

Multi-Agent Reinforcement Learning

Deep Reinforcement Learning

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Slides by: Leonard Hinckeldey

This lecture is based on

Multi-Agent Reinforcement Learning: Foundations and Modern Approaches

by Stefano V. Albrecht, Filippos Christianos and
Lukas Schäfer

MIT Press, 2024

Download book, slides, and code at:

www.marl-book.com



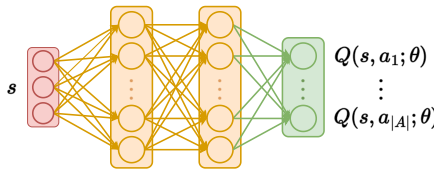
Lecture Outline

- Deep-Q learning
- Moving target problem
- Addressing correlations in consecutive experiences
- Policy gradient algorithms
- Concurrent training

Deep Q-Learning

Deep Q-Learning

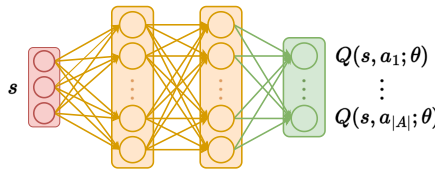
For **deep** Q-learning, we use a neural network to approximate the Q function.



- To train this we could define a loss function $\mathcal{L}(\theta) = (y^t - Q(s^t, a^t; \theta))$
- But unlike supervised learning, we are not given y^t beforehand

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- To train this we could define a loss function $\mathcal{L}(\theta) = (y^t - Q(s^t, a^t; \theta))$
- But unlike supervised learning, we are not given y^t beforehand
- We can use the Q-learning update rule to define our y^t

$$y^t = \begin{cases} r^t & \text{if } s^{t+1} \text{ is terminal} \\ r^t + \gamma \max_{a'} Q(s^{t+1}, a'; \theta) & \text{otherwise} \end{cases}$$

Naive Deep Q-Learning Pseudo-Code

Algorithm Deep Q-learning

```
1: Initialize value network  $Q$  with random parameters  $\theta$ 
2: for every episode do
3:   for time step  $t = 0, 1, 2, \dots$  do
4:     Observe current state  $s^t$ 
5:     With probability  $\epsilon$ : choose random action  $a^t \in A$ 
6:     Otherwise: choose  $a^t \in \arg \max_a Q(s^t, a; \theta)$ 
7:     Apply action  $a^t$ ; observe reward  $r^t$  and next state  $s^{t+1}$ 
8:     if  $s^{t+1}$  is terminal then
9:       Target  $y^t \leftarrow r^t$ 
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11:      Target  $y^t \leftarrow r^t + \gamma \max_{a'} Q(s^{t+1}, a'; \theta)$ 
12:      Loss  $\mathcal{L}(\theta) \leftarrow (y^t - Q(s^t, a^t; \theta))^2$ 
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This naive application of neural networks to RL algorithms has some problems.

The Moving Target Problem

Problem

Moving target problem arises from the bootstrapped targets:

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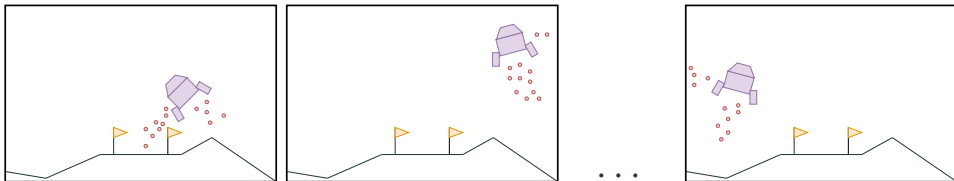
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Solution

One solution is to use a **target network** with parameters $\bar{\theta}$ that are updated less often than our Q network's parameters θ

Correlation of Consecutive Experiences

- Most ML algorithms using NNs assume i.i.d. data
- In RL, we collect data by interacting with an MDP with $s^{t+1} \sim \mathcal{T}(s^t, a^t) \rightarrow$ **not** i.i.d



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Solution

We train on samples of previous experiences stored in a **replay buffer**.

Deep Q-Networks

Deep Q-networks (DQN) is a foundational deep RL algorithm based on Q-learning.

- Tabular value function \longrightarrow neural network
- Moving target problem \longrightarrow target networks
- Correlated experiences \longrightarrow replay buffer

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- Update the "main" value network parameters θ by minimizing the DQL loss $\mathcal{L}(\theta)$
- Update the **target network** parameters in regular intervals $\bar{\theta} \leftarrow \theta$

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Note

Replay buffers can only be used for **off-policy** algorithms. We use experiences collected from previous (different) policies, which change as we update θ .

Algorithm Deep Q-networks (DQN)

- 1: Initialize value network Q with random parameters θ
 - 2: Initialize target network with parameters $\bar{\theta} = \theta$
 - 3: Initialize an empty replay buffer $\mathcal{D} = \{\}$
 - 4: **for** every episode **do**
 - 5: **for** time step $t = 0, 1, 2, \dots$ **do**
 - 6: Observe current state s^t
 - 7: With probability ϵ : choose random action $a^t \in A$
 - 8: Otherwise: choose $a^t \in \arg \max_a Q(s^t, a; \theta)$
 - 9: Apply action a^t ; observe reward r^t and next state s^{t+1}
 - 10: Store transition (s^t, a^t, r^t, s^{t+1}) in replay buffer \mathcal{D}
 - 11: Sample random mini-batch of B transitions (s^k, a^k, r^k, s^{k+1}) from \mathcal{D}
 - 12: **if** s^{k+1} is terminal **then**
 - 13: Targets $y^k \leftarrow r^k$
 - 14: **else**
 - 15: Targets $y^k \leftarrow r^k + \gamma \max_{a'} Q(s^{k+1}, a'; \bar{\theta})$
 - 16: Loss $\mathcal{L}(\theta) \leftarrow \frac{1}{B} \sum_{k=1}^B (y^k - Q(s^k, a^k; \theta))^2$
 - 17: Update parameters θ by minimising the loss $\mathcal{L}(\theta)$
 - 18: In a set interval, update target network parameters $\bar{\theta}$
-

Overestimation Bias

Problem

DQN tends to **overestimate** values with 1-step targets

$$y^k \leftarrow r^k + \gamma \max_{a'} Q(s^{k+1}, a'; \bar{\theta})$$

- Using the **max** operator, select the maximum value estimate for the target
- Since our value estimates do not necessarily reflect the true value function, the **max** operation will likely select an overestimated action-value estimate
- This can slow down the convergence of the algorithm as the agent spends too much time exploring states with overestimated values

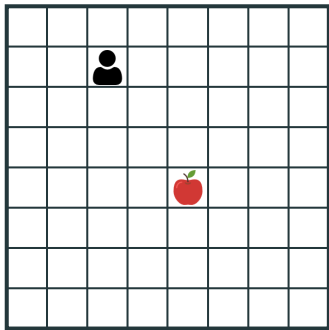
Solution

Double Q-learning reduces this overestimation bias by decoupling the action selection from value estimation using separate function approximations.

- This can be achieved with minimal changes in DQN
- DDQN uses the primary Q-network (with parameters θ) to select actions while using the target network (with parameters $\bar{\theta}$) to estimate action values
- The target thus becomes:

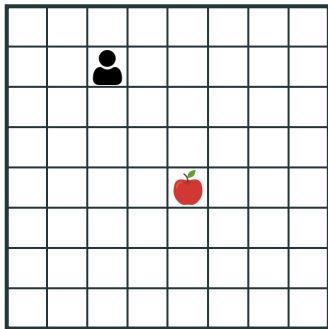
$$y^t = \begin{cases} r^t & \text{if } s^{t+1} \text{ is terminal} \\ r^t + \gamma Q(s^{t+1}, \arg \max_{a'} Q(s^{t+1}, a'; \theta); \bar{\theta}) & \text{otherwise} \end{cases}$$

Deep Q-learning in Practice

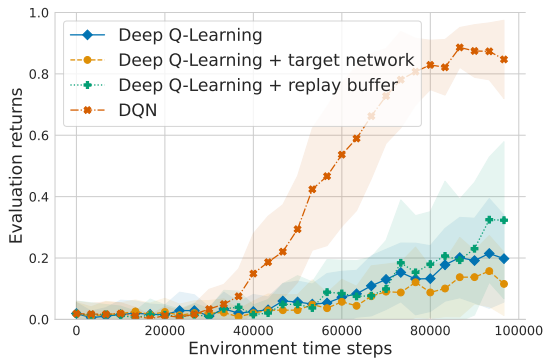


(a) Single-agent level-based foraging environment

Deep Q-learning in Practice

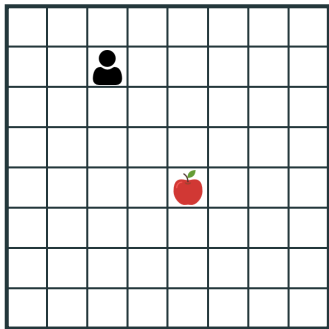


(a) Single-agent level-based foraging environment

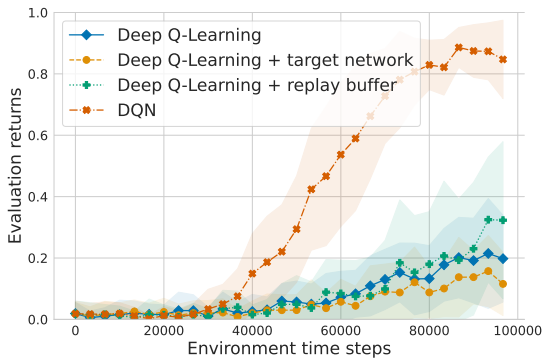


(b) Learning curves

Deep Q-learning in Practice



(a) Single-agent level-based foraging environment



(b) Learning curves

Note that in isolation, neither the addition of **target networks** nor of a **replay buffer** are sufficient to stably train the agent with deep Q-learning in this environment.

Policy Gradient Algorithms

Policy Gradients

We considered a parameterized value function, but we can also directly parameterize the policy π .

- Use a NN for policy π with parameters ϕ
- Policy network receives state s as input and outputs a scalar value for each action
- Scalars $l(s, a)$ represent the preference of the policy to select action a in state s

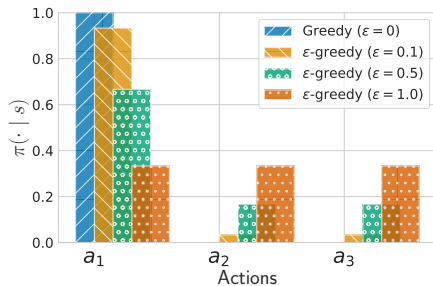
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- Scalars $l(s, a)$ represent the preference of the policy to select action a in state s
- Preferences are then transformed into a probability distribution across the action space using a **softmax** function:

$$\pi(a \mid s; \phi) = \frac{e^{l(s,a;\phi)}}{\sum_{a' \in A} e^{l(s,a';\phi)}}$$

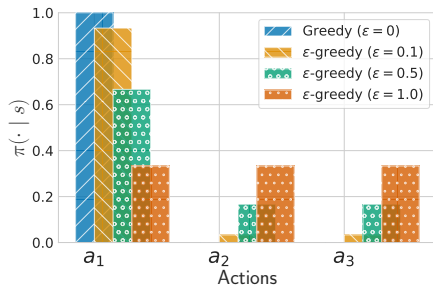
Advantages of Learning a Policy



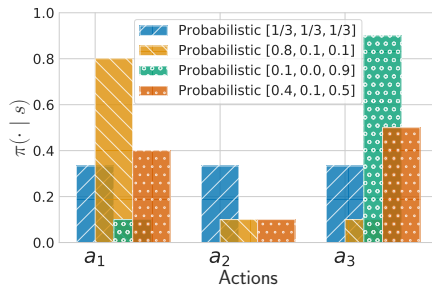
ϵ -greedy policies

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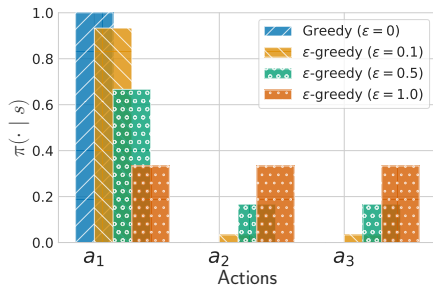
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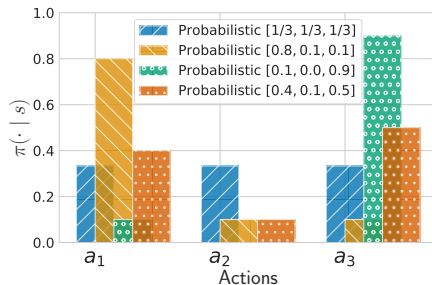
Probabilistic policies

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- Policy gradient algorithms allow us to represent **any** probabilistic policy
- Policy gradients are also effective for representing **continuous** action spaces

Policy Gradient Theorem

How to update parameters ϕ of the policy? Using the **policy gradient theorem**, we can express the gradient of the performance of a policy with respect to the parameter ϕ of the policy.

$$\nabla_{\phi} J(\phi) \propto \sum_{s \in S} \Pr(s \mid \pi) \sum_{a \in A} Q^{\pi}(s, a) \nabla_{\phi} \pi(a \mid s; \phi)$$

- J is a function measuring the quality of policy π
- $\Pr(s \mid \pi)$ is the state-visitation distribution for policy π
- $Q^{\pi}(s, a)$ is the value for a given action and state under π
- The J function is similar to a loss function, with the difference that we aim to **maximize** rather than minimize it

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Note

The policy gradient theorem assumes that $\Pr(s \mid \pi)$ are given under the currently optimized policy $\pi \rightarrow$ this needs **on-policy** data

Policy Gradient Theorem – Continued

The assumption that $Pr(s \mid \pi)$ is **on-policy** becomes apparent through a simple derivation.

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Policy Gradient Theorem – Continued

Intuition

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We can interpret the components of the policy gradient theorem as follows:

- $\nabla_{\phi} \pi(a | s; \phi)$: gradient of ϕ pointing in the direction that most increase the probability of taking action a when in state s

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- $Q^{\pi}(s, a)$: expected returns as a quality measure of taking action a in state s
- $\frac{1}{\pi(a | s; \phi)}$: normalization coefficient to account for varying probabilities of different actions under the policy

REINFORCE: Monte Carlo

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- The algorithm minimizes the following loss for an episodic history $h = \{s^0, a^0, r^0, \dots, s^{T-1}, a^{T-1}, r^{T-1}, s^T\}$:

$$\mathcal{L}(\phi) = -\frac{1}{T} \sum_{t=0}^{T-1} \left(\sum_{\tau=1}^{t-1} \gamma^{\tau-1} \mathcal{R}(s^\tau, a^\tau, s^{\tau+1}) \right) \log \pi(a^t | s^t; \phi)$$

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- Note this loss has a negative sign, as we want to **maximize** expected returns

Algorithm REINFORCE

- 1: Initialize policy network π with random parameters ϕ
 - 2: **for** every episode **do**
 - 3: **for** time step $t = 0, 1, 2, \dots, T - 1$ **do**
 - 4: Observe current state s^t
 - 5: Sample action $a^t \sim \pi(\cdot \mid s^t; \phi)$
 - 6: Apply action a^t ; observe reward r^t and next state s^{t+1}
 - 7: Loss $\mathcal{L}(\phi) \leftarrow -\frac{1}{T} \sum_{t=0}^{T-1} \left(\sum_{\tau=t}^{T-1} \gamma^{\tau-t} r^\tau \right) \log \pi(a^t \mid s^t; \phi)$
 - 8: Update parameters ϕ by minimizing the loss $\mathcal{L}(\phi)$
-

Baseline to Reduce Variance

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Solution

- Reduce variance by subtracting a **baseline** from the return estimates
- Common choice of baseline is state-value function $V(s)$ which can be trained to minimize the loss $\mathcal{L}(\theta) = \frac{1}{T} \sum_{t=1}^{T-1} (u(h^t) - V(s^t; \theta))^2$
- The REINFORCE policy loss then becomes

$$\mathcal{L}(\phi) = -\frac{1}{T} \sum_{t=0}^{T-1} (u(h^t) - V(s^t; \theta)) \log \pi(a^t | s^t; \phi)$$

Baselines Continued

The inclusion of the baseline does not affect the gradients in expectation:

$$\nabla_{\phi} J(\phi) \propto \sum_{s \in S} \Pr(s \mid \pi) \sum_{a \in A} (Q^{\pi}(s, a) - b(s)) \nabla_{\phi} \pi(a \mid s; \phi)$$

Baselines Continued

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Baselines Continued

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Actor-Critic Algorithms

The policy gradient theorem allows us to optimize our parameterized policy; we still need to approximate expected returns.

- MC methods have high variance and require an entire episode to compute estimates
- **Actor-critic** algorithms aim to reduce variance and update more often by using **bootstrapped return estimates**

$$\begin{aligned}\mathbb{E}_{\pi} [u(h^t) \mid s^t] &= \mathbb{E}_{\pi} [R(s^t, a^t, s^{t+1}) + \gamma u(h^{t+1}) \mid s^t, a^t \sim \pi(\cdot \mid s^t)] \\ &= \mathbb{E}_{\pi} [R(s^t, a^t, s^{t+1}) + \gamma V(s^{t+1}) \mid s^t, a^t \sim \pi(\cdot \mid s^t)]\end{aligned}$$

- We now train a **critic** (value function approximator) alongside the **actor** (parameterized policy) which acts as a baseline

Balancing Bias and Variance

Problem

MC estimates have high **variance** and bootstrapping introduces **bias** since the value function might not yet approximate the true expected returns.

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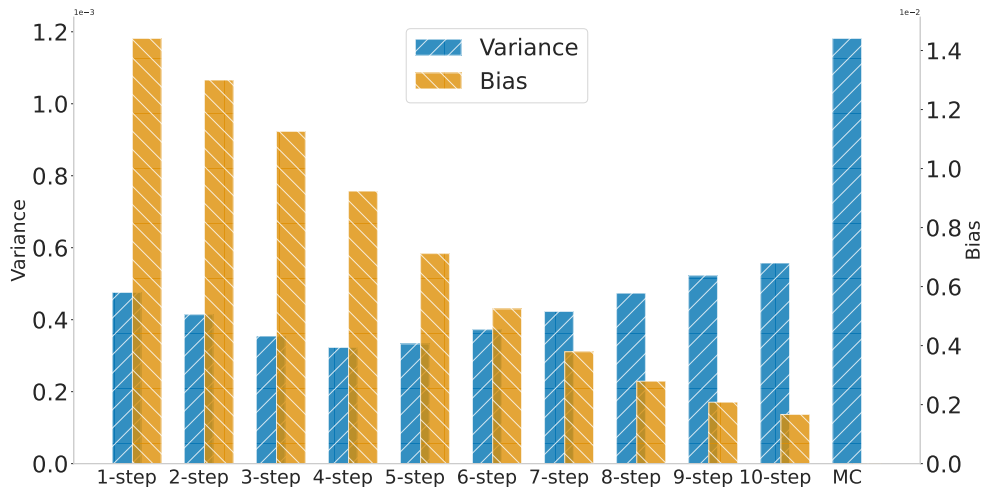
Solution

N-step returns allow us to balance between bias and variance:

$$\mathbb{E}_{\pi}[u(h^t) \mid s^t] = \mathbb{E}_{\pi} \left[\left(\sum_{\tau=0}^{N-1} \gamma^{\tau} \mathcal{R}(s^{t+\tau}, a^{t+\tau}, s^{t+\tau+1}) \right) + \gamma^N V(s^{t+N}) \middle| s^t, a^{\tau} \sim \pi(\cdot \mid s^{\tau}) \right]$$

$N = 1 \rightarrow$ one-step bootstrapped returns ... $N = T \rightarrow$ MC returns

Balancing Bias and Variance – Continued



Advantage Actor Critic (A2C)

Advantage actor-critic is a foundation actor-critic algorithm which uses the **advantage** of a policy to guide the policy gradients:

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We can compute the advantage using only a state-value function:

$$Adv(s^t, a^t) = Q(s^t, a^t) - V(s^t) = \begin{cases} r^t - V(s^t) & \text{if } s^{t+1} \text{ is terminal} \\ r^t + \gamma V(s^{t+1}) - V(s^t) & \text{otherwise} \end{cases}$$

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- We can also still use N-step returns to reduce bias during the advantage computation

Algorithm A2C

```
1: Initialize actor network  $\pi$  with random parameters  $\phi$ 
2: Initialize critic network  $V$  with random parameters  $\theta$ 
3: for every episode do
4:   for time step  $t = 0, 1, 2, \dots$  do
5:     Observe current state  $s^t$ 
6:     Sample action  $a^t \sim \pi(\cdot \mid s^t; \phi)$ 
7:     Apply action  $a^t$ ; observe reward  $r^t$  and next state  $s^{t+1}$ 
8:     if  $s^{t+1}$  is terminal then
9:       Advantage  $Adv(s^t, a^t) \leftarrow r^t - V(s^t; \theta)$ 
10:      Critic target  $y^t \leftarrow r^t$ 
11:    else
12:      Advantage  $Adv(s^t, a^t) \leftarrow r^t + \gamma V(s^{t+1}; \theta) - V(s^t; \theta)$ 
13:      Critic target  $y^t \leftarrow r^t + \gamma V(s^{t+1}; \theta)$ 
14:      Actor loss  $\mathcal{L}(\phi) \leftarrow -Adv(s^t, a^t) \log \pi(a^t \mid s^t; \phi)$ 
15:      Critic loss  $\mathcal{L}(\theta) \leftarrow (y^t - V(s^t; \theta))^2$ 
16:      Update parameters  $\phi$  by minimizing the actor loss  $\mathcal{L}(\phi)$ 
17:      Update parameters  $\theta$  by minimizing the critic loss  $\mathcal{L}(\theta)$ 
```

Proximal Policy Optimization (PPO)

Problem

Policy gradient methods can cause significant shifts in the policy with a single update, which can worsen the policy!

Solution

Limit the change of the policy in a single update → **trust region** of a policy

Proximal policy optimization (PPO) computes an efficient surrogate objective to limit the change in the policy when executing multiple updates:

$$\mathcal{L}(\phi) = - \min \left(\begin{array}{l} \rho(s^t, a^t) \text{Adv}(s^t, a^t), \\ \text{clip}(\rho(s^t, a^t), 1 - \epsilon, 1 + \epsilon) \text{Adv}(s^t, a^t) \end{array} \right)$$

PPO Surrogate Objective

$$\mathcal{L}(\phi) = -\min \left(\begin{array}{l} \rho(s^t, a^t) \text{Adv}(s^t, a^t), \\ \text{clip}(\rho(s^t, a^t), 1 - \epsilon, 1 + \epsilon) \text{Adv}(s^t, a^t) \end{array} \right)$$

- ρ represents the importance sampling ratio $\rho(s, a) = \frac{\pi(a|s; \phi)}{\pi_\beta(a|s)}$
- π_β represents the behavior policy followed to select action a^t in state s^t
- ϵ is a hyperparameter that determines the allowed change of the policy

PPO Surrogate Objective

$$\mathcal{L}(\phi) = -\min \left(\begin{array}{l} \rho(s^t, a^t) \text{Adv}(s^t, a^t), \\ \text{clip}(\rho(s^t, a^t), 1 - \epsilon, 1 + \epsilon) \text{Adv}(s^t, a^t) \end{array} \right)$$

- ρ represents the importance sampling ratio $\rho(s, a) = \frac{\pi(a|s; \phi)}{\pi_\beta(a|s)}$
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Importance sampling ratios serve multiple purposes:

- Correct for differences in data distributions of π_β and π
- Measure of divergence between π_β and $\pi \rightarrow$ can be clipped to limit divergence

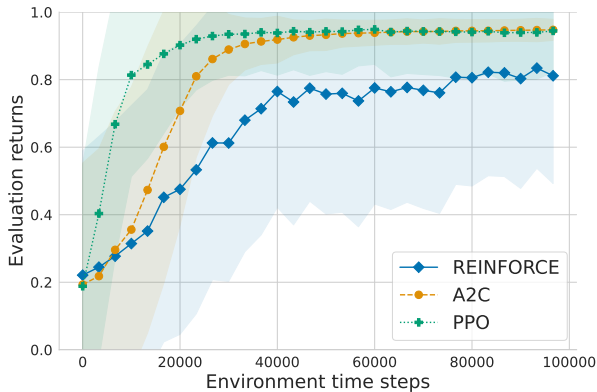
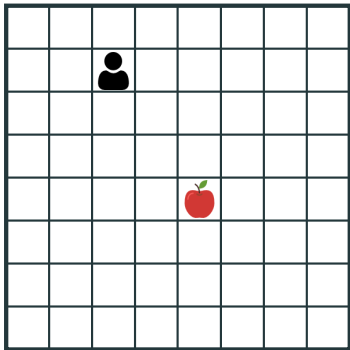
PPO Pseudocode

Algorithm Simplified proximal policy optimization (PPO)

```
1: Initialize actor network  $\pi$  with random parameters  $\phi$ 
2: Initialize critic network  $V$  with random parameters  $\theta$ 
3: for every episode do
4:   for time step  $t = 0, 1, 2, \dots$  do
5:     Observe current state  $s^t$ 
6:     Sample action  $a^t \sim \pi(\cdot \mid s^t; \phi)$ 
7:     Apply action  $a^t$ ; observe reward  $r^t$  and next state  $s^{t+1}$ 
8:      $\pi_\beta(a^t \mid s^t) \leftarrow \pi(a^t \mid s^t; \phi)$ 
9:     for epoch  $e = 1, \dots, N_e$  do
10:       $\rho(s^t, a^t) \leftarrow \pi(a^t \mid s^t; \phi) \div \pi_\beta(a^t \mid s^t)$ 
11:      if  $s^{t+1}$  is terminal then
12:        Advantage  $Adv(s^t, a^t) \leftarrow r^t - V(s^t; \theta)$ 
13:        Critic target  $y^t \leftarrow r^t$ 
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16:        Critic target  $y^t \leftarrow r^t + \gamma V(s^{t+1}; \theta)$ 
17:      Actor loss  $\mathcal{L}(\phi) \leftarrow -\min \left( \begin{array}{l} \rho(s^t, a^t) Adv(s^t, a^t), \\ \text{clip}(\rho(s^t, a^t), 1 - \epsilon, 1 + \epsilon) Adv(s^t, a^t) \end{array} \right)$ 
18:      Critic loss  $\mathcal{L}(\theta) \leftarrow (y^t - V(s^t; \theta))^2$ 
19:      Update parameters  $\phi$  by minimising the actor loss
20:      Update parameters  $\theta$  by minimising the critic loss  $\mathcal{L}(\theta)$ 
```

- PPO executes multiple epochs of updates!
- For first epoch, $\pi = \pi_\beta$
 $\Rightarrow \rho = 1$
- After first epoch, $\pi \neq \pi_\beta$
 \Rightarrow needs ρ to correct for off-policy data

Policy Gradient Algorithms in Practice



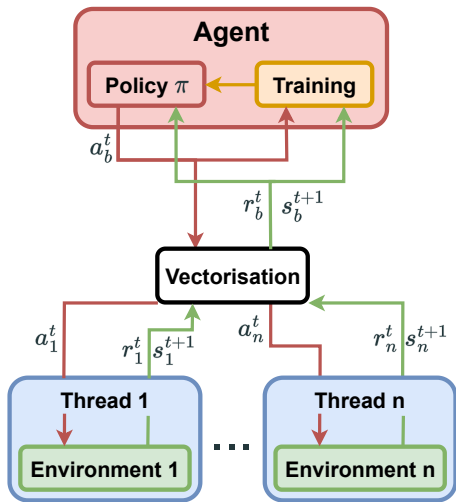
Concurrent Training

Concurrent Training of Policies

Policy gradient algorithms rely on on-policy data, which raises the question of how to deal with correlation in the data and how to increase sample efficiency.

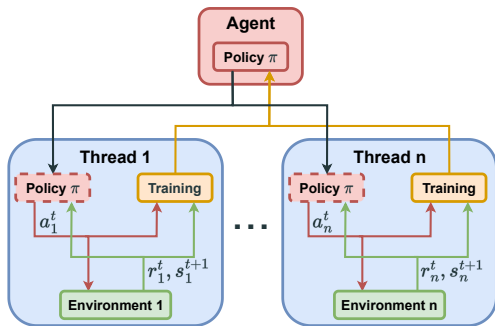
- Concurrent training of policies speeds up training by getting more samples (in parallel), leading to better gradients and breaking of correlation of data
- There are many ways to achieve this, but there are two simple methods commonly used, **synchronous training** and **asynchronous training**

Synchronous Training



- Initiates separate instances of the environment in separate threads
- At each timestep, the agent receives a batch of states and rewards from each thread
- The agent then independently chooses an action for each thread/environment
- Aggregate gradients across batch of experiences → more stable and efficient optimization

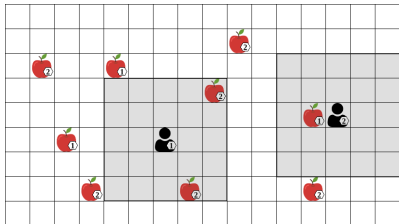
Asynchronous Training



- Asynchronous training parallelizes the optimization of the agent.
- Each thread separately computes the loss and gradients and optimizes the parameters of the agent's network
- Once gradients are computed, the central agent's network is updated
- Asynchronous training is particularly effective if multiple accelerators (e.g. GPUs) are available

Observation, States, and Histories in Practice

We have thus far only considered algorithms that condition on the entire states of the environment, but in practice we often have **partial observability**.

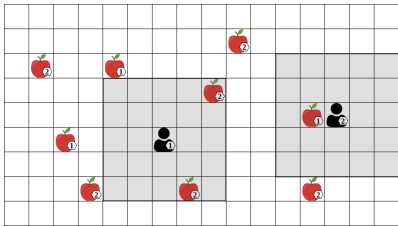


We want to condition value functions and policies on observation history

$$h^t = (o^0, \dots, o^t)$$

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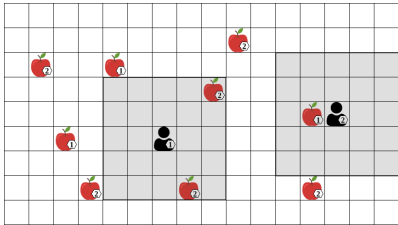
- Feedforward NN assume constant input size, this would require zero-padding vectors to the maximum episode length
- Zero-padding requires knowledge about maximum episode length and results in high-dimensional and sparse inputs

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$$h^t = (o^0, \dots, o^t)$$

- Feedforward NN assume constant input size, this would require zero-padding vectors to the maximum episode length
- Zero-padding requires knowledge about maximum episode length and results in high-dimensional and sparse inputs
- To avoid this we can use RNNs that process sequences of observations with one observation at a time while maintaining the previous history in the hidden state

Summary

We covered:

- Deep Q-learning
- Moving-target problem and correlations of consecutive experiences
- Policy gradient algorithms
- Concurrent training of policies
- Observation, states, and histories under partial observability

Next we'll cover:

- Multi-agent deep reinforcement learning