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# Notes on stability and causality in the z-domain

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In this article, we demonstrate the relationship between z-domain zero-pole plot and the time-domain plot of a signal.

## Case 1: Pole lies on the unit circle

$$X(z) = \frac{z^{-1}}{1 + z^{-1}}$$

```
b1 = [0 1]; a1=[1 1];  
[d,n]=impseq(0,-8,8);  
x1 = filter(b1,a1,d);
```

## Case 2: Pole lies inside the unit circle

$$X(z) = \frac{z^{-1}}{1 + (1/2)z^{-1}}$$

```
b2 = [0 1]; a2=[1 0.5];  
x2 = filter(b2,a2,d);
```

## Case 3: Pole lies outside the unit circle

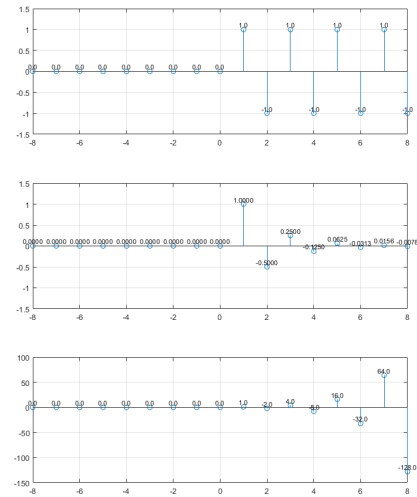
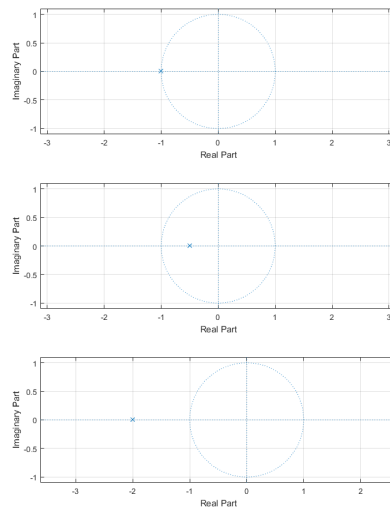
$$X(z) = \frac{z^{-1}}{1 + 2z^{-1}}$$

```
b3 = [0 1]; a3 = [1 2];  
x3 = filter(b3,a3,d);
```

## Plots

```
figure('units','normalized','outerposition',[0,0,1,1])  
subplot(3,2,1); zplane(b1,a1);  
grid  
subplot(3,2,2); stem(n,x1);  
grid
```

```
ylim([-1.5,1.5])
lbls=sprintfc('%0.1f',x1);
text(n,x1,lbls,'HorizontalAlignment','center',...
'VerticalAlignment','bottom','FontSize',8)
subplot(3,2,3); zplane(b2,a2);
grid
subplot(3,2,4); stem(n,x2);
grid
ylim([-1.5,1.5])
lbls=sprintfc('%0.4f',x2);
text(n,x2,lbls,'HorizontalAlignment','center',...
'VerticalAlignment','bottom','FontSize',8)
subplot(3,2,5); zplane(b3,a3);
grid
subplot(3,2,6); stem(n,x3);
grid
ylim([-150,100])
lbls=sprintfc('%0.1f',x3);
text(n,x3,lbls,'HorizontalAlignment','center',...
'VerticalAlignment','bottom','FontSize',8)
```



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