

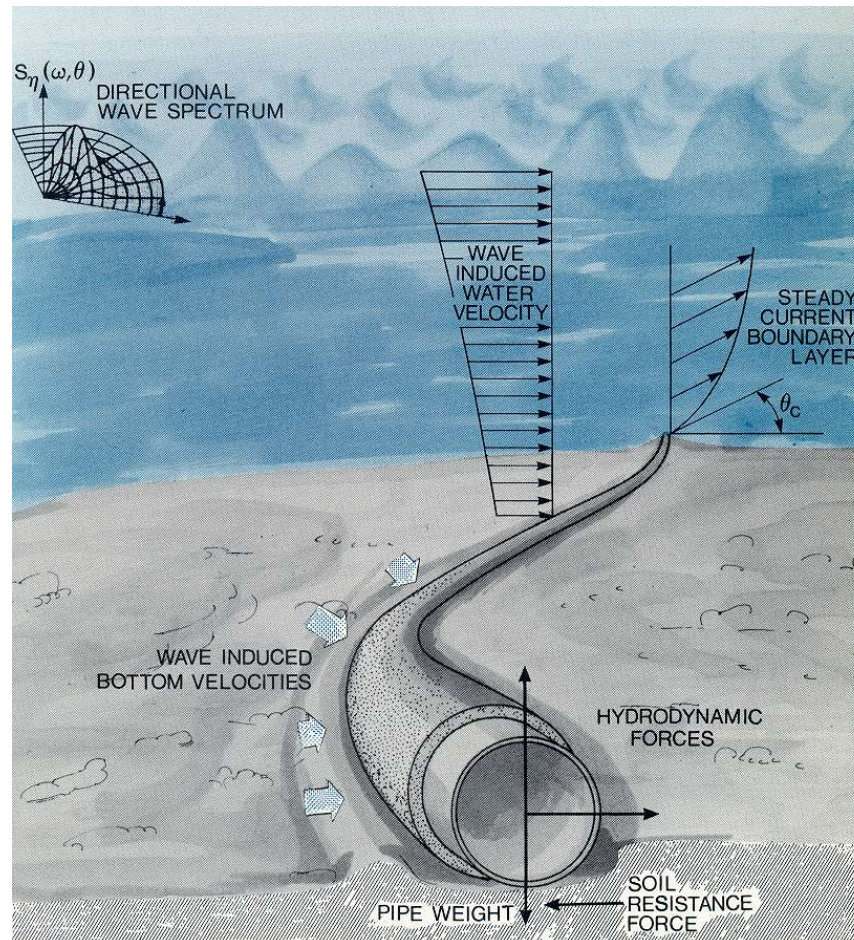
TMR 4585 Specialization Course UWT Pipeline Stability

**by
Prof. Svein Sævik,
Trondheim, 2019**

Contents

- On bottom stability
- Acceptance criteria
- Design currents and waves
- Hydrodynamic forces on pipeline
- Lateral pipe/soil resistance
- Weight coating
- Design for stability; DNV-RP-F109
- Example of stability design
- Software
- Simplified static model
- Linear wave theory

On-Bottom Stability



Acceptance criteria

- **DNV, OS-F101 and RP-F109**
- **Serviceability limit state, SLS**

Acceptance criteria – lateral stability

- **The pipe shall not move from its as-installed position**
- **Limited lateral displacement**
 - Local buckling, fatigue and fracture of pipe
 - Deterioration/wear of coating
 - Geometrical limitations of supports
 - Distance from other pipelines, structures or obstacles

Pipeline stability – Design criteria

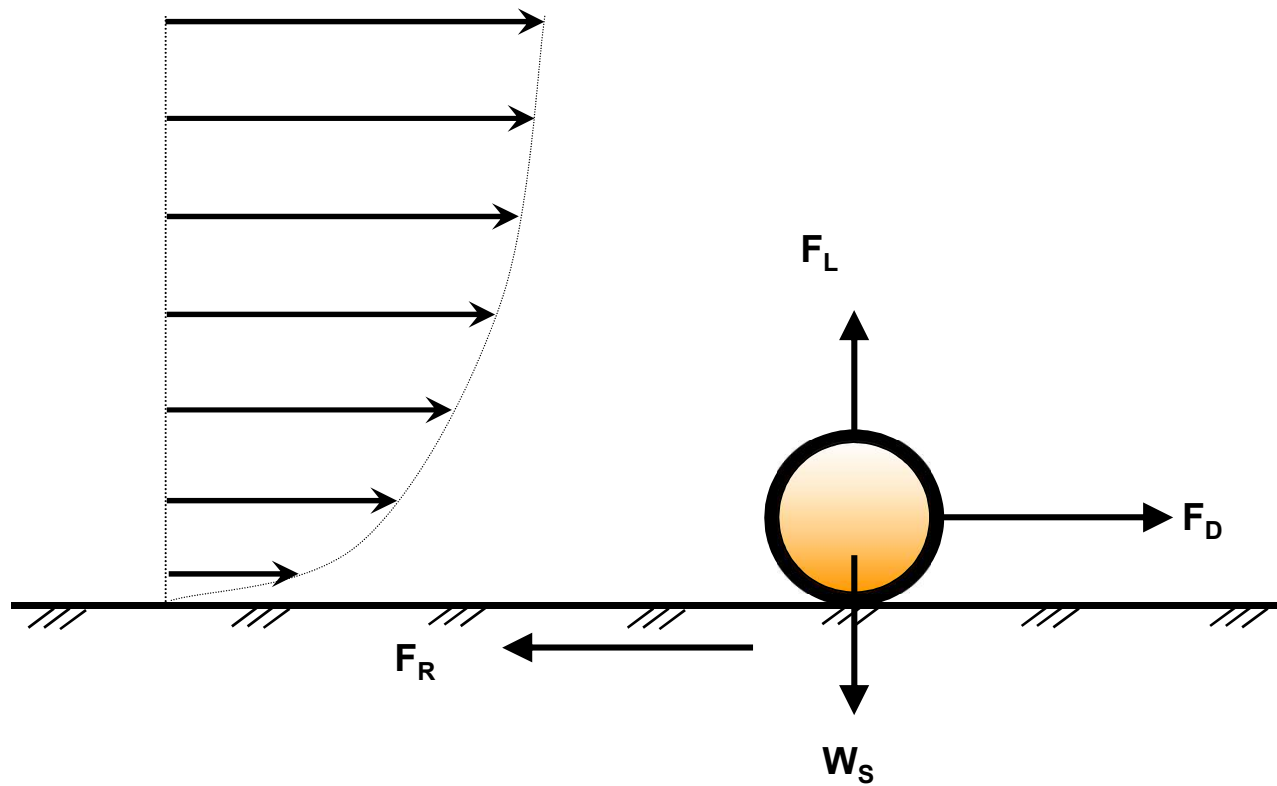
- **Stability design in accordance with DNV RP F109 “On-Bottom Stability Design of Submarine Pipelines”.**
- **Design Criteria**
 - Maximum allowable lateral displacement in operational condition:
 - ✓ Zone 1 10 Diameter
 - ✓ Zone 2 (safety zone) 1 Diameter
 - For installation condition a minimum specific gravity $(W_s+B)/B = 1.1$ is required (W_s = submerged weight, B =buoyancy)

Acceptance criteria – vertical stability

- **Vertical stability on and in soil**

- Sinking
 - ✓ waterfilled condition
- Floating (buried pipe)
 - ✓ Gasfilled condition

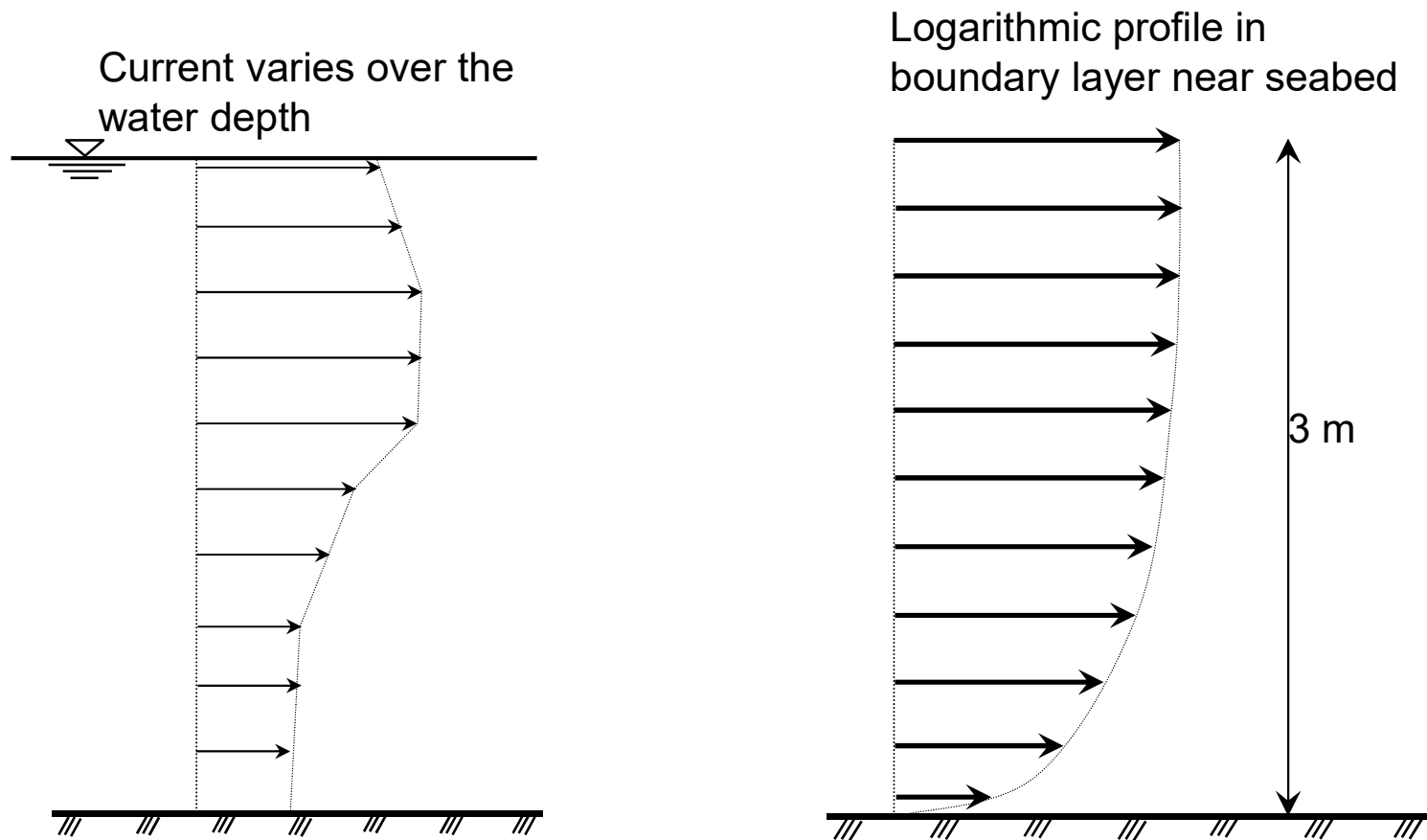
Pipeline stability – Physical model



Design current

- **Steady current will expose the pipe for a constant force**
 - Tidal current
 - Wind induce current
 - Storm surge induced current
 - Density driven current

Design current

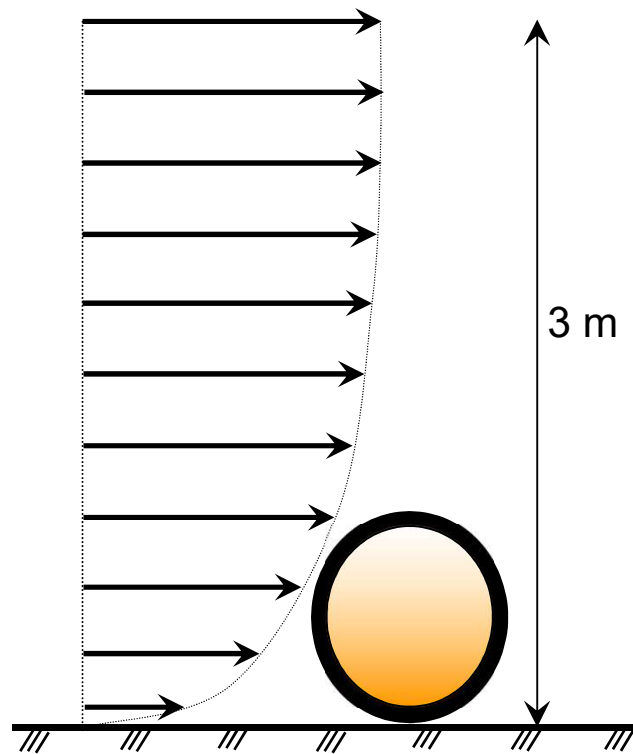


Logarithmic velocity profile

- **Average velocity acting over a pipe diameter**
- **Effect of bottom boundary layer**
 - Depends on soil properties, roughness of the surface of the seabed

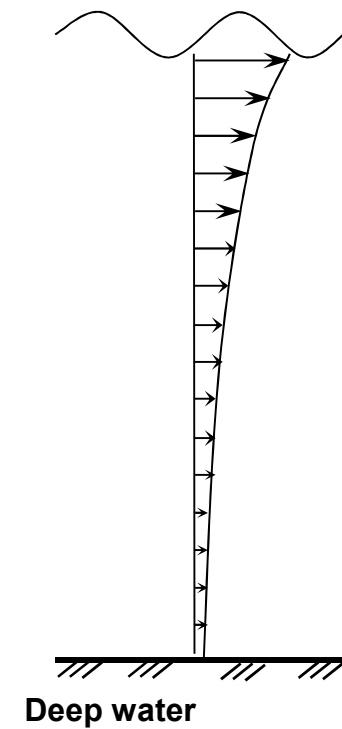
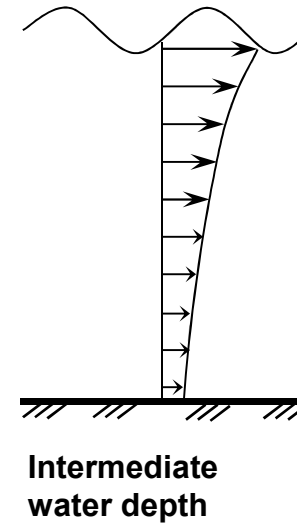
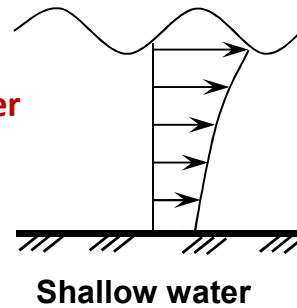
Design current

Logarithmic profile in
boundary layer near seabed

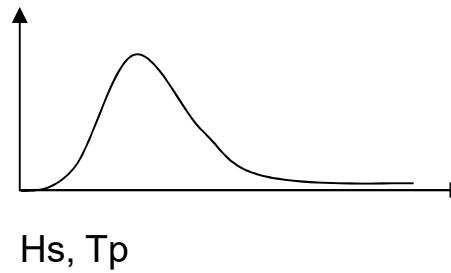
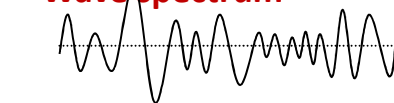


Design Waves

- Effect on pipelines depends on water depth



- Wave spectrum



Design currents and waves

- **Comination of current and waves**

- Operational conditions

- ✓ 100 year return period for waves combined with 10 years returnperiod for current
 - 10 year return period for waves combined with 100 years returnperiod for current

- Temporary conditions

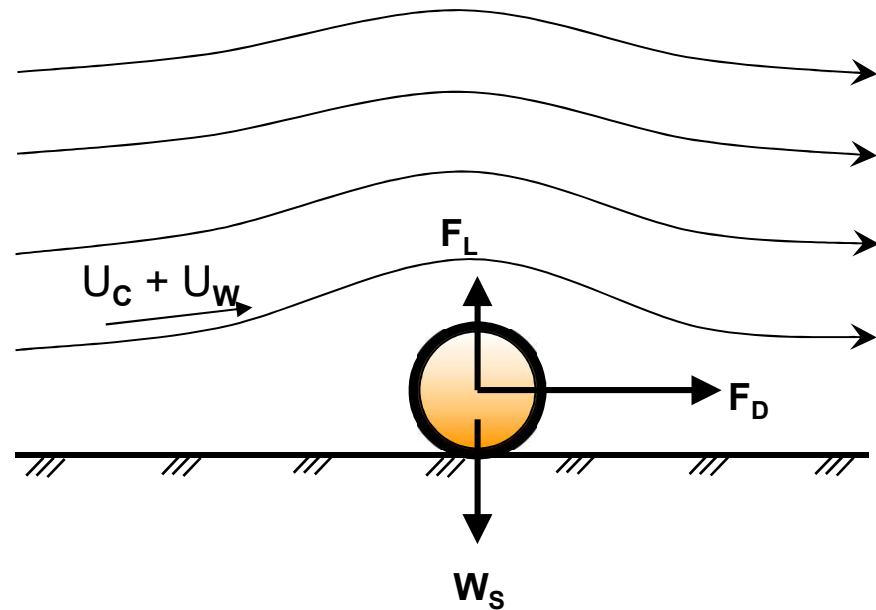
- ✓ 10 year return period for waves combined with 1 years returnperiod for current
 - 1 year return period for waves combined with 10 years returnperiod for current

Hydrodynamic forces

- Morison equation describes drag force F_D and lift force F_L in terms of the wave velocity U_w and current velocity U_c

$$F_D = \frac{1}{2} \rho D C_D (U_c + U_w) |U_c + U_w|$$

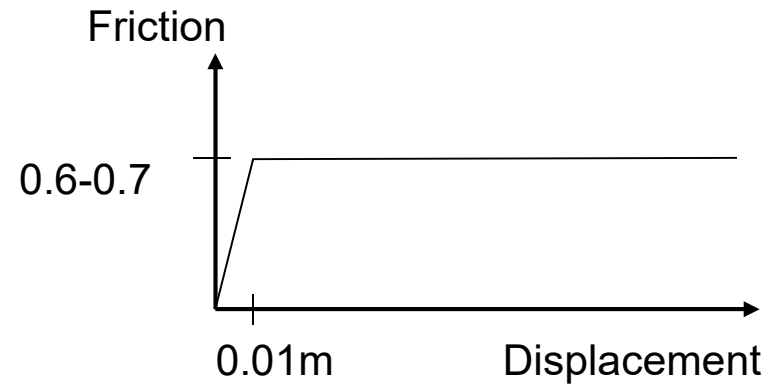
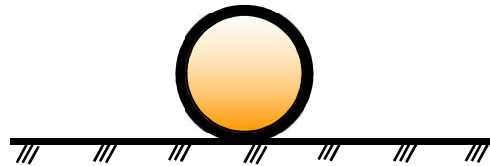
$$F_L = \frac{1}{2} \rho D C_L (U_c + U_w)^2$$



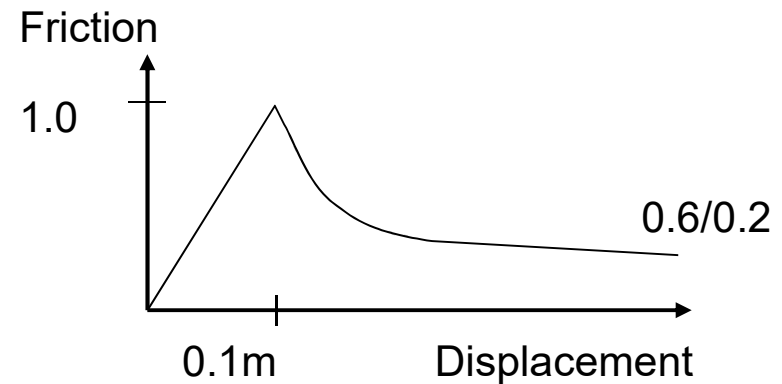
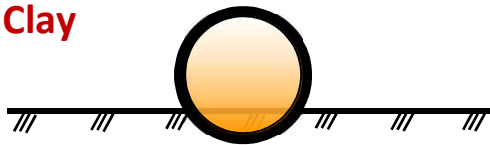
Lateral pipe/soil resistance

Soil types

- **Rock**



- **Sand / Clay**

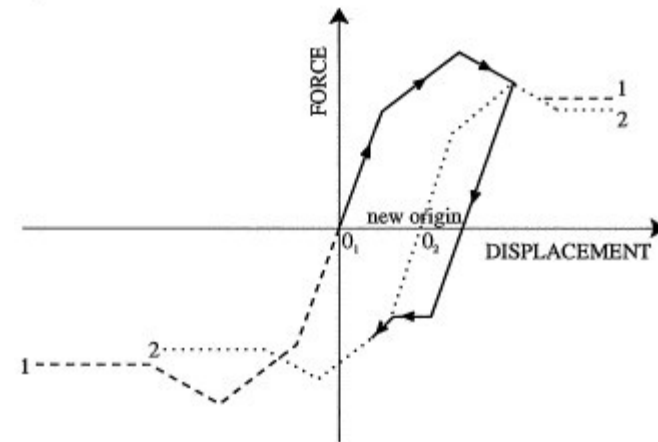
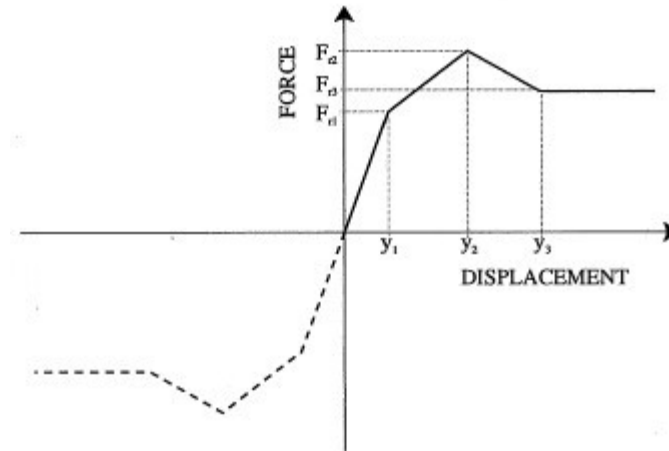


Lateral pipe/soil resistance

- Sand / Clay



- Resistance depends on penetration
- Time depended resistance



Stability criterion

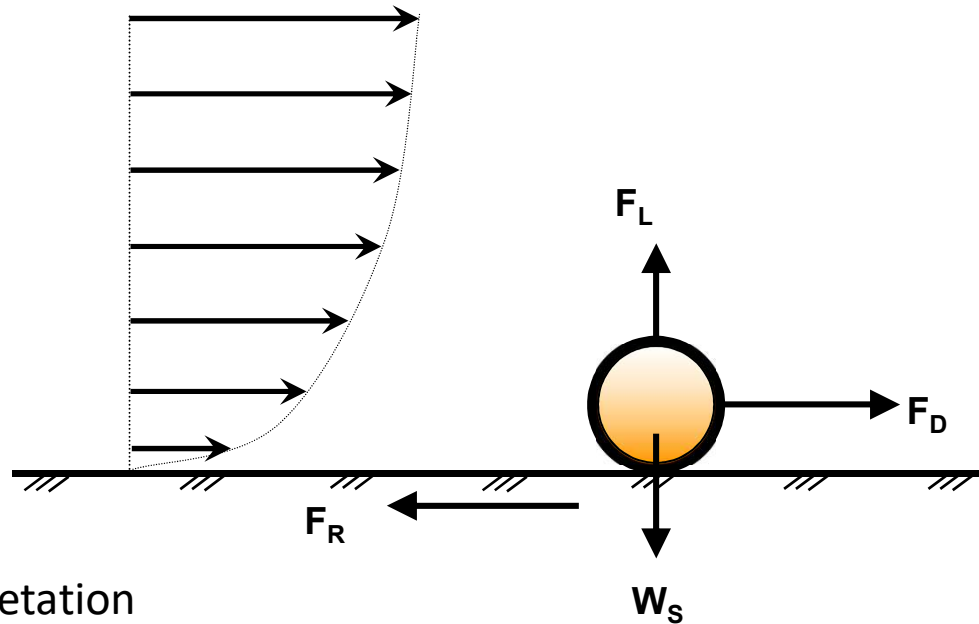
Lateral pipe/soil resistance:

$$F_R = \mu \cdot (W_S - F_L) + F_p$$

where

μ = friction coefficient

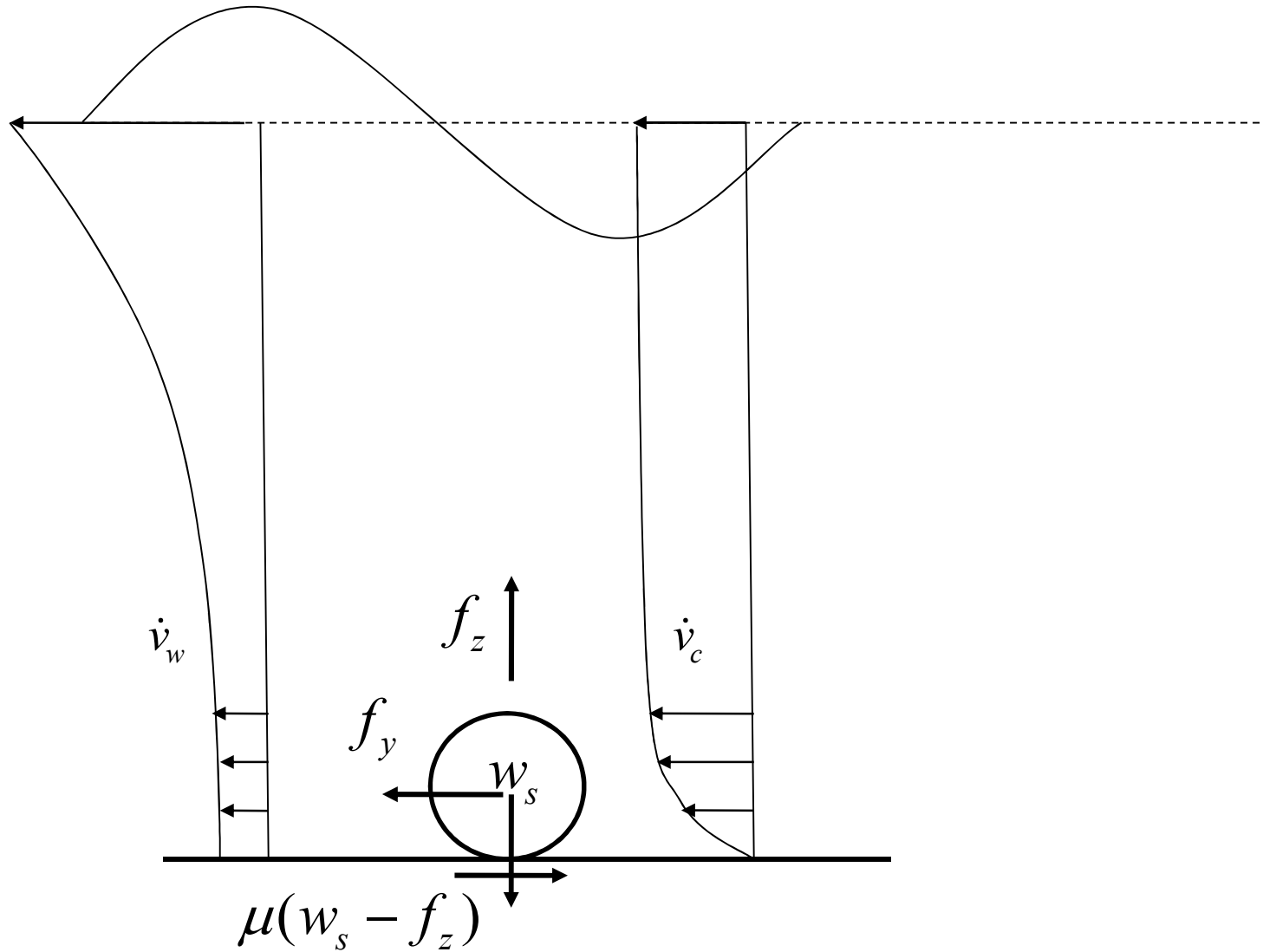
F_p = resistance due to penetration



The pipe is stable if:

$$F_R > F_D$$

Pipeline stability – simplified static model



Pipeline stability – Simple static model

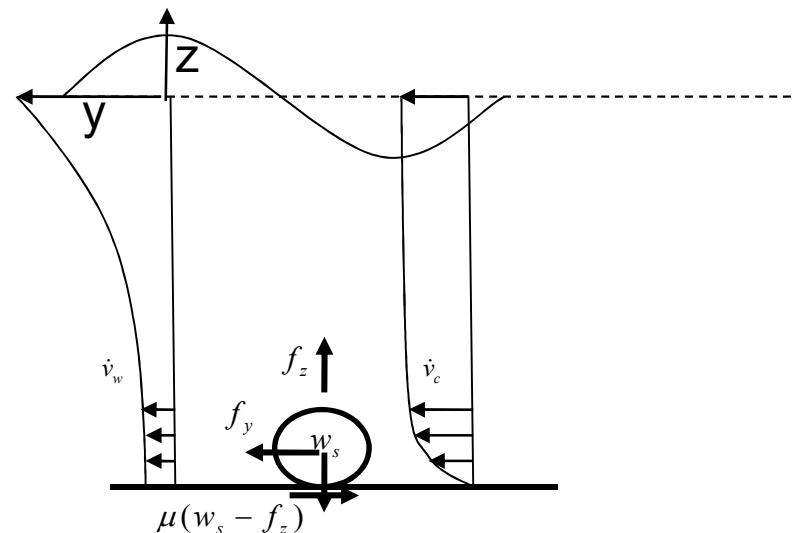
The Morison equation:

$$f_y = \frac{1}{2} \rho_w C_D D |\dot{v}_c + \dot{v}_{w0} \sin \omega t| (\dot{v}_c + \dot{v}_{w0} \sin \omega t) + \frac{\pi}{4} \rho_w D^2 C_M \ddot{v}_{w0} \cos \omega t$$

$$f_z = \frac{1}{2} \rho_w C_L D (\dot{v}_c + \dot{v}_{w0} \sin \omega t)^2$$

Equilibrium (Coulomb friction):

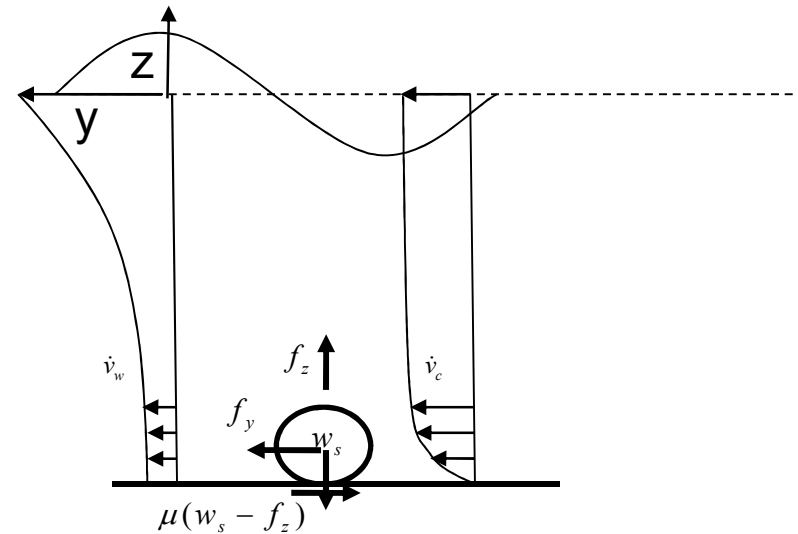
$$S_f f_y \leq \mu(w_s - f_z)$$



Pipeline stability – linear wave theory

- **How to find wave induced velocity:**
 - Linear wave theory
- **Linear wave theory assumptions:**
 - The water is incompressible and the density is constant in time and space
 - The motion of the water particle is rotation free so there exist a scalar function $\phi(y, z, t)$ from which the velocity components can be derived by differentiation:

$$v_y = \frac{\partial \phi}{\partial y} \quad v_z = \frac{\partial \phi}{\partial z} \Rightarrow \nabla^2 \phi = 0$$



Pipeline stability – linear wave theory

- There is no motion through the sea floor at $z = -d$ or ($z = -\infty$): $\frac{\partial \phi}{\partial z} = 0$

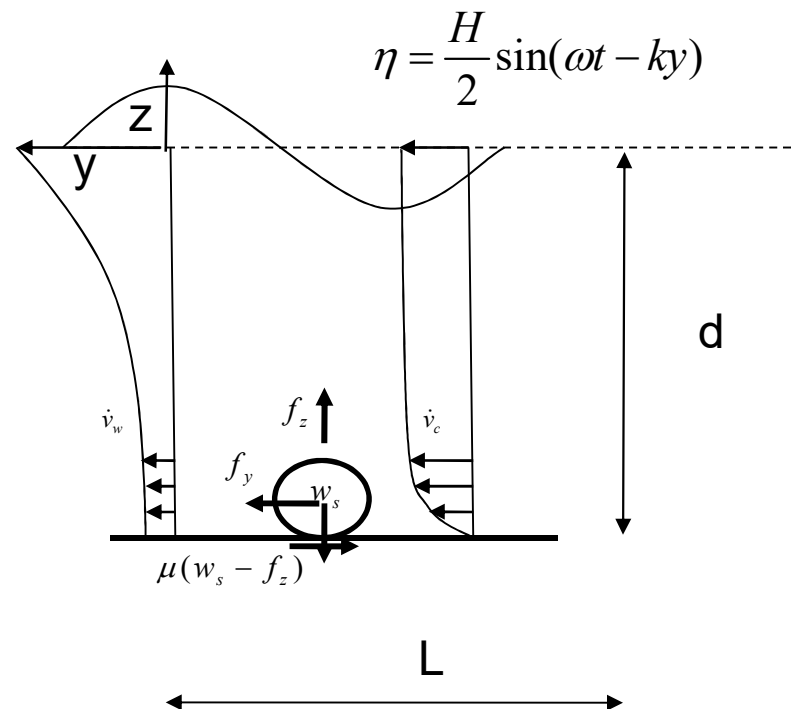
- One water particle that initially was positioned at the free surface will remain so:

$$\dot{w} = \frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial y} \dot{y}$$

- The pressure on the free surface is constant and equal to the atmospheric pressure using the Bernoulli equation:

$$g\eta + \frac{\partial \phi}{\partial t} + \frac{1}{2} \left(\left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right) = 0$$

- The wave elevation is small (compared to wave length): $\eta \ll d$



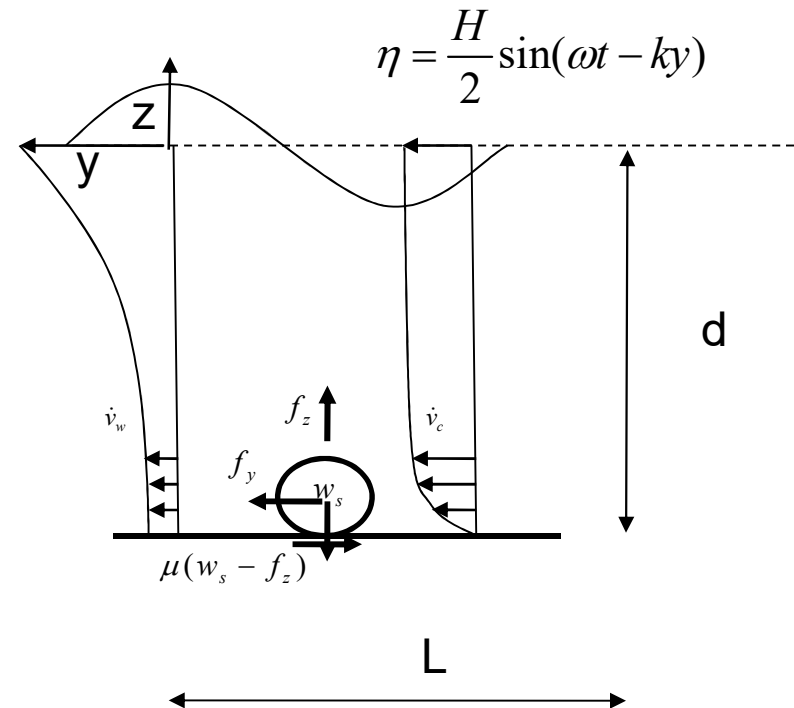
Pipeline stability – linear wave theory

- Three cases:
 - ✓ Shallow water $d/L < 1/20$
 - ✓ Finite water depth $1/20 < d/L < 1/2$
 - ✓ Deep water $d/L > 1/2$
- For deep water (T =wave period, H =wave height, L =wave length):

$$L = 1.56T^2$$

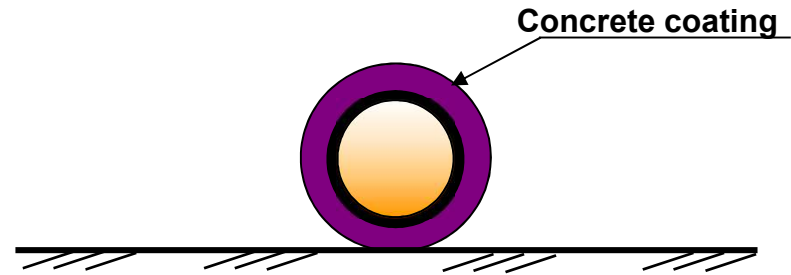
$$\dot{v} = \underbrace{\omega \frac{H}{2} e^{kz}}_{\dot{v}_{w0}} \sin(\omega t - ky), \quad \omega = \frac{2\pi}{T}, \quad k = \frac{2\pi}{L}$$

$$\ddot{v} = \underbrace{\omega^2 \frac{H}{2} e^{kz}}_{\ddot{v}_{w0}} \cos(\omega t - ky), \quad \omega = \frac{2\pi}{T}, \quad k = \frac{2\pi}{L}$$

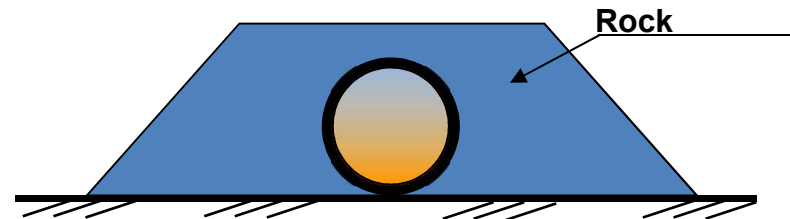


Methods for stabilisation

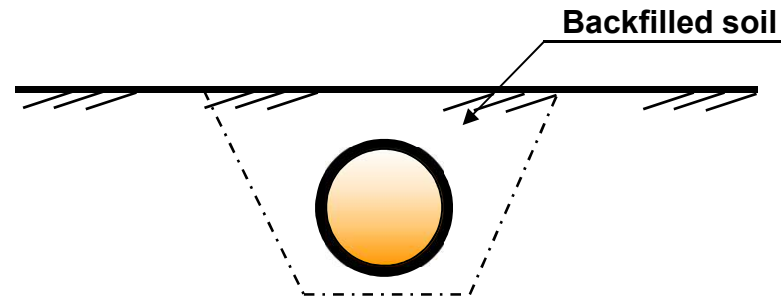
- **Pipe Weight**



- **Rock Dumping**



- **Trenching**



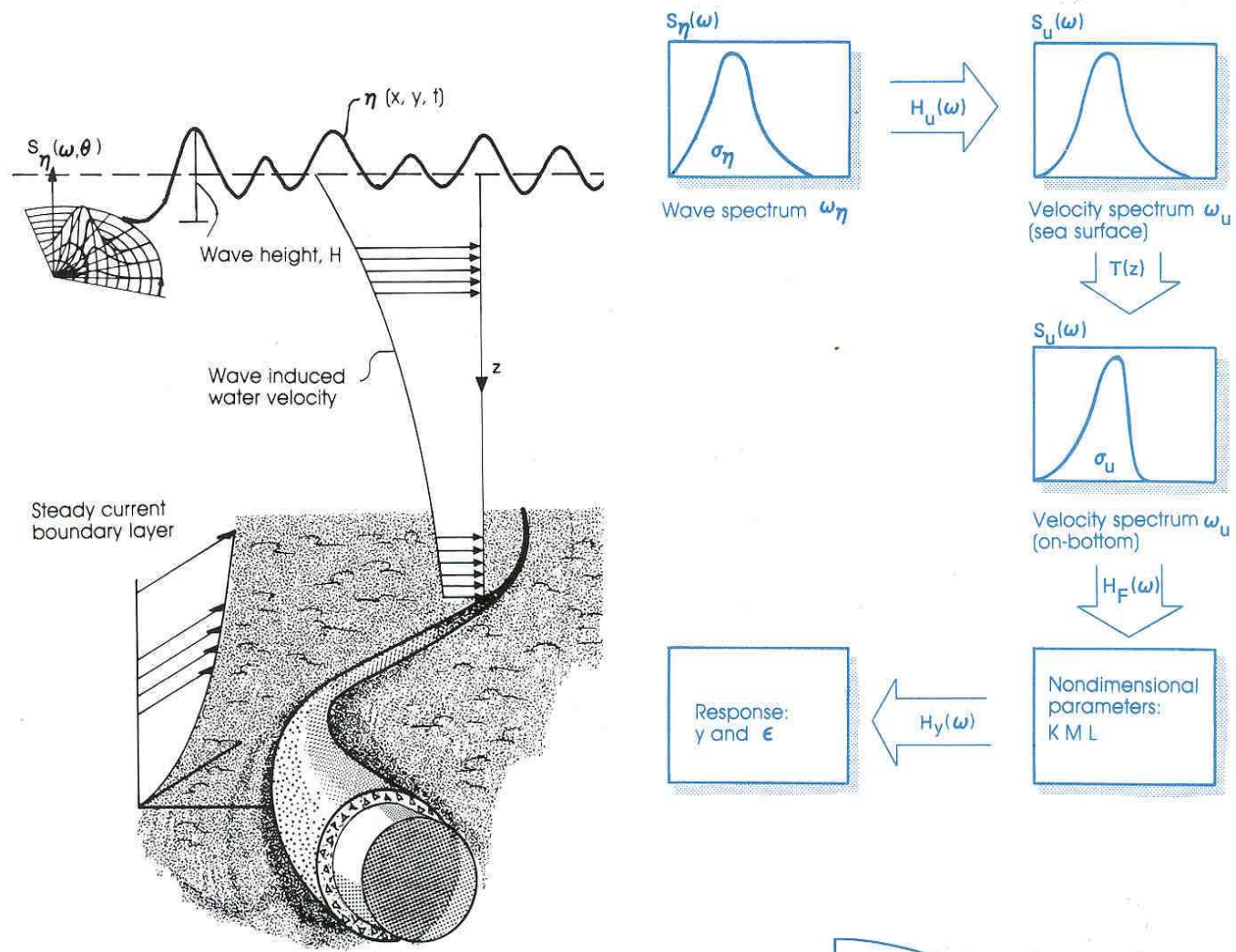
Methods for stabilisation



Improvement for on-bottom stability

- **Rock or concrete mattresses support c/c 500-1000m**
 - Depends on pipe geometry
 - Induce axial membrane forces due to lateral displacement
- **Trench**
 - Reduced loading
 - Increased resistance

Pipe, program overview



Pipeline stability – Environmental data (Example)

Determine weight coating requirements for a multiphase pipeline

Environmental data:

- **Wave data**

- 1 year return period $H_s = 8.8$ m $T_p = 13.8$ s
- 10 year return period $H_s = 10.7$ m $T_p = 14.9$ s
- 100 year return period $H_s = 12.5$ m $T_p = 15.9$ s

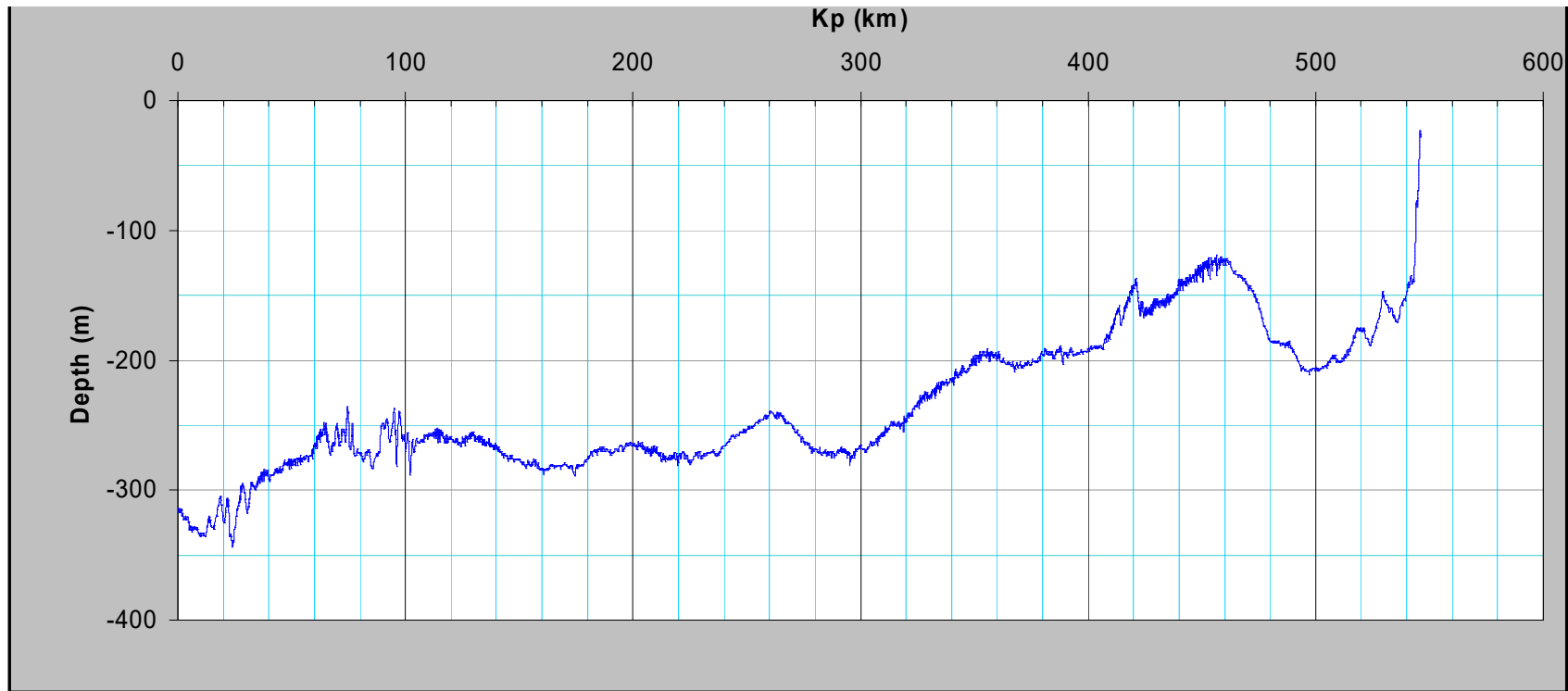
- **Current data**

- Installation condition $U_c = 0.39$ m/s
- Operation condition $U_c = 0.44$ m/s

Soil data:

- **Soft clay with undrained shear strength of 7 kPa**
- **Sand with density of 1900 kg/m³**

Pipeline stability – Profile (Example)



Pipeline stability – Coating design (Example)

32"-38" multiphase pipeline

Nominal size	KP		Concrete coating thickness (mm)	Concrete coating density (kg/m ³)	Submerged empty pipe weight (N/m)
	From (km)	To (km)			
32"	0	30	45	2250	2420
32"	30	175	45	2250	2110
34"	175	300	45	2250	2240
36"	300	425	45	2250	2530
38"	425	540	45	2250	2880
38"	540	550	55	2250	4690

Important parameters

- **Submerged weight**
 - Steel thickness
 - Polypropylene Coating (Insulation)
 - Concrete density

Important parameters

- **Hydrodynamic diameter**
 - Drag forces increases
 - Penetration reduces

Important parameters

- **Soil condition**

- Sandy soil
- Clayey soil
- Carbonate soils

- **Initial penetration**

- Laying
- Increase the soil-resistance

Important parameters

- **Directional environmental conditions**

- Reduce the loading

- **Gap**

- Drag forces will be reduced
- Lift force will be reduced
- Contact to the seabed increased

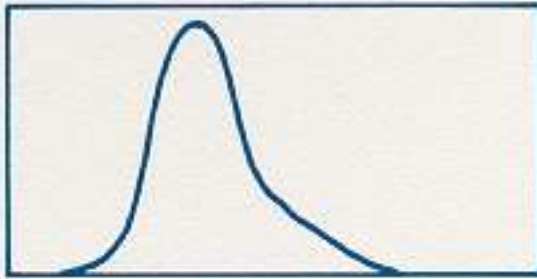
Software for on-bottom stability

- **PONDUS (time domain)**
- **PIPE (Generalized Method, data base)**
- **PRCI**
 - AGA, level I, II and III

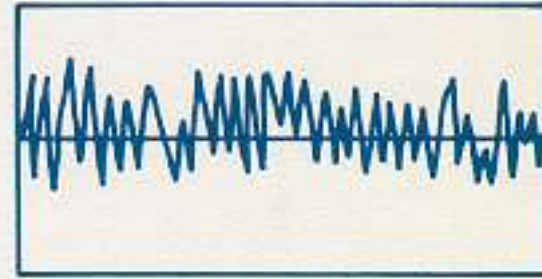
Time domain analysis

PONDUS program system overview:

$S_{\eta}(\omega)$

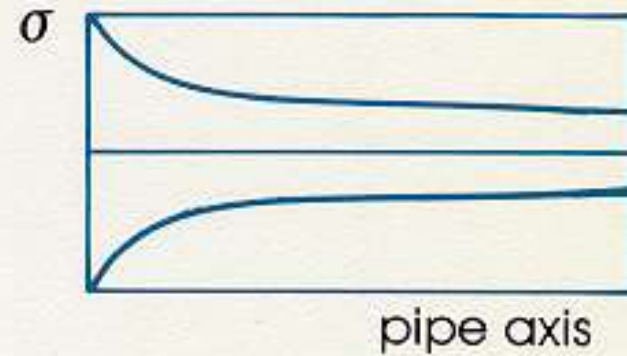
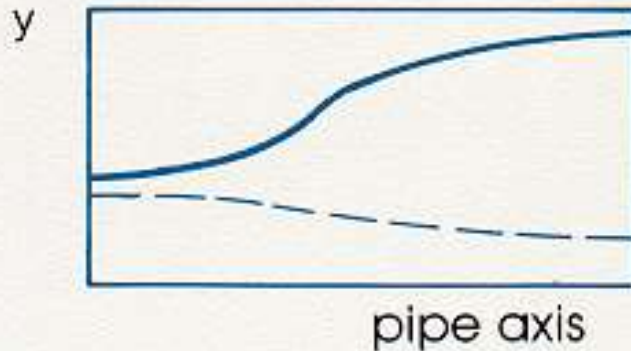


$U(t)$



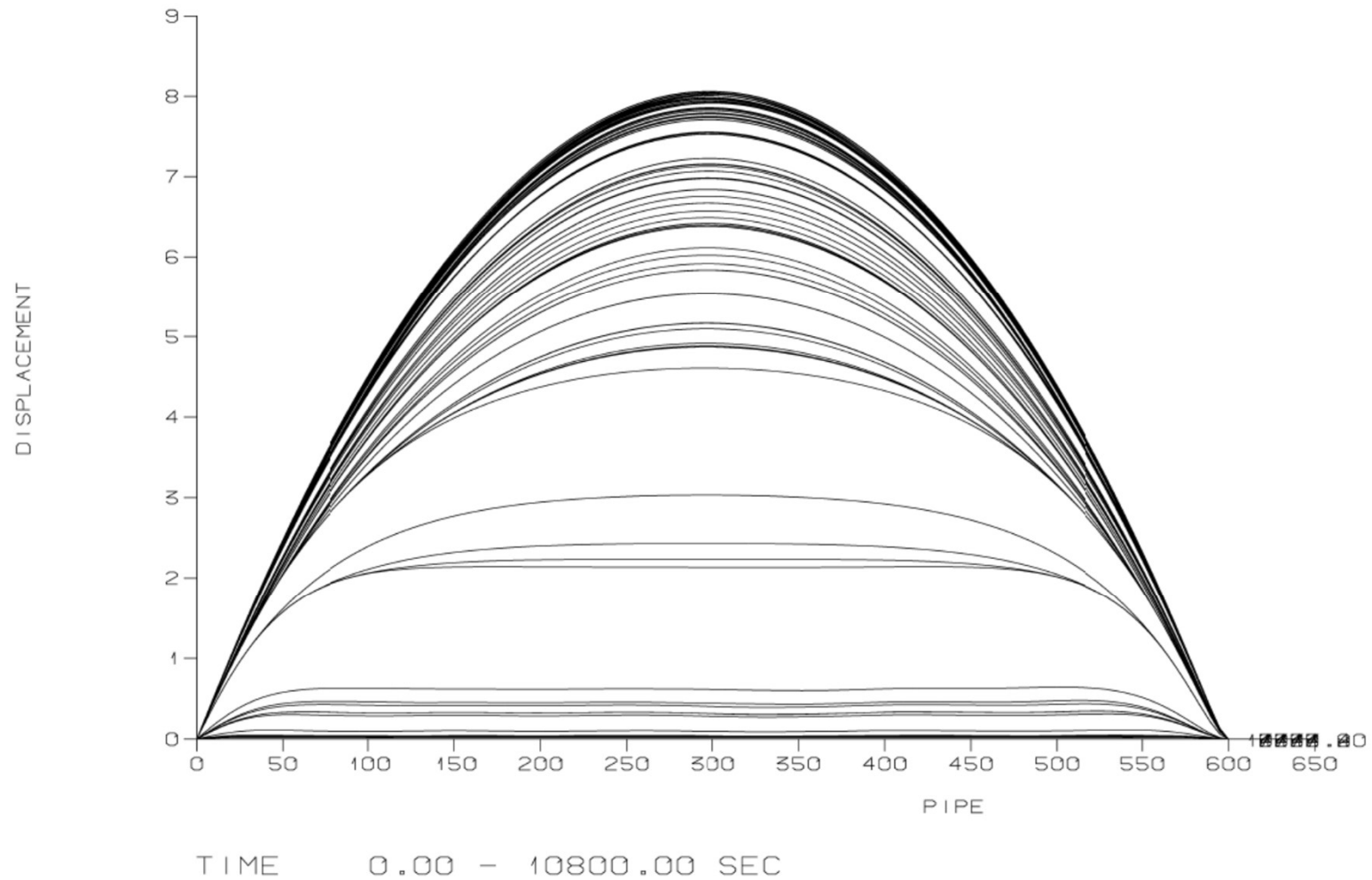
- transformation from wave elevation spectrum to timeseries of water velocities at specified grid points along the pipeline section
- preprocessing of time series
 - time dependent hydrodynamic force coefficient
 - space and timewise rearranging of timeseries

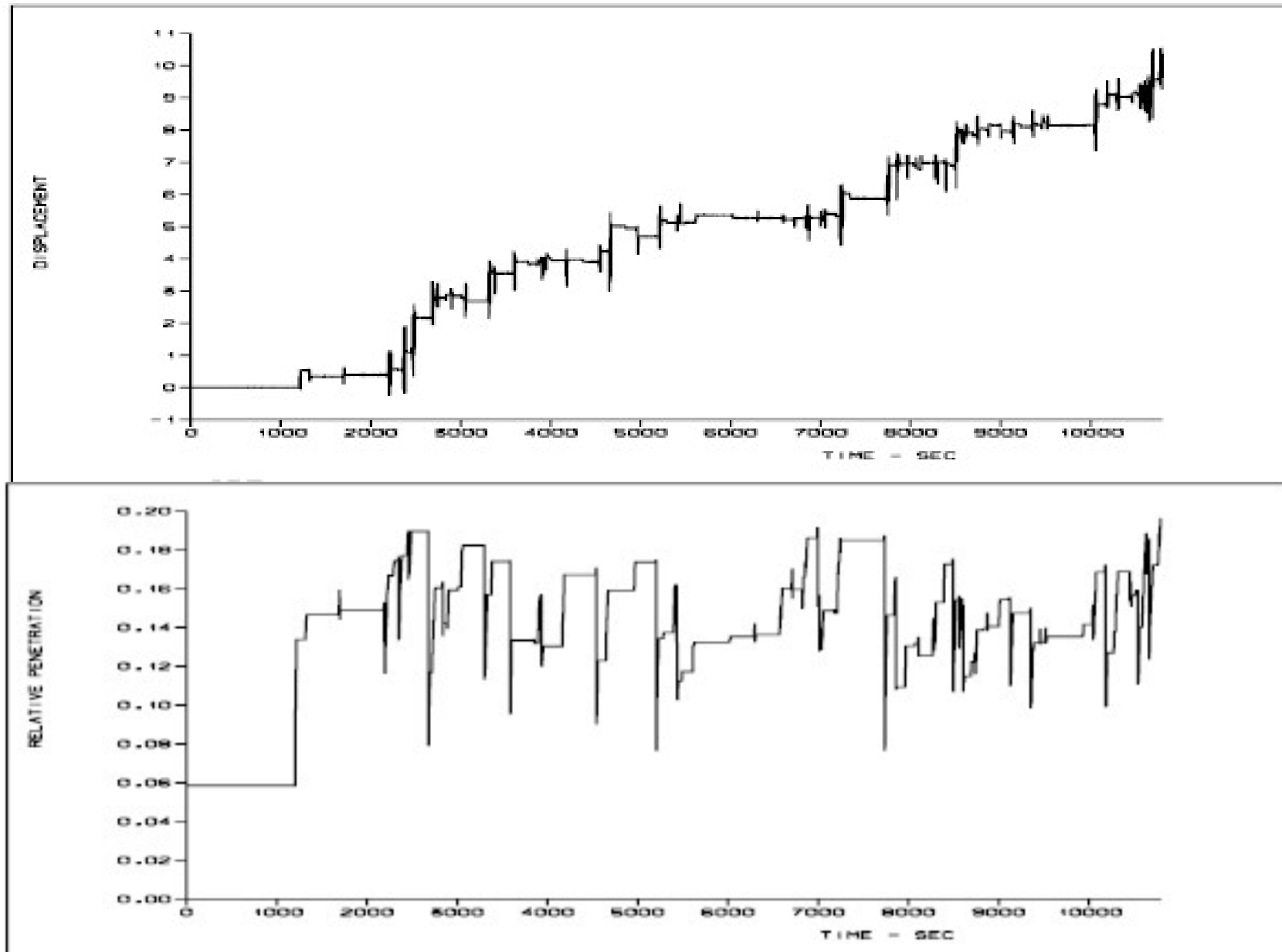
Time domain analysis



- dynamic response calculations (lateral direction)
- extreme response calculation (displacement/stress)
- simulation for a 3 hour stationary sea state
- nonlinear time dependent soil model
- nonlinear material models available

Example, timedomain analysis





Generalized method

- The equation of motion for a pipeline subjected to hydrodynamic forces and soil forces is to a large extent governed by a set of non-dimensional parameters.

$$\frac{y_0}{D} = f(K, L, M, N, G, \tau)$$

- The dimensionless equation shows that the scaled displacement, $y' = y/D$, will be the same for different pipeline/seastate cases, provided that the non-dimensional parameters are the same.

Non-Dimensional Parameters

- Significant Keulegan-Carpenter number
- Current to wave velocity ratio
- Acceleration parameter
- Pipe weight parameter
- Sand soil density parameter
- Clay soil parameter
- Shear strength parameter

$$K = \frac{U_s T_u}{D}$$

$$M = \frac{V}{U_s}$$

$$N = \frac{U_s}{g T_u}$$

$$L = \frac{W_s}{1/2 \rho D U_s^2}$$

$$G_s = \frac{\gamma_s}{\gamma_w}$$

$$G_c = \frac{S_u}{D \gamma_s}$$

$$S = \frac{W_s}{D S_u}$$

On-Bottom Stability

- **Calculates the necessary submerged weight for a pipeline to meet specified criteria**
- **Gives load-effect prediction for a given specified submerged weight (or concrete coating thickness), i.e. accumulation of displacements during a sea state with specified probability of occurrence.**

Static Stability

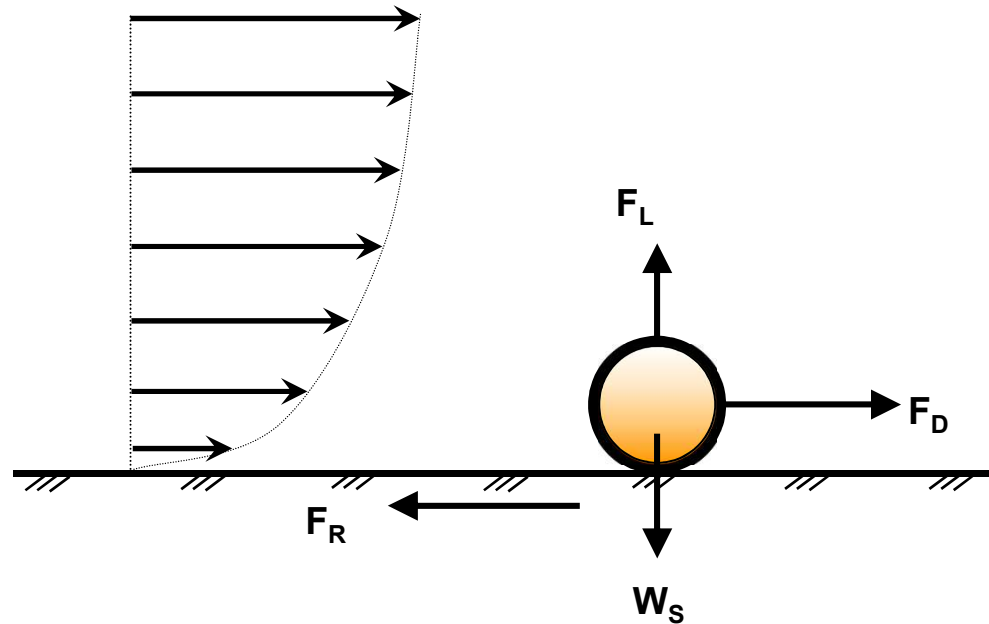
- **Design for no allowable movement (or very limited movement)**
- **A balance between external maximal horizontal hydrodynamic force and the soil resistance, including effects from lift forces.**
- **The soil resistance forces are based on the recent soil models and account for penetration effects.**
- **Two approaches:**
 - Find the necessary pipe weight to ensure stability for a given trench geometry or on bottom
 - Find the pipe penetration to ensure stability of a given pipe in a given trench or on bottom.

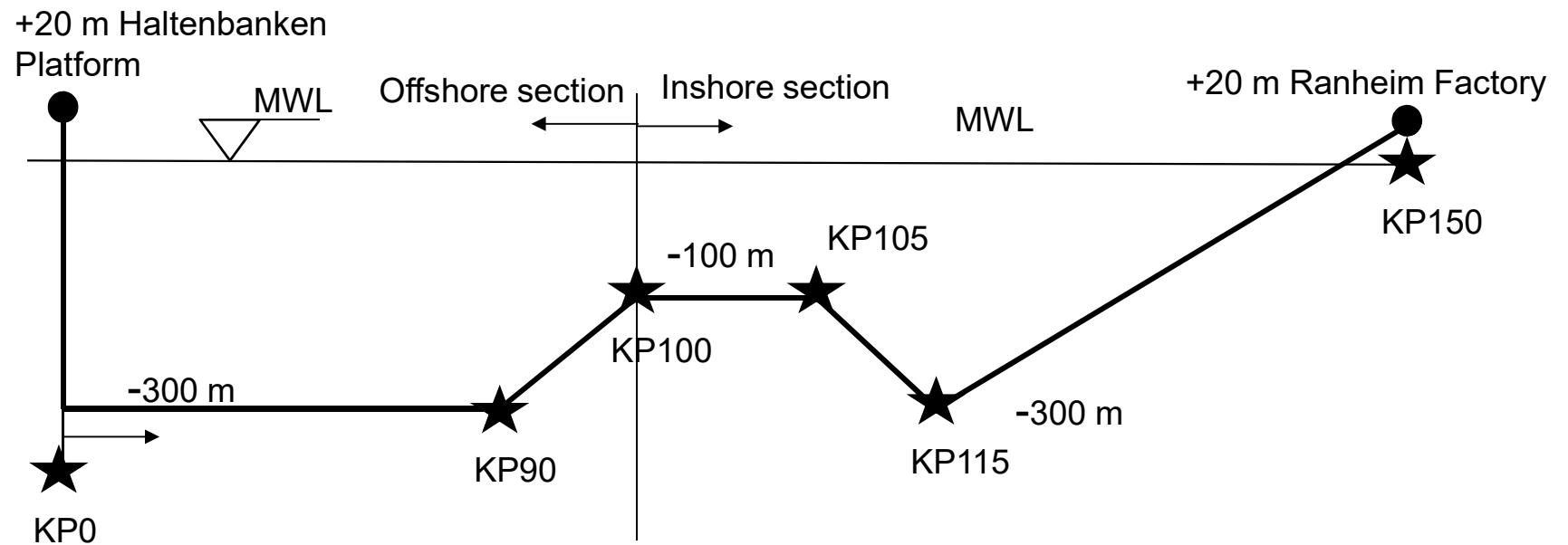
Final remarks

The pipe is stable if:

$$F_R > F_D$$

$$F_R = \mu \cdot (W_S - F_L) + F_p$$





DnV-RP-F109-On bottom stability Design of Submarine Pipelines OCTOBER 2010

On-bottom stability may follow one of three distinct approaches:

- 1) Ensuring absolute stability, Ref. Section 3.6: This approach is based on force equilibrium ensuring that the hydrodynamic loads are less than the soil resistance under a design extreme oscillatory cycle in the sea state considered for design.
- 2) Ensuring no break-out, Ref. Section 3.5. This approach allows some small displacements under the largest waves in a sea state. However, maximum displacement is small, less than about one half diameter which ensures that the pipe does not move out of its cavity, i.e. the pipe is virtually stable. This approach may take advantage of the build-up of passive resistance during the small displacements that the pipe will experience. There will be no accumulated displacement and maximum displacement can be considered to be independent of time.
- 3) Allowing accumulated displacement, Ref. Section 3.5. In this approach one specifies a certain, larger, allowable displacement during the sea state considered in design. The pipe will then several times during the sea state break out of its cavity and the calculated displacement should be assumed to be proportional with time, i.e. number of waves in the sea state considered. One should also in this context note that the displacement is an accumulated damage and that a sea state less severe than the one considered in design may also move the pipe, i.e. add to the damage.

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[Previous](#)[Next](#)

3. Design Methods

3.1 Introduction

The purpose of this section is to provide design methods and acceptance criteria for vertical and lateral stability of pipelines.

A design equation is presented for vertical stability, i.e. sinking, in sea water.

Design in order to ensure vertical stability of pipelines resting on the seabed or buried in soil is presented in general terms.

For lateral on-bottom stability, three design methods are presented in detail:

- dynamic lateral stability analysis
- a generalised lateral stability method based on data base results from dynamic analyses/simulations
- an absolute lateral static stability method.

The dynamic lateral stability analysis gives general requirements to a time domain simulation of pipe response, including hydrodynamic loads from an irregular sea-state and soil resistance forces.

The generalised lateral stability method and the absolute lateral static stability method give detailed specific design results for two approaches to stability design.

The generalised lateral stability method is based on an allowable displacement in a design spectrum of oscillatory wave-induced velocities perpendicular to the pipeline at the pipeline level. The design spectrum is characterised by spectrally derived characteristics U_s (oscillatory velocity), T_y (period) and the associated steady current velocity V . As a special case a “virtually stable” case is considered whereby the displacement is limited to about one half pipe diameter and is such that it does not reduce the soil resistance and the displacements do not increase no matter how long the sea-state is applied for.

The absolute lateral static stability method is a “design wave” approach, i.e. it ensures absolute static stability for a single design (extreme) wave-induced oscillation. The design oscillation is characterised by oscillatory velocity amplitude U^* and period T^* and the associated steady component V^* . Often $V^* = V$, however some hydrodynamic models account for a local mean velocity V^* within a wave-induced oscillation and this may be different to the overall mean velocity V .

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2. Design

2.1 Target failure probability

Excessive lateral displacement due to the action of hydrodynamic loads is considered to be a *serviceability limit state* SLS with the target safety levels given in DNV-OS-F101., Ref. /1/.

If this displacement leads to significant strains and stresses in the pipe itself, these load effects should be dealt with in accordance with e.g. DNV-OS-F101.

2.2 Load combinations

The characteristic load condition shall reflect the most probable extreme response over a specified design time period.

For permanent operational conditions and temporary phases with duration in excess of 12 months, a 100-year return period applies, i.e. the characteristic load condition is the load condition with 10^{-2} annual exceedance probability. When detailed information about the joint probability of waves and current is not available, this condition may be approximated by the most severe condition among the following two combinations:

- 1) The 100-year return condition for waves combined with the 10-year return condition for current.
- 2) The 10-year return condition for waves combined with the 100-year return condition for current.

For a temporary phase with duration less than 12 months but in excess of three days, a 10-year return period for the actual seasonal environmental condition applies. An approximation to this condition is to use the most severe condition among the following two combinations:

- 1) The seasonal 10-year return condition for waves combined with the seasonal 1-year return condition for seasonal current.
- 2) The seasonal 1-year return condition for waves combined with the seasonal 10-year return condition for current.

DNV-RP-F109– On-Bottom Stability Design of Submarine Pipelines

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The current velocity may be reduced to take account of the effect of the bottom boundary layer and directionality:

$$V(z) = V(z_r) \cdot \frac{\ln(z + z_0) - \ln z_0}{\ln(z_r + z_0) - \ln z_0} \cdot \sin \theta_c \quad (3.2)$$

Table 3-1 Seabed roughness		
<i>Seabed</i>	<i>Grain size d_{50} [mm]</i>	<i>Roughness z_0 [m]</i>
Silt and clay	0.0625	$\approx 5 \cdot 10^{-6}$
Fine sand	0.25	$\approx 1 \cdot 10^{-5}$
Medium sand	0.5	$\approx 4 \cdot 10^{-5}$
Coarse sand	1.0	$\approx 1 \cdot 10^{-4}$
Gravel	4.0	$\approx 3 \cdot 10^{-4}$
Pebble	25	$\approx 2 \cdot 10^{-3}$
Cobble	125	$\approx 1 \cdot 10^{-2}$
Boulder	500	$\approx 4 \cdot 10^{-2}$

For a clayey seabed the seabed roughness parameter of silt should be used.

The mean perpendicular current velocity over a pipe diameter applies:

$$V_c = V_c(z_r) \cdot \left(\frac{\left(1 + \frac{z_0}{D}\right) \cdot \ln\left(\frac{D}{z_0} + 1\right) - 1}{\ln\left(\frac{z_r}{z_0} + 1\right)} \right) \cdot \sin \theta_c \quad (3.3)$$

DNV-RP-F109– On-Bottom Stability Design of Submarine Pipelines

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$$S_{\eta\eta}(\omega) = \alpha \cdot g^2 \cdot \omega^{-5} \cdot \exp\left(-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right) \cdot \gamma^{\exp\left(-0.5\left(\frac{\omega-\omega_p}{\sigma \cdot \omega_p}\right)^2\right)} \quad (3.4)$$

The Generalised Phillips' constant is given by:

$$\alpha = \frac{5}{16} \cdot \frac{H_s^2 \cdot \omega_p^4}{g^2} \cdot (1 - 0.287 \cdot \ln \gamma) \quad (3.5)$$

The spectral width parameter is given by:

$$\sigma = \begin{cases} 0.07 & \text{if } \omega \leq \omega_p \\ 0.09 & \text{else} \end{cases} \quad (3.6)$$

DNV-RP-F109– On-Bottom Stability Design of Submarine Pipelines

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In lieu of other information, the peak-enhancement factor may be taken as:

$$\gamma = \begin{cases} 5.0 & \varphi \leq 3.6 \\ \exp(5.75 - 1.15\varphi) & 3.6 < \varphi < 5.0; \\ 1.0 & \varphi \geq 5.0 \end{cases} \quad \varphi = \frac{T_p}{\sqrt{H_s}} \quad (3.7)$$

The Pierson-Moskowitz spectrum appears for $\gamma = 1.0$.

The JONSWAP spectrum describes wind sea conditions that are reasonable for the most severe sea states. However, moderate and low sea states, not dominated by limited fetch, are often composed of both wind-sea and swell. A two peak (bi-modal) spectrum should be considered to account for swell if considered important. See e.g. Ref. /3/.

The wave induced velocity spectrum at the sea bed $S_{UU}(\omega)$ may be obtained through a spectral transformation of the waves at sea level using a first order wave theory:

$$S_{UU}(\omega) = G^2(\omega) \cdot S_{\eta\eta}(\omega) \quad (3.8)$$

The transfer function G transforms sea surface elevation to wave induced flow velocities at sea bed and is given by:

$$G(\omega) = \frac{\omega}{\sinh(k \cdot d)} \quad (3.9)$$

DnV-RP-F109-On bottom stability Design of Submarine Pipelines

OCTOBER 2010

where d is the water depth and k is the wave number established by iteration from the transcendent:

$$\frac{\omega^2}{g} = k \cdot \tanh(k \cdot d) \quad (3.10)$$

The spectral moments of order n is defined as:

$$M_n = \int_0^{\infty} \omega^n \cdot S_{UU}(\omega) d\omega \quad (3.11)$$

Significant flow velocity amplitude at pipe level is:

$$U_s = 2\sqrt{M_0} \quad (3.12)$$

It is not recommended to consider any boundary layer effect on the wave induced velocity.

Mean zero up-crossing period of oscillating flow at pipe level is:

$$T_u = 2\pi \sqrt{\frac{M_0}{M_2}} \quad (3.13)$$

Assuming linear wave theory, U_s may be taken from Figure 3-2 and T_u from Figure 3-3 in which:

$$T_n = \sqrt{\frac{d}{g}} \quad (3.14)$$

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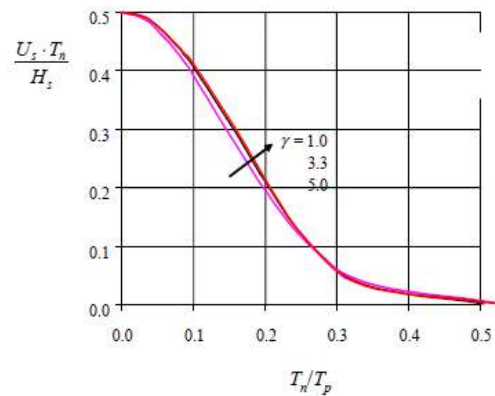


Figure 3-2
Significant flow velocity amplitude U_s at sea bed level

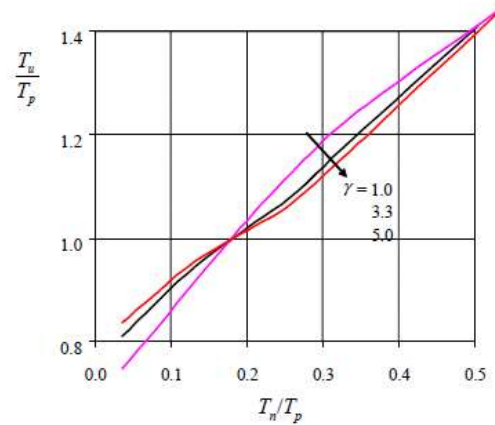


Figure 3-3
Mean zero up-crossing period of oscillating flow T_u at sea bed level

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The ratio between the design single oscillation velocity amplitude and the design spectral velocity at τ oscillations is:

$$k_U = \frac{U^*}{U_s} = \frac{1}{2} \cdot \left(\sqrt{2 \cdot \ln \tau} + \frac{0.5772}{\sqrt{2 \cdot \ln \tau}} \right) \quad (3.15)$$

Duration 3hrs:

$$\tau = \frac{3 \cdot 3600}{T^*}$$

The ratio between design single oscillation velocity period and the average zero up-crossing period (seabed level) is site specific. In absence of other data, this can be taken as:

$$k_T = \frac{T^*}{T_u} = \begin{cases} k_t - 5 \cdot (k_t - 1) \cdot T_n / T_u & \text{for } T_n / T_u \leq 0.2 \\ 1 & \text{for } T_n / T_u > 0.2 \end{cases}$$

$$k_t = \begin{cases} 1.25 & \text{for } \gamma = 1.0 \\ 1.21 & \text{for } \gamma = 3.3 \\ 1.17 & \text{for } \gamma = 5.0 \end{cases} \quad (3.16)$$

DnV-RP-F109-On bottom stability Design of Submarine Pipelines OCTOBER 2010

3.6.2 Design criterion

A pipeline can be considered to satisfy the absolute static stability requirement if:

$$\gamma_{SC} \cdot \frac{F_Y^* + \mu \cdot F_Z^*}{\mu \cdot w_s + F_R} \leq 1.0 \quad (3.38)$$

and

$$\gamma_{SC} \cdot \frac{F_Z^*}{w_s} \leq 1.0 \quad (3.39)$$

3.6.3 Safety factors

The safety factors γ_{SC} to be used for absolute stability in regular winter sea states are listed in Tables 3.5 and 3.6.

Table 3-5 Safety factors, winter storms in North Sea			
	<i>Low</i>	<i>Normal</i>	<i>High</i>
Sand and rock	0.98	1.32	1.67
Clay	1.00	1.40	1.83

Table 3-6 Safety factors, winter storms in Gulf of Mexico and Southern Ocean			
	<i>Low</i>	<i>Normal</i>	<i>High</i>
Sand and rock	0.95	1.41	1.99
Clay	0.97	1.50	2.16

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3.6.4 Loads

Peak horizontal and vertical loads are:

$$F_Y^* = r_{tot,y} \cdot \frac{1}{2} \cdot \rho_w \cdot D \cdot C_Y^* \cdot (U^* + V^*)^2 \quad (3.40)$$

$$F_Z^* = r_{tot,z} \cdot \frac{1}{2} \cdot \rho_w \cdot D \cdot C_Z^* \cdot (U^* + V^*)^2 \quad (3.41)$$

Maximum wave induced water particle velocity, including reduction due to directionality and spreading, U^* and T^* can be taken from Eqs. (3.15) and (3.16).

Current velocity, including reduction due to directionality and the boundary layer, V^* , can be taken from Section 3.4.2.

Peak load coefficients C_Y^* and C_Z^* are taken from Tables 3.9 and 3.10. Load reductions due to a permeable

DnV-RP-F109-On bottom stability Design of Submarine Pipelines OCTOBER 2010

The ratio between the design single oscillation velocity amplitude and the design spectral velocity amplitude for τ oscillations is:

$$k_U = \frac{U^*}{U_s} = \frac{1}{2} \cdot \left(\sqrt{2 \cdot \ln \tau} + \frac{0.5772}{\sqrt{2 \cdot \ln \tau}} \right) \quad (3.15)$$

Duration 3hrs:

$$\tau = \frac{3 \cdot 3600}{T^*}$$

The ratio between design single oscillation velocity period and the average zero up-crossing period (both at seabed level) is site specific. In absence of other data, this can be taken as:

$$k_T = \frac{T^*}{T_u} = \begin{cases} k_t - 5 \cdot (k_t - 1) \cdot T_n / T_u & \text{for } T_n / T_u \leq 0.2 \\ 1 & \text{for } T_n / T_u > 0.2 \end{cases}$$

$$k_t = \begin{cases} 1.25 & \text{for } \gamma = 1.0 \\ 1.21 & \text{for } \gamma = 3.3 \\ 1.17 & \text{for } \gamma = 5.0 \end{cases} \quad (3.16)$$