


# Sequential Decisions

**Mark van der Wilk**

Department of Computing  
Imperial College London

 @markvanderwilk  
m.vdwilk@imperial.ac.uk

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# Sequential Decision Making

- ▶ Last time: Principle for taking an action based on beliefs.
- ▶ This time: How do we plan for decisions made after each other?
- ▶ Key idea: At each point, apply MEU, given all the knowledge you have at that point
- ▶ Consequence: After making an earlier decision, you can **learn**. This information has value.
- ▶ This is key reason why things become complicated, but principle is no more complex!

## Example: Commuting I

Paula has an exam at 9am today, and it is 8:35am. She will not be admitted if she is late, so her loss function is

$$L(t, a) = \begin{cases} 0 & \text{if } t \leq 25 \text{ min} \\ 100 & \text{if } t > 25 \text{ min} \end{cases}. \quad (1)$$

Paula decides to take a taxi. The taxi driver knows of routes A and B. Route A can either be clear and fast ( $Q = 0$ ) or jammed and slow ( $Q = 1$ ). The time for route B follows a Gaussian.

$$t_A = \begin{cases} 10 \text{ min} & \text{if } Q = 0 \\ 50 \text{ min} & \text{if } Q = 1 \end{cases}, \quad P(Q) = \begin{cases} 0.5 & \text{for } Q = 0 \\ 0.5 & \text{for } Q = 1 \end{cases}, \quad (2)$$

$$p(t_B) = \mathcal{N}(t_B; 25, 5^2). \quad (3)$$

## Example: Commuting II

The driver suggests the following strategy:

- ▶ Drive the first 2.5 minutes of route A to see whether it is clear (i.e. to observe  $Q$ ).
- ▶ At that point, decide whether to keep going with route A, or to drive back 2.5 minutes and take route B.

Question: Should Paula:

- ▶ Ask the driver to take route A?
- ▶ Ask the driver to take route B?
- ▶ Ask the driver to continue with his strategy?

# Decision Trees

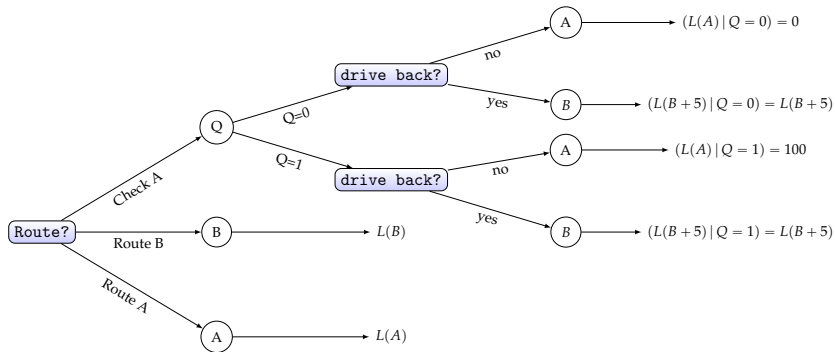
- ▶ Arrange sequence of observations and decisions into a **tree**
  - ▶ Helps us to compute utility based on decisions
  - ▶ Helps us to find sequence of optimal decisions
- 
- ▶ Rectangle: Decision nodes.
  - ▶ Circle: Observation of a random variable.
  - ▶ Leaf nodes: Outcome of utility.
  - ▶ Nodes are functions of the outcome of parent RVs.

# Finding optimal decisions

Simple process:

- ▶ Start at leaf nodes and move towards root.
- ▶ RV nodes: Calculate expectation over utility values coming in.
- ▶ Decision nodes: Choose action corresponding to child with maximum utility. Pass on this utility.

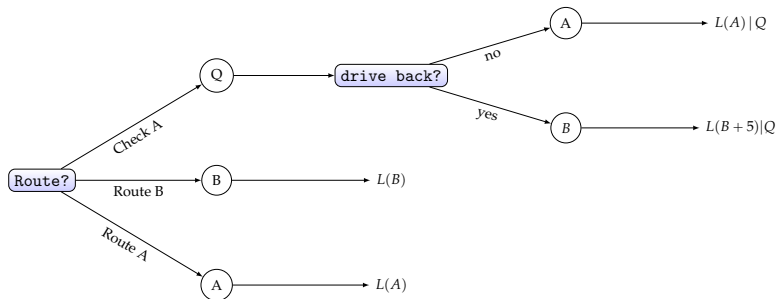
# Example: Commuting



- ▶ We can split up outcomes of RVs explicitly
- ▶ This allows us to find explicit numerical values at all points.
- ▶ Note:  $(L(B + 5) | Q = 0) = L(B + 5)$ , since  $B \perp\!\!\!\perp Q$ .

Board: solution.

# Example: Commuting



- ▶ Can make graph smaller by not writing out all outcomes of RVs.
- ▶ Decisions become *functions* of parent outcomes.

Board: solution



# Exploration-Exploitation Trade-Off

- ▶ We saw that “Check A” was the best option.
- ▶ Spending effort to gather information, can help us to make better decisions later.
- ▶ Uncertainty determines how much exploration can gain.
- ▶ (Would exploration be worth it if  $P(Q = 1) = 0.99$ ?)
- ▶ ►► Exploration-exploitation trade-off.

# Conclusion

- ▶ Sequential decision making is the same as earlier.
- ▶ Can deal with complexity using decision trees.
- ▶ Exploration-exploitation trade-off.

Recommended reference: MacKay [1] chapter 36.

Further reading: Russell [2] chapter 17.

There's a whole theory of MDPs out there!

# References I

- [1] D. J. C. MacKay. *Information Theory, Inference, and Learning Algorithms*. Cambridge University Press, 2003.
- [2] S. J. Russell. *Artificial intelligence a modern approach*. Pearson Education, Inc., 2010.