CS222 Homework 4

Algorithm Analysis & Deadline: 2020-11-04 Wednesday 23:59

Exercises for Algorithm Design and Analysis by Li Jiang, 2020 Autumn Semester

1. Given a integer set S, which has n elements in it, you need to divide this set into 2 subarray, where the subarray S1 has m elements, and the subarray S2 has (n-m) elements. the target is minimizing $abs(sum(S1) - sum(S2))$.

   Implement your algorithm with C/C++/Python. Please attach your source code named as *Code-P1.**. The file `Data-P1.txt` is a test case, includes an 1-D array of integer with random length, and your program needs to output the final sum. You need to briefly describe your algorithm and find the final sum of `Data-P1.txt` by your program.

   textbfExample:

   Given input array: $arr[] = 1, 6, 11, 5$. The algorithm should return 1. The subset $S1 = [1, 5, 6], S2 = [11]$.

   Solution. *Uncomment this block to write your proof.*

2. **Bookshelf:** Tim has n books and he wants to make a bookshelf to them. The pages' width of the i-th book is w_i and the thickness is t_i.

   Tim puts the books on the bookshelf in the following way. He selects some books and put them vertically. Then the rest of the books are put horizontally above the vertical books. Obviously, the total thickness of the books put vertically must be greater than the sum of widths of the horizontal books. As long as tim wants to make the bookshelf as small as possible, please help him to find the minimum total thickness of the vertical books.

   To simplify the problem, the thickness of each book is either 1 or 2. And all the numbers in this problem are positive integers.

   Design an algorithm based on dynamic programming and implement it in C/C++/Python. The file `Data-P2.txt` is a test case, where the first line contains an integer n. Each of the next n lines contains two integers t_i and w_i denoting two attributes of the i-th book. Source code should be named as *Code-P2.** .You need to briefly describe your algorithm and find the result of `Data-P2.txt` by your program.

   Example:

   Given $n = 5$ books, and $\{(t_i, w_i)|1 \leq i \leq 5\} = \{(1, 12), (1, 3), (2, 15), (2, 5), (2, 1)\}$. The algorithm should return 5.

   Solution. *Uncomment this block to write your proof.*

3. Recall the *String Similarity* problem in class, in which we calculate the edit distance between two strings in a sequence alignment manner.

   You are to find the lowest aligning cost between 2 DNA sequences, in which the cost matrix is defined as

	-	A	T	G	C
-	0	1	2	1	3
A	1	0	1	5	1
T	2	1	0	9	1
G	1	5	9	0	1
C	3	1	1	1	0

where (-, A) means adding (or removing) one A, etc.

   (a) Implement Hirschberg's algorithm with C/C++/Python. Please attach your source code named as *Code-P3.**. Your program will be tested against random inputs. Your program should be able to output two sequences after editing.

   (b) Using your program, find the edit distance between the two DNA sequences found in attachments `Data-P3a.txt` and `Data-P3b.txt`.

Solution. *Uncomment this block to write your proof.*

4. Considering you are playing a game with your friend, there are n coins placed in one row with the value of v1, v2, ..., vn, respectively. You and your friend can take one coin from row head or row tail sequentially. If you are the first to choose the coin, write an algorithm to ensure that you can get the maximum profit. Suppose that your friend is as smart as you.
Implement your algorithm with C/C++/Python. Please attach your souce code named as *Code-P4.**. The file `Data-P4.txt` is a test case, includes an 1-D array of unsigned integer with random length, and your program needs to output the the coin value list that you choose, and the maximum profit you will get. You need to briefly describe your algorithm and find the final answer of `Data-P4.txt` by your program.

textbfExample:

Given input array: $arr[] = 8, 15, 3, 7$. The algorithm should return your choice list [7, 15], and the final profit 22.

Solution. *Uncomment this block to write your proof.*

Remark: You need to upload your .pdf file and write the pseudocode.