

Reasoning with Uncertainty

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Reasoning & Logic

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- ▶ Beliefs in statements is subjective.
- ▶ The process to manipulate them is not.

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Is there a well-defined **process**
for reasoning under uncertainty?

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Probability satisfies requirements!

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Taking binary variables R, C to indicate rainyness and cloudiness, our background knowledge \mathcal{H} gives us beliefs in:

- ▶ $P(R = 1 \mid C = 0, \mathcal{H}) = r_0, P(R = 1 \mid C = 1, \mathcal{H}) = r_1$
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We can derive our belief whether it will rain:

$$\begin{aligned} P(R = 1 | F, \mathcal{H}) &= \sum_c P(R = 1 | C = c, \mathcal{H}) P(C = c | F, \mathcal{H}) \\ &= r_1 c + r_0 (1 - c) \end{aligned} \tag{3}$$

If we had full certainty about statements, e.g. $r_1 = 1, c = 1$, then this is *modus ponens*.

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$$P(C = 1 | \mathcal{H}) = \sum_r P(C = 1 | R = r, \mathcal{H}) P(R = r | \mathcal{H}) \quad (4)$$

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Full certainty (e.g. $r_1 = 1, d = 1$) \rightarrow *modus tollens*.

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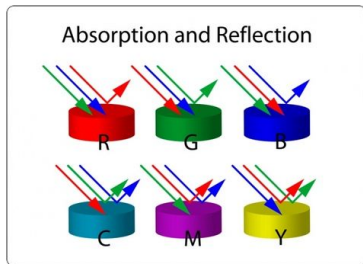
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Note: we can't choose $P(C | \mathcal{H})$ independently of $P(R = 0 | \mathcal{H})$, so this is to show the reduction to logic only.

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- ▶ We observe light L , reflected off an object with colour C , under illumination I .
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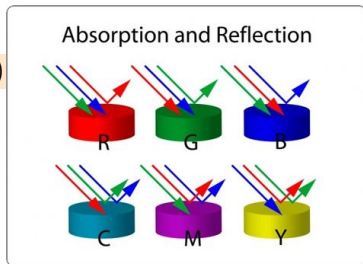


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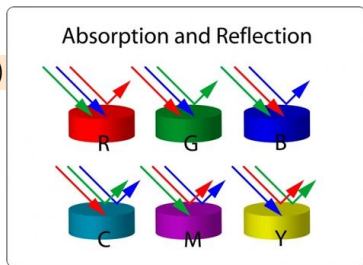
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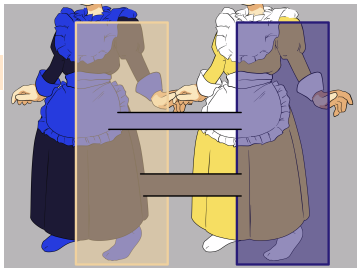
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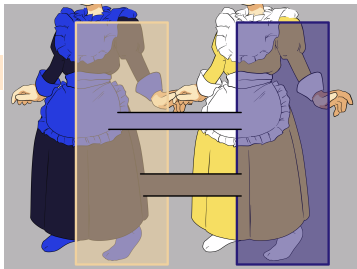


Table: Left: $P(C|L = \ell, I)$, right: $P(C|L, \mathcal{H}_k)$ for $k \in \{1, 2, 3\}$.

	$I = B$	$I = Y$	$p_{B1} = 0.8$	$p_{Y2} = 0.8$	$p_{B3} = 0.5$
$C = WG$	0.9	0.1	0.74	0.26	0.5
$C = BB$	0.1	0.9	0.26	0.74	0.5

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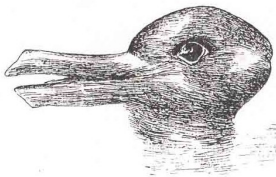
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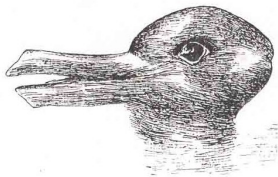
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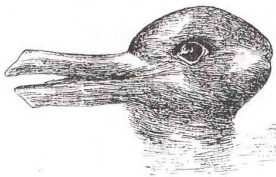
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- ▶ Wouldn't it be better if we kept track of all possibilities?

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- ▶ Further claim: Probability theory is the **only** way to reason under uncertainty!
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Beware of looking into this! You end up doing philosophy, which is interesting, but not helpful to get things done...

Further Reading: Dutch Books

Map beliefs to bets:

You take a bet if its expected value under your beliefs is positive.

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Dutch book theorem: No Dutch book if you follow probability.

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What do I know about what I don't observe?

- ▶ Use probability to represent subjective state of **uncertainty**.
- ▶ Reduction in uncertainty is **learning**!

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Inference

Given my **understanding** of how the world works

Given **incomplete information** about the world

What do I know about what I don't observe?

- ▶ Use probability to represent subjective state of **uncertainty**.
- ▶ Reduction in uncertainty is **learning**!
- ▶ Rules of probability describe learning **process**.

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The course will be mathematical! (but it's not a maths course)

- ▶ We will discuss proofs (although focussing on the big picture).
- ▶ We will analyse the behaviour of algorithms.
- ▶ Exam will require you to demonstrate ability to apply mathematical principles.
- ▶ BUT, hopefully you gain an intuition into the principles too.

Course outline

- I) Bayesian brainteasers (graphical models, tractable inference)
How do we put problems into the mathematical formalism?
What is a model? How do we formulate assumptions in models?
- II) Gaussian processes
Specifying models, computing beliefs.
- III) Decision Theory & Bayesian optimisation
Using uncertainty to take actions.
- IV) Approximate inference
What happens if we cannot compute our beliefs exactly?
- V) Modern applications, e.g. generative models
How is this used right now?

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Combining cues from disparate sources

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Probably not necessary if you want to focus on implementing ML models, or ML infrastructure.

What problems can we solve?

- ▶ Low data prediction (how (un)certain am I?)
- ▶ Experiment design (what data should I gather next?)
- ▶ Data fusion (How should I combine information sources?)
- ▶ Learning (How should my belief change to match the world?)
- ▶ Decision making (Should I take a risk or play it safe?)

Prerequisites

- ▶ Good understanding of Mathematics for Machine Learning, e.g.:
 - ▶ Linear algebra (eigendecompositions etc)
 - ▶ Probability and basic statistics
 - ▶ Vector calculus
 - ▶ Gradient-based optimisation
- ▶ Python coding

<https://mml-book.com>

Expectations

What is expected for the exam:

- ▶ Knowledge of topics discussed
- ▶ Awareness of why topics are relevant
- ▶ Derive methods using mathematical concepts discussed
- ▶ Analyse methods using mathematical concepts discussed

How to study & revise:

- ▶ Join the lectures in person, engage, share your questions
- ▶ Think about how theory applies in different settings
- ▶ Do the exercises

Highly Recommended Reading

Information Theory, Inference, and Learning Algorithms (MacKay, 2003)

- ▶ §2.1 (4pgs): Refresher of probability + notation we will use.
- ▶ §2.2 (1pg): Probability as belief.
- ▶ §2.3 (5pgs): Examples of Bayes rule. Exercises + solutions are very illustrative.

You really should read this.

See EdStem for links to books.

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- ▶ I would like
- ▶ I will look at EdStem questions once per week.

Questions?

Hopefully this gives you an overview of the course.

Questions?

References I

Cox, R. T. (1946). Probability, frequency and reasonable expectation. *American journal of physics*, 14(1):1–13.

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