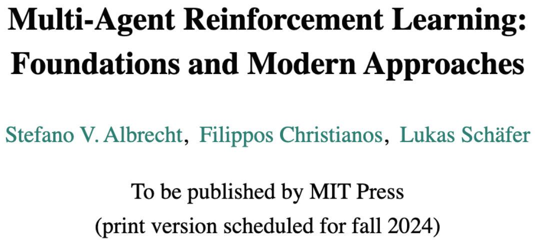
# Multi-Agent Reinforcement Learning

## Games: Models of Multi-Agent Interaction

Stefano V. Albrecht, Filippos Christianos, Lukas Schäfer
Slides by: Leonard Hinckeldey

## MARL Book



**Multi-Agent Reinforcement Learning: Foundations and Modern Approaches**

Stefano V. Albrecht, Filippos Christianos, Lukas Schäfer

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This lecture is based on *Multi-Agent Reinforcement Learning: Foundations and Modern Approaches* by Stefano V. Albrecht, Filippos Christianos and Lukas Schäfer

The book can be downloaded for free here.

## Lecture Outline

Part 1: Game Models

- Normal-form games
- Stochastic games
- Partially observable stochastic games

Part 2: Modeling Communication

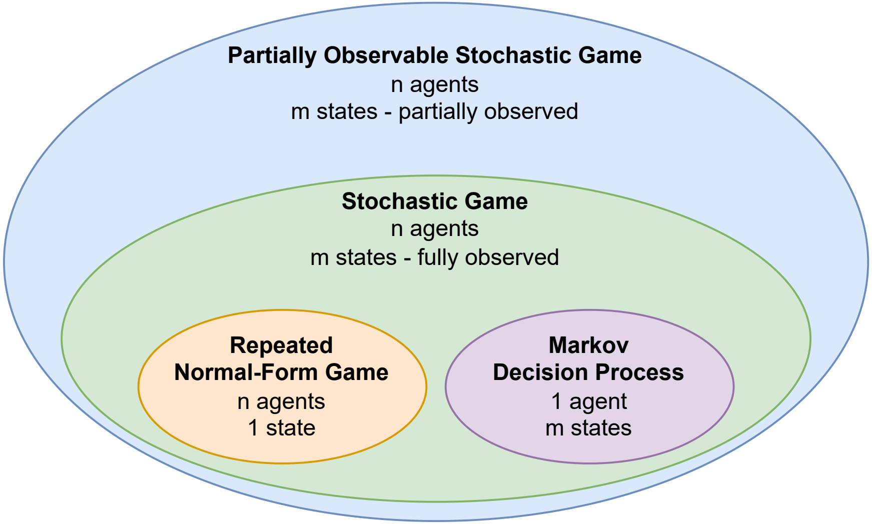
- Communication as an action
- Communication with observation functions

Part 3: Assumptions

- Game theory vs MARL assumptions

## Game Models

## Hierarchy of Games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n agents
m states - partially observed
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## Normal-Form Games

**Normal-form** games define a **single** interaction between two or more agents, providing a simple kernel for more general games to build upon.

**Normal-form** games are defined as a 3 tuple $(I, \{A_i\}_{i \in I}, \{\mathcal{R}_i\}_{i \in I})$:

- $I$ is a finite set of agents $I = \{1, ..., n\}$
- For each agent $i \in I$:
  - $A_i$ is a finite set of actions
  - $\mathcal{R}_i$ is the reward function $\mathcal{R}_i : A \to \mathbb{R}$ where $A = A_1 \times ... \times A_n$ (set of **joint** actions).

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In a normal-form game, there are **no time steps or states**. Agents choose an action and observe a reward.

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2. The resulting actions from all agents form a **joint action**, $a = (a_1, ..., a_n)$
3. Each agent receives a reward based on their **individual** reward function and the **joint action**, $r_i = \mathcal{R}_i(a)$

## Classes of Games

Games can be classified based on the relationship between the agents' reward functions.

- In **zero-sum games**, the sum of the agents' reward is always 0 i.e. $\sum_{i \in I} \mathcal{R}_i(a) = 0, \forall a \in A$
- In **common-reward** games, all agents receive the same reward ($R_i = R_j; \forall i, j \in I$)
- In **general-sum** games, there are no restrictions on the relationship between reward functions.

## Matrix Games

Normal-from games with **2** agents are also called **matrix games** because they can be represented using reward matrices.

| | R | P | S |
|---|---|---|---|
| R | 0,0 | -1,1 | 1,-1 |
| P | 1,-1 | 0,0 | -1,1 |
| S | -1,1 | 1,-1 | 0,0 |

**Figure:** 1. Rock-Paper-Scissors

| | A | B |
|---|---|---|
| A | 10 | 0 |
| B | 0 | 10 |

**Figure:** 2. Coordination Game

| | C | D |
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**Figure:** 3. Prisoner's Dilemma

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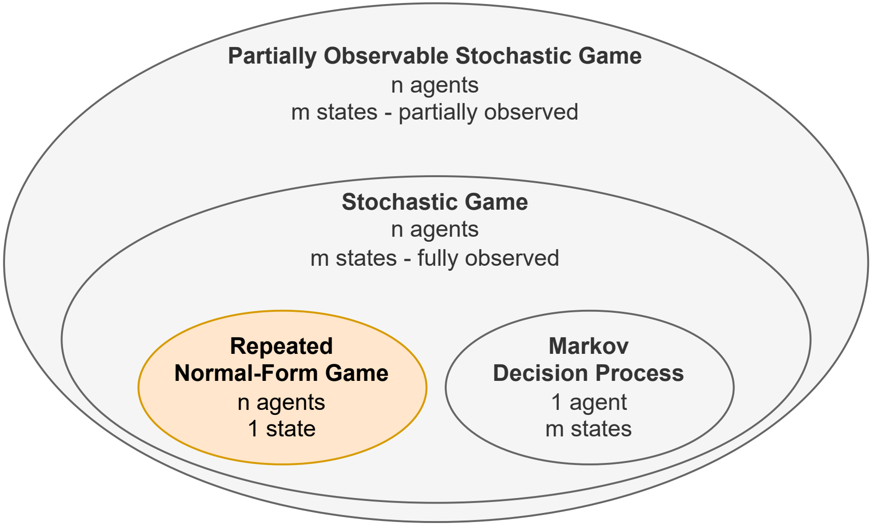
Note general sum is a **superset** class for all games

## Repeated Normal-Form Games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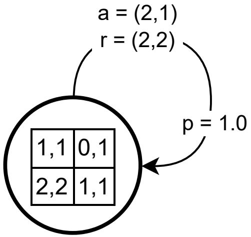
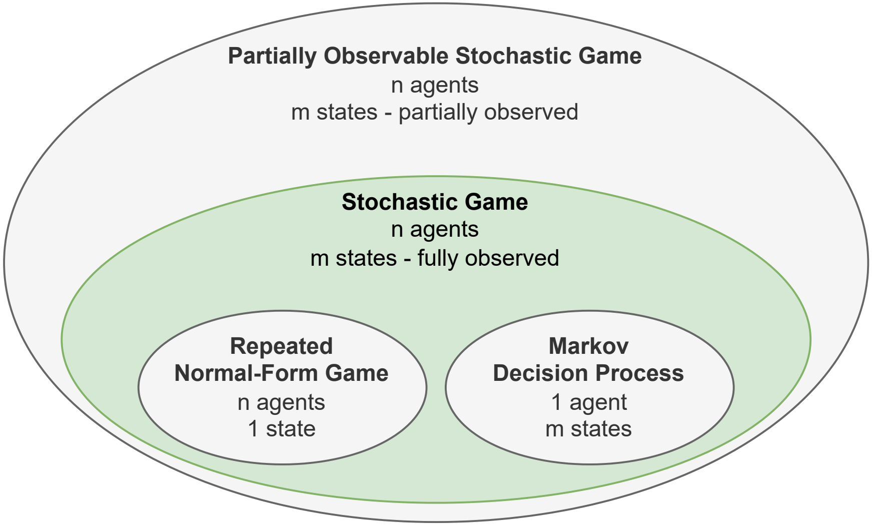
## Repeated Normal-Form Games

To extend normal-form games to **sequential** multi-agent interaction, we can repeat the same game over $T$ timesteps.



- At each time step $t$ an agent $i$ samples an action $a_i^t$
- The policy is now conditioned on a **joint-action** history $\pi_i(a_i^t|h^t)$ where $h^t = (a^o, ..., a^{t-1})$
- In special cases $h^t$ contains $n$ last joint actions. E.g. in a tit-for-tat strategy (Axelrod and Hamilton 1981), the policy is conditioned only on $a^{t-1}$

## Stochastic Games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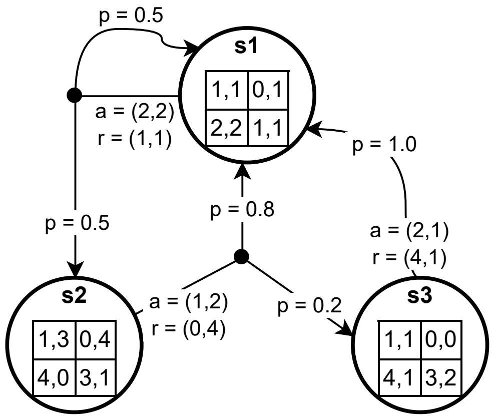
## Stochastic Games

Stochastic games introduce the notion of **states** and are defined as a 6 tuple $(I, S, \{A_i\}_{i \in I}, \{\mathcal{R}_i\}_{i \in I}, \mathcal{T}, \mu)$

- $I$ is a finite set of agents
- $S$ is a finite set of states with subset of terminal states $\bar{S} \subset S$
- For each agent $i \in I$:
  - $A_i$ is a set finite set of actions
  - $\mathcal{R}_i$ is the reward function $\mathcal{R}_i : S \times A \times S \to \mathbb{R}$ where $A$ is the set of **joint** actions $A = A_1 \times ... \times A_n$
- $\mu$ is the initial state distribution $\mu : S \to [0, 1]$
- $\mathcal{T}$ is the state transition function $\mathcal{T} : S \times A \times S \to [0, 1]$

## Stochastic Games - Continued



- Each **state** can be viewed as a **non-repeated normal-form game**
- Stochastic games can also be classified into: zero-sum, common-reward or general-sum
- The figure on the left shows a general-sum case

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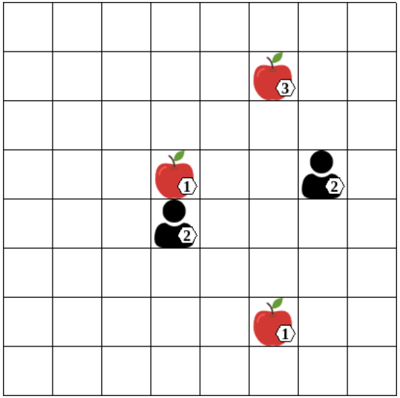
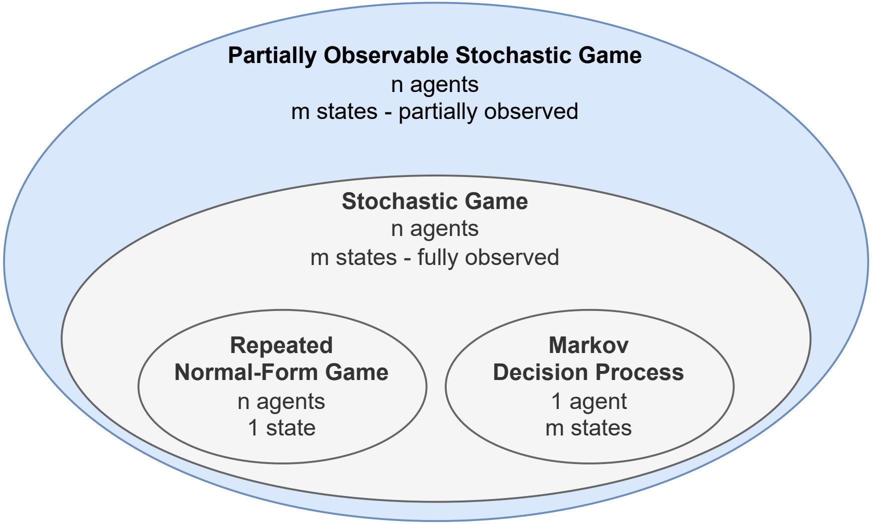
## Stochastic Game Level-Based Foraging Example



- $s \in S$: vector of x-y positions for agents/items, binary collection flags, levels for agents/items
- $a_i \in A_i$: move in four directions, collect item, or no operation (noop)
- $\mathcal{T}$: joint actions update state, e.g., two agents collecting an item switch its flag
- $\mathcal{R}$:
  - common-reward: +1 reward for any item collected by any agent
  - general-sum: +1 reward only for agents directly involved in item collection

# POSG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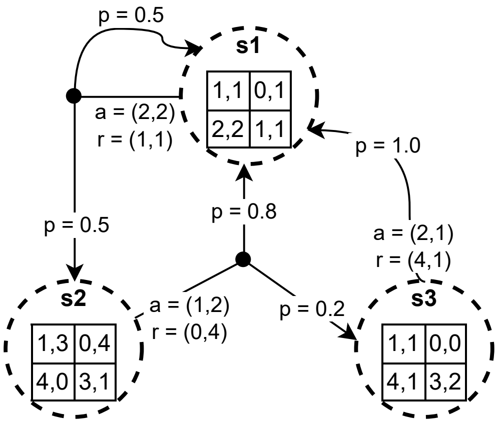
## Partially Observable Stochastic Games (POSG)

- At the top of the game model hierarchy, the most **general** model is the POSG
- POSGs represent complex decision processes with **incomplete information**
- Unlike in stochastic games, agents receive **observations** providing **incomplete information** about the state and agents’ actions
- POSGs apply to scenarios where agents have limited sensing capabilities.
  - Autonomous driving
  - Strategic games (e.g. card games) with private, hidden information

## POSG Definition

POSG is defined in the same way stochastic games are, with two additions. Thus it is defined as a 8 tuple $(I, S, \{A_i\}_{i \in I}, \{\mathcal{R}_i\}_{i \in I}, \mathcal{T}, \mu, \{O_i\}_{i \in I}, \{\mathcal{O}_i\}_{i \in I})$



For each agent $i$ we additionally define:

- a finite set of observation $O_i$
- an observation function $\{\mathcal{O}_i\}_{i \in I}$ such that $\mathcal{O}_i : A \times S \times O_i \to [0, 1]$ and $\forall a \in A, s \in S : \sum_{o_i \in O_i} \mathcal{O}_i(a, s, o_i) = 1$

## POSG Process

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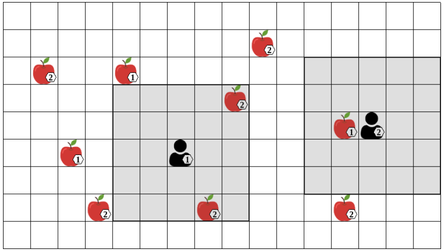
5. This is done until a terminating state $s^t \in \bar{S}$ is reached or a maximum number of time steps is completed

## The Observation Function

The observation function in POSG can represent diverse observability conditions. For example:

- modeling noise by adding uncertainty in the possible observation
- to limit the visual region of agents (see LBF example)

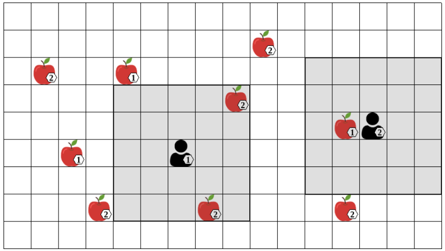


- Here, the agent has access to a subset of joint action and states
- $o_i^t = (\bar{s}^t, \bar{a}^t)$ where $\bar{s}^t \subset s^t, \bar{a}^t \subset a^t$

## Belief States

In partially observable settings, it becomes more challenging to infer optimal actions. For example:

- Optimal action for agent 1 is to move left towards level 1 apple
- But level 1 apple is not directly observable
- Agent 1 can hold a **belief states** $b_i^t$, providing a probability distribution over possible state $s \in S$
- Agent 1 might have seen the level-1 apple previously and can thus 'remember' its location



## Single Agent Belief Update

To simplify, let's consider the single-agent perspective:

- The initial belief state is given by $b_i^0 = \mu$
- After taking action $a_i^t$ and observing $o_i^{t+1}$, the belief state $b_i^t$ is updated to $b_i^{t+1}$ using a Bayesian update:

$$b_i^{t+1}(s') \propto \sum_{s \in S} b_i^t(s) \mathcal{T}(s'|s, a_i^t) \mathcal{O}_i(o_i^{t+1}|a_i^t, s')$$

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In MARL this type of update is typically **unfeasible**:

- High-dimensional state spaces make storage and updates of beliefs intractable
- In MARL for POSG, agents assumed not to know $(S, \mathcal{T}, \mathcal{O}_i)$
- Deep learning can be used to approximate state information (see later lectures)

## Modeling Communication

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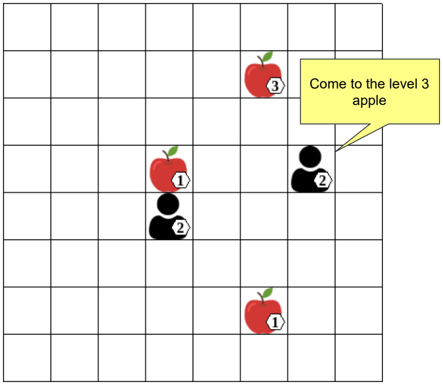

- Using games, we can model more complex agent interactions, such as communication
- We can view communication as a type of action that other agents can observe without affecting the state of the environment
- Agents learn communication meanings through trials and observations, identical to environment actions
- This can lead to the evolution of a shared language or protocol

## Communication Actions

To model communication, we can extend the action set of agents:

$$A_i = X_i \times M_i$$

- Where $M_i$ is a set of possible messages $\{m1, m2, m3, ...\}$ and $X_i$ is the set of environment actions
- The action $a_i$ can thus be expressed as $(x_i, m_i) \in A_i$

## Communication in Stochastic Games

- Agents observe the current state $s_t$ and previous joint action $a_{t-1}$
- Communication action $m^i_{t-1}$ by agent $i$ is part of $a_{t-1}$ and observed by all agents
- State transitions are independent of the joint communication actions $M = \times_{i \in I} M_i$

$$\forall s, s' \in S \forall a \in A, m \in M : T(s'|s, a) = T(s'|s, \langle (a_1, m_1), \dots, (a_n, m_n) \rangle)$$

## Communication in POSG

- In POSG we can use the observation function $\mathcal{O}_i$ to model noisy or unreliable communication
- We can define the observation as $o_i^t = [\bar{s}^t, w_1^{t-1}, ..., w_n^{t-1}]$
  - $\bar{s}^t$ is some partial information about the state
  - $w_j^{t-1}$ is a message from the agent $j$ at time step $t-1$ which has been augmented by $\mathcal{O}_i$
  - E.g. $w_j^{t-1} = f(m_j^{t-1})$ where $f(m_j^{t-1}) = m_j^{t-1} + \eta$, and $\eta$ is some random noise component.
- You could also model $\mathcal{O}_i$ to hide messages such that $w_1^{t-j} = \emptyset$ if agent $i$ is too far from agent $j$

## Assumptions of Games

## Game Theory Assumption

- In game theory, we typically assume that all agents know all components of the game **(complete knowledge games)**
- Agents know all agents' **action spaces and reward functions**
- Knowledge of other agents' reward functions may be used for informing the agent's **best response** action (we will cover this in more depth in the next lecture)
- Knowledge of the transition function ($T$) allows for predicting state changes and **planning** actions multiple steps ahead

## MARL Assumptions

- In MARL, we assume **limited knowledge**, i.e. no knowledge of the $\mathcal{T}$ and no knowledge of other agents $\mathcal{R}$
- Additional assumption can be added and specific knowledge of the game can be held **mutually** or **asymmetrically**
- We usually assume the **number of agents to be fixed**, although recent research has looked at *open* multi-agent systems, this will not be covered in these lectures

## Dictionary: Reinforcement Learning $\leftrightarrow$ Game Theory

| RL | | GT |
|---|---|---|
| environment | $\leftrightarrow$ | game |
| agent | $\leftrightarrow$ | player |
| reward | $\leftrightarrow$ | payoff, utility |
| policy | $\leftrightarrow$ | strategy |
| deterministic X | $\leftrightarrow$ | pure X |
| probabilistic X | $\leftrightarrow$ | mixed X |
| joint X | $\leftrightarrow$ | X profile |

- **Environment/Game**: Model with actions, observations, rewards, state dynamics.
- **Agent/Player**: Decision-maker, possibly with specific roles.
- **Reward/Payoff, Utility**: Scalar value received after an action
- **Policy/Strategy**: Assigns probabilities to actions; 'pure strategy' may refer to actions
- **Deterministic X/Pure X**: Assigns probability 1 to $X$ e.g. $X$ = equilibrium or policy
- **Probabilistic X/Mixed X**: Assigns probabilities $\leq 1$ to $X$
- **Joint X/X Profile**: Tuple representing collective aspects, e.g., rewards or policies

## Summary

We covered:

- Game models
- Modelling agent communication
- Assumptions of game models

Next we’ll cover:

- Solution concepts for games