Multi-Agent Reinforcement Learning

Multi-Agent Deep Reinforcement Learning – Part 1

Stefano V. Albrecht, Filippos Christianos, Lukas Schäfer

The MARL Book

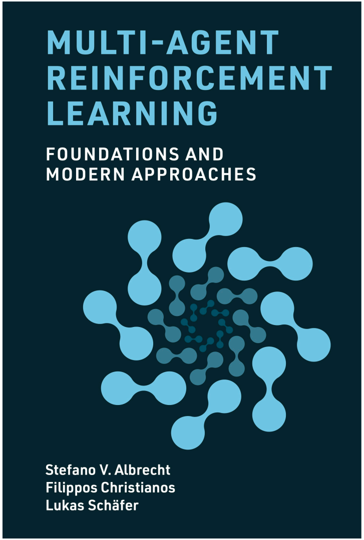
This lecture is based on

Multi-Agent Reinforcement Learning: Foundations and Modern Approaches

by Stefano V. Albrecht, Filippos Christianos and Lukas Schäfer

MIT Press, 2024

Download book, slides, and code at:
www.marl-book.com



Lecture Outline

- Training and execution modes
- Independent learning with deep reinforcement learning
- Multi-agent policy gradient algorithms
- Policy gradient algorithms
- Value decomposition in common-reward games

Training and Execution Modes

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We often distinguish between training and execution modes in MARL:

- Training: what information is available to agents during learning?
- Execution: what information is available to agents for action selection?

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We have already seen two single-agent reductions of the MARL problem:

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We have already seen two single-agent reductions of the MARL problem:

- **Independent learning**: each agent learns its policy independently
  $\rightarrow$ decentralized training and decentralized execution

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We have already seen two single-agent reductions of the MARL problem:

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- **Central learning**: learn single policy over the joint action space
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- **Central learning**: learn single policy over the joint action space
  $\rightarrow$ centralized training and centralized execution

But we can also have centralised training with decentralised execution (CTDE)!

Independent Learning with Deep Reinforcement Learning

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Reminder

In the independent learning framework, each agent i learns its policy π_i using only its local history of observations, treating the effects of other agents' actions as part of the environment.

- From the perspective of the individual agent, the environment transition function looks like this:

$$\mathcal{T}_i(s^{t+1}|s^t, a_i) \propto \sum_{a_{-i} \in A_{-i}} \mathcal{T}(s^{t+1}|s^t, \langle a_i, a_{-i} \rangle) \prod_{j \neq i} \pi_j(a_j|s^t)$$

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How about we do this with deep RL? We have already seen several single-agent deep RL algorithms: DQN, REINFORCE, A2C, PPO, etc.

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How about we do this with deep RL? We have already seen several single-agent deep RL algorithms: DQN, REINFORCE, A2C, PPO, etc.

Independent Deep Q-Networks

Algorithm Independent deep Q-networks

```
Initialize n value networks with random parameters θ_1, ..., θ_n
Initialize n target networks with parameters θ̄_1 = θ_1, ..., θ̄_n = θ_n
Initialize a replay buffer for each agent D_1, D_2, ..., D_n
for time step t = 0, 1, 2, ... do
    Collect current observations o_1^t, ..., o_n^t
    for agent i = 1, ..., n do
        With probability ε: choose random action a_i^t
        Otherwise: choose a_i^t ∈ arg max_{a_i} Q(h_i^t, a_i; θ_i)
    Apply actions (a_1^t, ..., a_n^t); collect rewards r_1^t, ..., r_n^t and next observations o_1^{t+1}, ..., o_n^{t+1}
    for agent i = 1, ..., n do
        Store transition (h_i^t, a_i^t, r_i^t, h_i^{t+1}) in replay buffers D_i
        Sample random mini-batch of B transitions (h_i^k, a_i^k, r_i^k, h_i^{k+1}) from D_i
        if s^{k+1} is terminal then
            Targets y_i^k ← r_i^k
        else
            Targets y_i^k ← r_i^k + γ max_{a'_i ∈ A_i} Q(h_i^{k+1}, a'_i; θ̄_i)
        Loss L(θ_i) ← (1/B) Σ_{k=1}^{B} ( y_i^k − Q(h_i^k, a_i^k; θ_i) )^2
        Update parameters θ_i by minimizing the loss L(θ_i)
        In a set interval, update target network parameters θ̄_i
```

Independent Deep Q-Networks

Algorithm Independent deep Q-networks

Initialize n value networks with random parameters $\theta_1, \ldots, \theta_n$
Initialize n target networks with parameters $\overline{\theta}_1 = \theta_1, \ldots, \overline{\theta}_n = \theta_n$
Initialize a replay buffer for each agent $D_1, D_2, \ldots, D_n$
for time step $t = 0, 1, 2, \ldots$ **do**
    Collect current observations $o_1^t, \ldots, o_n^t$
    for agent $i = 1, \ldots, n$ **do**
        With probability ϵ: choose random action a_i^t
        Otherwise: choose $a_i^t \in \arg\max_{a_i} Q(h_i^t, a_i; \theta_i)$
    Apply actions $(a_1^t, \ldots, a_n^t)$; collect rewards $r_1^t, \ldots, r_n^t$ and next observations $o_1^{t+1}, \ldots, o_n^{t+1}$
    for agent $i = 1, \ldots, n$ **do**
        Store transition $(h_i^t, a_i^t, r_i^t, h_i^{t+1})$ in replay buffers D_i
        Sample random mini-batch of B transitions $(h_i^k, a_i^k, r_i^k, h_i^{k+1})$ from D_i
        if s^{k+1} is terminal **then**
            Targets $y_i^k \leftarrow r_i^k$
        else
            Targets $y_i^k \leftarrow r_i^k + \gamma \max_{a_i' \in A_i} Q(h_i^{k+1}, a_i'; \overline{\theta_i})$
        Loss $\mathcal{L}(\theta_i) \leftarrow \frac{1}{B} \sum_{k=1}^{B} \left(y_i^k - Q(h_i^k, a_i^k; \theta_i) \right)^2$
        Update parameters θ_i by minimizing the loss $\mathcal{L}(\theta_i)$
        In a set interval, update target network parameters $\overline{\theta}_i$

- Almost identical to DQN from Chapter 8 but with n agents!

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  for agent $i = 1, \ldots, n$ **do**
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    Update parameters θ_i by minimizing the loss $\mathcal{L}(\theta_i)$
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- Almost identical to DQN from Chapter 8 but with n agents!
- Replay buffer contains off-policy experiences due to changing policies
- In MARL, the policies of all agents are changing → training can be unstable

Independent Advantage Actor-Critic

Algorithm Independent A2C with synchronous environments

Initialize n actor networks with random parameters $\phi_1, \ldots, \phi_n$
Initialize n critic networks with random parameters $\theta_1, \ldots, \theta_n$
Initialize K parallel environments
for time step $t = 0 \ldots$ **do**
  Batch of observations for each agent and environment: $\begin{bmatrix} o_1^{t,1} \ldots o_1^{t,K} \\ \ddots \\ o_n^{t,1} \ldots o_n^{t,K} \end{bmatrix}$
  Sample actions $\begin{bmatrix} a_1^{t,1} \ldots a_1^{t,K} \\ \ddots \\ a_n^{t,1} \ldots a_n^{t,K} \end{bmatrix} \sim \pi(\cdot \mid h_1^t; \phi_1), \ldots, \pi(\cdot \mid h_n^t; \phi_n)$
  Apply actions; collect rewards $\begin{bmatrix} r_1^{t,1} \ldots r_1^{t,K} \\ \ddots \\ r_n^{t,1} \ldots r_n^{t,K} \end{bmatrix}$ and observations $\begin{bmatrix} o_1^{t+1,1} \ldots o_1^{t+1,K} \\ \ddots \\ o_n^{t+1,1} \ldots o_n^{t+1,K} \end{bmatrix}$
  for agent $i = 1, \ldots, n$ **do**
    if $s^{t+1,k}$ is terminal **then**
      Advantage $Adv(h_i^{t,k}, a_i^{t,k}) \leftarrow r_i^{t,k} - V(h_i^{t,k}; \theta_i)$
      Critic target $y_i^{t,k} \leftarrow r_i^{t,k}$
    else
      Advantage $Adv(h_i^{t,k}, a_i^{t,k}) \leftarrow r_i^{t,k} + \gamma V(h_i^{t+1,k}; \theta_i) - V(h_i^{t,k}; \theta_i)$
      Critic target $y_i^{t,k} \leftarrow r_i^{t,k} + \gamma V(h_i^{t+1,k}; \theta_i)$
    Actor loss $\mathcal{L}(\phi_i) \leftarrow \frac{1}{K} \sum_{k=1}^{K} Adv(h_i^{t,k}, a_i^{t,k}) \log \pi(a_i^{t,k} \mid h_i^{t,k}; \phi_i)$
    Critic loss $\mathcal{L}(\theta_i) \leftarrow \frac{1}{K} \sum_{k=1}^{K} \left(y_i^{t,k} - V(h_i^{t,k}; \theta_i) \right)^2$
    Update parameters ϕ_i by minimizing the actor loss $\mathcal{L}(\phi_i)$
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    Apply actions; collect rewards $\begin{bmatrix} r_1^{t,1} \ldots r_1^{t,K} \\ \ddots \\ r_n^{t,1} \ldots r_n^{t,K} \end{bmatrix}$ and observations $\begin{bmatrix} o_1^{t+1,1} \ldots o_1^{t+1,K} \\ \ddots \\ o_n^{t+1,1} \ldots o_n^{t+1,K} \end{bmatrix}$
    for agent $i = 1, \ldots, n$ **do**
        if $s^{t+1,k}$ is terminal **then**
            Advantage $Adv(h_i^{t,k}, a_i^{t,k}) \leftarrow r_i^{t,k} - V(h_i^{t,k}; \theta_i)$
            Critic target $y_i^{t,k} \leftarrow r_i^{t,k}$
        else
            Advantage $Adv(h_i^{t,k}, a_i^{t,k}) \leftarrow r_i^{t,k} + \gamma V(h_i^{t+1,k}; \theta_i) - V(h_i^{t,k}; \theta_i)$
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- Almost identical to single-agent A2C from Chapter 8
- Similar adaptation can be done for independent REINFORCE and independent PPO

Challenges of Multi-Agent Reinforcement Learning

Reminder

MARL algorithms suffer from multi-agent specific challenges:

- **Non-stationarity**: exacerbated due to changing policies of all agents
- **Equilibrium selection**: how to converge to a stable equilibrium?
- **Multi-agent credit assignment**: how to attribute rewards to agents' actions?
- **Scaling to many agents**: how to efficiently scale to large numbers of agents?

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Centralised training with decentralised execution (CTDE) can help address some of these challenges.

Multi-Agent Policy Gradient Algorithms

The Policy-Gradient Theorem

Reminder

Follow this gradient to optimise the parameters ϕ of the policy π to maximise the expected return:

$$
\begin{aligned}
\nabla_\phi J(\phi) &\propto \sum_{s \in S} \Pr(s \mid \pi) \sum_{a \in A} Q^\pi(s, a) \nabla_\phi \pi(a \mid s; \phi) \\
&= \mathbb{E}_{s \sim \Pr(\cdot \mid \pi), a \sim \pi(\cdot \mid s; \phi)} \left[Q^\pi(s, a) \nabla_\phi \log \pi(a \mid s; \phi) \right]
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\end{aligned}
$$

Does this also hold for MARL? Yes, with minor modifications!

The Multi-Agent Policy-Gradient Theorem

Solution

In MARL, the expected returns of agent i under its policy π_i depends on the policies of all other agents π_{-i} $\rightarrow$ the multi-agent policy gradient theorem defines an expectation over the policies of **all** agents:

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$$\nabla_{\phi_i} J(\phi_i) \propto \mathbb{E}_{\hat{h} \sim \Pr(\hat{h}|\pi), a_i \sim \pi_i, a_{-i} \sim \pi_{-i}} \left[Q_i^\pi(\hat{h}, \langle a_i, a_{-i} \rangle) \nabla_{\phi_i} \log \pi_i(a_i \mid h_i = \sigma_i(\hat{h}); \phi_i) \right]$$

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$\rightarrow$ Derive policy update rules by finding estimators for expected returns $Q_i^{\pi}(\hat{h}, \langle a_i, a_{-i} \rangle)$.

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$\rightarrow$ Derive policy update rules by finding estimators for expected returns $Q_i^{\pi}(\hat{h}, \langle a_i, a_{-i} \rangle)$.

We have already seen independent A2C that estimates $Adv(h_i, a_i) \propto Q_i^{\pi}(\hat{h}, \langle a_i, a_{-i} \rangle)$.

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$\rightarrow$ Derive policy update rules by finding estimators for expected returns $Q_i^{\pi}(\hat{h}, \langle a_i, a_{-i}\rangle)$.

We have already seen independent A2C that estimates $Adv(h_i, a_i) \propto Q_i^{\pi}(\hat{h}, \langle a_i, a_{-i}\rangle)$.

But can we do better? Perhaps by leveraging more information?

Note

In actor-critic algorithms, only the policy/actor is used during execution and the critic is used only during training $\rightarrow$ the critic can be conditioned on centralised information z without compromising decentralised execution.

Centralized Critics

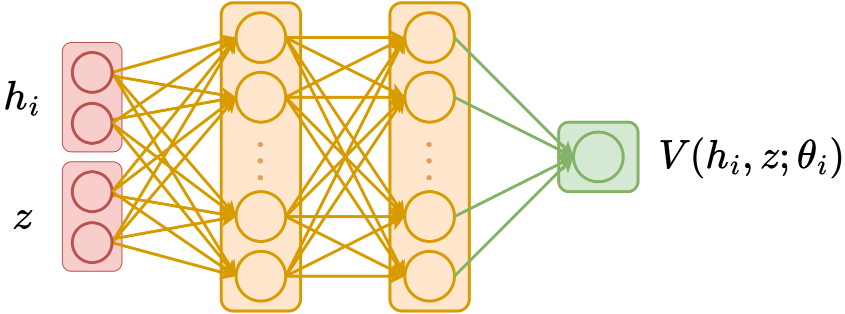
Note

In actor-critic algorithms, only the policy/actor is used during execution and the critic is used only during training $\rightarrow$ the critic can be conditioned on centralised information z without compromising decentralised execution.

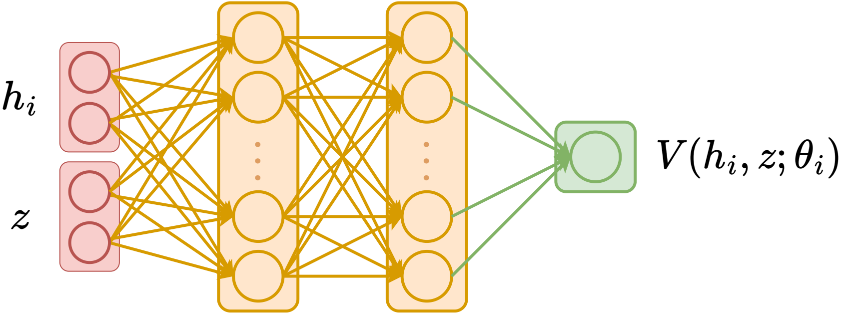
This might include:

- Global state s
- Joint action a
- Joint observation history h
- ...

Centralized Critics



Centralized Critics



Now we can integrate centralized critics into multi-agent policy gradient algorithms.

Centralized Advantage Actor-Critic

Algorithm Centralized A2C with synchronous environments

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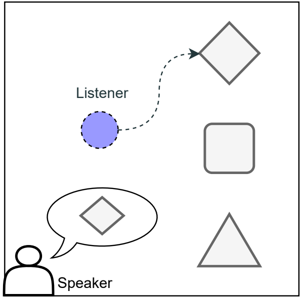
Initialize n critic networks with random parameters $\theta_1, \ldots, \theta_n$
Initialize K parallel environments
for time step $t = 0 \ldots$ **do**
  Batch of observations for each agent and environment: $\begin{bmatrix} o_1^{t,1} \ldots o_1^{t,K} \\ \ddots \\ o_n^{t,1} \ldots o_n^{t,K} \end{bmatrix}$
  Batch of centralized information for each environment: $\begin{bmatrix} z^{t,1} \ldots z^{t,K} \end{bmatrix}$
  Sample actions $\begin{bmatrix} a_1^{t,1} \ldots a_1^{t,K} \\ \ddots \\ a_n^{t,1} \ldots a_n^{t,K} \end{bmatrix} \sim \pi(\cdot \mid h_1^t; \phi_1), \ldots, \pi(\cdot \mid h_n^t; \phi_n)$
  Apply actions; collect rewards $\begin{bmatrix} r_1^{t,1} \ldots r_1^{t,K} \\ \ddots \\ r_n^{t,1} \ldots r_n^{t,K} \end{bmatrix}$, observations $\begin{bmatrix} o_1^{t+1,1} \ldots o_1^{t+1,K} \\ \ddots \\ o_n^{t+1,1} \ldots o_n^{t+1,K} \end{bmatrix}$, and centralized information $\begin{bmatrix} z^{t+1,1} \ldots z^{t+1,K} \end{bmatrix}$
  for agent $i = 1, \ldots, n$ **do**
    if $s^{t+1,k}$ is terminal **then**
      $Adv(h_i^{t,k}, z^{t,k}, a_i^{t,k}) \leftarrow r_i^{t,k} - V(h_i^{t,k}, z^{t,k}; \theta_i)$
      Critic target $y_i^{t,k} \leftarrow r_i^{t,k}$
    else
      $Adv(h_i^{t,k}, z^{t,k}, a_i^{t,k}) \leftarrow r_i^{t,k} + \gamma V(h_i^{t+1,k}, z^{t+1,k}; \theta_i) - V(h_i^{t,k}, z^{t,k}; \theta_i)$
      Critic target $y_i^{t,k} \leftarrow r_i^{t,k} + \gamma V(h_i^{t+1,k}, z^{t+1,k}; \theta_i)$
    Actor loss $\mathcal{L}(\phi_i) \leftarrow \frac{1}{K} \sum_{k=1}^{K} Adv(h_i^{t,k}, z^{t,k}, a_i^{t,k}) \log \pi(a_i^{t,k} \mid h_i^{t,k}; \phi_i)$
    Critic loss $\mathcal{L}(\theta_i) \leftarrow \frac{1}{K} \sum_{k=1}^{K} \left(y_i^{t,k} - V(h_i^{t,k}, z^{t,k}; \theta_i)\right)^2$
    Update parameters ϕ_i by minimizing the actor loss $\mathcal{L}(\phi_i)$
    Update parameters θ_i by minimizing the critic loss $\mathcal{L}(\theta_i)$

- Simple extension of independent A2C.
- Centralized information z is added to the critic input.

Centralized Critics in Action



(a) Speaker-listener game

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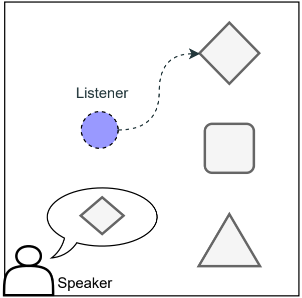

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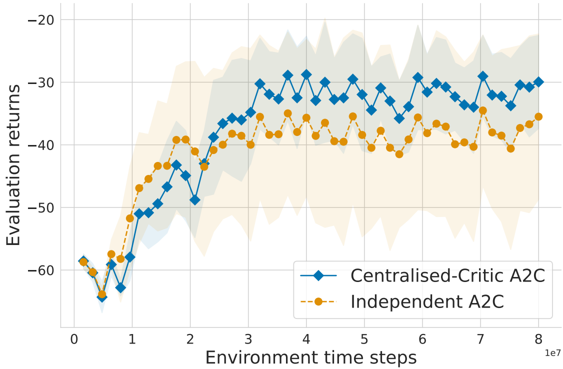
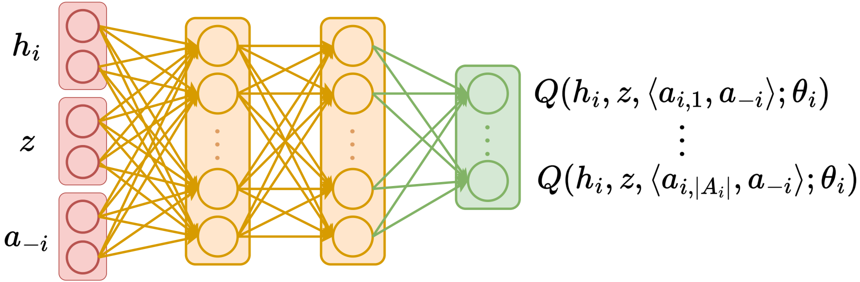


(b) Training curves

Agents with centralized critics converge to higher returns than agents with independent critics in the partially observable speaker-listener game.

Centralized Action-Value Critics

Similarly, we can learn an action-value function that receives additional centralized information z.



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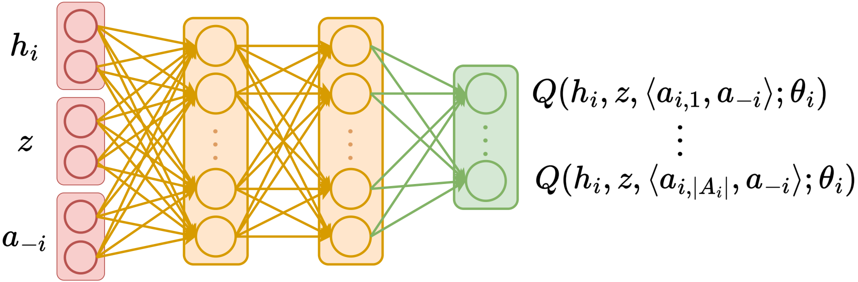

But what for? The centralized action-value critic can reason about the joint action space!

Counterfactual Multi-Agent Policy Gradient

For example, we can compute a counterfactual advantage for agent i

$$Adv_i(h_i, z, a) = Q(h_i, z, a; \theta) - \underbrace{\sum_{a'_i \in A_i} \pi(a'_i \mid h_i; \phi_i) Q(h_i, z, \langle a'_i, a_{-i} \rangle; \theta)}_{\text{counterfactual baseline}}$$

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Identify contribution of agent i's action a_i to received rewards $\Rightarrow$ help to address the credit assignment problem in common-reward games

The Equilibrium Selection Problem

Problem

Many multi-agent games have multiple equilibria. In such games, it difficult for all agents to agree on and stably converge to a single equilibrium. This is known as the equilibrium selection problem.

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	A	B
A	4,4‡	0,3
B	3,0	2,2†

Figure: Stag Hunt

	A	B	C
A	11‡	-30	0
B	-30	7†	0
C	0	6	5

Figure: Climbing

The Stag Hunt and Climbing matrix games have multiple equilibria.

†: Pareto-dominated equilibria

‡: Pareto-optimal equilibria

Example for the Equilibrium Selection Problem

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How can we overcome this problem and robustly converge to the optimal equilibrium?

Pareto Actor-Critic for Equilibrium Selection

Both shown matrix games are no-conflict games where agents agree on the optimal policy:

$$\arg\max_{\pi} U_i(\pi) = \arg\max_{\pi} U_j(\pi) \quad \forall i, j \in I$$

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We can use this property! Agent i during training assumes that all other agents follow the policy π^+_{-i} that is best for agent i, i.e. $\pi^+_{-i} \in \arg\max_{\pi_{-i}} U_i(\pi_i, \pi_{-i})$.

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During training, agent i optimises its policy π_i by minimising the following loss:

$$\mathcal{L}(\phi_i) = -\mathbb{E}_{a_i^t \sim \pi_i, a_{-i}^t \sim \pi^+_{-i}} \left[\log \pi(a_i^t \mid h_i^t; \phi_i) \left(Q^{\pi^+}(h_i^t, z^t, \langle a_i^t, a_{-i}^t \rangle; \theta_i^q) - V^{\pi^+}(h_i^t, z^t; \theta_i^v) \right) \right]$$

Pareto Actor-Critic for Equilibrium Selection in Climbing Game

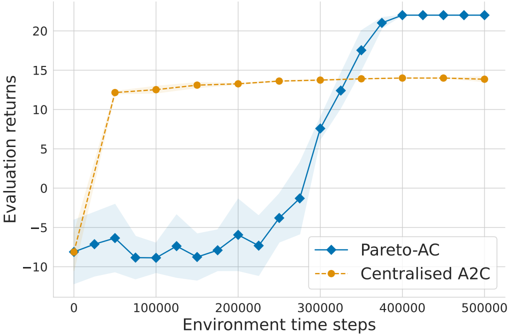
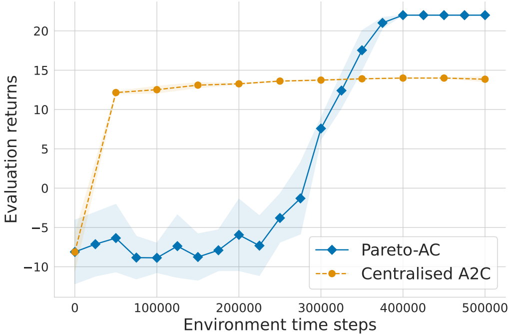


Figure: Learning curves in the Climbing game.



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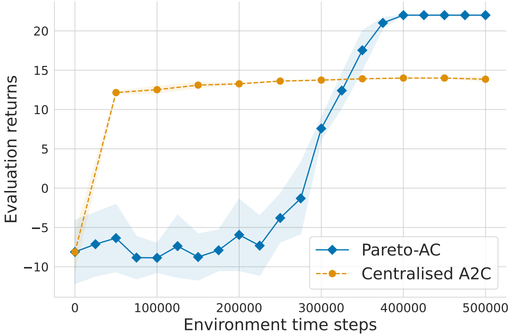

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Value Decomposition in Common-Reward Games

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How can we overcome this limitation and leverage the benefits of centralized value functions in value-based MARL algorithms?

Value Decomposition

We will focus on value decomposition methods for common-reward games. These methods aim to decompose a centralized action-value function of all agents

$$Q(h^t, z^t, a^t; \theta) = \mathbb{E}\left[\sum_{\tau=t}^{\infty} \gamma^{\tau-t} r^\tau \mid h^t, z^t, a^t\right]$$

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This decomposition has several benefits:

- Agents benefit from centralized information during training
- Simplify learning by decomposing the centralized value function
- Agents learn their individual utility functions to represent their contribution to the centralized value function, helping to address the **credit assignment problem**

Individual-Global-Max Property

How do we ensure that decentralized action selection with respect to the agents' individual utility functions leads to effective joint actions with respect to the decomposed centralized action-value function?

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Solution

Let $\hat{h}$ be a full history with joint-observation histories $h = \sigma(\hat{h})$, individual observation histories, $h_i = \sigma_i(\hat{h})$, and centralized information z. The individual-global-max (IGM) property is satisfied if and only if:

$$\forall a = (a_1, \ldots, a_n) \in A : a \in A^*(h, z; \theta) \iff \forall i \in I : a_i \in A_i^*(h_i; \theta_i)$$

with $A^*(h, z; \theta) = \arg\max_{a \in A} Q(h, z, a; \theta)$ and $A_i^*(h_i; \theta_i) = \arg\max_{a_i \in A_i} Q(h_i, a_i; \theta_i)$.

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Note

It is not guaranteed that for a given environment, there exists a decomposition of the centralized action-value function that satisfies the IGM property.

Linear Value Decomposition

Value decomposition networks (VDN) uses a simple linear decomposition of the centralized action-value function:

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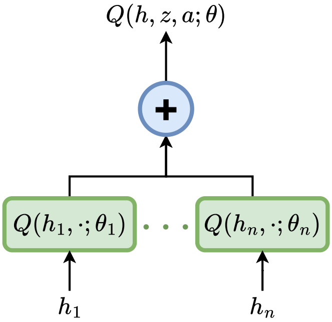
This decomposition satisfies the IGM property and we can jointly optimise the parameters of all networks by minimising the following loss on sampled batches of experiences $\mathcal{B}$:

$$\mathcal{L}(\theta) = \frac{1}{B} \sum_{(h^t, a^t, r^t, h^{t+1}) \in \mathcal{B}} \left(r^t + \gamma \sum_{i \in I} \max_{a_i \in A_i} Q(h_i^{t+1}, a_i; \bar{\theta}_i) - \sum_{i \in I} Q(h_i^t, a_i^t; \theta_i) \right)^2$$

with $\bar{\theta}_i$ denoting the parameters of agent i's target network.

Algorithm Value decomposition networks (VDN)

Initialize n utility networks with random parameters $\theta_1, \ldots, \theta_n$
Initialize n target networks with parameters $\bar{\theta}_1 = \theta_1, \ldots, \bar{\theta}_n = \theta_n$
Initialize a shared replay buffer D
for time step $t = 0, 1, 2, \ldots$ **do**
    Collect current observations $o_1^t, \ldots, o_n^t$
    for agent $i = 1, \ldots, n$ **do**
        With probability ϵ: choose random action a_i^t
        Otherwise: choose $a_i^t \in \arg\max_{a_i} Q(h_i^t, a_i; \theta_i)$
    Apply actions; collect shared reward r^t and next observations $o_1^{t+1}, \ldots, o_n^{t+1}$
    Store transition (h^t, a^t, r^t, h^{t+1}) in shared replay buffer D
    Sample mini-batch of B transitions (h^k, a^k, r^k, h^{k+1}) from D
    if s^{k+1} is terminal **then**
        Targets $y^k \leftarrow r^k$
    else
        Targets $y^k \leftarrow r^k + \gamma \sum_{i \in I} \max_{a'_i \in A_i} Q(h_i^{k+1}, a'_i; \bar{\theta}_i)$
    Loss $\mathcal{L}(\theta) \leftarrow \frac{1}{B} \sum_{k=1}^{B} \left(y^k - \sum_{i \in I} Q(h_i^k, a_i^k; \theta_i) \right)^2$
    Update parameters θ by minimizing the loss $\mathcal{L}(\theta)$
    In a set interval, update target network parameters $\bar{\theta}_i$ for each agent i



Monotonic Value Decomposition

A more general decomposition (that also ensures the IGM property) can be formulated by assuming that the centralized action-value function is a (strictly) monotonically increasing function with respect to any individual utility function:

$$\forall i \in I, \forall a \in A : \frac{\partial Q(h, z, a; \theta)}{\partial Q(h_i, a_i; \theta_i)} > 0$$

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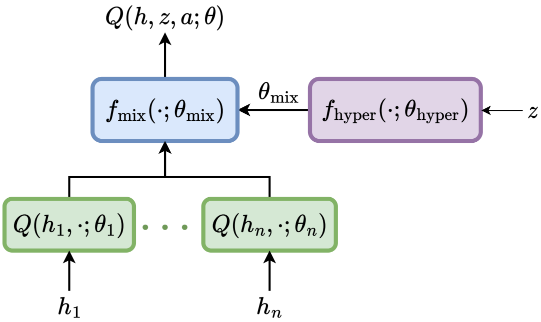
The QMIX algorithm implements this assumption using a mixing function f_{mix} that aggregates individual utilities to approximate the centralized action-value function:

$$Q(h, z, a, \theta) = f_{\text{mix}}\left(Q(h_1, a_1; \theta_1), \ldots, Q(h_n, a_n; \theta_n); \theta_{\text{mix}}\right)$$

QMIX Architecture



The centralized action-value function is monotonic with respect to individual utilities if all weights of f_{mix} are positive $\rightarrow$ ensure positive weights by obtaining the parameters of the mixing function from a hypernetwork f_{hyper} conditioned on centralized information z

QMIX Optimisation

The parameters of all utility functions and the hypernetwork are jointly optimised by minimising the following loss on batches of experiences $\mathcal{B}$ sampled from a replay buffer:

$$\mathcal{L}(\theta) = \frac{1}{B} \sum_{(h^t, z^t, a^t, r^t, h^{t+1}, z^{t+1}) \in \mathcal{B}} \left(r^t + \gamma \max_{a \in A} Q(h^{t+1}, z^{t+1}, a; \bar{\theta}) - Q(h^t, z^t, a^t; \theta) \right)^2$$

with the following decomposed value estimates:

$$Q(h^t, z^t, a^t, \theta) = f_{\text{mix}} \left(Q(h_1^t, a_1^t; \theta_1), \ldots, Q(h_n^t, a_n^t; \theta_n); \theta_{\text{mix}} \right)$$

$$\max_{a \in A} Q(h^{t+1}, z^{t+1}, a; \bar{\theta}) = f_{\text{mix}} \left(\max_{a_1 \in A_1} Q(h_1^{t+1}, a_1; \bar{\theta}_1), \ldots, \max_{a_n \in A_n} Q(h_n^{t+1}, a_n; \bar{\theta}_n); \bar{\theta}_{\text{mix}} \right)$$

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A	1	5
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Figure: Linear game

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	Linear game	Monotonic game	Climbing game
Linearly decomposable	✓	✗	✗
Monotonically decomposable	✓	✓	✗

Value Decomposition in Linearly Decomposable Matrix Game

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(a) True rewards

	0.12	**4.12**
0.88	1.00	5.00
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(b) VDN decomposition

	-0.21	**0.68**
0.19	1.00	5.00
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- VDN and QMIX are able to learn the true centralized action-value function
- The learned decompositions are not unique and can vary between different runs
- Individual utility values, in particular for QMIX, can be difficult to interpret (besides larger values indicating higher return estimates)

Value Decomposition in Monotonically Decomposable Matrix Game

	A	B
A	0	0
B	0	10

(a) True rewards

	−1.45	**3.45**
−0.94	−2.43	2.51
4.08	2.60	**7.53**

(b) VDN decomposition

	−4.91	**0.82**
−4.66	0.00	0.00
1.81	0.00	**10.00**

(c) QMIX decomposition

Value Decomposition in Monotonically Decomposable Matrix Game

	A	B
A	0	0
B	0	10

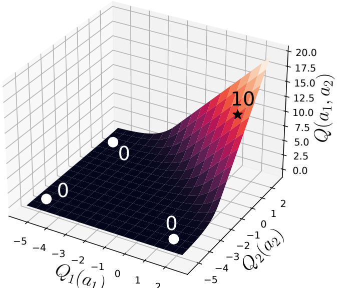
(a) True rewards

	-1.45	**3.45**
-0.94	-2.43	2.51
4.08	2.60	**7.53**

(b) VDN decomposition

	-4.91	**0.82**
-4.66	0.00	0.00
1.81	0.00	**10.00**

(c) QMIX decomposition



Only QMIX is able to represent the non-linear but monotonic relationship between the individual utility functions and the centralized action-value function.

Value Decomposition in Climbing Game

	A	B	C
A	11	-30	0
B	-30	7	0
C	0	6	5

Figure: True rewards

In the Climbing game, neither VDN nor QMIX are able to learn the true centralized action-value function and converge to sub-optimal policies.

	-4.56	-4.15	$\mathbf{3.28}$
-4.28	-8.84	-8.43	-1.00
-6.10	-10.66	-10.25	-2.82
$\mathbf{5.31}$	0.75	1.16	$\mathbf{8.59}$

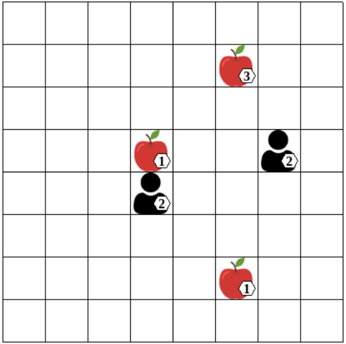
(a) VDN decomposition

	-16.60	$-\mathbf{0.24}$	-4.68
-7.44	-11.16	-11.16	-11.16
7.65	-11.15	2.34	-1.37
$\mathbf{11.27}$	-4.95	$\mathbf{8.72}$	5.01

(b) QMIX decomposition

Value Decomposition in LBF

So far, we looked at simple single-step matrix games. We will now compare IDQN, VDN and QMIX in a common-reward level-based foraging task where two agents need to collect three items in a 8×8 grid world:



Value Decomposition in LBF

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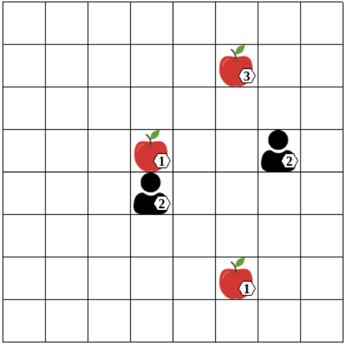

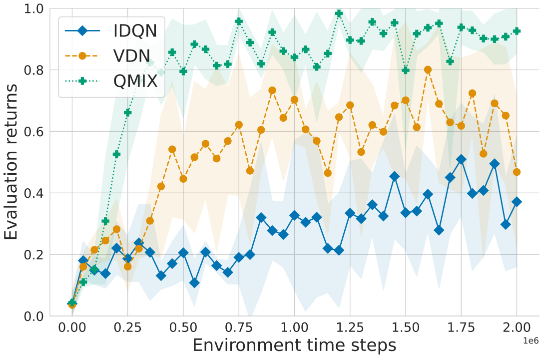

Summary

We covered:

- Training and execution modes
- Independent learning with deep reinforcement learning
- Multi-agent policy gradient algorithms
- Value decomposition in common-reward games

Next we'll cover:

- Agent modeling with deep learning
- Parameter and experience sharing
- Policy self-play in zero-sum games
- Population-based training