Closed-Form Inference

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Probabilistic Models

Previously we saw:

- How to specify probabilistic models.
- Graphical models to derive conditional independencies.
- Finding efficient posteriors using CIs.
- How the inference process worked (discrete variables).

Today: Final bit of background mathematics, before we start solving problems with these techniques.

- How to do Bayesian inference with continuous distributions.
- Why Bayesian inference is difficult, and why approximations are needed.

Some Notation

- Usually, I refer to both a random variable and outcome as a lower-case letter (e.g. x), and let context determine the usage.
- As in MML, pmfs/pdfs are determined by their arguments, e.g. $p(x)$. To be more explicit I may write $p(x) = p_X(x)$.
- For conditional distributions especially, you should keep track of which variables are fixed, and which are free.
  - E.g. posterior $p(x|y) = p_{X|Y}(x|y)$, function is over x, and y is fixed.
  - More explicitly: $p_{X|Y}(\cdot|y)$.
- When writing general rules, I don't distinguish pdfs and pmfs.
  - With slight abuse of notation, can think of a pmf $p(X = x) = \sum_{x'=1}^{K} \delta(x - x')$.
  - Integrals turn to sums if you carefully work through.

Closed-form Inference

Inference

The procedure of drawing conclusions from observations.

In Bayesian statistics: Computing some conditional distribution (posterior).

Closed-form Expressions

A mathematical expression consisting of a finite number of standard operations (pow, exp, log, trig, etc).

See `https://en.wikipedia.org/wiki/Closed-form_expression`.

Closed-form Inference

An inference problem where all relevant quantities (e.g. posteriors) can be computed in closed-form.

Exchangeable Models

Prior Likelihood terminiology