

MATHEMATICAL FINANCE

Market Depth and Liquidity Provision Profitability Uniswap V3/V2

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Abstract

This paper compares the market depth and liquidity provision profitability of the automated market makers (AMMs) Uniswap V3 and Uniswap V2 empirically. Due to the complexity of the algorithms, the ETH-USDC pools of both AMMs are used to make this comparison. Specifically, the market depth is measured following the framework of (Robinson and Liao, 2022). To compare the liquidity provision profitability, the framework developed by (O'Neill, 2022) is used to split returns into Fee Yield, Inventory Holding Returns and Adverse Selection Costs (Impermanent Loss). This mathematical model has been adjusted to remove the unrealistic assumption made that no liquidity is added nor removed during the period of the liquidity position. The results show that the mean market depth of Uniswap V3 is a factor of 15x higher than the mean market depth of Uniswap V2 under a price impact of $\pm 2\%$. This difference diminishes, the higher the price impact is. This is due to the concentrated liquidity structure of Uniswap V3. Furthermore, results show that the mean Fee Yields of Uniswap V3 liquidity positions are higher than the mean Fee Yields of Uniswap V2 liquidity positions. Nevertheless, due to high Adverse Selection Costs associated with Uniswap V3, the average total returns of Uniswap V2 liquidity positions are higher. When distinguishing between active and passive liquidity provision strategies, a similar conclusion can be drawn taking Inventory Holding Costs and Adverse Selection Costs into account. Although, for passive strategies, we can not conclude that the mean Fee Yields differ significantly. So-called gas fees are ignored in this paper.



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1 Introduction

Decentralised Finance (DeFi) is a financial ecosystem that does not rely on traditional intermediaries such as brokerages, exchanges, or banks by using smart contracts on a blockchain (usually Ethereum) (Fang, Azmi, Hor, and Win, 2021b). The main goal of the DeFi movement is to create a more efficient, accessible and transparent financial ecosystem (Abdulhakeem, Hu, et al., 2021). The financial services in DeFi are provided via Decentralised Applications (Dapps), the majority of which are deployed on the Ethereum blockchain.

Uniswap is an example of a Dapp, specifically it is an open-source protocol for providing liquidity and trading ERC20 tokens¹ on the Ethereum blockchain (Uniswap, 2020). Uniswap is an example of a so-called Decentralised Exchange (DEX). It uses the concept of an automated market maker (AMM) to determine the price of an asset, in contrast to a Centralised Exchange (CEX) such as Coinbase that uses traditional bid and ask orders placed on order books to determine the price of an asset (Aigner and Dhaliwal, 2021).

In Uniswap, token pairs can be swapped for each other using liquidity pools, which contain both tokens (Neuder, Rao, Moroz, and Parkes, 2021). Uniswap is a so-called Constant Product Market Maker (CPMM) that makes use of the simple but powerful formula $x * y = k$, where x and y represent the quantity of two tokens in a liquidity pool, and k represents the constant product. The $x * y = k$ formula is called the reserve-curve and implicitly creates a range of prices for the two tokens depending on each token's available quantities (Fang, Azmi, Hor, and Win, 2021a). Liquidity providers add tokens to these liquidity pools and are rewarded through fees that traders pay (Neuder et al., 2021).

In Uniswap V2, released in May 2020, liquidity providers add liquidity on the full range of possible prices $(0, \infty)$, and earn rewards based on their fraction of the total liquidity in the pool (Adams, 2021). However, the majority of this liquidity is never put to use. This would make Uniswap V2 not as capital efficient and so less beneficial to liquidity providers. The introduction of Uniswap V3 in May 2021 would be a more capital efficient AMM by making use of concentrated liquidity provision. Instead of providing liquidity on the full range of possible prices, liquidity providers could provide liquidity on a specified range of prices. They would earn fees only when the price is within the specified range. In general, the smaller the range set by the liquidity provider, the higher the potential fees earned per a given amount of tokens deposited. However, a smaller range is also associated with a higher risk of earning no fees at all, since the price might easily fall out of the specified range. Updating the range is possible at anytime, but comes with transaction costs including so-called gas fees. Active liquidity management strategies seem therefore in-

¹"ERC20 is an abbreviation of Ethereum Request for Comment, number 20. It is the standard for smart contract tokens created using Ethereum." (Reiff, 2022)

evitable when providing liquidity on Uniswap V3. Uniswap Labs claims that liquidity providers can provide liquidity with up to 4000x capital efficiency relative to Uniswap V2, earning higher returns on their capital (Uniswap, 2021). Since deciding on these active liquidity management strategies is rather complex, it might be that the reverse holds in practice. Therefore, this paper aims to answer the following research question: "Does the Uniswap V3 AMM offer more market depth where liquidity providers earn higher returns on their capital than in Uniswap V2?" To answer this research question, we will focus solely on the the USDC-ETH pairs, as this is one of the largest pairs with a TVL of approximately \$600m in Uniswap V3 as of 19-08-2022. It is important to mention that gas fees are ignored in this paper.

2 Literature Review and Hypothesis Development

There is barely any research on the market microstructure of decentralised exchanges due to the early stage of DeFi that we are currently in. In contrast, the market microstructure of centralised exchanges have been researched extensively by (Glosten and Milgrom, 1985), (Foucault, Pagano, Roell, and Röell, 2013), (Kyle, 1985) and many more. As mentioned before, Uniswap V2 and Uniswap V3 have only been active since May 2020 and May 2021 respectively. Adams (2021) released a paper about the core concepts of Uniswap V3 in March 2021. In the paper, they touch on the theoretically improved capital efficiency and benefits for the liquidity providers. Neuder et al. (2021) is the first paper that formalizes the problem of active liquidity provision on Uniswap V3 and studies three classes of strategies for liquidity providers: uniform, proportional, and optimal (via a constrained optimization problem). Neuder et al. (2021) claims that their simple liquidity provision strategies can yield near-optimal utility and earn over 200x more than Uniswap V2 liquidity provision. Although the developed framework is promising for further research, the paper makes several unrealistic assumptions regarding transaction costs. Furthermore, the paper ignores the risk of impermanent loss, defined as the risk of losing money during large and sustained movement in the underlying token price compared to simply holding the tokens outside of the AMM. In addition, the paper ignores the fact that prices might fall back into the liquidity providers specified range. A stochastic optimal control approach towards finding the optimal liquidity provision strategy might be a better approach. Elsts (2021) formalized the mathematics involved in providing liquidity on Uniswap V3. Robinson and Liao (2022) compared the market depth of Uniswap V3 to centralised exchanges, and built a mathematical framework to measure liquidity depth in Uniswap V2 and Uniswap V3. This paper builds on their framework and aims to use it to compare the market depth of Uniswap V2 and Uniswap V3 instead of a comparison between Uniswap and decentralised exchanges.

Austin and Liao (2022) compared the returns of non-actively managed liquidity positions on Uniswap V2 and V3. They claim that on average, non-rebalancing V3 positions outperform V2 positions by around 54% in fee returns. Actively managed V3 positions are ignored, as well as the impact of Impermanent Loss on the total returns. This paper includes the effect of Impermanent Loss by slightly adjusting the framework developed by O'Neill (2022). This mathematical model has been adjusted to remove the unrealistic assumption that no liquidity is added nor removed during the period of the liquidity position. Furthermore, this paper does include actively managed Uniswap V3 positions in order to make a fair empirical comparison. Barbon and Rinaldo (2021) compared the quality of decentralised and centralised cryptocurrency markets and used a similar approach to split returns.

As mentioned before, Adams (2021) shows a theoretical improvement of capital efficiency and improved benefits for liquidity providers in Uniswap V3

compared to Uniswap V2. Since this paper is released by Uniswap Labs themselves, the paper might be biased in favor of Uniswap V3 and should be looked at with a critical eye. Nevertheless, their theoretical evidence on capital efficiency and liquidity provider profitability seems to contain no flaws. Loesch, Hindman, Richardson, and Welch (2021) shows that most liquidity providers on Uniswap lose money due to the impermanent loss. They claim that only 3 of the 17 pools analyzed earned fees that exceeded the impermanent loss, and only by a small margin. This would suggest that Uniswap V3 might be not as profitable in practice after all. Once again, this paper is published by Bancor, another decentralised exchange, and might therefore be biased towards their own decentralised exchange. Taking this and the rest of the literature into account, our hypothesis is that Uniswap V3 is likely to offer more market depth and is more profitable for liquidity providers than Uniswap V2, but not as much as the theory suggests.

3 Methodology

3.1 Uniswap AMM Design

Recall from the introduction that Uniswap is Constant Product Market Maker (CPMM). An AMM consists usually of three (or two if you wish) main players: The liquidity providers, the liquidity demanders (traders) and a formula called reserve-curve that takes the role of the market makers in traditional financial markets. Both Uniswap V2 and V3 are designed around the reserve-curve $x * y = k$, where x and y represent the quantity of two tokens in a liquidity pool, and k represents the constant product that is constant by design. That is, permitted swaps are fully determined by the reserve-curve (Neuder et al., 2021). Suppose one wants to trade some amount of token Y for some amount of token X , then one needs to deposit amount Δy to the liquidity pool and receives an amount of Δx in returns such that $(x + \Delta x)(y + \Delta y) = k$ holds. The reserve-curve implicitly defines therefore the price of token X (expressed in token Y) and vice versa; $p_x(x, y) = dy/dx = k/x^2$ (Neuder et al., 2021). Hence, a liquidity demander has to trade such that the reserve-curve remains constant. The liquidity providers are the only ones that can change the value of the reserve-curve by minting and burning tokens. For Uniswap V2, burning and minting can only be done in a constant 50/50 ratio. In return for their service, the liquidity providers earn a fee on each swap executed by a trader times the fraction of total liquidity they own (Xu, Paruch, Cousaert, and Feng, 2021). Due to the simplicity of the Uniswap V2 reserve-curve, liquidity providers provide liquidity on the entire range of possible prices $(0, \infty)$ and a fee of 0.3%. Since tokens are expected to trade close to particular prices over time, the majority of liquidity is never put to use in Uniswap V2.

To allow for concentrated liquidity provision in Uniswap V3, its real reserve-curve is defined by $(x + \frac{L}{\sqrt{p_b}})(y + L\sqrt{p_a}) = L^2$, where $L = \sqrt{K}$ for a specific position. A position is defined here as liquidity concentrated to a finite range $[p_a, p_b]$. A position only needs to maintain enough reserves to support trading within its range, and therefore can act like a constant product pool with larger reserves (we call these the virtual reserves) within that range (Adams, 2021). Hence, the real reserve-curve is essentially a transformation of the Uniswap V2 (virtual) reserve-curve such that it remains constant for a position.

Besides the difference in the construction of the reserve-curve between Uniswap V2 and V3, Uniswap V3 allows for different fee structures. Where Uniswap V2 has a uniform fee of 0.30% for every swap in all pools, Uniswap V3 allows pools to have a 0.05%, 0.30%, or 1% fee level.

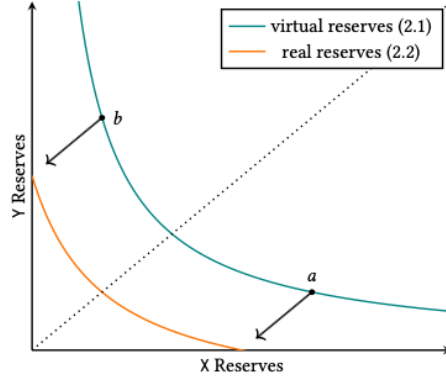


Figure 1: Real Reserves vs Virtual Reserves (Adams, 2021)

3.2 Uniswap Market Depth

Let us define market depth as how much of token X can be traded for token Y at a given price level. The higher the market depth, the better the market is able to withstand large volume transactions. Theoretically, liquidity providers can provide the same market depth in Uniswap V3 as in Uniswap V2 for a given price impact while putting less capital at risk, and hence Uniswap V3 would be more capital efficient. (Robinson and Liao, 2022) constructed a framework for calculating this market depth for Uniswap V2 and Uniswap V3. This framework will be used to calculate the market depth for Uniswap V2 and V3 empirically.

3.2.1 Market Depth Uniswap V2

Let $m(\delta)$ be market depth defined as the amount of token X that can be exchanged for token Y for a given δ percent of price impact (e.g. $\delta = 2\%$). Recall that the reserve-curve for V2 is defined by $x * y = L^2$. As the reserve-curve implicitly defines the price relation, one could rewrite the reserve-curve as $p = \frac{L^2}{x^2}$, $p = \frac{y^2}{L^2}$, $x = \frac{L}{\sqrt{p}}$, or $y = L\sqrt{p}$.

We know that after a trade of Δx units of token X for Δy units of token Y , the reserve-curve becomes

$$(x + \Delta x)(y + \Delta y) = L^2.$$

Note from the definition of the market depth we are interested in Δx for a given price impact δ . Let $p_0 = y/x$ and $p_1 = (y + \Delta y)/(x + \Delta x)$ be the price before and after the trade respectively. Then we solve for Δx s.t. $p_1/p_0 - 1 = \delta$. That is, we solve for Δx s.t.

$$\frac{y + \Delta y}{x + \Delta x} \frac{x}{y} - 1 = \delta.$$

Substitute into the post-trade equation, we get

$$(x + \Delta x)^2(\delta + 1)p_0 = L^2$$

Market depth in units of token X is therefore given by

$$m(\delta) \equiv |\Delta x| = \left| \frac{L}{\sqrt{(1 + \delta)p_0}} - \frac{L}{\sqrt{p_0}} \right|$$

3.2.2 Market Depth Uniswap V3

In a similar fashion as for Uniswap V2, the market depth of Uniswap V3 can be derived. However, this is more complicated as the liquidity distribution is an aggregation of individual liquidity provider positions. Therefore, we prefer working with ticks rather than prices. Again, we follow the approach of (Robinson and Liao, 2022), with some additional steps from (Elsts, 2021) for clarification. Let i be an index for lower tick boundary of a position and s be the tick spacing ($s = 60$ for 0.3% pools, $s = 10$ for 0.05% pools). Let p_0 be the current price and p_a, p_b the prices of the upper and lower price of a position. That is, $p_a = 1.0001^i$ and $p_b = 1.0001^{i+s}$.

Recall that the real-reserve-curve for a position is given by

$$\left(x + \frac{L}{\sqrt{p_b}}\right)(y + L\sqrt{p_a}) = L^2$$

The real-reserve-curve is constructed such that the position needs to maintain exactly enough reserves to fully support trading within its price boundaries $[p_a, p_b]$. The real reserves of token X shrink as the value of token X in terms of token Y increases and are fully depleted at the upper price boundary p_b (Heimbach, Schertenleib, and Wattenhofer, 2022). For the real-reserves of token Y, the opposite holds. Note that when p_0 is outside of the range, the position is inactive and the liquidity provider does not earn any fees from traders. Hence we distinguish 3 cases:

Case 1: $p_0 \leq p_a$, the position is fully in X (position inactive);

$$\begin{aligned} \left(x + \frac{L}{\sqrt{p_b}}\right)L\sqrt{p_a} &= L^2 \\ x\sqrt{p_a} + L\frac{\sqrt{p_a}}{\sqrt{p_b}} &= L \\ x &= \frac{L}{\sqrt{p_a}} - \frac{L}{\sqrt{p_b}} \\ x &= L\frac{\sqrt{p_b} - \sqrt{p_a}}{\sqrt{p_a} \cdot \sqrt{p_b}} \end{aligned}$$

The liquidity of the position is:

$$L_x(p_a, p_b) = x \frac{\sqrt{p_a} \cdot \sqrt{p_b}}{\sqrt{p_b} - \sqrt{p_a}}$$

Case 2: $p_0 \geq p_b$, the position is fully in Y (position inactive);

$$\begin{aligned}\frac{L}{\sqrt{p_b}} (y + L\sqrt{p_a}) &= L^2 \\ \frac{y}{\sqrt{p_b}} + L\frac{\sqrt{p_a}}{\sqrt{p_b}} &= L \\ y &= L (\sqrt{p_b} - \sqrt{p_a})\end{aligned}$$

The liquidity of the position is:

$$L_y(p_a, p_b) = \frac{y}{\sqrt{p_b} - \sqrt{p_a}}$$

Case 3: $p_0 \in (p_a, p_b)$, position is in Y and X (position active); This one is slightly more involved, as we can not simply set x or y equal to 0. (Elsts, 2021) has an intuitive way of obtaining x and y ; since both tokens contribute to the liquidity of the position equally, the liquidity L_x provided by token X in one side of the range (p_0, p_b) must be equal to the liquidity L_y provided by token Y in the other side of the range (p_a, p_0) . Hence we get $L_x(p_0, p_b) = L_y(p_a, p_0)$, which is equivalent to;

$$x \frac{\sqrt{p_0} \cdot \sqrt{p_b}}{\sqrt{p_b} - \sqrt{p_0}} = \frac{y}{\sqrt{p_0} - \sqrt{p_a}}$$

Solving for x and y separately yields the real reserves for this case;

$$\begin{aligned}x &= \frac{L}{\sqrt{p_0}} - \frac{L}{\sqrt{p_b}} = L \frac{\sqrt{p_b} - \sqrt{p_0}}{\sqrt{p_0 p_b}} \\ y &= L (\sqrt{p_0} - \sqrt{p_a})\end{aligned}$$

All cases can be summarised together can be written as

$$\begin{aligned}x &= \frac{L}{\sqrt{z}} - \frac{L}{\sqrt{p_b}} \\ y &= L (\sqrt{z} - \sqrt{p_a})\end{aligned}$$

$$\text{where } z = \begin{cases} p_a & p_0 \leq p_a \\ p_0 & p_a < p_0 < p_b \\ p_b & p_0 \geq p_b \end{cases}$$

Let δ be percentage change in price and d be the associated change in ticks such that

$$\delta = \frac{p_1}{p_0} - 1 = \frac{1.0001^{i_0+d}}{1.0001^{i_0}} - 1$$

Let the amount of token X between i and $i+s$ be given by

$$L_x(i, i+s) = \frac{L}{\sqrt{z}} - \frac{L}{\sqrt{p_b}}$$

and the amount of token Y between i and $i+s$ by

$$L_y(i, i+s) = L (\sqrt{z} - \sqrt{p_a})$$

with z defined above. Then the market depth for Uniswap V3 expressed in units of token x is

$$m(\delta) = \frac{1}{s} \sum_{i=i_0}^{i_0+d(\delta)} \left| L_x(i, i+s) + p_0^{-1} L_y(i, i+s) \right|$$

where i_0 is the current tick associated with p_0 and d is the tick-equivalent of the percentage price change δ .

3.3 Returns

To measure the profitability of Uniswap V2 and V3 liquidity positions, we need to calculate the returns. Specifically, we are interested in the composition of these returns. Therefore, we follow the methodology developed by (O'Neill, 2022), that splits returns in three components; Adverse Selection Costs (Impermanent Loss), Inventory Holding Returns and Fee Yields.

Uniswap V2 and V3 have a different way of storing fees for liquidity providers. In Uniswap V2, fees are deposited to the pool as liquidity. Hence, liquidity of the pools grows over time, even when mints are absent. In Uniswap V3, fees are stored separately and held in a different type of token; the so-called liquidity token. This makes it way easier to keep track of fees in Uniswap V3. For Uniswap V2 we use the methodology developed by (Austin and Liao, 2022) to derive the fees.

3.3.1 Return Splits

As mentioned before, we follow the methodology of (O'Neill, 2022) to split the return into three components. However, (O'Neill, 2022) ignores mints and burns in the derivation of this return split. As we want to calculate the return split as accurate as possible, we adjust their derivation such that we are able to calculate the actual return split, rather than an estimation. Since, Uniswap V3 keeps track of the fees separately (and does not add them to the total liquidity of the pool), the return split for V3 is easier to derive.

(O'Neill, 2022) developed the following framework for splitting returns:

Let $V_0 = x_0 + y_0 P_0 = x_0 + y_0 \frac{x_0}{y_0} = 2x_0$ be the portfolio value (expressed in token X) of the initial minted liquidity $t = 0$, and $V_T = x_T + y_T P_T = 2x_T$ the portfolio value expressed in token X of the burned liquidity of the position. Let F_T be the total fees earned by the corresponding liquidity provider. Then we have

$$R_{TOTAL} = \frac{V_T + F_T}{V_0} - 1$$

Let $V_{T, FIXED} = x_0 + y_0 P_T$ be the portfolio value (expressed in token X) of what the staked tokens would be worth if the liquidity provider held them passively outside of the AMM. Then the LP's total return in can be rewritten such that it is broken down into three components:

$$R_{TOTAL} = \underbrace{\left(\frac{V_{T, FIXED}}{V_0} - 1 \right)}_{\text{Inventory Holding Return}} + \underbrace{\left[\left(\frac{V_T}{V_0} - 1 \right) - \left(\frac{V_{T, FIXED}}{V_0} - 1 \right) \right]}_{\text{Adverse Selection Cost}} + \underbrace{\frac{F_T}{V_0}}_{\text{Fee Yield}}$$

where the three terms are defined as;

- **Inventory Holding Return** is a change in the value of the liquidity providers inventory purely caused by movements in the token prices.
- **Adverse Selection Costs (Impermanent Loss)** is how much worse the liquidity provider is from depositing her tokens to the AMM compared to simply holding her tokens outside of the AMM.
- **Fee Yield** is the return earned in fees by providing liquidity on the portfolio value V_0

The return split derivation for Uniswap V2 works in the exact same way, although we need to calculate the Fee Yield as described in section 3.3.2.

3.3.2 Fee Yields V2

(Austin and Liao, 2022) developed the following framework for calculating the V2 Fee Yields:

Let x_t, y_t be the amount of token X and token Y in a Uniswap V2 liquidity pool at time t . Let p_t be the price of token Y expressed in units of token X, $p_t = \frac{y_t}{x_t}$. Let k_t denote the liquidity that exists in the pool at time t . Obviously, $x_t * y_t = k_t$ is held in every period due to the design of the AMM. For that reason, $x_t = k_t / y_t = \sqrt{k_t / p_t}$ and $y_t = \sqrt{k_t p_t} * k_t$ can only be changed due to fee accrual in absent of mints and burns. Let $v_t \equiv v(k_t, p_t)$ be the portfolio value of the liquidity pool expressed in token X as the numeraire at time t . When expressed in terms of the amount of reserve token X and Y, the portfolio value is given by $v(k_t, p_t) = x_t + y_t p_t^{-1}$. By applying the reserve-curve formula, this can be rewritten as

$$v_t \equiv v(k_t, p_t) = \sqrt{\frac{k_t}{p_t}} + \sqrt{k_t p_t} p_t^{-1}$$

Suppose no mints or burns occur between t and $t + 1$, then the simple fee return of the liquidity pool is given by

$$r_{t+1}^{fee} = \frac{v(k_{t+1}, p_{t+1}) - v(k_t, p_{t+1})}{v(k_t, p_t)}$$

Note that $\frac{v(k_{t,p+1})}{v_t}$ is the gross return of the liquidity pool portfolio value without taking fees into account. Obviously, mints and burns usually occur in between time t and $t + 1$, so we need to account for this. The evolution of liquidity value follows $k_{t+1} = k_t + \kappa_{t+1} + \phi_{t+1}$, where κ_{t+1} is the net mints minus burns that occurs in between t and $t + 1$ and ϕ_{t+1} is the fee accrual due to transactions in this period, i.e. $\phi_{t+1} = \sum_i \lambda |s_i| \forall$ swap s_i that occurs between t and $t + 1$ and fee tier λ . Let $k'_{t+1} \equiv k_{t+1} - \kappa_{t+1}$. This would be the liquidity at $t + 1$ when disregarding the mints and burns. Hence, the mint-burn adjusted simple fee

return on a liquidity pool is given by:

$$r_{t+1}^{fee} = \frac{v(k'_{t+1}, p_{t+1}) - v(k_t, p_{t+1})}{v(k_t, p_t)}.$$

4 Data Description

To compare the market depth and liquidity provider profitability between Uniswap V2 and V3, we analyze data from the period 2021-05-06 (introduction of Uniswap V3) to 2022-06-01. Specifically, we gather data on "Mints" (adds of liquidity), "Burns" (removals of liquidity), "Swaps" (trades), "Pool" (state of the pool, i.e. prices, liquidity) and "Fees" (V3 fees) via the Uniswap subgraph API. As mentioned before, we solely look at the Uniswap V2 0.3% ETH-USDC pool and the Uniswap V3 0.3%, 0.05% ETH-USDC pools. We have chosen to exclude the 0.01% ETH-USDC pool due to the computational complexity of the market depth algorithm in combination with its minimal share on the TVL of the ETH-USDC together. A link to the Python scripts used for this paper can be found here [Code](#), this includes the specific API requests used to gather the desired data. Please note that from now on Uniswap V3 data is an aggregation of the 0.3% and 0.05% pools unless stated differently. An overview of the data can be found below.

BurnsV3 # 74485	MintsV3 # 74485	SwapsV3 # 2099853	FeesV3 # 61607	PoolV3 # 394	BurnsV2 # 4700	MintsV2 # 3600	SwapsV2 # 416000	PoolV2 # 394
Origin Timestamp	Origin Timestamp	Origin Timestamp Token1Price	Origin Timestamp	Timestamp	Origin Timestamp	Origin Timestamp	Origin Timestamp Token1Price	Timestamp
TickLower TickUpper Amount	TickLower TickUpper Amount Amount0 Amount1		TickLower TickUpper Deposited0 Deposited1 Withdrawn0 Withdrawn1 FeesToken0 FeesToken1		Liquidity Amount0 Amount1	Liquidity Amount0 Amount1		
Amount0 Amount1								

Table 1: Data Description

Some data cleaning has been done on the data gathered. For the specifics, we refer to the data-cleaning file of the Code. The most important transformations include;

- Getting rid of inaccurate data.
- Getting rid of amounts equal to 0 for burns, mints, fees. The value equal to 0 corresponds with a liquidity provider checking her balance.
- Getting rid of inconsistent data by merging data frames.
- Converting numbers to the right human-readable data types.

4.1 Descriptive Analysis

To get a feeling for the data, we start with some descriptive analysis of the data. Specifically, we look at the daily frequencies of mints, burns and swaps for Uniswap V2 and V3. An overview of the mean daily frequencies can be seen below:

	V2	V3
Swaps	3670.5	5343.1
Mints	9.6	186.3
Burns	12.1	189.1

Table 2: Mean Daily Frequencies

In appendix 8.1 the underlying distributions of the data can be found. Observe that the mean daily swap frequencies are relatively similar in Uniswap V2 and V3. That is, the order of magnitude is the same. Although, the mean number of daily swap frequency for V2, $V2 = 3616.87$, is statistically significantly lower than for V3, $V3 = 5343.14$; $t(779) = -12.0618$, $p = 0.0000^2$ (table 7). Looking at the mean daily mint and burn frequencies, we observe a different story. Uniswap V3 has an extremely higher mean daily mint and burn frequency, that is, the order of magnitudes differ as well. The mean daily burn and mint frequencies of V2 are statistically significantly lower than of V3 (tables 5, 6). This can be explained by the more active liquidity management strategies used by liquidity providers in Uniswap V3. When the price falls outside the specified range, one is likely to update its range to keep earning fees. This rebalancing strategy is not necessary in Uniswap V2.

When plotting the number of swaps per mints (or burns) per day, one could easily see this major difference between Uniswap V2 and V3 throughout the entire timeline.

²p-value extremely close to 0, but not exactly 0

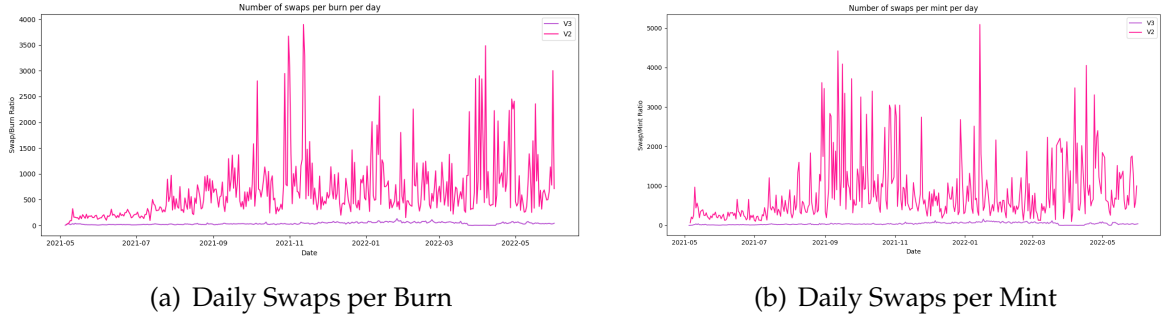


Figure 2: Daily Number of Swaps per Mint(Right)/Burn(Left)

The number of mints per burn per day for Uniswap V2 and V3 are very similar as can be seen in the plot below. Due to the continuous rebalancing strategies in Uniswap V3, the ratio is slightly more consistent than for Uniswap V2 over time.

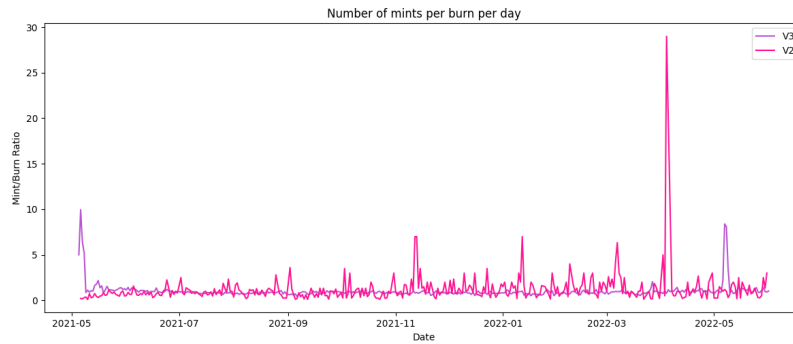


Figure 3: Daily Number of Mints per Burns

Furthermore, it is important to mention that the amount of unique liquidity providers in Uniswap V2 is 1321 against 19895 in Uniswap V3. The amount of unique liquidity demanders in Uniswap V2 is 136949 against 400289 in Uniswap V3. It is noticeable that the number of unique liquidity providers that are active in Uniswap V3 is significantly larger than in Uniswap V2, especially in comparison with the amount of unique liquidity demanders. In other words, Uniswap V3 seems to be more popular among liquidity providers.

5 Results

5.1 Market Depth

To compare the market depth between Uniswap V2 and V3, we calculate the market depths over time for both AMMs. Recall that the market depth is defined as the amount of token X that can be exchanged for token Y for a given δ percent of price impact. In our case token X is USDC and token Y ETH. The plot below shows how the market depth changes throughout the sample period for a given $\delta = \pm 2\%$ of price impact.

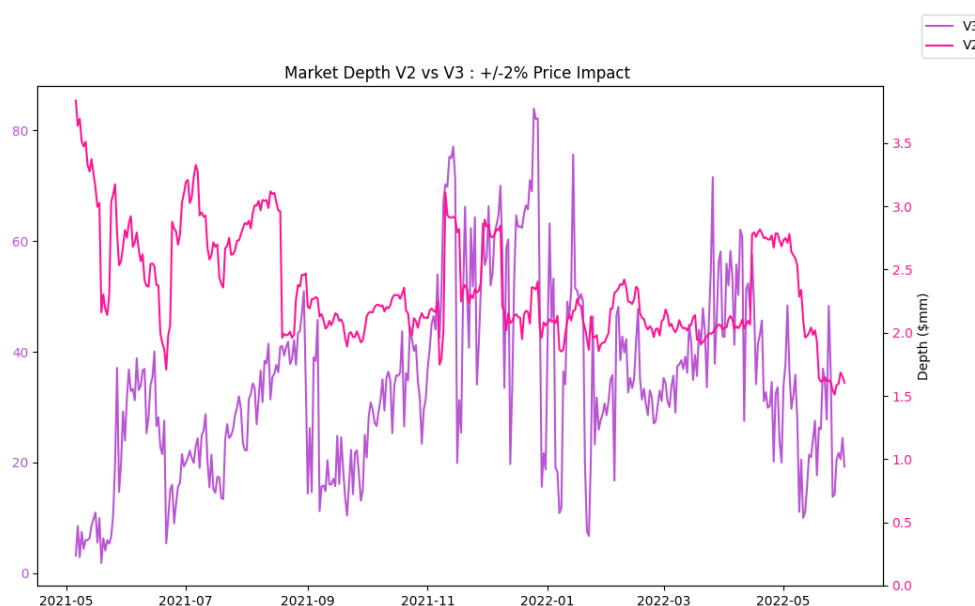


Figure 4: Market Depth in millions of USD for $\delta = \pm 2\%$ Uniswap V3 and V2 over time

Note that a double y-axis is used in order to plot both market depths on the same plot. The mean market depth for Uniswap V2 and Uniswap V3 is \$2.35m vs \$34.6m respectively. We could safely say that the average market depth for $\delta = \pm 2\%$ of V2, $V2 = 2.352261$, is statistically significantly lower than for V3, $V3 = 33.989686$; $t(780) = -38.1830$, $p = 0.0000^3$, a factor of around 15x (table 14). A summary of the underlying distributions of the market depth for a given $\delta = \pm 2\%$ can be found in tables 15 and 16. With a correlation coefficient of -0.090 the market depths are slightly negatively correlated. Since both market depths are expected to be highly correlated with Ethereum related news, this

³p-value extremely close to 0, but not exactly 0

is an interesting observation. A possible explanation could be that although both market depths move in the same direction if the cause of the movement is Ethereum related, the market depths move in the opposite direction if liquidity providers prefer providing liquidity in Uniswap V2 or in Uniswap V3, depending on the popularity of the AMM at that moment. Right after the introduction of Uniswap V3, one could see from the plot that the market depth of V2 heavily declines, whereas the market depth of Uniswap V3 increases. This is such an example of movement in the opposite direction caused by the popularity of the AMM at a specific moment in time. At the 8th of November 2021, ETH reached an all time high of 4811 USD. This caused both market depths to move heavily upwards. This would be an example of market depth movement in a similar direction caused by Ethereum related news. The Terra collapse on 7th of May 2022 is another example of such an event, where both market depths heavily declined.

So far, we have focused only on the market depth for a given $\delta = \pm 2\%$ of price impact. It would be interesting to see how the market depths of Uniswap V2 and V3 change for different values of δ . Specifically, it would be interesting to look at the ratio between the market depths for different values of δ . The plot below shows the mean market depths for a range of price impacts δ together with the corresponding V3/V2 market depth ratio.

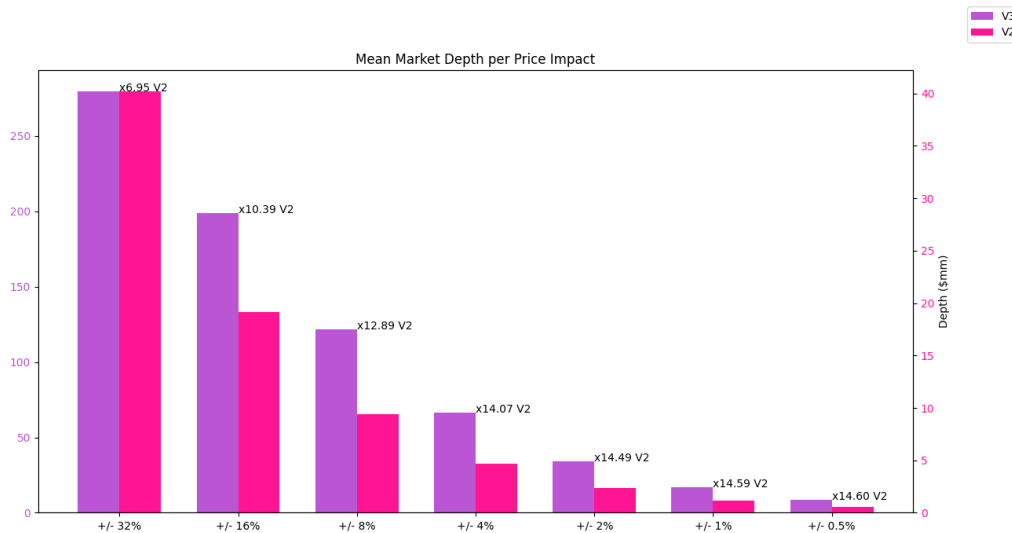


Figure 5: Mean Market Depth in millions of USD for a range of values δ including V3/V2 Market Depth Ratio

Note from the plot that the higher the price impact δ , the smaller the mean market depth ratio. Although the higher price impacts δ are rather unrealistic,

it does show quite nicely the effect of concentrated liquidity. As liquidity is extremely high around the current price p_0 in Uniswap V3, there is much of token X needed in order to cause a small price change. However, larger price movements require less and less additional amounts of token X . Since liquidity in Uniswap V2 is evenly distributed over the entire range of prices $(0, \infty)$, the market depth grows linearly with δ .

5.2 Liquidity Provision Profitability

To compare the liquidity provision profitability in Uniswap V3 and V2, we are mainly interested in the Fee Yields. As mentioned before, for Uniswap V2 we follow the framework of (Austin and Liao, 2022) to derive these Fee Yields. For Uniswap V3 this is much easier, as fees are stored separately rather than added to the reserves. The plot below shows how the Fee Yields are distributed in Uniswap V2 and Uniswap V3.

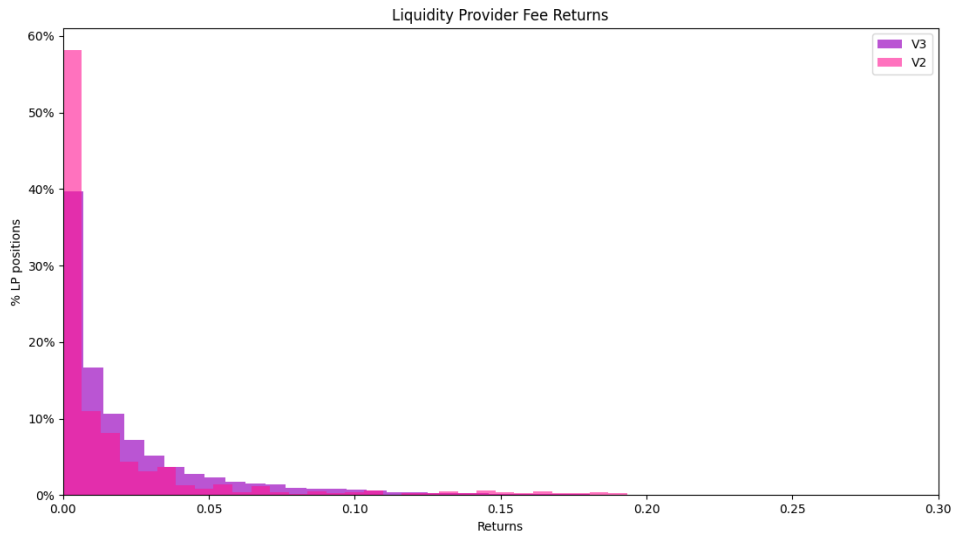


Figure 6: Liquidity Provision Fee Yields V3 vs V2

A summary of the underlying distributions of the Fee Yields can be found in table 3. We observe that the distribution of Fee Yields in Uniswap V2 and V3 are relatively similar. Nevertheless, we do observe some key differences. First, we notice that a much larger share of liquidity positions in Uniswap V2 have Fee Yields of less than 1% compared to Uniswap V3. Furthermore, a larger share of liquidity positions in Uniswap V3 have Fee Yields between 1% and 10% compared to Uniswap V2. Barely any Uniswap V3 liquidity positions have a higher Fee Yield than 10%, whereas there is a significantly larger share of Uniswap V2 liquidity positions that have Fee Yields above 10%. These differences in the distribution of Fee Yields might be caused by the different strategies used by liquidity providers in both AMMs. Therefore we examine the distribution of liquidity position duration in the plot below.

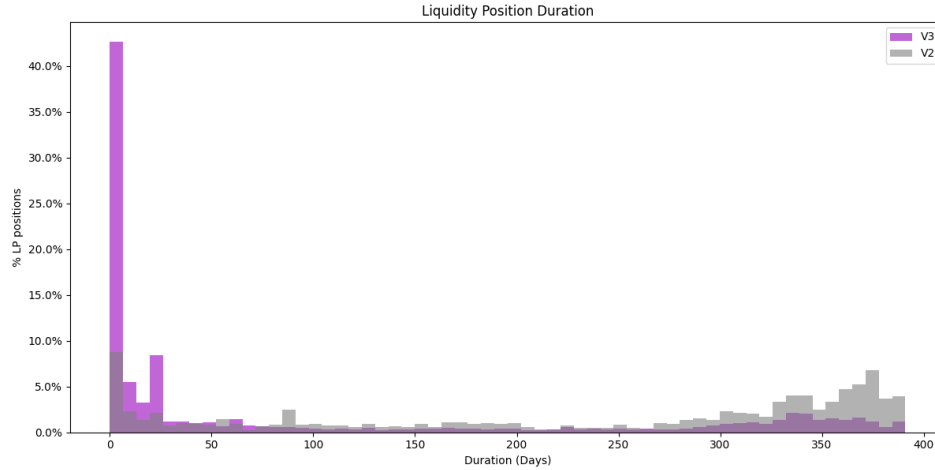


Figure 7: Liquidity Provision Duration V3 vs V2

It is important to mention that we have included positions that were still active when creating this plot, by taking the final date of the sample period as closing date. The actual duration for some positions will therefore be larger than the plot currently indicates. A summary of the underlying data can be found in tables 17 and 18. From the plot it is clear that the Uniswap V3 liquidity position duration distribution is extremely skewed to the right, whereas the Uniswap V2 liquidity position duration distribution has a much more uniform shape. This is consistent with the literature that suggests that liquidity providers in Uniswap V3 require a much more active liquidity management strategy in order to stay profitable. The average duration for V3 liquidity positions, $V3 = 96.71$, is statistically significantly lower than for V2 liquidity positions, $V2 = 233.943333$; $t(73043) = -59.1474$, $p = 0.0000^4$ (table 19). A position with a long duration, will have more time to generate fees than the exact same position with a smaller duration. Hence, a position with a longer duration will have a Fee Yield that is at least as large as the exact same position with a smaller duration. The liquidity position duration distributions can therefore be directly linked to the distributions of Fee Yields (figure 6). The long tail of the Uniswap V2 Fee Yields distribution can be explained by the long term passive strategies of liquidity providers. The opposite would be an explanation for the smaller tail of the Uniswap V3 Fee Yields distribution. Since Uniswap V3 liquidity providers provide concentrated liquidity, short term positions will likely generate higher Fee Yields than a similar position in Uniswap V2, under the assumption that a short term Uniswap V3 position is always active. This would be an explanation for the left-hand-side of the Fee Yield distribution.

⁴p-value extremely close to 0, but not exactly 0

Simple fee returns are not the only important type of returns to consider. Although it does give an indication of the capital efficiency of the AMM. However, in the end of the day, liquidity providers are interested in their total return of their position, which includes so-called Adverse Selection Costs (Impermanent Loss) and Inventory Holding Returns. We therefore split the total return in three components according to (O'Neill, 2022) to get a better understanding of the composition of the total returns.

Recall the following from section 3.3.1. Let $V_{T, FIXED} = x_0 + y_0 P_T$ be the portfolio value (expressed in token X) of what the staked tokens would be worth if the liquidity provider held them passively outside of the AMM. Then the liquidity providers' total return can be rewritten such that it is broken down into three components:

$$R_{TOTAL} = \underbrace{\left(\frac{V_{T, FIXED}}{V_0} - 1 \right)}_{\text{Inventory Holding Return}} + \underbrace{\left[\left(\frac{V_T}{V_0} - 1 \right) - \left(\frac{V_{T, FIXED}}{V_0} - 1 \right) \right]}_{\text{Adverse Selection Cost}} + \underbrace{\frac{F_T}{V_0}}_{\text{Fee Yield}}$$

Where the three components are shortly defined in the methodology section, below we give some context to the three terms.

- **Inventory Holding Return** is a change in the value of the liquidity providers inventory purely caused by movements in the token prices. In traditional financial markets, where the market makers take the role of the liquidity providers, market makers face little inventory risk. They face little inventory risk, as they typically buy and sell their inventory almost simultaneously and hence are only exposed to price changes for an extremely small amount of time. This is different for liquidity providers in AMMs. Due to the construction of AMMs, liquidity providers are exposed to price changes during the entire duration of their position. As we have seen in figure 7, the average duration for Uniswap V2 and Uniswap V3 is approximately 234 days and 97 days respectively. Hence, they face a much greater inventory risk than traditional market makers. When providing liquidity on an AMM such as Uniswap, one should therefore be aware of this inventory risk. Hence, the Inventory Holding Return reflects the profit or loss on the inventory the liquidity provider holds.

- **Adverse Selection Costs (Impermanent Loss)** is how much worse the liquidity provider is from depositing her tokens to the AMM compared to simply holding her tokens outside of the AMM. Let us look at an artificial example of Impermanent Loss in Uniswap V2 from (Binance, 2022).

Asha, as a liquidity provider, deposits 1 ETH and 100 USDC to the ETH/USDC pool. The total dollar value of her deposits are 200 USD, as the ratio of deposits implicitly reveals the prices in an AMM. Let us assume that the total ETH/USDC pool consists of 10 ETH and 1000 USDC such that the total liquidity in the pool is 10000. Hence, Asha owns a 10% share of the pool. Suppose that the general price of ETH suddenly increases to 400 USDC. Then arbitrage

traders will trade USDC for ETH until the ratio reflects the general (correct) price while the pools liquidity remains 10000. Hence, there is now a total of 5 ETH and 2000 USDC in the pool. If Asha now decides to withdraw her tokens from the ETH/USDC pool, she will receive $10\% * 5 \text{ ETH} = 0.5 \text{ ETH}$ and $10\% * 2000 \text{ USDC} = 200 \text{ USDC}$, a total dollar value of 400 USD. Recall that her initially deposited dollar value was 200 USD. It seems like she made a great deal by providing her tokens to the ETH/USDC pool. But what would have been her total dollar value if she had simply hold her 1 ETH and 100 USDC outside of the ETH/USDC pool? Then her total dollar value would have been 500 USD. Hence, not such a great deal after all, she suffered from Impermanent Loss. Due to arbitrage traders, the quantities of the tokens have changed in an adverse way, which explains why the Impermanent Loss is also known as Adverse Selection Costs in traditional models of market making. The Adverse Selection Costs are always smaller or equal than 0, as arbitrage traders will only trade in their advantage as arbitrage opportunities exist.

- **Fee Yield** is the return earned in fees by providing liquidity on the portfolio value V_0 . The Fee Yield can be seen as a compensation for the risks taken by the liquidity provider. The Fee Yield is always bigger or equal than 0, and strictly bigger than 0 if any swaps have been executed.

The return split distribution for Uniswap V2 and Uniswap V3 can be seen below.

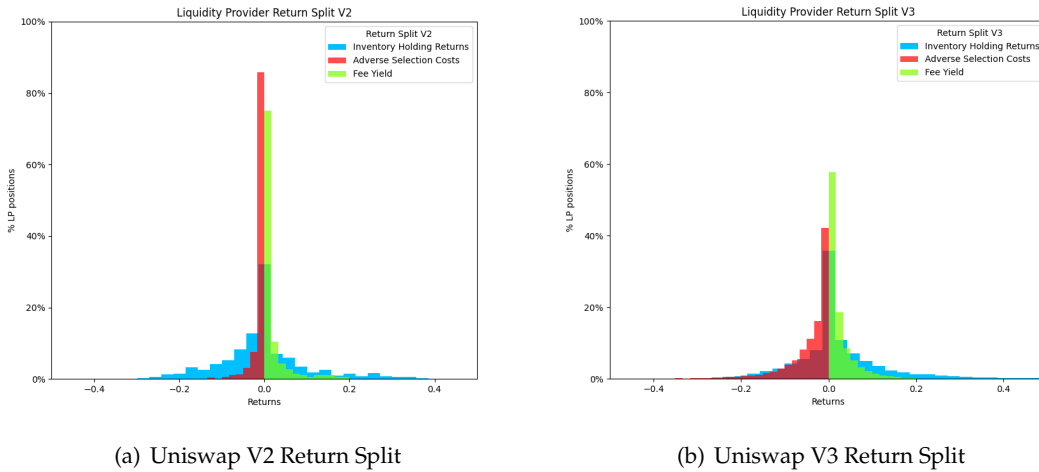


Figure 8: Return Split Distribution

From looking at the plot we observe that the distribution of Inventory Holding Returns are relatively similar between Uniswap V2 and Uniswap V3. Never-

theless, the mean inventory holding returns of V2, $V2 = 0.008755$, is statistically significantly lower than for V3, $V3 = 0.024141$; $t(15843) = -3.7381$, $p = 0.0001$ (table 22). Furthermore, we note that the distributions for the Adverse Selection Costs and Fee Yield differ significantly. In Uniswap V2 the majority of liquidity positions have small Fee Yield as well as small Adverse Selection Costs. The opposite holds for liquidity positions in Uniswap V3, there are more liquidity positions that have higher Fee Yields, but also lower (more negative) Adverse Selection Costs. This gives evidence to the claim that the Fee Yields are in fact a compensation for the risks that are taken by the liquidity provider. As Uniswap V3 suffers from a higher risk of Impermanent Loss, the Fee Yields are expected to be higher as well. To get an even better understanding of the composition of returns as well as the total returns, we summarise the data below.

V3	FeeYield	AdSelCost	InvHolReturn	Total
mean	0.02325	-0.04131	0.01893	0.00087
std	0.03358	0.06826	0.11465	0.09905
min	0.00000	-1.63148	-0.49518	-0.69521
25%	0.00294	-0.05320	-0.01821	-0.03305
50%	0.01084	-0.01729	0.00000	0.00012
75%	0.02865	-0.00077	0.04245	0.03732
max	0.39567	0.00000	1.19986	0.64752
V2	FeeYield	AdSelCost	InvHolReturn	Total
mean	0.01745	-0.00719	0.00124	0.01150
std	0.03382	0.01652	0.10618	0.10616
min	0.00000	-0.12988	-0.29235	-0.31037
25%	0.00043	-0.00554	-0.04113	-0.02885
50%	0.00448	-0.00058	-0.00069	0.00003
75%	0.01652	-0.00001	0.02862	0.04317
max	0.66411	0.00000	0.66411	0.65806

Table 3: Return Split Summary

The table shows that the mean fee yield of V2, $V2 = 0.021404$, is statistically significantly lower than for V3, $V3 = 0.025359$; $t(15843) = -3.2515$, $p = 0.0006$ (table 20). However, the mean Adverse Selection costs of V2, $V2 = -0.006136$, is statistically significantly higher than for V3, $V3 = -0.047170$; $t(15843) = 17.0171$, $p = 0.0000^5$ (table 21) as well. The mean Inventory Holding Return for both is positive, with a median around 0. When looking at the Total Return, we see that the mean of Uniswap V2 is statistically significantly higher (table 23). Hence, taking Adverse Selection Costs and Inventory Holding Returns into account, a liquidity provider is likely better off staking her tokens in Uniswap V2. Nevertheless, the median of both Total Returns is around 0. That is, almost half of the Liquidity Providers make losses from providing liquidity in Uniswap V2 as well as in Uniswap V3.

⁵p-value extremely close to 0, but not exactly 0

5.2.1 Active vs Passive Strategies

As the liquidity provision strategies are considered important (in theory) when providing liquidity, we now distinguish between active and passive liquidity provision strategies. A liquidity position is considered an active strategy if it falls in the 1st quantile of the duration distribution, and a liquidity position is considered a passive strategy if it falls in the 4th quantile of the duration distribution (table 17, 18). We summarise the data below.

	Active				Passive			
V3	FeeYield	AdSelCos	InvHol	Tot	FeeYield	AdSelCos	InvHol	Tot
Mean	0.00960	-0.02424	0.00308	-0.01156	0.04101	-0.06833	0.04799	0.02067
Std	0.01598	0.03542	0.04873	0.05203	0.04729	0.09236	0.16047	0.13528
Min	0.00000	-0.69778	-0.29712	-0.33244	0.00000	-0.91433	-0.43039	-0.63610
25%	0.00127	-0.03479	0.00000	-0.02647	0.00800	-0.09084	-0.03198	-0.04315
50%	0.00427	-0.01196	0.00000	-0.00111	0.02268	-0.03522	0.00628	0.00885
75%	0.01163	-0.00077	0.00662	0.00453	0.05915	-0.00917	0.09618	0.09048
Max	0.23238	0.00000	0.69458	0.32468	0.39567	0.00000	1.15120	0.62452
V2	Active				Passive			
Mean	0.00086	-0.00021	-0.00004	0.00061	0.03948	-0.00865	0.00483	0.03566
Std	0.00250	0.00084	0.02425	0.02538	0.05860	0.02110	0.012726	0.13801
Min	0.00000	-0.00760	-0.09448	-0.09560	0.00000	-0.09926	-0.024301	-0.24059
25%	0.00000	-0.00010	-0.00663	-0.00595	0.00151	-0.00700	-0.06116	-0.04130
50%	0.00010	-0.00001	-0.00014	-0.00011	0.00998	-0.00135	-0.00932	0.00024
75%	0.00067	0.00000	0.00214	0.00243	0.04888	-0.00006	0.03742	0.09014
Max	0.02238	0.00000	0.17218	0.17943	0.22068	0.00000	0.66411	0.65806

Table 4: Passive / Active strategies Returns

From the table we notice that the mean Fee Yield of active V2 liquidity providers, V2= 0.000863, is statistically significantly lower than for V3, V3= 0.009602 ; $t(3922) = -7.5322$, $p = 0.0000$ ⁶ (table 24). The mean Fee Yield of passive V2 liquidity providers is V2= 0.039481, the mean fee yield of passive V3 liquidity providers is V3= 0.041006 and we can not reject the H_0 hypothesis that the mean Fee Yields are the same; $t(3948) = -0.4561$, $p = 0.6483$ (table 25). Just as important as the Fee Yields are the Adverse Selection Costs as shown in the previous section. The average Adverse Selection Costs of active as well as of passive liquidity providers, is statistically significantly higher than for V3 (tables 26, 27). The observation that the Adverse Selection costs of passive liquidity providers in V2 is significantly higher than of V3 is rather interesting, as we could not reject the H_0 that the Fee Yields are the same. Since we expect Fee Yields to be a compensation for the risks taken by the liquidity provider, which includes Adverse Selection, this is in contradiction with the theory. Hence,

⁶p-value extremely close to 0, but not exactly 0

taking Inventory Holding Costs and Adverse Selection Costs into account, a liquidity provider with a passive strategy is better off (on average) providing liquidity in Uniswap V2 when looking at the total return (table 29). The same holds for liquidity providers with an active strategy as the average Total Return of active V2 liquidity providers, $V2 = 0.000613$, is statistically significantly higher than for V3, $V3 = -0.011556$; $t(3922) = 3.2045$, $p = 0.0007$ (table 28). In general the results show that passive liquidity provision strategies are more profitable than active liquidity provision strategies.

6 Discussion

As mentioned before, the data used in this paper is gathered from the Uniswap subgraph API. From actively participating in the Uniswap Labs Discord community, it became clear that not all researchers / DeFi enthusiasts trusted every result coming from the Uniswap subgraph API. Gathering the data directly from the blockchain itself would be a better alternative. Nevertheless, only the most pure data from the API has been used in this paper. So-called "derived data"⁷ from the API has not been used, as this data has especially been questioned by many DeFi enthusiasts participating in the Uniswap Labs Discord community. Furthermore, it is extremely important to mention that the Uniswap V3 fees are so-called collected fees. That is, only when a liquidity provider has decided to collect her fees, it is registered as Fee Yield in this paper. This implies that the actual Fee Yields for Uniswap V3 might be higher, which of course would change the results on liquidity provision profitability. However, it is not an unrealistic assumption that liquidity providers prefer to retrieve their fees after closing their position. In addition, as we only consider the USDC-ETH pools in this paper, it is not entirely fair to draw any general conclusions about the market depth and liquidity provision profitability of Uniswap V2 and V3. However, it should be acknowledged that the results of the USDC-ETH pool are likely to give a good indication of Uniswap V2 and V3 in general, as it is one of the largest pools present. Finally, transaction fees including so-called gas fees are ignored in this paper for simplicity. For that reason, the total returns will turn out lower in practice than reported.

⁷Data which is derived from pure Mints-, Burns-, Swap-s, Pool- and Fees- data by subgraph itself.

7 Conclusion

In this paper we compared the market depth, as well as the liquidity provision profitability of Uniswap V2 and V3. Recall that the research question is given by:

"Does Uniswap V3 offer more market depth where liquidity providers earn higher returns on their capital than in Uniswap V2?"

Firstly, we compared the market depths of both AMMs. The results show that the mean market depth of Uniswap V3 is a factor of 15x higher than the mean market depth of Uniswap V2 under a price impact of $\pm 2\%$. This difference diminishes, the higher the price impact is. This can be explained by the concentrated liquidity structure of Uniswap V3. Then, we compared the liquidity provision profitability of both AMMs by splitting the returns into three components; Fee Yield, Inventory Holding Returns and Adverse Selection Costs (Impermanent Loss). The results show that the mean Fee Yields of Uniswap V3 liquidity positions are higher than the mean Fee Yields of Uniswap V2 liquidity positions. Nevertheless, due to high Adverse Selection Costs associated with Uniswap V3, the average total returns of Uniswap V2 liquidity positions are higher. We distinguished between active and passive strategies to get an even better understanding of the liquidity provision profitability of both AMMs. The results show that the mean Fee Yields of active Uniswap V2 liquidity positions are higher than the mean Fee Yields of active Uniswap V3 liquidity positions. However, for passive strategies, we can not conclude that the mean Fee Yields differ. Again, taking Inventory Holding Costs and Adverse Selection Costs into account, a liquidity provider with a passive strategy is better off providing liquidity in Uniswap V2 when looking at the mean total return. The same holds for liquidity providers with an active strategy. In general, passive liquidity provision strategies outperform active liquidity provision strategies on average.

Hence, we conclude that Uniswap V3 offers a higher market depth than Uniswap V2. Despite the higher market depth of Uniswap V3 (which is favourable for liquidity demanders), the results show that Uniswap V2 is more profitable for liquidity providers when looking at the total returns, for both active as well as passive liquidity provision strategies. When solely looking at the Fee Yields, this can not be concluded for passive liquidity provision strategies, but can be concluded for active liquidity provision strategies.

For further research it would be interesting to look into active liquidity provision strategies for Uniswap V3. Specifically, one could approach this by a stochastic optimal control problem, where the main parameters would be the upper and lower bounds together with a reset indicator. If one is more interested in empirical analysis, it would be interesting to extend this research paper by including more liquidity pools/pairs for comparison. Another idea would

be to zoom in even further on the market depths, specifically by looking into the relationship between the market depth and volatility of price returns. A final and last suggestion would be to look at the results from the perspective of the liquidity demander, instead of the perspective of liquidity providers. Due to the higher market depth of Uniswap V3, liquidity demanders will likely be better off in Uniswap V3. However, no empirical studies have shown this yet.

8 Appendix: Tables

8.1 Descriptive Statistics

Independent t-test results	Results
Difference (V2 Mints - V3 Mints) =	-176.6882
Degrees of freedom =	761.0000
t =	-18.6601
Two side test p value =	0.0000
Difference <0 p value =	0.0000
Difference >0 p value =	1.0000

Table 5: t-test mean Mints

Independent t-test results	Results
Difference (V2 Burns - V3 Burns) =	-176.9833
Degrees of freedom =	761.0000
t =	-26.1598
Two side test p value =	0.0000
Difference <0 p value =	0.0000
Difference >0 p value =	1.0000

Table 6: t-test mean Burns

Independent t-test results	Results
Difference (V2 Swaps - V3 Swaps) =	-1726.2714
Degrees of freedom =	779.0000
t =	-12.0618
Two side test p value =	0.0000
Difference <0 p value =	0.0000
Difference >0 p value =	1.0000

Table 7: t-test mean Swaps

Mean	9.589189
Std	10.386508
Min	1.000000
25%	3.000000
50%	6.000000
75%	12.000000
Max	56.000000

Table 8: V2 Mints Distribution

Mean	186.277354
Std	181.849517
Min	5.000000
25%	87.000000
50%	131.000000
75%	223.000000
Max	2079.000000

Table 9: V3 Mints Distribution

Mean	12.545946
Std	20.491503
Min	1.000000
25%	4.000000
50%	6.000000
75%	12.000000
Max	238.000000

Table 10: V2 Burns Distribution

Mean	189.529262
Std	128.603884
Min	1.000000
25%	102.000000
50%	143.000000
75%	245.000000
Max	715.000000

Table 11: V3 Burns Distribution

Mean	3616.865979
Std	1643.439620
Min	1428.000000
25%	2452.000000
50%	3050.000000
75%	4406.250000
Max	11309.000000

Table 12: V2 Swaps Distribution

Mean	5343.137405
Std	2298.019214
Min	4.000000
25%	4201.000000
50%	5291.000000
75%	6674.000000
Max	16891.000000

Table 13: V3 Swaps Distribution

8.2 Market Depth

Independent t-test results	Results
Difference (V2 Depth - V3 Depth) =	-31.6374
Degrees of freedom =	780.0000
t =	-38.1830
Two side test p value =	0.0000
Difference <0 p value =	0.0000
Difference >0 p value =	1.0000

Table 14: t-test mean Market Depth $\delta = +/-2\%$

Mean	33.989686
Std	16.378464
Min	1.861756
25%	21.724994
50%	33.332417
75%	42.724256
Max	83.962340

Table 15: V3 Market Depth Distribution for $\delta = +/-2\%$

Mean	2.352261
Std	0.424747
Min	1.511643
25%	2.042004
50%	2.203252
75%	2.713710
Max	3.836693

Table 16: V2 Market Depth Distribution for $\delta = +/-2\%$

8.3 Duration

Mean	234 days 10:25:12
Std	138 days 12:03:49
Min	0 days 00:00:00
25%	94 days 01:13:17
50%	299 days 05:43:19
75%	358 days 01:14:57
Max	391 days 17:42:07

Table 17: V2 Duration Distribution

Mean	97 days 03:46:19
Std	135 days 15:22:07
Min	0 days 00:00:00
25%	0 days 18:53:00
50%	16 days 20:23:37
75%	191 days 09:02:18
Max	391 days 23:54:13

Table 18: V3 Duration Distribution

Independent t-test results	Results
Difference (V3 Duration - V2 Duration) =	-137.2294
Degrees of freedom =	73043.0000
t =	-59.1474
Two side test p value =	0.0000
Difference <0 p value =	0.0000
Difference >0 p value =	1.0000

Table 19: t-test Duration

8.4 Liquidity Provision Profitability

Independent t-test results	Results
Difference (V2 Fee Yield - V3 Fee Yield) =	-0.0040
Degrees of freedom =	15843.0000
t =	-3.2515
Two side test p value =	0.0012
Difference <0 p value =	0.0006
Difference >0 p value =	0.9994

Table 20: t-test mean Fee Yields

Independent t-test results	Results
Difference (V2 AdSelCos - V3 AdSelCos) =	0.0410
Degrees of freedom =	15843.0000
t =	17.0171
Two side test p value =	0.0000
Difference <0 p value =	1.0000
Difference >0 p value =	0.0000

Table 21: t-test mean Adverse Selection Costs

Independent t-test results	Results
Difference (V2 InvHolRet - V3 InvHolRet) =	-0.0154
Degrees of freedom =	15843.0000
t =	-3.7381
Two side test p value =	0.0002
Difference <0 p value =	0.0001
Difference >0 p value =	0.9999

Table 22: t-test mean Inventory Holding Return

Independent t-test results	Results
Difference (V2 Tot - V3 Tot) =	0.0217
Degrees of freedom =	15843.0000
t =	6.0319
Two side test p value =	0.0000
Difference <0 p value =	1.0000
Difference >0 p value =	0.0000

Table 23: t-test mean Total Returns

8.4.1 Active vs Passive Returns

Independent t-test results	Results
Difference (V2 Active Fee Yield - V3 Active Fee Yield) =	-0.0087
Degrees of freedom =	3922.0000
t =	-7.5322
Two side test p value =	0.0000
Difference <0 p value =	0.0000
Difference >0 p value =	1.0000

Table 24: t-test mean active Fee yields

Independent t-test results	Results
Difference (V2 Passive Fee Yield - V3 Passive Fee Yield) =	-0.0015
Degrees of freedom =	3948.0000
t =	-0.4561
Two side test p value =	0.6483
Difference <0 p value =	0.3242
Difference >0 p value =	0.6758

Table 25: t-test mean passive Fee yields

Independent t-test results	Results
Difference (V2 Active AdSelCos - V3 Active AdSelCos) =	0.0240
Degrees of freedom =	3922.0000
t =	9.3463
Two side test p value =	0.0000
Difference <0 p value =	1.0000
Difference >0 p value =	0.0000

Table 26: t-test mean active Adverse Selection Costs

Independent t-test results	Results
Difference (V2 Passive AdSelCos - V3 Passive AdSelCos) =	0.0597
Degrees of freedom =	3948.0000
t =	9.5243
Two side test p value =	0.0000
Difference <0 p value =	1.0000
Difference >0 p value =	0.0000

Table 27: t-test mean passive Adverse Selection Costs

Independent t-test results	Results
Difference (V2 Active Tot - V3 Active Tot) =	0.0122
Degrees of freedom =	3922.0000
t =	3.2045
Two side test p value =	0.0014
Difference <0 p value =	0.9993
Difference >0 p value =	0.0007

Table 28: t-test mean active Total Returns

Independent t-test results	Results
Difference (V2 Passive Tot - V3 Passive Tot) =	0.0150
Degrees of freedom =	3948.0000
t =	1.5882
Two side test p value =	0.1123
Difference <0 p value =	0.9438
Difference >0 p value =	0.0562

Table 29: t-test mean passive Total Returns

9 References

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