Multi-Agent Reinforcement Learning

Introduction

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Multi-Agent Reinforcement Learning: Foundations and Modern Approaches

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The book can be downloaded for free at www.marl-book.com.

Lecture Outline

Part 1: Introduction

- Multi-agent systems
- Advantages of MARL vs SARL in multi-agent systems
- Challenges of MARL

Part 2: Reinforcement Learning Basics

- Markov decision processes
- Discounted returns
- Dynamic programming and temporal-difference learning

Part 1: Introduction

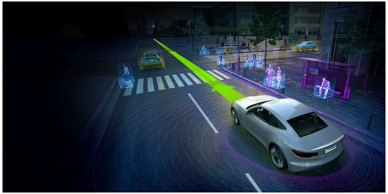
What is MARL?

Multi-agent reinforcement learning (MARL) is about finding optimal decision policies for two or more artificial agents interacting in a common environment.

- Applying reinforcement learning (RL) algorithms to multi-agent systems
- Goal is to learn optimal policies for two or more agents

MARL Applications

Competitive games



Autonomous driving

Multi-robot warehouses

Automated trading

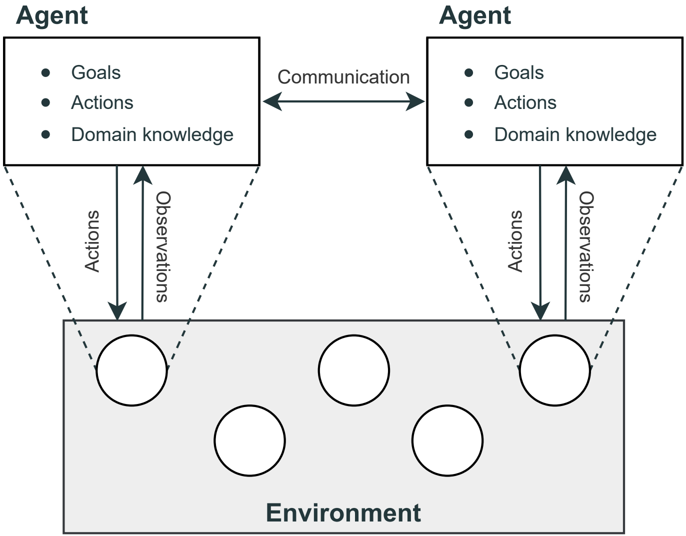
Mutli-Agent Systems

A multi-agent system consists of:

- **Environment:** The environment is a physical or virtual world whose state evolves and is influenced by the agents' actions within the environment.
- **Agents:** An agent is an entity which receives information about the state of the environment and can choose actions to influence the state.

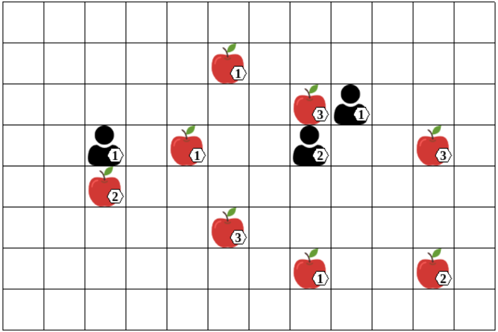
  $\Rightarrow$ Agents are goal-directed, e.g. maximizing returns

Multi-Agent Systems

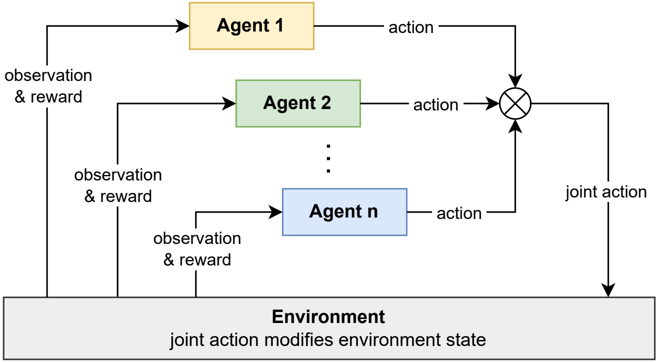


Example: Level-Based Foraging

- Three agents with varying skill levels
- Goal: to collect all apples
- Items can be collected if a group of one or more agents are located next to the item and the sum of agents' levels is greater than or equal to the item level
- Action space $A = \{up, down, left, right, collect, noop\}$



MARL for Solving Multi-Agent Systems



- Goal: learn optimal policies for a set of agents in a multi-agent system
- Each agent chooses an action based on its policy $\Rightarrow$ joint action
- Environment transitions into new state; agents receive rewards + new observations

Why MARL?

Why should we use MARL to find optimal solutions to multi-agent systems rather than controlling multiple ’agents’ using a single-agent RL (SARL) algorithm?

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Decomposing a larger problem

- For example, controlling 3 agents each with 6 actions (see LBF example), the action space becomes $6^3 = 216$.
- Using MARL, we decompose this into three more tractable problems.

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Decentralized decision making

- There are many real-world scenarios where it is beneficial for each agent to make decisions independently.
- E.g. autonomous driving is impractical for frequent long-distance data exchanges between a central agent and the vehicle.

Challenges of MARL

New **challenges** arise in MARL:

- Non-stationarity caused by multiple learning agents

We will explore these challenges in upcoming lectures!

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New **challenges** arise in MARL:

- Non-stationarity caused by multiple learning agents
- Optimality of policies and equilibrium selection
- Multi-agent credit assignment
- Scaling in number of agents

We will explore these challenges in upcoming lectures!

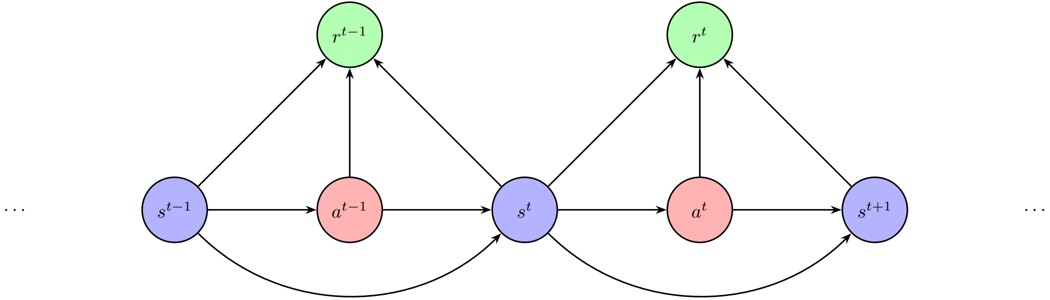
Part 2: Reinforcement Learning Basics

Back to RL Basics

RL Problem = **Decision Process Model** *e.g. MDP, POMDP, multi-armed bandit* + **Learning Objective** *e.g. discounted return with specific discount factor*

- RL algorithms learn solutions for sequential decision problems via repeated environment **interaction**
- Sequential decision process is defined formally as Markov decision process (MDP)
- Goal is to learn an optimal decision policy for a specific learning objective

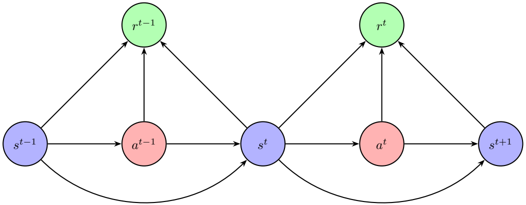
MDP As a Graph



MDP Definition

MDP is defined as a tuple $(S, A, \mathcal{R}, \mathcal{T}, \mu)$:

- S: Finite set of states with subset of terminal states $\bar{S} \subset S$
- A: Finite set of actions
- $\mathcal{R}$: Reward function $\mathcal{R} : S \times A \times S \to \mathbb{R}$
- $\mathcal{T}$: State transition function $\mathcal{T} : S \times A \times S \to [0, 1]$
- μ: Initial state distribution $\mu : S \to [0, 1]$ such that $\sum_{s \in S} \mu(s) = 1$ and $\forall s \in \hat{S} : \mu(s) = 0$



MDP Assumptions

Markov property

- Future states are temporally independent of past states and actions, given the current state and action.
- $Pr(s^{t+1}, r^t | s^t, a^t, s^{t-1}, a^{t-1}, ..., s^0, a^0) = Pr(s^{t+1}, r^t | s^t, a^t)$

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Full observability

- The agent can see the entire state of the environment.
- In practice, agent may only have partial view.

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Stationarity

- The dynamics of the environment are assumed to be stationary.
- i.e. $\mathcal{T}$ and $\mathcal{R}$ remain constant through time.

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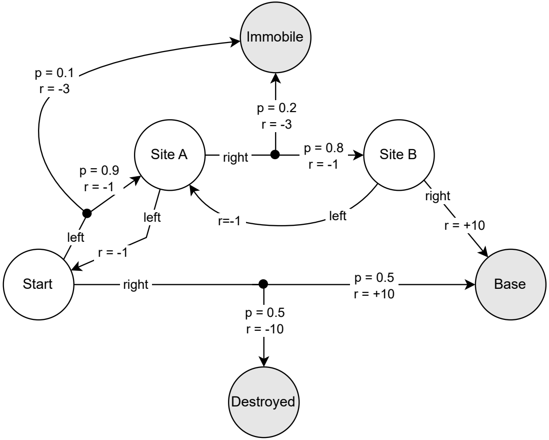
Incomplete knowledge of MDP

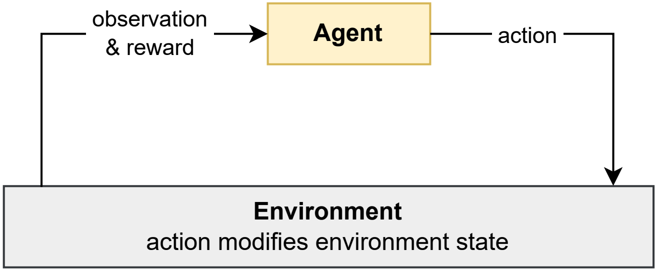
- The agent may only have knowledge of the action and state spaces (A, S)
- The transition and reward function $(\mathcal{T}, \mathcal{R})$ are usually to be **unknown**.

Mars Rover MDP Example



- *Start* is the initial state s^0
- Two possible actions $A = \{right, left\}$
- Goal is to get to *Base*
- Rewards given by $\mathcal{R}(s, a, s')$ are shown as r.
- State transition probabilities given by $\mathcal{T}(s, a, s)$, are shown as p

RL for Optimizing Policies in MDPs



Value-Based RL

These methods indirectly update the policy by learning value functions.

Policy-Based RL

These methods update a parameterized policy function directly.

Expected Discounted Returns

Using RL, we aim to maximize the expected **return**.

- Returns (u) are the sum of rewards received over time
- If MDP is non-terminating (i.e. $t \to \infty$), we **discount** returns to ensures finite (discounted) returns

$$\mathbb{E}_{\pi}[u_t] = \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t r^t\right]$$

- γ is the discount factor, such that $\gamma \in [0, 1]$
- π is the behavior policy that determines which actions are chosen.

State-Value Functions

State-value functions $V^\pi(s)$ give the 'value' of state s when following the policy π.

$$V^\pi(s) = \mathbb{E}_\pi \left[u^t | s^t = s\right]$$

The return (u) can be recursively defined as:

$$\begin{aligned} u^t &= r^t + \gamma(r^{t+1} + \gamma r^{t+2} + ...) \\ &= r^t + \gamma u^{t+1} \end{aligned}$$

Therefore:

$$V^\pi(s) = \mathbb{E}_\pi \left[r^t + \gamma u^{t+1} | s^t = s\right]$$

The Bellman Equation

$$\begin{aligned} V^\pi(s) &= \mathbb{E}_\pi\left[r^t + \gamma u^{t+1} \mid s^t = s\right] \\ &= \sum_{a \in A} \pi(a \mid s) \sum_{s' \in S} \mathcal{T}(s' \mid s, a)\left[\mathcal{R}(s, a, s') + \gamma \mathbb{E}_\pi\left[u^{t+1} \mid s^{t+1} = s'\right]\right] \\ &= \sum_{a \in A} \pi(a \mid s) \sum_{s' \in S} \mathcal{T}(s' \mid s, a)\left[\mathcal{R}(s, a, s') + \gamma V^\pi(s')\right] \end{aligned}$$

The last equation is known as the Bellman equation in honor of Richard Bellman.

- The value of being in state s while following a fixed policy π is equivalent to the immediate reward ($\mathcal{R}(s, a, s') \to r$) received when taking action a in state s ($\pi(a \mid s)$) plus the discounted value of the next state s'.

State-Action Value Function

State-Action value functions $Q^\pi(s, a)$ are an extension on the *State* value functions. They condition the expected return on the current state and the action taken.

$$Q^\pi(s, a) = \mathbb{E}_\pi \left[u^t | s^t = s, a^t = a \right]$$

The *state-action* value Bellman equation is therefore:

$$Q^\pi(s, a) = \sum_{a \in A} \pi(a|s) \sum_{s' \in S} \mathcal{T}(s'|s, a) \left[\mathcal{R}(s, a, s') + \gamma Q^\pi(s') \right]$$

Greedy Policies

A policy can act *greedily* i.e. choosing actions which maximize the immediate reward and the value of the next state.

Greedy π using *state value* functions:

$$\pi(s) = \arg\max_{a \in A} \sum_{s',r} \mathcal{T}(s', r|s, a) \left[r + \gamma V(s')\right]$$

Or using the *state-action value function*:

$$\pi(s) = \arg\max_{a \in A} Q(s, a)$$

Optimal Greedy Policy

A greedy policy with respect to a value function is optimal only when using the **optimal value function.**

An **optimal value function** for a MDP can be defined as:

$$V^*(s) = \max_{\pi'} V^{\pi'}(s), \quad \forall s \in S$$
$$Q^*(s, a) = \max_{\pi'} Q^{\pi'}(s, a), \quad \forall s \in S, a \in A$$

Therefore, the optimal policy is:

$$\pi^*(s) = \arg\max_{a \in A} Q^*(s, a)$$

Dynamic Programming

- Dynamic Programming (DP) is a family of algorithms to compute **optimal value functions** and **optimal policies in MDPs** (Bellman 1957; Howard 1960).
- In DP, we **assume complete knowledge** of the MDP, including the transition and reward function $(\mathcal{T}, \mathcal{R})$.
- Given complete knowledge, we can use the **Bellman equation** to find optimal value functions and policies.

Policy Iteration

Policy iteration is a DP algorithm that alternates between two steps:

- **Policy evaluation**: compute value function V^{π} for current policy π
- **Policy improvement**: improve current policy π with respect to V^{π}

$$\pi^0 \rightarrow V^{\pi^0} \rightarrow \pi^1 \rightarrow V^{\pi^1} \rightarrow \pi^2 \rightarrow \ldots \rightarrow V^* \rightarrow \pi^*$$

Policy Iteration Pseudo Code

Algorithm Policy Iteration

Initialize π randomly, initialise $V(s)$ arbitrarily for all $s \in S$ except $V(\text{terminal}) = 0$
repeat
    Policy Evaluation:
    repeat
        for each state s in S **do**
            $V(s) \leftarrow \sum_{s'} \mathcal{T}(s'|s, \pi(s))[\mathcal{R}(s, \pi(s), s') + \gamma V(s')]$
    until $V(s)$ converges for all $s \in S$
    Policy Improvement:
    policy_stable $\leftarrow$ true
    for each state s in S **do**
        old_action $\leftarrow \pi(s)$
        $\boxed{\pi(s) \leftarrow \operatorname{arg\,max}_a \sum_{s'} \mathcal{T}(s'|s, a)[\mathcal{R}(s, a, s') + \gamma V(s')]}$
        if old_action $\neq \pi(s)$ **then**
            policy_stable $\leftarrow$ false
until policy_stable **return** V, π

Value Iteration

Value Iteration uses the **Bellman optimality equation**

- Combines iterative policy evaluation and improvement into one single update equation.

Bellman Optimality Equation as update operator:

$$V^{k+1}(s) \leftarrow \max_{a \in A} \sum_{s' \in S} \mathcal{T}(s'|s,a)[\mathcal{R}(s,a,s') + \gamma V^k(s')], \quad \forall s \in S$$

- The max operator makes this the Bellman *optimality* equation.
- The equation expresses the value of a state as the maximum expected return achievable by taking the best action and then following the optimal policy.

Value Iteration Pseudo Code

Algorithm Value Iteration

Initialize: $V(s) = 0, \forall s \in S$

repeat

$\quad \boxed{\forall s \in S : V(s) \leftarrow \max_{a \in A} \sum_{s' \in S} \mathcal{T}(s'|s, a) \left[\mathcal{R}(s, a, s') + \gamma V(s')\right]}$

until V converged **return** optimal policy π^* with:

$\forall s \in S : \arg\max_{a \in A} \sum_{s' \in S} \mathcal{T}(s'|s, a) \left[\mathcal{R}(s, a, s') + \gamma V(s')\right]$

Temporal-Difference Learning

Temporal-Difference (TD) learning is a family of RL algorithms that learn optimal policies and value functions based on data collected via environment **interactions**.

- These algorithms learn which actions yield the best returns by **trial and error** and exploring different actions and states

  $\Rightarrow$ Learns without **model** of environment's **reward** and **transition** functions $(\mathcal{R}, \mathcal{T})$

  $\Rightarrow$ Can learn **online**, updating the policy while interacting with the environment

Temporal Difference Update

The update for temporal-difference learning relies on value functions:

$$V(s^t) \leftarrow V(s^t) + \alpha \left[\mathcal{X} - V(s^t)\right]$$

or

$$Q(s^t, a^t) \leftarrow (s^t, a^t) + \alpha \left[\mathcal{X} - (s^t, a^t)\right]$$

- $\mathcal{X}$ is the update target, serving as an estimate of the current state value.
- α is the learning rate

Temporal-Difference Update

In SARSA (a basic TD algorithm), we use the experience tuple $(s^t, a^t, r^t, s^{t+1}, a^{t+1})$ to construct a target:

$$\mathcal{X} = r^t + \gamma Q(s^{t+1}, a^{t+1})$$

(The immediate reward plus the discounted value of the next state) - note the resemblance to the Bellman equation.

$$Q^\pi(s,a) = \sum_{a \in A} \pi(a \mid s) \sum_{s' \in S} \mathcal{T}(s' \mid s, a) \boxed{[\mathcal{R}(s,a,s') + \gamma Q^\pi(s',a')]}$$

SARSA

The SARSA update rule thus becomes:

$$Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha[r^t + \gamma Q(s^{t+1}, a^{t+1}) - Q(s^t, a^t)]$$

- Note the TD error $r^t + \gamma Q(s^{t+1}, a^{t+1}) - Q(s^t, a^t)$.
- The TD update is **bootstrapped**
- Using the value **estimates** of the next state ($Q(s^{t+1}, a^{t+1})$) to update the current state value ($Q(s^t, a^t)$)

SARSA Pseudo Code

Algorithm SARSA

Initialize $Q(s, a) = 0$ for all $s \in S, a \in A$
for every episode **do**
    Observe initial state s^0
    With probability ϵ: choose random action $a^0 \in A$
    Otherwise: choose action $a^0 \in \arg\max_a Q(s^0, a)$
    for $t = 0, 1, 2, \ldots$ **do**
        Apply action a^t, observe reward r' and next state s^{t+1}
        With probability ϵ: choose random action $a^{t+1} \in A$
        Otherwise: choose action $a^{t+1} \in \arg\max_a Q(s^{t+1}, a)$
        $\boxed{Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha[r^t + \gamma Q(s^{t+1}, a^{t+1}) - Q(s^t, a^t)]}$

Convergence of SARSA

SARSA is guaranteed to converge to the optimal **state-value function**, for all $S \in S$ and $a \in A$, if:

- All *state-action* pairs are explored infinitely many times:

$$\forall s \in S, a \in A : \sum_{k=0}^{\infty} \mathbb{I}(s, a) \rightarrow \infty$$

- The learning rate α is reduced over time according to the "standard stochastic approximation conditions":

$$\forall s \in S, a \in A : \sum_{k=0}^{\infty} \alpha_k(s, a) \rightarrow \infty \quad \text{and} \quad \sum_{k=0}^{\infty} \alpha_k(s, a)^2 < \infty$$

ϵ-Greedy Policies

Using a **greedy policy** would **violate the convergence condition** of SARSA (infinite exploration of S and A).

- Intuitively, we must explore a wide range of states and actions to find state action combinations that yield high returns
- One solution is to add an **exploration** parameter $\epsilon \in [0, 1]$. This gives us a stochastic **epsilon-greedy** policy.

$$\pi(a|s) = \begin{cases} 1 - \epsilon + \frac{\epsilon}{|A|} & \text{if } a \in \arg\max_{a'} Q(s, a') \\ \frac{\epsilon}{|A|} & \text{otherwise} \end{cases}$$

- With probability $1 - \epsilon$, the policy chooses the greedy action, and with probability ϵ chooses an action uniformly at random.

Q-Learning

Q-learning (Watkins & Dayan 1992) is a popular TD algorithm which uses the Bellman optimality equation to update its value function estimates.

- By using the Bellman optimality equation, Q-learning directly learns the **optimal state-action value function**
- Q-learning is **off-policy**, meaning the policy followed to gather experiences is different from the optimized policy
- We use the ϵ-greedy policy to collect experiences

Q-Learning Update

The target in Q-learning uses the *max* operator to target the optimal Q-values directly.

$$\mathcal{X} = r^t + \gamma \max_{a' \in A} Q(s^{t+1}, a')$$

The Q-learning update is thus:

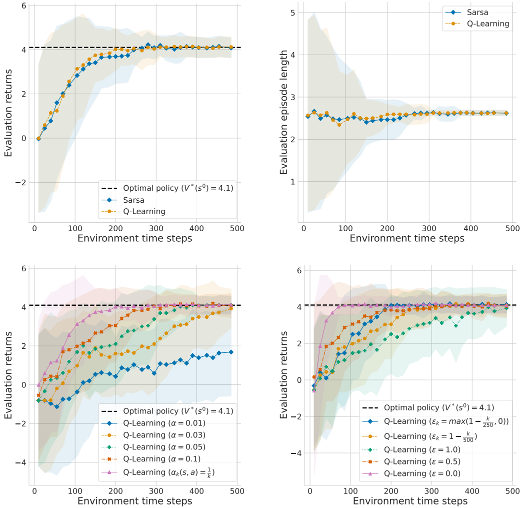
$$Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha \left[r^t + \gamma \max_{a' \in A} Q(s^{t+1}, a') - Q(s^t, a^t) \right]$$

Q-Learning Pseudo Code

Algorithm Q-Learning

```
Initialize $Q(s, a) = 0$ for all $s \in S, a \in A$
for every episode do
    for $t = 0, 1, 2, \ldots$ do
        Observe current state $s^t$
        With probability $\epsilon$: choose random action $a^t \in A$
        Otherwise: choose action $a^t \in \arg\max_a Q(s^t, a)$
        Apply action $a^t$, observe reward $r^t$ and next state $s^{t+1}$
        $\boxed{Q(s^t, a^t) \leftarrow Q(s^t, a^t) + \alpha \left[r^t + \gamma \max_{a'} Q(s^{t+1}, a') - Q(s^t, a^t)\right]}$
```

Evaluating RL Algorithms



Y-axis:

- Average **discounted evaluation returns**. This shows us how our greedy policy would perform if we stopped learning at time step T.
- In some cases, **undiscounted returns** are easier to interpret and may be used instead.

X-axis:

- Cumulative training steps across episodes.
- Number of episodes can also be used. This might, however, distort the learning speed.

Summary

We covered:

- Multi-Agent Systems and the case for MARL
- MDPs
- Value Functions
- Dynamic Programming
- Temporal Difference Learning (SARSA and Q-Learning)

Next we'll cover:

- Games: Models of Multi-Agent Interaction

Reading

Richard Bellman: Dynamic Programming. Princeton University Press, 1957

Ronald Howard: Dynamic Programming and Markov Processes. John Wiley, 1960

Richard Sutton and Andrew Barto: Reinforcement learning: An introduction (2nd edition). MIT Press, 2018