Multi-Agent Reinforcement Learning

Multi-Agent Deep Reinforcement Learning – Part 2

Stefano V. Albrecht, Filippos Christianos, Lukas Schäfer

This lecture is based on

Multi-Agent Reinforcement Learning:
Foundations and Modern Approaches

by Stefano V. Albrecht, Filippos Christianos and
Lukas Schäfer

MIT Press, 2024

Download book, slides, and code at:
www.marl-book.com

Lecture Outline

- Agent modeling with deep learning
- Parameter and experience sharing
- Policy self-play in zero-sum games
- Population-based training

Agent Modeling with Deep Learning

Agents Modeling – Motivation

In MARL, agents need to consider the policies of other agents to coordinate their actions.

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Approaches presented so far account for the action selection of other agents through:

- Distribution of training data is dependent on the policies of all agents
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Problem

Can we provide agents with more **explicit** information about the policies of other agents so they can learn to coordinate better, e.g. by learning best-response policies?

Recap: Agent Modeling

Reminder

In Chapter 6, we have seen approaches that model other agents' policies:

- Learn models of other agents to predict their actions
- Compute optimal action (best-response) against agent models

| Observations (past states, actions, ...) | → | **Agent model** | → | Prediction (action, goal, beliefs, ...) |

S. Albrecht, P. Stone. Autonomous Agents Modelling Other Agents: A Comprehensive Survey and Open Problems. *Artificial Intelligence*, 2018

Recap: Tabular Agent Modeling

In Chapter 6, we modeled other agents' policies as stationary distributions by maintaining tables of applied action frequencies for each states

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Problem

Similar to tabular value functions, **tabular agent models** are limited due to their inability to generalise across states.

Solution

As for value functions, we can use **deep learning** to learn generalisable agent models!

Reminder

We have already seen joint-action learning with agent models (JAL-AM)

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- Agents learn value functions conditioned on the joint action: $Q_i(s, a)$

- Using the value function and agent models, agent i can compute its expected action values under the current models of other agents:

$$AV_i(s, a_i) = \sum_{a_{-i} \in A_{-i}} Q_i(s, \langle a_i, a_{-i} \rangle) \prod_{j \in I \setminus \{i\}} \hat{\pi}_j(a_j \mid s)$$

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- Use AV_i to select optimal actions and as learning update targets

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Joint-Action Learning with Deep Agent Models

How to do this with deep learning for partially observable environments (POSG)?

- Represent agent i's model of agent j as a neural network: $\hat{\pi}_j^i(a_j \mid h_i; \phi_j^i)$
- Given the observation history of agent i, its model $\hat{\pi}_j^i(a_j \mid h_i; \phi_j^i)$ for agent j outputs a probability distribution over its actions

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- Agent i can train its model for agent j by minimizing the cross-entropy loss between the predicted policy $\hat{\pi}_j^i$ and the observed actions of agent j:

$$\mathcal{L}(\phi_j^i) = \mathbb{E}_{a_j^t \sim \pi_j(h_j^t)} \left[-\log \hat{\pi}_j^i(a_j^t \mid h_i^t; \phi_j^i) \right]$$

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- Then, agent i can compute expected action values:

$$AV(h_i, a_i; \theta_i) = \sum_{a_{-i} \in A_{-i}} Q(h_i, \langle a_i, a_{-i} \rangle; \theta_i) \prod_{j \neq i} \hat{\pi}_j^i(a_j \mid h_i; \phi_j^i)$$

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Solution

Approximate *AV* with only *K* joint actions sampled from the agent models:

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To optimise the centralized joint-action-value function of agent *i*, we then minimize the following loss over batches of experiences sampled from a replay buffer:
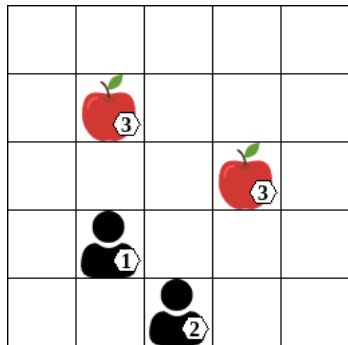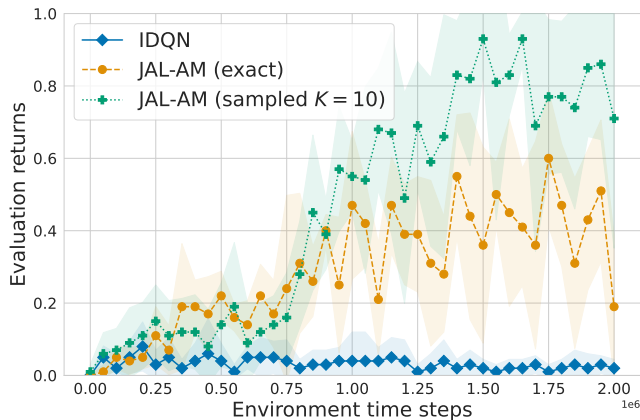
Joint-Action Learning with Deep Agent Models
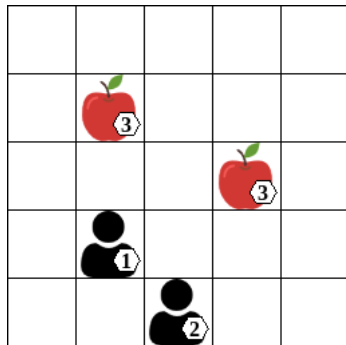
(a) Environment

(a) Environment

(b) Learning curve

Learning Compact Representations of Agent Policies

- JAL-AM combines agent models and centralized value functions to compute best-response policies.
- Can we integrate agent models into multi-agent policy gradient algorithms, e.g. by conditioning policies on agent models?

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Problem

To condition policies (and value functions) of agents on the policies of other agents, we need **compact** representations of the policies of other agents. How can we learn such representations?
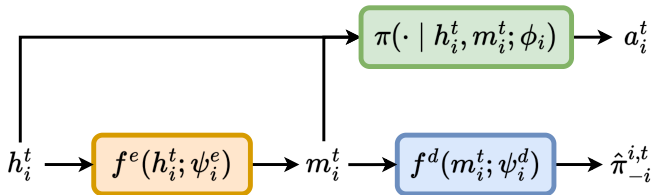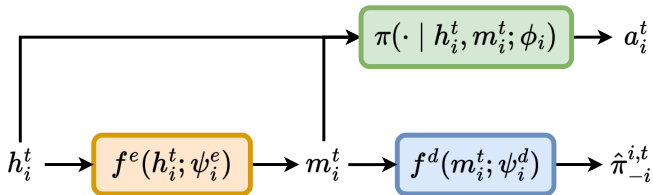
Agent *i* trains encoder-decoder architecture with ...

- Encoder f^e with parameters ψ_i^e: given observation history h_i^t of agent *i*, output compact representation m_i^t of the policies of other agents

- Decoder f^d with parameters ψ_i^d: given compact representation m_i^t, predict the policies $\hat{\pi}_{-i}^{i,t}$ of other agents

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- Decoder f^d with parameters ψ_i^d: given compact representation m_i^t, predict the policies $\hat{\pi}_{-i}^{i,t}$ of other agents

Then, agent *i* can condition its policy on the obtained compact representations m_i^t.

The encoder and decoder are jointly trained to minimize the cross-entropy loss for the predicted action probabilities and true actions of all other agents:

$$\mathcal{L}(\psi_i^e, \psi_i^d) = \sum_{j \neq i} - \log \hat{\pi}_j^{i,t}(a_j^t) \text{ with } \hat{\pi}_j^{i,t} = f^d \left(f^e(h_i^t; \psi_i^e); \psi_i^d \right)_j$$
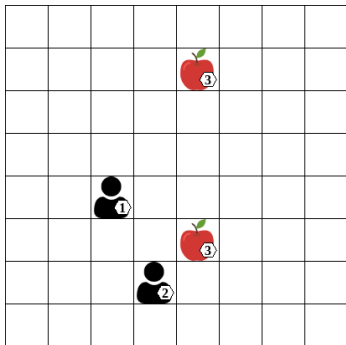
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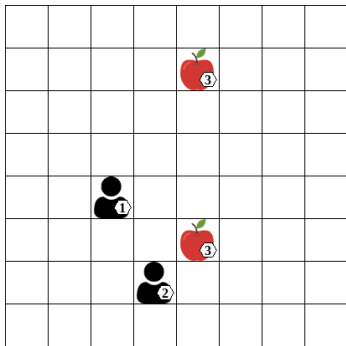
Solution

Encoder-decoder agent models can be integrated into any MARL algorithm by conditioning trained value functions and policies on the obtained policy representations.
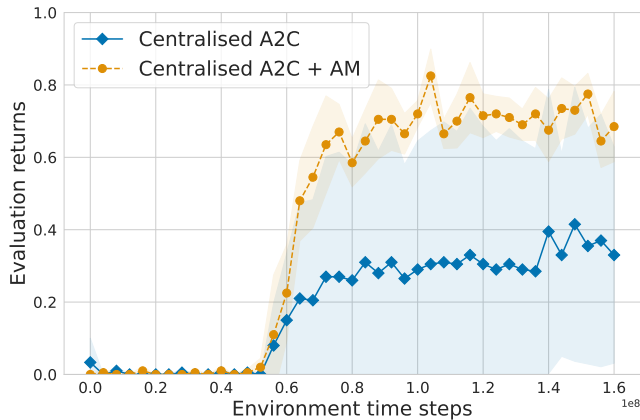
(a) Environment

(a) Environment

(b) Learning curve

$$\pi(\cdot \mid h_i^t, m_i^t; \phi_i) \rightarrow a_i^t$$

$$h_i^t \rightarrow f^e(h_i^t; \psi_i^e) \rightarrow m_i^t \rightarrow f^d(m_i^t; \psi_i^d) \rightarrow \text{???}$$
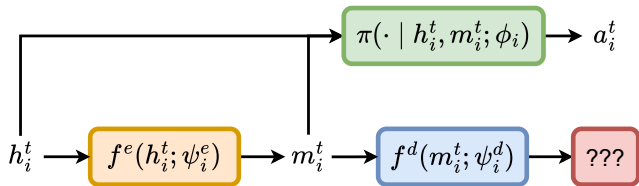
Note

So far, we used the ground truth actions as information to encode by using them as targets for the decoder. Instead or in addition, we could use

- Observations – try capture information that other agents have access
- Rewards – try to predict the objectives that other agents optimise for
- …

Parameter and Experience Sharing

Parameter and Experience Sharing – Motivation

Problem

Training agents with MARL becomes difficult for environments with many agents due to the increased number of parameters to train, resulting in unstable or slow training. How can we reduce these challenges?

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Solution

We will look at two approaches to improve the efficiency of training many agents:

- **Parameter sharing**: Agents share their network parameters with each other
- **Experience sharing**: Agents share experiences with each other

Environments with Homogeneous Agents

- Parameter and experience sharing assume that agents are trying to learn similar or identical policies → not applicable to all environments
- We call agents in such environments homogeneous

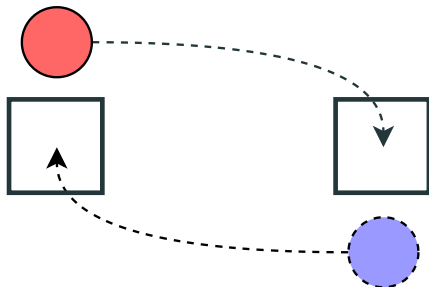## Environments with Homogeneous Agents

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  - Strongly homogeneous agents: All agents have the same optimal policy, i.e. $\pi_1^* = \ldots = \pi_n^*$

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  - Strongly homogeneous agents: All agents have the same optimal policy, i.e. $\pi_1^* = \ldots = \pi_n^*$
  - Weakly homogeneous agents: Agents can be permuted and their expected returns remain the same under the permutation $\sigma : I \mapsto I$:

$$U_i(\pi) = U_{\sigma(i)} \left(\langle \pi_{\sigma(1)}, \pi_{\sigma(2)}, \ldots, \pi_{\sigma(n)} \rangle \right), \ \forall i \in I$$
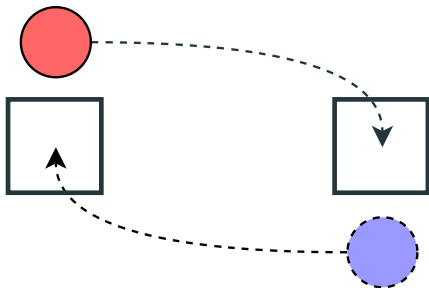
Weakly homogeneous agents:

Agents need to learn similar policies.

Environments with Homogeneous Agents – Examples

Weakly homogeneous agents:

Strongly homogeneous agents:

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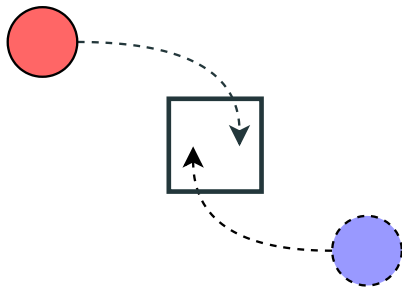
Agents need to learn identical policies.

Parameter Sharing

Sharing network parameters across agents is a common practice to make MARL training more efficient. We can share parameters across value functions, policies, or both.

No parameter sharing   Parameter sharing

Value functions

$h_1^t \longrightarrow V(\cdot; \theta_1)$
$\vdots \longrightarrow \vdots$
$h_n^t \longrightarrow V(\cdot; \theta_n)$

$h_i^t \longrightarrow V(\cdot; \theta)$

Policies

$h_1^t \longrightarrow \pi(\cdot; \phi_1)$
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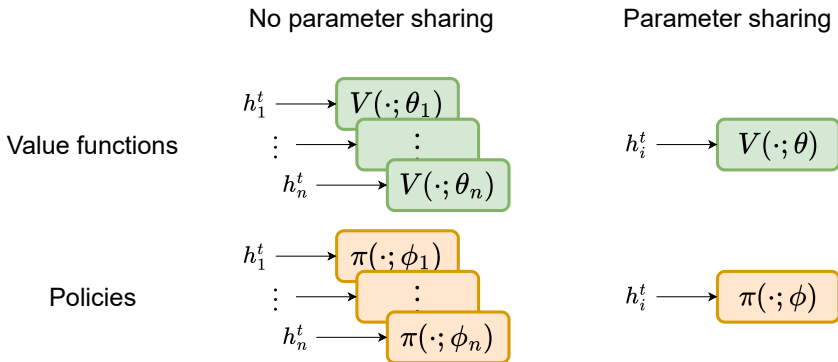
Parameter Sharing

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Parameter sharing has two primary benefits:

- **Scalability**: the number of parameters remains constant independent of the number of agents $\rightarrow$ less computational cost

- **Efficiency**: shared parameters are updated using the experiences of all agents $\rightarrow$ more training data for the shared parameters

The downside is that (naive) parameter sharing assumes strongly homogeneous agents.
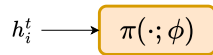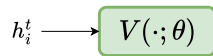
Parameter sharing

$$h_i^t \longrightarrow \boxed{V(\cdot; \theta)}$$

$$h_i^t \longrightarrow \boxed{\pi(\cdot; \phi)}$$

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> **Note**
>
> The experiences of agent j is **off-policy** data for agent i → experience sharing needs to use off-policy MARL algorithms or correct for the differences in data distributions.

Deep Q-Networks with Shared Experience Replay

We can extend IDQN with experience sharing by following the steps below:

- Collect the experience of all agents in a shared replay buffer $\mathcal{D}_{\text{shared}}$
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Note

DQN is an off-policy algorithm so it is theoretically sound to use the experience of other agents that have different policies.

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SEAC policy loss (based on IA2C) with importance sampling (IS) weight correction:

policy loss on own data

$$\mathcal{L}(\phi_i) = -\left(r_i^t + \gamma V(h_i^{t+1}; \theta_i) - V(h_i^t; \theta_i)\right) \log \pi(a_i^t \mid h_i^t; \phi_i)$$

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$$

IS correction

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Hyperparameter λ determines weighting for the loss over the experience of other agents. The same IS weight correction can be applied to the critic loss.

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Policy Self-Play in Zero-Sum Games

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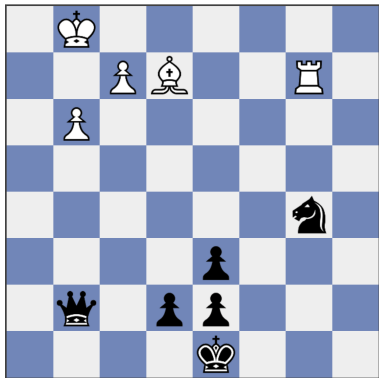
(a) Chess

(b) Go

The Challenge of Zero-Sum Board Games

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These games are challenging decision-making problems due to

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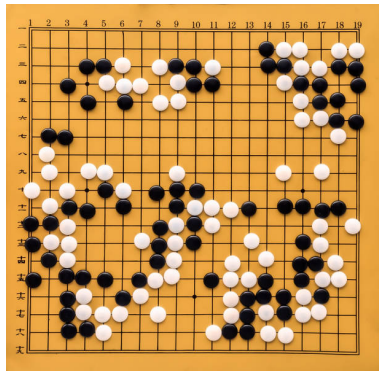
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Fortunately, we can exploit the structure of these games to develop effective algorithms.

s^0

a_1^0

s^1

a_2^1

s^2

a_1^2

s^3

Agent 1's turn

Agent 2's turn

Tree Search for Zero-Sum Games
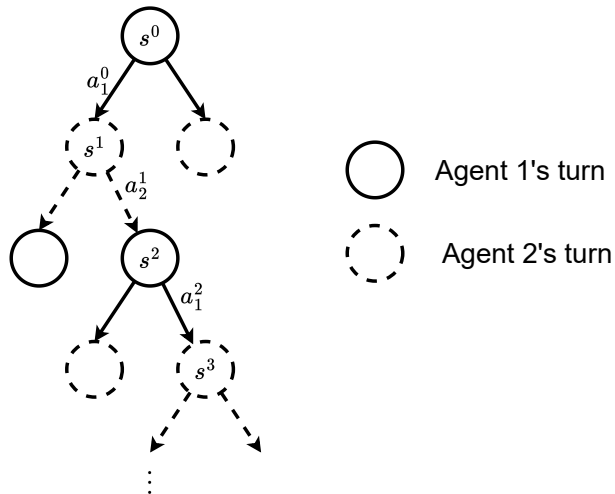
We can view turn-based zero-sum games as trees where

- Nodes represent game states
- Edges represent actions
- Leaves represent terminal states

and in each node either agent 1 or agent 2 makes a move.
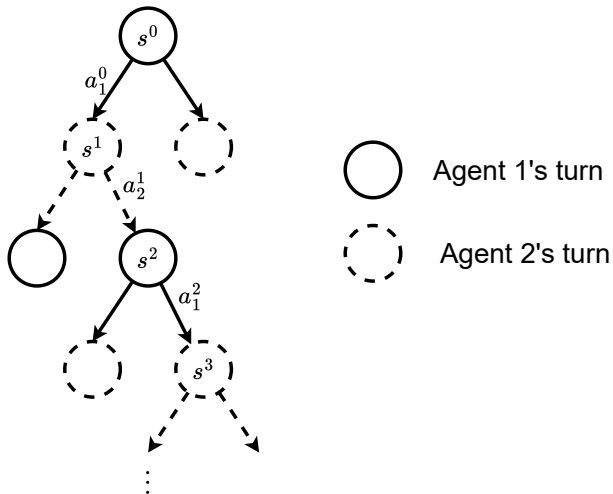
Problem

The tree can grow very large depending on its

- **Depth**: number of time steps until terminal states

- **Breadth**: number of actions available in each state

$\rightarrow$ makes search computationally expensive

Monte Carlo Tree Search (MCTS)

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- **Simulation** simulate a game from the current state until a leaf node is reached
- **Expansion:** expand the tree by adding a new node for the reached state if it does not exist yet
- **Backpropagation:** update the estimated values of the nodes visited during the selection step

Monte Carlo Tree Search – Simulation

MCTS maintains two statistics for each visited state-action pair:

- Value estimates $Q(s, a)$
- Visitation counts $N(s, a)$

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- Visitation counts $N(s, a)$

MCTS generates k simulations from the current state by sampling actions according to a policy until a leaf node is reached. Leaf nodes are previously unvisited or terminal states.

Monte Carlo Tree Search – Simulation

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To sample actions, MCTS commonly uses ϵ-greedy policies with respect to action-value function Q, or the upper confidence bound (UCB) policy:

$$
\hat{a}^\tau = \begin{cases} \hat{a} & \text{if } N(\hat{s}^\tau, \hat{a}) = 0 \\ \arg\max_{\hat{a} \in A} \left(Q(\hat{s}^\tau, \hat{a}) + \sqrt{\frac{2 \ln N(\hat{s}^\tau)}{N(\hat{s}^\tau, \hat{a})}} \right) & \text{otherwise} \end{cases}
$$
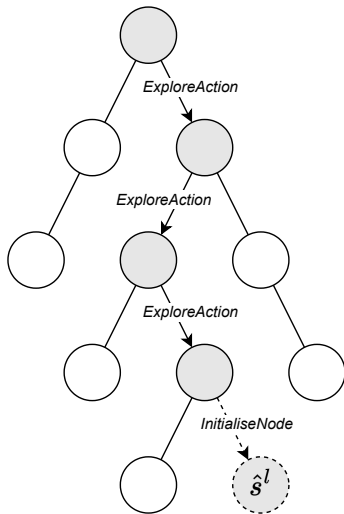
If leaf node is reached $\rightarrow$ expand the search tree by adding a new node for the reached state $\hat{s}^l$
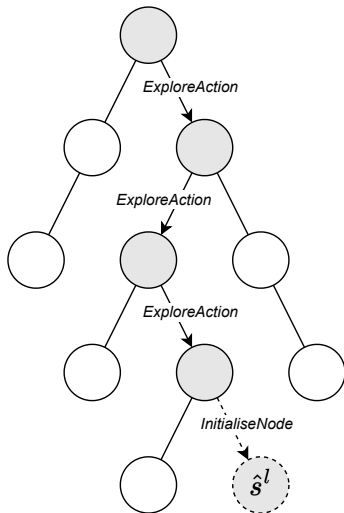
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The new node is initialized with

- $N(\hat{s}^l, \hat{a}) = 0$ for all actions $\hat{a}$
- An initial value estimate $Q(\hat{s}^l, \hat{a})$ for all actions $\hat{a}$, e.g. from a learned value function, heuristic, or random samples of outcomes.

Once a value estimate u for the leaf node is obtained $\rightarrow$ backpropagate rewards and value estimates starting from the leaf node up to the root node.

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For each visited state-action pair $(\hat{s}^\tau, \hat{a}^\tau)$, we increment the visitation count and update the value:

$$Q(\hat{s}^\tau, \hat{a}^\tau) \leftarrow Q(\hat{s}^\tau, \hat{a}^\tau) + \frac{1}{N(\hat{s}^\tau, \hat{a}^\tau)} \left[u - Q(\hat{s}^\tau, \hat{a}^\tau)\right]$$

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(Any RL TD update rule can be used to update the value estimates.)

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Following k simulations form the current state s^t, the best action is selected. This process can be done by choosing the action with:

- highest value estimate: $BestAction(s^t) = \arg\max_{\hat{a} \in A} Q(s^t, \hat{a})$
- highest visitation count: $BestAction(s^t) = \arg\max_{\hat{a} \in A} N(s^t, \hat{a})$

Monte Carlo Tree Search – Pseudocode

Algorithm Monte Carlo tree search (MCTS) for MDPs

1: Repeat for every episode:
2: **for** $t = 0, 1, 2, 3, \ldots$ **do**
3:     Observe current state s^t
4:     **for** k simulations **do**
5:         $\tau \leftarrow t$
6:         $\hat{s}^\tau \leftarrow s^t$ ▷ Perform simulation
7:         **while** $\hat{s}^\tau$ is non-terminal and $\hat{s}^\tau$-node exists in tree **do**
8:             $\hat{a}^\tau \leftarrow$ *ExploreAction*$(\hat{s}^\tau)$
9:             $\hat{s}^{\tau+1} \sim \mathcal{T}(\cdot \mid \hat{s}^\tau, \hat{a}^\tau)$
10:            $\hat{r}^\tau \leftarrow \mathcal{R}(\hat{s}^\tau, \hat{a}^\tau, \hat{s}^{\tau+1})$
11:            $\tau \leftarrow \tau + 1$
12:         **if** $\hat{s}^\tau$-node does not exist in tree **then**
13:            *InitializeNode*$(\hat{s}^\tau)$ ▷ Expand tree
14:         **while** $\tau > t$ **do**   ▷ Backpropagate
15:            $\tau \leftarrow \tau - 1$
16:            *Update*$(Q, \hat{s}^\tau, \hat{a}^\tau)$
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MCTS assumes known transition function $\mathcal{T}$ and reward function $\mathcal{R}$.

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Can learn estimates of these functions from data to simulate possible outcomes of the game.

Self-Play Monte Carlo Tree Search

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In zero-sum games with symmetrical roles and egocentric observations, agents can use the same policy to control both players → learn a policy in self-play
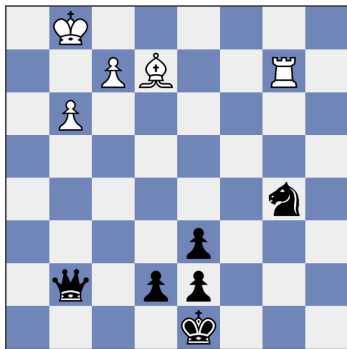
(a) Agent 1 perspective
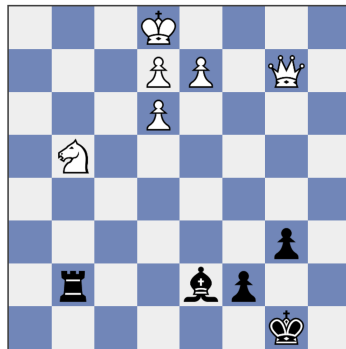
(b) Agent 2 perspective

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For each episode, a triplet (s, π, z) of data is stored where

- s are the states

- π are policy distributions computed by *BestAction*

- z is the game outcome (+1 for win, -1 for loss, 0 for draw)

AlphaZero – Self-Play MCTS with Deep Learning

The network is randomly initialized and trained using sampled batches of data to minimise the following combined loss:

$$\mathcal{L}(\theta) = \mathcal{L}_{\text{value}} + \mathcal{L}_{\text{policy}} + c \, ||\theta||^2$$

$$\mathcal{L}_{\text{value}} = \mathbb{E}_{(s,\pi,z)\sim\mathcal{D}}\Big[(V(s;\theta) - u)^2\Big]$$

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For exploration, AlphaZero combines a UCB policy with the learned policy:

$$\hat{a}^\tau = \begin{cases} \hat{a} & \text{if } N(\hat{s}^\tau, \hat{a}) = 0 \\ \arg\max_{\hat{a}\in A}\left(Q(\hat{s}^\tau, \hat{a}) + C(\hat{s}^\tau)P(\hat{s}^\tau, \hat{a})\frac{\sqrt{N(\hat{s}^\tau)}}{1+N(\hat{s}^\tau,\hat{a})}\right) & \text{otherwise} \end{cases}$$

with the additional exploration rate $C(\hat{s}^\tau)$.

Population-Based Training

Population-Based Training – Self-Play for General-Sum Games

Problem

With MCTS, we focused on policy self-play in **two-agent zero-sum** games. Can we extend the idea of self-play to **general-sum** games with **more than two agents**?

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Population-based training is a generalisation of self-play to general-sum games:

- Maintain a **population of policies** representing possible strategies of the agent
- Evolve populations so they become more effective against the populations of other agents
- We denote the population of policies for agent i at generation k as Π_i^k.

First, initialize a population of policies for each agent (e.g. random policies).

Population-Based Training – Overview

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For each generation, population-based training then follows two steps:

- **Evaluation:** evaluate the performance of the policies in the population by playing games against policies of the current populations of all other agents

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Policy space response oracles (PSRO) is a population-based MARL algorithm with the following steps:

1. Initialize populations of random policies for each agent
2. Construct a **meta-game** M^k at generation k from the populations of all agents as a stateless single-step general-sum game with
   - Actions: policies of agent populations, i.e. $A_i = \Pi_i^k$
   - Rewards: returns of joint policies $\langle \pi_1, \ldots, \pi_n \rangle$, i.e. $\mathcal{R}_i(\pi_1, \ldots, \pi_n) = U_i(\pi_1, \ldots, \pi_n)$.

M^k	$\pi_2^{(1)}$	$\pi_2^{(2)}$	$\cdots$	$\pi_2^{(k)}$
$\pi_1^{(1)}$	$0,1$	$1,2$	$\cdots$	$0,3$
$\pi_1^{(2)}$	$2,1$	$0,1$	$\cdots$	$1,1$
$\vdots$	$\vdots$	$\vdots$	$\cdot$	$\vdots$
$\pi_1^{(k)}$	$5,1$	$0,1$	$\cdots$	$4,3$

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Problem

Need to compute the expected returns of each agent for any joint policy $\langle \pi_1, \ldots, \pi_n \rangle$ in the meta-game. How can we do this?

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Problem

Need to compute the expected returns of each agent for any joint policy $\langle \pi_1, \ldots, \pi_n \rangle$ in the meta-game. How can we do this?

Solution

Compute average returns of each agent over multiple episodes of the underlying game with respective joint policy $\rightarrow$ converges to expected returns in the limit.

M^k	$\pi_2^{(1)}$	$\pi_2^{(2)}$	$\cdots$	$\pi_2^{(k)}$
$\pi_1^{(1)}$	$0, 1$	$1, 2$	$\cdots$	$0, 3$
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$\vdots$	$\vdots$	$\vdots$	$\cdot$	$\vdots$
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δ_1^k/δ_2^k

Compute a **solution to the meta-game** M^k following some solution concept, e.g. a Nash equilibrium.

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Compute a **solution to the meta-game** M^k following some solution concept, e.g. a Nash equilibrium.

Solution to the meta-game: distributions δ_i^k over policies in the population of agent i for each agent $i \in I$ that satisfies the solution concept.

M^k	$\pi_2^{(1)}$	$\pi_2^{(2)}$	$\cdots$	$\pi_2^{(k)}$	π_2'
$\pi_1^{(1)}$	$0,1$	$1,2$	$\cdots$	$0,3$	?
$\pi_1^{(2)}$	$2,1$	$0,1$	$\cdots$	$1,1$	?
$\vdots$	$\vdots$	$\vdots$	$\cdot$	$\vdots$	?
$\pi_1^{(k)}$	$5,1$	$0,1$	$\cdots$	$4,3$	?
π_1'	?	?	?	?	?

Each agent i determines an effective **oracle policy** π_i' against the solution distribution of the other agents and adds this policy to its population:

$$\Pi_i^{k+1} = \Pi_i^k \cup \{\pi_i'\}$$

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For example, agent i might determine its best-response policy

$$\pi_i' \in \arg\max_{\pi_i} \mathbb{E}_{\pi_{-i} \sim \delta_{-i}^k}[U_i(\langle \pi_i, \pi_{-i} \rangle)]$$

by training a policy π_i' using RL against sampled policies of the other agents.

Policy Space Response Oracles – Pseudocode

Algorithm Policy space response oracles (PSRO)

1: Initialize populations Π_i^1 for all $i \in I$ (e.g., random policies)
2: **for** each generation $k = 1, 2, 3, \ldots$ **do**
3:      Construct meta-game M^k from current populations $\{\Pi_i^k\}_{i \in I}$
4:      Use meta-solver on M^k to obtain distributions $\{\delta_i^k\}_{i \in I}$
5:      **for** each agent $i \in I$ **do**   ▷ Train best-response policies
6:          **for** each episode $e = 1, 2, 3, \ldots$ **do**
7:              Sample policies for other agents $\pi_{-i} \sim \delta_{-i}^k$
8:              Use single-agent RL to train π_i' wrt. π_{-i} in underlying game G
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Repeat this process until policy populations converge (new best-response policies are already in the respective populations), or for a fixed number of generations.   41

Policy Space Response Oracles in Rock-Paper-Scissors

If PSRO computes exact Nash equilibria solutions to the meta-game, and computes exact best-response policies, then the population distributions of PSRO converge to the Nash equilibrium of the underlying game.

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For example for rock-paper-scissors for two agents, with initial populations deterministically choosing rock and paper, respectively:

k	Π_1^k	Π_2^k	δ_1^k	δ_2^k	π_1'	π_2'
1	$\underline{R}$	$\underline{P}$	1	1	S	P
2	R,$\underline{S}$	P	$(0,1)$	1	S	R
3	R,S	$\underline{R}$,P	$(\frac{2}{3},\frac{1}{3})$	$(\frac{2}{3},\frac{1}{3})$	P	R/P
4	R,$\underline{P}$,S	R,P	$(0,\frac{2}{3},\frac{1}{3})$	$(\frac{1}{3},\frac{2}{3})$	R	S
5	R,P,S	R,P,$\underline{S}$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$	R/P/S	R/P/S

StarCraft II is a real-time strategy game for two or more players in which players have to collect resources, build infrastructure and armies to defeat their opponents.

Figure: StarCraft II game, image source: https://deepmind.google/discover/blog/alphastar-mastering-the-real-time-strategy-game-starcraft-ii/.

AlphaStar – GrandMaster in StarCraft II

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Similar as for two-player board games, StarCraft II is challenging due to

- **Sparse rewards:** players only receive a terminal reward at the end of the game
- **Large action space:** players choose between many actions constituting of a type (e.g., build, move, attack), which unit should execute the action, and the target of the action
- **Long horizon:** games can last for thousands of time steps

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- **Long horizon:** games can last for thousands of time steps
- **Partial observability:** players only observe a limited view of the game state
- **Diversity of strategies:** players choose between three available races offering many units and possible strategies

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Agents of each type are added to the league whenever they become effective (measured by win rates) against their respective opponents.

During population-based training, AlphaStar computes distributions over the policies in the league to train any policy π_i' against using prioritized fictitious self-play (PFSP):

$$\delta_i^k(\pi_i) \propto f\big(\Pr[\pi_i' \text{ wins against } \pi_i]\big)$$

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- f_{hard}: focus on the most difficult opponent policies for π_i'
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Win probabilities are estimated by running multiple matches of π_i' versus π_i.

During population-based training, AlphaStar computes distributions over the policies in the league to train any policy π_i' against using prioritized fictitious self-play (PFSP):

$$\delta_i^k(\pi_i) \propto f\big(\mathrm{Pr}[\pi_i' \text{ wins against } \pi_i]\big)$$

which is proportional to the probability of the agent winning against the policy π_i.

and $f : [0, 1] \rightarrow [0, \infty)$ is a weighting function with two components:

- f_{hard}: focus on the most difficult opponent policies for π_i'
- f_{var} focus on opponent policies that are at a similar level of performance as π_i'

Win probabilities are estimated by running multiple matches of π_i' versus π_i.

$\rightarrow$ reached "GrandMaster" level in StarCraft II (top 0.2% of ranked human players).

Summary

We covered:

- Agent modelling with deep learning
- Parameter and experience sharing
- Policy self-play in zero-sum games
- Population-based training