TMR 4585 Specialization Course UWT
Thermal expansion behaviour

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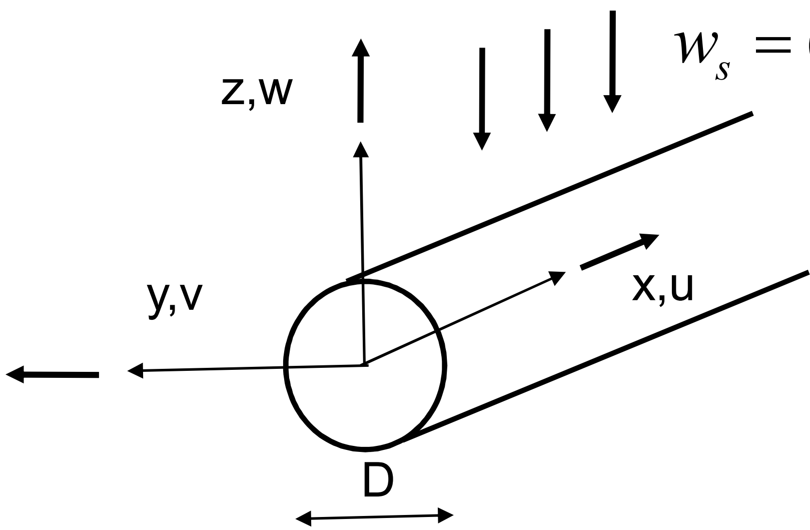
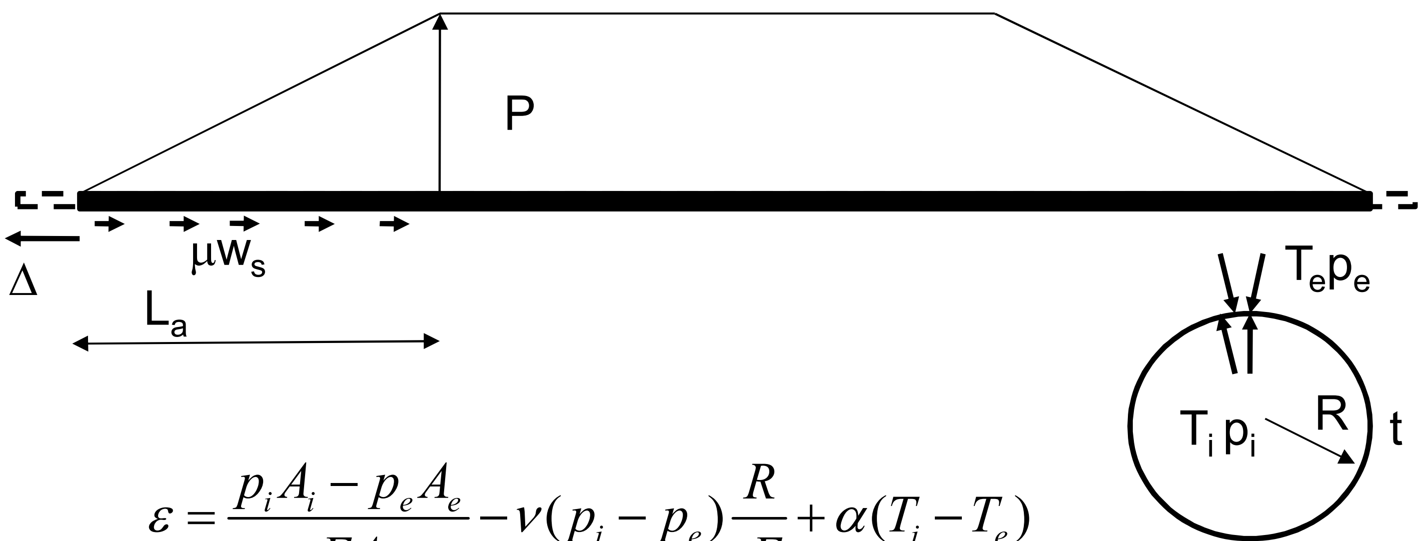
Pipeline thermal expansion
Coordinate system



$$w_s = (m_d g - \tfrac{\pi}{4}\rho_w g D^2)$$

Pipeline thermal expansion



$$\varepsilon = \frac{p_i A_i - p_e A_e}{EA} - \nu(p_i - p_e)\frac{R}{tE} + \alpha(T_i - T_e)$$

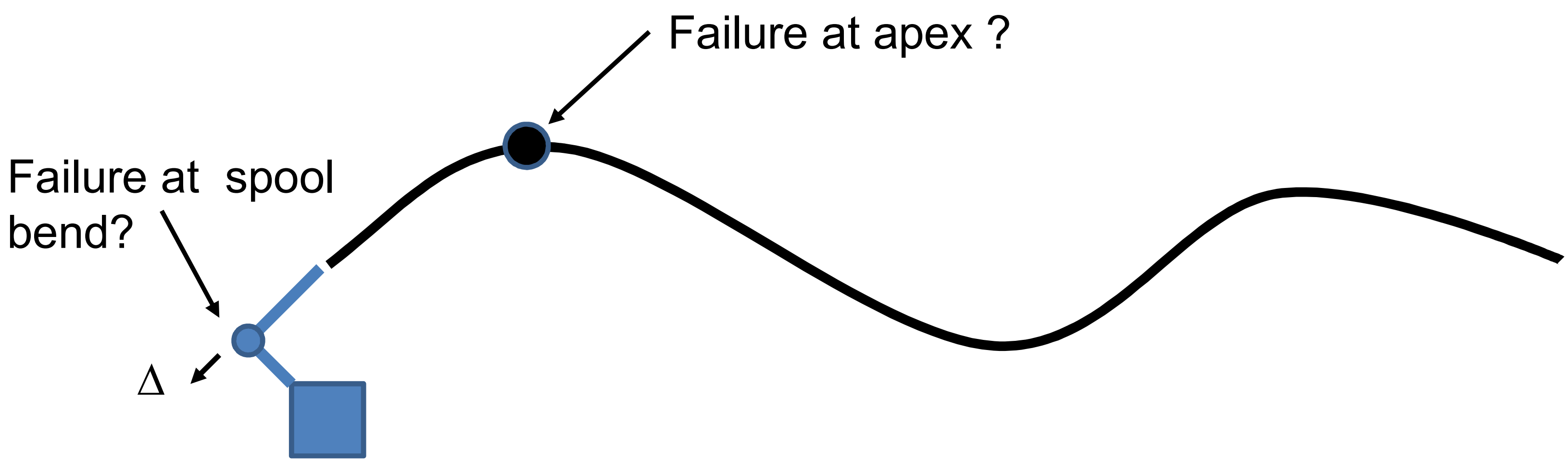
$$\varepsilon \cong \frac{\Delta p R}{2Et}(1-2\nu) + \alpha \Delta T$$

$$P \cong \Delta p \pi R^2 (1-2\nu) + 2\pi R t E \alpha \Delta T = \Delta p \pi R^2 (1-2\nu) + EA\alpha\Delta T$$

$$L_a = \frac{P}{\mu w_s} \qquad \Delta = \int_0^{L_a} \left(\frac{P}{EA} - \frac{\mu w_s x}{EA}\right)dx = \frac{Pl_a}{2EA}$$

What are the governing failure modes and critical sections?

- **Failure mode concerns at inline buckle apex and at expansion spool bends:**
  - ➢ Local buckling due to bending and external/internal pressure
  - ➢ Ovalization
  - ➢ Fracture
  - ➢ Low Cycle Fatigue

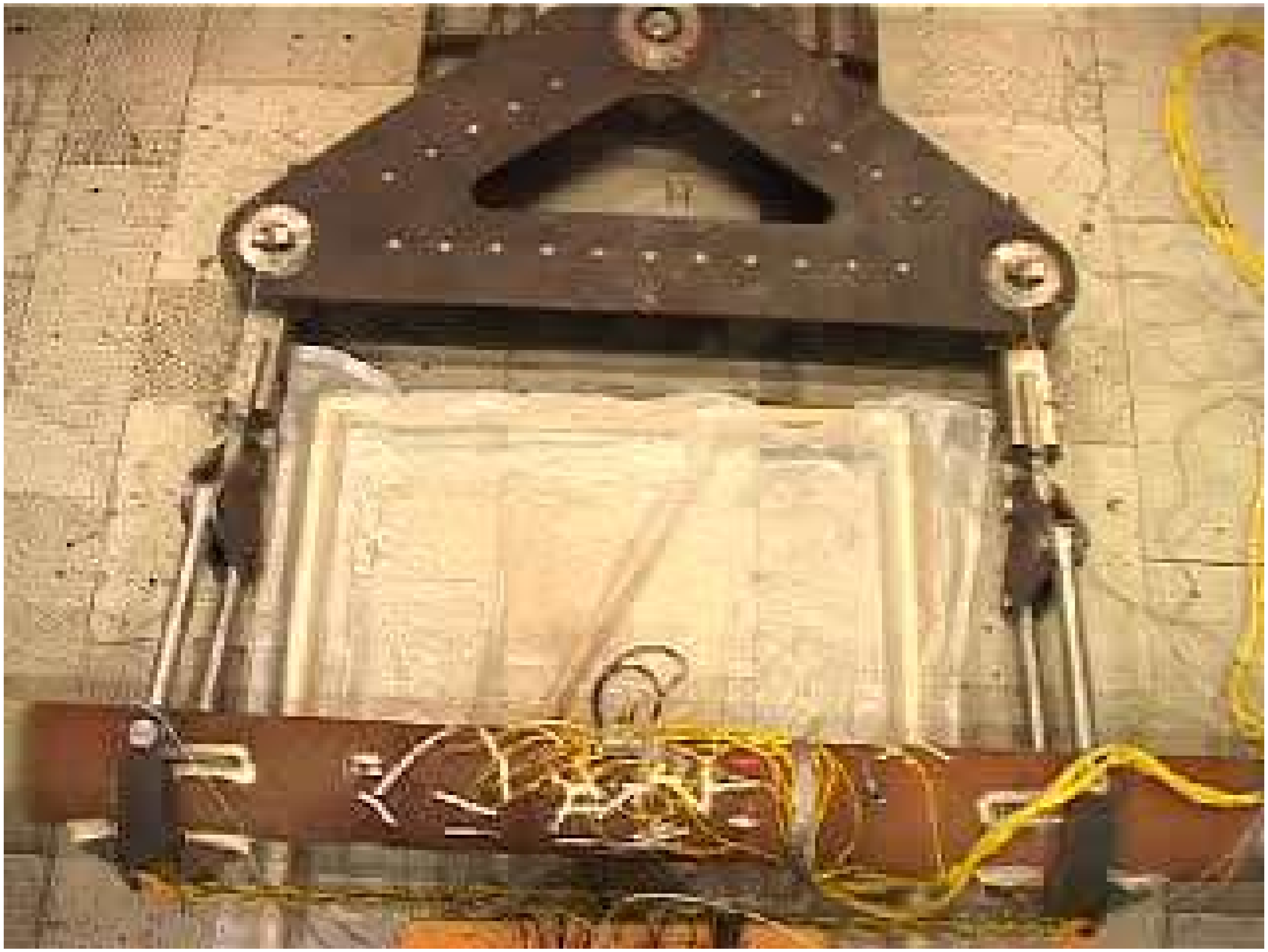


Bending induced local buckling at apex

Ceetron 3D Plugin

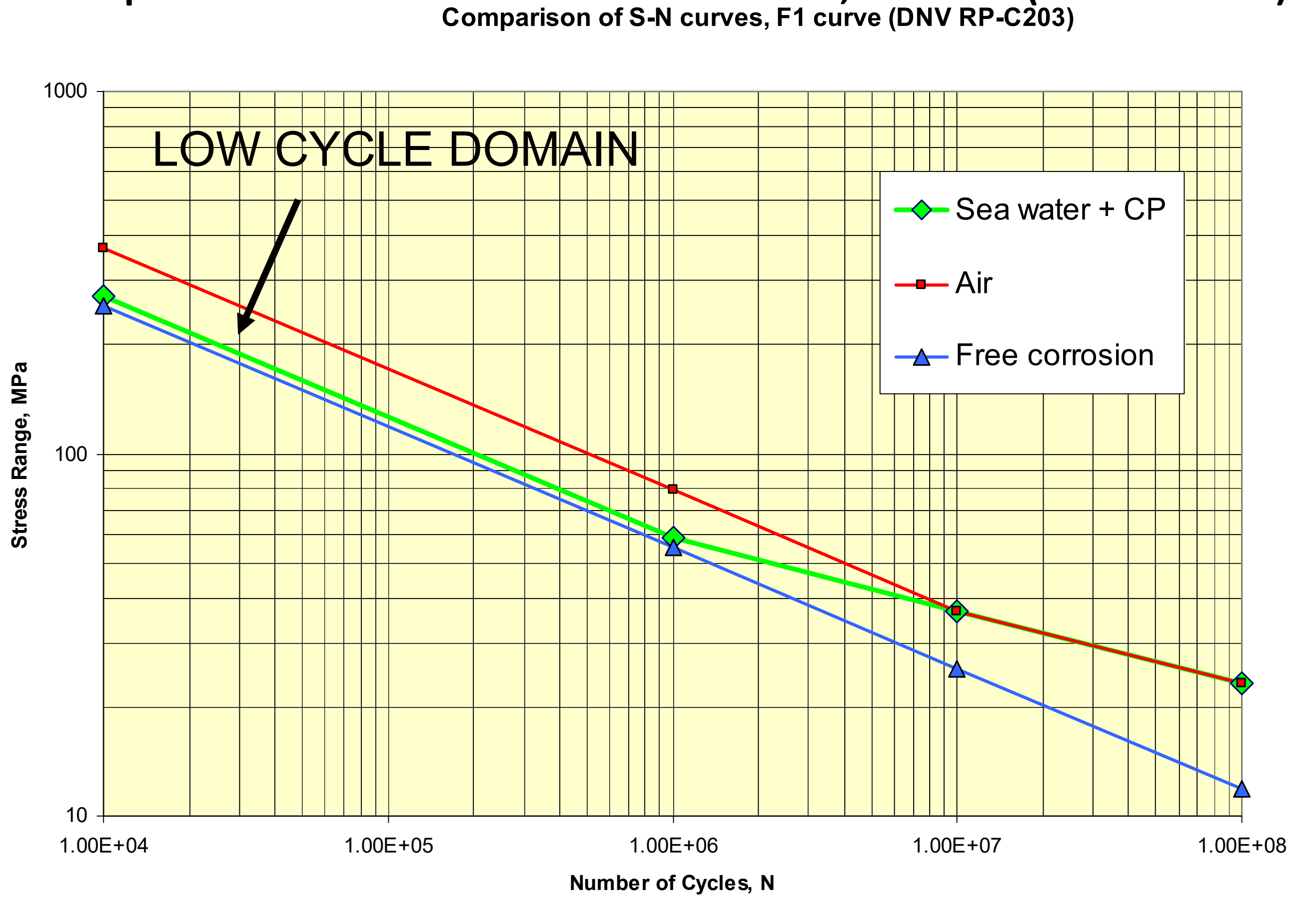
[bendingbuckling.vtf]

Fracture due to bending moment at buckling apex



Fatigue due to cyclic bending moment at buckling apex (cyclic shut-down and start-up)

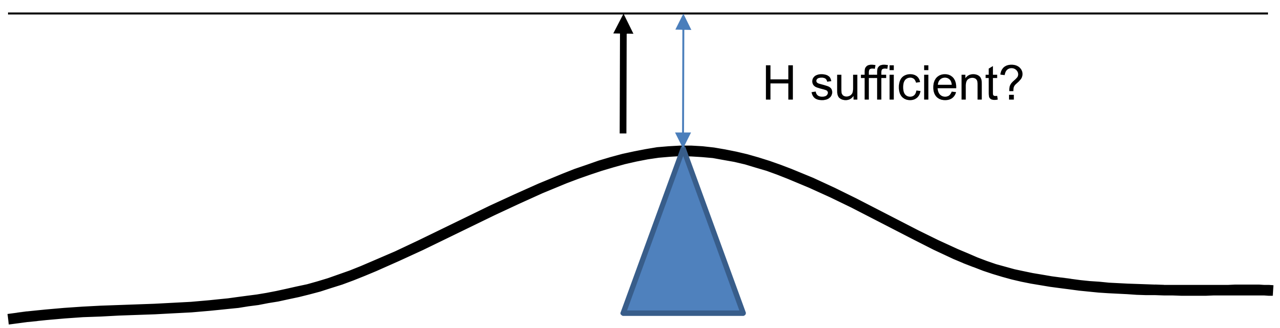
Comparison between different environments, Curve F1 (DNV RP-C203)



Pipeline global buckling

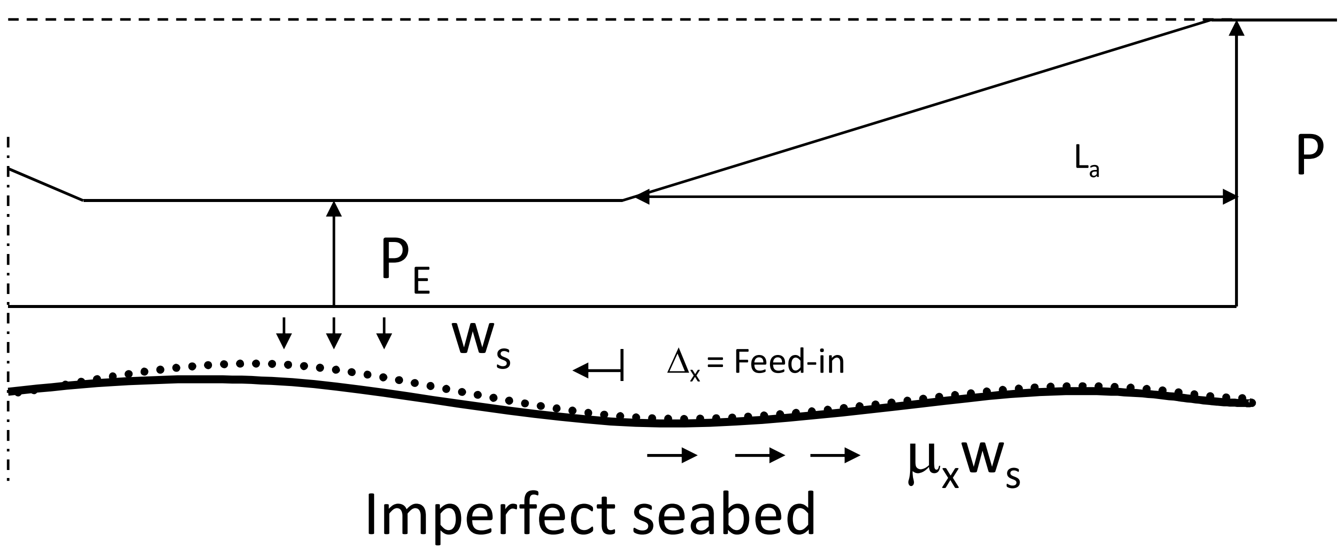
- Pipelines resting exposed on seabed – often referred to as ”snaking”

- Buried pipelines – buckling in vertical plane often referred to as ”upheaval buckling”



Pipeline global buckling – exposed pipelines (no burial)

- **Pipelines resting on seabed and exposed to high temperatures and pressures may buckle as a bar (Euler Buckling)**
- **Due to seabed irregularities the buckling may be localized at the point of max imperfections:**
  - The pipe first undergoes uplift at crown of imperfection
  - Having lost contact at the crown of imperfection the pipe will then buckle laterally
- **To avoid excessive strains it may be necessary to perform rock installation to increase axial friction and reduce feed-in**



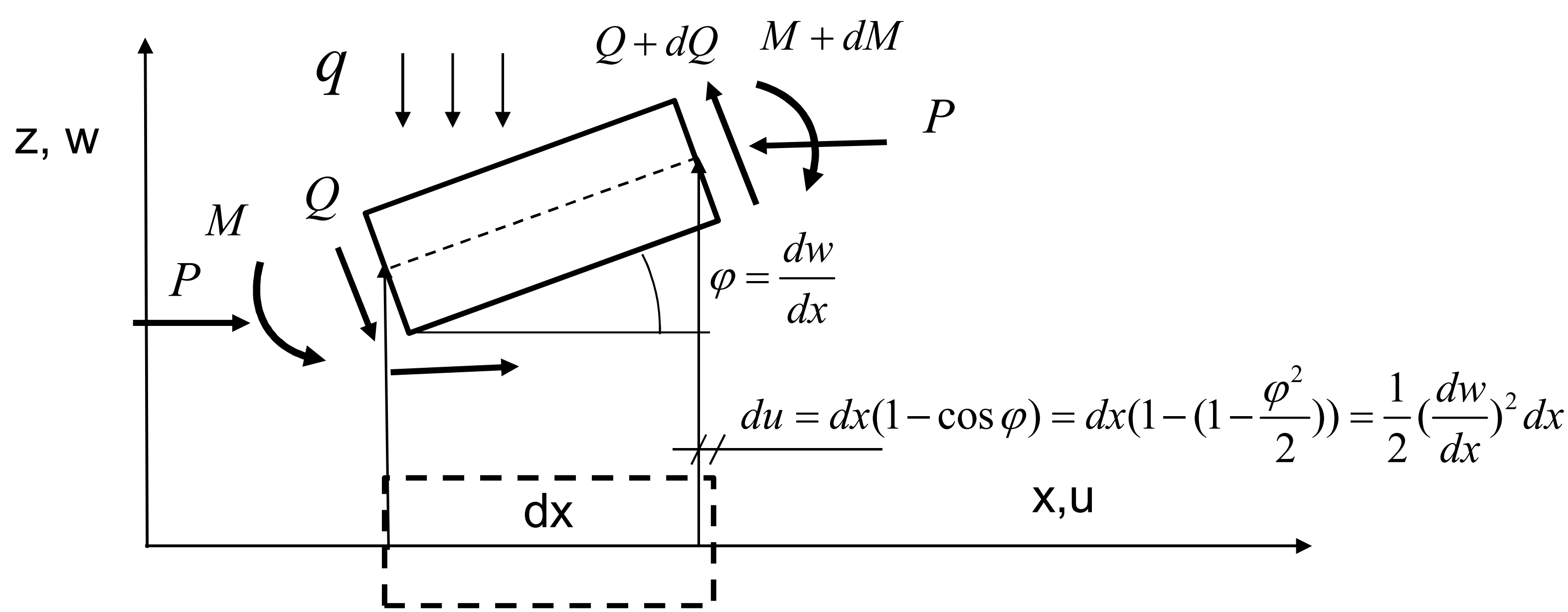
Pipeline global buckling- by FEM

Ceetron 3D Plugin

[roughsnaking.vtf]

Pipeline global buckling – analytical calculation

Differential equation of an Euler Bernoulli beam exposed to transverse load and an axial compressive force:



Pipeline global buckling – analytical calculation

Differential equation of an Euler Bernoulli beam exposed to transverse load and an axial compressive force based on equilibrium in moment and in z-direction:

$$\frac{d^4 w}{dx^4} + k^2 \frac{dw^2}{dx^2} = \frac{q}{EI} \qquad (M = -EI\frac{d^2 w}{dx^2} \qquad k^2 = \frac{P}{EI})$$

$$Q = P\frac{dw}{dx} - \frac{dM}{dx} = P\frac{dw}{dx} + EI\frac{dw^3}{dx^3}$$

$$w = w_h + w_p = C_1 + C_2 x + C_3 \cos kx + C_4 \sin kx + \frac{qx^2}{2P}$$

$$\Delta_x = \tfrac{1}{2}\int_0^l \left(\frac{dw}{dx}\right)^2 dx$$

Pipeline global buckling – analytical calculation

Pipeline resting on uneven seabed – uplift phase

Boundary conditions:

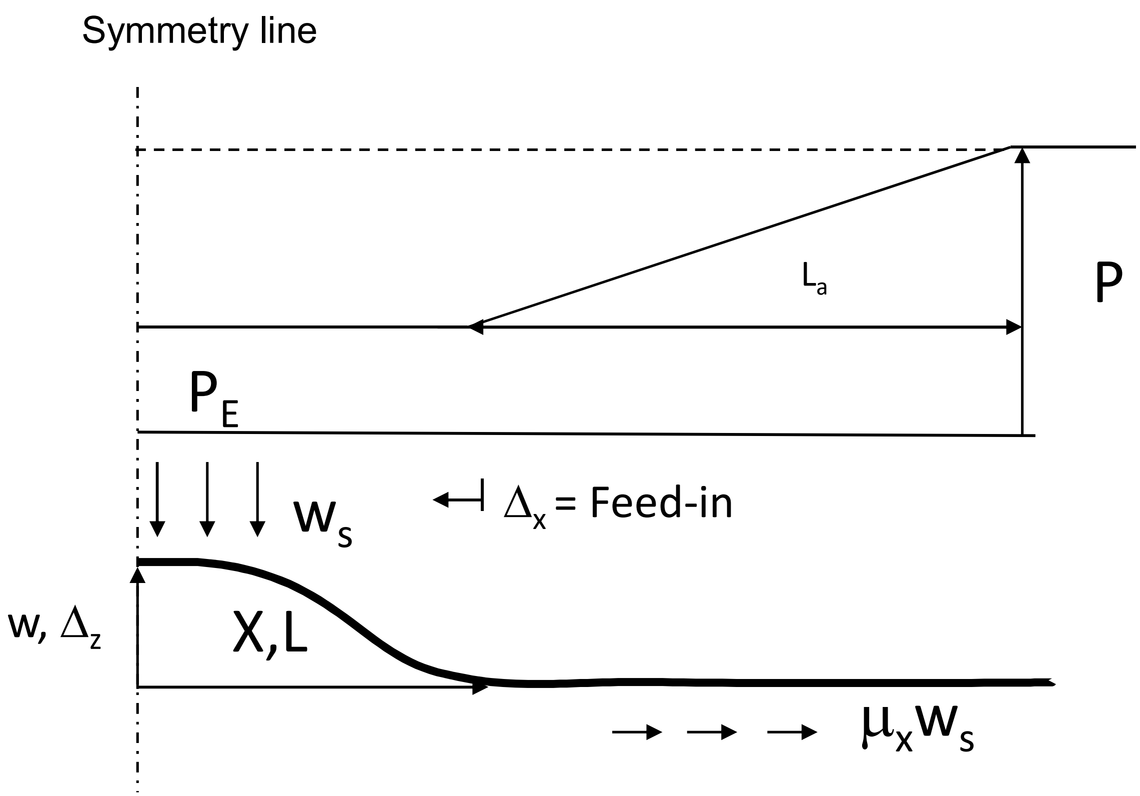
$$1: \left.\frac{dw}{dx}\right|_{x=0} = 0$$

$$2: \left.\frac{dw}{dx}\right|_{x=l} = 0$$

$$3: \left.w\right|_{x=l} = 0$$

$$4: \left.\frac{d^2w}{dx^2}\right|_{x=l} = 0$$

$$5: \left.Q\right|_{x=0} = 0$$



Pipeline global buckling – analytical calculation

Pipeline resting on uneven seabed – uplift phase:

Results:

$$1: \Delta_z = 15.7 \frac{w_s EI}{P_E{}^2} = 0.0385 \frac{w_s L^4}{EI}$$

$$2: P_E = 20.25 \frac{EI}{l^2} = 3.98 \sqrt{\frac{w_s EI}{\Delta_z}}$$

$$3: M_y \big|_{x=0} = M_{\max} = 5.6 \frac{w_s EI}{P_E}$$

$$4: Q_z \big|_{x=l} = ql$$

$$5: \Delta_x = 37.4 \frac{w_s^2 EI^{\frac{3}{2}}}{P_E^{\frac{7}{2}}}$$

Pipeline global buckling – analytical calculation

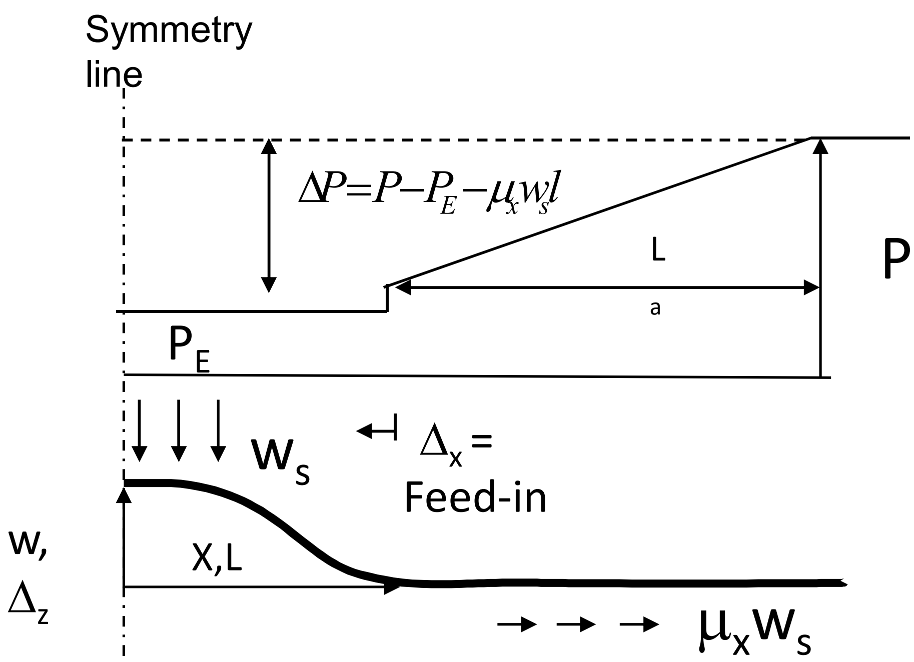
Pipeline resting on uneven seabed – uplift phase

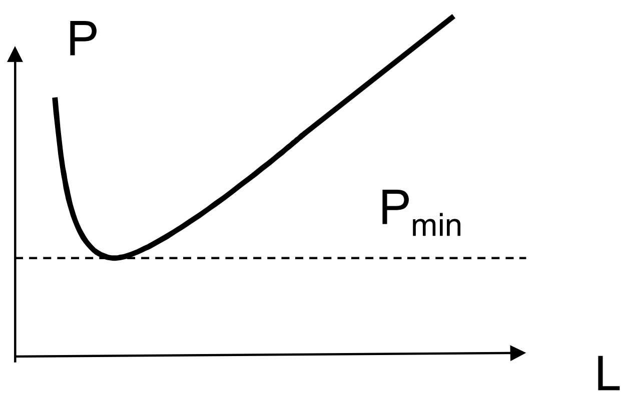
Results:

$$\Delta_x = \frac{\Delta P^2}{2EA\mu_x w_s} = 37.4\frac{w_s^2 EI^{\frac{3}{2}}}{P_E^{\frac{7}{2}}} = 1.022\cdot 10^{-3}\frac{w_s^2 l^7}{EI^2} \Rightarrow$$

$$P(l) = 20.25\frac{EI}{l^2} + \mu_x w_s l + 4.52\cdot 10^{-2}\frac{w_s}{EI}\sqrt{EA\mu_x w_s l^7}$$

$$P_{\max} \cong \Delta p\pi R^2(1-2\nu) + 2\pi RtE\alpha\Delta T = \Delta p\pi R^2(1-2\nu) + EA\alpha\Delta T$$





If $P_{min} > P_{max}$ => No buckling!!

Pipeline global buckling – analytical calculation

Pipeline resting on uneven seabed – Horizontal buckling

Boundary conditions:

$$1: \left.\frac{dv_1}{dx}\right|_{x=0} = 0$$

$$2: \left.v_1\right|_{x=l_1} = 0$$

$$3: \left.\frac{d^2v_1}{dx^2}\right|_{x=l_1} = \left.\frac{d^2v_2}{dx^2}\right|_{x=0}$$

$$4: \left.Q_{1y}\right|_{x=0} = 0$$

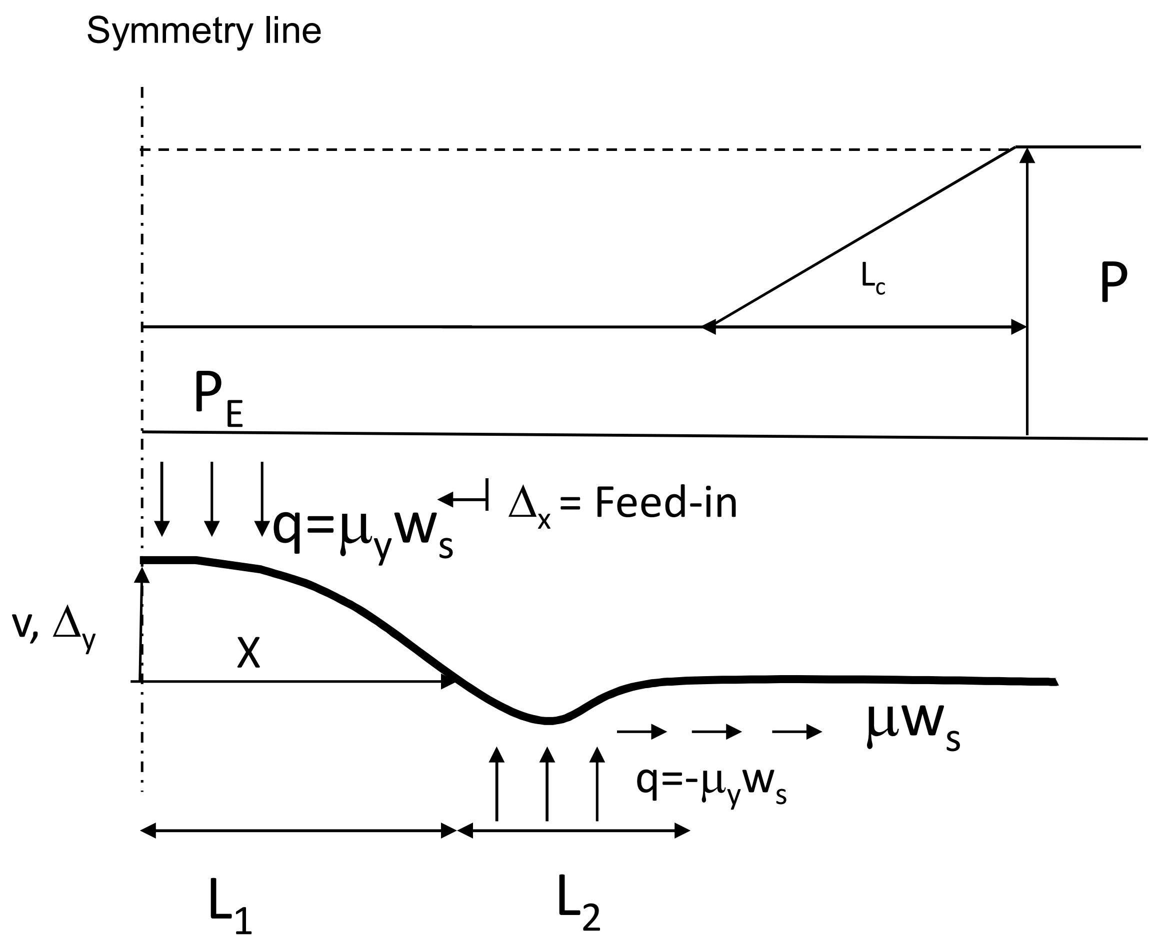
$$5: \left.v_2\right|_{x=0} = 0$$

$$6: \left.v_2\right|_{x=l} = 0$$

$$7: \left.\frac{dv_2}{dx}\right|_{x=l_2} = 0$$

$$8: \left.\frac{d^2v_2}{dx^2}\right|_{x=l_2} = 0$$

$$9: \left.Q_{1y}\right|_{x=l_1} = \left.Q_{2y}\right|_{x=0}$$



Pipeline global buckling – analytical calculation

Pipeline resting on uneven seabed – horizontal buckling:

Results:

$$1 : \Delta_{1y} = 11.94 \frac{\mu_y w_s EI}{P_E^{\,2}}$$

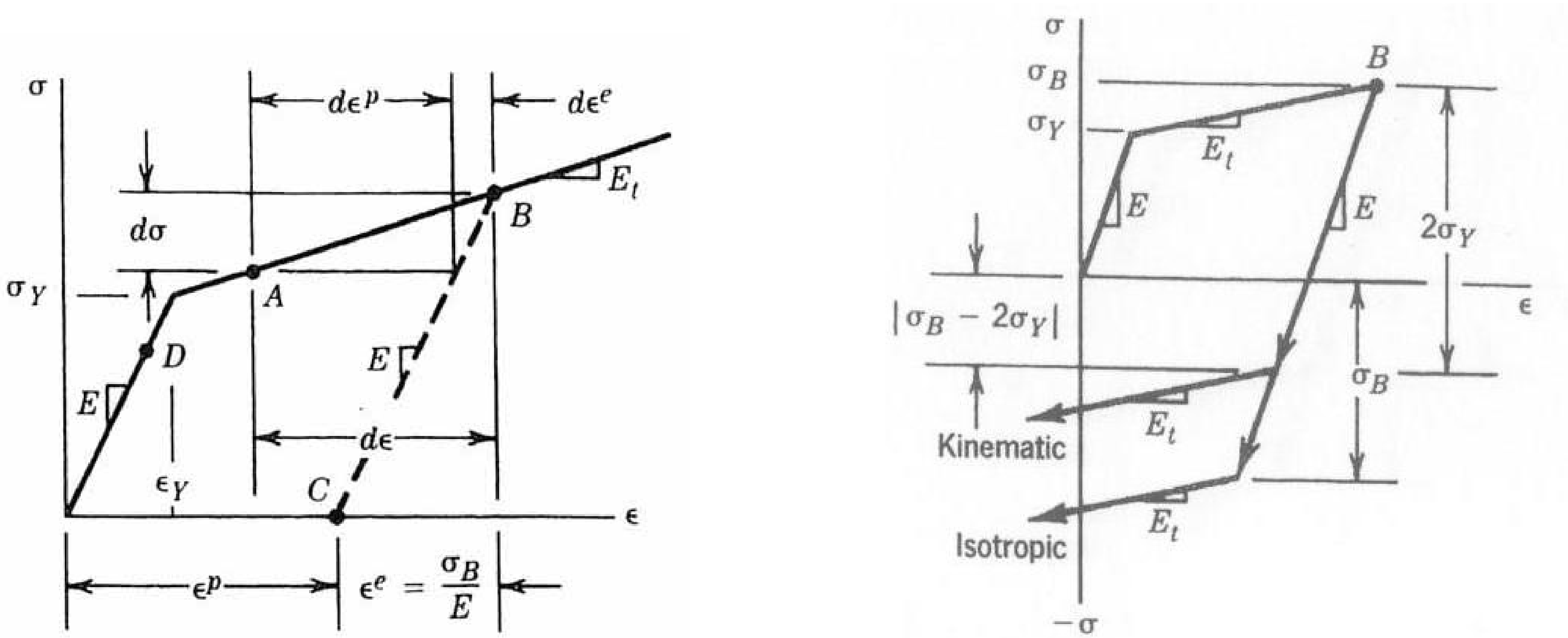
$$2 : P_E = 3.35 \sqrt{\frac{\mu_y w_s EI}{\Delta_{1y}}}$$

$$3 : M_{1y}\Big|_{x=0} = M_{\max} = 4.89 \frac{\mu_y w_s EI}{P_E}$$

$$4 : \Delta_x = 32.54 \frac{\mu_y^2 w_s^2 EI^{\frac{3}{2}}}{P_E^{\frac{7}{2}}}$$

Pipeline global buckling - Finite element analysis

- Large displacement kinematics (non-linear geometry)
- Pipe elements with elastoplastic material properties including the multiaxial stress effects (non-linear material)
- Seabed contact elements to describe the pipe/seabed interaction



$d\sigma = Ed\varepsilon^e$

a) Stress-strain plot in uniaxial stress, idealized as two straight lines, where σ_y is the stress at first onset of yielding.

b) Kinematic and isotropic hardening rules.

Figure 12.17 One-dimensional stress-strain relationships.

Elastic-plastic material behaviour in uniaxial stress :

1) The yield criterion : $|\sigma| = \sigma_Y$ (σ_Y tensile yield strength)

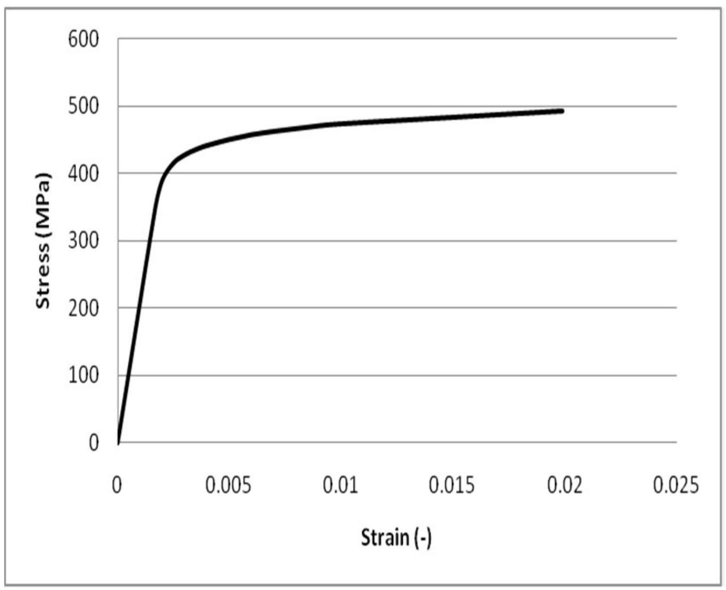
2) A hardening rule,

- unloading occurs from point B to point C (Fig. a)
- reverse loading from B: yield stress (?) (Fig. b) isotropic or kinematic

3) ***Flow rule*** $d\sigma = E_t\, d\varepsilon$, ($d\sigma = E\, d\varepsilon$ - before yielding occurs)

Material models

Steel X65



Soil/Roller properties