Reasoning with Uncertainty

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January 16, 2023

Reasoning & Logic

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- Beliefs in statements is subjective.
- The process to manipulate them is not.

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Is there a well-defined **process**
for reasoning under uncertainty?

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Probability satisfies requirements!

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Taking binary variables R, C to indicate rainyness and cloudiness, our background knowledge $\mathcal{H}$ gives us beliefs in:

- $P(R = 1 \,|\, C = 0, \mathcal{H}) = r_0$, $P(R = 1 \,|\, C = 1, \mathcal{H}) = r_1$
- $P(C = 1 | F, \mathcal{H}) = c$ (where F is what our friend told us)

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We can derive our belief whether it will rain:

$$\begin{aligned} P(R = 1|F, \mathcal{H}) &= \sum_c P(R = 1|C = c, \mathcal{H}) P(C = c|F, \mathcal{H}) \\ &= r_1 c + r_0(1 - c) \end{aligned} \tag{3}$$

If we had full certainty about statements, e.g. $r_1 = 1, c = 1$, then this is *modus ponens*.

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We can derive our belief of whether there will be clouds:

$$P(C = 1|\mathcal{H}) = \sum_r P(C = 1|R = r, \mathcal{H})P(R = r|\mathcal{H}) \tag{4}$$

$$P(C = 1|R = r, \mathcal{H}) = \frac{P(R = r|C = 1, \mathcal{H})P(C = 1|\mathcal{H})}{\sum_c P(R = r|C = c, \mathcal{H})P(C = c|\mathcal{H})} \tag{5}$$

Full certainty (e.g. $r_1 = 1, d = 1$) $\rightarrow$ *modus tollens*.

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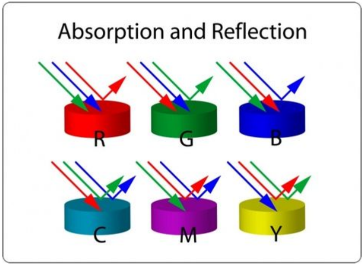
Full certainty (e.g. $r_1 = 1, d = 1$) → *modus tollens*.
Note: we can't choose $P(C|\mathcal{H})$ independently of $P(R = 0|\mathcal{H})$, so this is to show the reduction to logic only.

Example: What colour is an object?

- We observe light L, reflected off an object with colour C, under illumination I.
- Given we observe light, what colour is the object?



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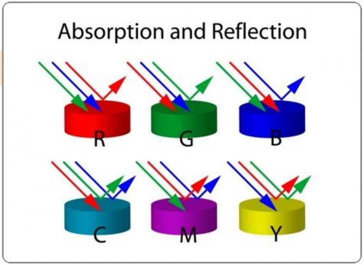

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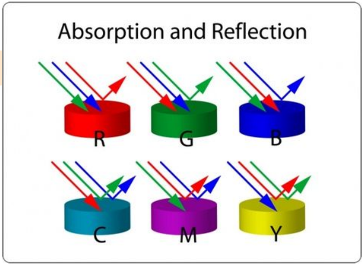

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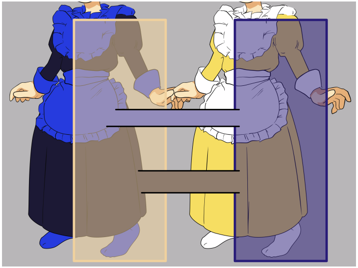

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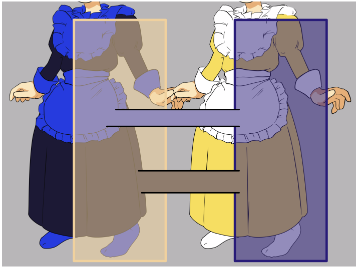

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Table: Left: $P(C|L=\ell, I)$, right: $P(C|L,\mathcal{H}_k)$ for $k \in \{1,2,3\}$.

	$I=B$	$I=Y$	$p_{B1}=0.8$	$p_{Y2}=0.8$	$p_{B3}=0.5$
$C=\text{WG}$	0.9	0.1	0.74	0.26	0.5
$C=\text{BB}$	0.1	0.9	0.26	0.74	0.5

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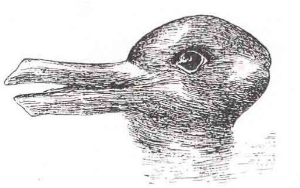

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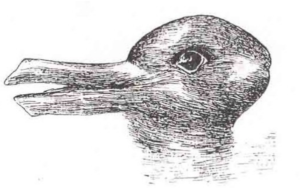

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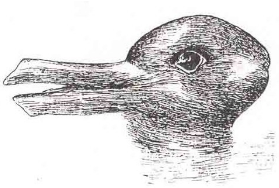

- Guess: Our brain jumps to one interpretation, even if the belief is only mildly stronger.
- Wouldn't it be better if we kept track of all possibilities?

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Beware of looking into this! You end up doing philosophy, which is interesting, but not helpful to get things done...

Further Reading: Dutch Books

Map beliefs to bets:
You take a bet if its expected value under your beliefs is positive.

- Example: For a coin for which you believe $P(\text{heads}) = p$, where you get a payout of **€1 if you win**, you will pay up to **€p to take the bet**.

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- Result: You always lose €0.8.

Dutch book theorem: No Dutch book if you follow probability.

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- Reduction in uncertainty is **learning**!

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What do I know about what I don't observe?

- Use probability to represent subjective state of **uncertainty**.
- Reduction in uncertainty is **learning**!
- Rules of probability describe learning **process**.

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The course will be mathematical! (but it's not a maths course)

- We will discuss proofs (although focussing on the big picture).
- We will analyse the behaviour of algorithms.
- Exam will require you to demonstrate ability to apply mathematical principles.
- BUT, hopefully you gain an intuition into the principles too.

Course outline

I) Bayesian brainteasers (graphical models, tractable inference)
How do we put problems into the mathematical formalism?
What is a model? How do we formulate assumptions in models?

II) Gaussian processes
Specifying models, computing beliefs.

III) Decision Theory & Bayesian optimisation
Using uncertainty to take actions.

IV) Approximate inference
What happens if we cannot compute our beliefs exactly?

V) Modern applications, e.g. generative models
How is this used right now?

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  Combining cues from disparate sources

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Probably not necessary if you want to focus on implementing ML models, or ML infrastructure.

What problems can we solve?

- Low data prediction (how (un)certain am I?)
- Experiment design (what data should I gather next?)
- Data fusion (How should I combine information sources?)
- Learning (How should my belief change to match the world?)
- Decision making (Should I take a risk or play it safe?)

Prerequisites

- Good understanding of Mathematics for Machine Learning, e.g.:
  - Linear algebra (eigendecompositions etc)
  - Probability and basic statistics
  - Vector calculus
  - Gradient-based optimisation
- Python coding

`https://mml-book.com`

Expectations

What is expected for the exam:

- Knowledge of topics discussed
- Awareness of why topics are relevant
- Derive methods using mathematical concepts discussed
- Analyse methods using mathematical concepts discussed

How to study & revise:

- Join the lectures in person, engage, share your questions
- Think about how how theory applies in different settings
- Do the exercises

Highly Recommended Reading

Information Theory, Inference, and Learning Algorithms (MacKay, 2003)

- §2.1 (4pgs): Refresher of probability + notation we will use.
- §2.2 (1pg): Probability as belief.
- §2.3 (5pgs): Examples of Bayes rule. Exercises + solutions are very illustrative.

You really should read this.

See EdStem for links to books.

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- I would like
- I will look at EdStem questions once per week.

Questions?

Hopefully this gives you an overview of the course.

Questions?

References I

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