

# Multi-Agent Reinforcement Learning

## Introduction

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Stefano V. Albrecht, Filippos Christianos, Lukas Schäfer

This lecture is based on

## **Multi-Agent Reinforcement Learning: Foundations and Modern Approaches**

by Stefano V. Albrecht, Filippos Christianos and  
Lukas Schäfer

MIT Press, 2024

Download book, slides, and code at:

[www.marl-book.com](http://www.marl-book.com)



## Part 1: Introduction

- Multi-agent systems
- Multi-agent RL in multi-agent systems
- Challenges of MARL

## Part 2: Reinforcement Learning Basics

- Markov decision processes
- Discounted returns
- Dynamic programming and temporal-difference learning

## Part 1: Introduction

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# What is MARL?

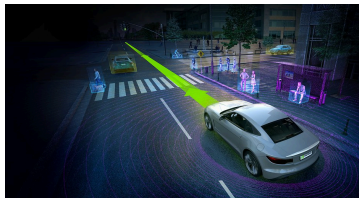
*Multi-agent reinforcement learning (MARL) is about finding optimal decision policies for two or more artificial agents interacting in a shared environment.*

- Applying reinforcement learning (RL) algorithms to multi-agent systems
- Goal is to learn optimal policies for two or more agents

# MARL Applications



Computer games



Autonomous driving



Multi-robot warehouses



Automated trading

A multi-agent system consists of:

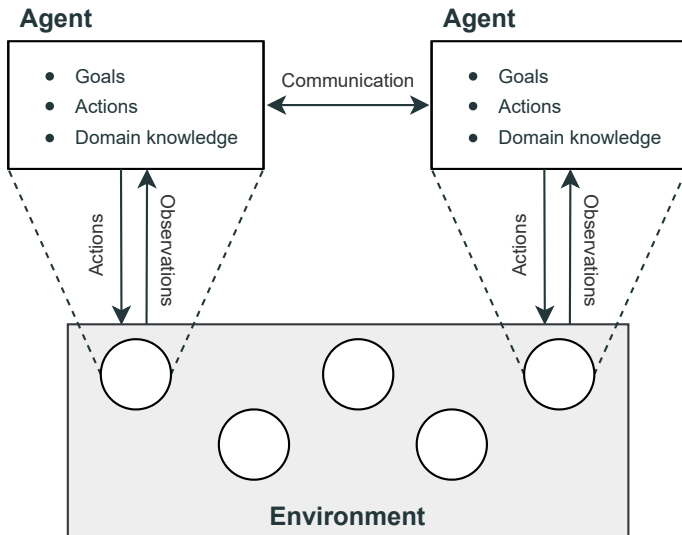
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- **Environment:** The environment is a physical or virtual world whose state evolves and is influenced by the agents' actions within the environment.
- **Agents:** An agent is an entity which receives information about the state of the environment and can choose actions to influence the state.  
⇒ Agents are goal-directed, e.g. maximizing returns

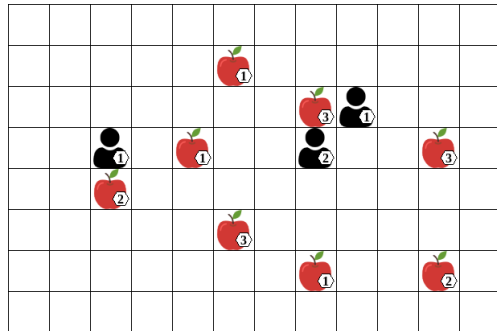


# Multi-Agent Systems

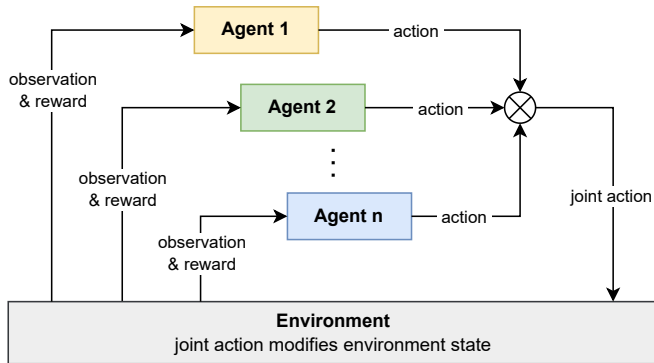


## Example: Level-Based Foraging

- Three agents (robots) with varying skill levels
- Goal: to collect all items (apples)
- Items can be collected if a group of one or more agents are located next to the item and the sum of agents' levels is greater than or equal to the item level
- Action space  
 $A = \{up, down, left, right, collect, noop\}$



# MARL for Solving Multi-Agent Systems



- **Goal:** learn optimal policies for a set of agents in a multi-agent system
- Each agent chooses an action based on its policy  $\Rightarrow$  joint action
- Joint action affects environment state; agents get rewards + new observations

# Why MARL?

Why should we use MARL to find optimal solutions to multi-agent systems rather than controlling multiple 'agents' using a single-agent RL (SARL) algorithm?

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## Decomposing a large problem

- In LBF example, controlling 3 robots each with 6 actions, the joint action space becomes  $6^3 = 216$ .  
⇒ Large action space for SARL!
- We can decompose this into three independent agents, each selecting from only 6 actions.  
⇒ Use MARL to train separate agent policies (more tractable)

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## Decentralized decision making

- There are many real-world scenarios where it is required for each agent to make decisions independently.
- E.g. autonomous driving is impractical for frequent long-distance data exchanges between a central agent and the vehicle.

# Challenges of MARL

New **challenges** arise in MARL:

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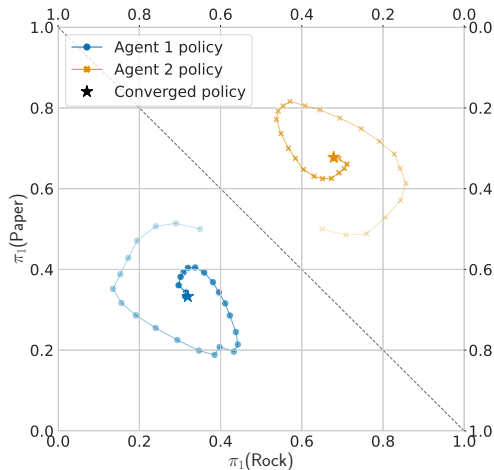
We will explore these challenges in upcoming lectures!

# Challenges of Multi-Agent Learning

## Non-stationary environment:

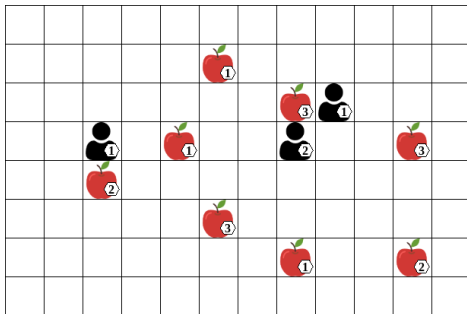
If multiple agents are learning, the environment becomes **non-stationary** from the perspective of individual agents

⇒ **Moving target:** each agent is optimizing against changing policies of other agents



# Multi-Agent Credit Assignment

**Multi-agent credit assignment:** which agent's actions contributed to received rewards?



- At time step  $t$  all agents perform *collect*, each receiving reward +1
- Whose actions led to the reward?
- The agent on the left did not contribute
- Learning agents must disentangle the contributions of actions!

## Part 2: Reinforcement Learning Basics

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# What is Reinforcement Learning?

## Reinforcement learning (RL)

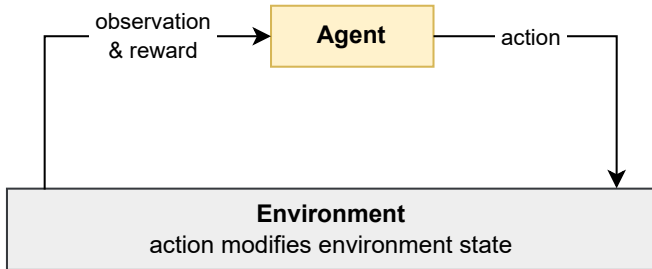
Learning to solve sequential decision problems via **repeated interaction with environment**

- What is a sequential decision problem?
- What does it mean to “solve” the problem?
- What is learning by interaction?

# What is Reinforcement Learning?

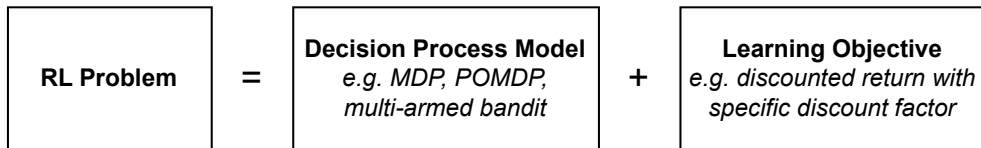
Agent takes actions in environment

- Take action, observe new state and reward from environment
  - Goal is to maximise total rewards received
- ⇒ Learning: find best actions by *trying* them



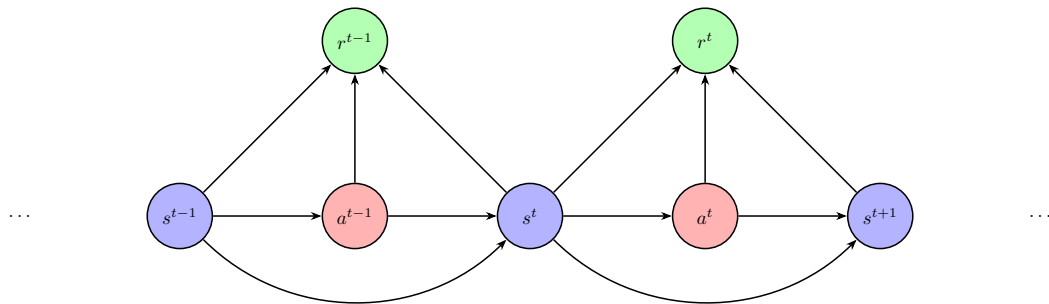


# RL Problem Definition



- Sequential decision process is defined formally as **Markov decision process** (MDP)
- Goal is to learn an optimal decision policy for a specific learning objective

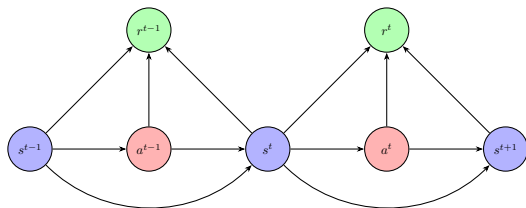
# Markov Decision Process – Graph



# Markov Decision Process – Definition

MDP is defined as:

- $S$ : Finite set of states, with subset of terminal states  $\bar{S} \subset S$
- $A$ : Finite set of actions
- $\mathcal{R}$ : Reward function  $\mathcal{R} : S \times A \times S \rightarrow \mathbb{R}$
- $\mathcal{T}$ : State transition function  
 $\mathcal{T} : S \times A \times S \rightarrow [0, 1]$
- $\mu$ : Initial state distribution  $\mu : S \rightarrow [0, 1]$



# MDP Assumptions

## Markov property

- Future states are temporally independent of past states and actions, given the current state and action.
- $Pr(s^{t+1}, r^t | s^t, a^t, s^{t-1}, a^{t-1}, \dots, s^0, a^0) = Pr(s^{t+1}, r^t | s^t, a^t)$

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- Agent can see the entire state of the environment.
- In many applications, agent may only have partial view.

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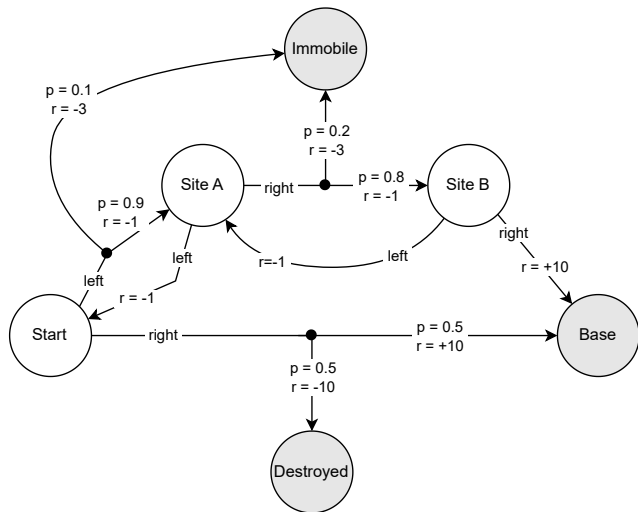
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## Incomplete knowledge of MDP

- Agent may only have knowledge of the action and state spaces ( $\mathcal{A}$ ,  $\mathcal{S}$ )
- Transition and reward functions ( $\mathcal{T}$ ,  $\mathcal{R}$ ) are usually assumed to be **unknown**.

# Mars Rover MDP Example



- *Start* is the initial state  $s^0$
- Two possible actions  
 $A = \{right, left\}$
- Goal is to get to *Base* terminal state
- Rewards given by  $\mathcal{R}(s, a, s')$  are shown as  $r$
- State transition probabilities given by  $\mathcal{T}(s, a, s')$ , are shown as  $p$



# Expected Discounted Returns

Using RL, learn policy  $\pi$  that maximizes the expected **return**:

- Returns are the sum of rewards received over time
- If MDP is non-terminating (i.e.  $t \rightarrow \infty$ ), we **discount** returns to ensure finite (discounted) returns

$$\mathbb{E}_{\pi}[u_t] = \mathbb{E}_{\pi} \left[ \sum_{t=0}^{\infty} \gamma^t r^t \right]$$

- $\gamma$  is the discount factor, such that  $\gamma \in [0, 1]$
- $\pi$  is the policy that determines which actions are chosen

# State-Value Functions

**State-value** functions  $V^\pi(s)$  give the 'value' of state  $s$  when following the policy  $\pi$ :

$$V^\pi(s) = \mathbb{E}_\pi [u^t | s^t = s]$$

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The return ( $u$ ) can be recursively defined as:

$$\begin{aligned} u^t &= r^t + \gamma(r^{t+1} + \gamma r^{t+2} + \dots) \\ &= r^t + \gamma u^{t+1} \end{aligned}$$

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Therefore:

$$V^\pi(s) = \mathbb{E}_\pi [r^t + \gamma u^{t+1} | s^t = s]$$

# The Bellman Equation

$$\begin{aligned} V^\pi(s) &= \mathbb{E}_\pi[r^t + \gamma u^{t+1} \mid s^t = s] \\ &= \sum_{a \in A} \pi(a \mid s) \sum_{s' \in S} \mathcal{T}(s' \mid s, a) [\mathcal{R}(s, a, s') + \gamma \mathbb{E}_\pi[u^{t+1} \mid s^{t+1} = s']] \\ &= \sum_{a \in A} \pi(a \mid s) \sum_{s' \in S} \mathcal{T}(s' \mid s, a) [\mathcal{R}(s, a, s') + \gamma V^\pi(s')] \end{aligned}$$

The last equation is known as the **Bellman equation** in honor of Richard Bellman.

- The value of being in state  $s$  while following a fixed policy  $\pi$  is equivalent to the immediate reward ( $\mathcal{R}(s, a, s') \rightarrow r$ ) received when taking action  $a$  in state  $s$  ( $\pi(a \mid s)$ ) plus the discounted value of the next state  $s'$ .

# State-Action Value Function

**State-action** value functions  $Q^\pi(s, a)$  are an extension on the *State* value functions. They condition the expected return on the current state and the action taken:

$$Q^\pi(s, a) = \mathbb{E}_\pi [u^t | s^t = s, a^t = a]$$

The *state-action* value Bellman equation is therefore:

$$Q^\pi(s, a) = \sum_{s' \in S} \mathcal{T}(s' | s, a) [\mathcal{R}(s, a, s') + \gamma V^\pi(s')]$$

# Greedy Policies

A policy can act **greedily**: choosing actions that have maximum estimated value.

Greedy  $\pi$  using *state value* functions:

$$\pi(s) = \arg \max_{a \in A} \sum_{s', r} \mathcal{T}(s', r | s, a) [r + \gamma V(s')]$$

Or using the *state-action value* function:

$$\pi(s) = \arg \max_{a \in A} Q(s, a)$$

# Optimal Greedy Policy

A greedy policy with respect to a value function is optimal only when using the **optimal value function**.

Optimal value function for a MDP is defined as:

$$V^*(s) = \max_{\pi'} V^{\pi'}(s), \quad \forall s \in S$$

$$Q^*(s, a) = \max_{\pi'} Q^{\pi'}(s, a), \quad \forall s \in S, a \in A$$

Therefore, the optimal policy is:

$$\pi^*(s) = \arg \max_{a \in A} Q^*(s, a)$$



# Dynamic Programming

- Dynamic Programming (DP) is a family of algorithms to compute **optimal value functions** and **optimal policies in MDPs** (Bellman 1957; Howard 1960).
- In DP, we **assume complete knowledge** of the MDP, including the transition and reward function  $(\mathcal{T}, \mathcal{R})$ .
- Given complete knowledge, we can use the **Bellman equation** to find optimal value functions and policies.

## Policy Iteration

**Policy Iteration** is a DP algorithm that alternates between two steps:

- **Policy evaluation:** compute value function  $V^\pi$  for current policy  $\pi$
- **Policy improvement:** improve current policy  $\pi$  with respect to  $V^\pi$

$$\pi^0 \rightarrow V^{\pi^0} \rightarrow \pi^1 \rightarrow V^{\pi^1} \rightarrow \pi^2 \rightarrow \dots \rightarrow V^* \rightarrow \pi^*$$

# Policy Iteration Pseudo Code

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## Algorithm Policy Iteration

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```
1: Initialize deterministic policy  $\pi$  randomly, and  $V(s) = 0$  for all  $s \in S$ 
2: repeat
3:   // Iterative Policy Evaluation:
4:   repeat
5:     for each state  $s$  in  $S$  do
6:        $V(s) \leftarrow \sum_{s'} \mathcal{T}(s'|s, \pi(s)) [\mathcal{R}(s, \pi(s), s') + \gamma V(s')]$ 
7:   until  $V(s)$  converges for all  $s \in S$ 
8:   // Policy Improvement:
9:   policy_stable  $\leftarrow$  true
10:  for each state  $s$  in  $S$  do
11:    old_action  $\leftarrow \pi(s)$ 
12:     $\pi(s) \leftarrow \arg \max_a \sum_{s'} \mathcal{T}(s'|s, a) [\mathcal{R}(s, a, s') + \gamma V(s')]$ 
13:    if old_action  $\neq \pi(s)$  then
14:      policy_stable  $\leftarrow$  false
15: until policy_stable return  $V, \pi$ 
```

# Value Iteration

Value Iteration uses the **Bellman optimality equation**

- Combines iterative policy evaluation and improvement into one update rule.

*Bellman Optimality Equation as update operator:*

$$V^{k+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} \mathcal{T}(s'|s, a) [\mathcal{R}(s, a, s') + \gamma V^k(s')], \quad \forall s \in S$$

- The max-operator makes this the Bellman *optimality* equation.
- The equation expresses the value of a state as the maximum expected return achievable by taking the best action and then following the optimal policy.

# Value Iteration Pseudo Code

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## Algorithm Value Iteration

---

- 1: Initialize:  $V(s) = 0, \forall s \in S$
  - 2: **repeat**
  - 3:    $\forall s \in S : V(s) \leftarrow \max_{a \in A} \sum_{s' \in S} \mathcal{T}(s'|s, a) [\mathcal{R}(s, a, s') + \gamma V(s')]$
  - 4: **until**  $V$  converged **return** optimal policy  $\pi^*$  with:
  - 5:  $\forall s \in S : \arg \max_{a \in A} \sum_{s' \in S} \mathcal{T}(s'|s, a) [\mathcal{R}(s, a, s') + \gamma V(s')]$
-

# Temporal-Difference Learning

**Temporal-Difference** (TD) learning is a family of RL algorithms that learn optimal policies and value functions based on data collected via environment **interactions**.

- These algorithms learn which actions yield the best returns by **trial and error** and exploring different actions and states
  - ⇒ Learns without **model** of environment's **reward** and **transition** functions ( $\mathcal{R}, \mathcal{T}$ )
  - ⇒ Can learn **online**, updating the policy while interacting with the environment

# Temporal-Difference Update

The update for temporal-difference learning uses value functions:

$$V(s^t) \leftarrow V(s^t) + \alpha [\mathcal{X} - V(s^t)]$$

or

$$Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha [\mathcal{X} - Q(s^t, a^t)]$$

- $\mathcal{X}$  is the update target, serving as an estimate of the current state value.
- $\alpha$  is the learning rate

# Temporal-Difference Update

In SARSA (a basic TD algorithm), we use the experience tuple  $(s^t, a^t, r^t, s^{t+1}, a^{t+1})$  to construct a target:

$$\mathcal{X} = r^t + \gamma Q(s^{t+1}, a^{t+1})$$

(The immediate reward plus the discounted value of the next state)

Note the resemblance to the Bellman equation:

$$Q^\pi(s, a) = \sum_{s' \in S} \mathcal{T}(s' | s, a) \left[ \mathcal{R}(s, a, s') + \gamma \sum_{a' \in A} \pi(a' | s') Q^\pi(s', a') \right]$$



The SARSA update rule thus becomes:

$$Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha[r^t + \gamma Q(s^{t+1}, a^{t+1}) - Q(s^t, a^t)]$$

- The TD update is **bootstrapped**: using the value estimates of the next state ( $Q(s^{t+1}, a^{t+1})$ ) to update the current state value ( $Q(s^t, a^t)$ )
- $(r^t + \gamma Q(s^{t+1}, a^{t+1}) - Q(s^t, a^t))$  is also called “TD error”

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## Algorithm SARSA

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- 1: Initialize  $Q(s, a) = 0$  for all  $s \in S, a \in A$
  - 2: **for** every episode **do**
  - 3:     Observe initial state  $s^0$
  - 4:     With probability  $\epsilon$ : choose random action  $a^0 \in A$
  - 5:     Otherwise: choose action  $a^0 \in \arg \max_a Q(s^0, a)$
  - 6:     **for**  $t = 0, 1, 2, \dots$  **do**
  - 7:         Apply action  $a^t$ , observe reward  $r'$  and next state  $s^{t+1}$
  - 8:         With probability  $\epsilon$ : choose random action  $a^{t+1} \in A$
  - 9:         Otherwise: choose action  $a^{t+1} \in \arg \max_a Q(s^{t+1}, a)$
  - 10:          $Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha[r^t + \gamma Q(s^{t+1}, a^{t+1}) - Q(s^t, a^t)]$
-

## Convergence of SARSA

SARSA is guaranteed to converge to the optimal **state-value function**, for all  $S \in \mathcal{S}$  and  $a \in A$ , if:

- All *state-action* pairs are explored infinitely many times:

$$\forall s \in S, a \in A : \sum_{k=0}^{\infty} \mathbb{I}(s, a) \rightarrow \infty$$

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- The learning rate  $\alpha$  is reduced over time according to the "standard stochastic approximation conditions":

$$\forall s \in S, a \in A : \sum_{k=0}^{\infty} \alpha_k(s, a) \rightarrow \infty \quad \text{and} \quad \sum_{k=0}^{\infty} \alpha_k(s, a)^2 < \infty$$

## $\epsilon$ -Greedy Policies

Using a **greedy policy** would **violate the convergence condition** of SARSA (infinite exploration of  $S$  and  $A$ ).

- Intuitively, we must explore a wide range of states and actions to find state action combinations that yield high returns
- One solution is to add an **exploration** parameter  $\epsilon \in [0, 1]$ . This gives us a stochastic **epsilon-greedy** policy

$$\pi(a|s) = \begin{cases} 1 - \epsilon + \frac{\epsilon}{|A|} & \text{if } a \in \arg \max_{a'} Q(s, a') \\ \frac{\epsilon}{|A|} & \text{otherwise} \end{cases}$$

- With probability  $1 - \epsilon$ , the policy chooses the greedy action, and with probability  $\epsilon$  chooses an action uniformly at random

**Q-learning** (Watkins & Dayan 1992) is a popular TD algorithm which uses the Bellman optimality equation to update its value function estimates.

- By using the Bellman optimality equation, Q-learning directly learns the **optimal state-action value function**
- Q-learning is **off-policy**, meaning the policy followed to gather experiences is different from the optimized policy
- We use the  $\epsilon$ -greedy policy to collect experiences

## Q-Learning Update

The target in Q-learning uses the *max* operator to target the optimal Q-values directly:

$$\mathcal{X} = r^t + \gamma \max_{a' \in A} Q(s^{t+1}, a')$$

The Q-learning update is thus:

$$Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha \left[ r^t + \gamma \max_{a' \in A} Q(s^{t+1}, a') - Q(s^t, a^t) \right]$$

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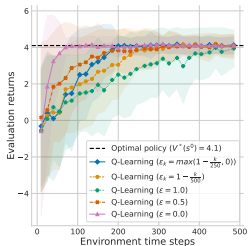
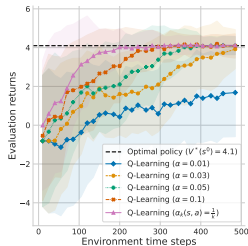
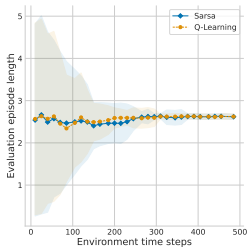
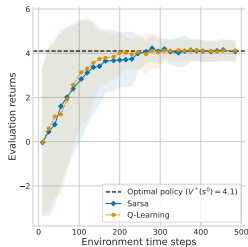
## Algorithm Q-Learning

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- 1: Initialize  $Q(s, a) = 0$  for all  $s \in S, a \in A$
  - 2: **for** every episode **do**
  - 3:     **for**  $t = 0, 1, 2, \dots$  **do**
  - 4:         Observe current state  $s^t$
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-



# Evaluating RL Algorithms



## Y-axis:

- Average **discounted** evaluation returns. This shows us how our greedy policy would perform if we stopped learning at time step T.
- In some cases, **undiscounted** returns are easier to interpret and may be used instead.

## X-axis:

- Cumulative training steps across episodes.
- Number of episodes can also be used. This might, however, distort the learning speed.

# Summary

## We covered:

- Intro to multi-agent systems and MARL
- MDPs
- Value functions
- Dynamic programming
- Temporal-difference learning

## Next we'll cover:

- Games: models of multi-agent interaction

Richard Bellman: Dynamic Programming. Princeton University Press, 1957

Ronald Howard: Dynamic Programming and Markov Processes. John Wiley, 1960

Richard Sutton and Andrew Barto: Reinforcement learning: An introduction (2nd edition). MIT Press, 2018