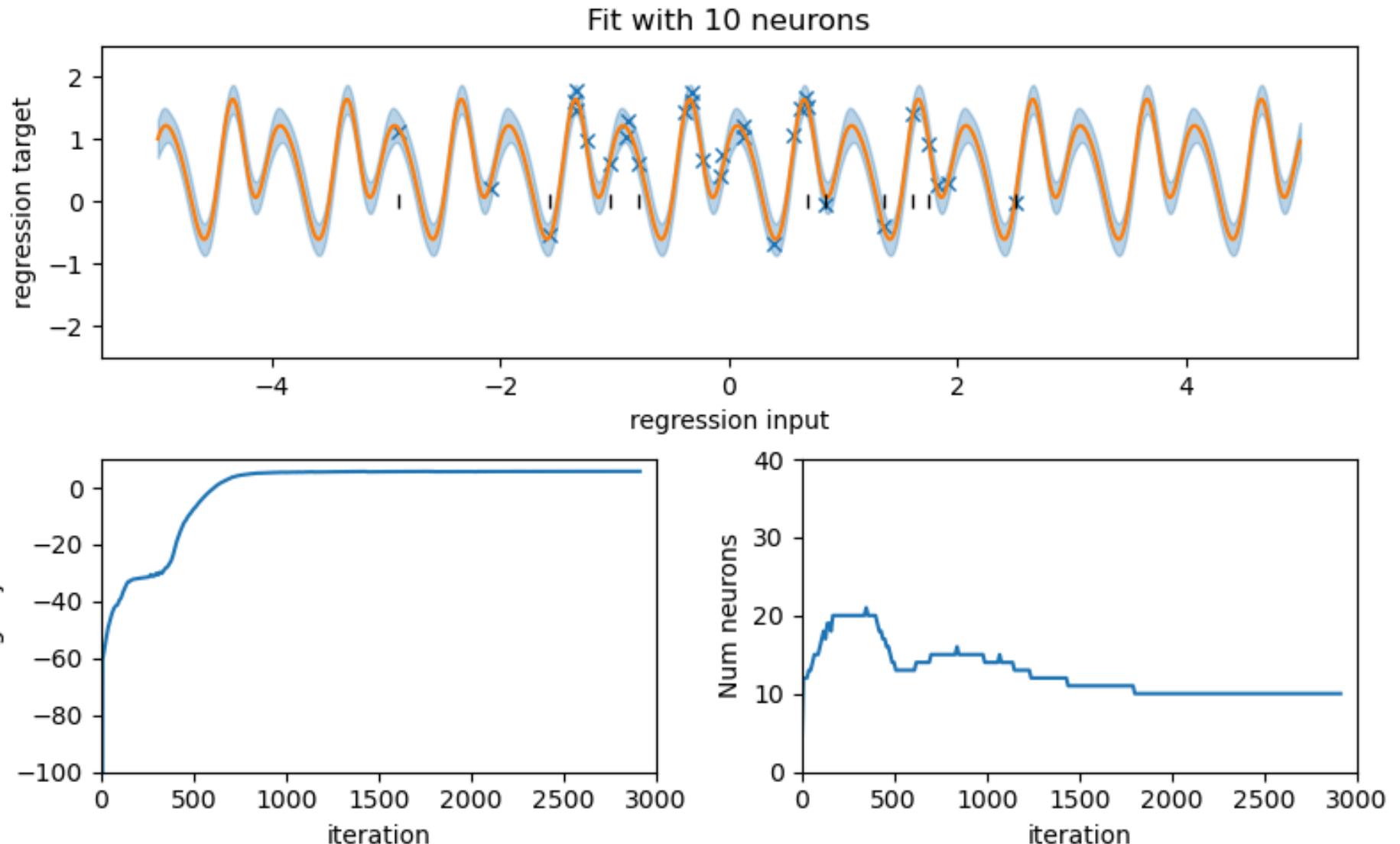
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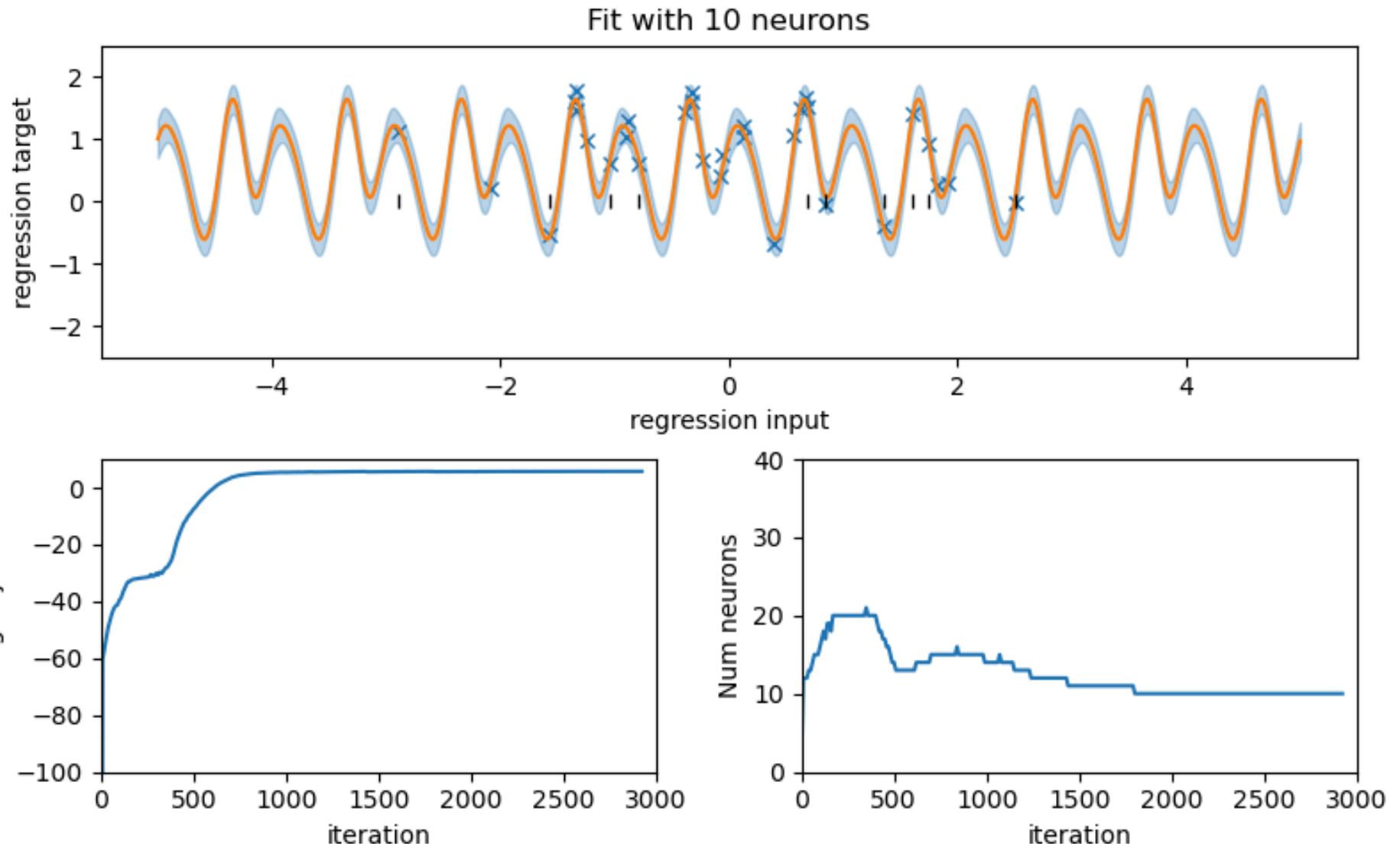
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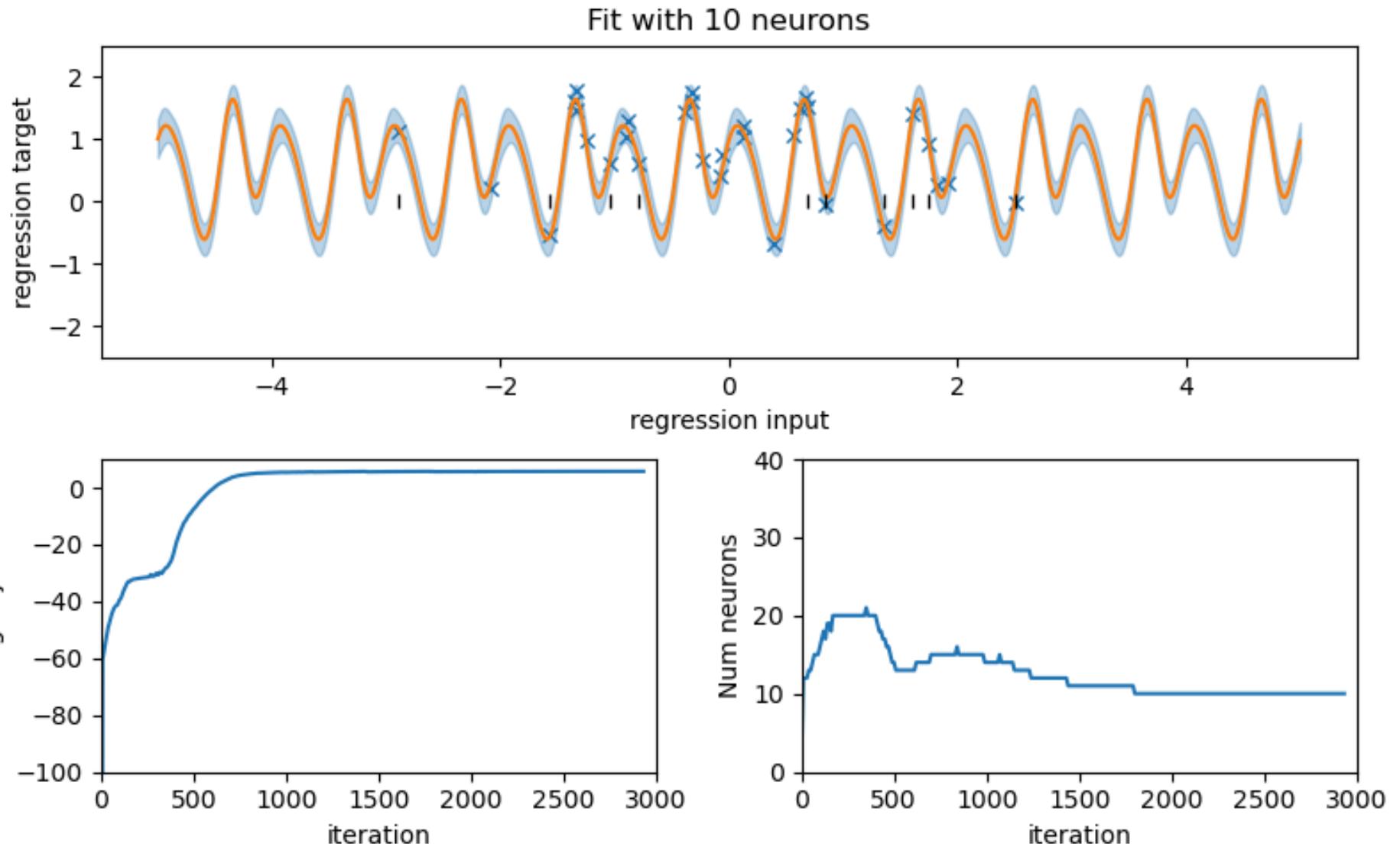
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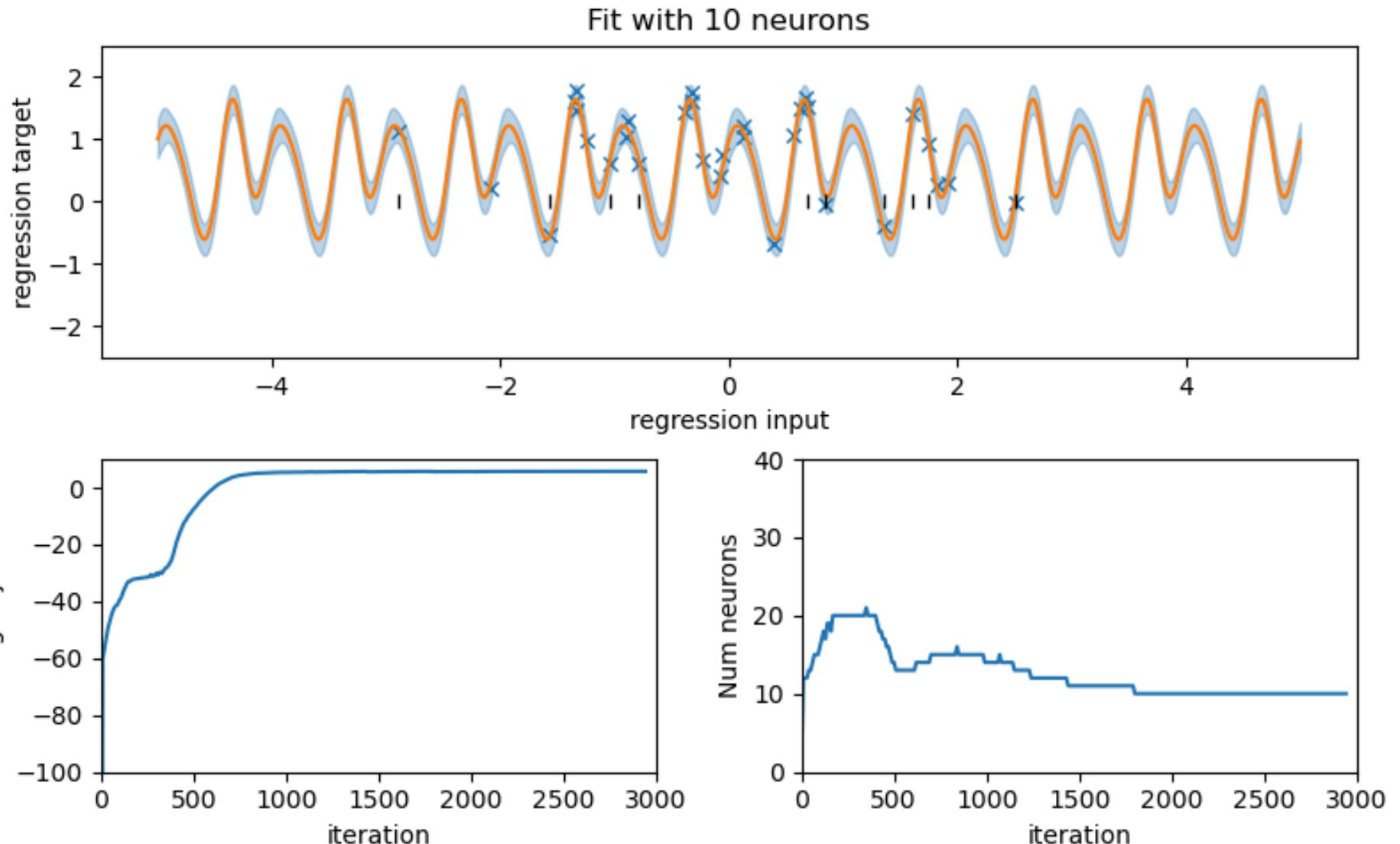
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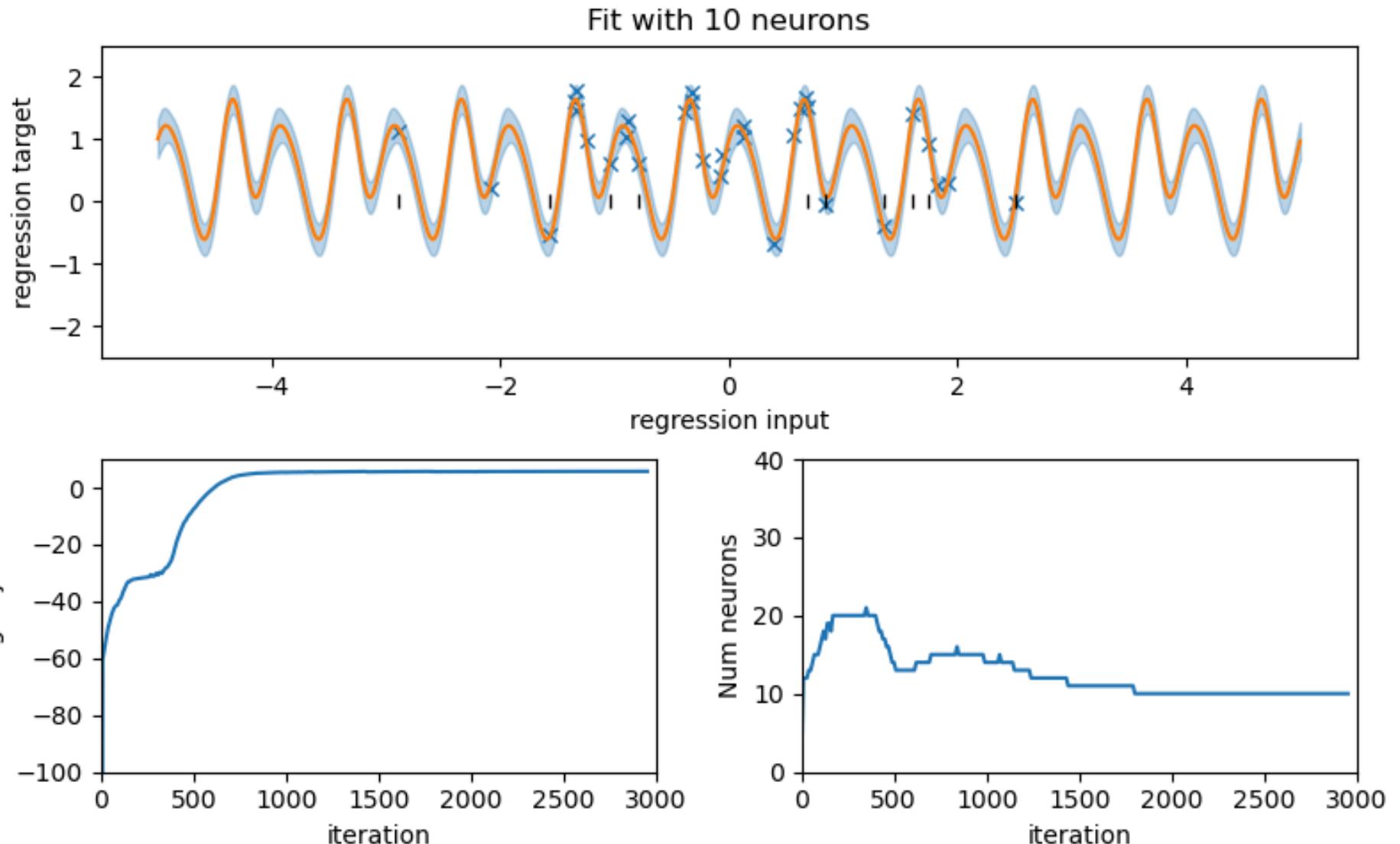
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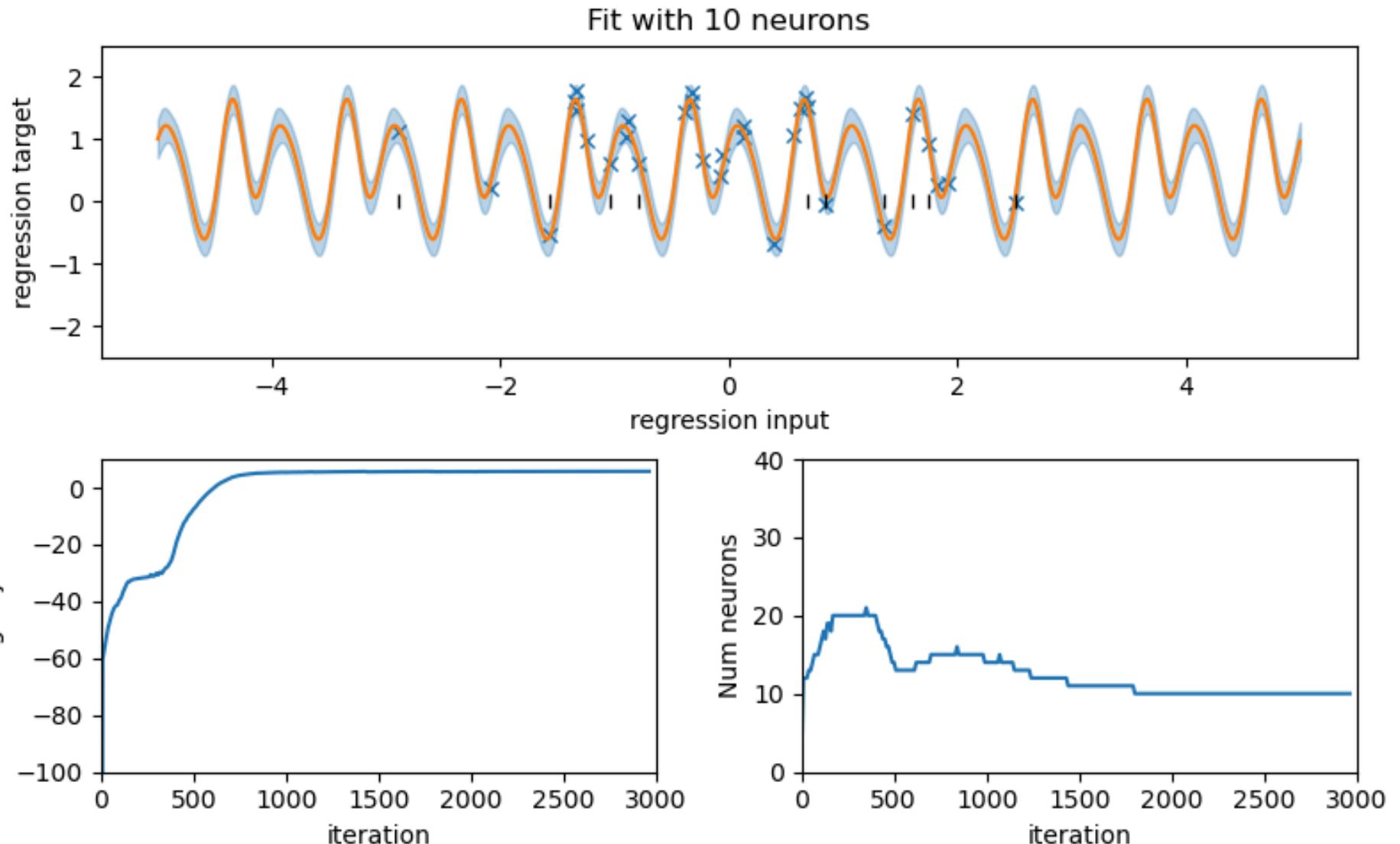
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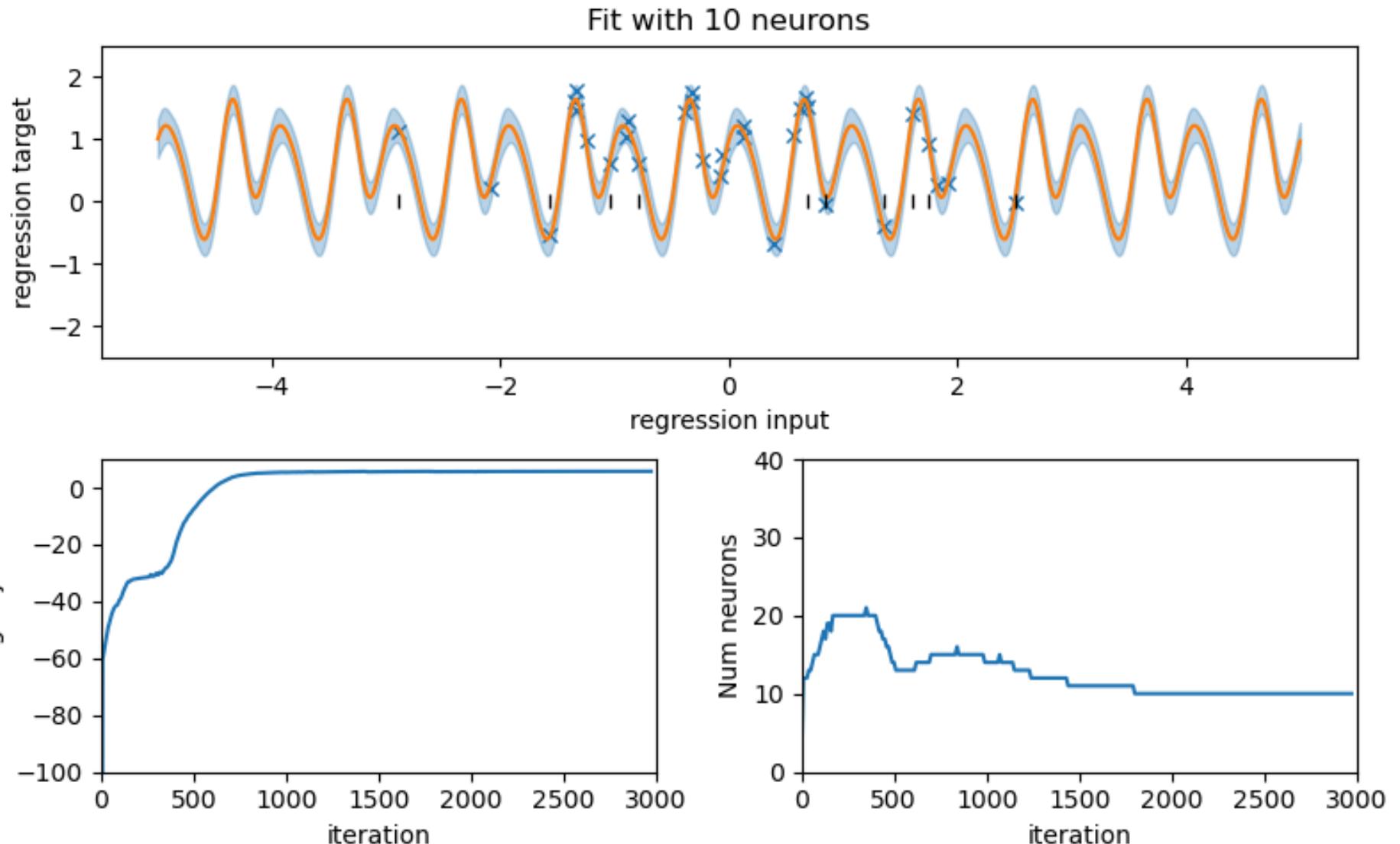
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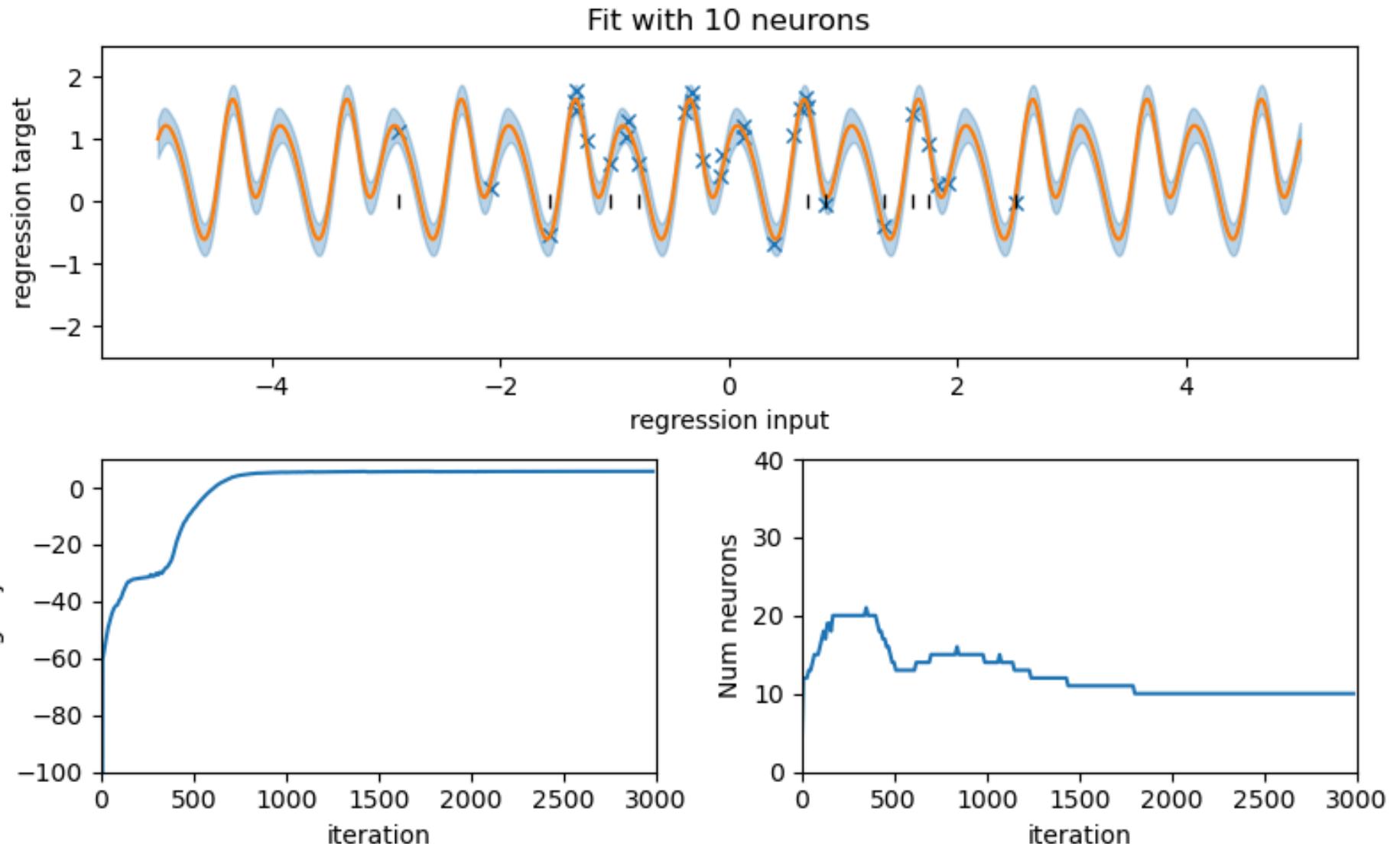
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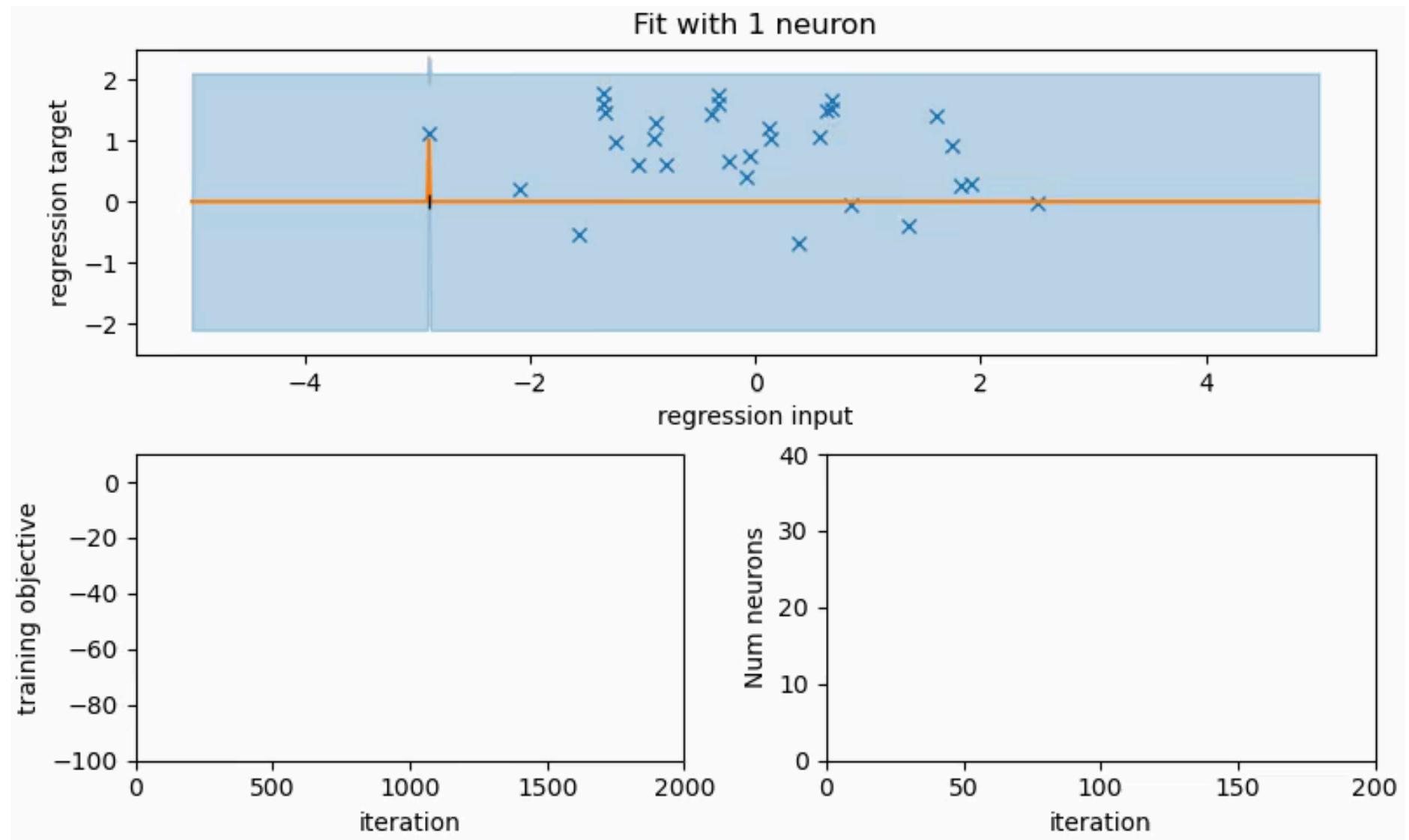


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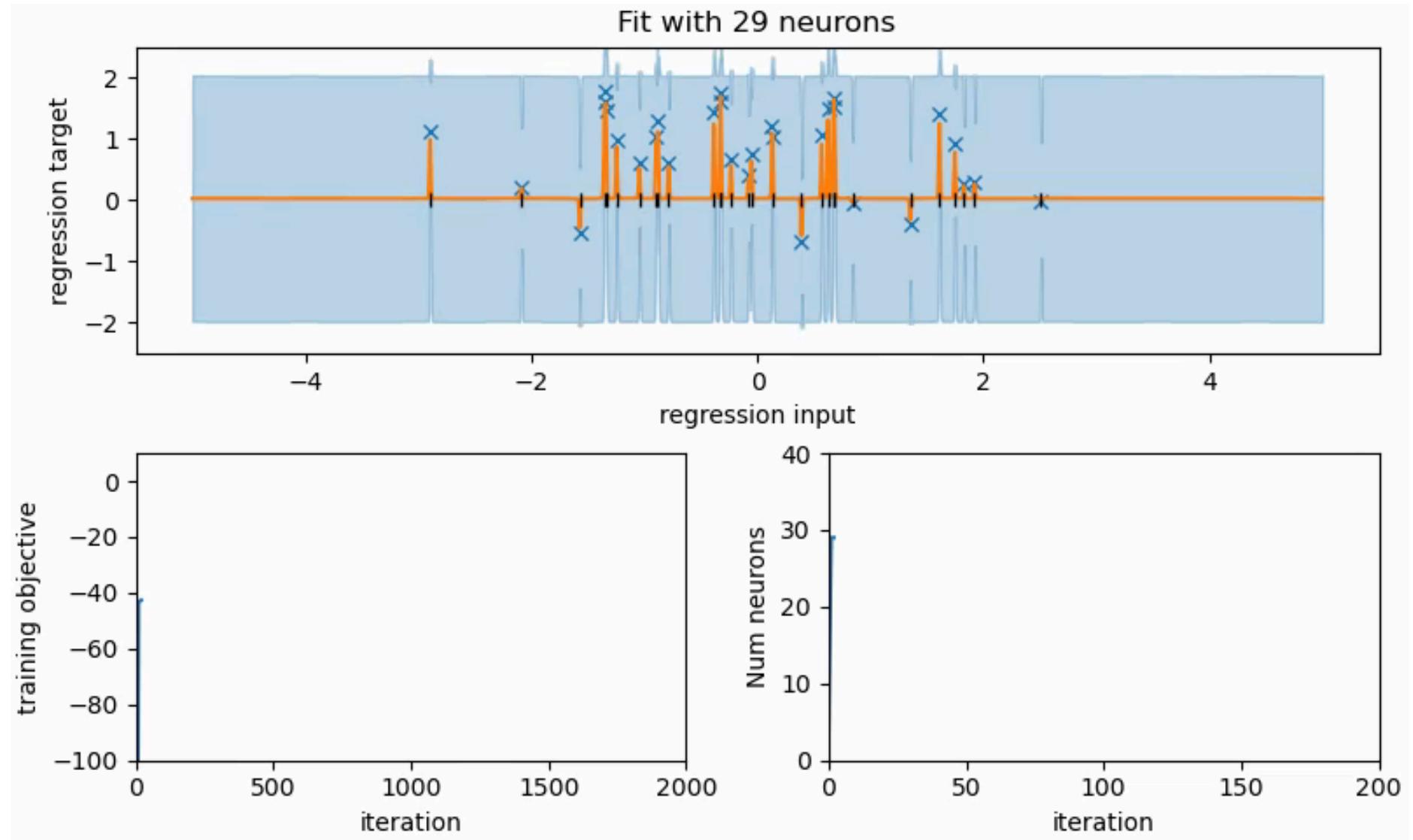
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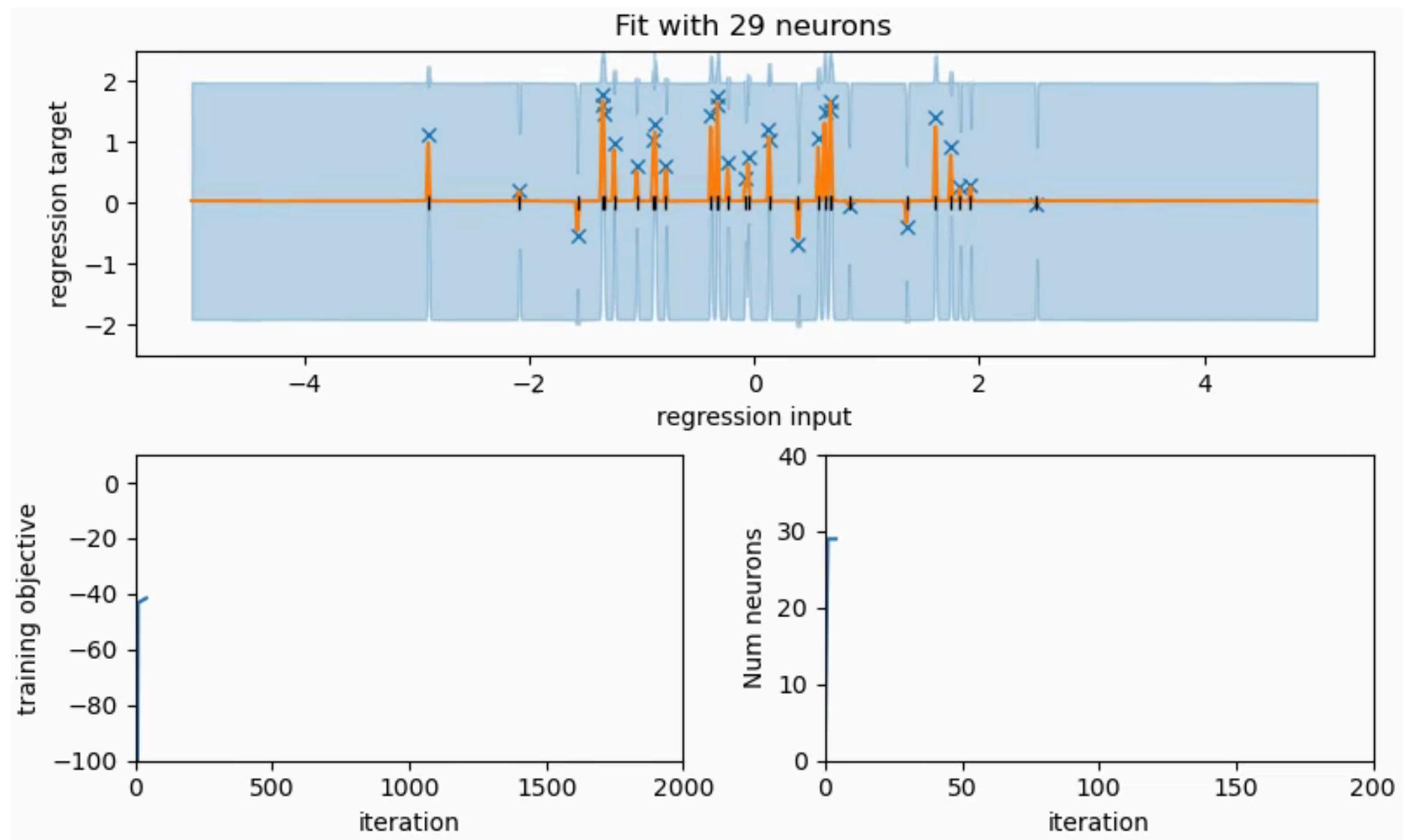
Memorising first, then pruning



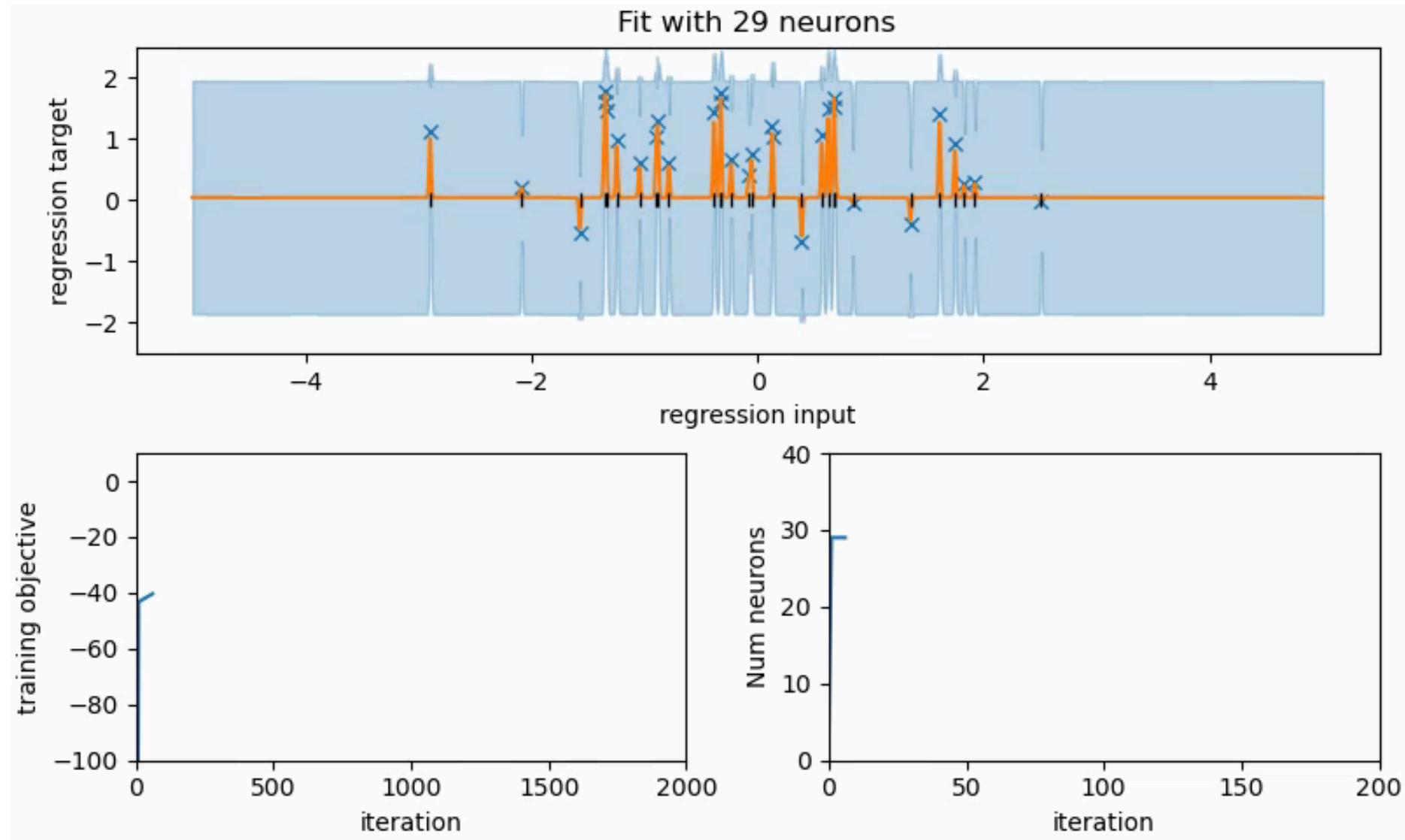
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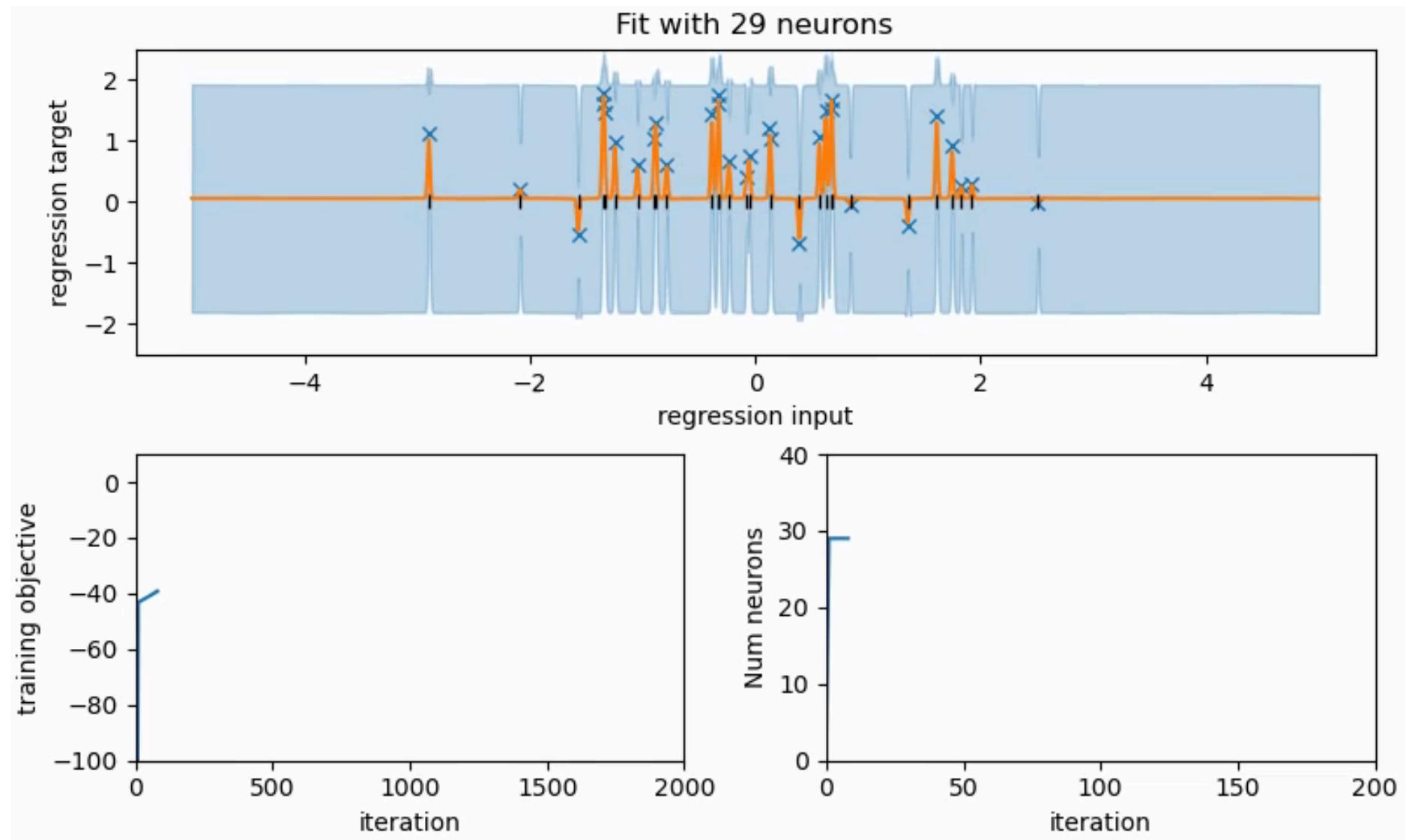
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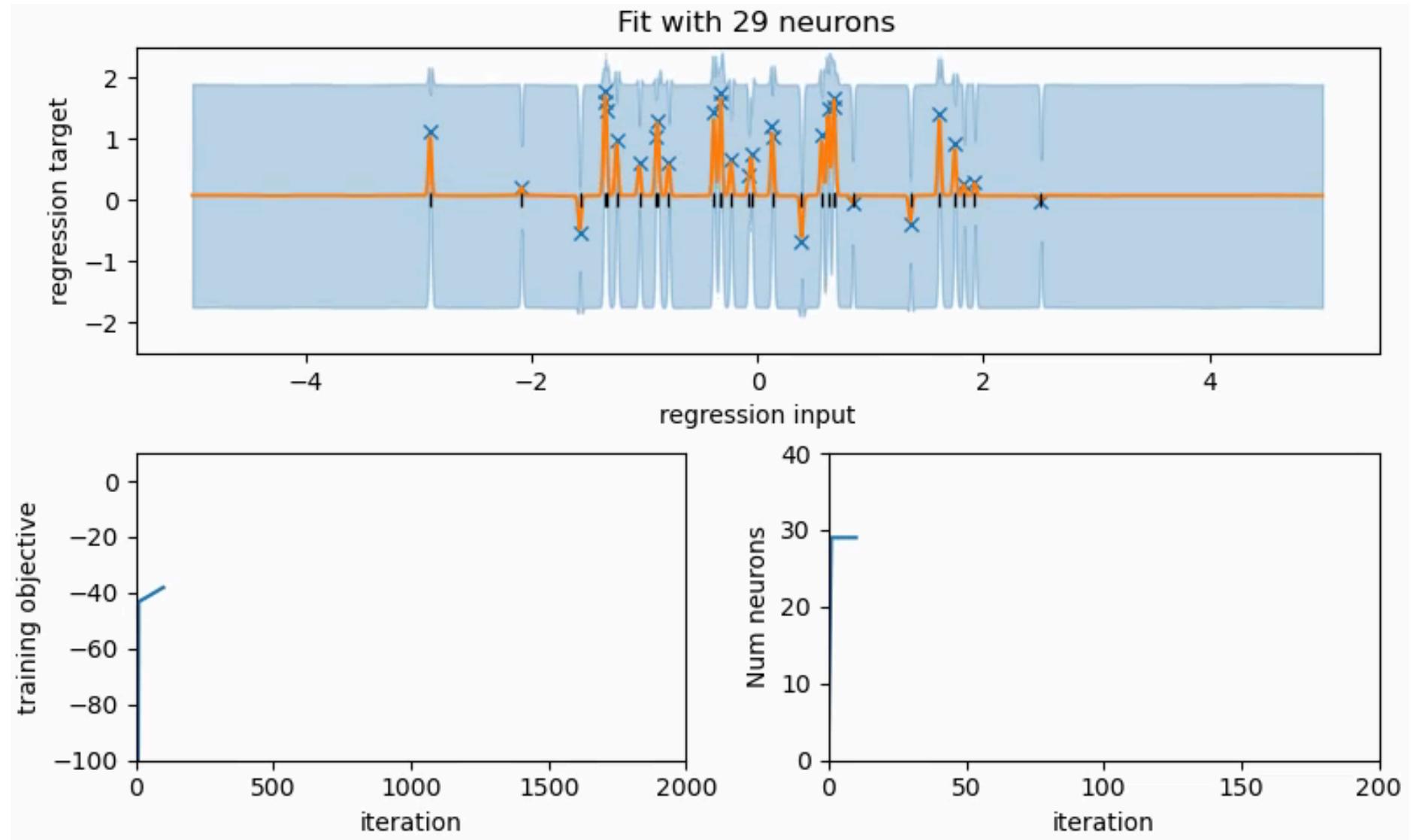
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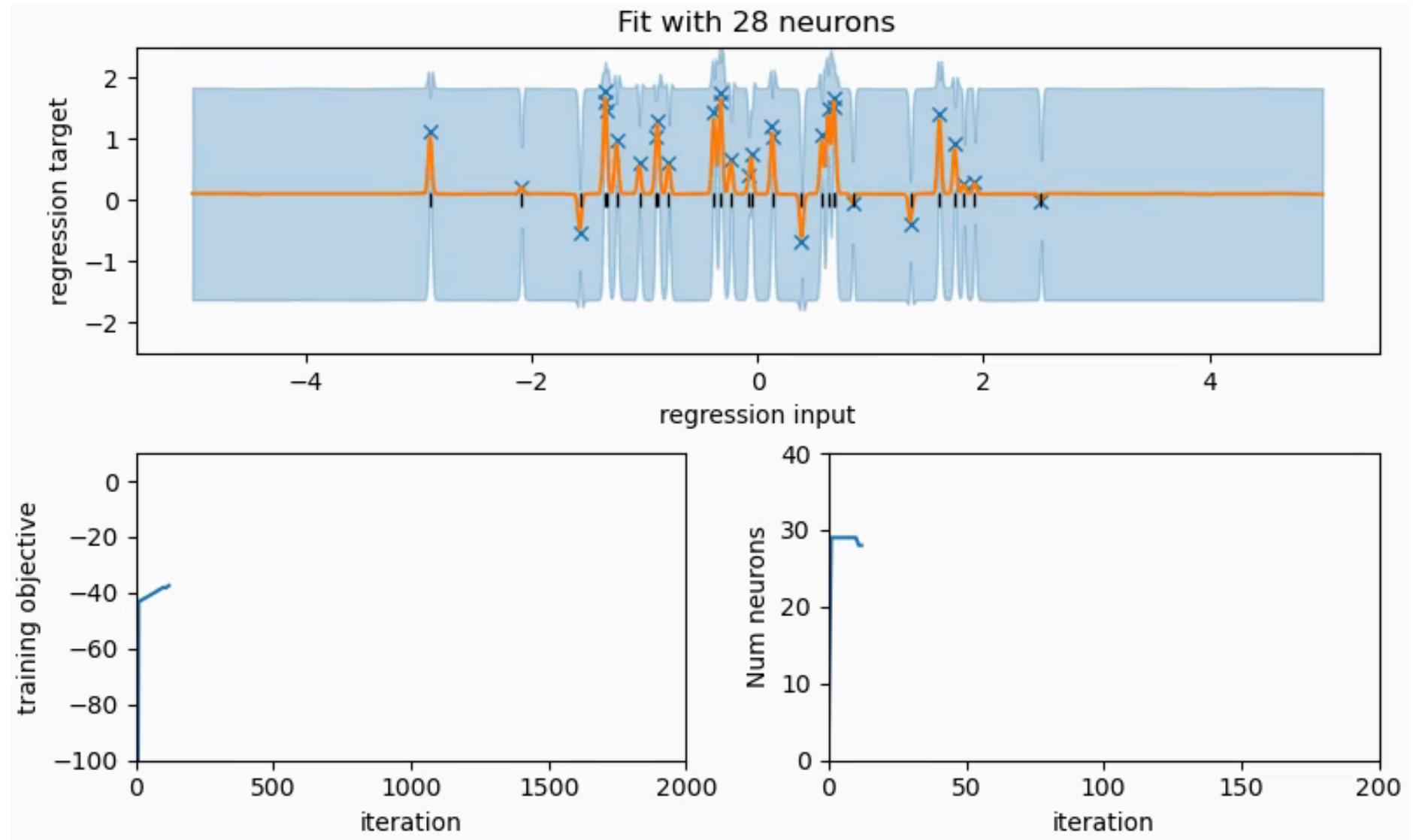
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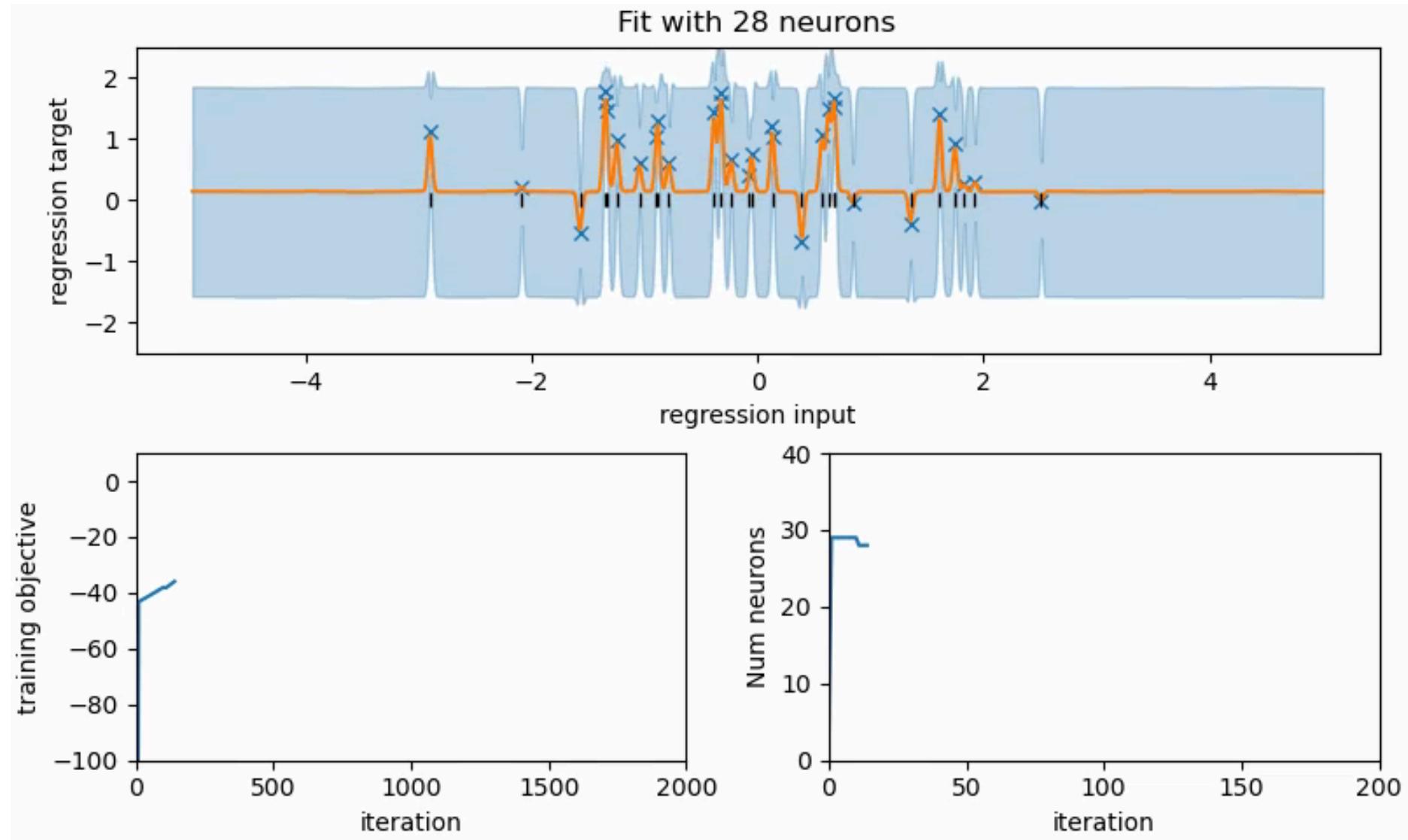
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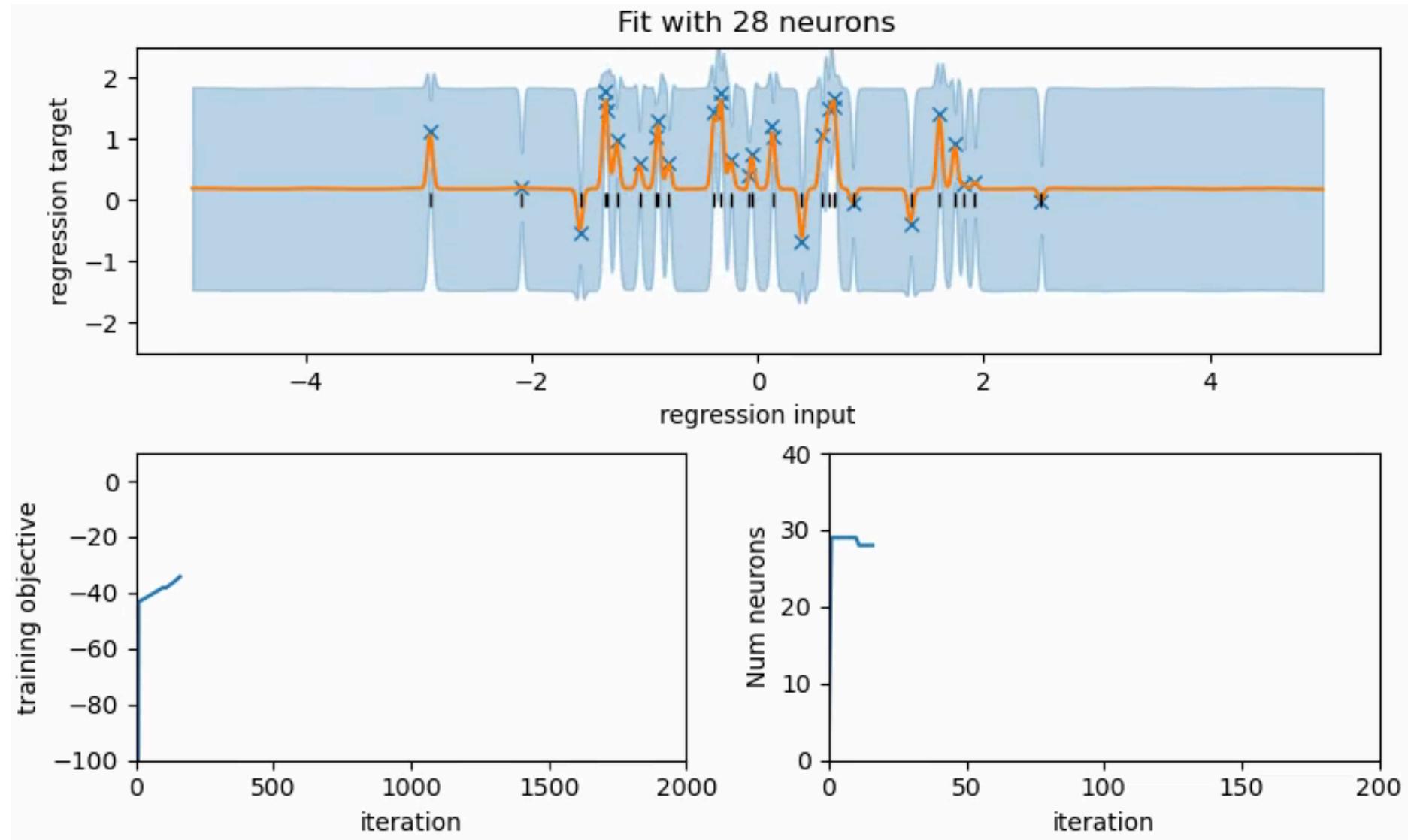
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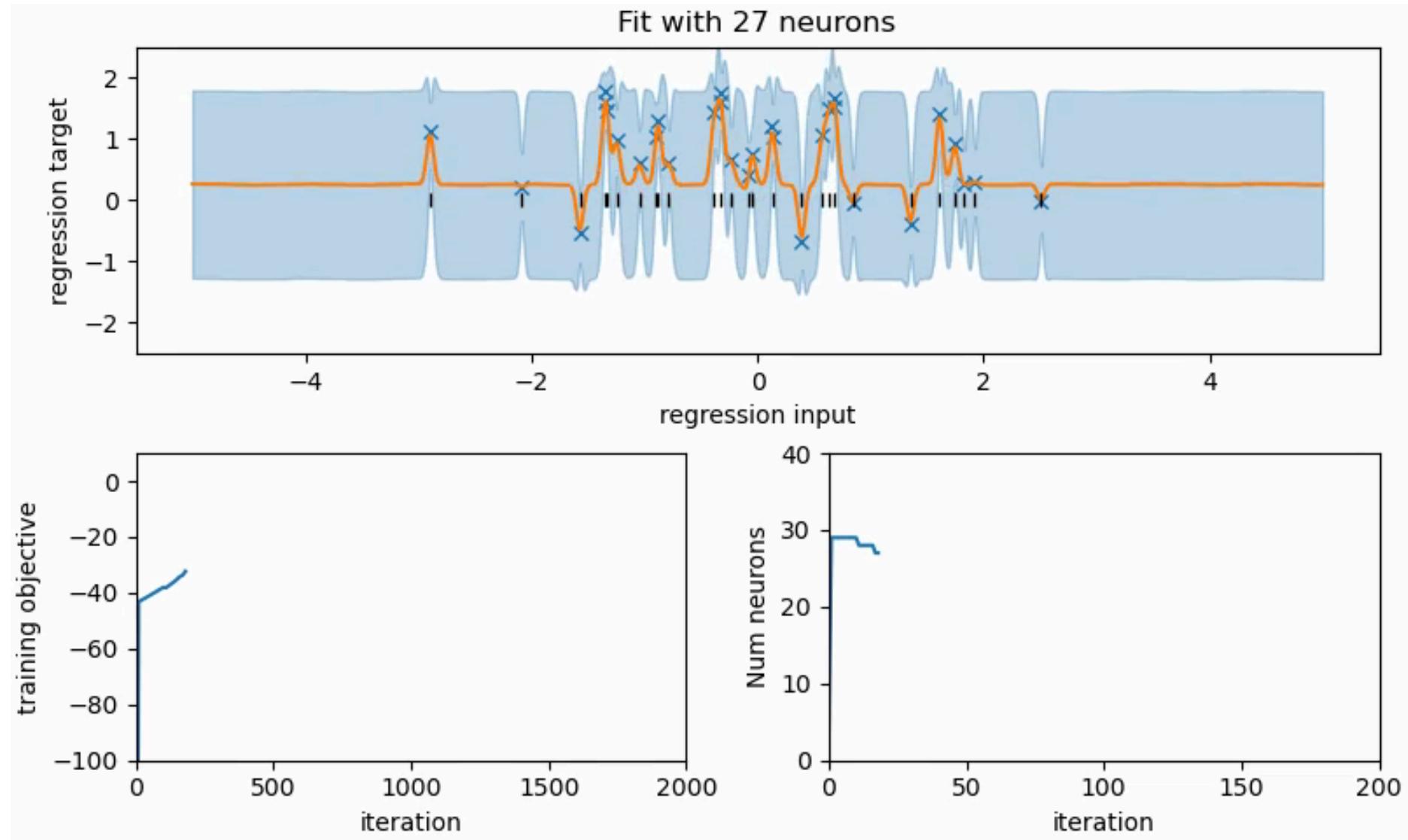
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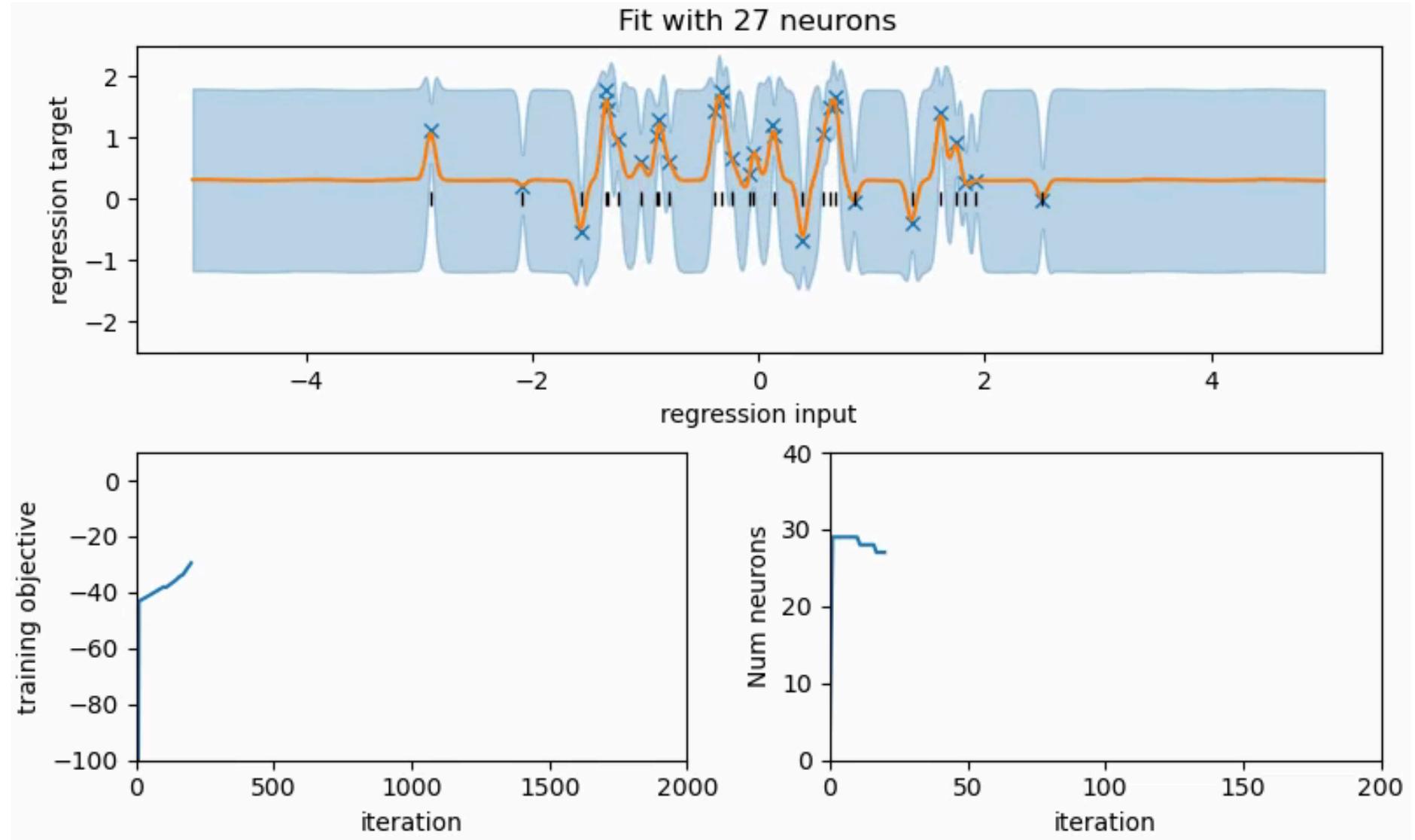
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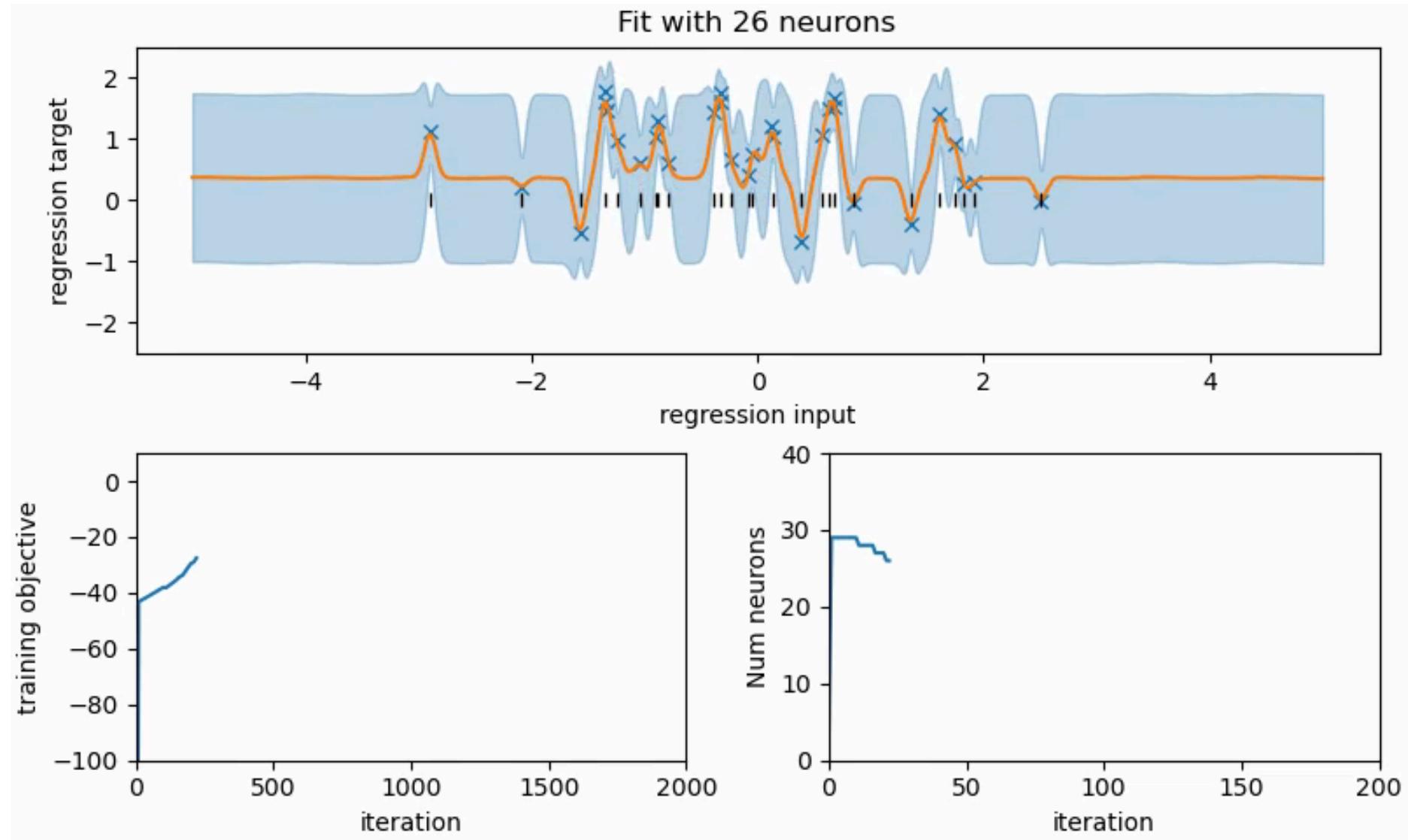
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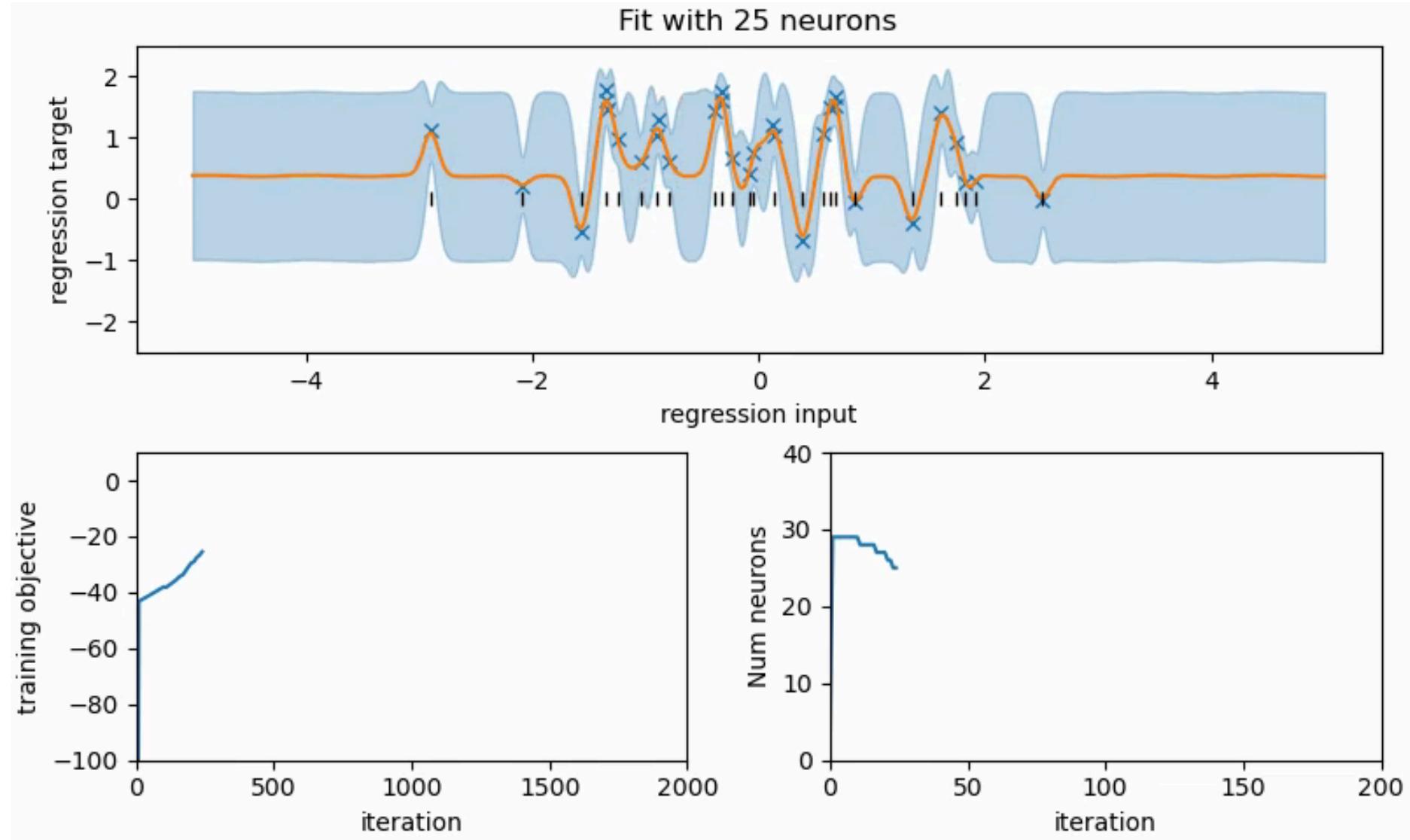
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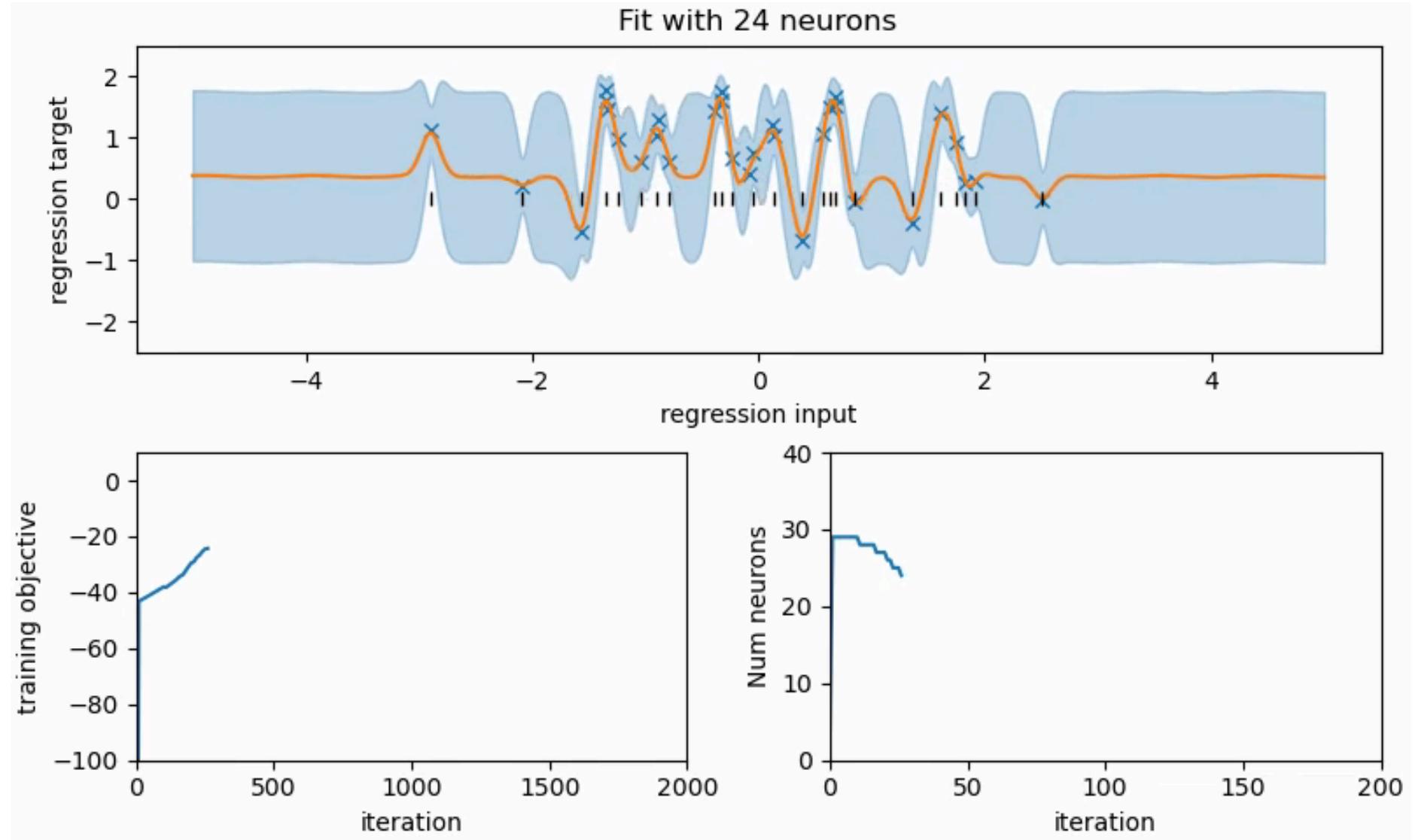
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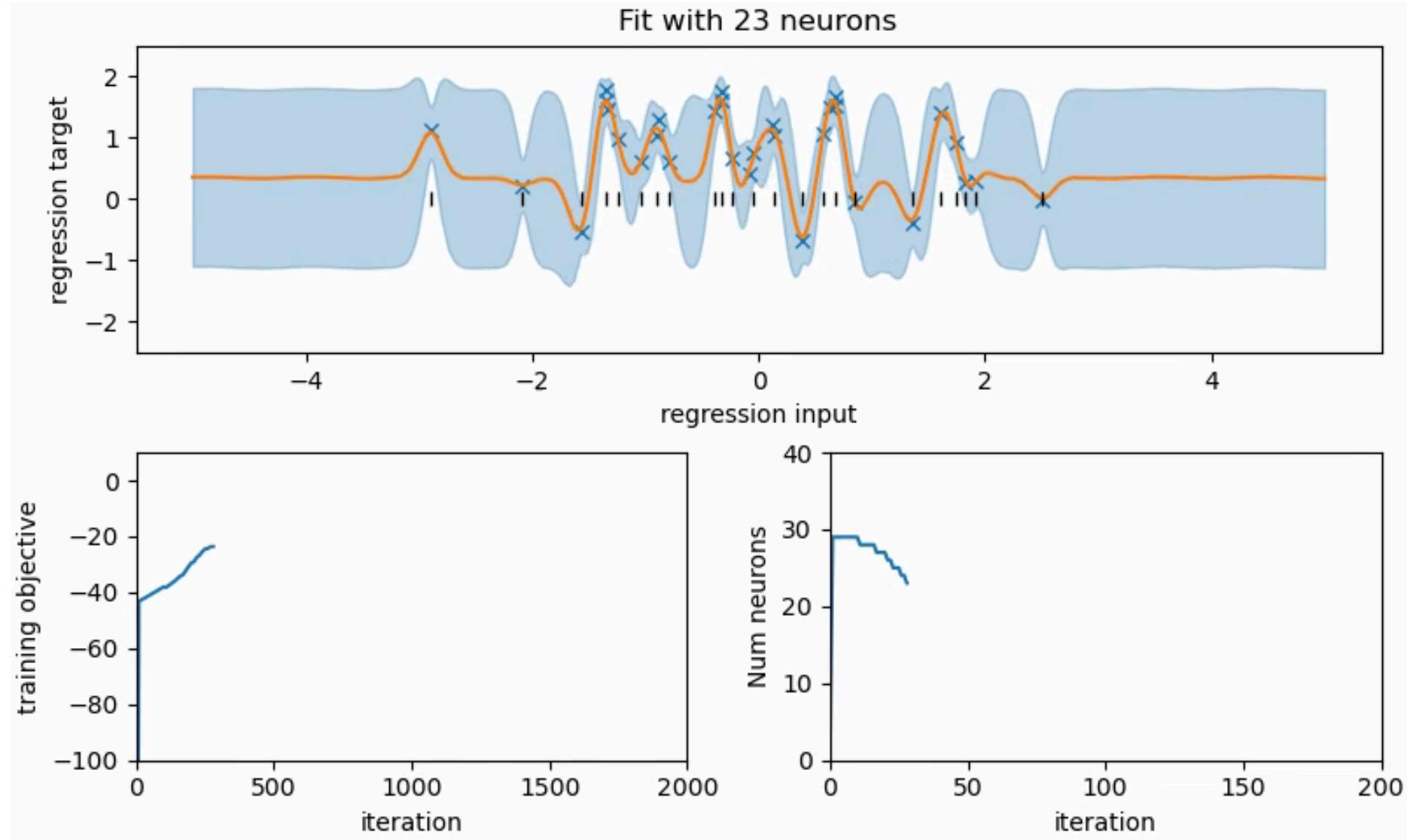
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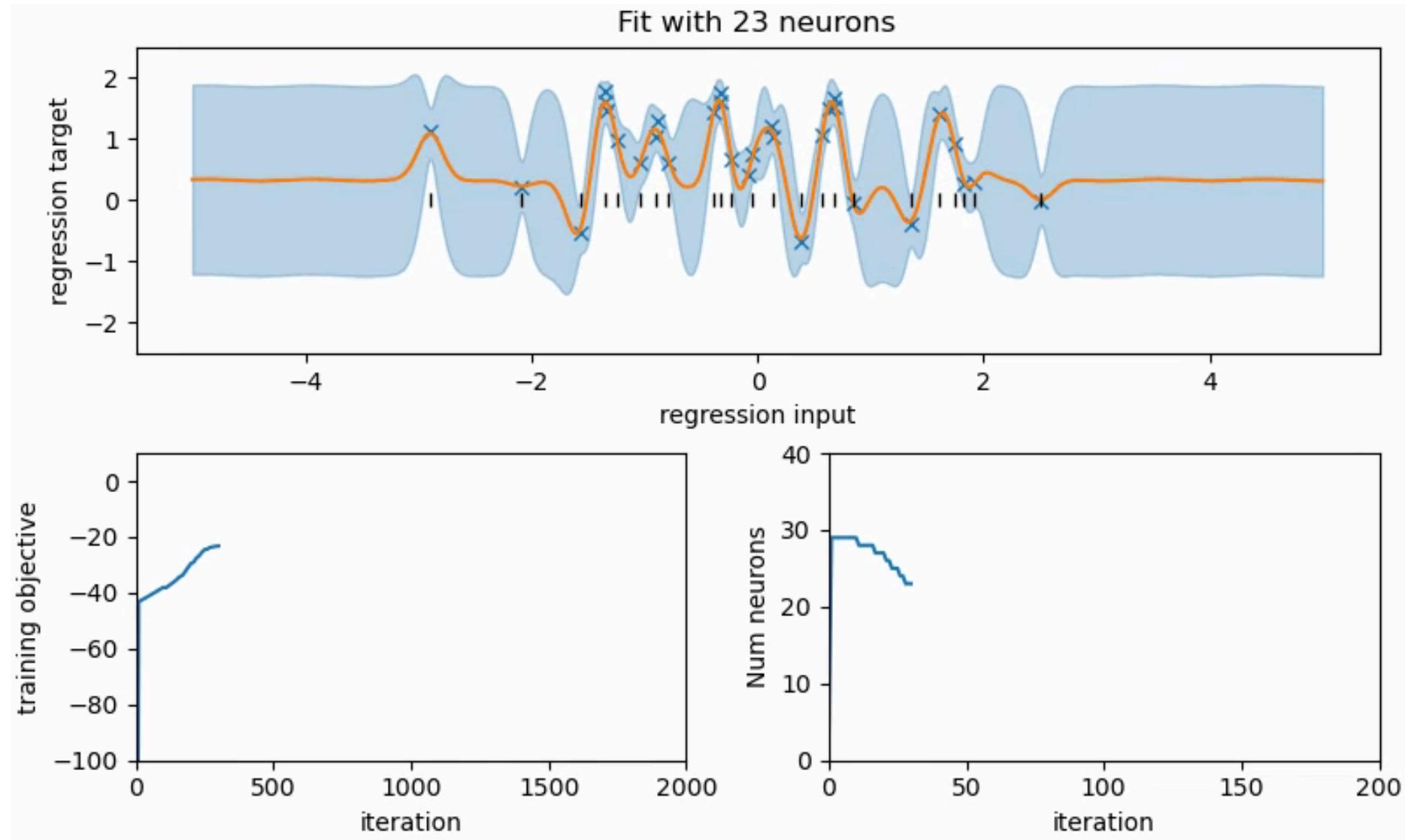
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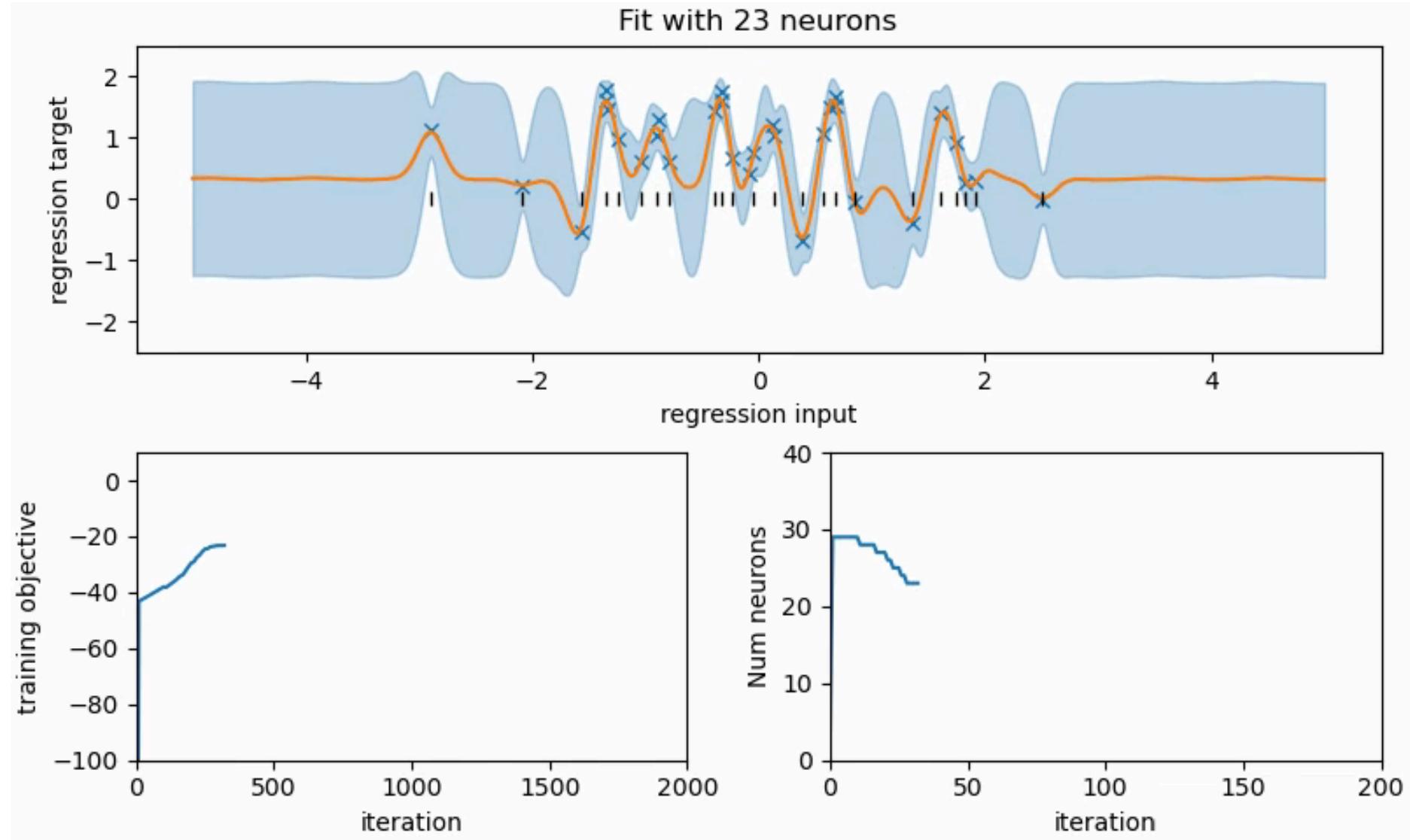
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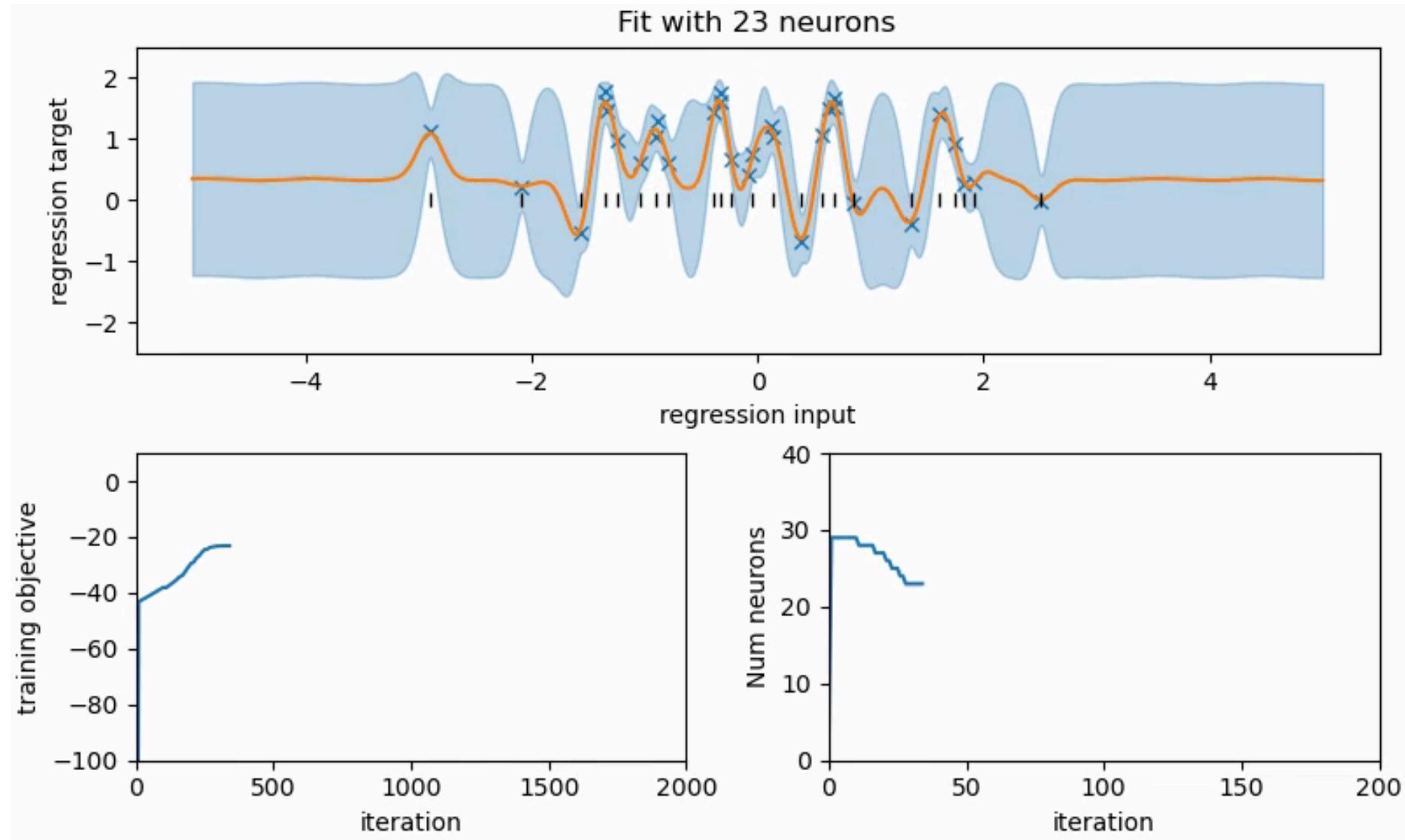
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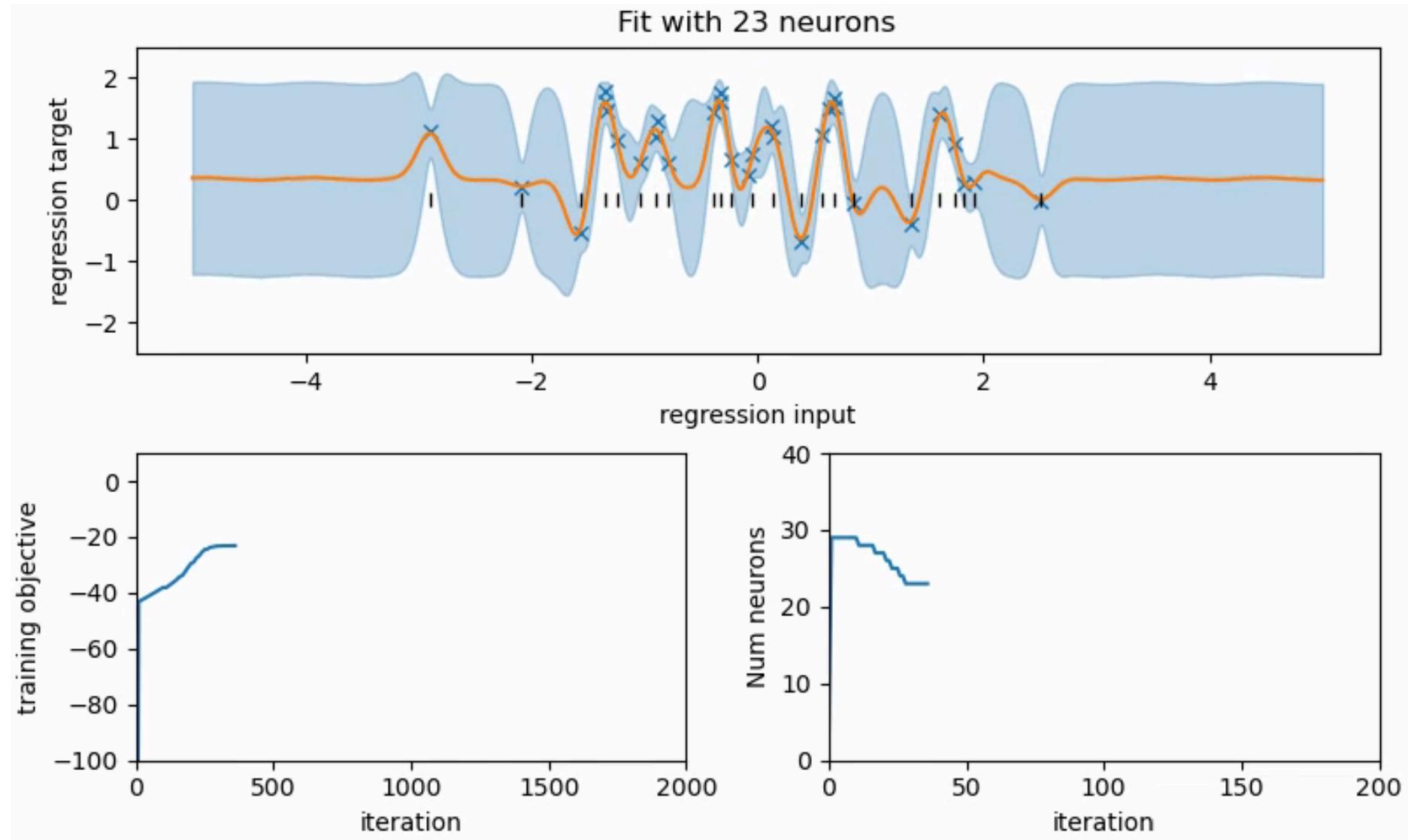
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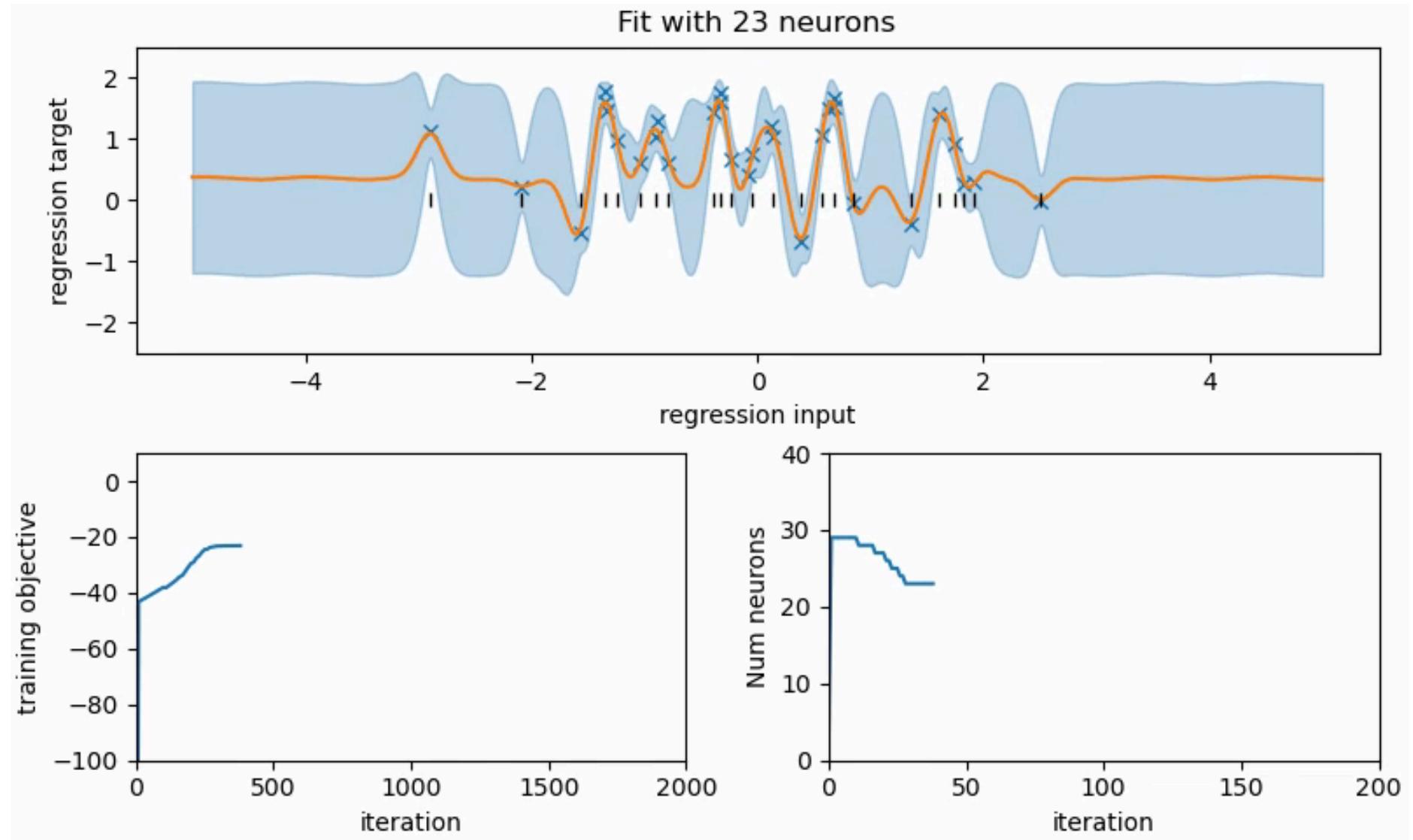
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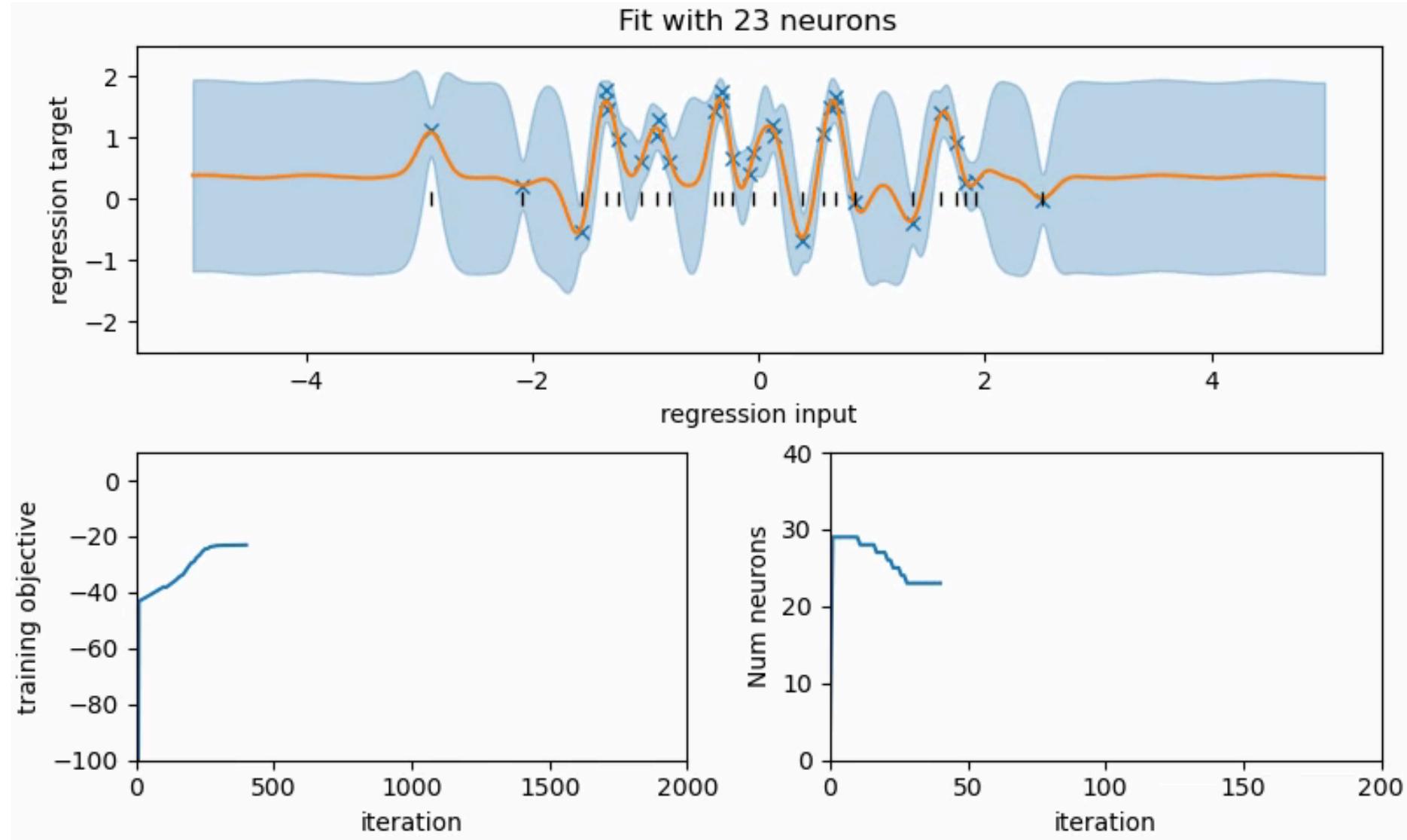
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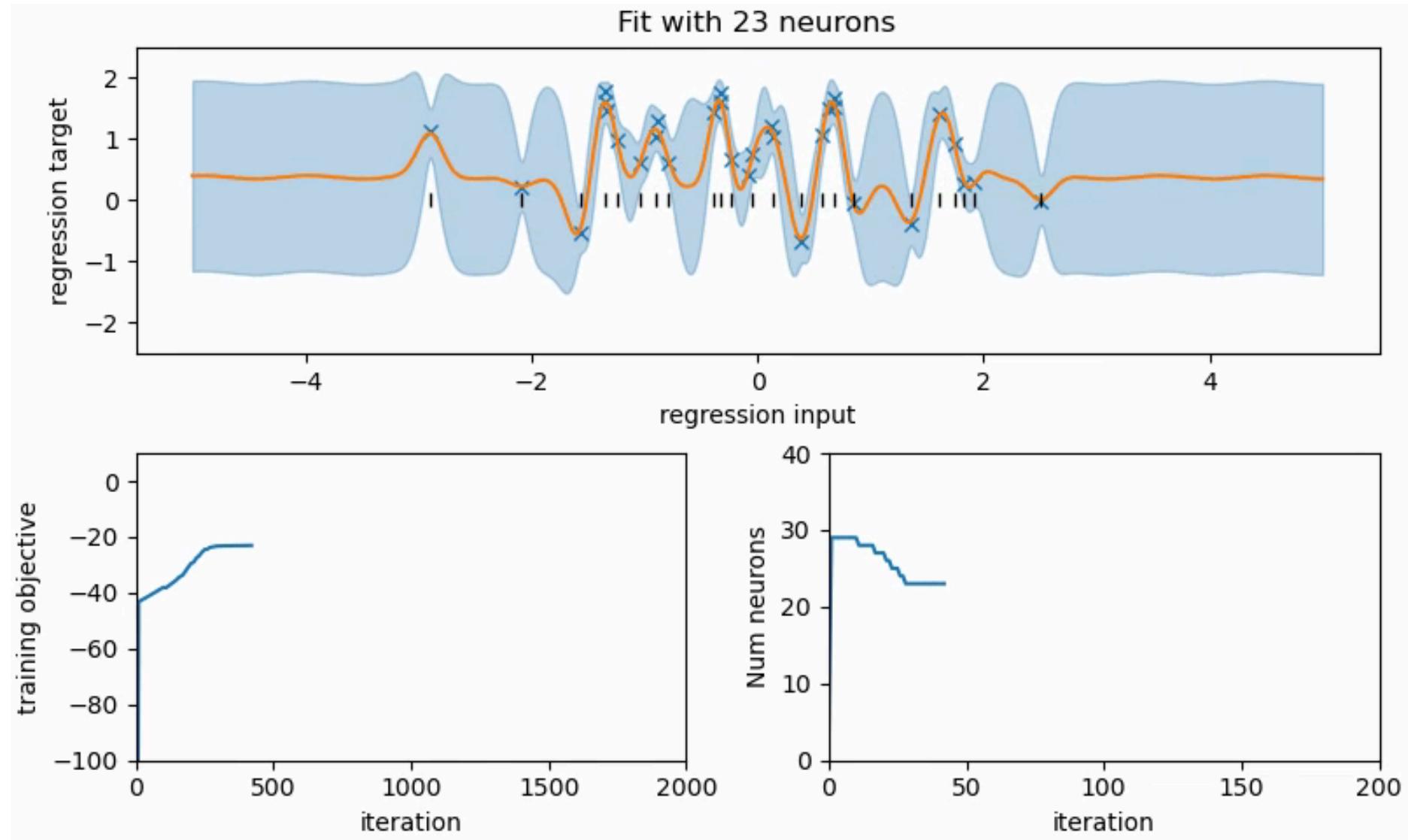
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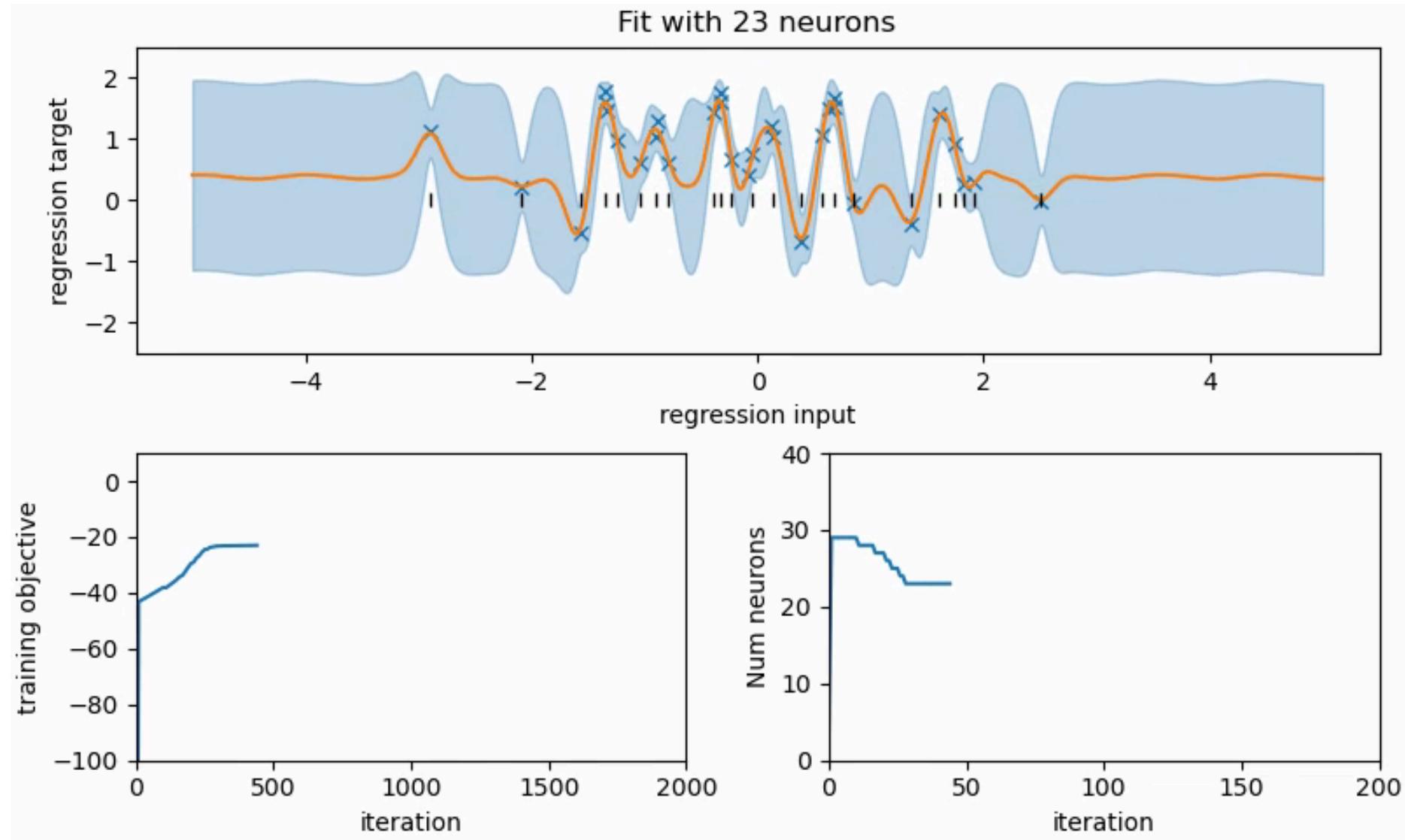
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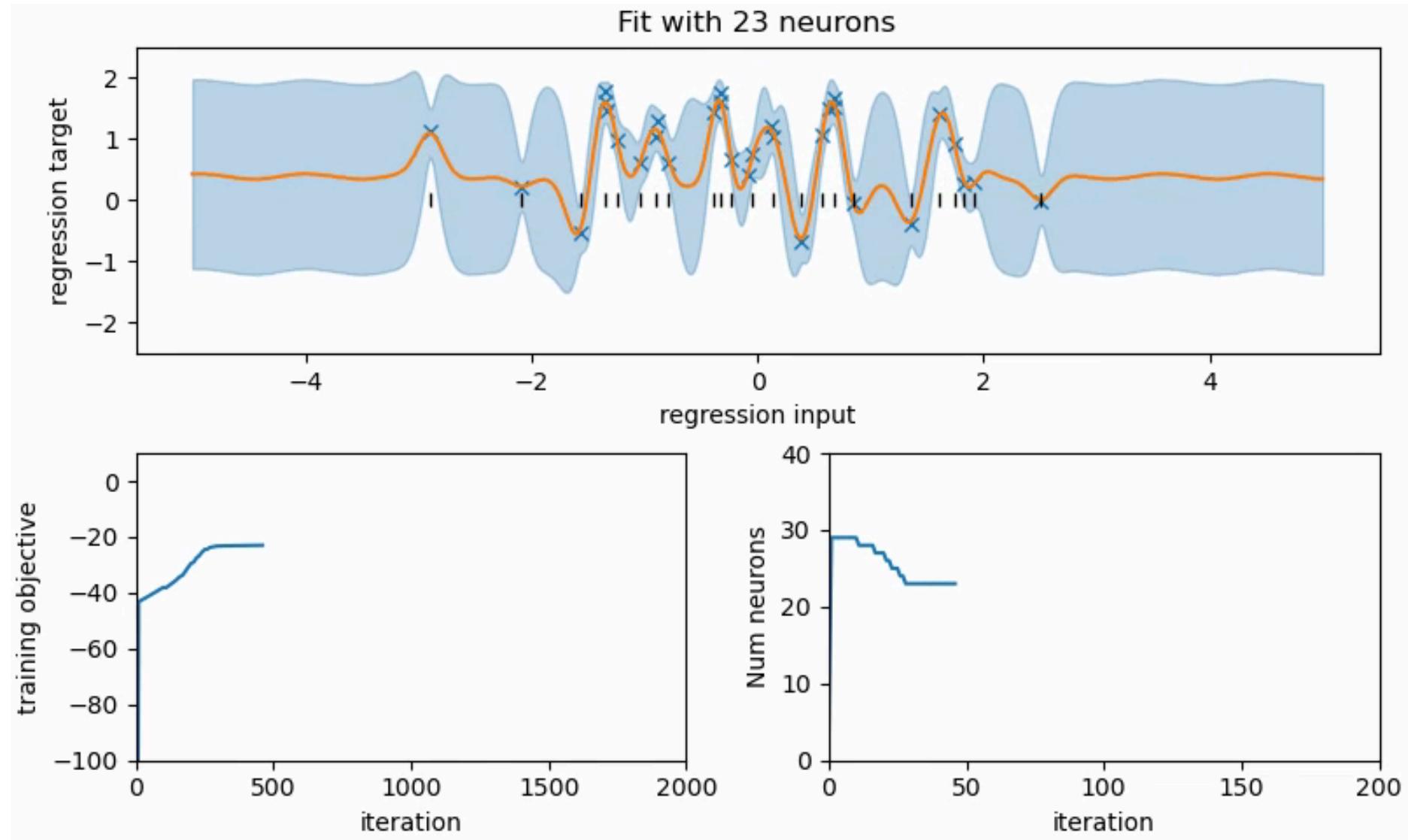
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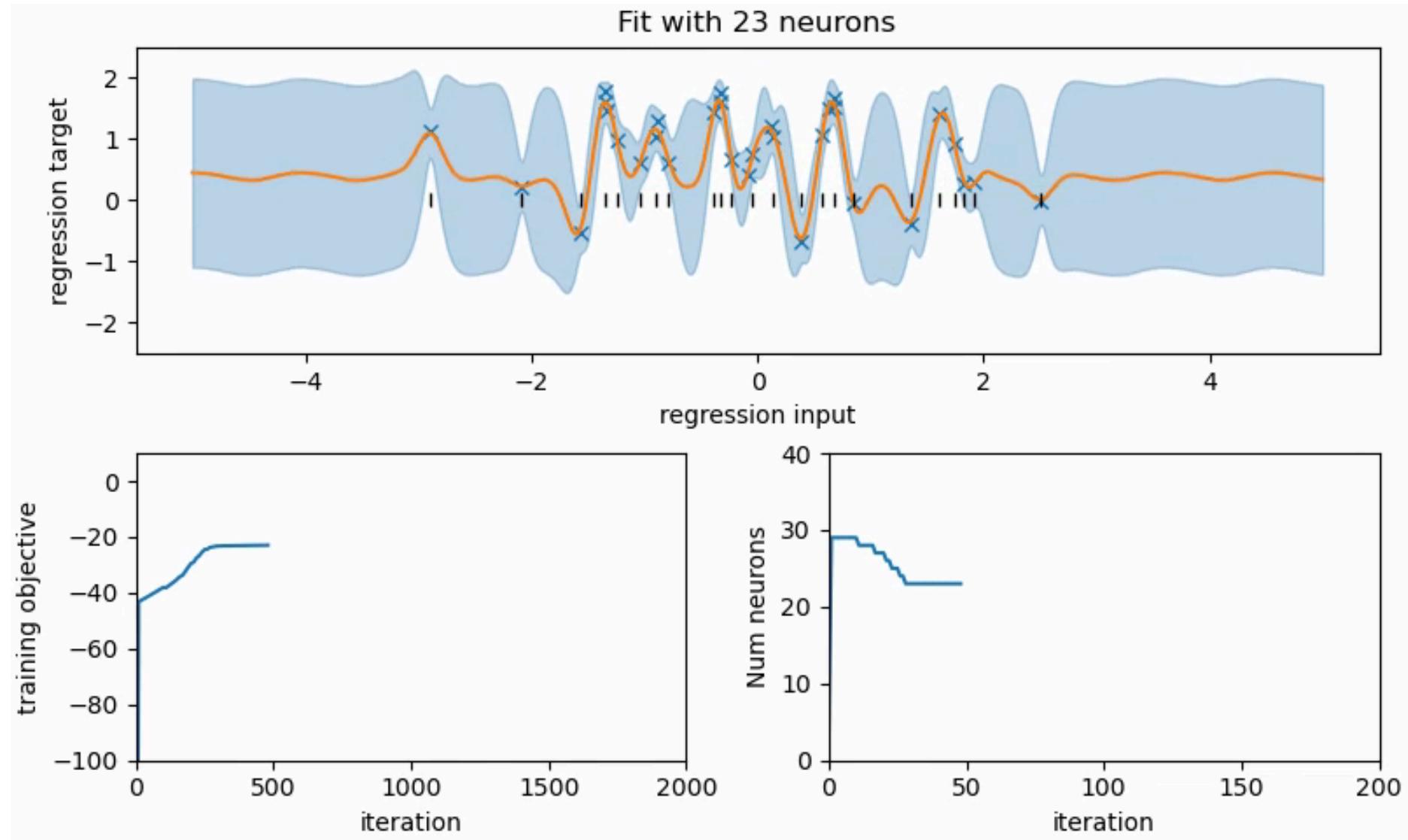
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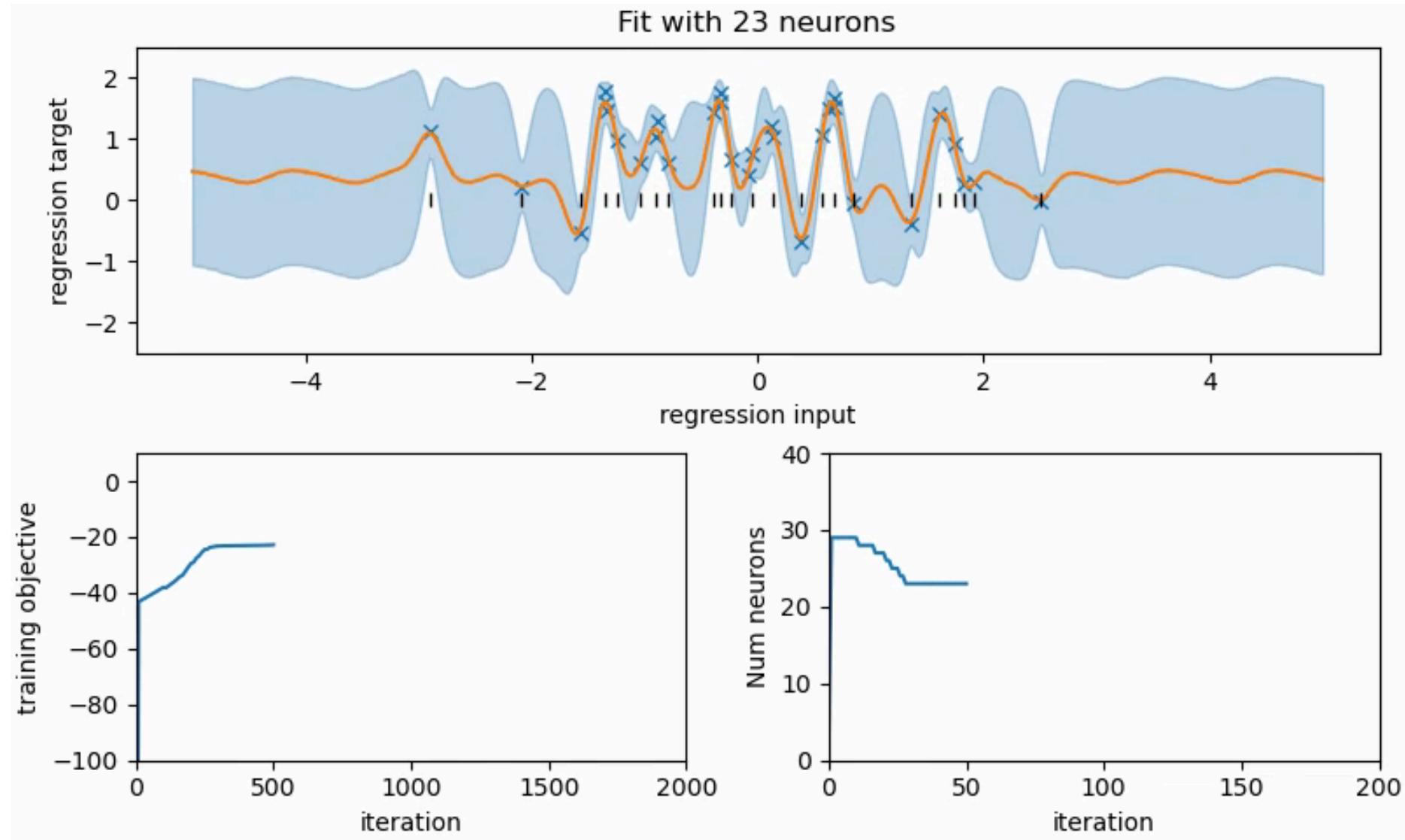
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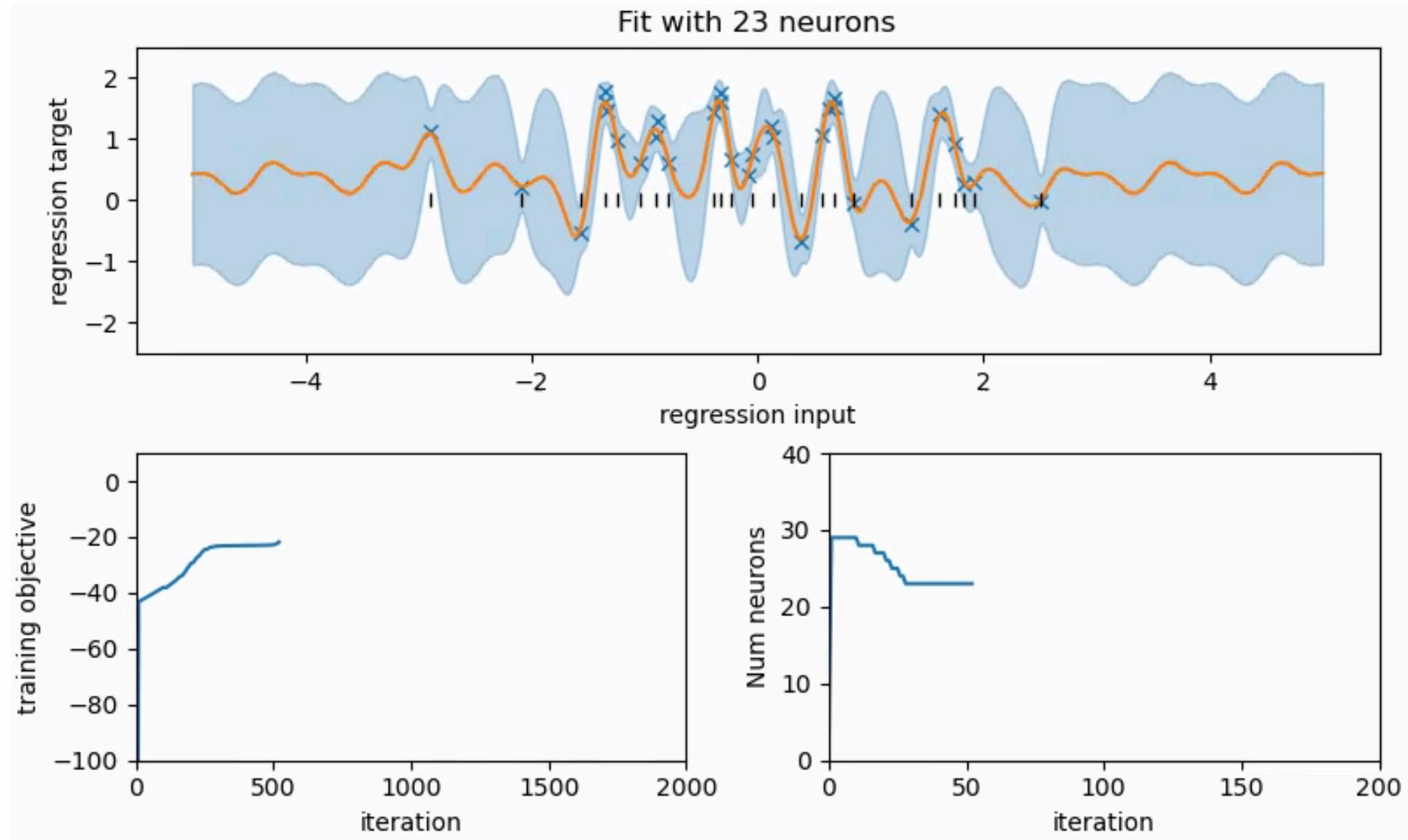
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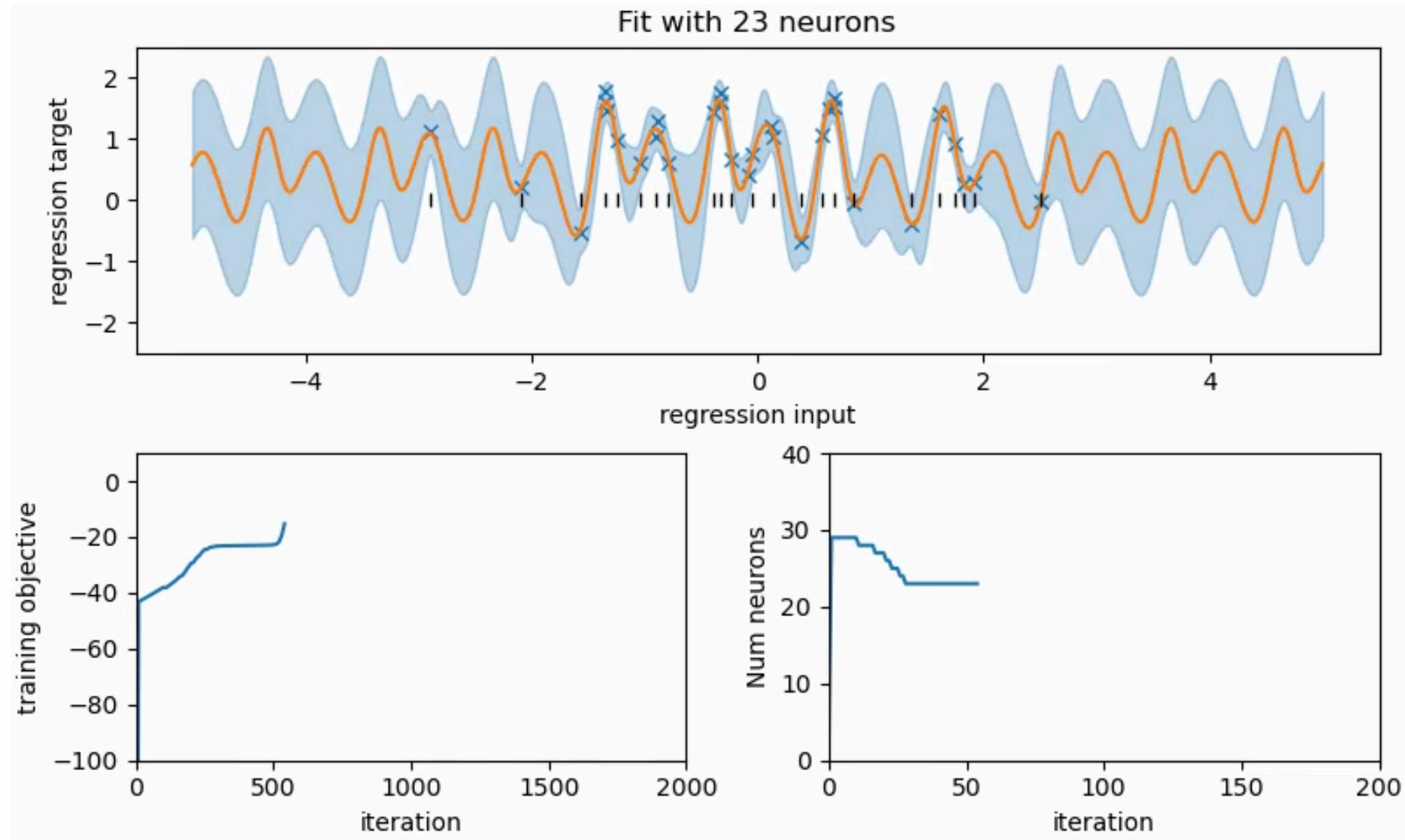
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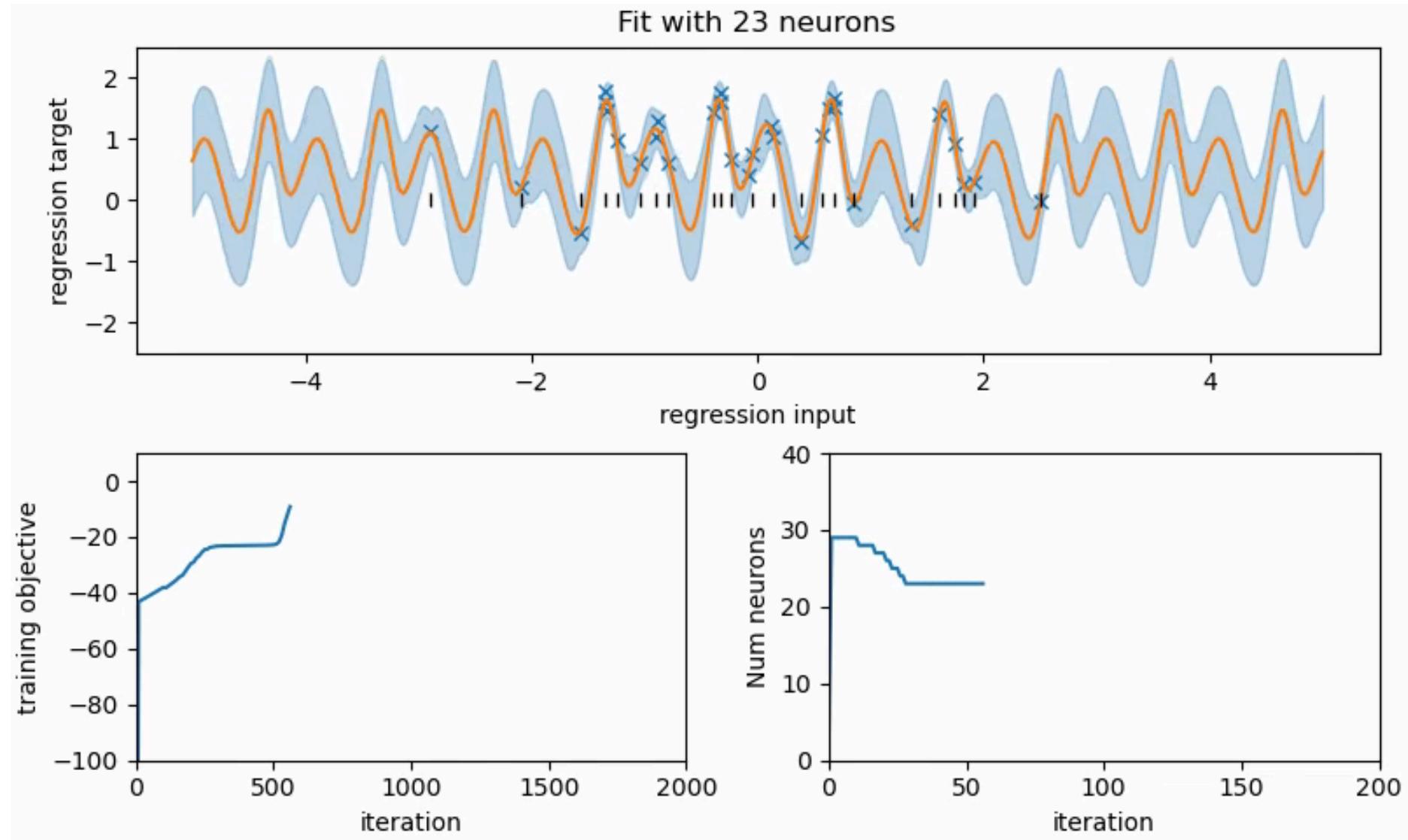
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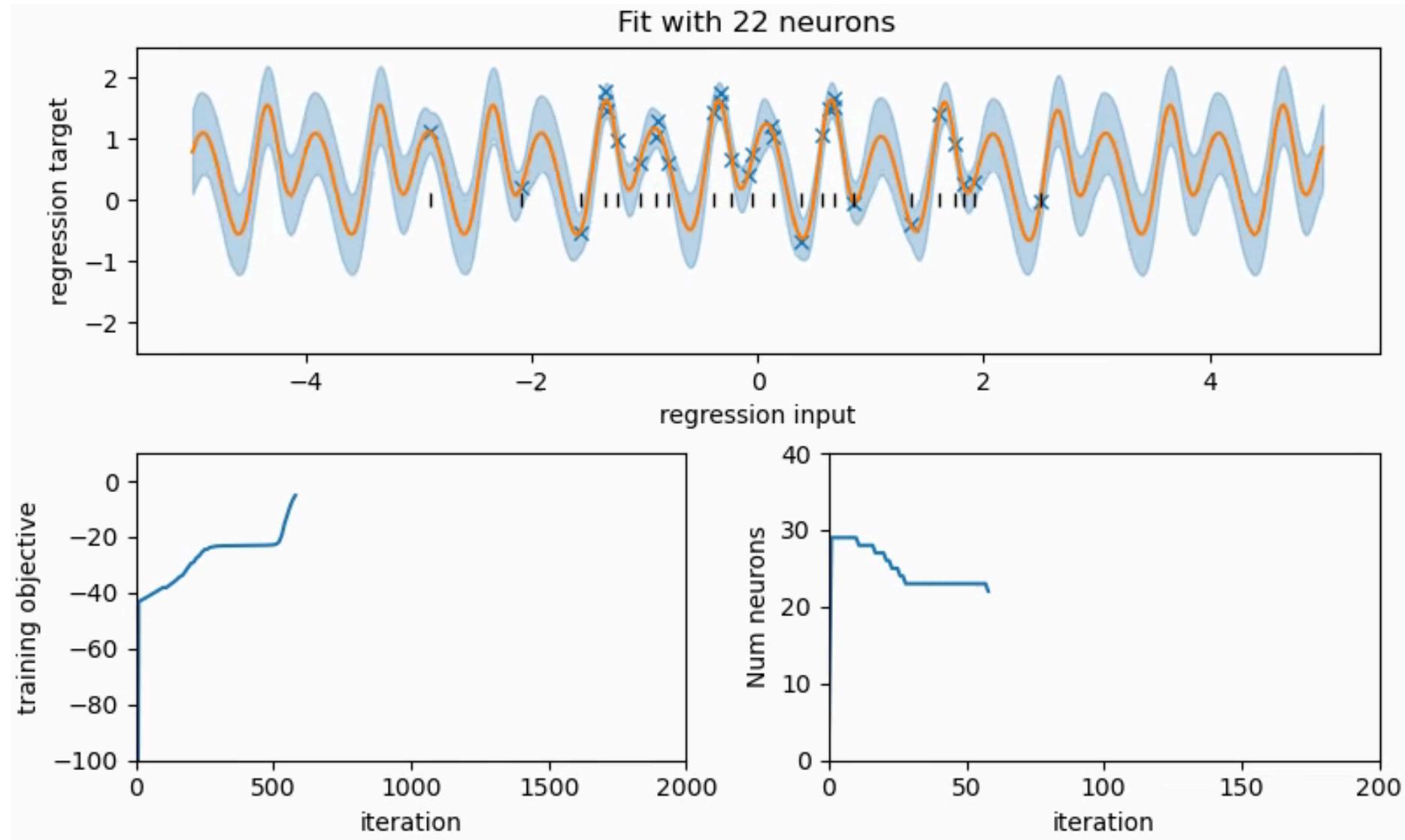
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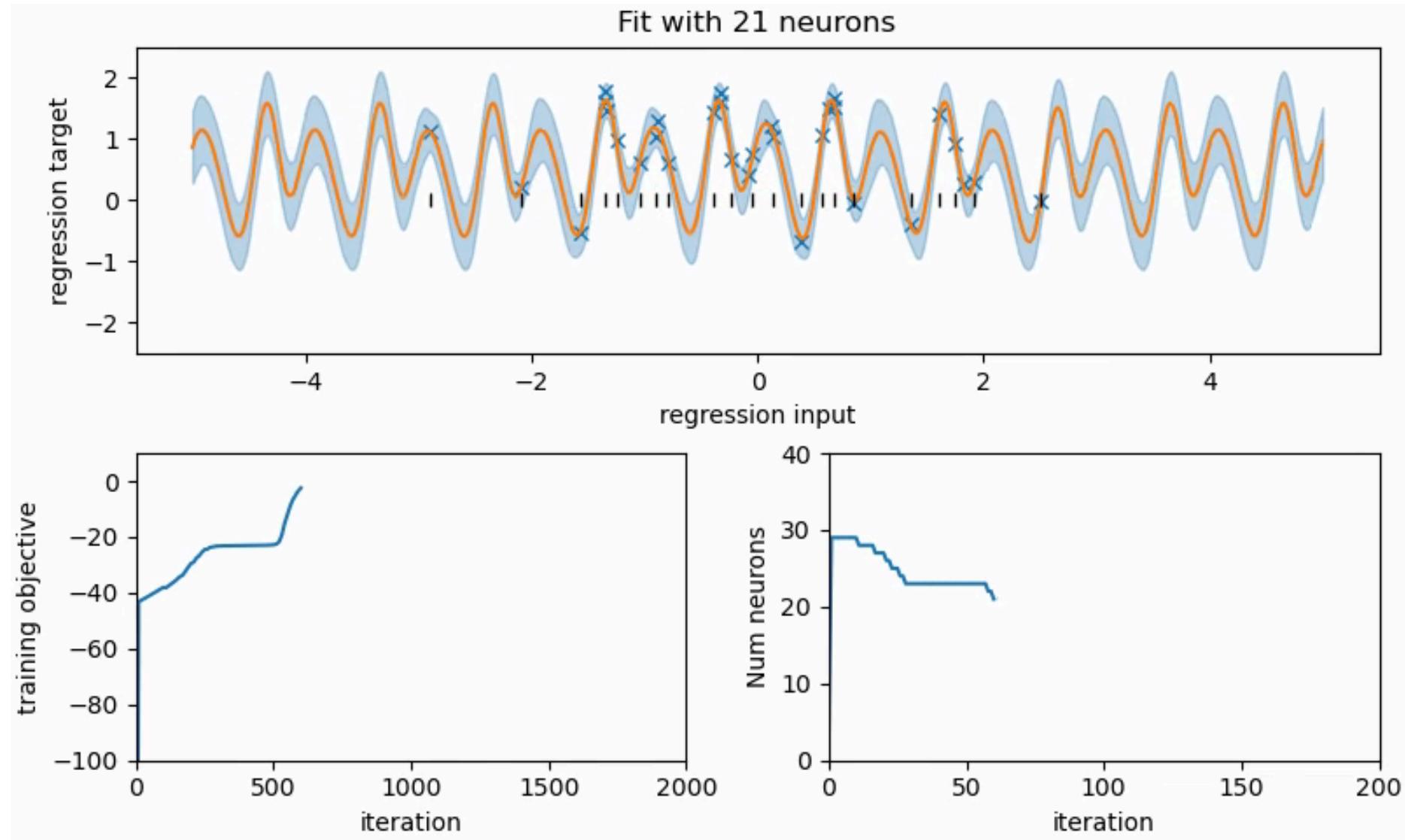
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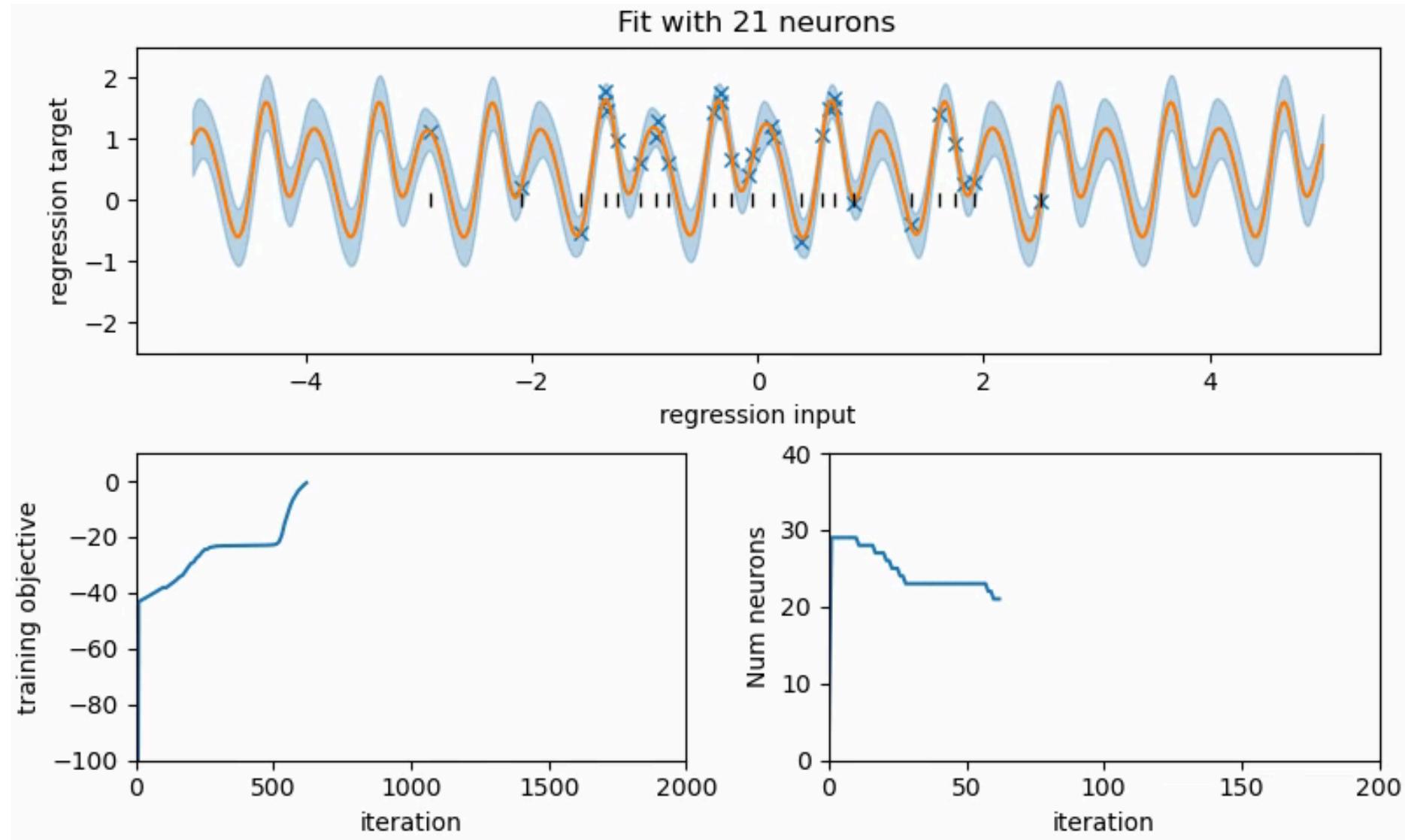
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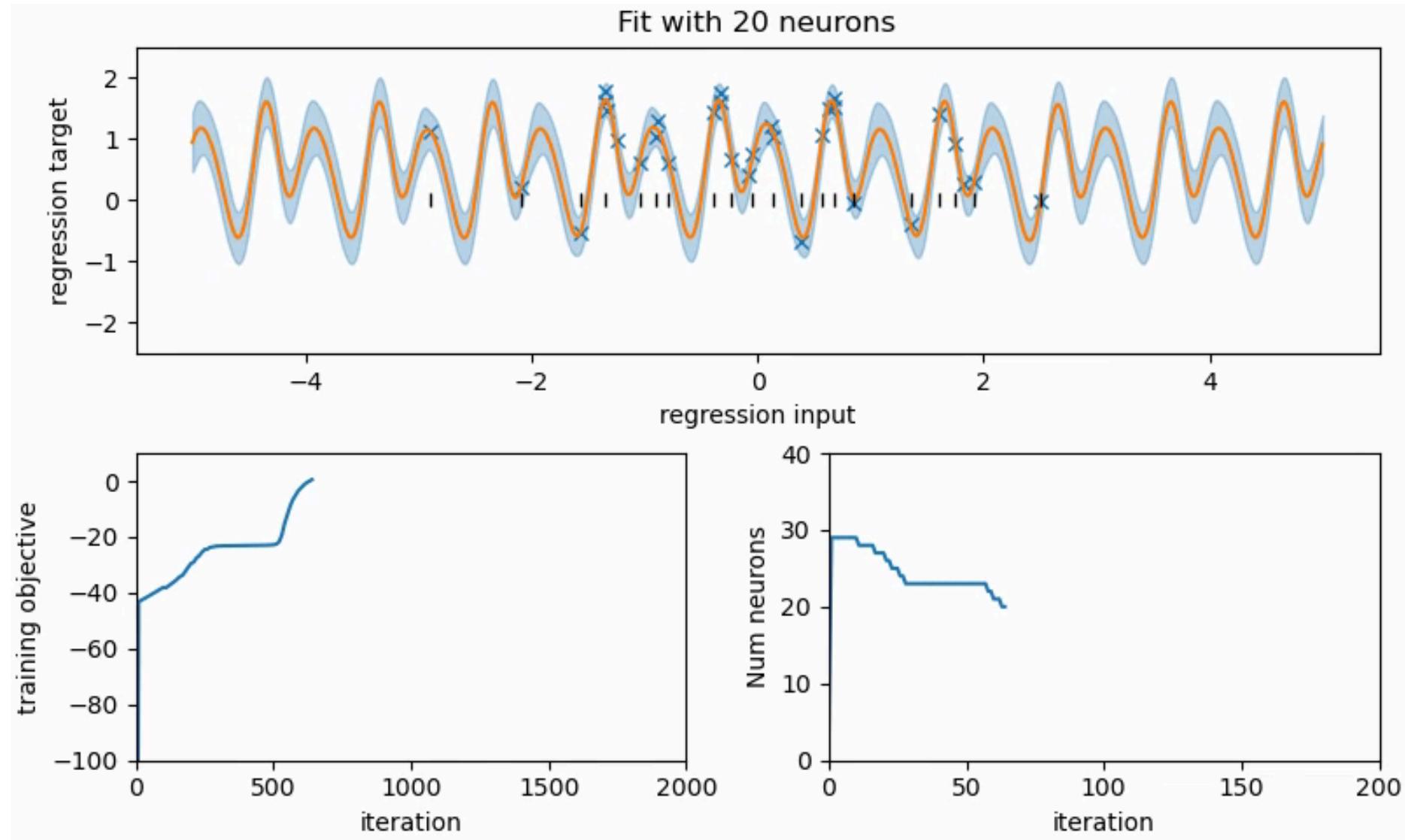
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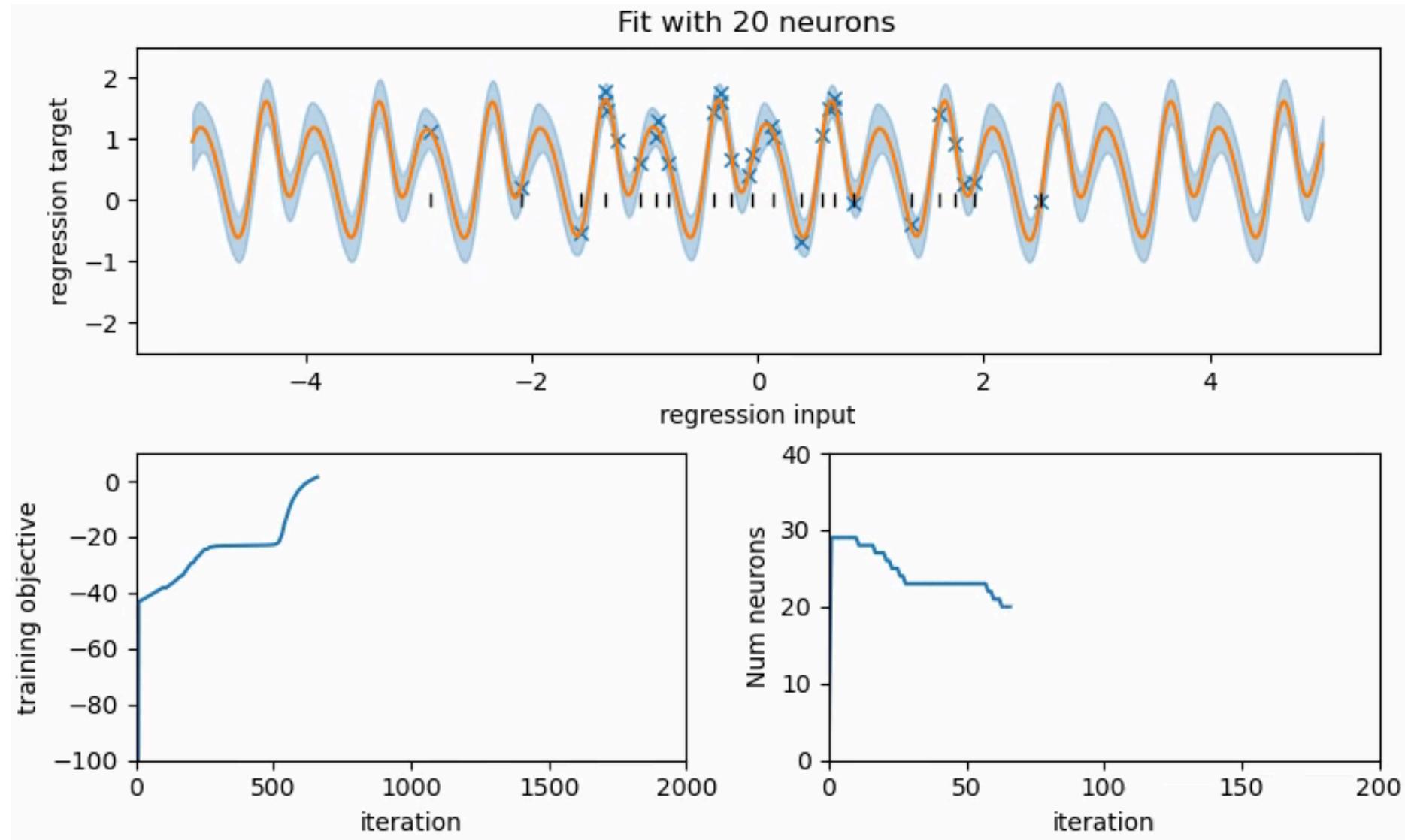
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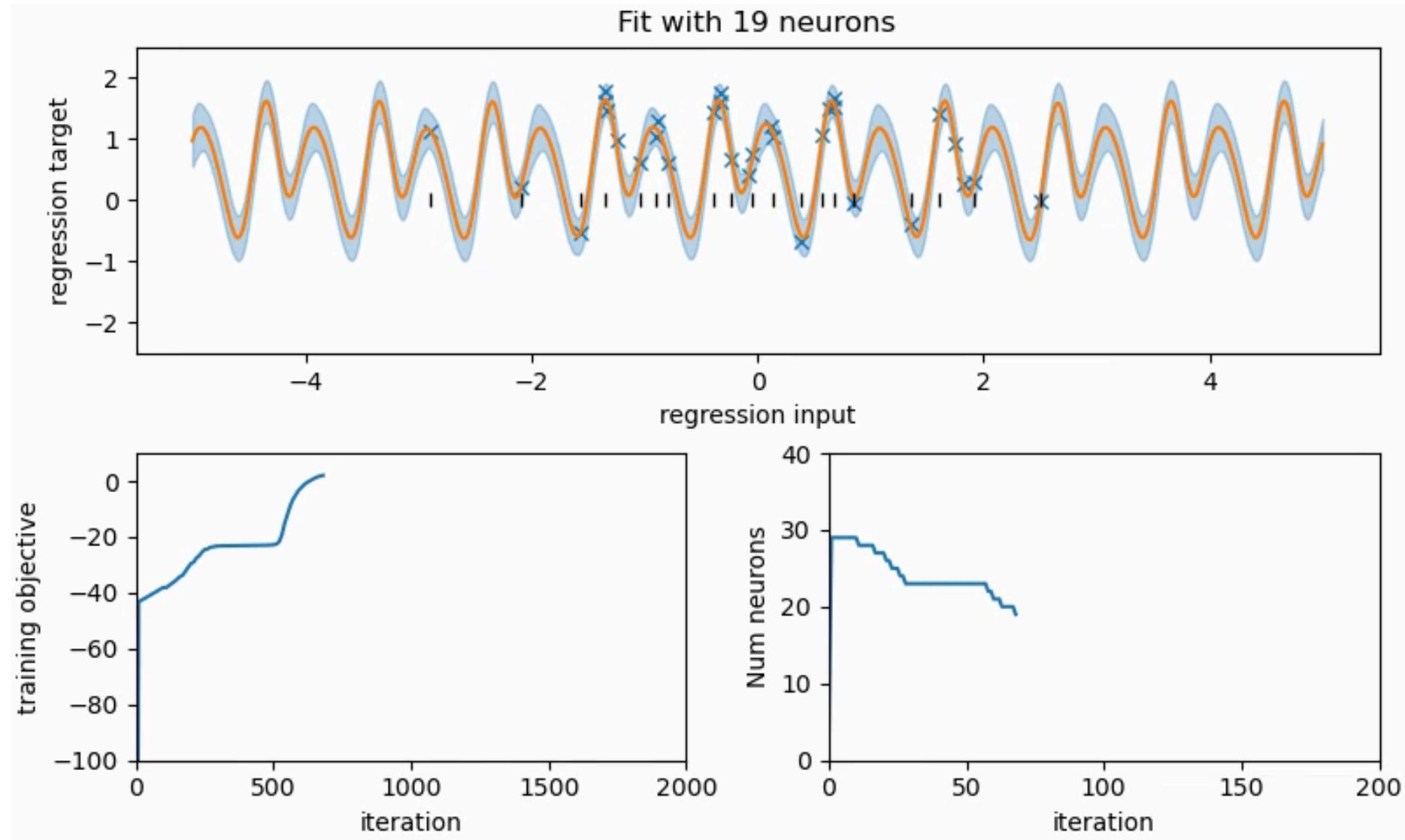
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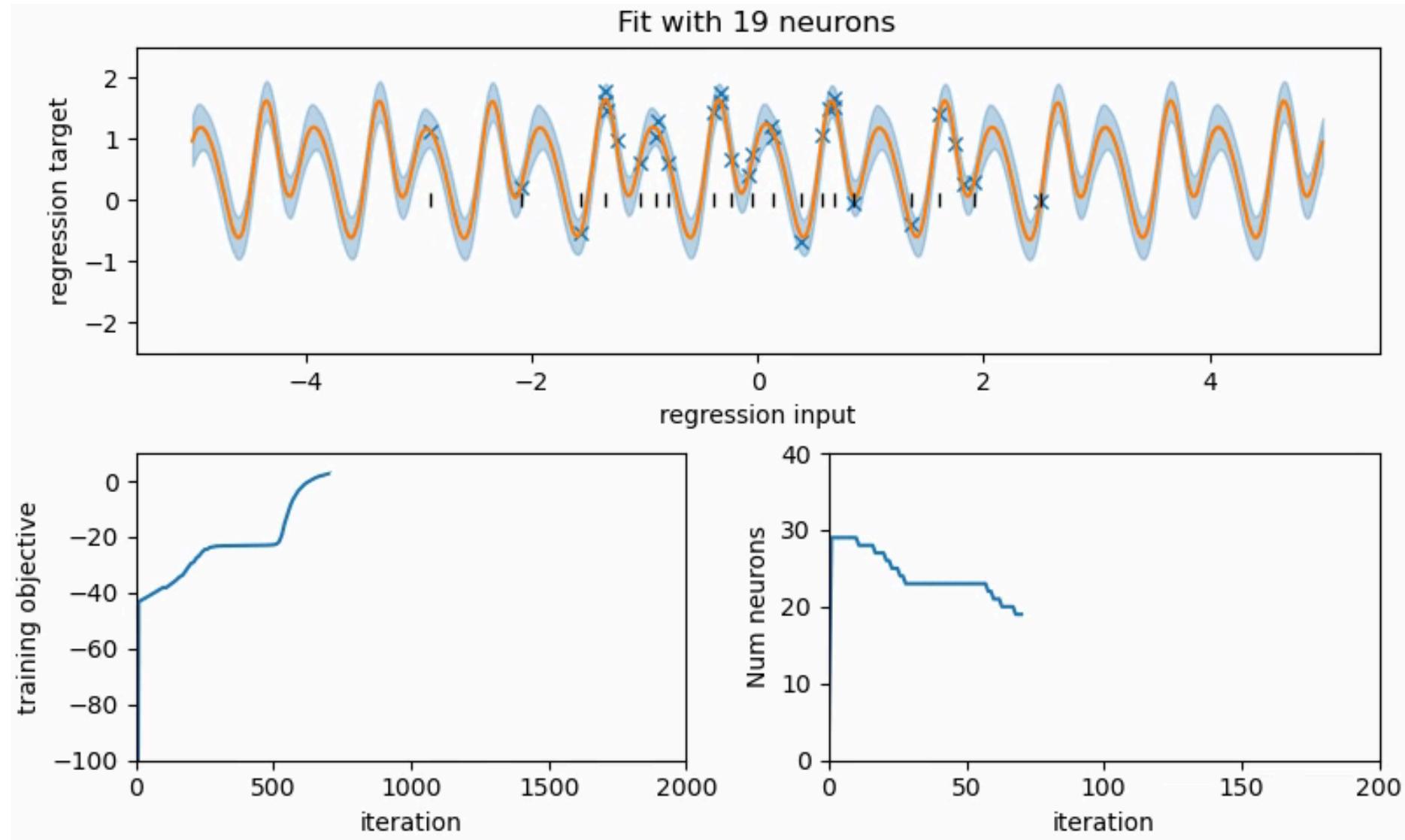
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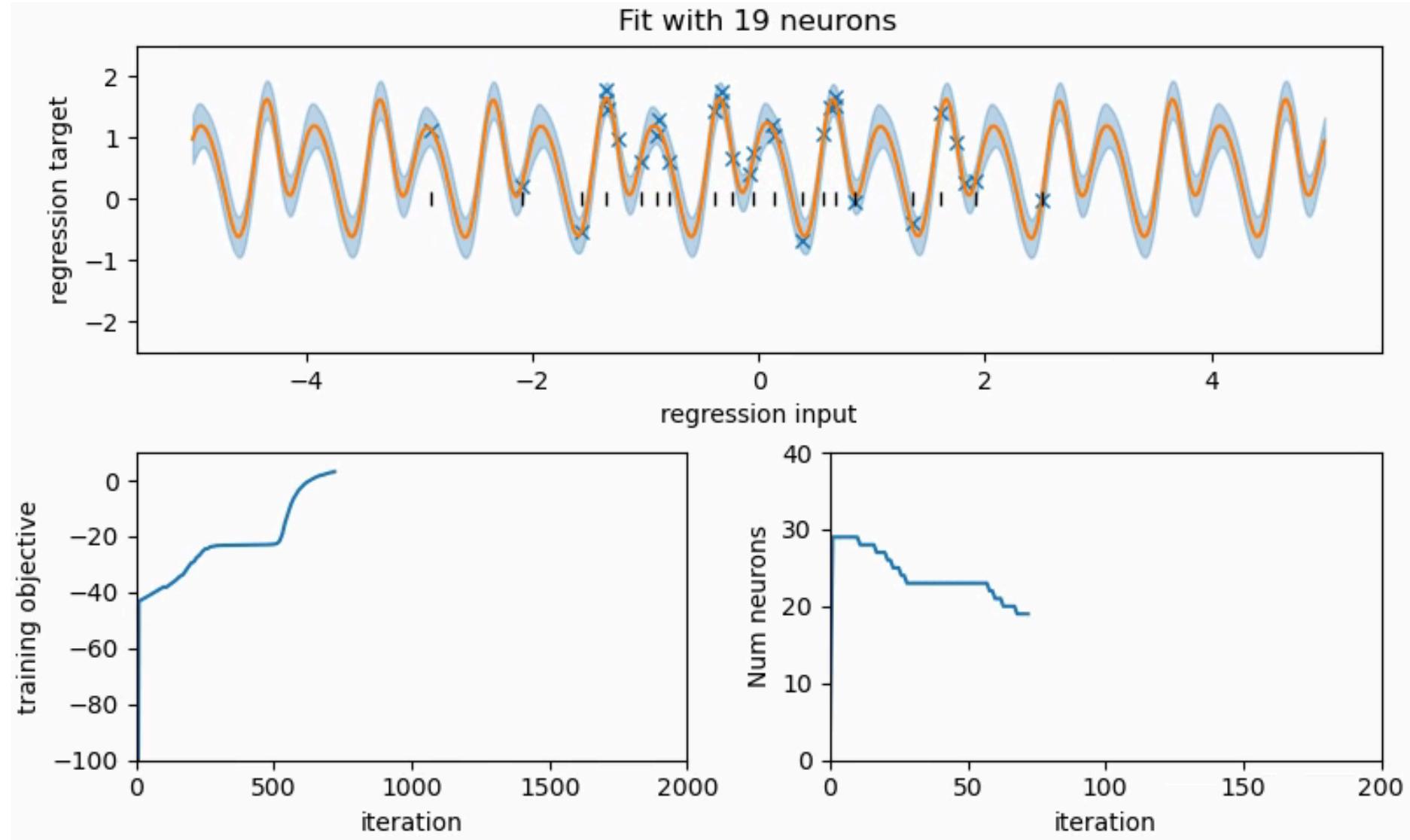
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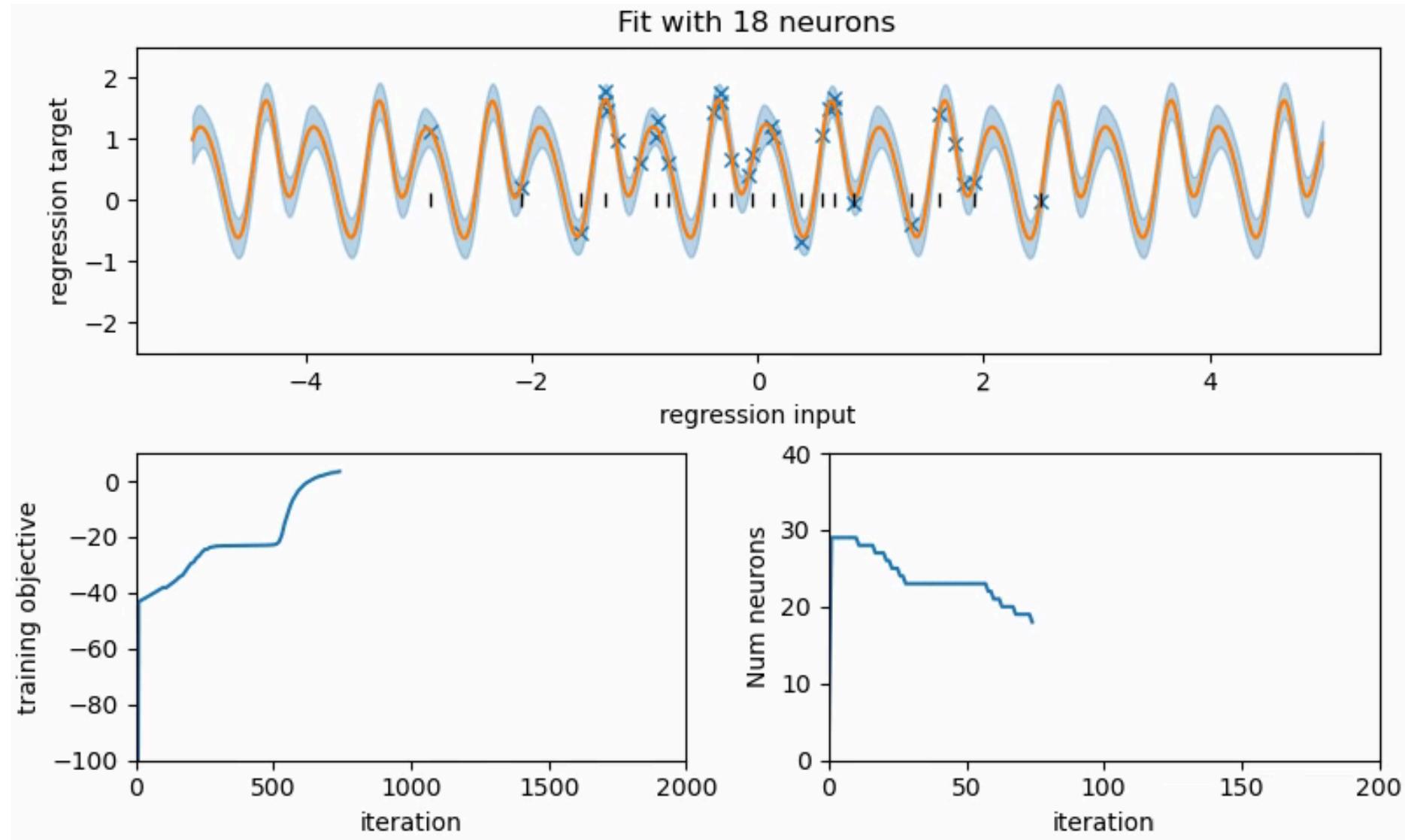
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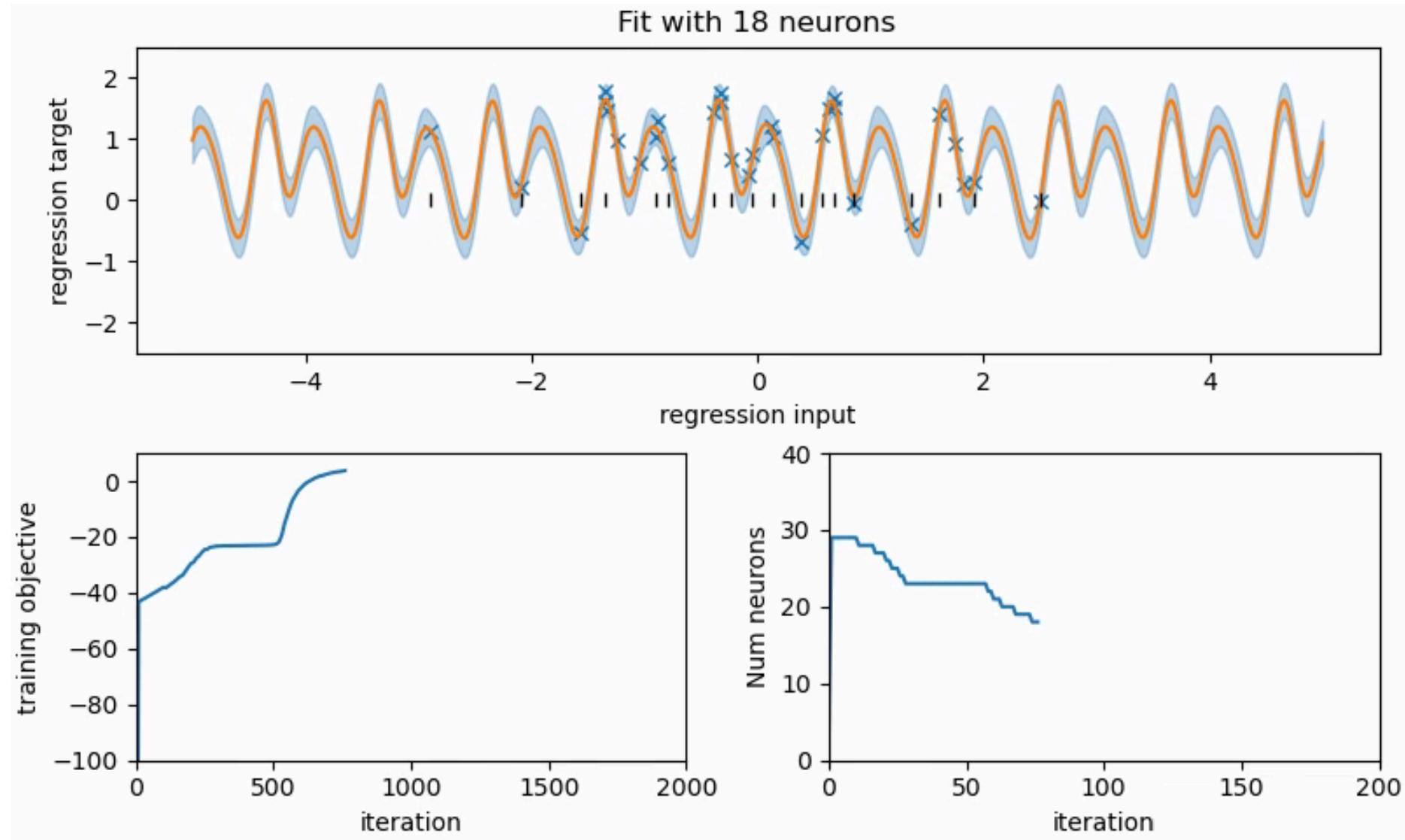
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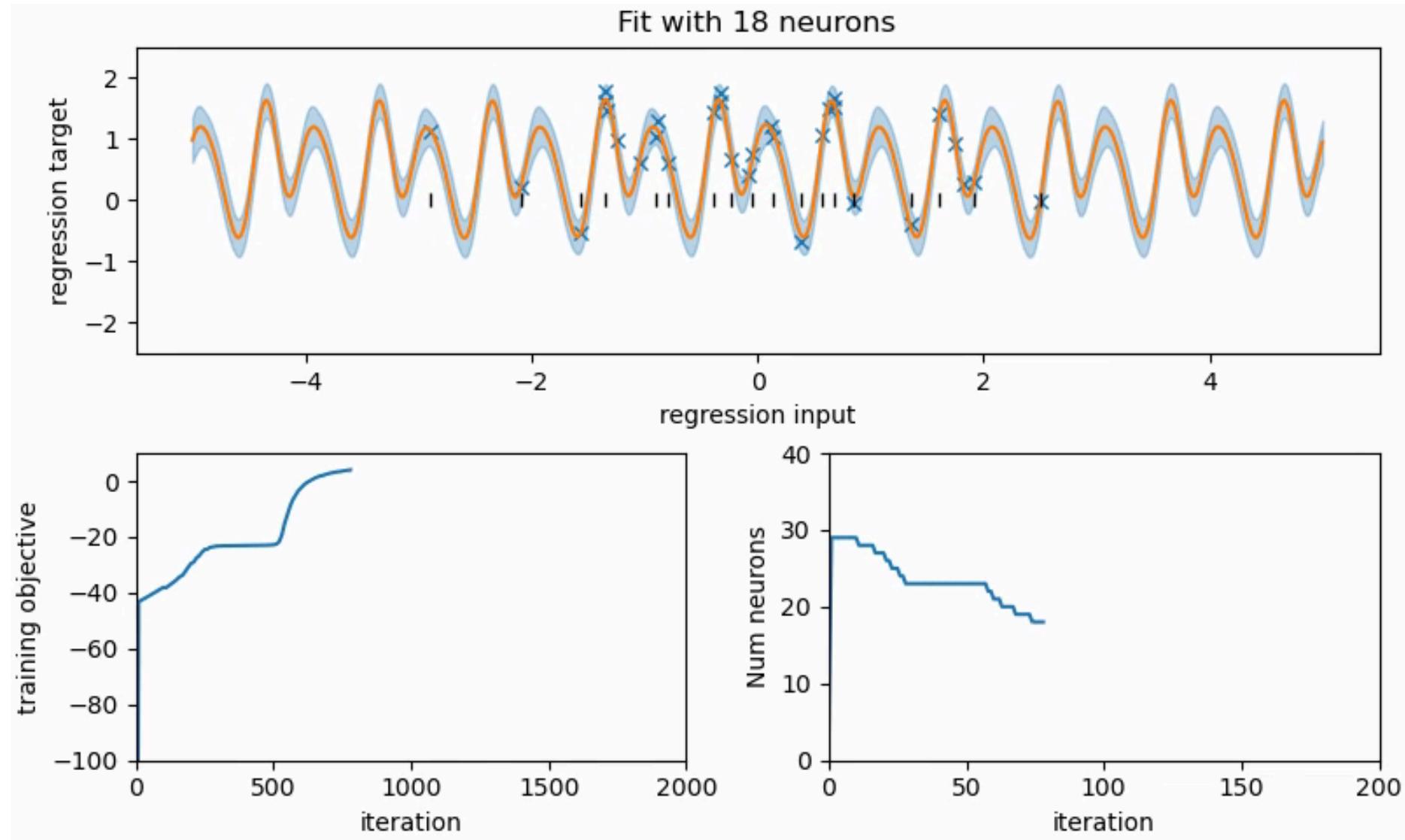
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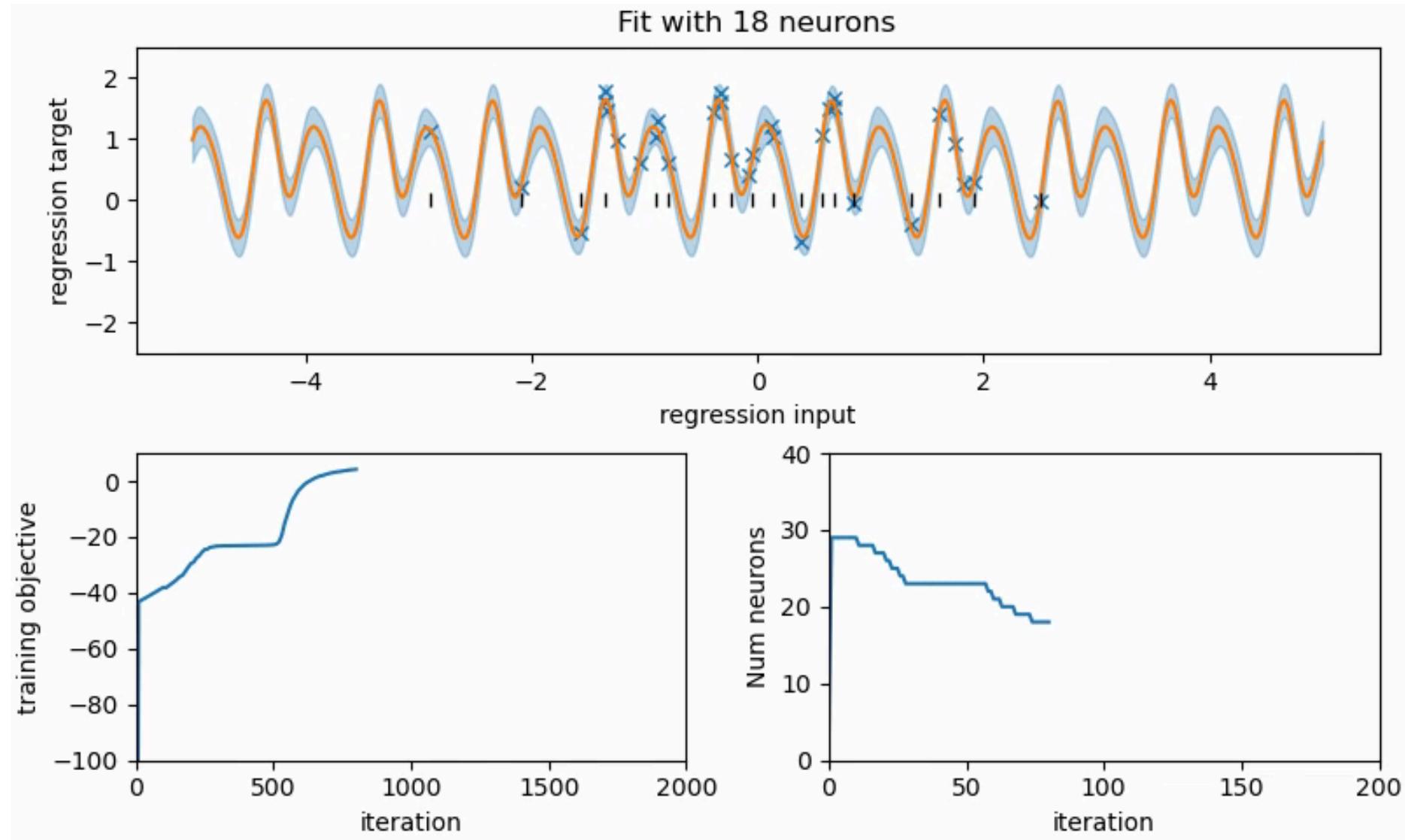
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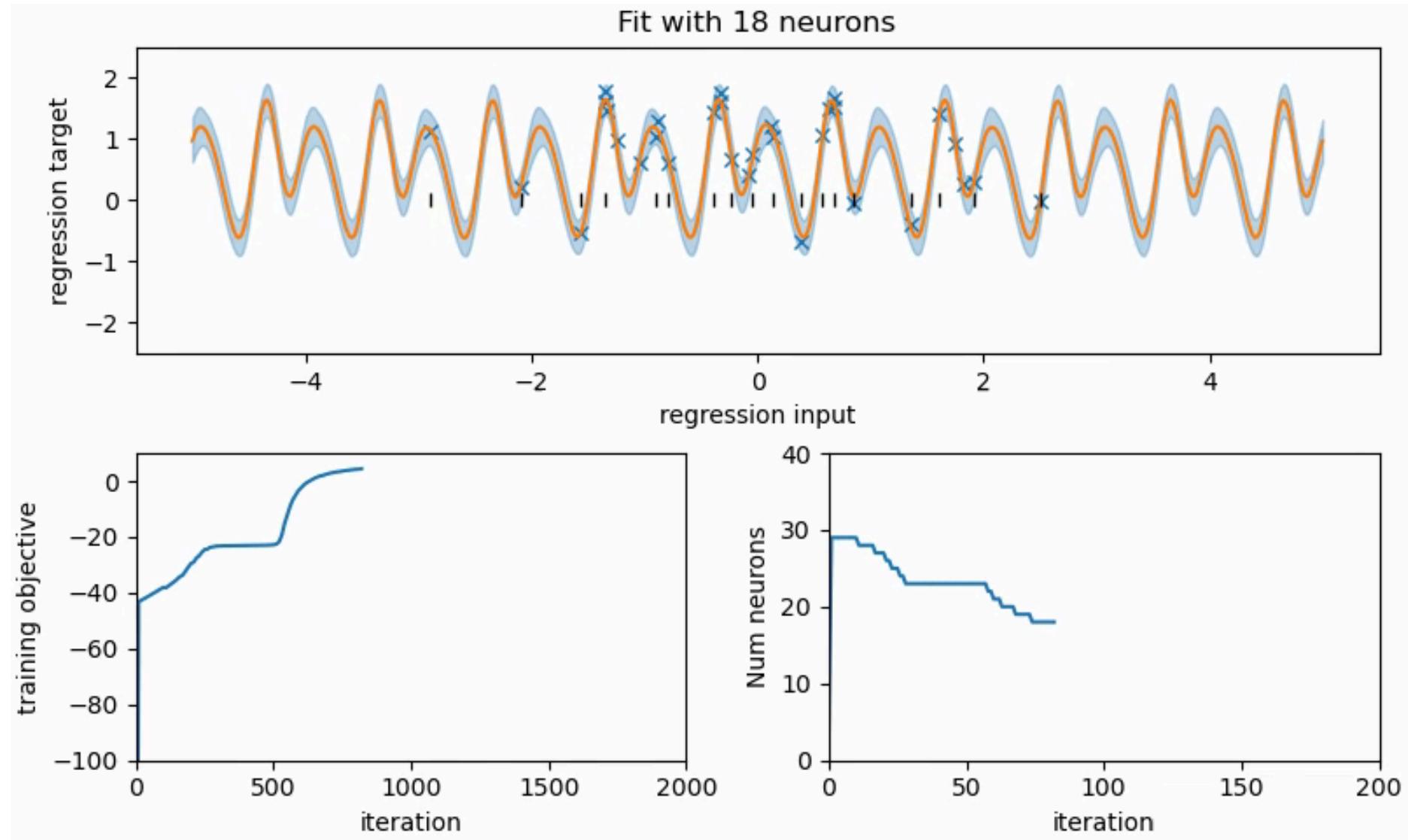
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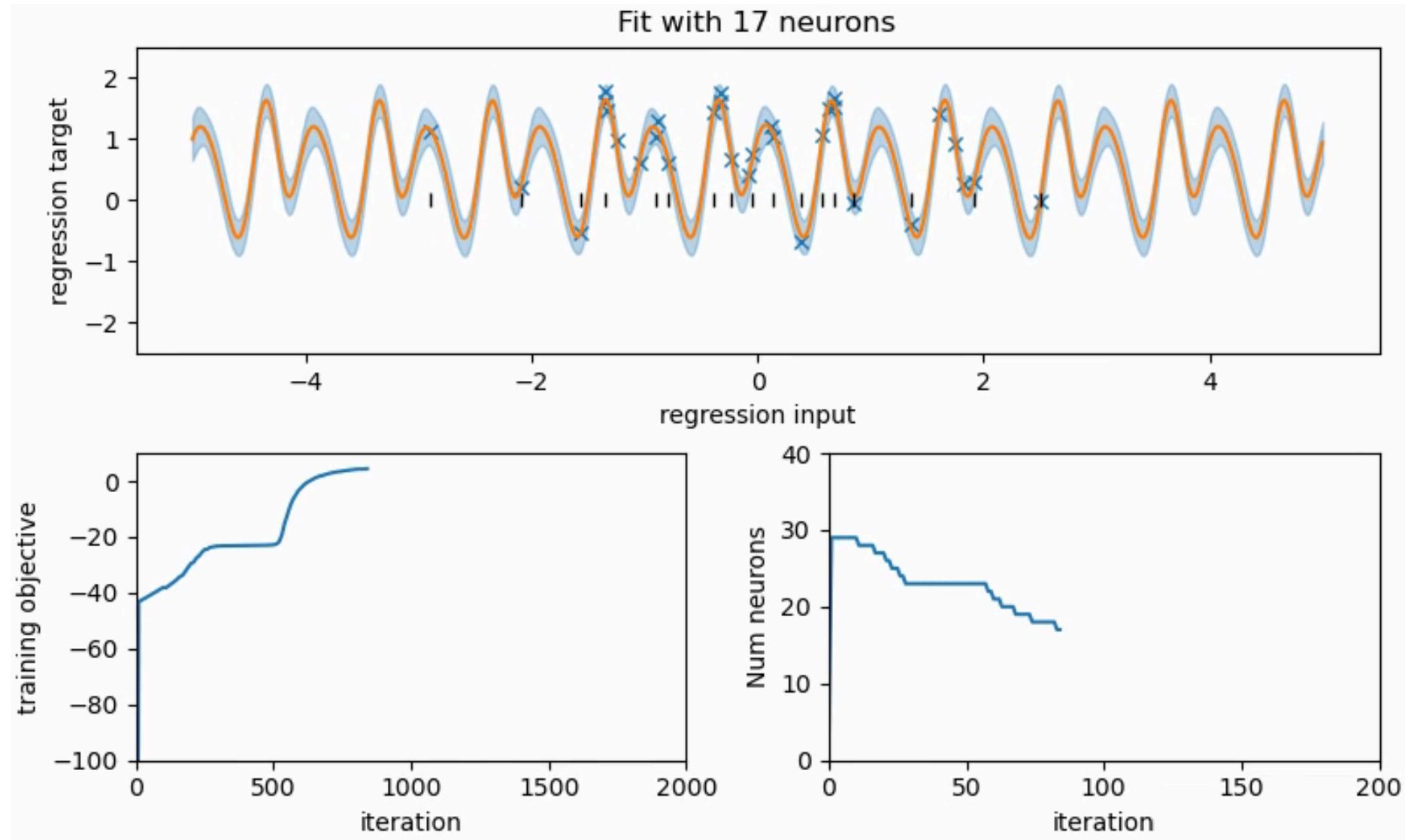
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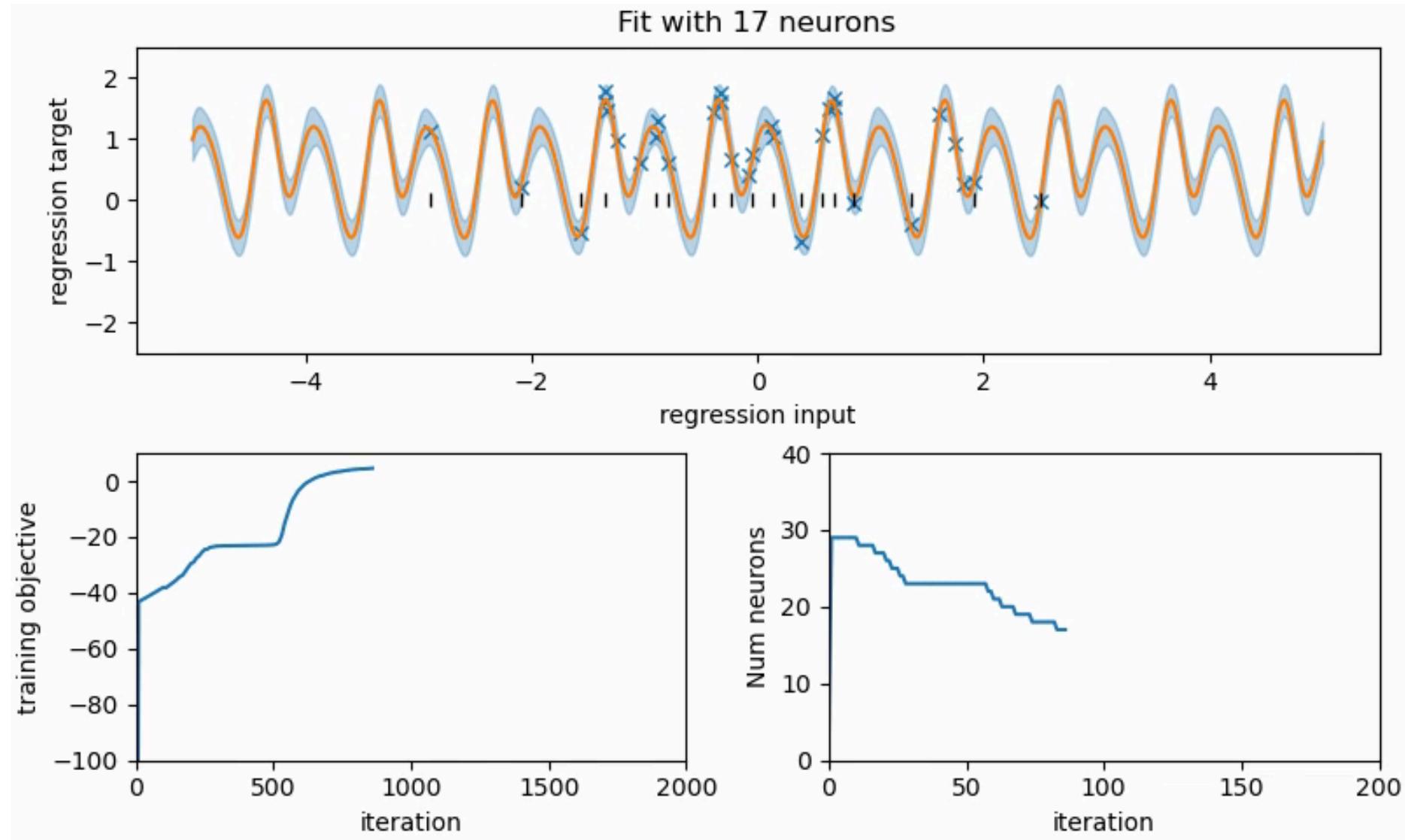
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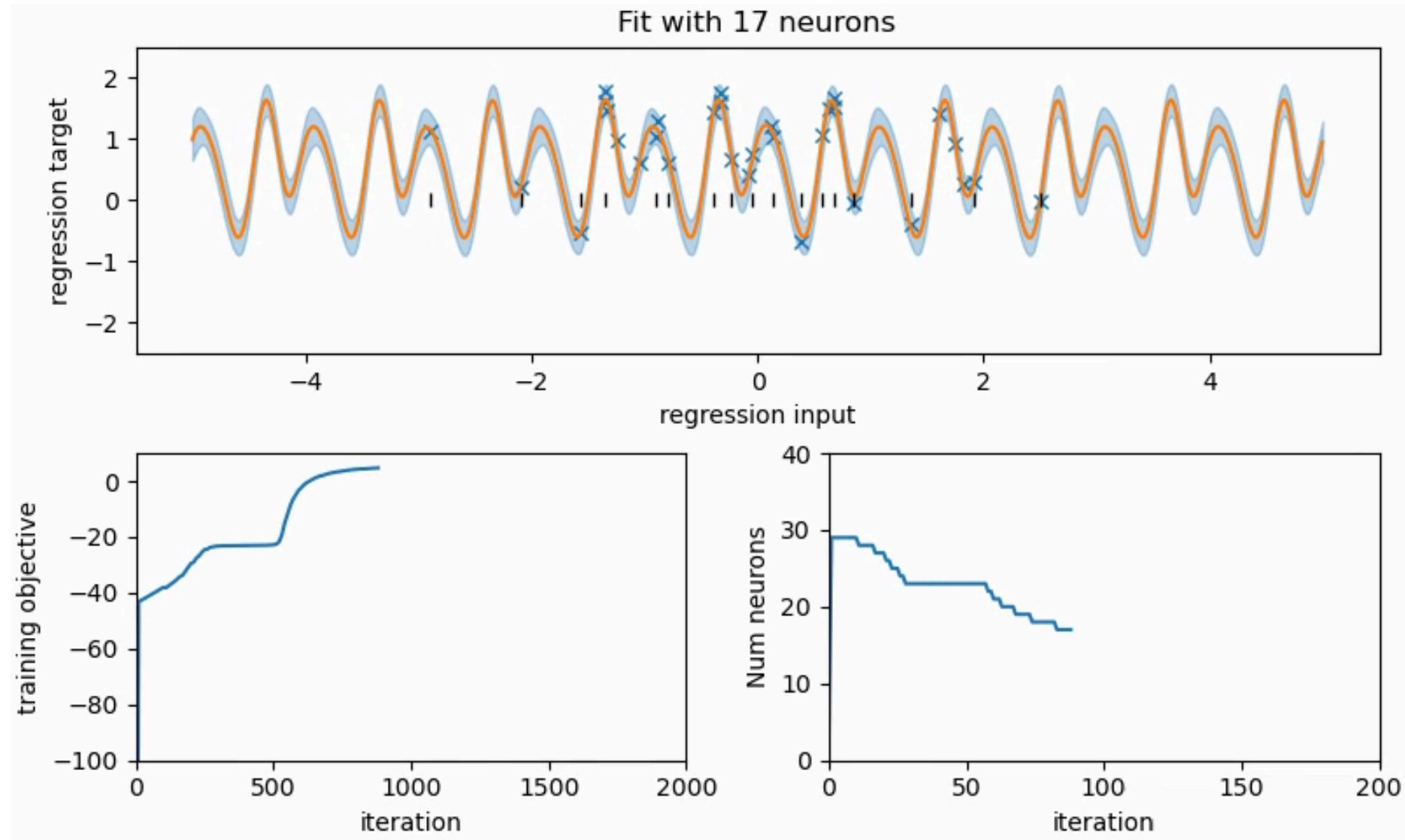
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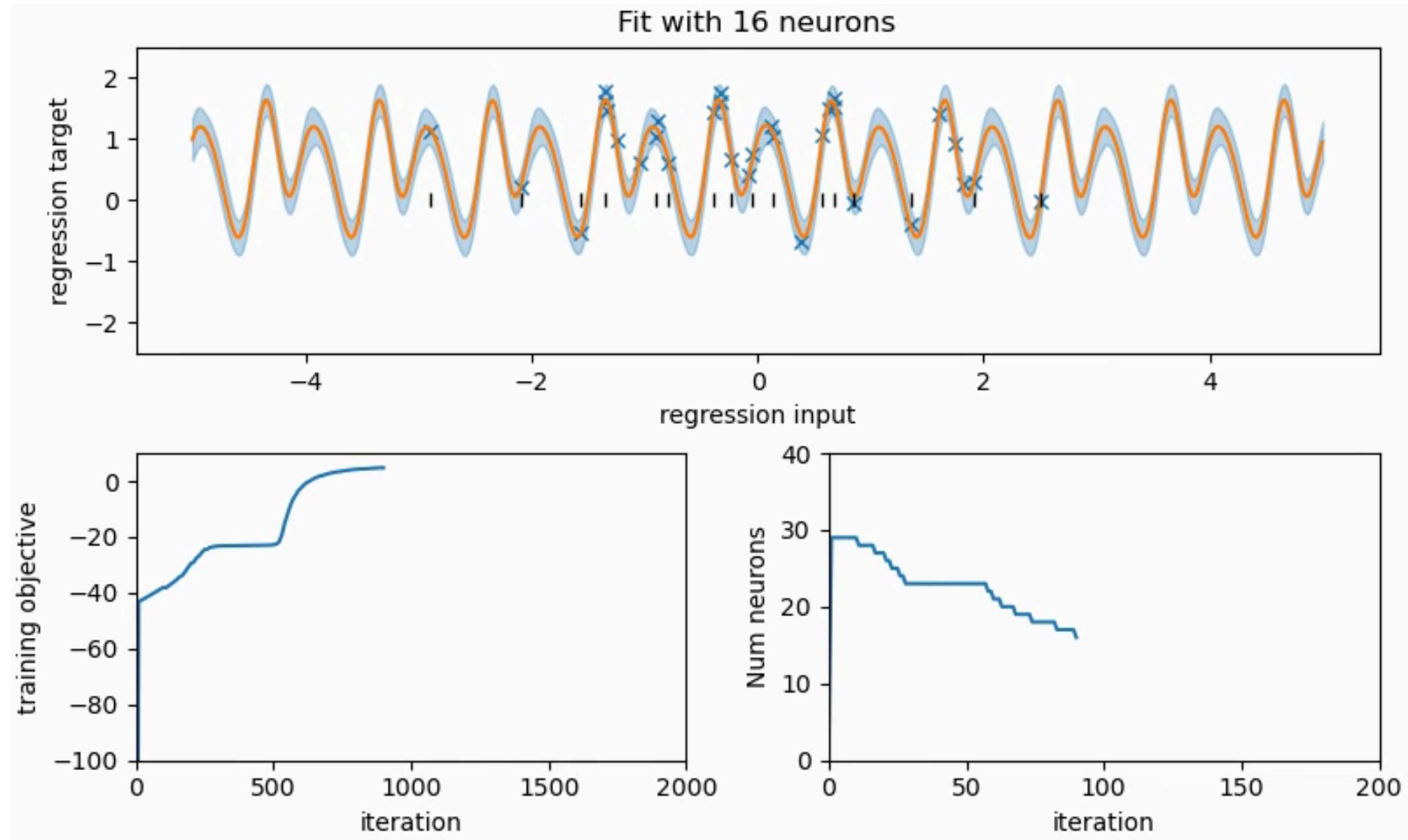
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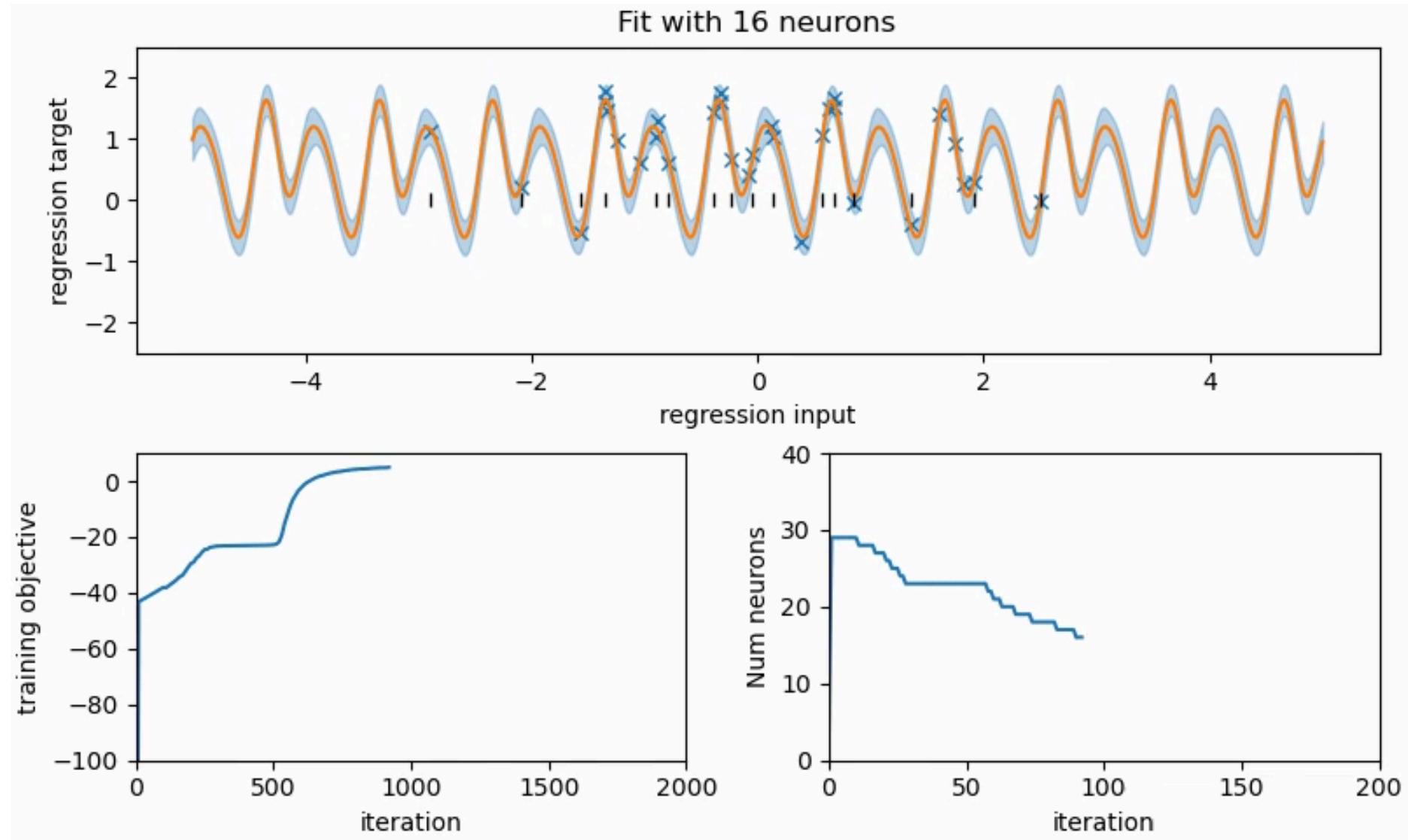
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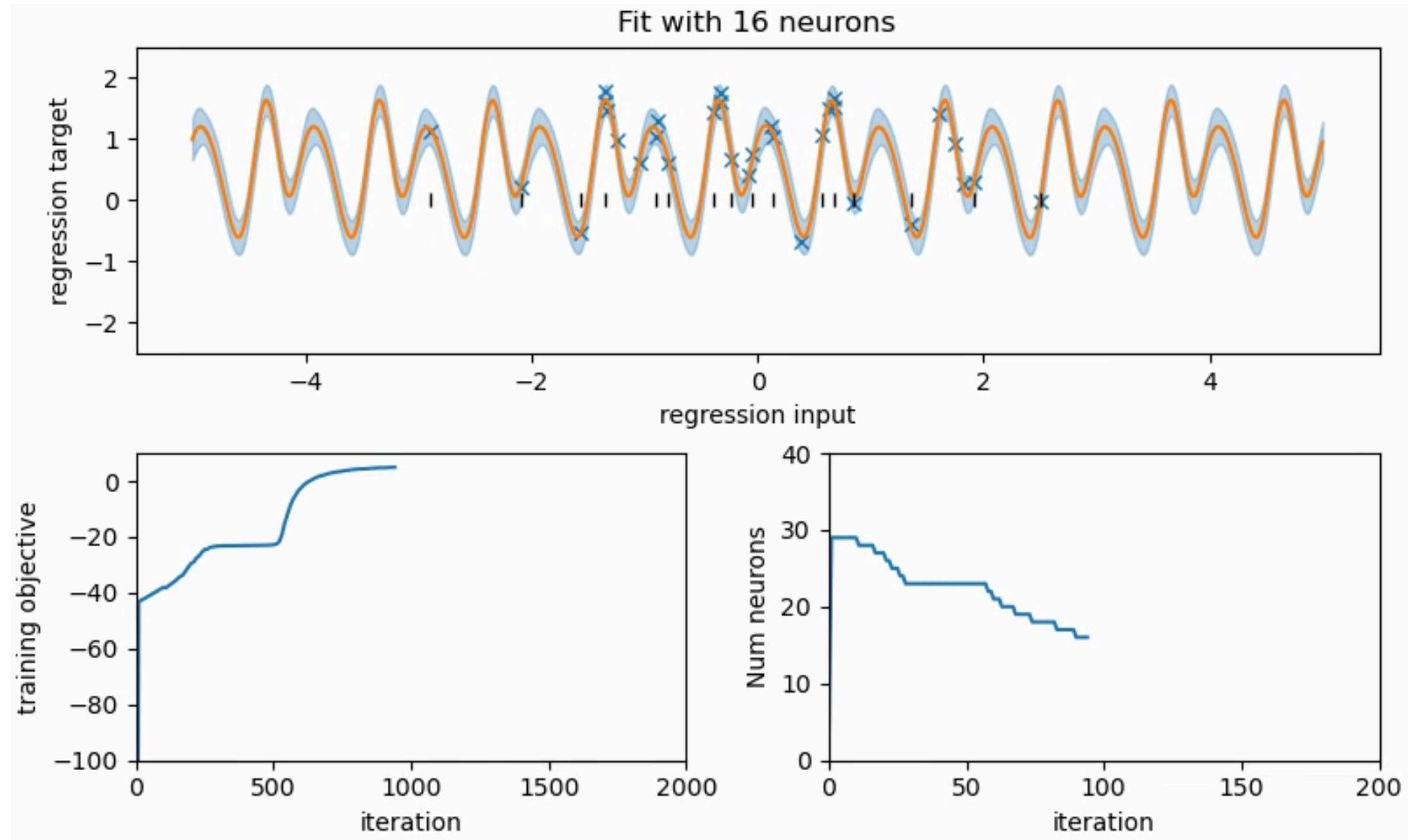
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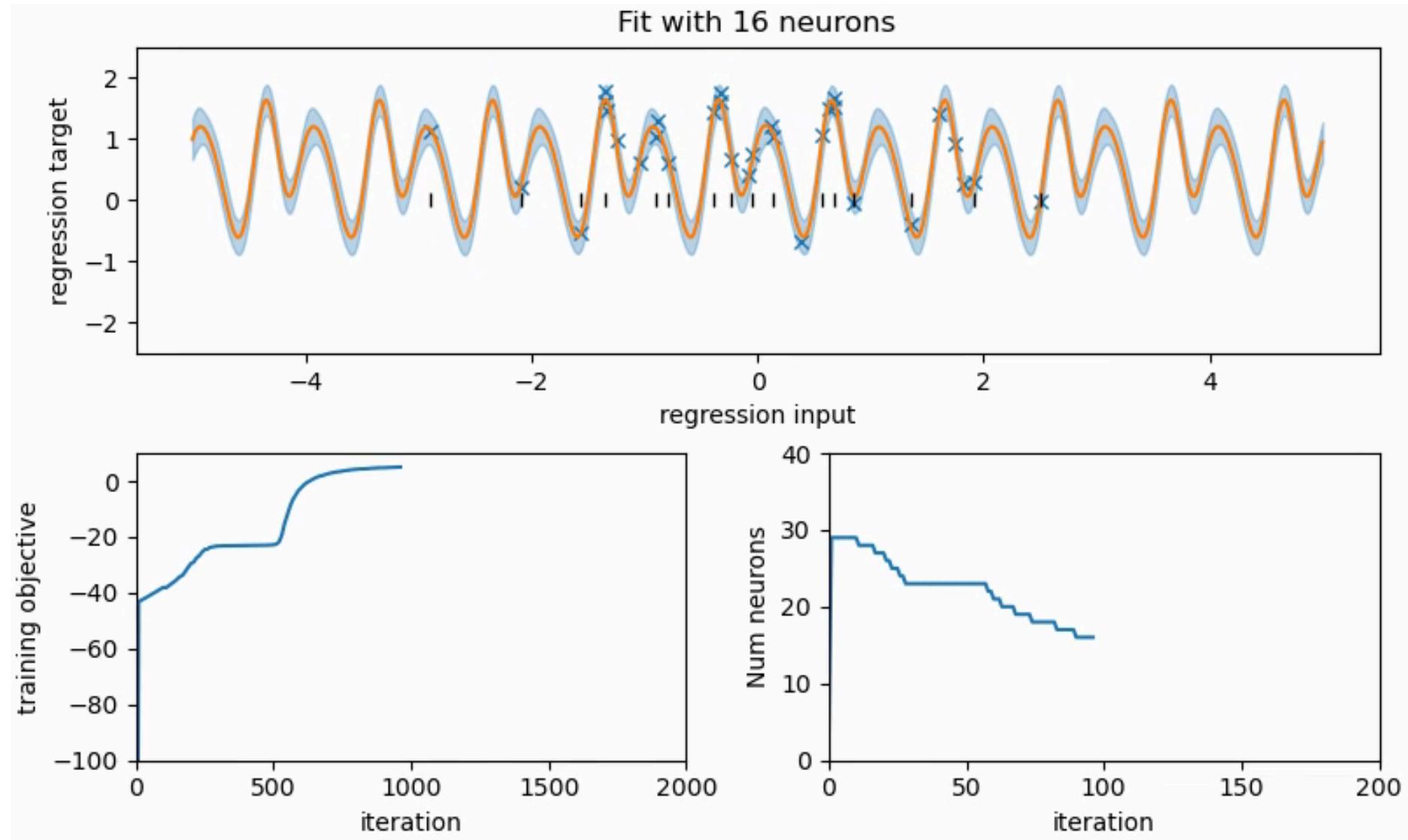
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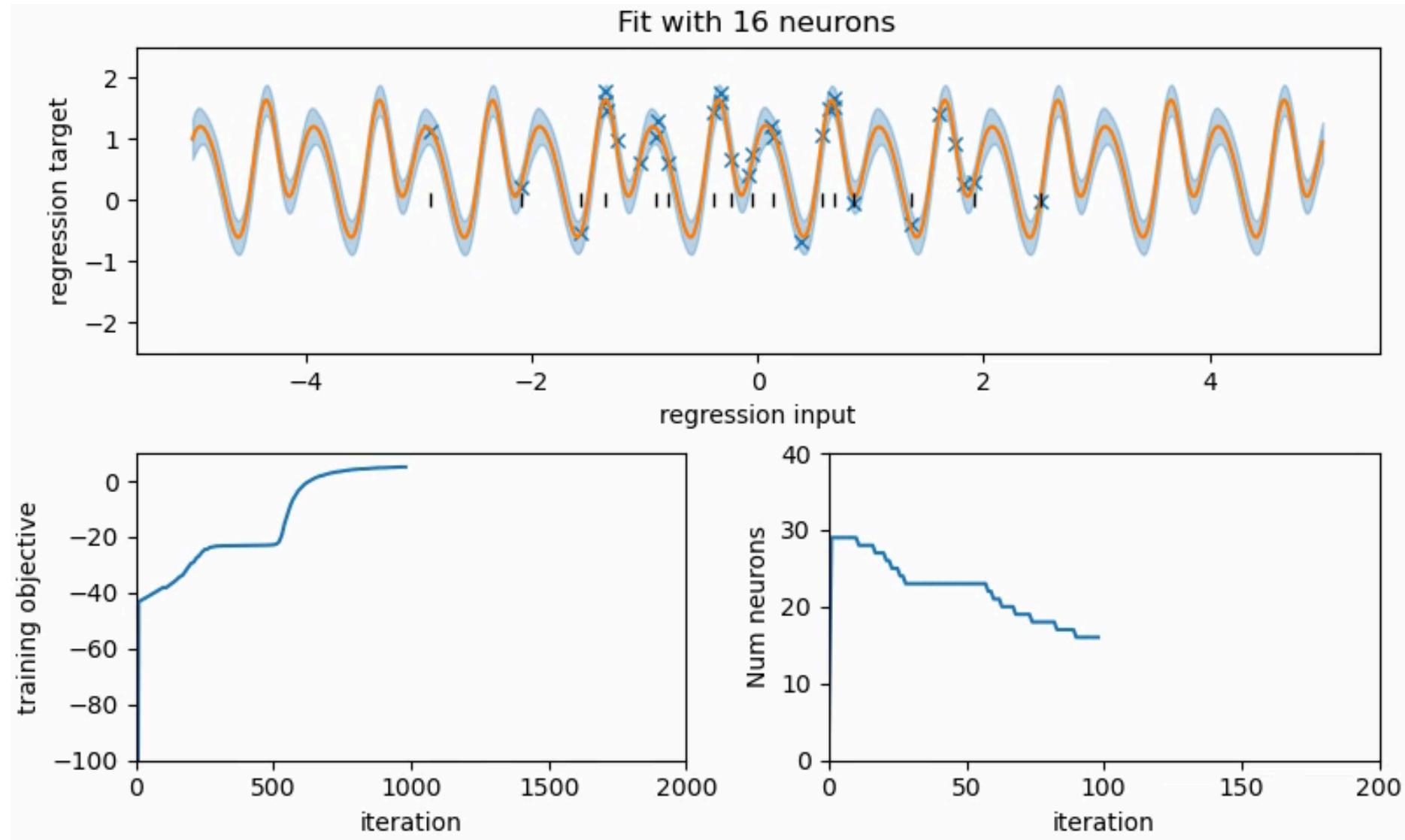
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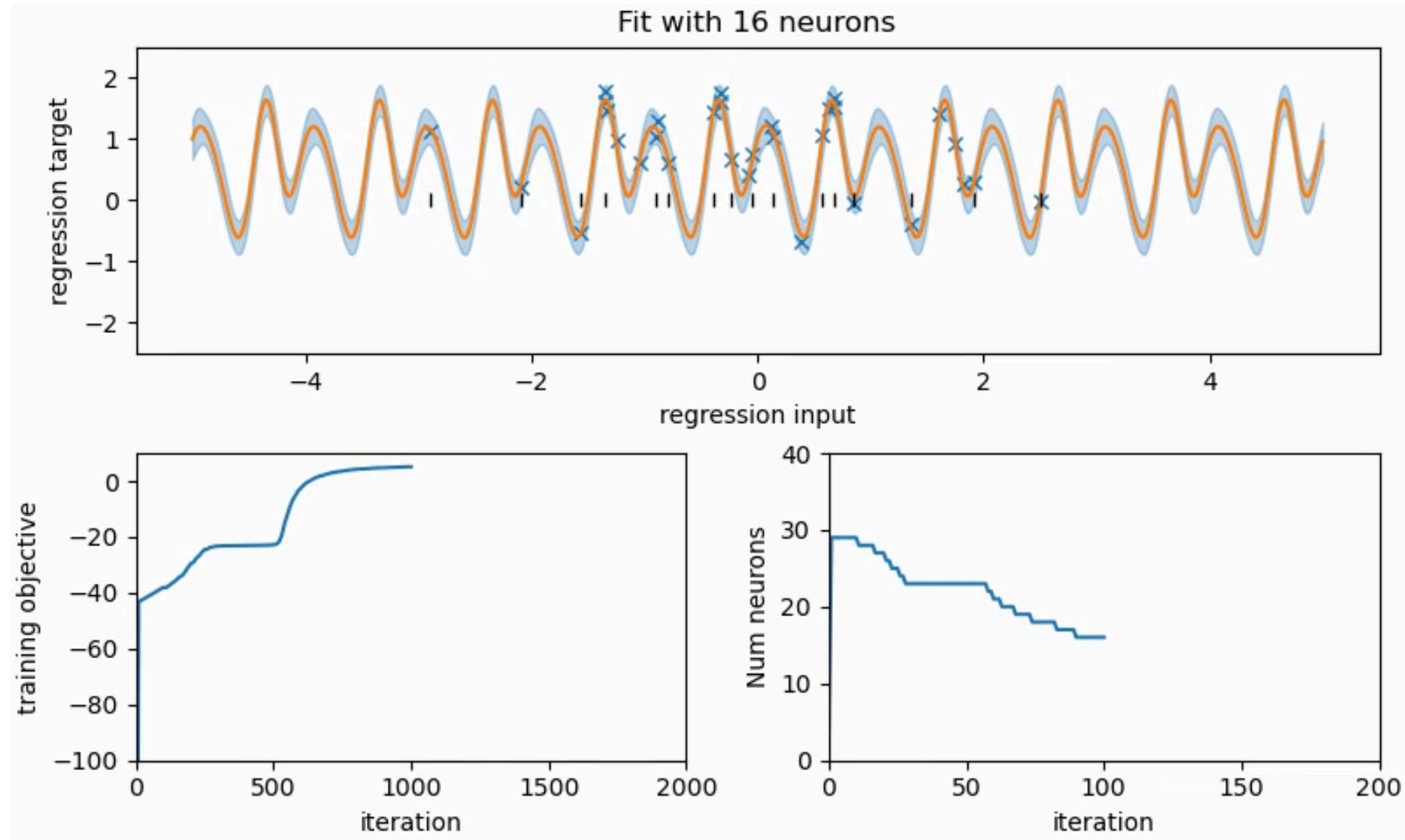
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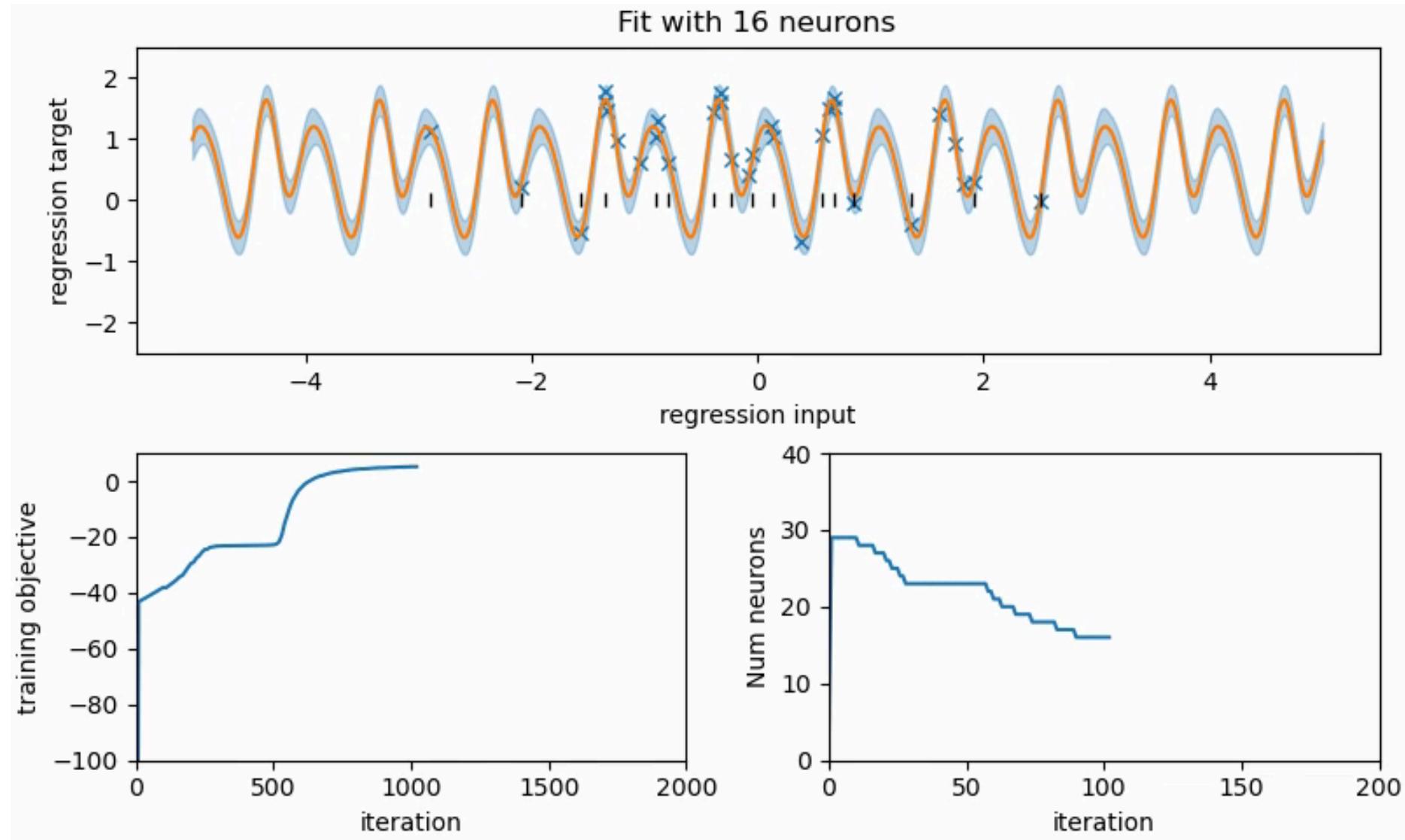
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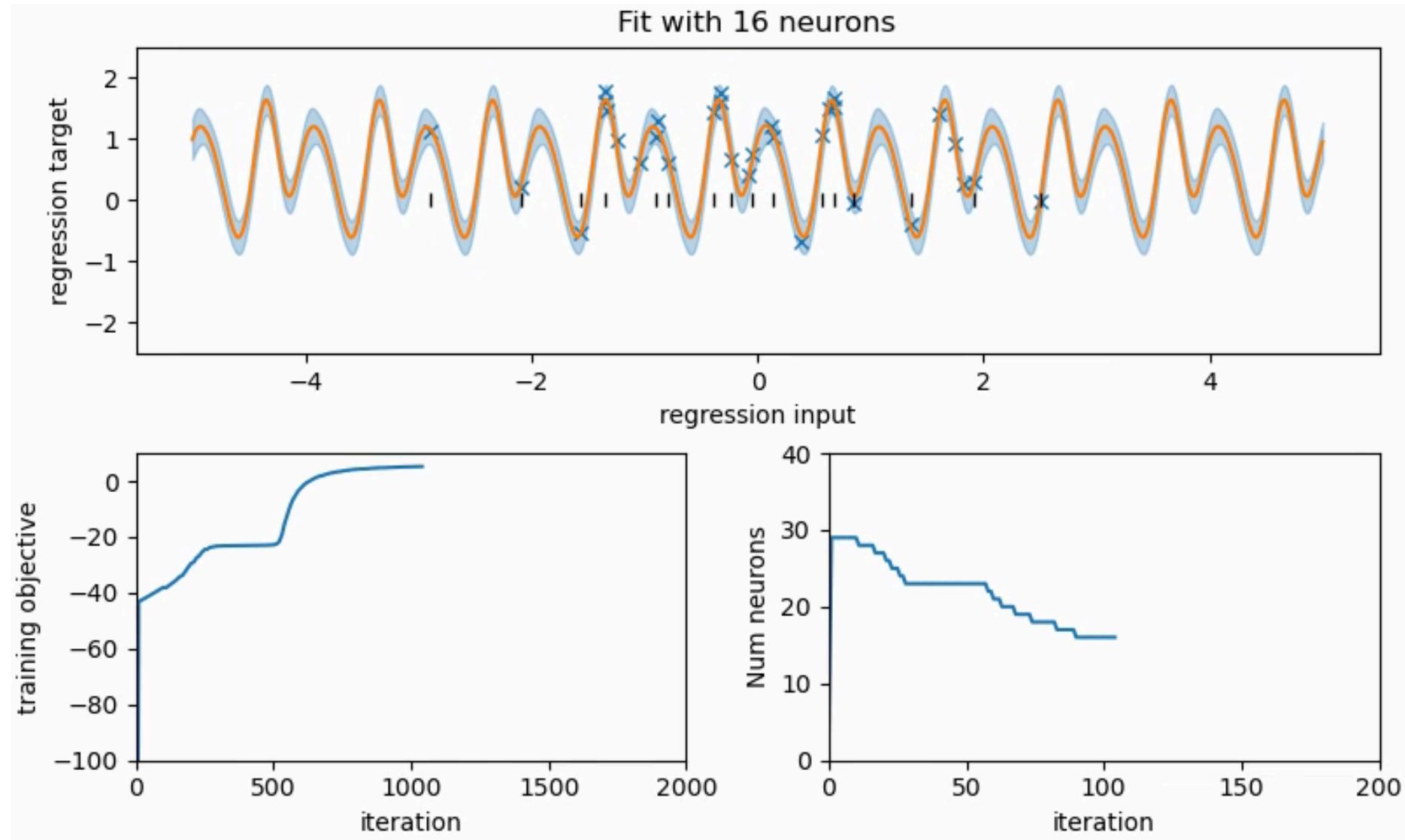
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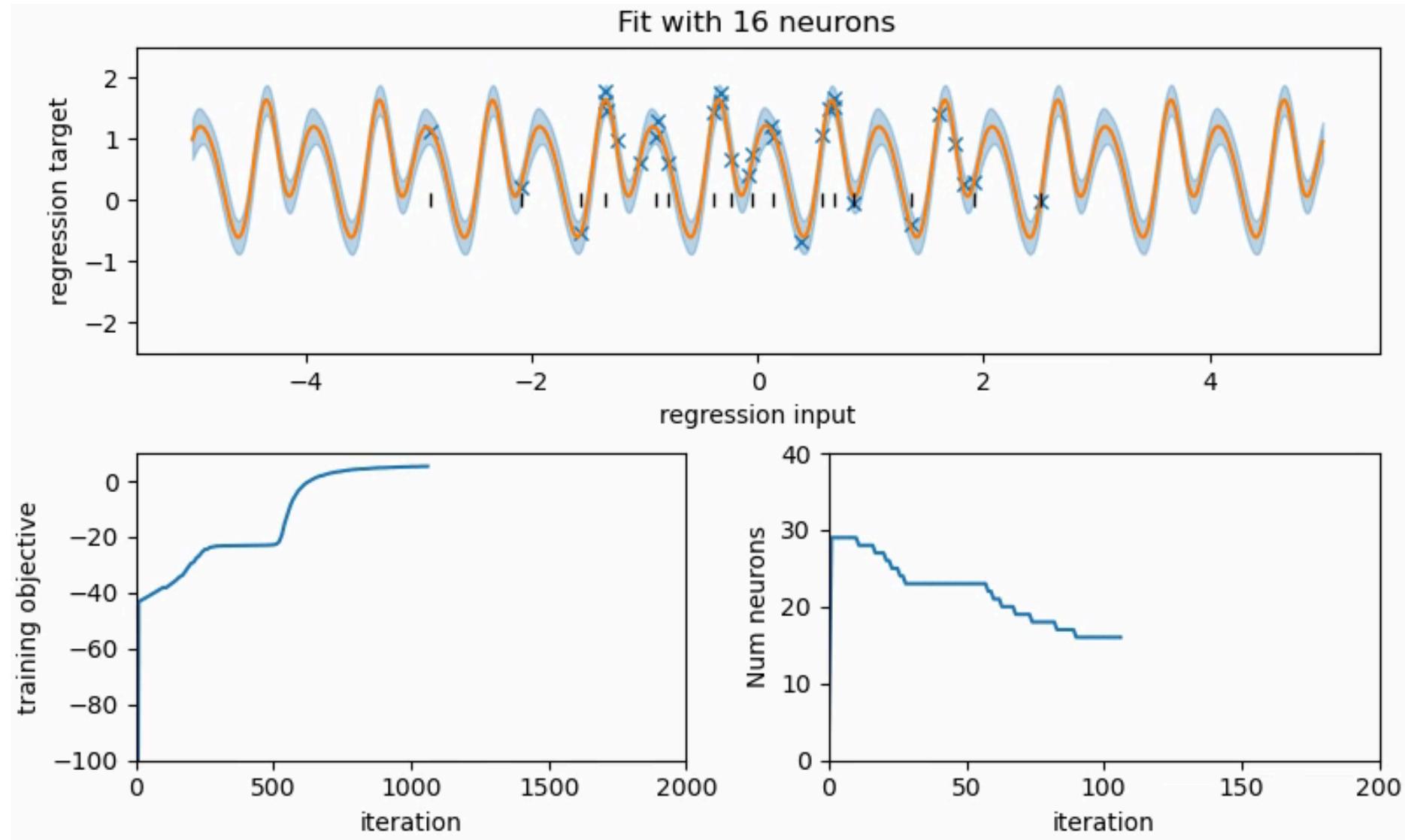
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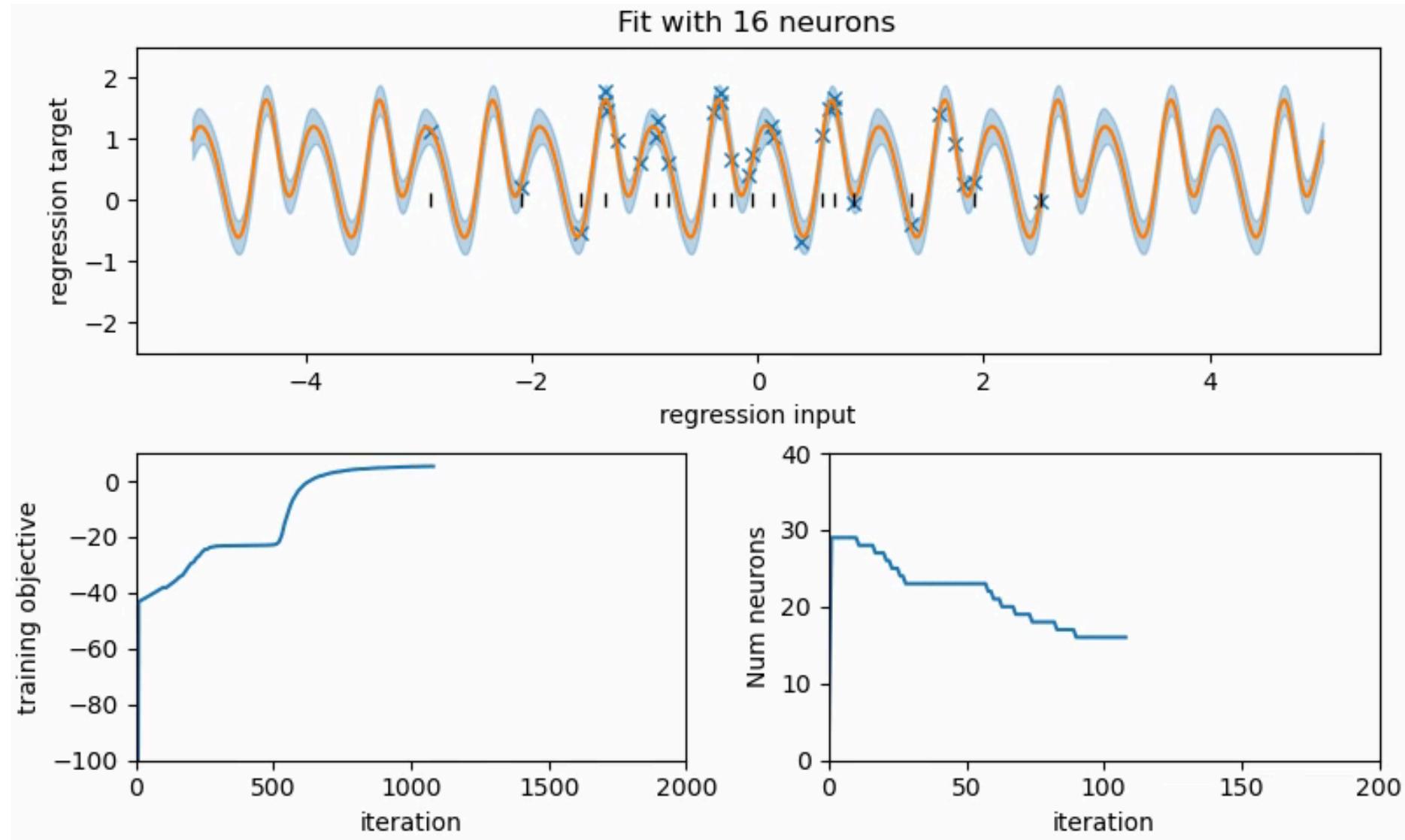
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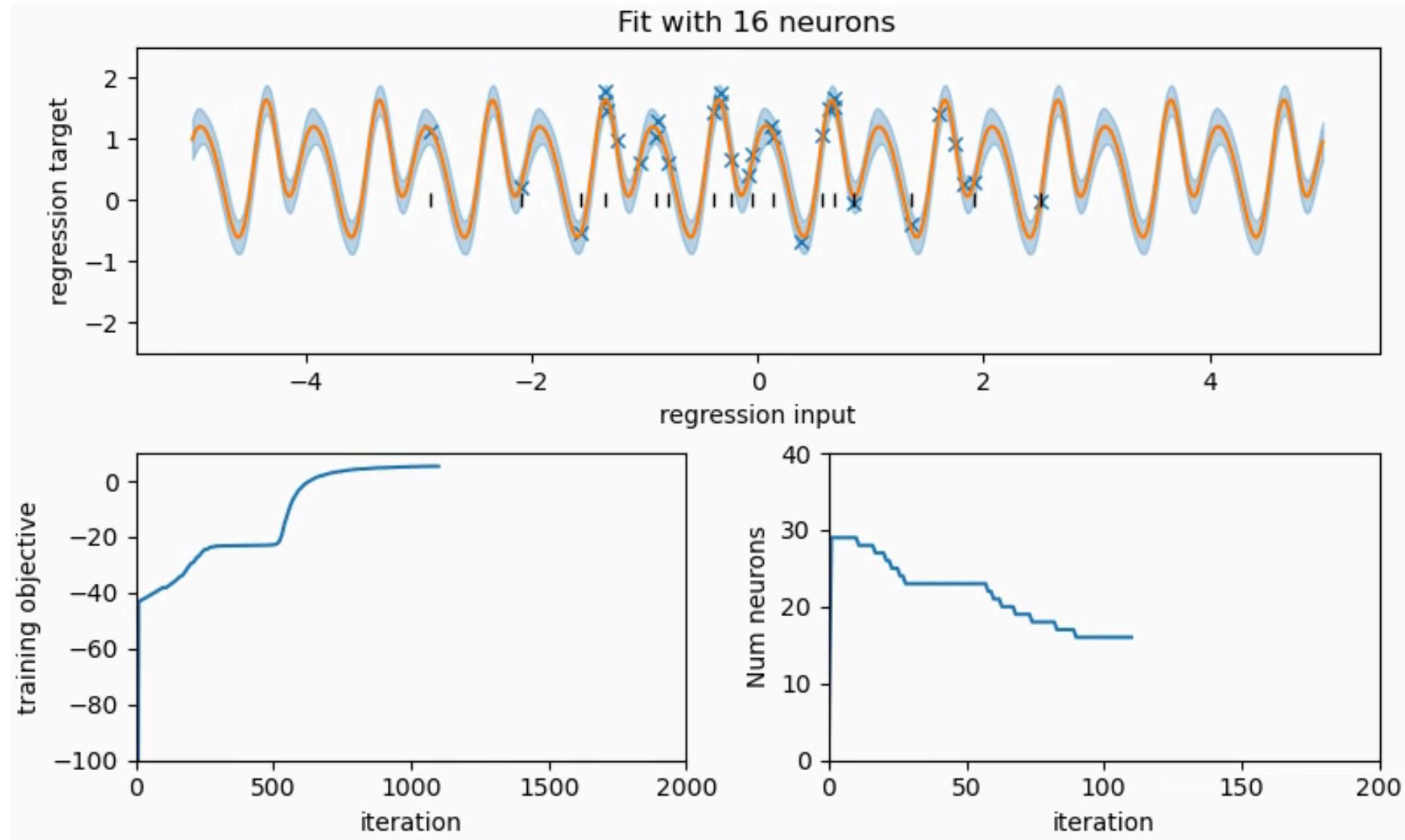
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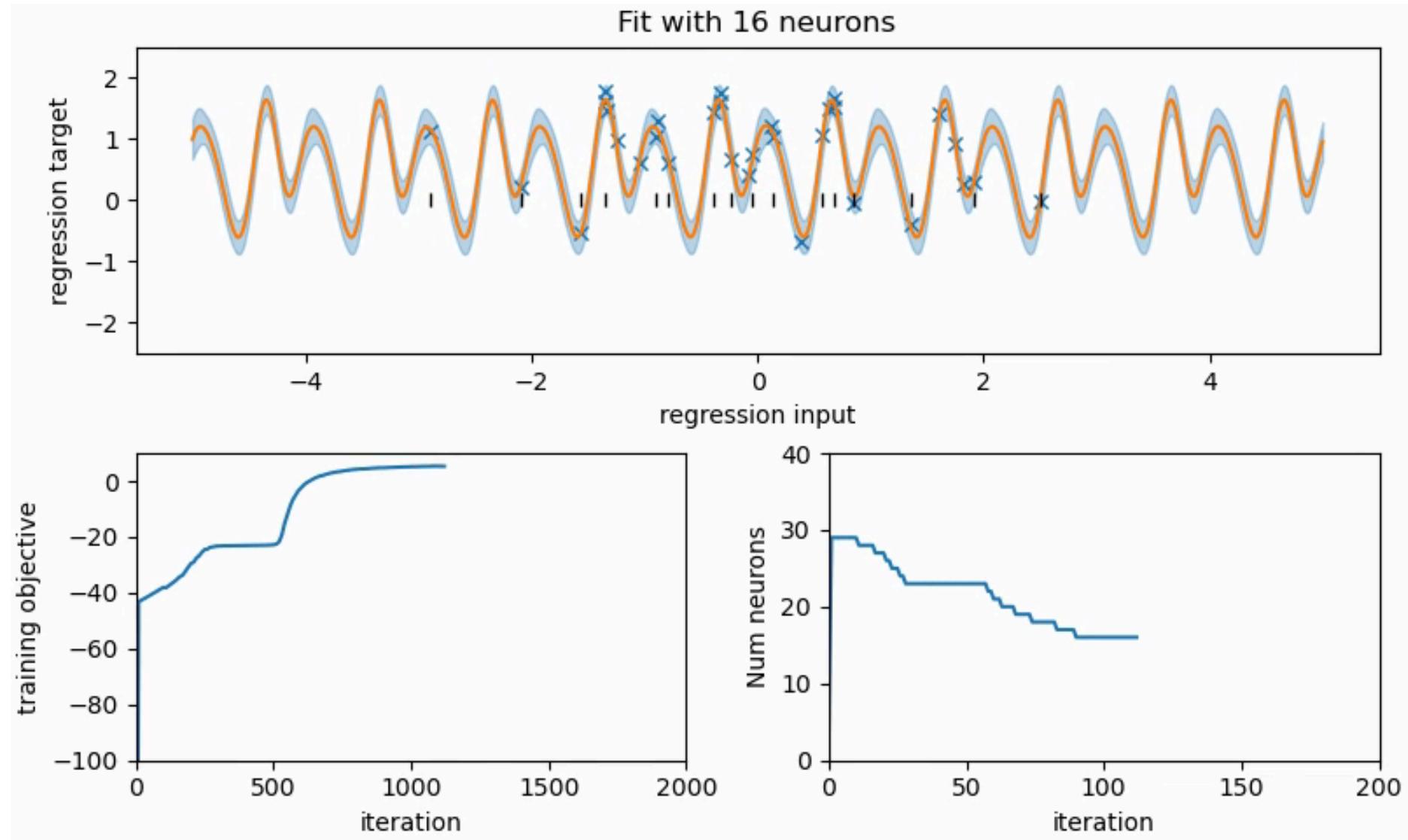
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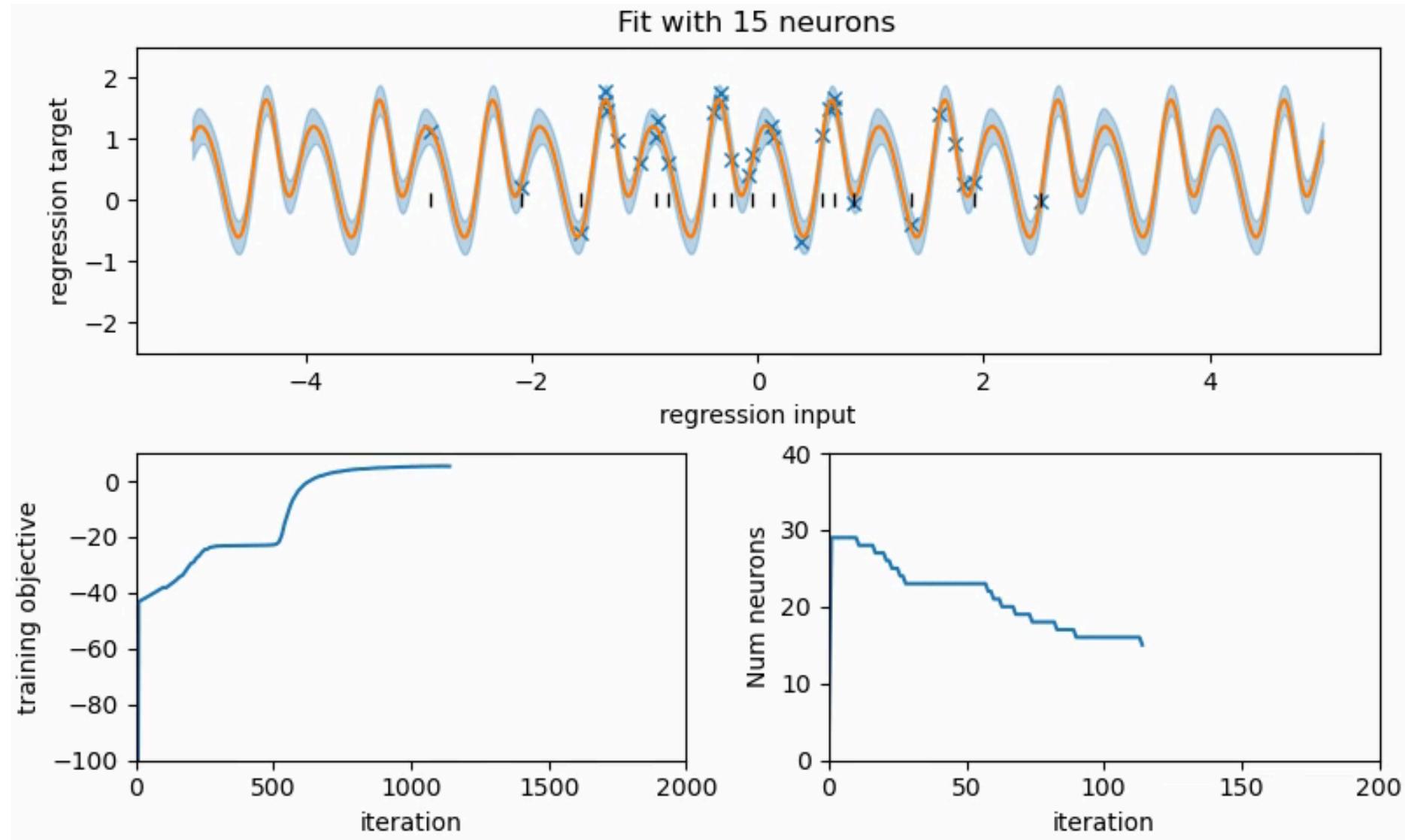
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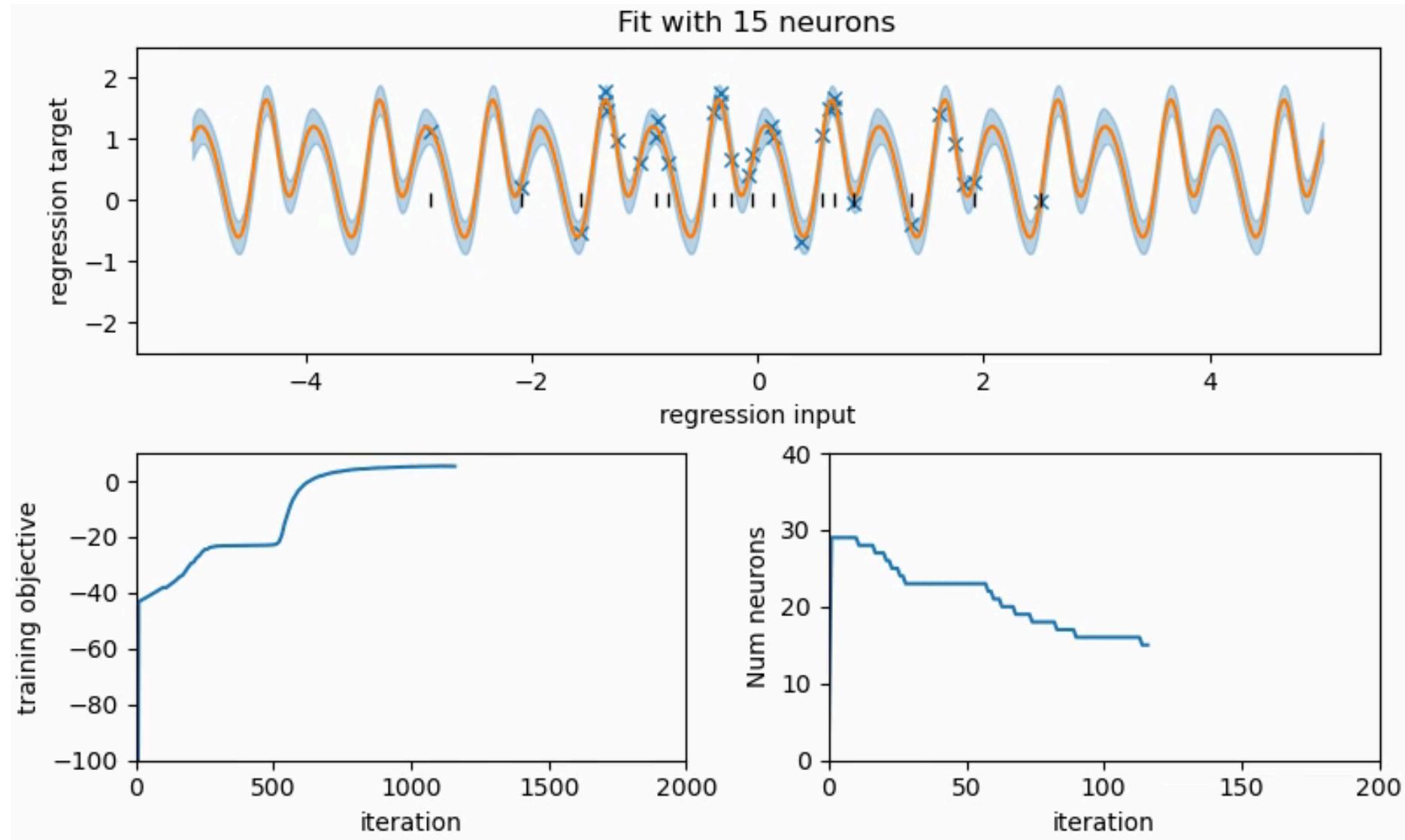
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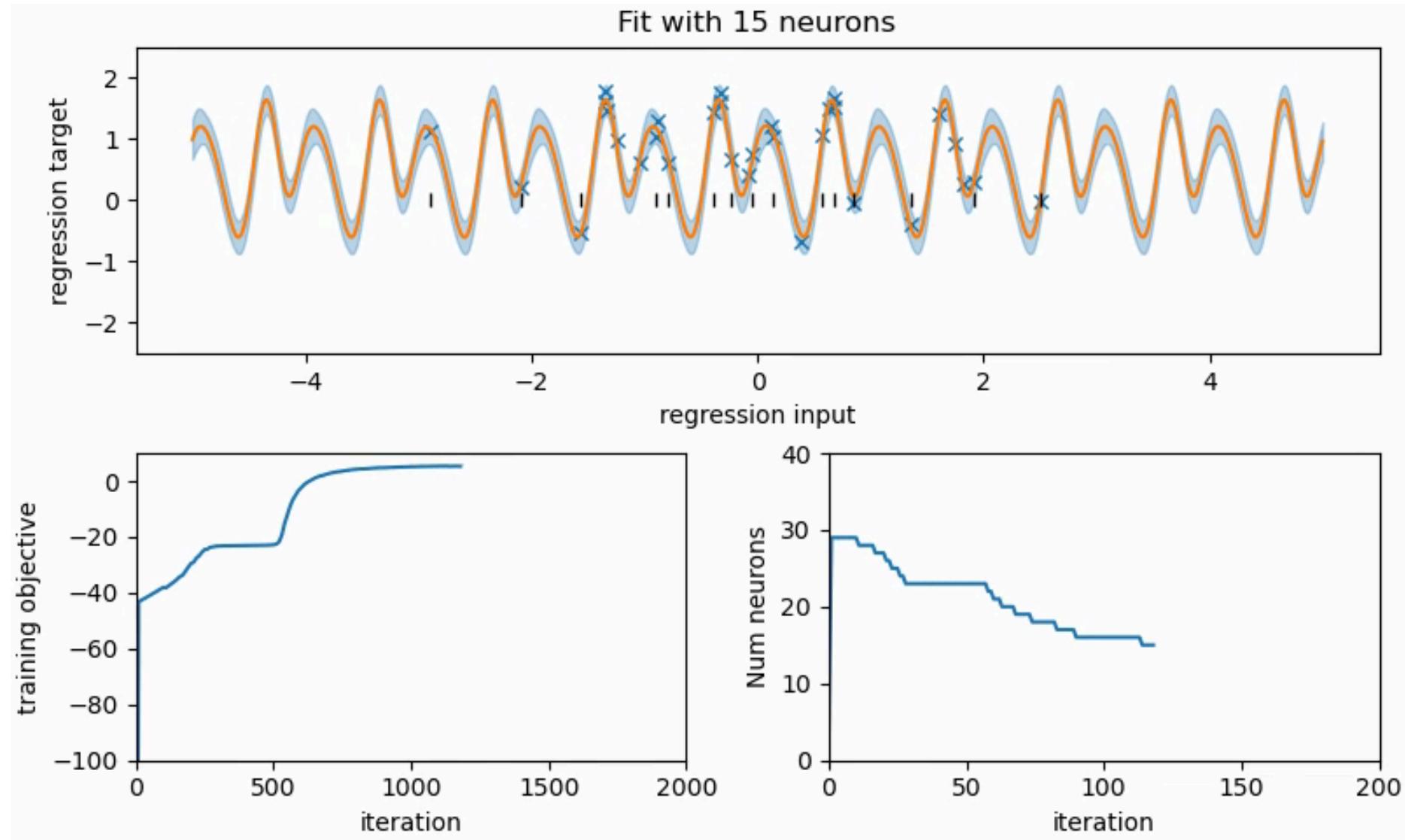
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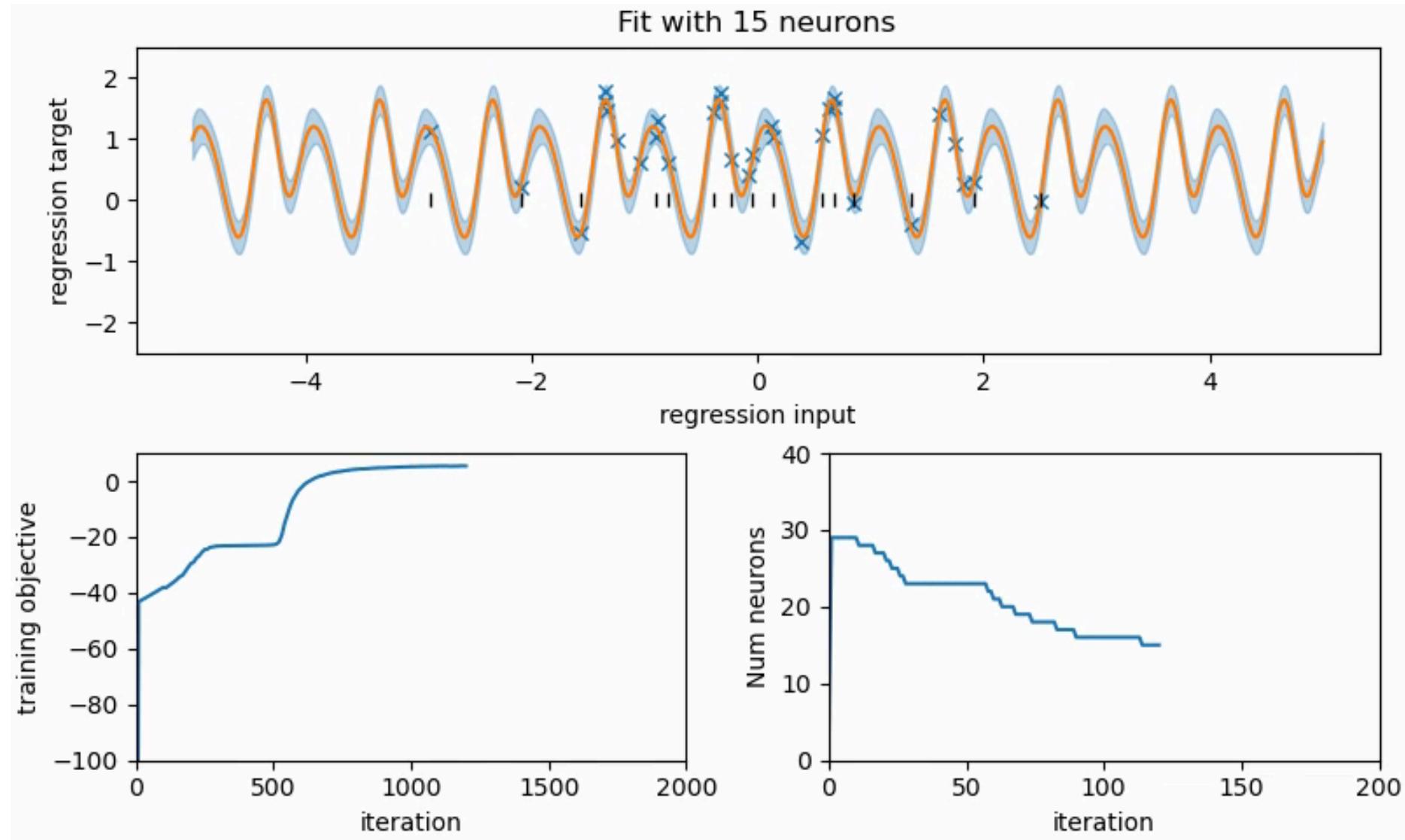
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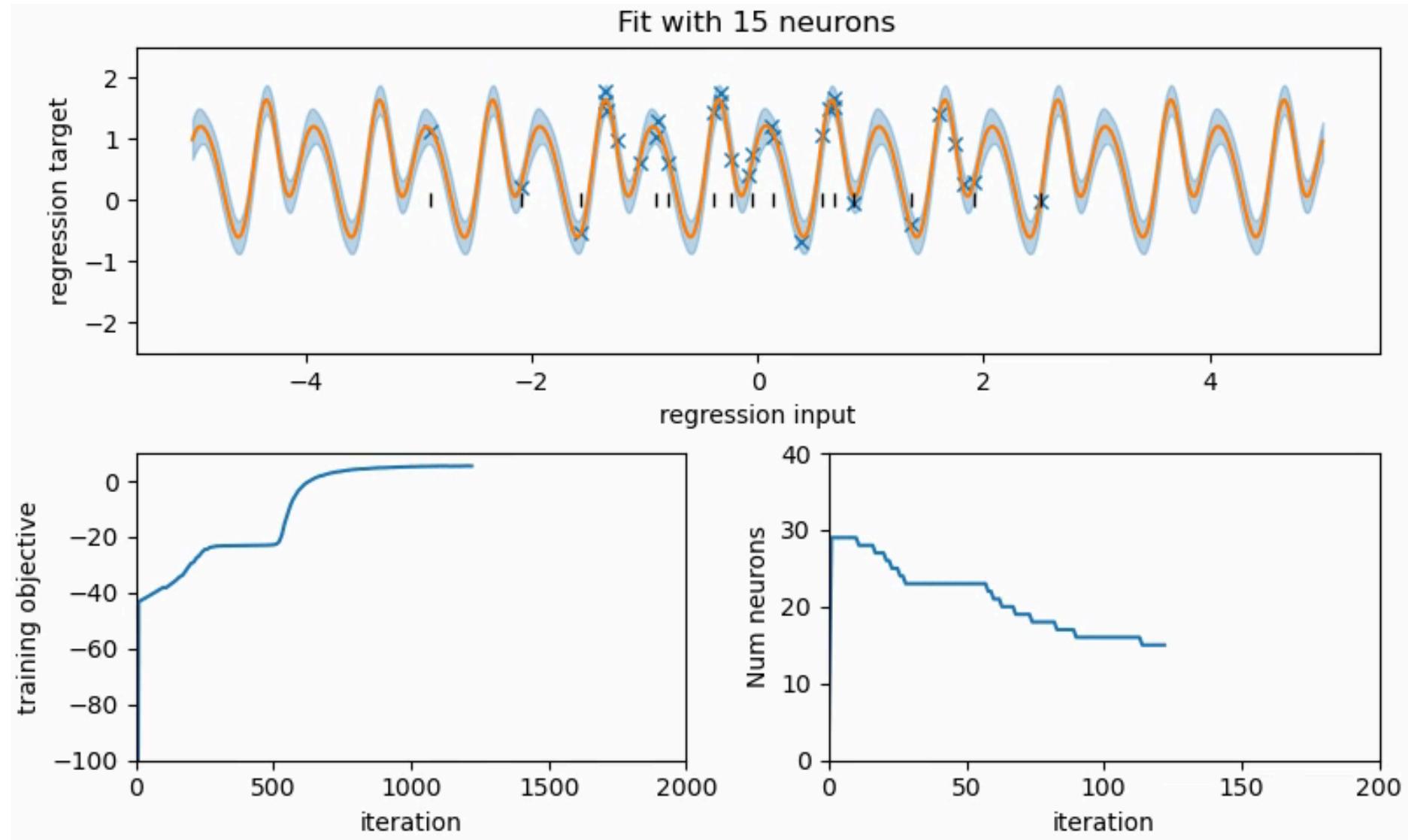
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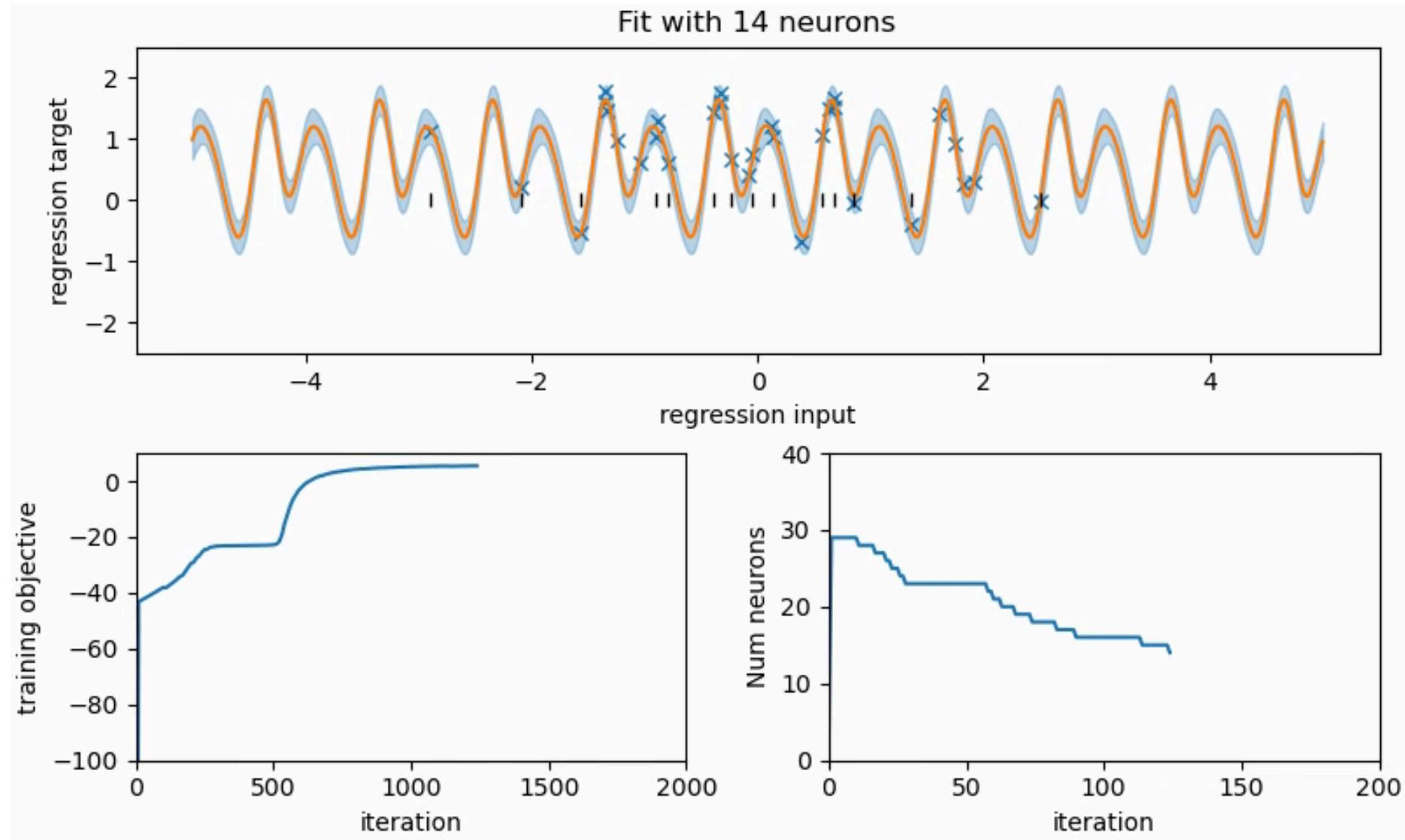
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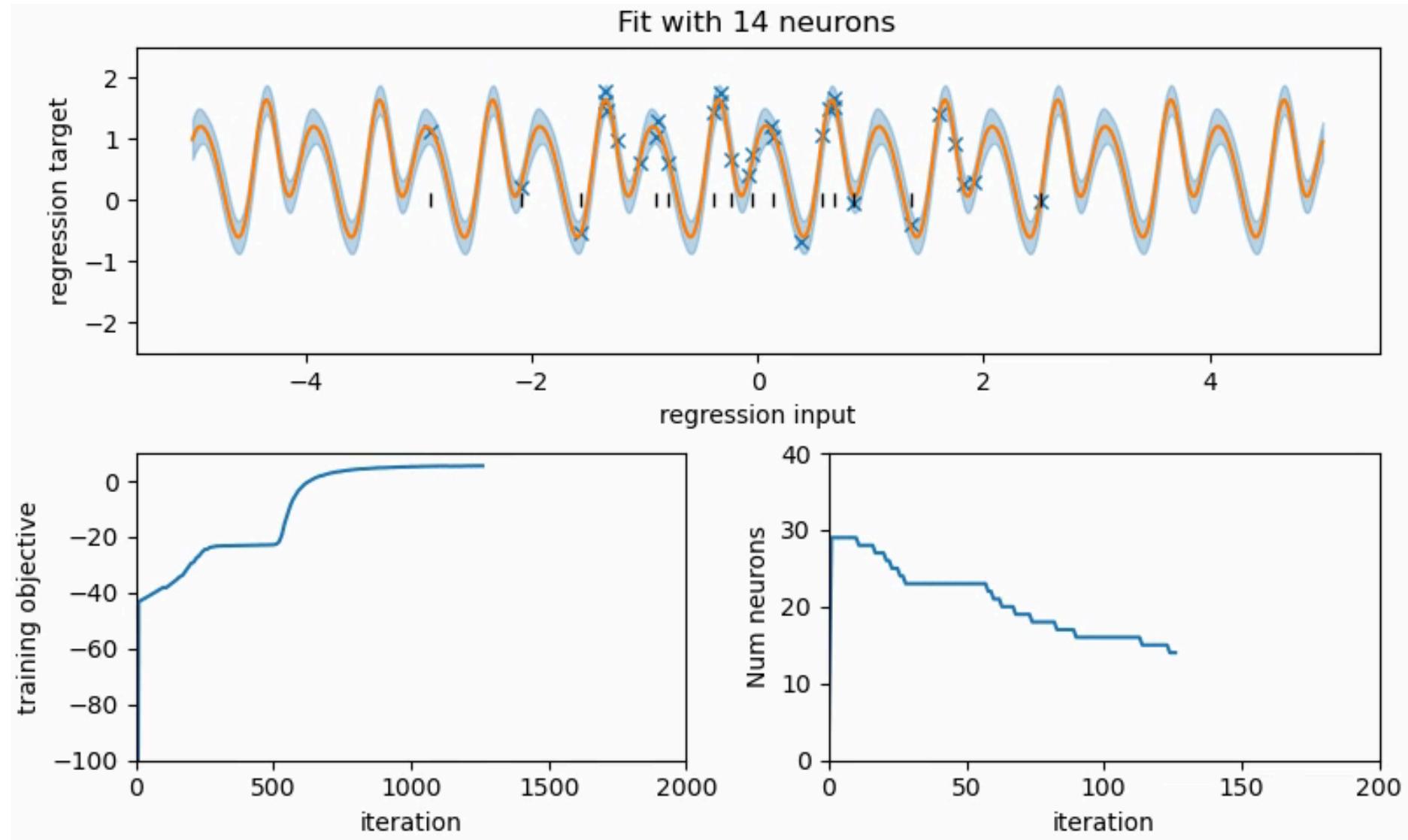
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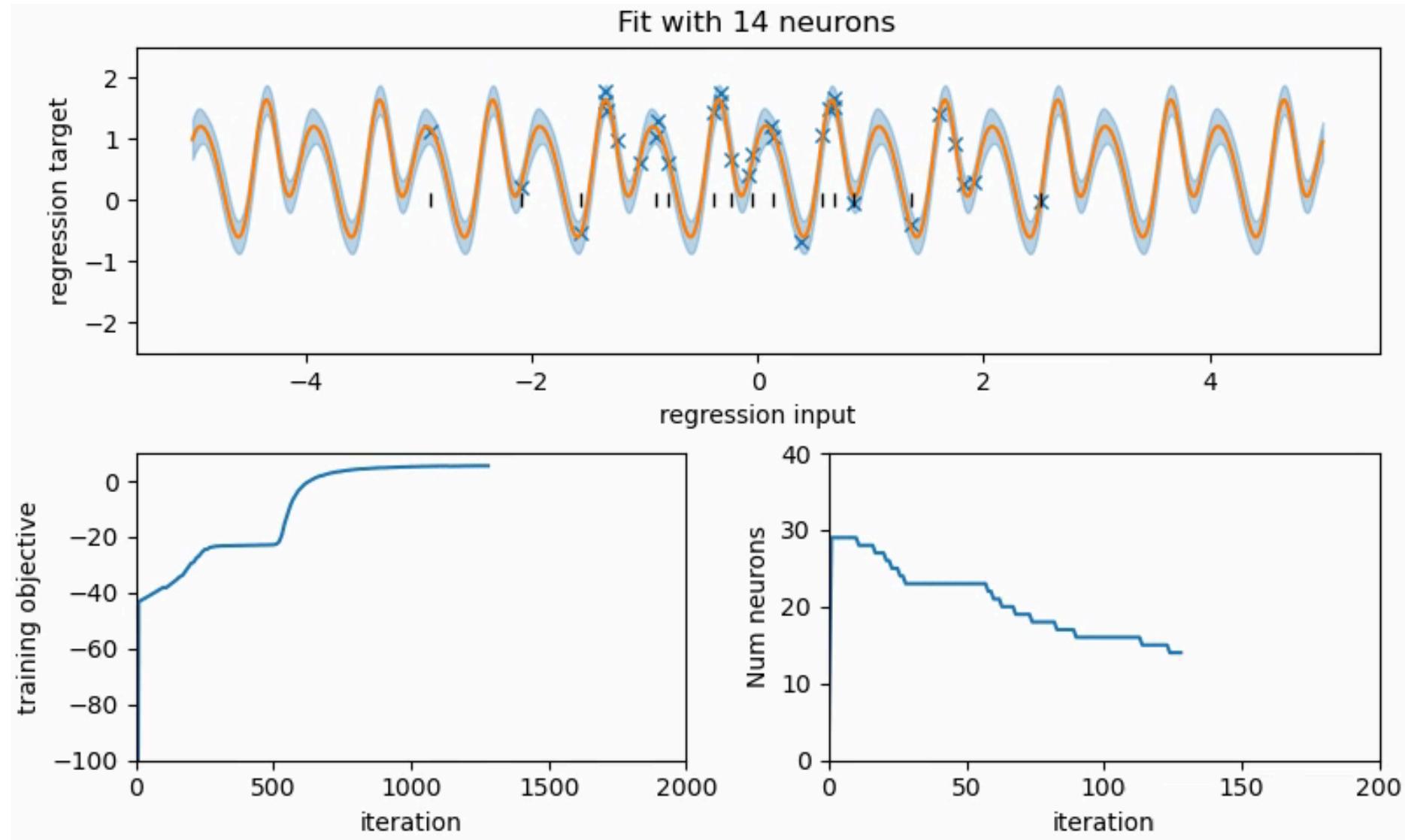
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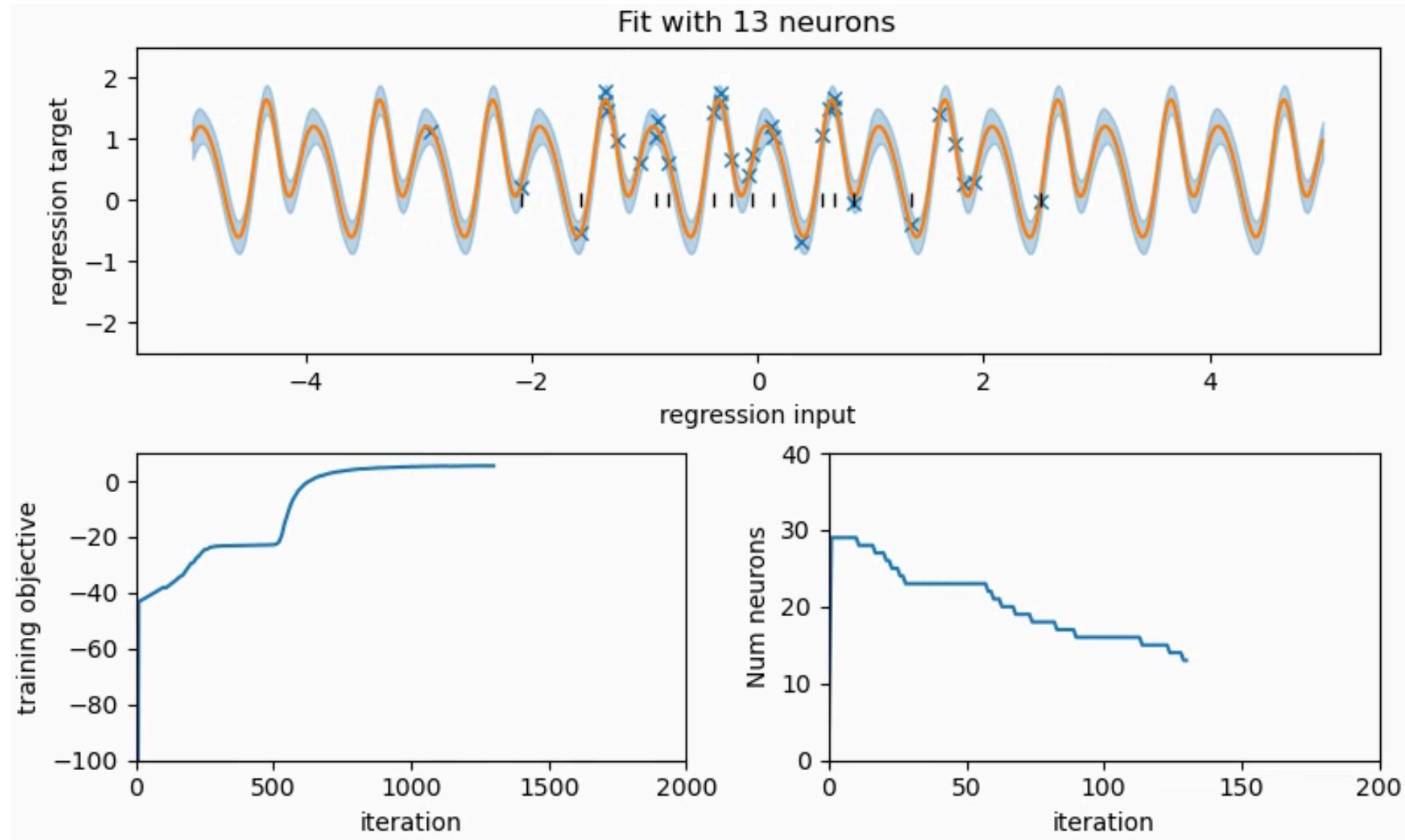
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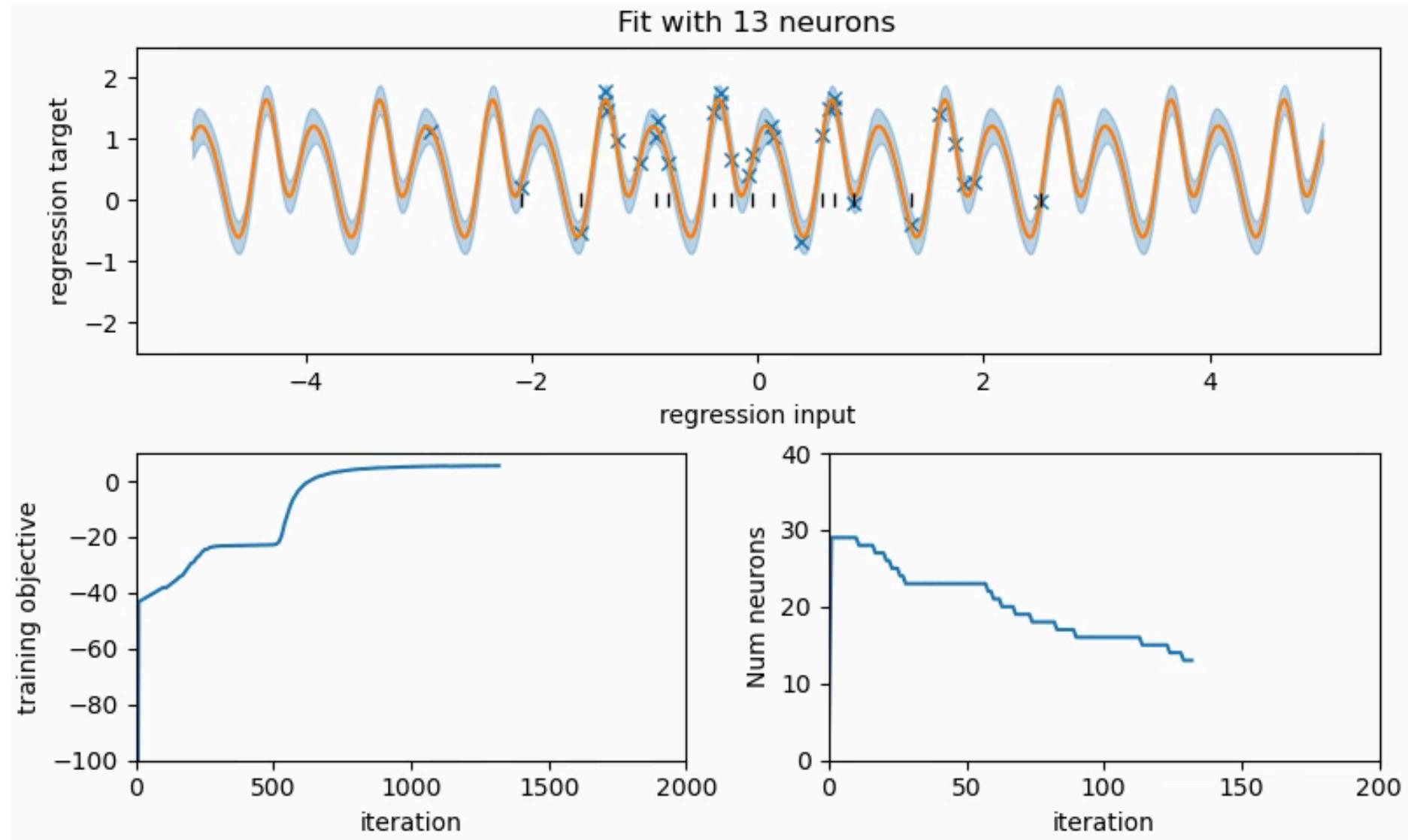
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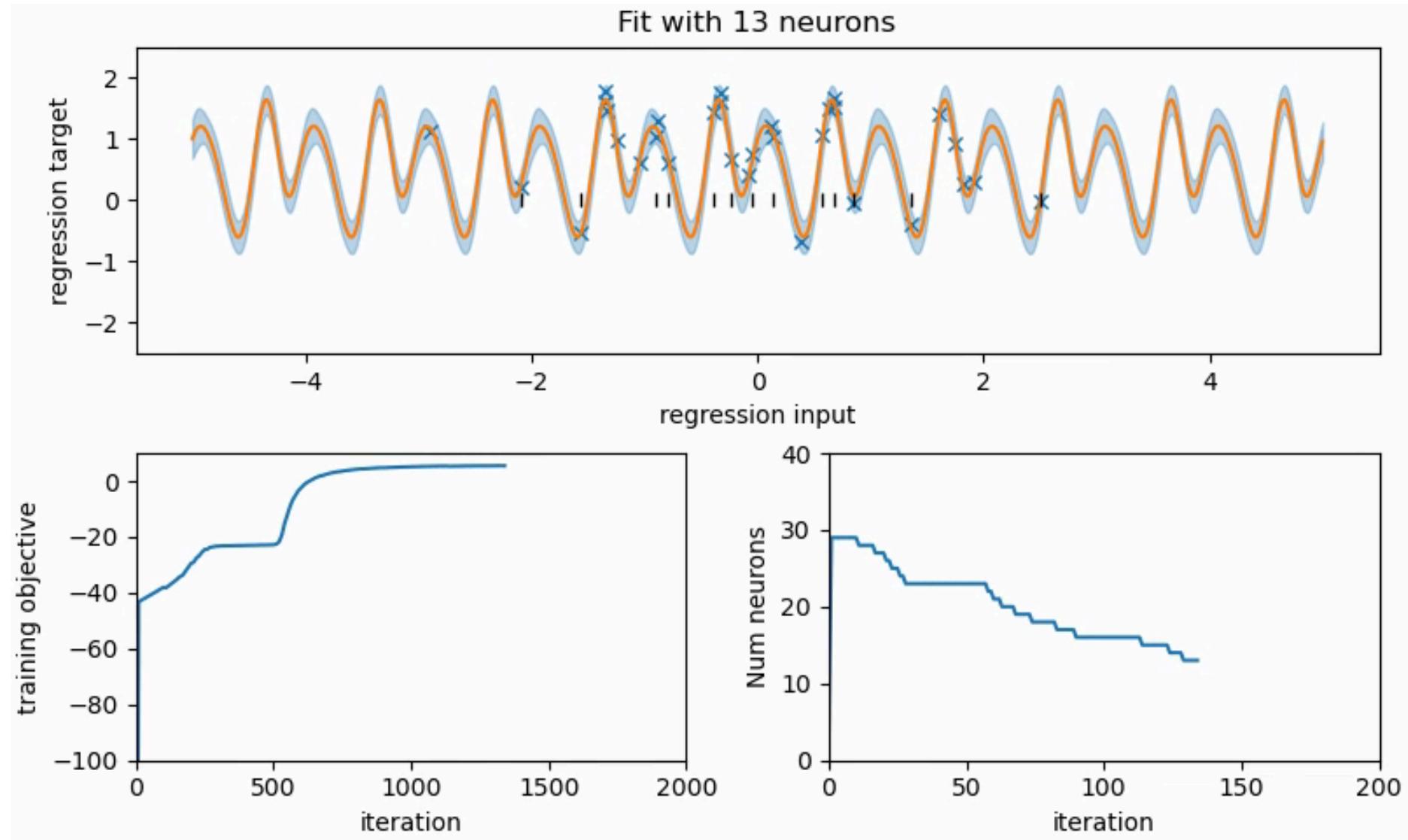
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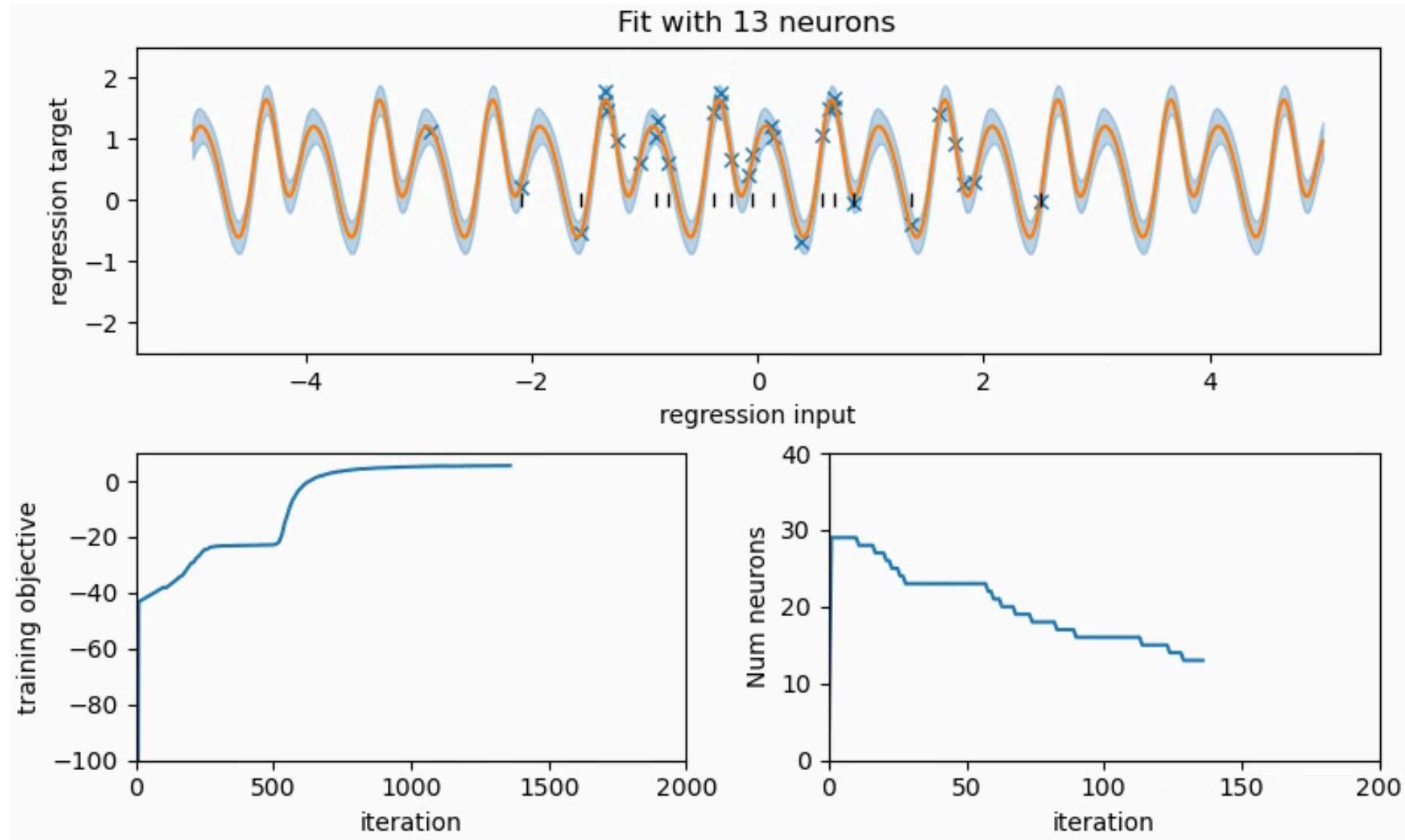
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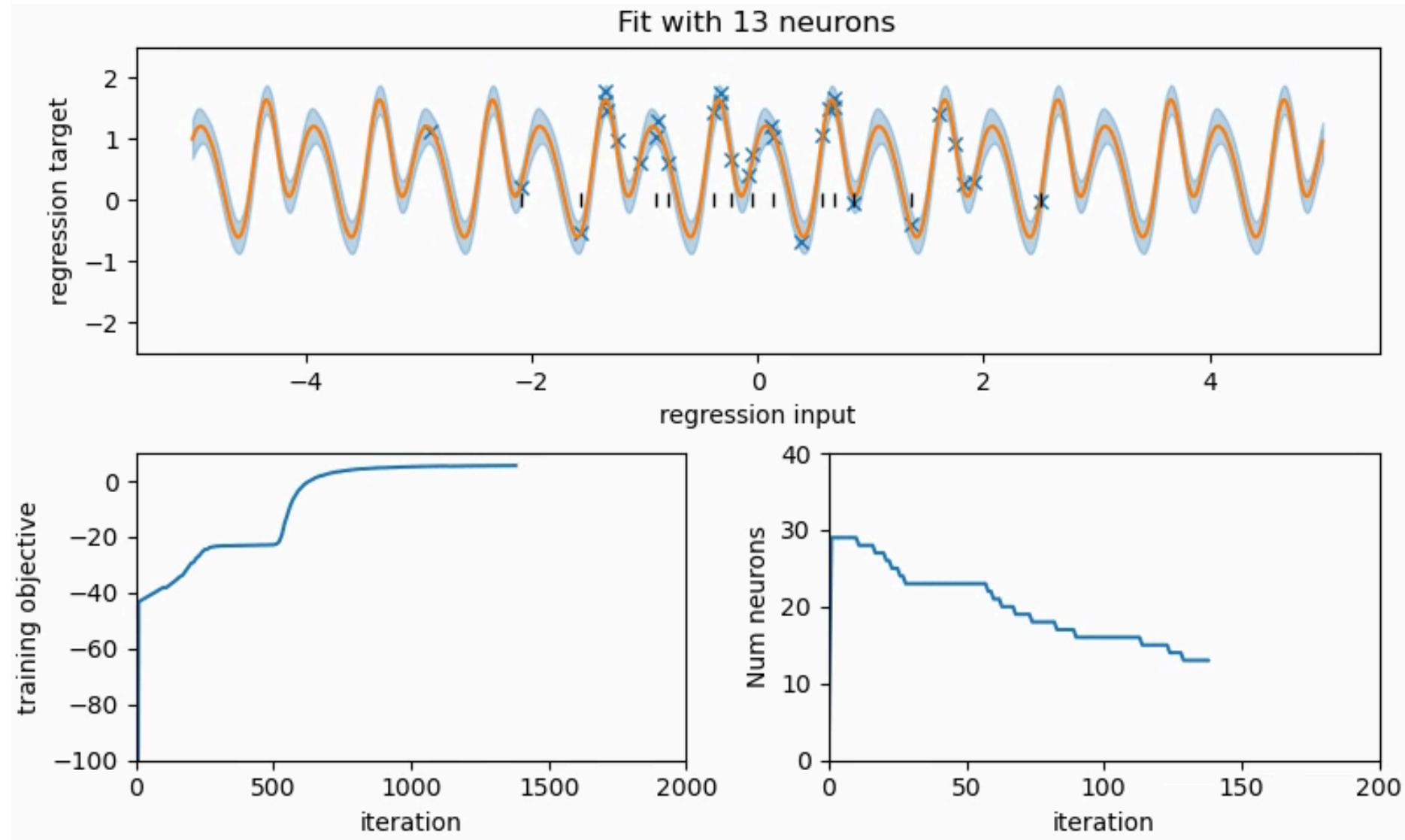
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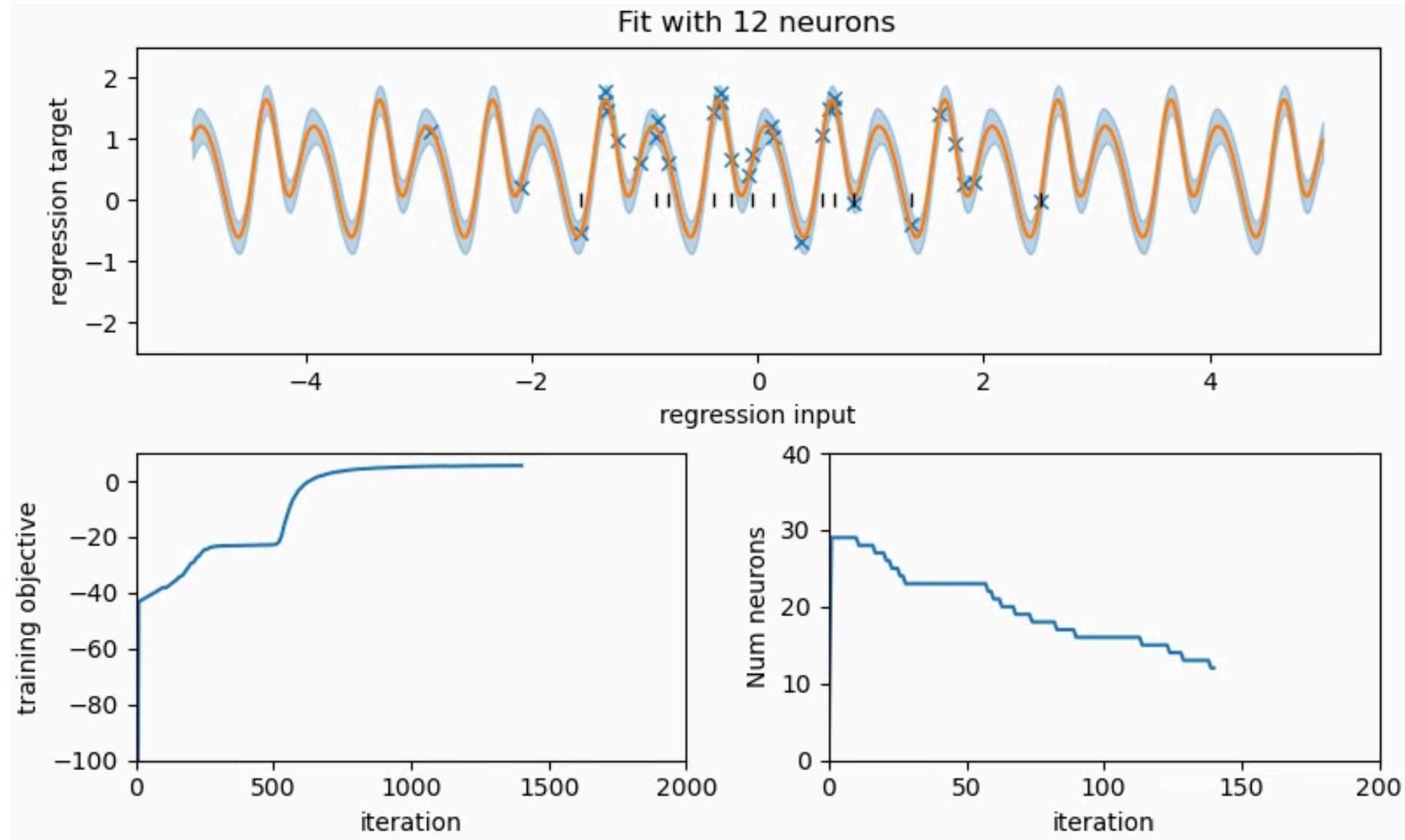
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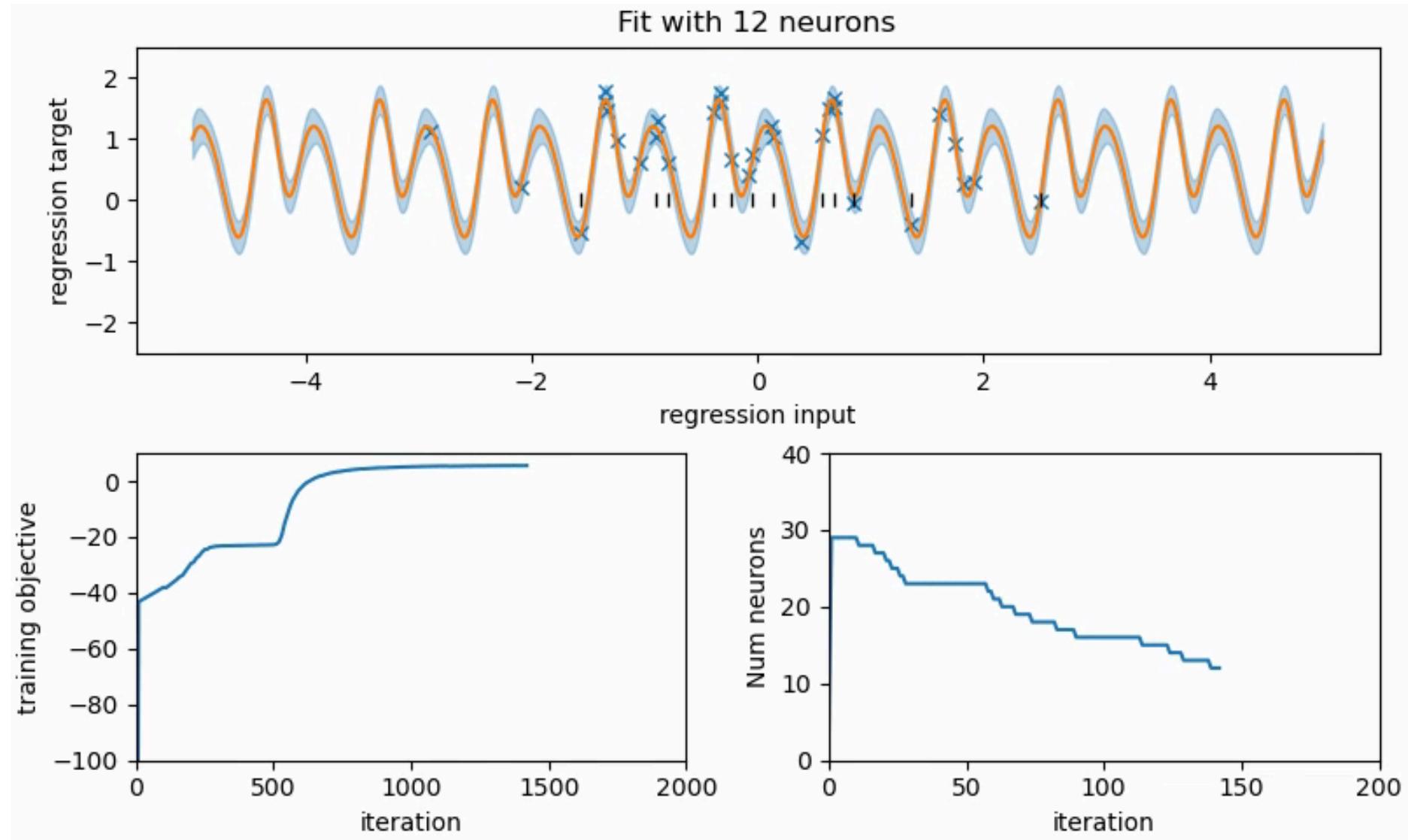
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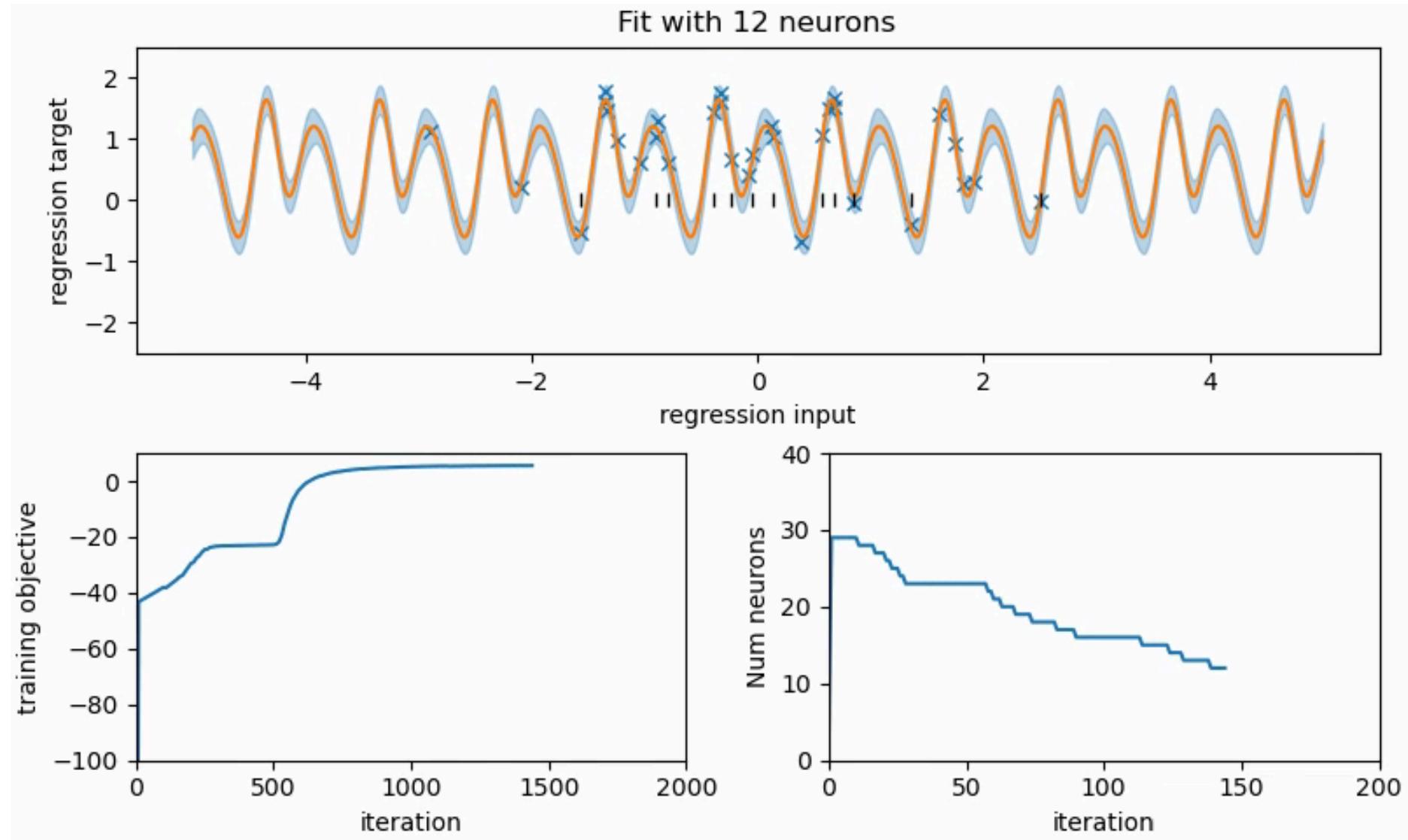
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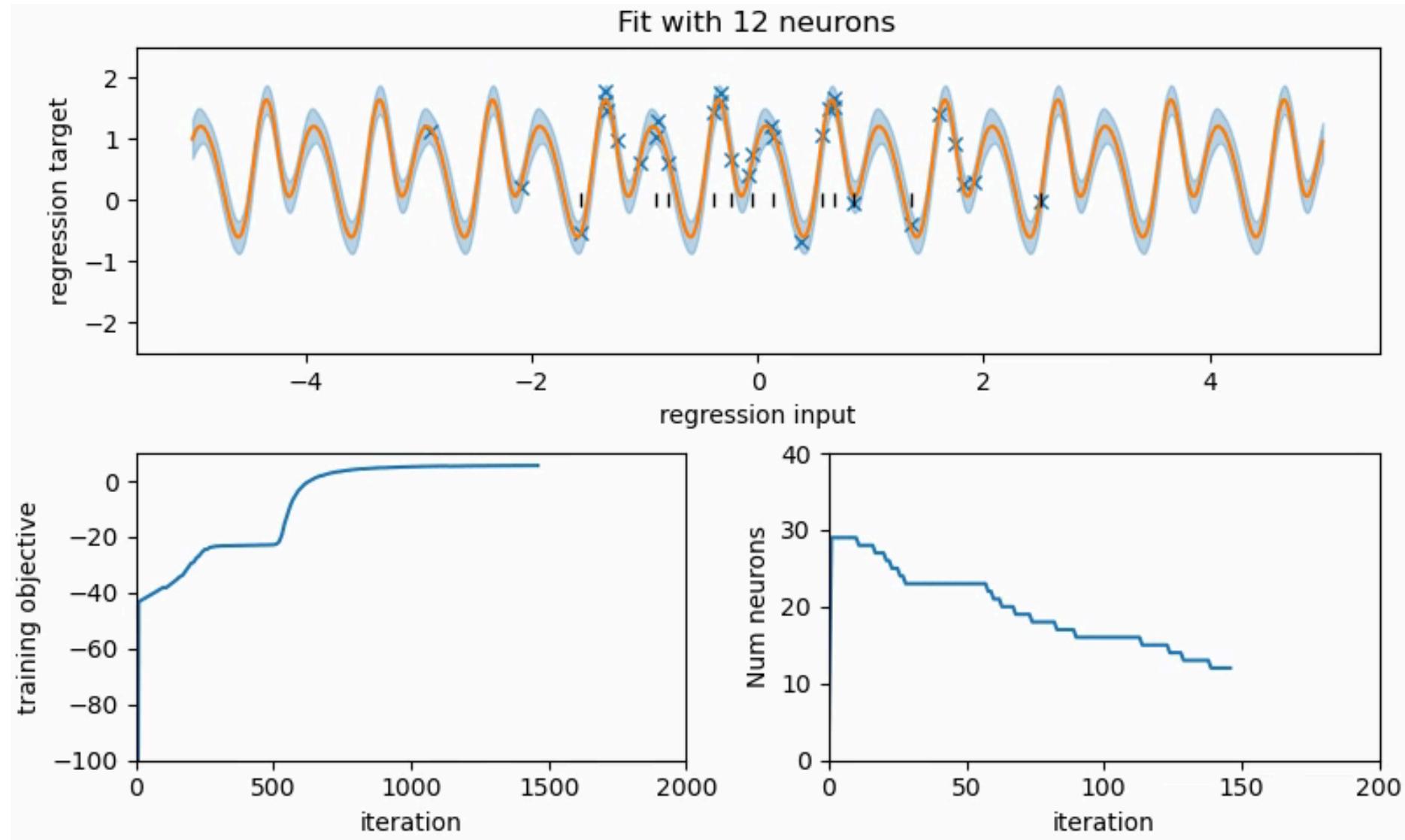
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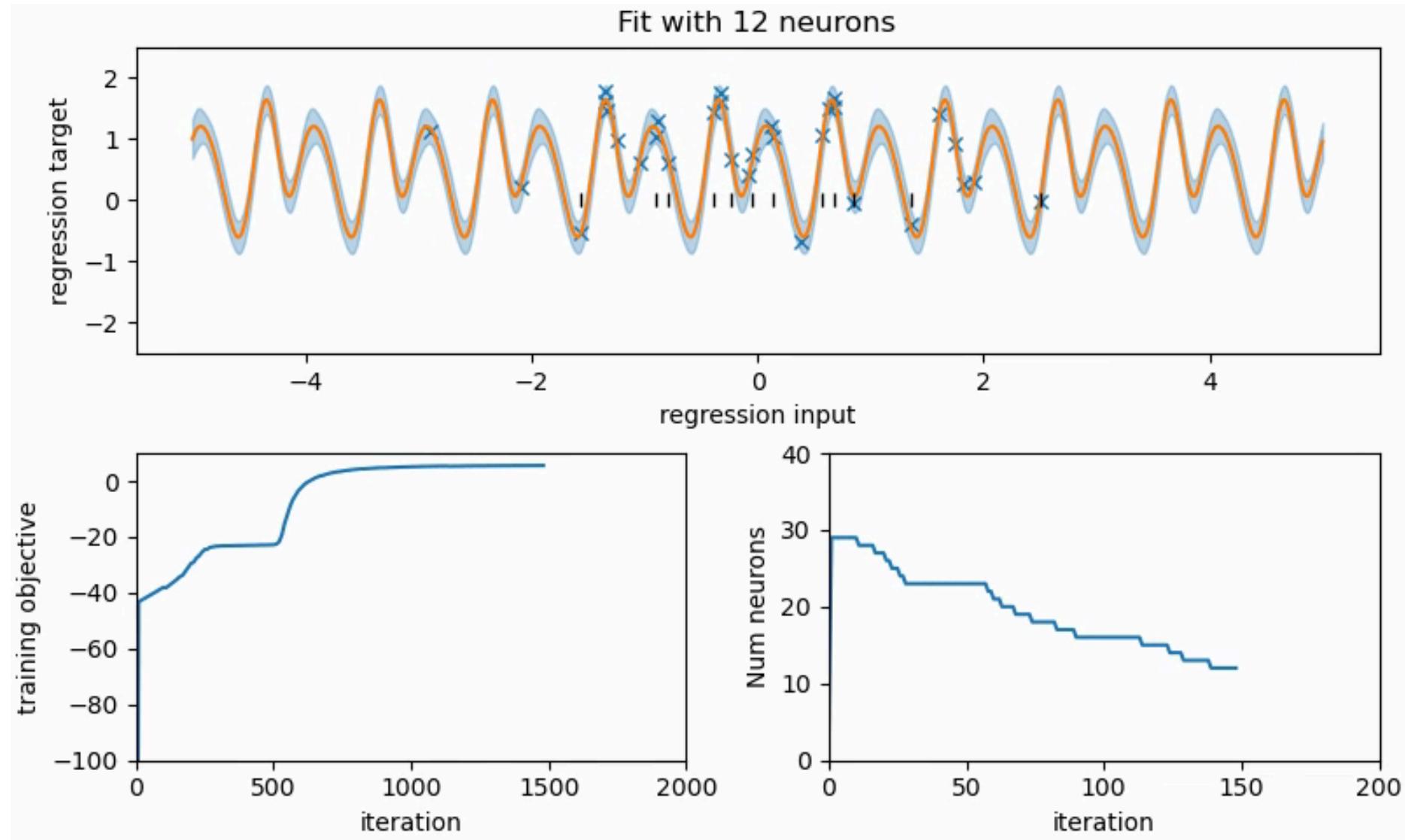
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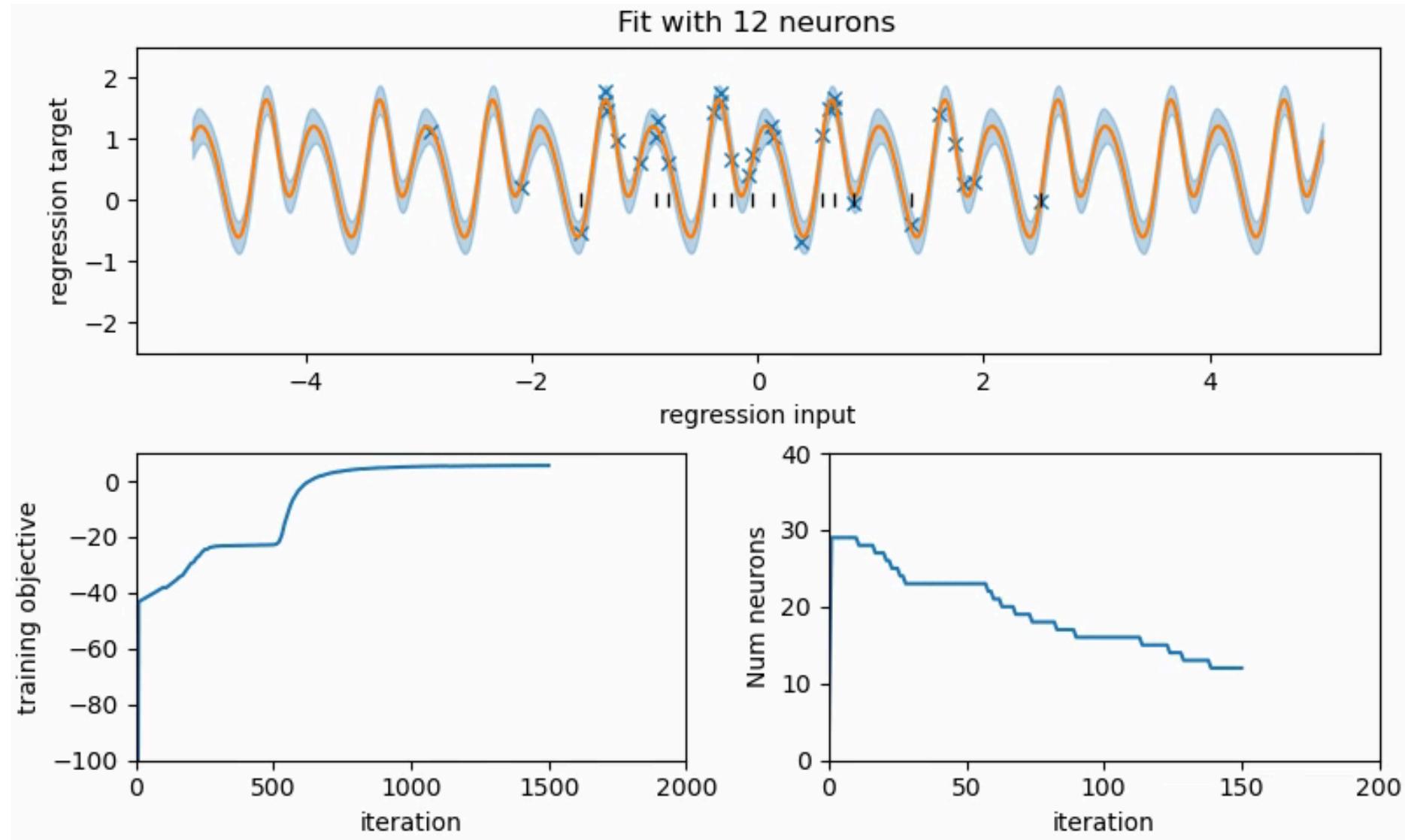
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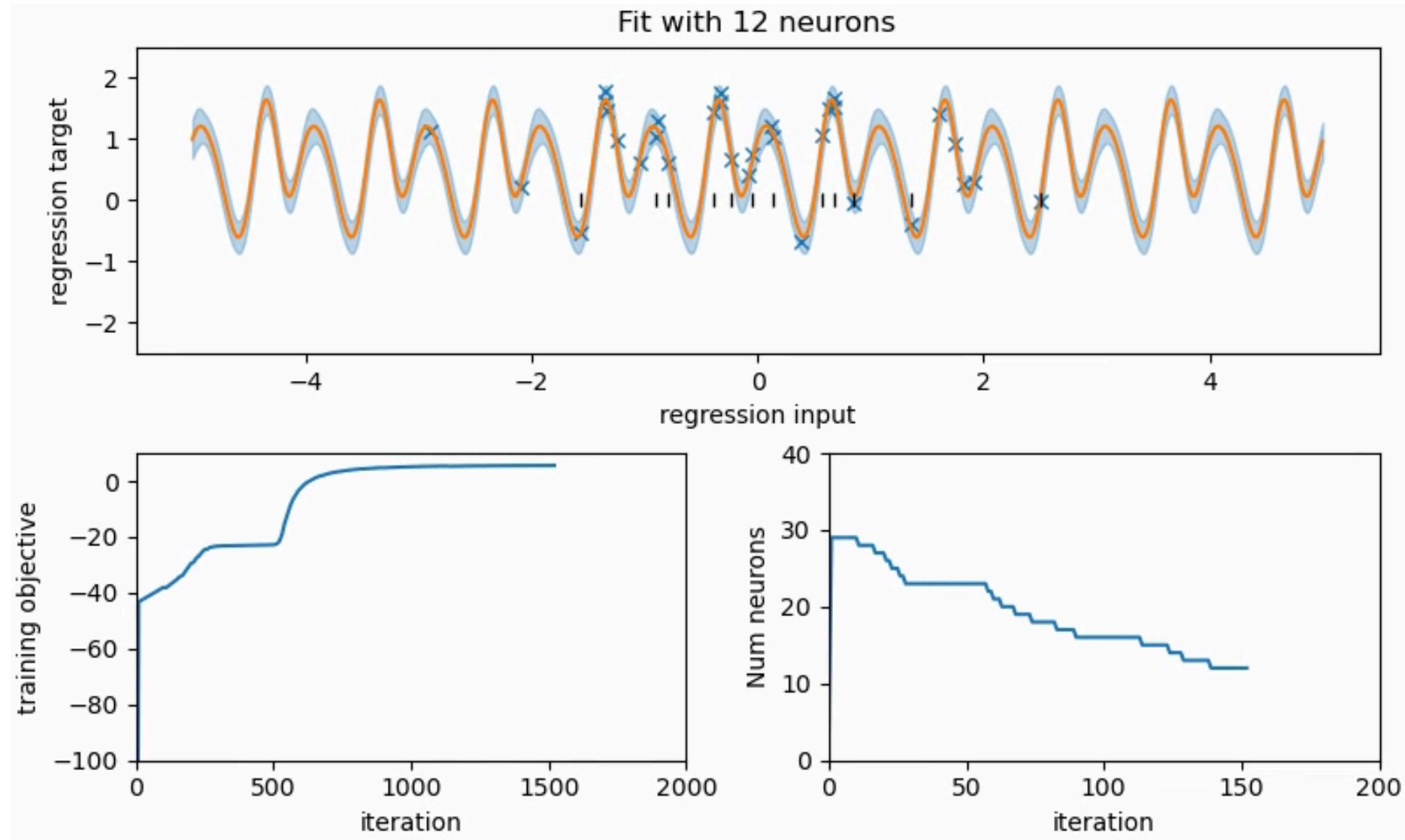
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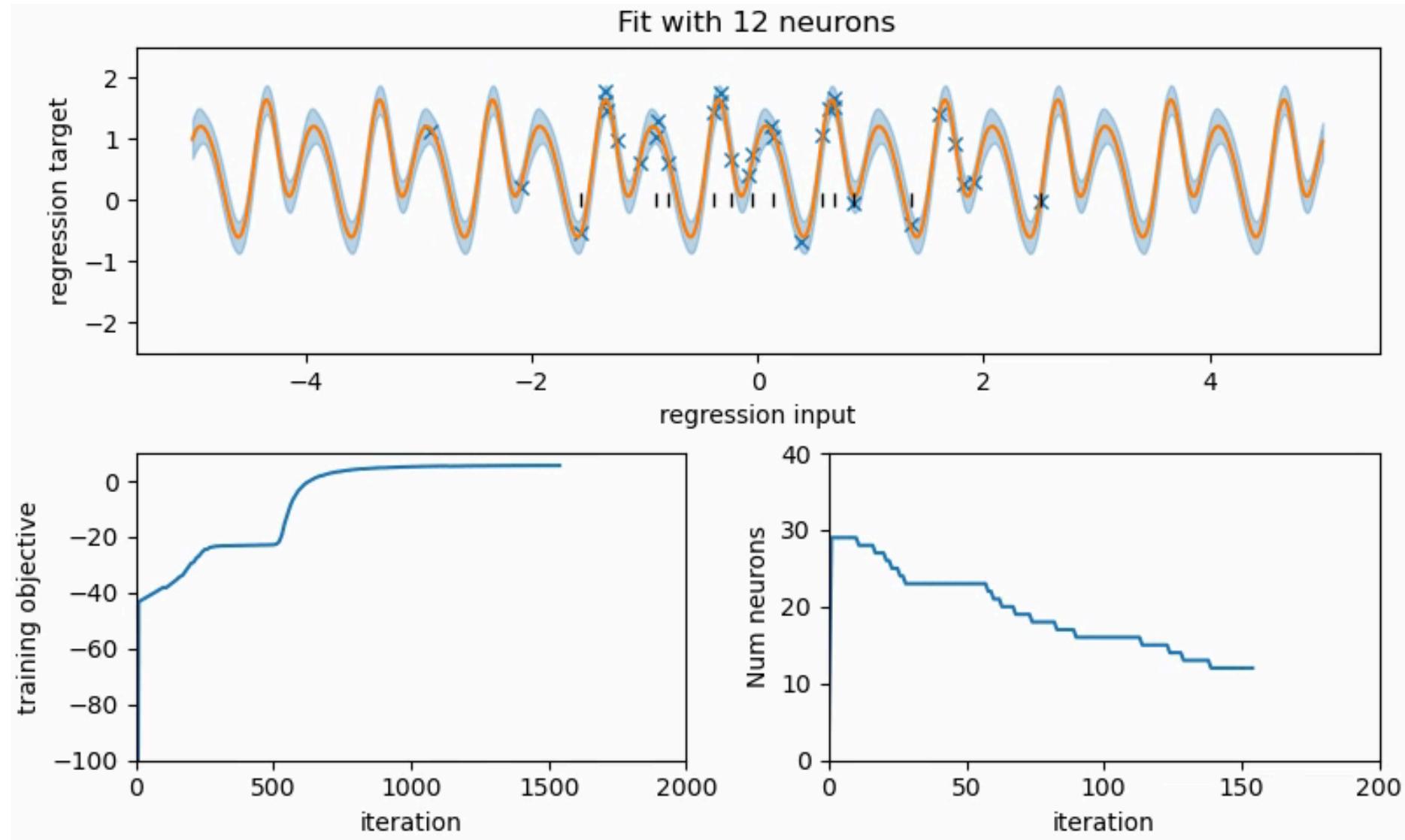
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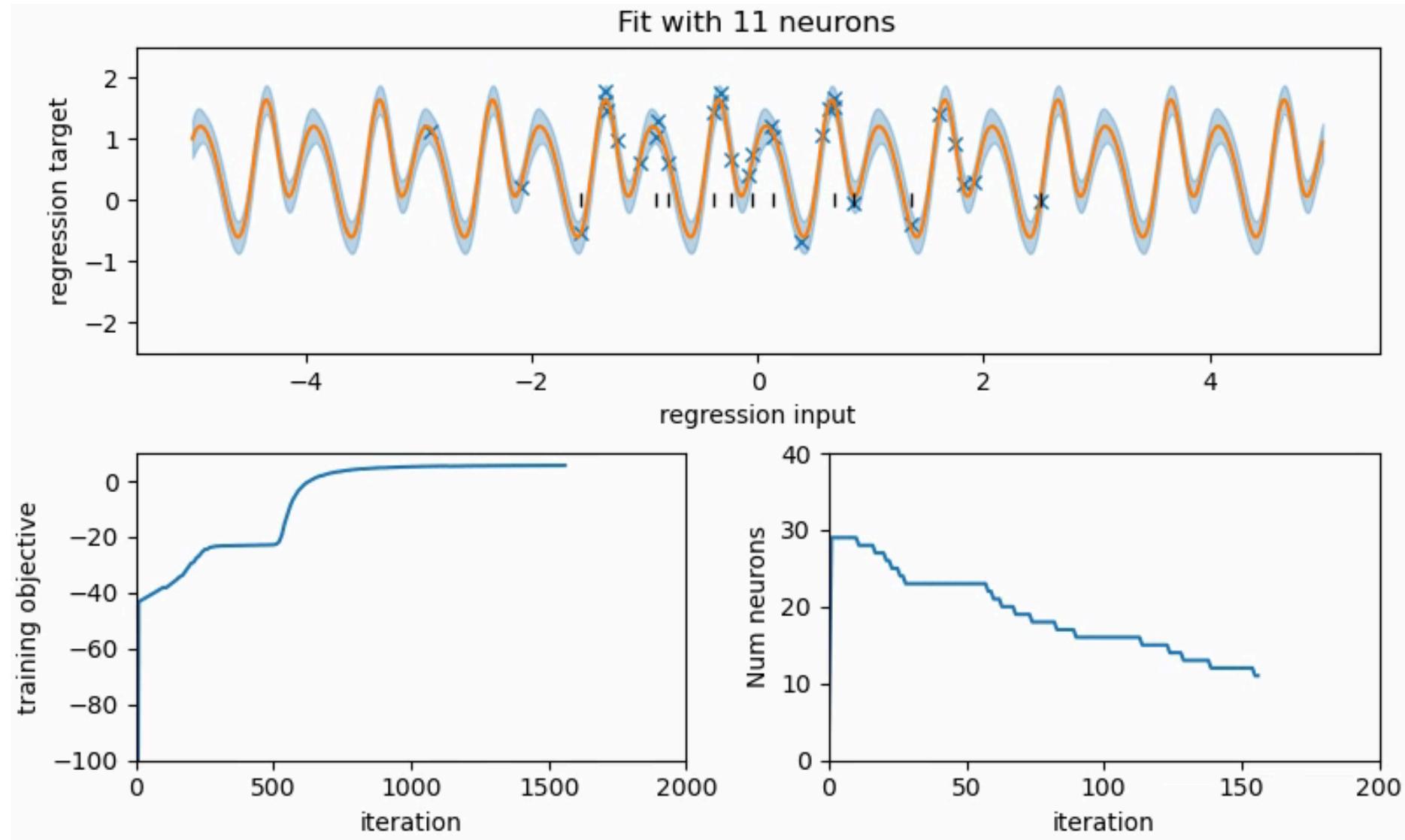
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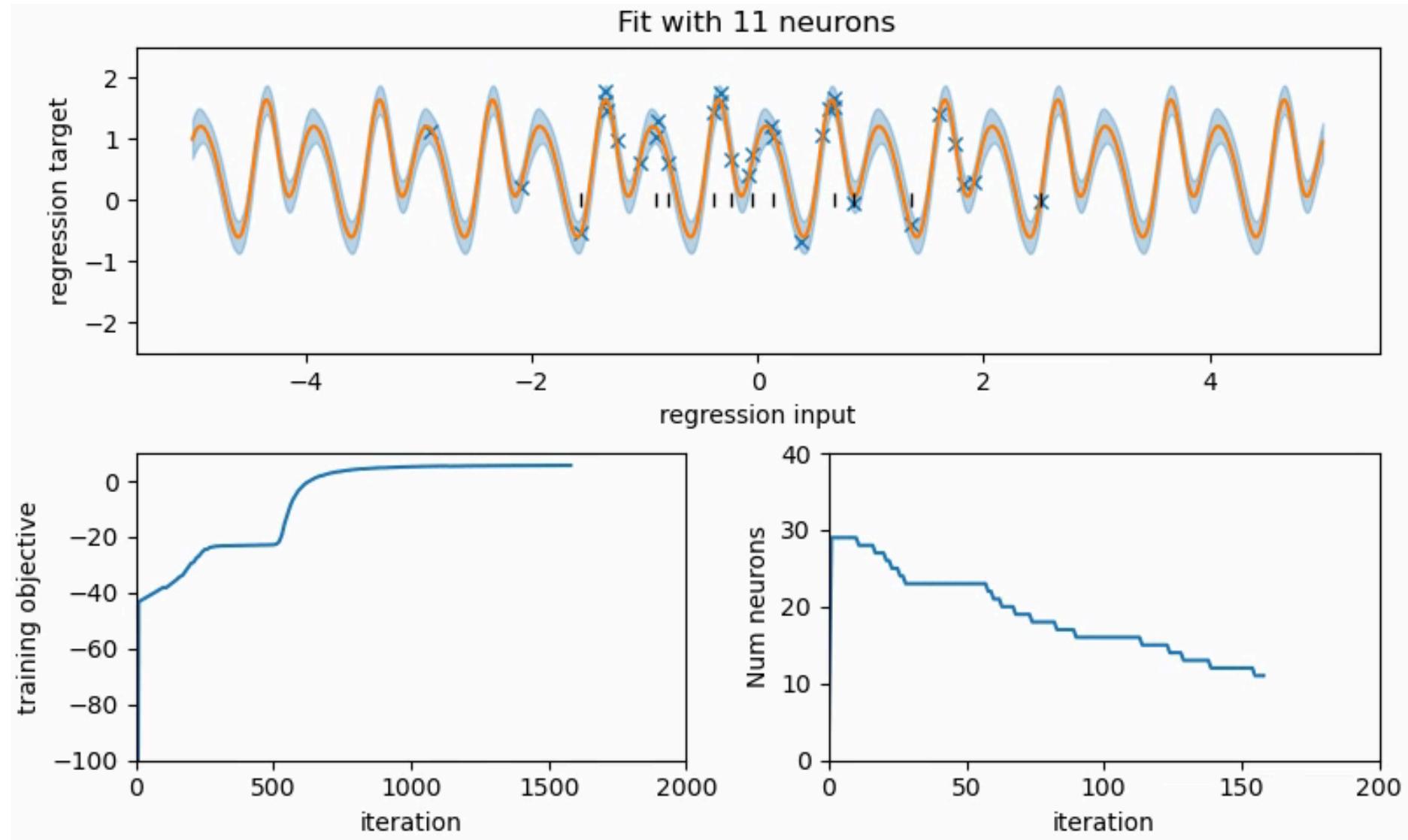
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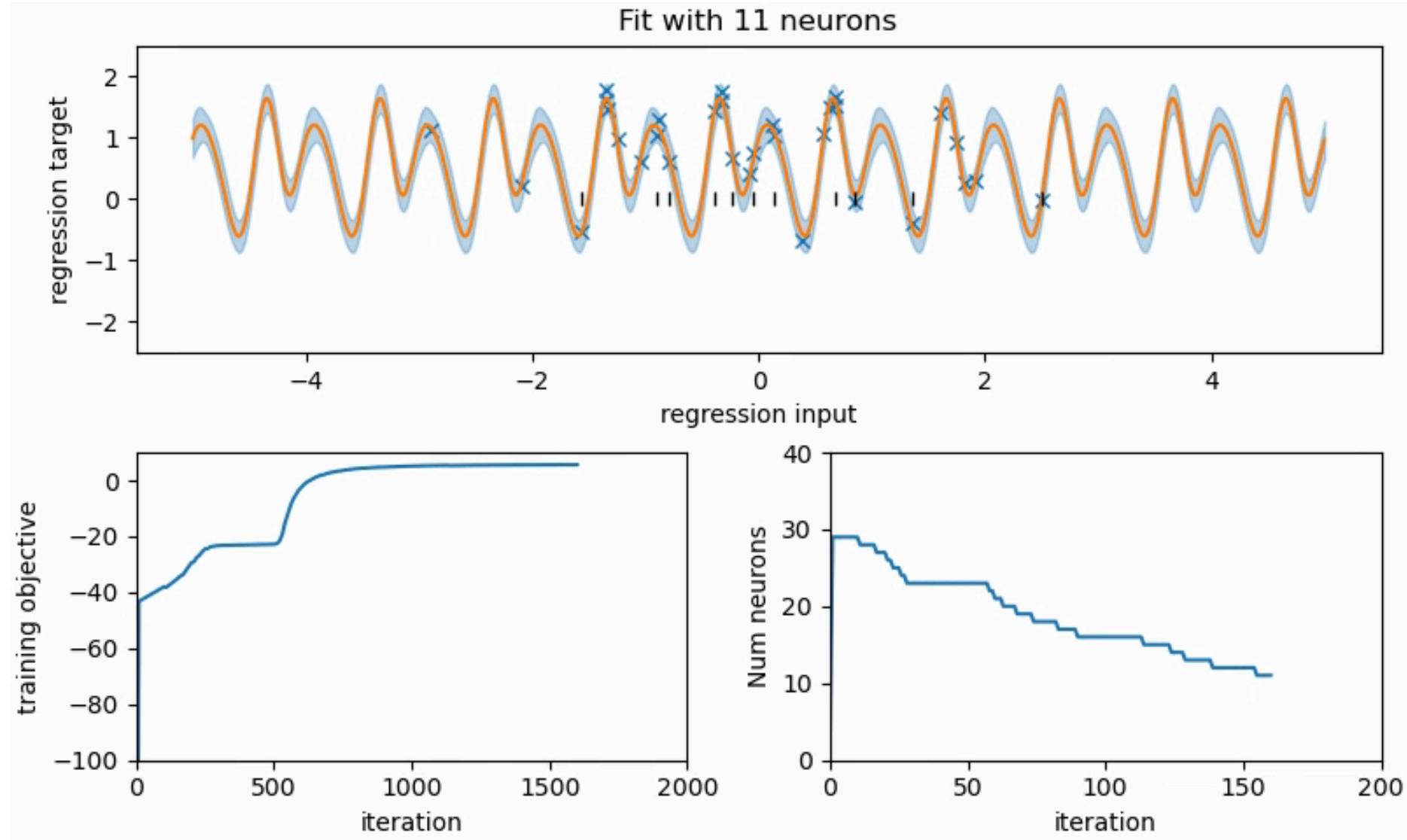
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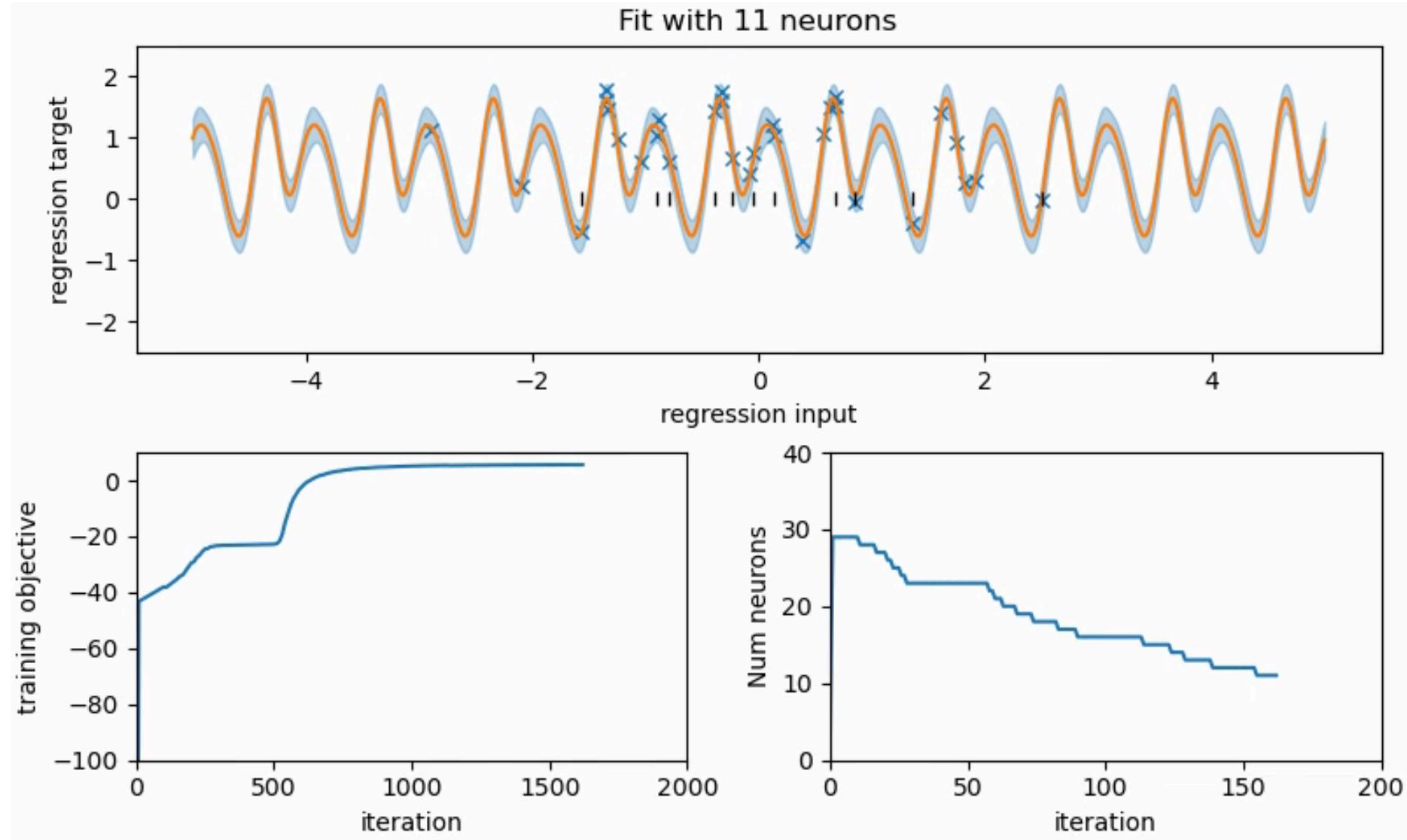
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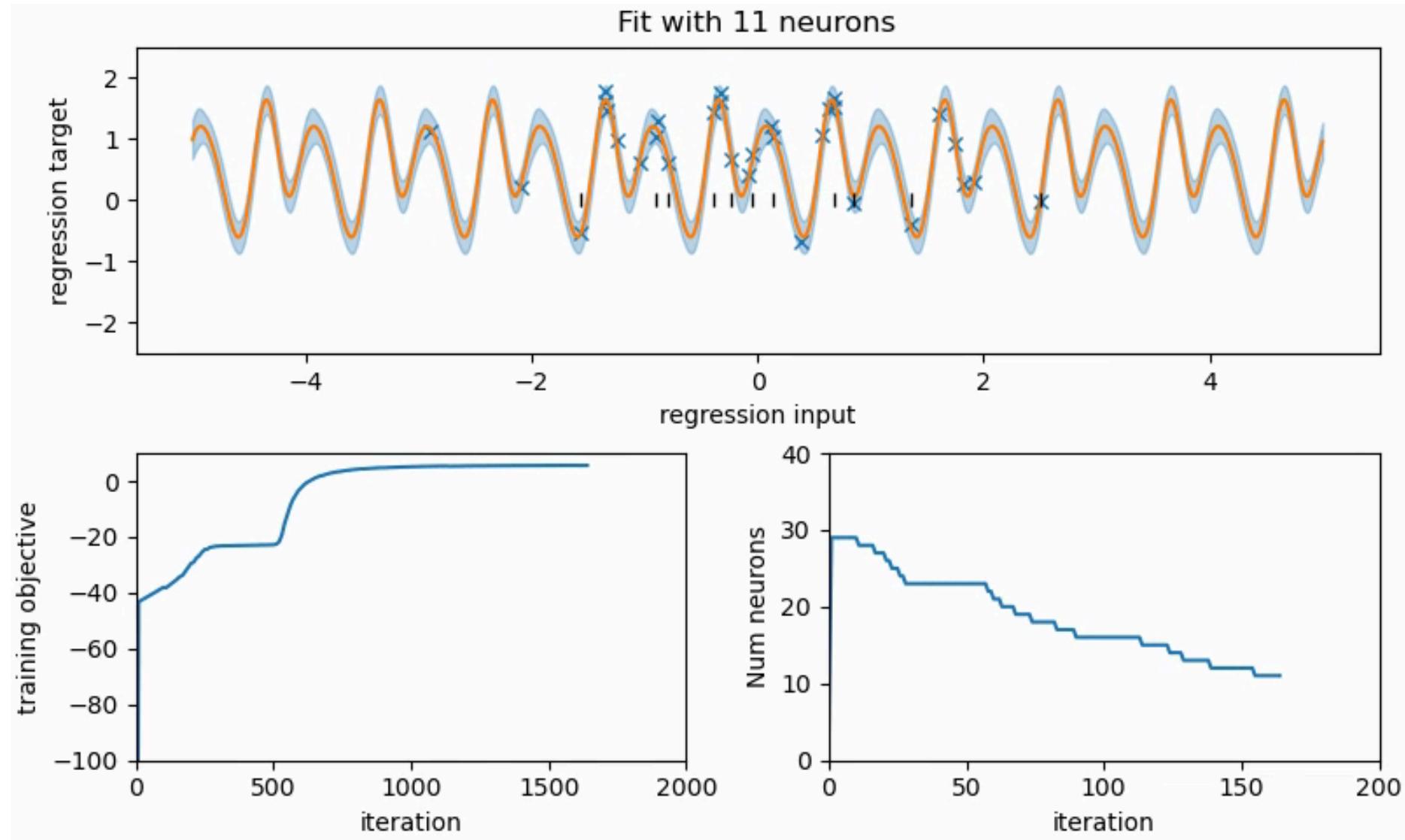
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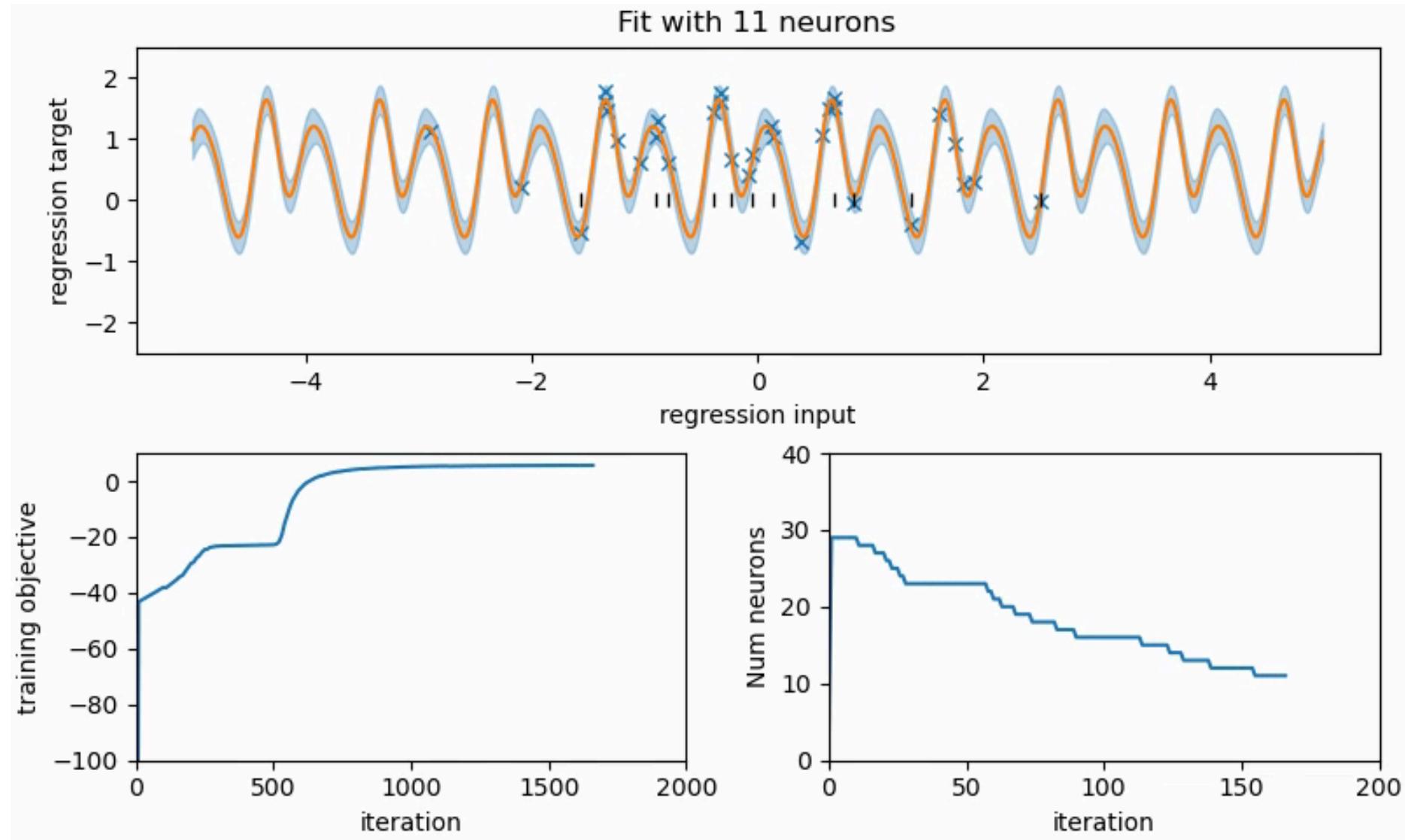
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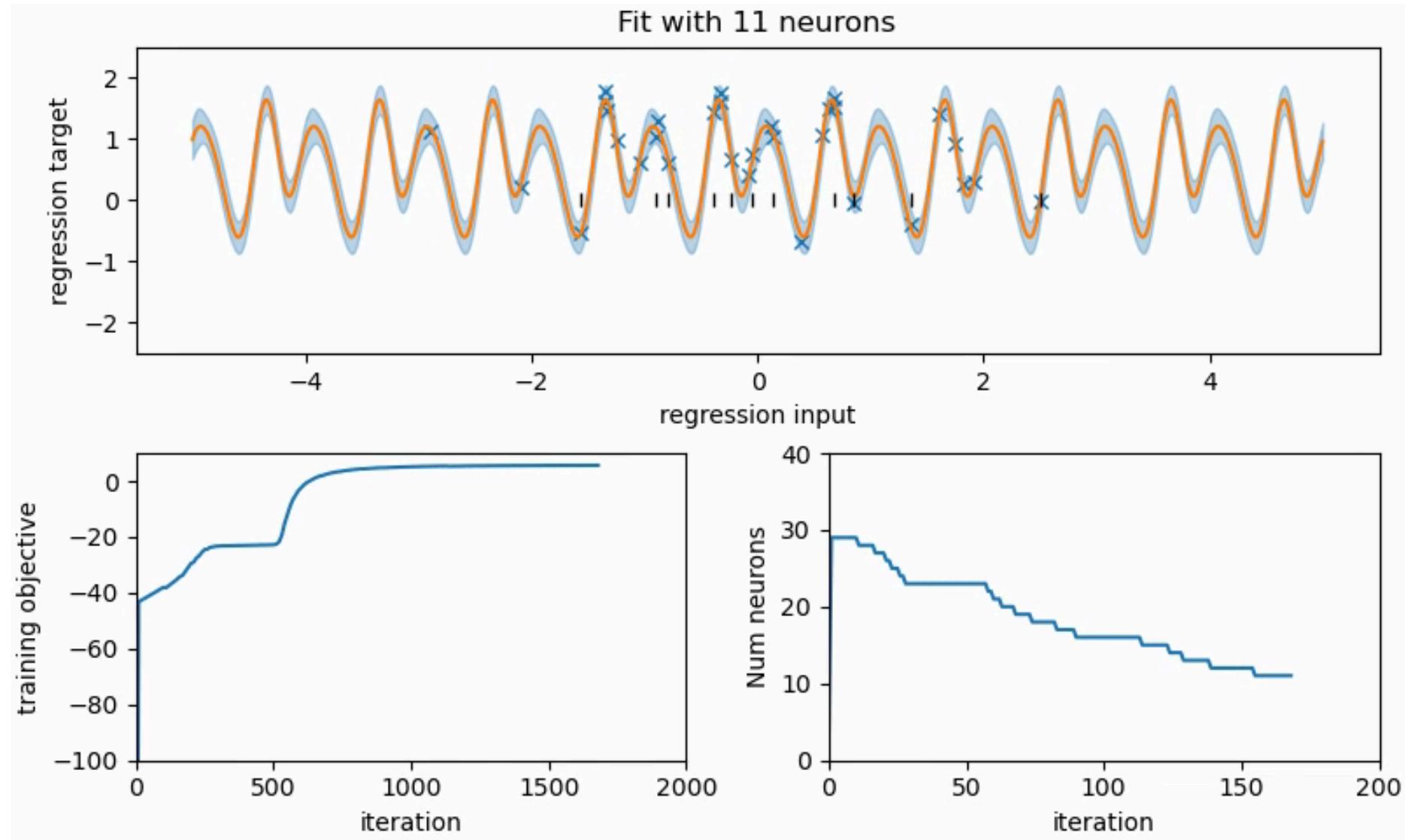
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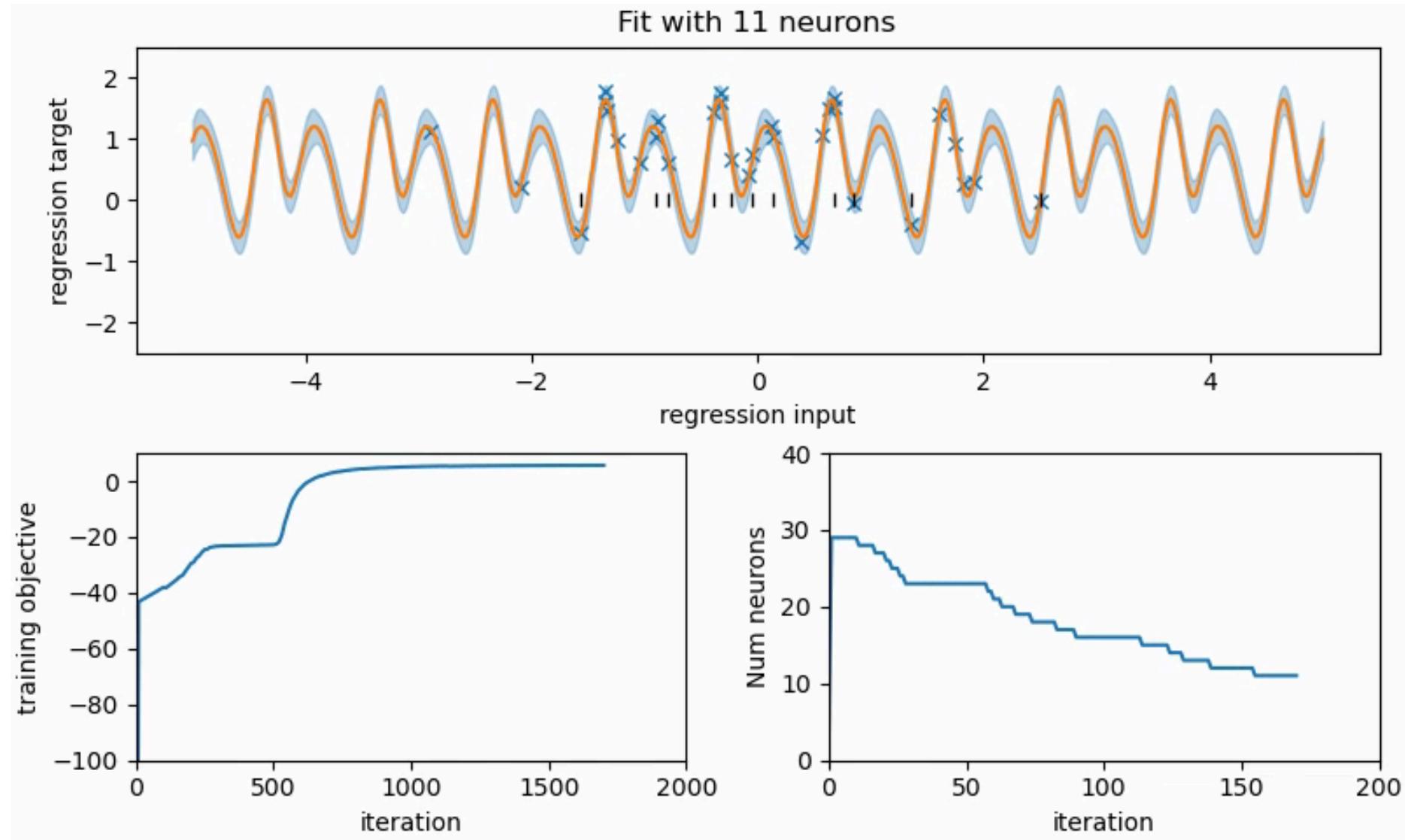
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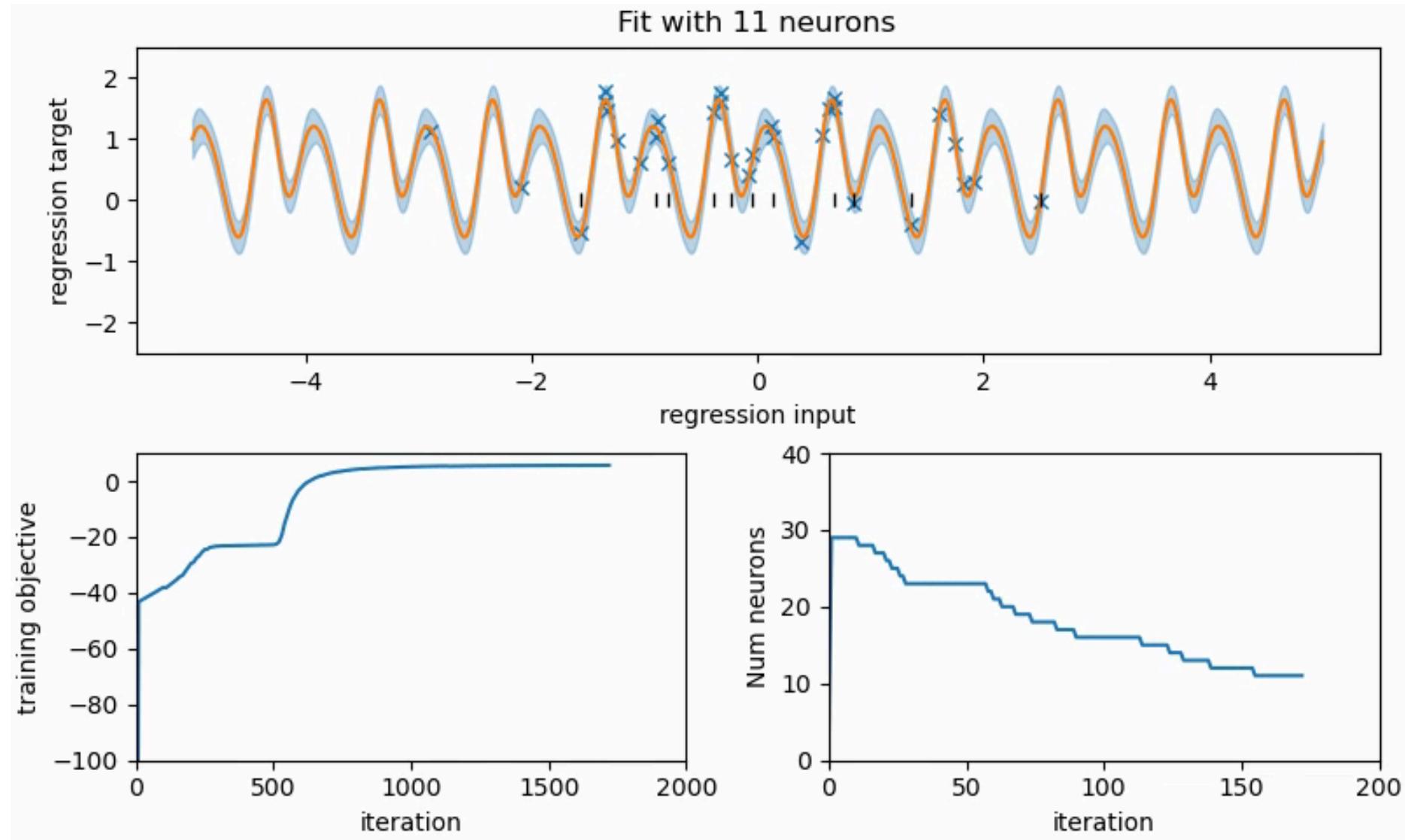
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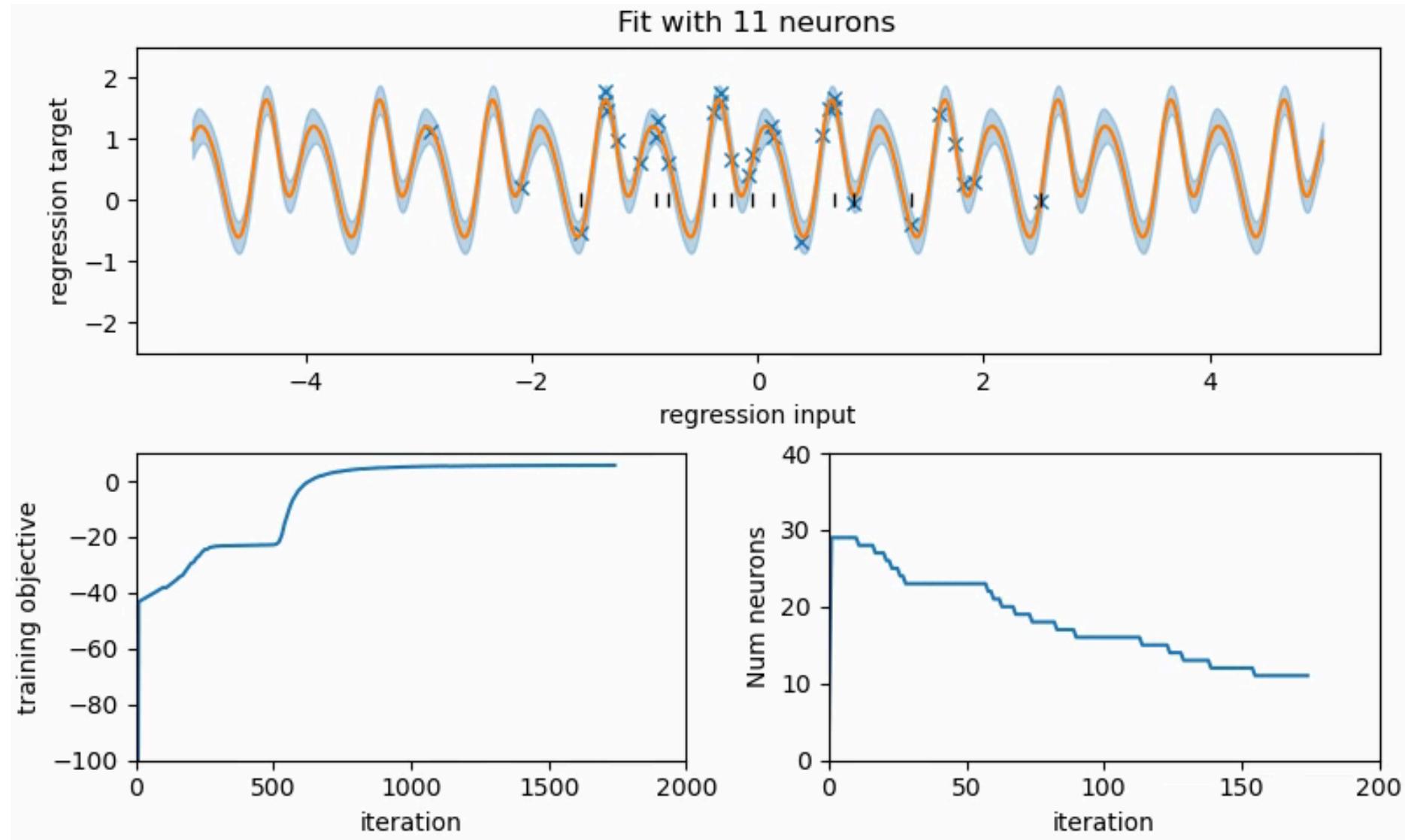
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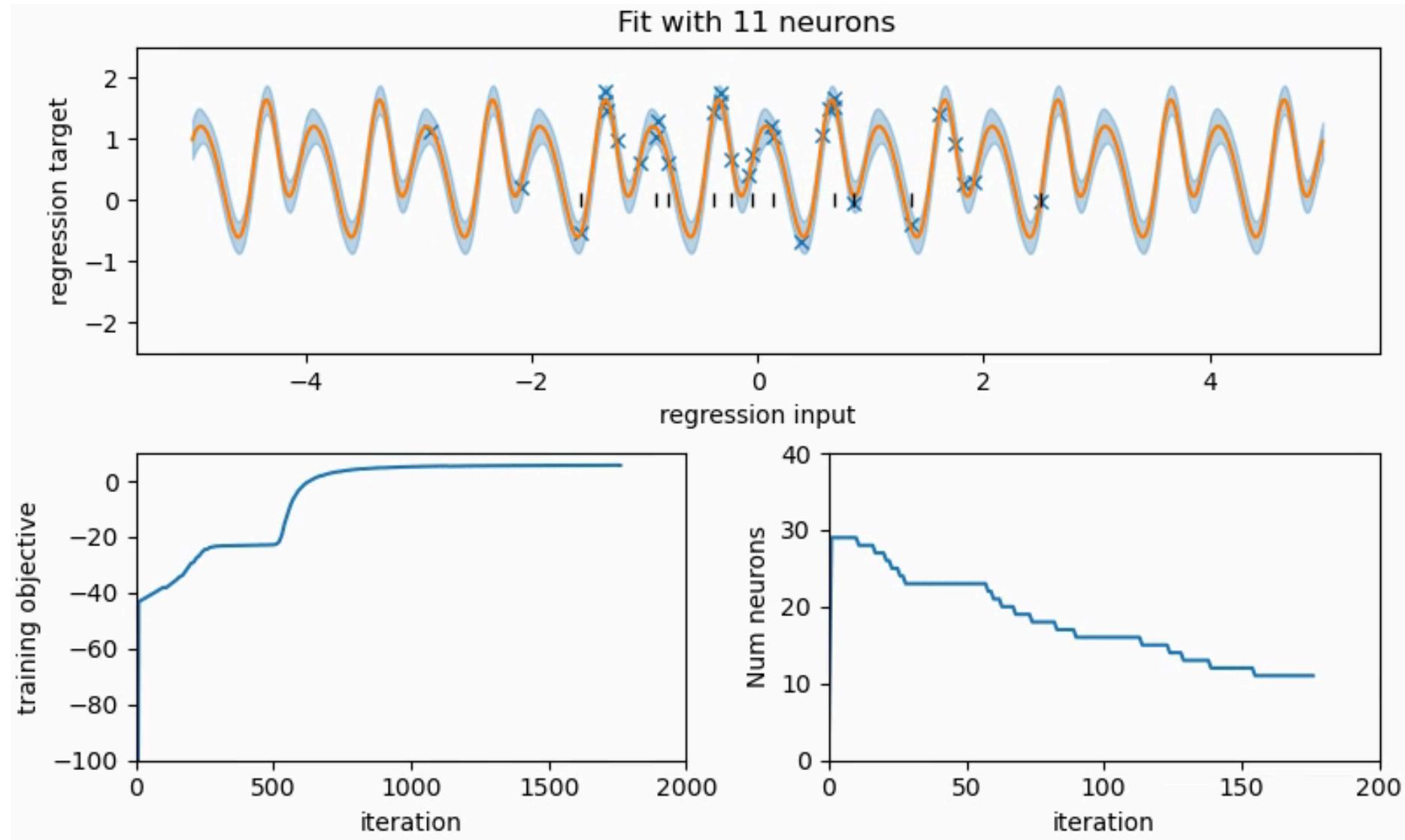
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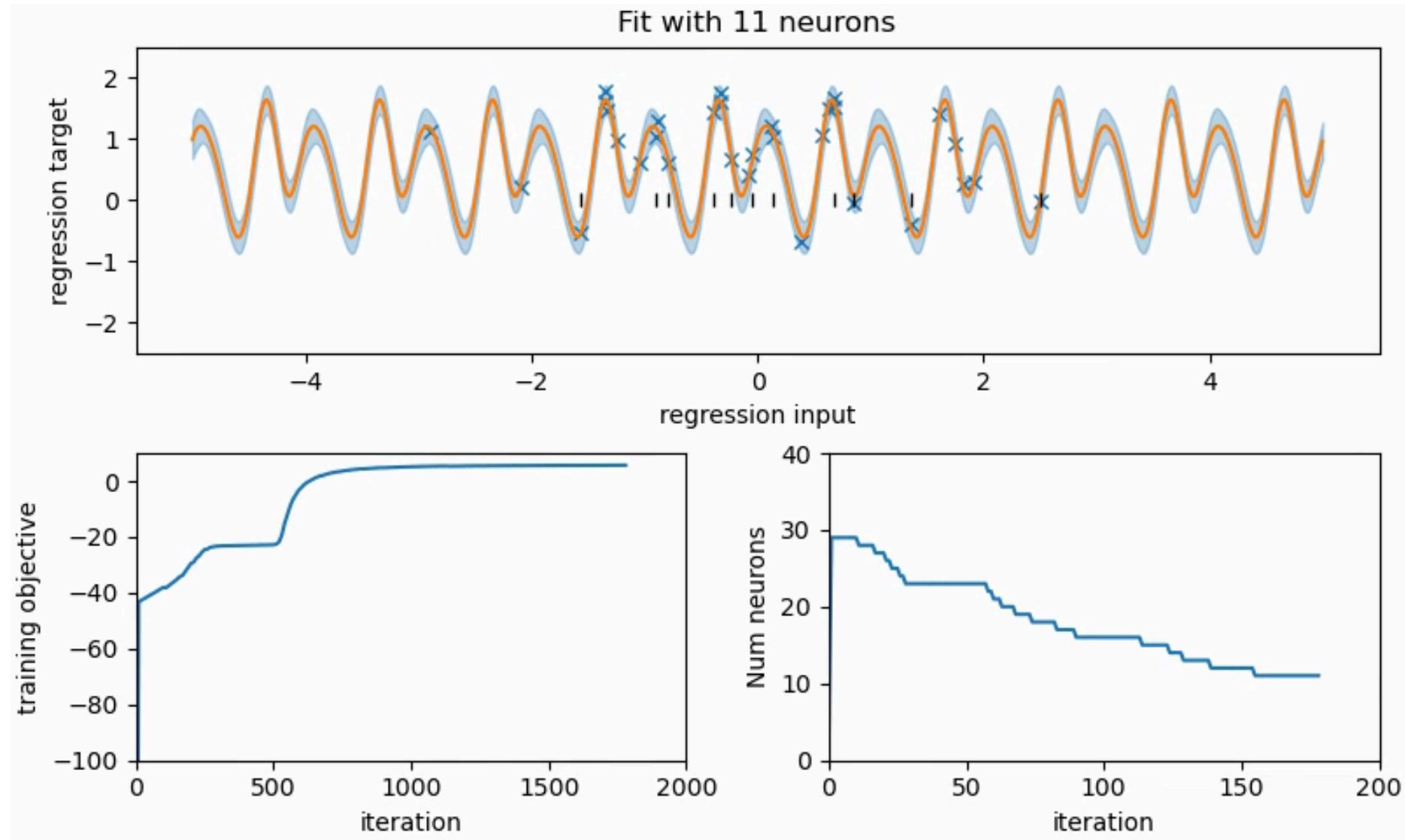
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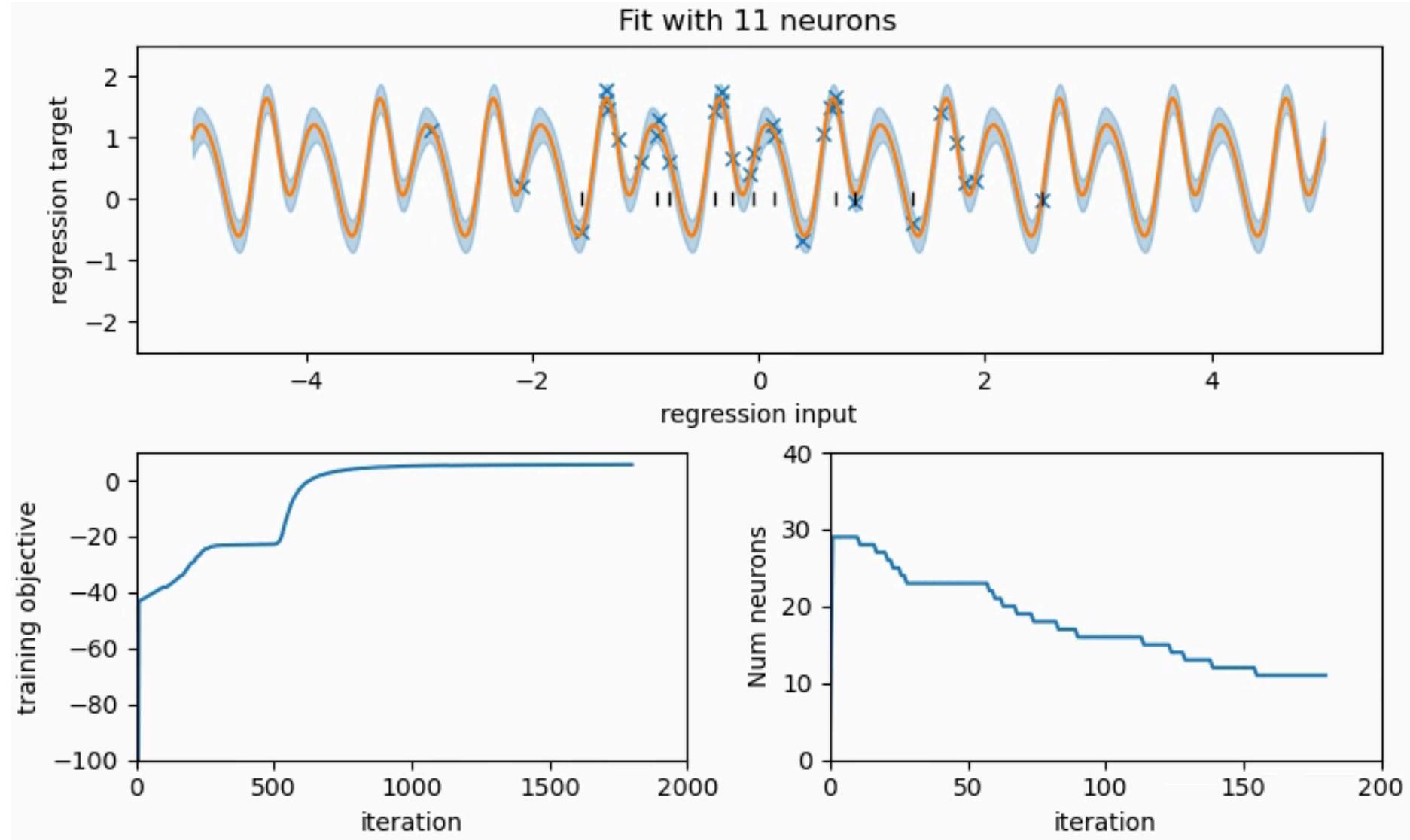
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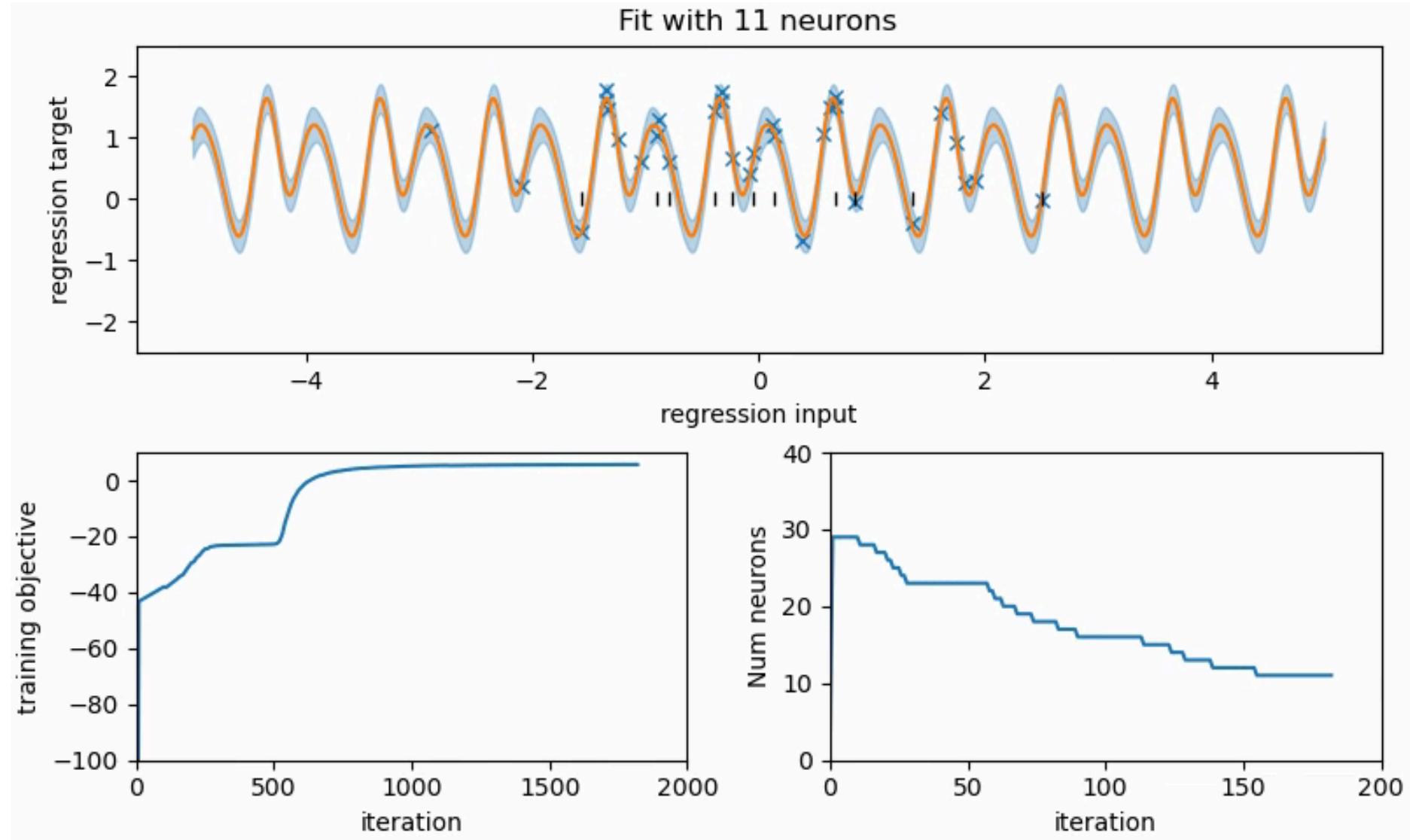
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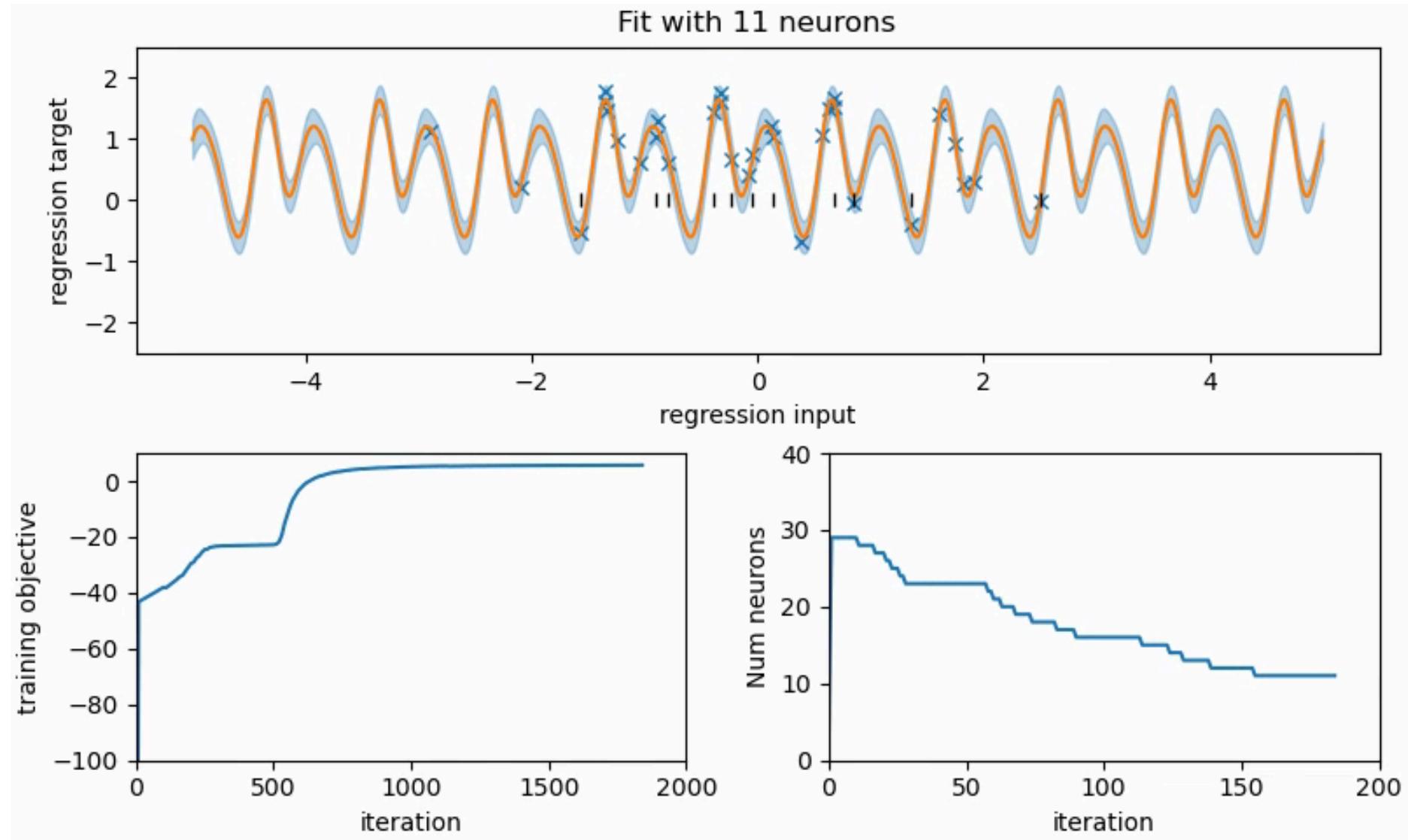
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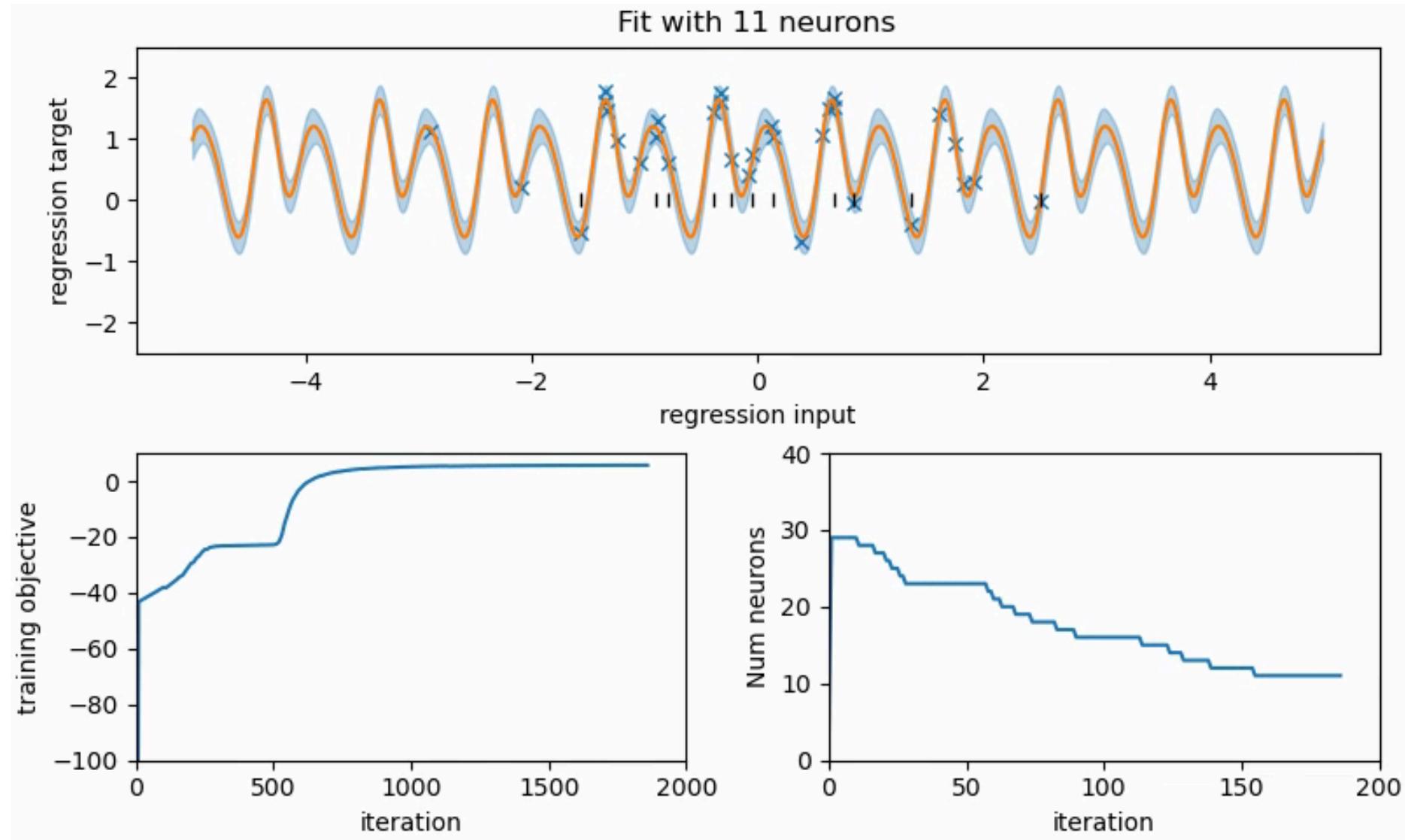
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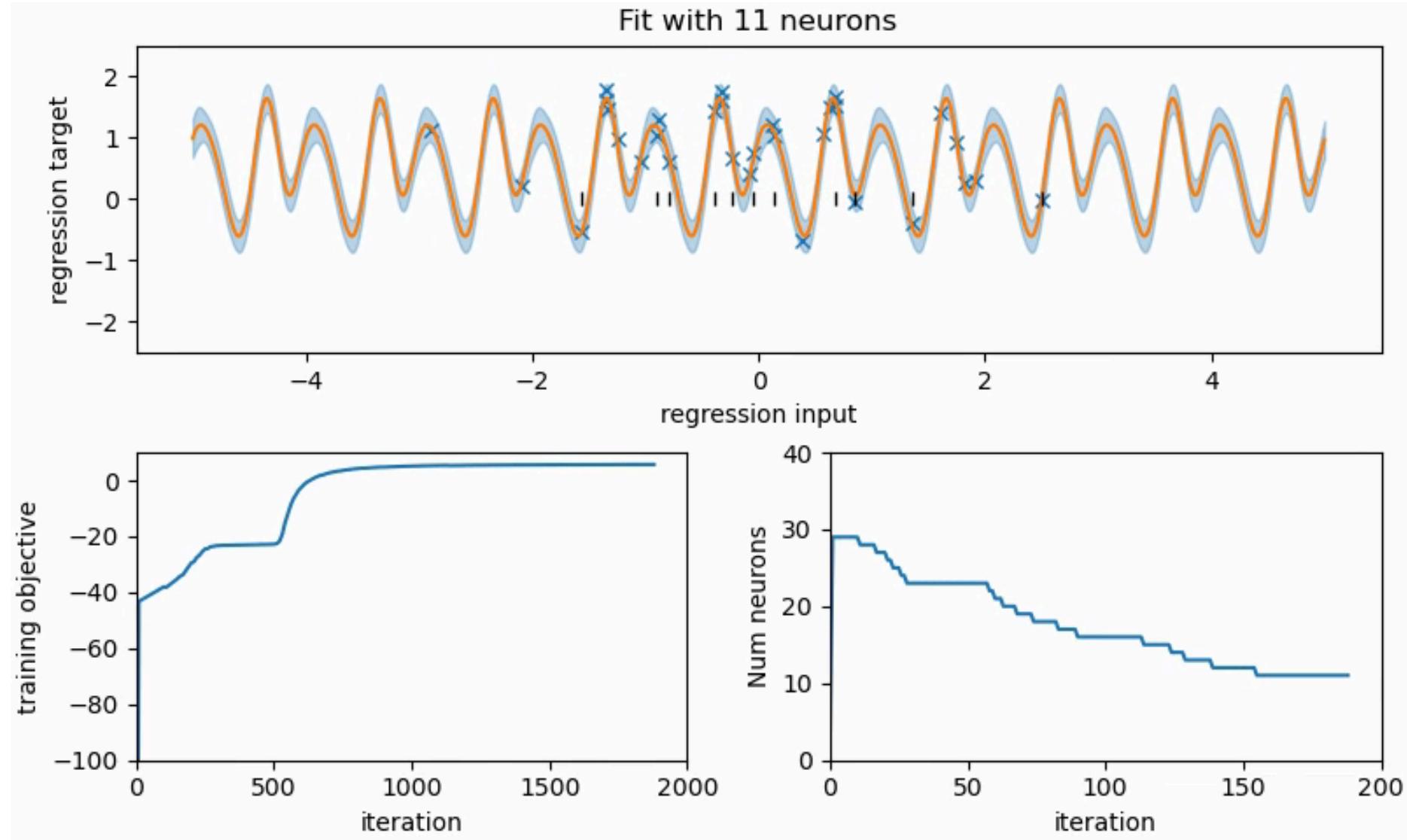
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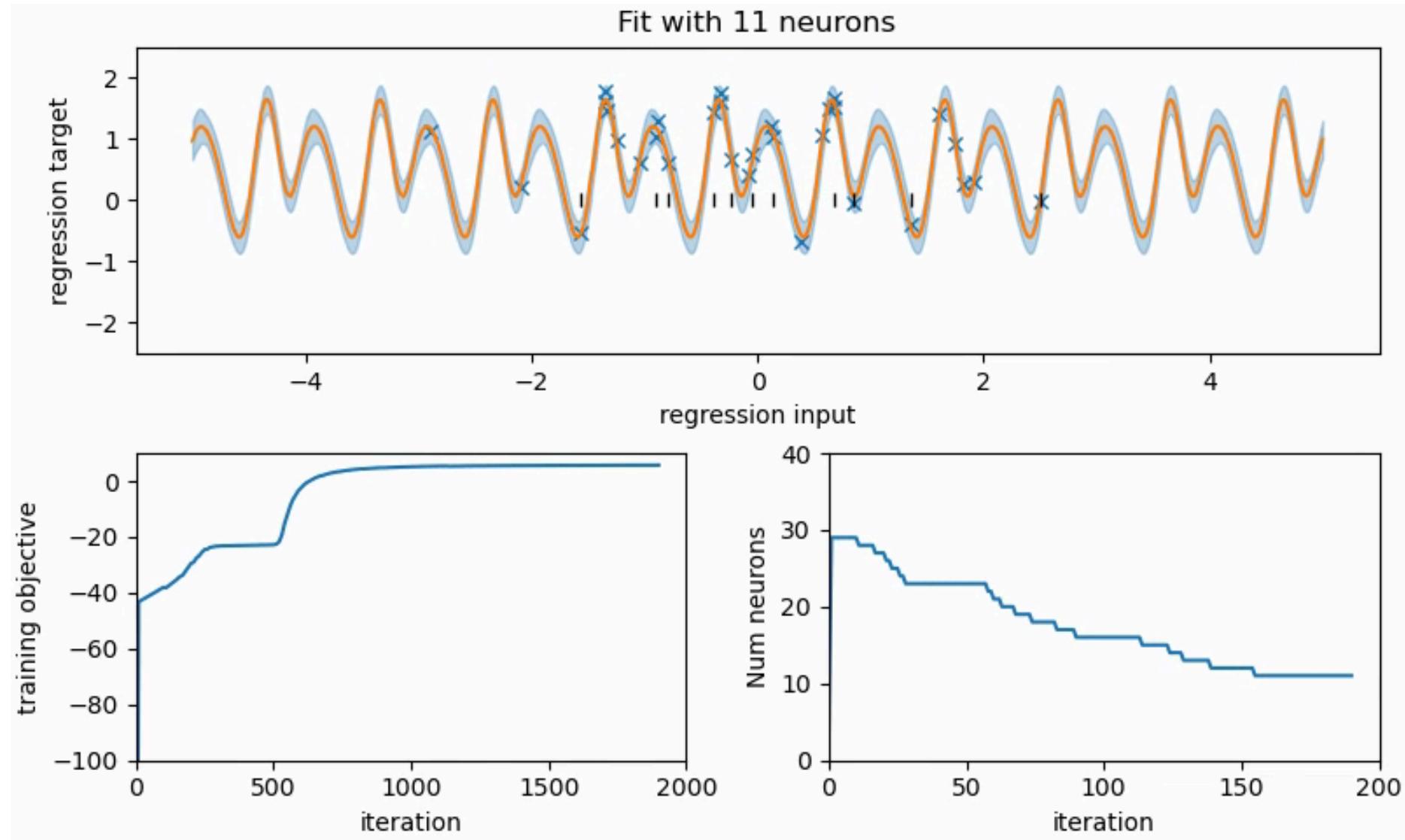
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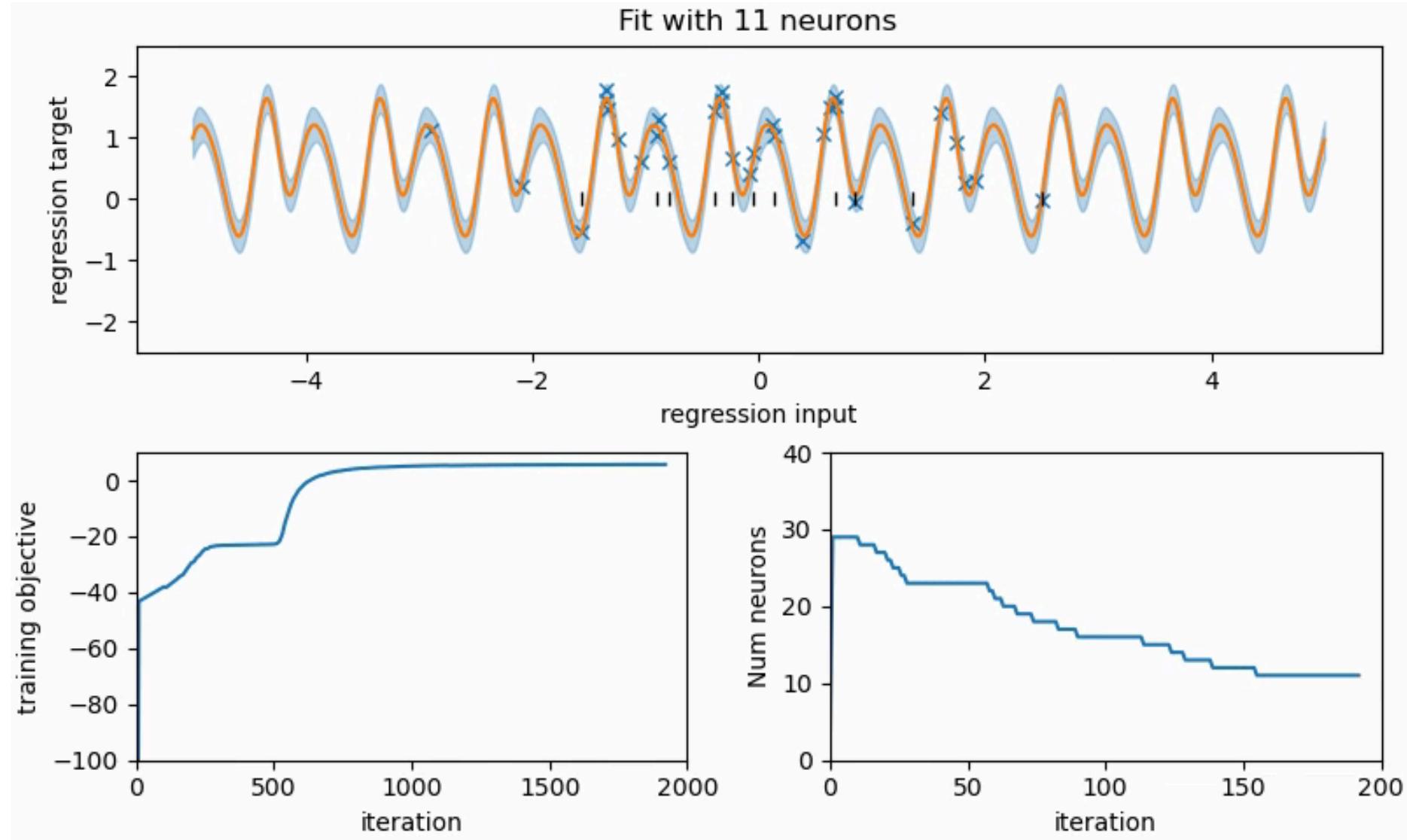
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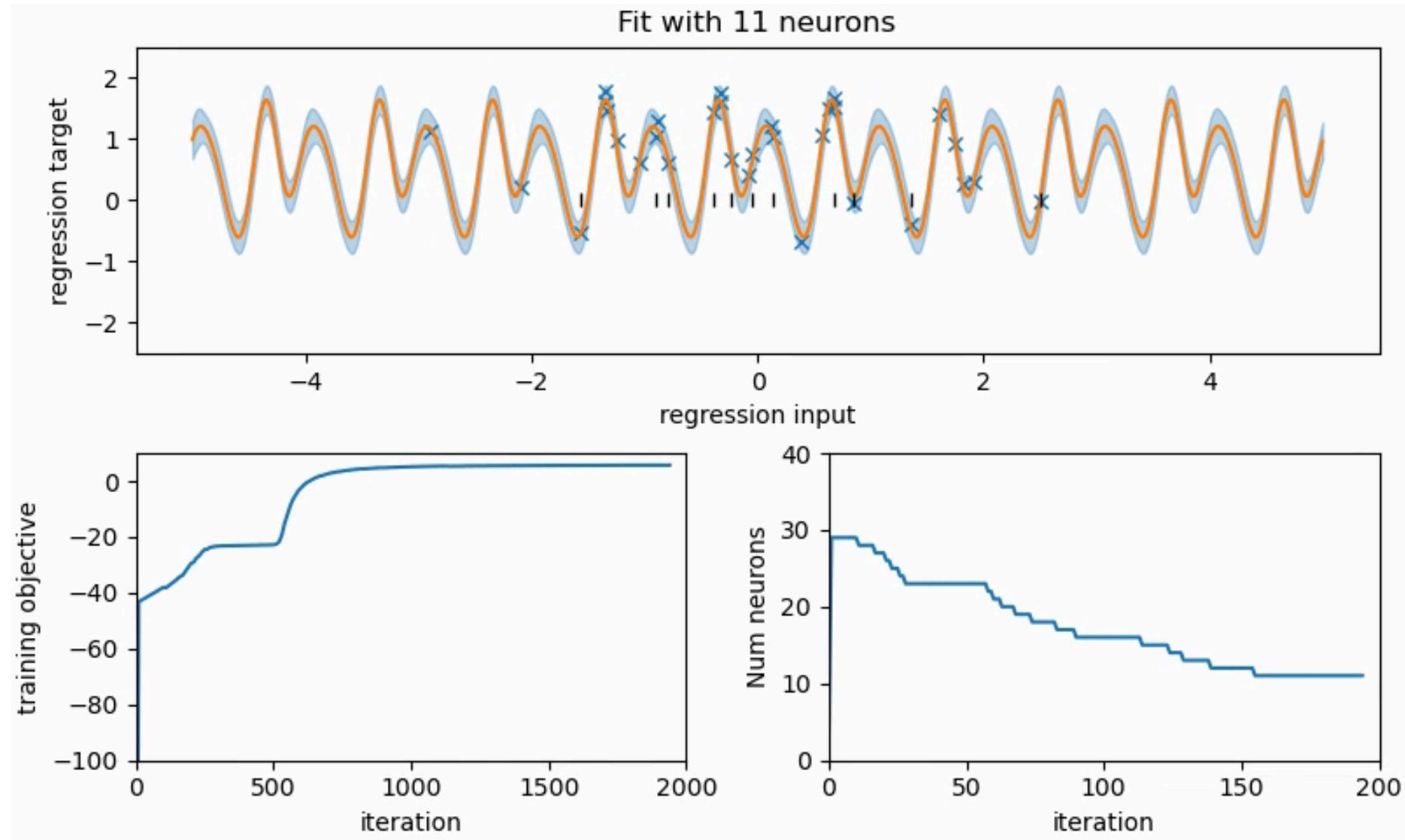
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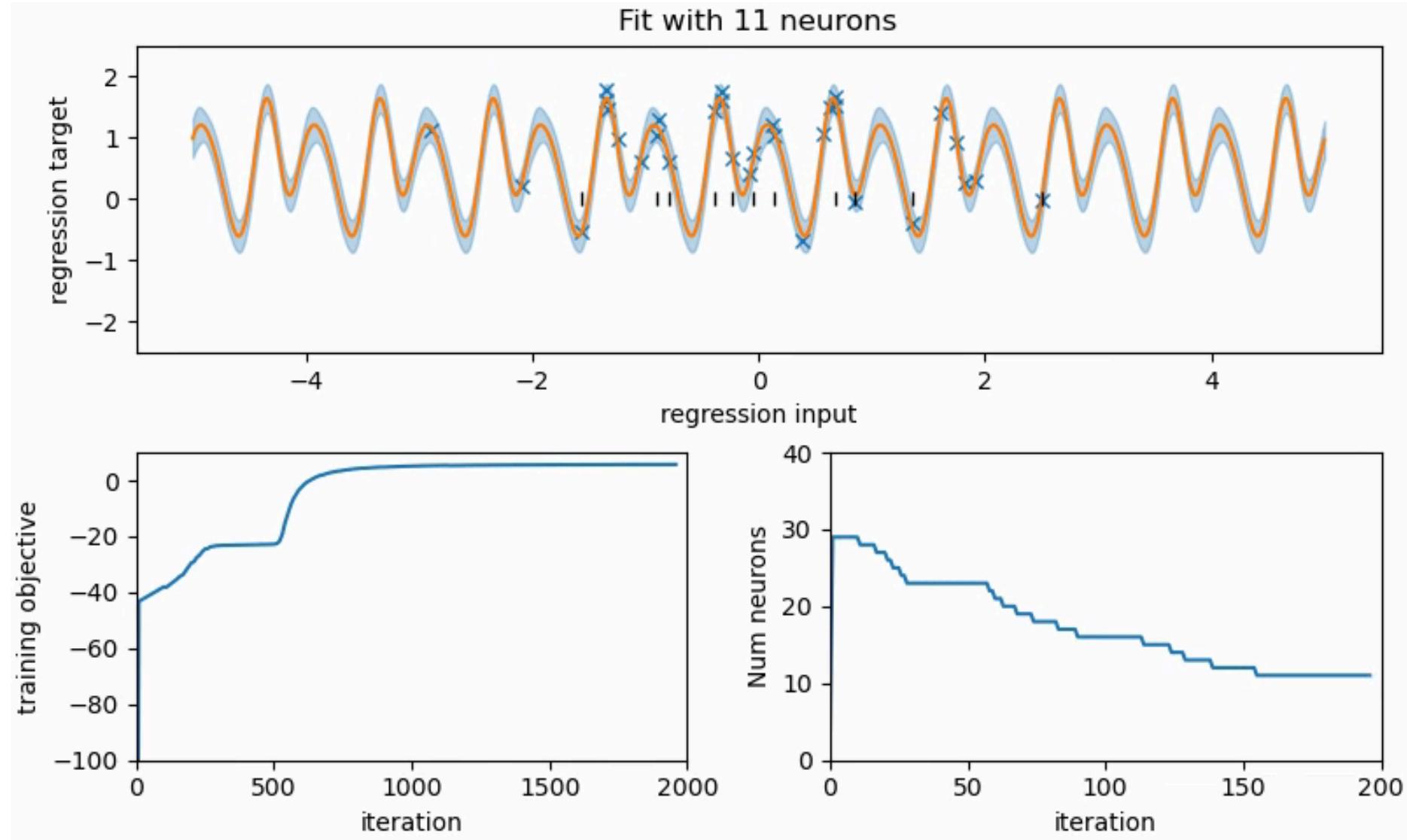
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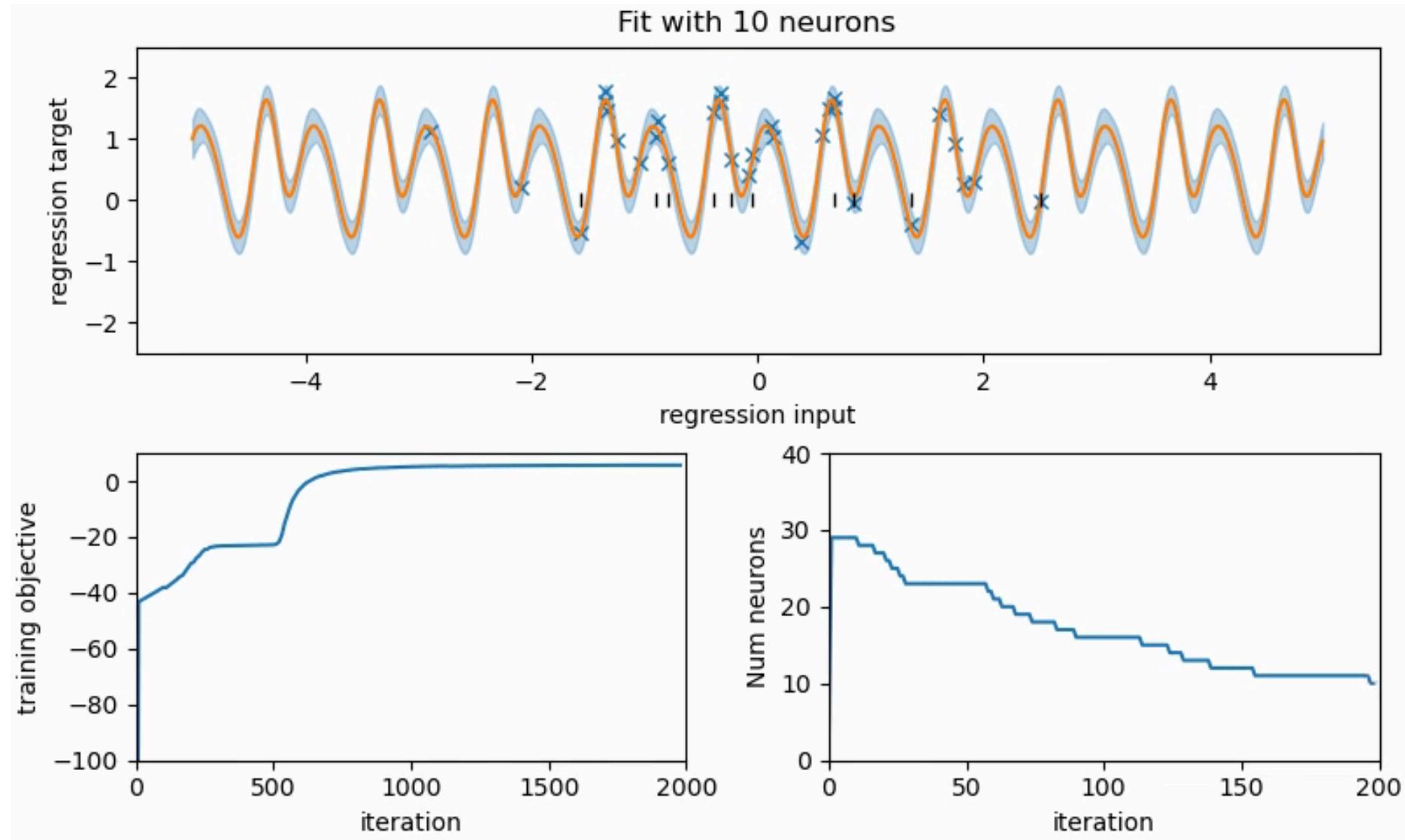
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Memorising first, then pruning



Conclusion

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We can have our cake and eat it

We can *define* an infinite-sized model, but near-perfectly
approximate it with *just* the right amount of computational
resources!

Designing a Neural Network

 **New procedures for training neural networks!**

Can we automatically find:

-

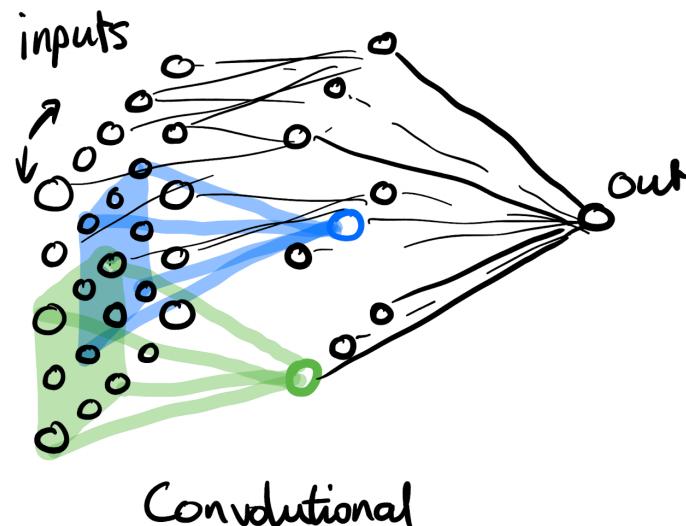
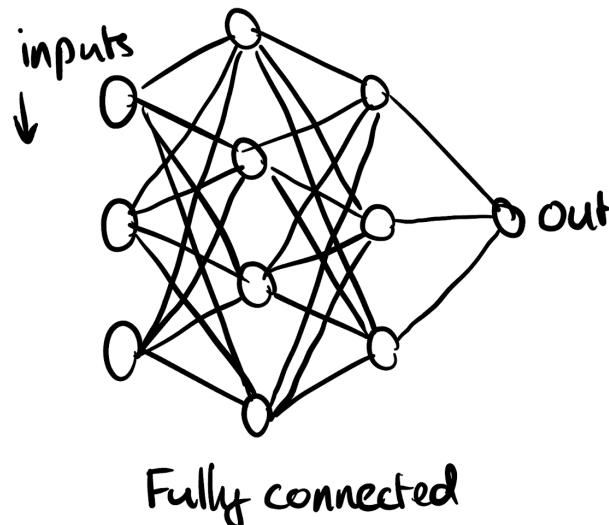
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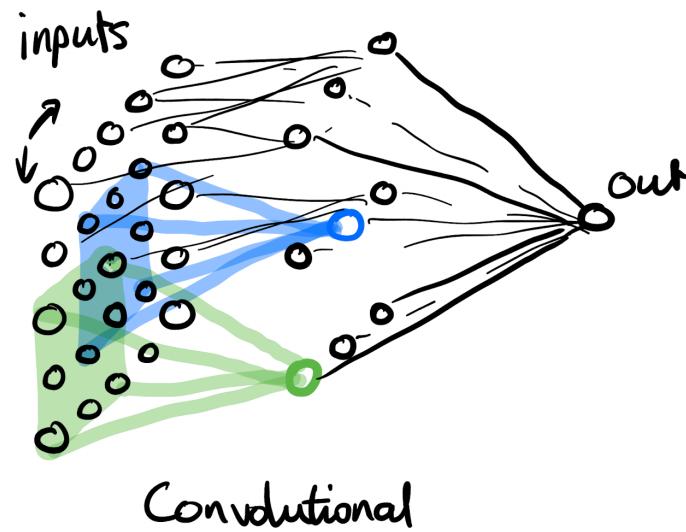
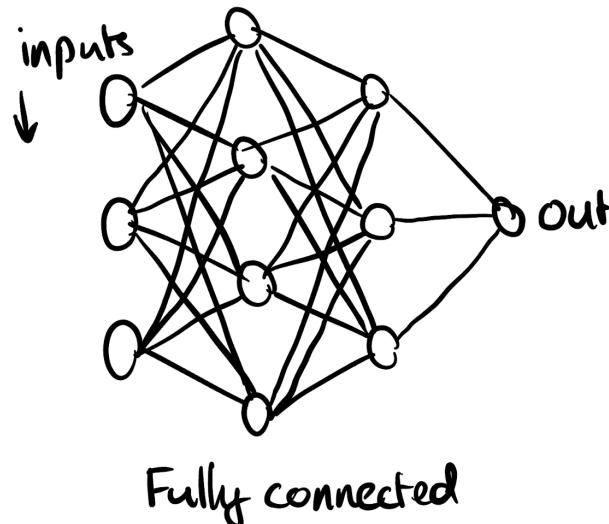
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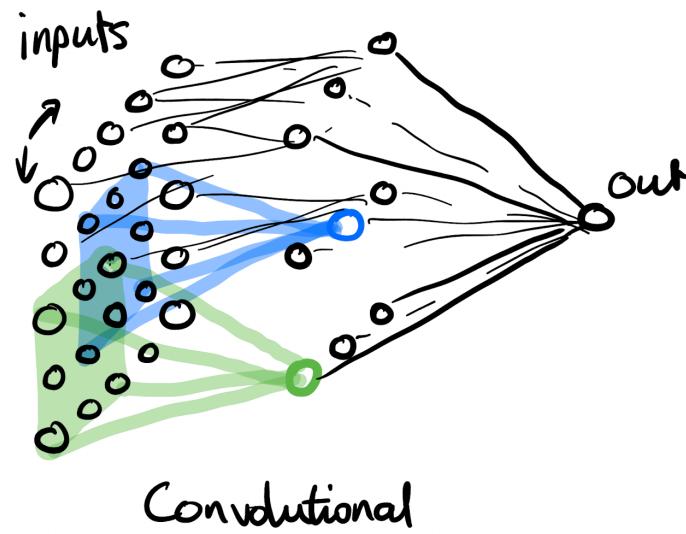
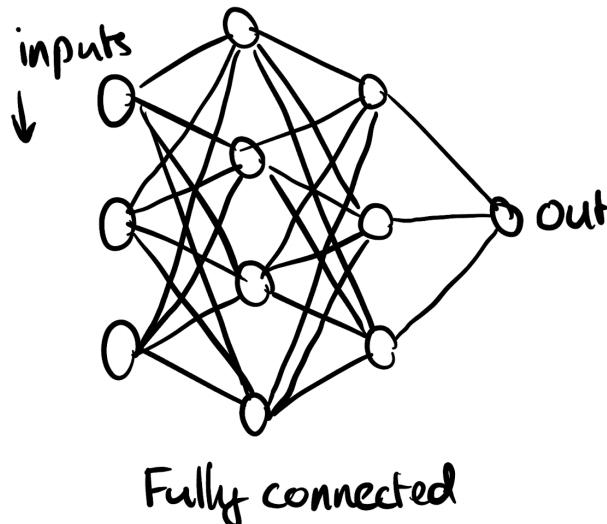
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Designing a Neural Network

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More efficient, more adaptive, more automatic!

Papers

Gaussian processes:

- Background of some ideas described in my thesis
(van der Wilk, 2019)
- Proof of accuracy of variational approximation (basis for when to stop adding inducing variables / basis functions)
(Burt et al., 2019; 2020)
- Adaptive model size for continual learning
(Pescador-Barrios et al., 2024)
- Overall narrative of this talk (online soon!)

Bayesian Model Selection in Neural Networks:

- Bayesian Model Selection (Laplace approximation) *recovers* ResNets, without explicit human design
(Ouderaa et al., 2023)
- See more by Tycho van der Ouderaa!

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