

# Multi-Agent Reinforcement Learning

Multi-Agent Reinforcement Learning: Foundational Algorithms

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Stefano V. Albrecht, Filippos Christianos, Lukas Schäfer

This lecture is based on

## **Multi-Agent Reinforcement Learning: Foundations and Modern Approaches**

by Stefano V. Albrecht, Filippos Christianos and  
Lukas Schäfer

MIT Press, 2024

Download book, slides, and code at:

[www.marl-book.com](http://www.marl-book.com)

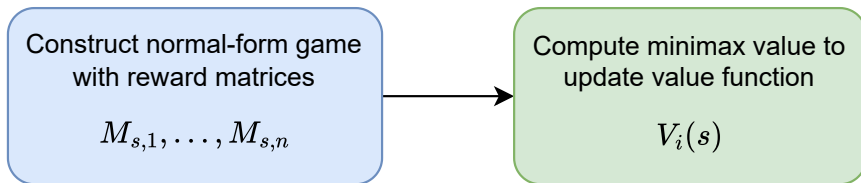


- Dynamic programming for games: value iteration
- Temporal-difference learning for games: joint-action learning
- Agent modeling
- Policy-based learning
- No-regret learning

# Dynamic Programming for Games: Value Iteration

Shapley (1953) proposed **value iteration** to compute **minimax** joint policy in zero-sum stochastic games with two agents

- Algorithm makes two sweeps over states  $s \in S$  and agents  $i \in I$ :



- Converges to minimax values  $V_i^*(s)$  of the stochastic game

# Value Iteration Pseudocode

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**Algorithm** Value iteration for stochastic games

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- 1: Initialize:  $V_i(s) = 0$  for all  $s \in S$  and  $i \in I$
- 2: Repeat until all  $V_i$  have converged:

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- 2: Repeat until all  $V_i$  have converged:
- 3: **for all** states  $s \in S$ , agents  $i \in I$ , joint actions  $a \in A$  **do**

$$M_{s,i}(a) \leftarrow \sum_{s' \in S} \mathcal{T}(s' \mid s, a) [\mathcal{R}_i(s, a, s') + \gamma V_i(s')]$$

# Value Iteration Pseudocode

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- 4: **for all** states  $s \in S$ , agents  $i \in I$  **do**

$$V_i(s) \leftarrow \text{Value}_i(M_{s,1}, \dots, M_{s,n}) \quad // \text{ Minimax value for agent } i$$

# Obtaining Minimax Policies for the Stochastic Game

To obtain minimax policies  $\pi_1, \dots, \pi_n$  for the stochastic game:

- Given converged state minimax values  $V_i^*$  and a state  $s$
- Construct the normal-form game  $M_{s,1}^*, \dots, M_{s,n}^*$
- Compute the minimax policies  $\pi_1^*, \dots, \pi_n^*$  of this normal-form game
- Set action probabilities  $\pi_i(a_i|s) \leftarrow \pi_i^*(a_i)$  for all  $a_i \in A_i$



# From Dynamic Programming to Temporal-Difference Learning

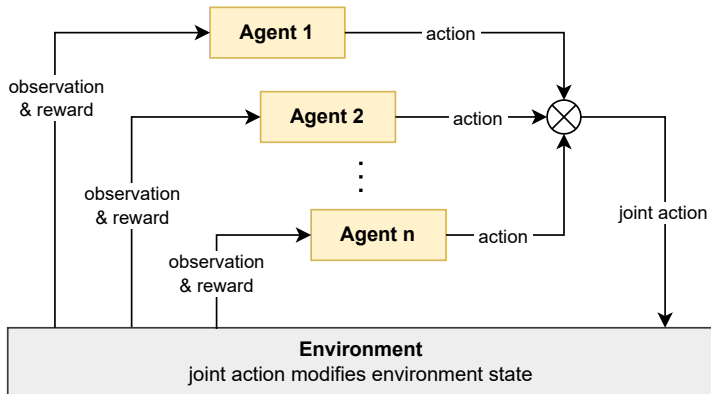
## Problem

Dynamic programming requires full knowledge of game

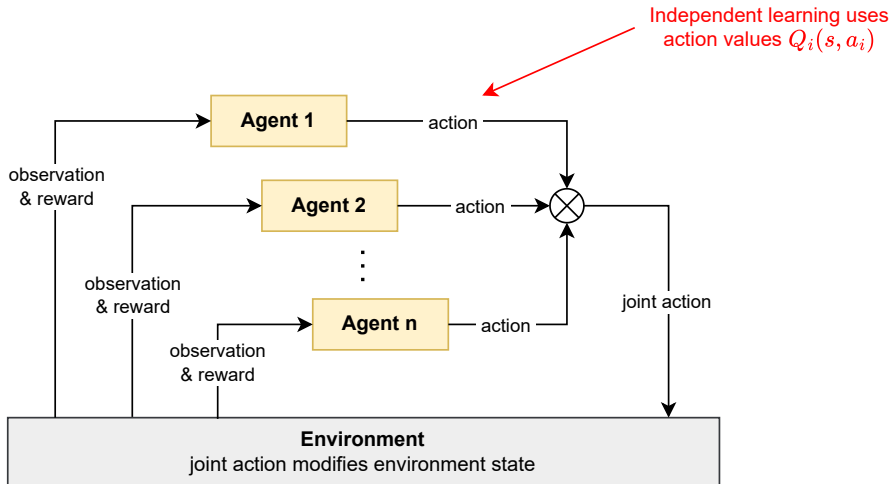
- Including reward functions  $\mathcal{R}_i$  and state transition function  $\mathcal{T}$
- May not be available!

Can we *learn* equilibrium joint policy via *temporal-difference learning*?

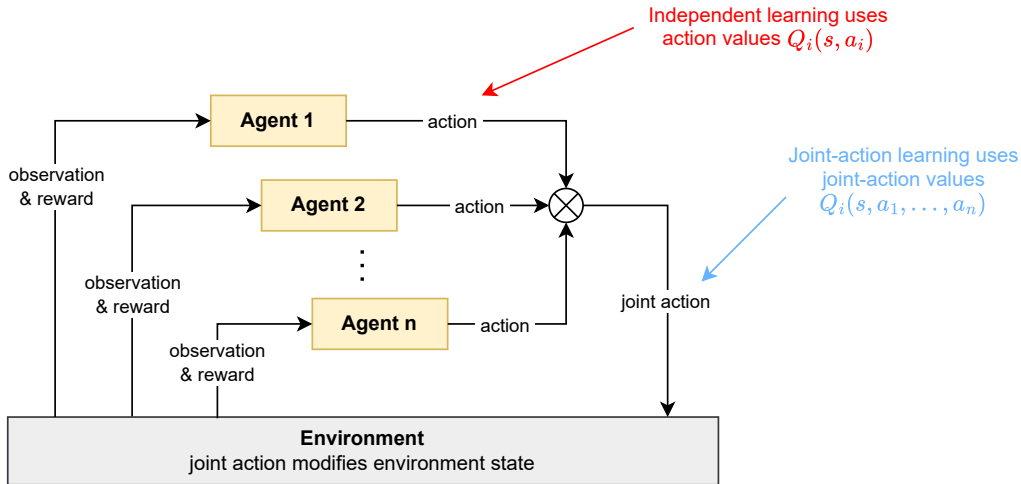
# Temporal-Difference Learning for Games: Joint-Action Learning



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$Q_i(s, a_1, \dots, a_n)$  is not enough to find optimal action for agent  $i$

- Cannot evaluate  $\max_{a_i} Q_i(s, a_1, \dots, a_n)$

$\Rightarrow$  Optimal action depends on actions of other agents!

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$\Rightarrow$  Optimal action depends on actions of other agents!

We have to define:

1. How to select action from  $Q_i$ ?
2. How to update  $Q_i$ ?

# Joint-Action Learning with Game Theory

**Idea:** joint-action value functions define a **normal-form game**:

- Each agent stores Q-function  $Q_j$  for every agent  $j \in I$   
(assumes agents can observe all agents' actions and rewards)

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(assumes agents can observe all agents' actions and rewards)
- Define reward functions for normal-form game in state  $s$  as

$$\mathcal{R}_j(a_1, \dots, a_n) = Q_j(s, a_1, \dots, a_n)$$



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- Define reward functions for normal-form game in state  $s$  as

$$\mathcal{R}_j(a_1, \dots, a_n) = Q_j(s, a_1, \dots, a_n)$$

- We can *solve* the normal-form game defined by

$$\Gamma_s = \left( \mathcal{R}_1 = Q_1(s, \cdot), \dots, \mathcal{R}_n = Q_n(s, \cdot) \right)$$

# Joint-Action Learning with Game Theory

**Solution** of  $\Gamma_s$  is a joint policy  $\pi_s^* = (\pi_{s,1}^*, \dots, \pi_{s,n}^*)$  with certain properties

- e.g. compute minimax solution or Nash equilibrium of  $\Gamma_s$   
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**Value** of  $\Gamma_s$  for agent  $j$  is its expected reward under joint policy  $\pi_s^*$

$$Value_j(\Gamma_s) = \sum_{a \in A} Q_j(s, a) \pi_s^*(a)$$

$\Rightarrow$  Update  $Q_j$  towards update target:  $r_j + \gamma Value_j(\Gamma_{s'})$

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**Algorithm** Joint-action learning with game theory (JAL-GT)

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*// Algorithm controls agent  $i$*

- 1: Initialize:  $Q_j(s, a) = 0$  for all  $j \in I$  and  $s \in S, a \in A$
  - 2: Repeat for every episode:
  - 3: **for**  $t = 0, 1, 2, \dots$  **do**
  - 4:   Observe current state  $s^t$
  - 5:   With probability  $\epsilon$ : choose random action  $a_i^t$
  - 6:   Otherwise: solve  $\Gamma_{s^t}$  to get policies  $(\pi_1, \dots, \pi_n)$ , then sample action  $a_i^t \sim \pi_i$
  - 7:   Observe joint action  $a^t = (a_1^t, \dots, a_n^t)$ , rewards  $r_1^t, \dots, r_n^t$ , next state  $s^{t+1}$
  - 8:   **for all**  $j \in I$  **do**
  - 9:      $Q_j(s^t, a^t) \leftarrow Q_j(s^t, a^t) + \alpha \left[ r_j^t + \gamma \text{Value}_j(\Gamma_{s^{t+1}}) - Q_j(s^t, a^t) \right]$
-

**Minimax-Q** solves  $\Gamma_s$  via minimax solution (Littman, 1994)

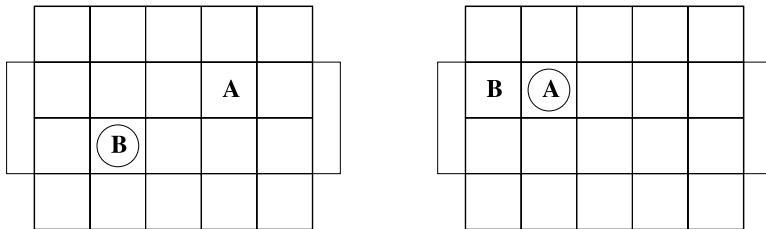
- Converges to unique minimax values in two-agent zero-sum stochastic games
- Minimax profile can be computed with linear programming (LP)

**Nash-Q** solves  $\Gamma_s$  via Nash equilibrium (Hu and Wellman, 2003)

**CE-Q** solves  $\Gamma_s$  via correlated equilibrium (Greenwald and Hall, 2003)

- Converges to equilibrium under highly restrictive conditions  
⇒ Problem: often no unique equilibrium value in general-sum games
- Compute CE with LP, compute NE with quadratic programming

## Example: Minimax-Q in Grid Soccer (Littman, 1994)



- Episodes start in left state with random ball assignment
- Agent wins episode if it moves the ball into opponent goal
- Agent loses ball to opponent if it moves into opponent's location

Against *unknown* opponent, optimal policy must randomise (right state; why?)

## Example: Minimax-Q in Grid Soccer (Littman, 1994)

	minimax Q		independent Q	
	% won	ep. len.	% won	ep. len.
vs. random				
vs. hand-built				
vs. optimal				

- random: uniform-random opponent policy
- hand-built: manual opponent policy
- optimal: Q-learning opponent policy trained against final policy of minimax Q / independent Q

## Example: Minimax-Q in Grid Soccer (Littman, 1994)

	minimax Q		independent Q	
	% won	ep. len.	% won	ep. len.
vs. random	99.3	13.89		
vs. hand-built	53.7	18.87		
vs. optimal	37.5	22.73		

- minimax Q learns “safe” policy that works against any opponent  
⇒ minimax policy guarantees minimum average 50% win
- lower % win against optimal because minimax Q did not fully converge during training, so could be exploited by optimal opponent



## Example: Minimax-Q in Grid Soccer (Littman, 1994)

	minimax Q		independent Q	
	% won	ep. len.	% won	ep. len.
vs. random	99.3	13.89	99.5	11.63
vs. hand-built	53.7	18.87	76.3	30.30
vs. optimal	37.5	22.73	0	83.33

### Problem

- Independent Q-learning can learn strong performance
- **But:** overfits to opponent, does not generalise well to other opponents  
⇒ “optimal” opponent exploits deterministic policy learned by independent Q-learning, resulting in 0% wins

# Agent Modeling & Best Response

Game theory solutions are **normative**: they *prescribe* how agents *should* behave

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What if agents don't behave as prescribed by solution?

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Other approach: **agent modeling with best response**

- Learn models of other agents to predict their actions
- Compute optimal action (best response) against agent models



Many kinds of agent modeling:

- Policy reconstruction
- Type-based reasoning
- Classification
- Plan recognition
- Recursive reasoning
- Graphical methods
- Group modeling
- Implicit modeling

S. Albrecht, P. Stone. **Autonomous Agents Modelling Other Agents: A Comprehensive Survey and Open Problems.**  
*Artificial Intelligence*, 2018

# Policy Reconstruction & Best Response

**Policy reconstruction:** learn model  $\hat{\pi}_j \approx \pi_j$  from past observed actions

In general, can train model with **supervised learning** on data  $\{(s^\tau, a_j^\tau)\}_{\tau=1}^{t-1}$

- E.g. look-up table, neural network, finite state machine, ...
- Model should support incremental updating

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- Model should support incremental updating

Given models for other agents  $\hat{\pi}_{-i} = \{\hat{\pi}_j\}_{j \neq i}$ , compute **best response**

$$\pi_i \in \text{BR}_i(\hat{\pi}_{-i})$$

# Fictitious Play

Fictitious play (Brown 1951) algorithm for non-repeated normal-form games

Each agent  $i$  models other agents  $j$  as stationary distribution:

$$\hat{\pi}_j(a_j) = \frac{C(a_j)}{\sum_{a'_j \in A_j} C(a'_j)}$$

$C(a_j)$  is number of times agent  $j$  chose action  $a_j$  in prior episodes



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In each episode, agents choose best-response action:

$$\text{BR}_i(\hat{\pi}_{-i}) = \arg \max_{a_i \in A_i} \sum_{a_{-i} \in A_{-i}} \mathcal{R}_i(\langle a_i, a_{-i} \rangle) \prod_{j \neq i} \hat{\pi}_j(a_j)$$

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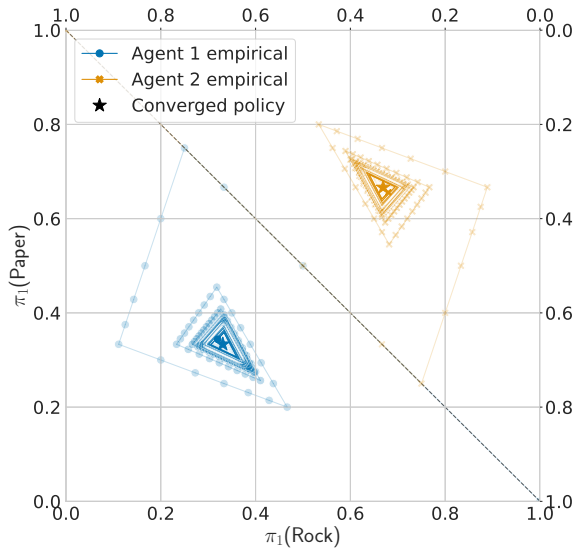
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## Fictitious Play Convergence

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- If in any episode the agents' actions form a NE, then they will always remain in the equilibrium
- If empirical distribution of agents' actions converges, then the distributions converge to a NE
- The empirical distributions converge in several game classes, e.g. in two-agent zero-sum finite games

# Fictitious Play in Rock-Paper-Scissors



## Fictitious Play in Rock-Paper-Scissors

Episode $e$	Joint action $(a_1^e, a_2^e)$	Agent model $\hat{\pi}_2$	Agent 1 action values
1	R,R	(0.33, 0.33, 0.33)	(0.00, 0.00, 0.00)
2	P,P	(1.00, 0.00, 0.00)	(0.00, 1.00, -1.00)
3	P,P	(0.50, 0.50, 0.00)	(-0.50, 0.50, 0.00)
4	P,P	(0.33, 0.67, 0.00)	(-0.67, 0.33, 0.33)
5	S,S	(0.25, 0.75, 0.00)	(-0.75, 0.25, 0.50)
6	S,S	(0.20, 0.60, 0.20)	(-0.40, 0.00, 0.40)
7	S,S	(0.17, 0.50, 0.33)	(-0.17, -0.17, 0.33)
8	S,S	(0.14, 0.43, 0.43)	(0.00, -0.29, 0.29)
9	S,S	(0.13, 0.38, 0.50)	(0.12, -0.38, 0.25)
10	R,R	(0.11, 0.33, 0.56)	(0.22, -0.44, 0.22)

# Joint-Action Learning with Agent Modeling

Extend fictitious play approach to stochastic games by using joint-action learning with agent models



# Joint-Action Learning with Agent Modeling

Extend fictitious play approach to **stochastic games** by using **joint-action learning with agent models**

Agents model other agents  $j$ , this time conditioned on states  $s$ :

$$\hat{\pi}_j(a_j | s) = \frac{C(s, a_j)}{\sum_{a'_j \in A_j} C(s, a'_j)}$$

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Given models  $\{\hat{\pi}_j\}_{j \neq i}$ , action values are defined as:

$$AV_i(s, a_i) = \sum_{a_{-i} \in A_{-i}} Q_i(s, \langle a_i, a_{-i} \rangle) \prod_{j \neq i} \hat{\pi}_j(a_j | s)$$

$\Rightarrow$  Use  $AV_i$  to select optimal actions and as learning update targets

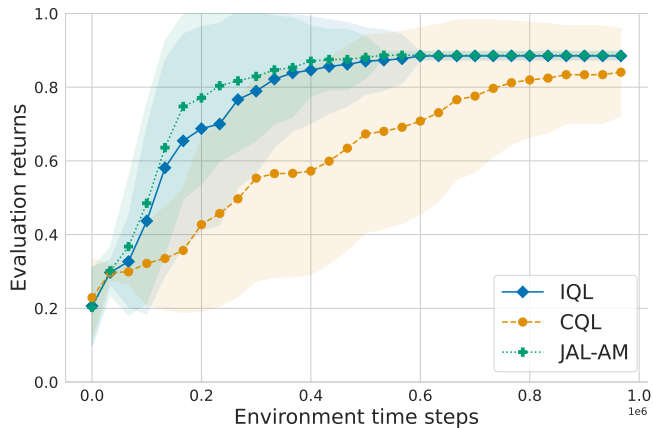
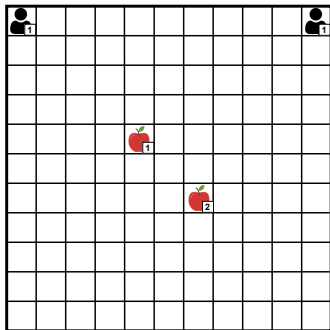
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## Algorithm Joint-action learning with agent modeling (JAL-AM)

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- 1: Initialize:
- 2:    $Q_i(s, a) = 0$  for all  $s \in S, a \in A$
- 3:   Agent models  $\hat{\pi}_j(a_j | s) = \frac{1}{|A_j|}$  for all  $j \neq i, a_j \in A_j, s \in S$
- 4: Repeat for every episode:
- 5:   **for**  $t = 0, 1, 2, \dots$  **do**
- 6:     Observe current state  $s^t$
- 7:     With probability  $\epsilon$ : choose random action  $a_i^t$
- 8:     Otherwise: choose best-response action  $a_i^t \in \arg \max_{a_i} AV_i(s^t, a_i)$
- 9:     Observe joint action  $a^t = (a_1^t, \dots, a_n^t)$ , reward  $r_i^t$ , next state  $s^{t+1}$
- 10:    Update agent models  $\hat{\pi}_j$  with new observations (e.g.,  $(s^t, a_j^t)$ )
- 11:     $Q_i(s^t, a^t) \leftarrow Q_i(s^t, a^t) + \alpha \left[ r_i^t + \gamma \max_{a'_i} AV_i(s^{t+1}, a'_i) - Q_i(s^t, a^t) \right]$

# JAL-AM in Level-Based Foraging



All algorithms so far derive policies from learned **action-value functions**

- Has important limitations, e.g. algorithms using best-response actions from values (e.g. fictitious play, JAL-AM) cannot represent probabilistic equilibria
- Fictitious play unable to represent uniform-random NE in Rock-Paper-Scissors

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**Policy-based learning** instead uses learning data to directly optimise policies

- Use *parameterised policies* that are differentiable
- Use *gradient-ascent techniques* to optimise parameters  
⇒ Can directly learn action probabilities in policies!

# Gradient Ascent in Expected Reward

**Gradient-ascent learning** in non-repeated normal-form games with two agents  $i, j$  and two actions

Reward matrices:

$$\mathcal{R}_i = \begin{bmatrix} r_{1,1} & r_{1,2} \\ r_{2,1} & r_{2,2} \end{bmatrix} \quad \mathcal{R}_j = \begin{bmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{bmatrix}$$

Policies with parameters  $\alpha, \beta \in [0, 1]$ :

$$\pi_i = (\alpha, 1 - \alpha) \quad \pi_j = (\beta, 1 - \beta)$$

## Gradient Ascent in Expected Reward

Update policy in direction of gradient in expected reward using step size  $\kappa > 0$ :

$$\alpha^{k+1} = \alpha^k + \kappa \frac{\partial U_i(\alpha^k, \beta^k)}{\partial \alpha^k}$$
$$\beta^{k+1} = \beta^k + \kappa \frac{\partial U_j(\alpha^k, \beta^k)}{\partial \beta^k}$$

Partial derivative of an agent's expected reward with respect to its policy:

$$\frac{\partial U_i(\alpha, \beta)}{\partial \alpha} = \beta u + (r_{1,2} - r_{2,2})$$
$$\frac{\partial U_j(\alpha, \beta)}{\partial \beta} = \alpha u' + (c_{2,1} - c_{2,2}).$$

where  $u = r_{1,1} - r_{1,2} - r_{2,1} + r_{2,2}$  and  $u' = c_{1,1} - c_{1,2} - c_{2,1} + c_{2,2}$



# Learning Dynamics of Infinitesimal Gradient Ascent

What joint policy will agents converge to?

⇒ Can analyse learning dynamics via dynamical systems theory!

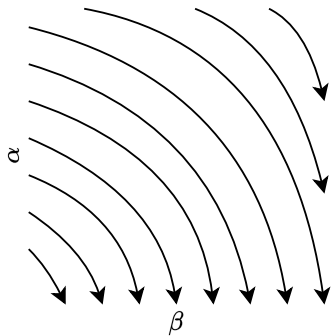
**Infinitesimal Gradient ascent (IGA):** use infinitesimal step size  $\kappa \rightarrow \infty$

Joint policy given by  $(\alpha(t), \beta(t))$  will follow continuous trajectory according to differential equation:

$$\begin{bmatrix} \frac{\partial \alpha}{\partial t} \\ \frac{\partial \beta}{\partial t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & u \\ u' & 0 \end{bmatrix}}_F \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} (r_{1,2} - r_{2,2}) \\ (c_{2,1} - c_{2,2}) \end{bmatrix}$$

# Learning Dynamics of Infinitesimal Gradient Ascent

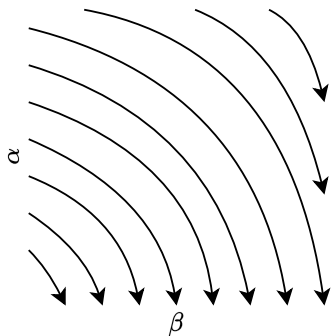
Learning dynamics of  $(\alpha, \beta)$  will follow one of three types of trajectories:



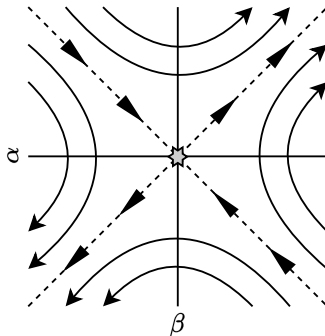
(a)  $F$  not invertible

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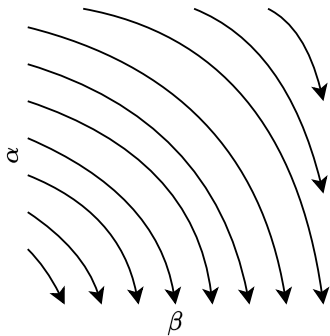
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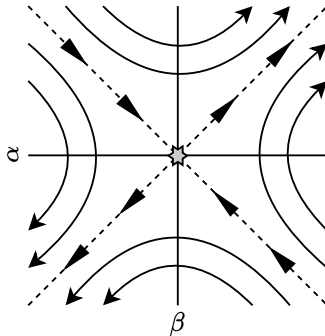
(b)  $F$  has purely real eigenvalues

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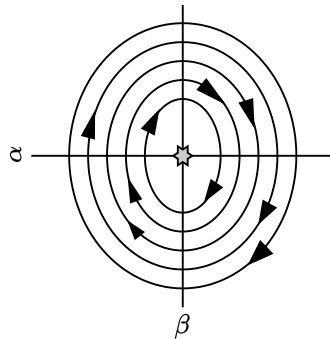
Learning dynamics of  $(\alpha, \beta)$  will follow one of three types of trajectories:



(a)  $F$  not invertible



(b)  $F$  has purely real eigenvalues



(c)  $F$  has purely imaginary eigenvalues

## IGA Convergence (Singh, Kearns, Mansour 2000)

- $(\alpha, \beta)$  does not converge in all cases

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- $(\alpha, \beta)$  does not converge in all cases
- If  $(\alpha, \beta)$  does not converge, then average rewards during learning converge to expected rewards of some NE
- If  $(\alpha, \beta)$  converges, then converged joint policy is a NE

## Win or Learn Fast – Variable Learning Rate

By using a variable step size  $\kappa$ , we can ensure that IGA policies *always* converge to NE

Win or Learn Fast (WoLF): (Bowling and Veloso 2002)

- learn fast (use larger  $\kappa$ ) when “losing”
- learn slow (use smaller  $\kappa$ ) when “winning”

Winning/losing depends on current expected reward compared to NE rewards



## Win or Learn Fast – Variable Learning Rate

Modify learning rule (analogous for agent  $j$ ):

$$\alpha^{k+1} = \alpha^k + l_i^k \kappa \frac{\partial U_i(\alpha^k, \beta^k)}{\partial \alpha^k}$$

with variable step size  $l_i^k \in [l_{\min}, l_{\max}] > 0$

$$l_i^k = \begin{cases} l_{\min} & \text{if } U_i(\alpha^k, \beta^k) > U_i(\alpha^e, \beta^k) \quad (\text{winning}) \\ l_{\max} & \text{otherwise} \quad (\text{losing}) \end{cases}$$

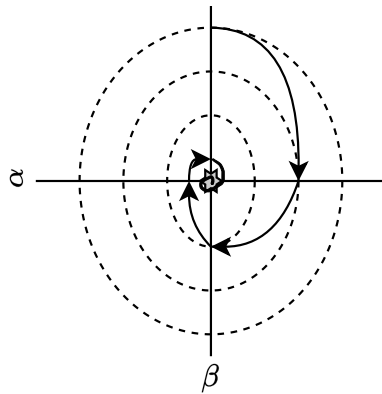
where  $\alpha^e$  is a policy from some NE

# WoLF-IGA Convergence

IGA does not converge if  $F$  is invertible and has imaginary eigenvalues

- In WoLF-IGA, trajectories of  $(\alpha, \beta)$  are piecewise elliptical, each quadrant tightens ellipse by factor  $\sqrt{\frac{l_{\min}}{l_{\max}}} < 1$
- Using variable learning rate, WoLF-IGA converges!

WoLF-IGA guaranteed to learn NE in two-agent two-action normal-form game



# Win or Learn Fast with Policy Hill Climbing (WoLF-PHC)

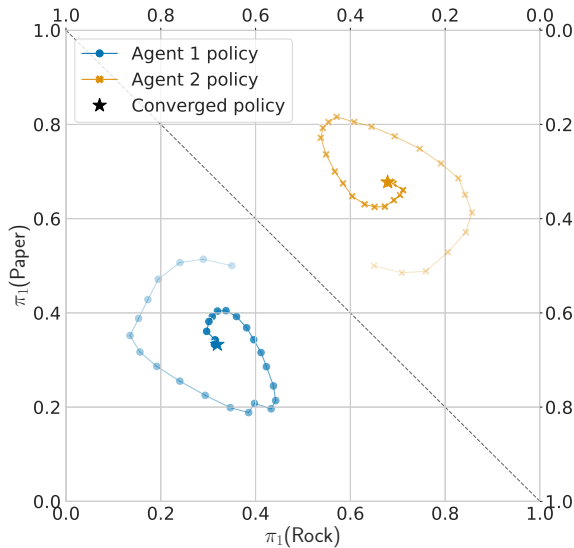
WoLF-PHC algorithm (Bowling and Veloso 2002) applies WoLF principle in stochastic games:

- Can learn in general-sum games with any number of agents and actions
- Does not require knowledge of reward functions and policies

To determine winning/losing, it compares average reward of current policy  $\pi_i$  to “average” policy  $\bar{\pi}_i$  over past policies:

$$\delta = \begin{cases} l_w & \text{if } \sum_{a'_i} \pi_i(a'_i | s) Q(s, a'_i) > \sum_{a'_i} \bar{\pi}_i(a'_i | s) Q(s, a'_i) \\ l_l & \text{otherwise} \end{cases}$$

# WoLF-PHC in Rock-Paper-Scissors



JAL-GT and JAL-AM algorithms use solution concepts and agent modeling to learn joint policies

**Now:** no-regret learning algorithms that use regret definitions to learn policies

- We consider two simple regret matching algorithms (Hart and Mas-Colell, 2000)
- In normal-form games, their empirical action distributions converge to set of (coarse) correlated equilibrium

# Unconditional Regret Matching

**Unconditional regret matching:** compute action probabilities proportional to (positive) average *unconditional* regrets of actions  $a_i \in A_i$

$$\text{Regret}_i^z(a_i) = \sum_{e=1}^z [\mathcal{R}_i(\langle a_i, a_{-i}^e \rangle) - \mathcal{R}_i(a^e)]$$

$a^e$  is joint action from past episodes  $e = 1, \dots, z$ .

Each agent  $i$  starts with a random initial policy  $\pi_i^1$ , then update  $\pi_i^z$  to

$$\pi_i^{z+1}(a_i) = \frac{[\bar{R}_i^z(a_i)]_+}{\sum_{a'_i \in A_i} [\bar{R}_i^z(a'_i)]_+} \quad \text{with } \bar{R}_i^z(a_i) = \frac{1}{z} \text{Regret}_i^z(a_i)$$

where  $[x]_+ = \max[x, 0]$ .

# Conditional Regret Matching

**Conditional regret matching:** compute action probabilities proportional to (positive) average *conditional* regrets with respect to most recent selected action  $a_i^z$

$$\text{Regret}_i^z(a'_i, a_i) = \sum_{e: a_i^e = a'_i} [\mathcal{R}_i(\langle a_i, a_{-i}^e \rangle) - \mathcal{R}_i(a^e)]$$

Each agent  $i$  starts with a random initial policy  $\pi_i^1$ , then update  $\pi_i^z$  to

$$\pi_i^{z+1}(a_i) = \begin{cases} \frac{1}{\eta} [\bar{R}_i^z(a_i^z, a_i)]_+ & \text{if } a_i \neq a_i^z \\ 1 - \sum_{a'_i \neq a_i^z} \pi_i^{z+1}(a'_i) & \text{otherwise} \end{cases} \quad \text{with } \bar{R}_i^z(a'_i, a_i) = \frac{1}{z} \text{Regret}_i^z(a'_i, a_i)$$

where  $\eta > 2 \cdot \max_{a \in A} |\mathcal{R}_i(a)| \cdot (|A_i| - 1)$  is a parameter.

# Regret Matching Convergence

In both types of regret matching, the average regrets are bounded by  $\kappa \frac{1}{\sqrt{z}}$  for some constant  $\kappa > 0$

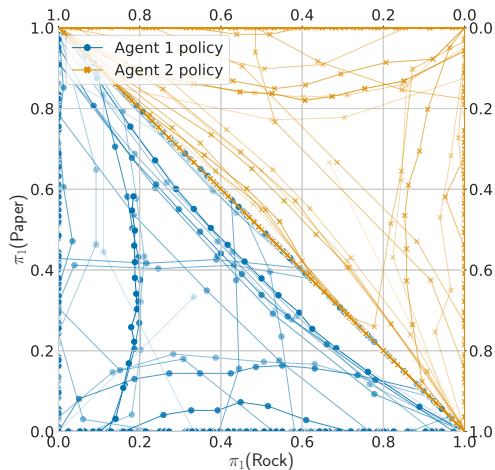
- For infinite episodes  $z \rightarrow \infty$ , the average regrets  $\bar{R}_i^z$  will be at most 0  
 $\Rightarrow$  Thus, agents learn no-regret joint policy!
- Does not require any assumptions about the actions of other agents

Empirical action distributions converge to:

- Unconditional regret matching  $\rightarrow$  coarse correlated equilibrium
- Conditional regret matching  $\rightarrow$  correlated equilibrium

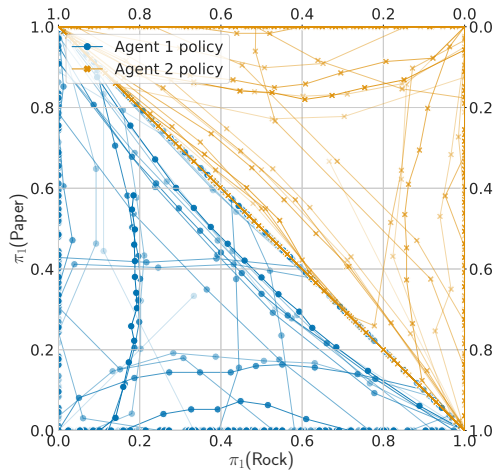


# Unconditional Regret Matching in Rock-Paper-Scissors

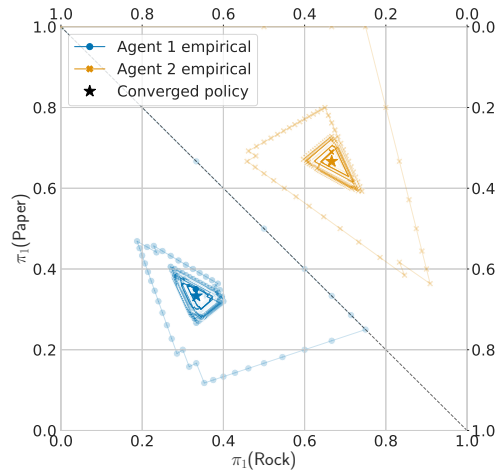


(a) Policies  $\pi_i^z$

# Unconditional Regret Matching in Rock-Paper-Scissors

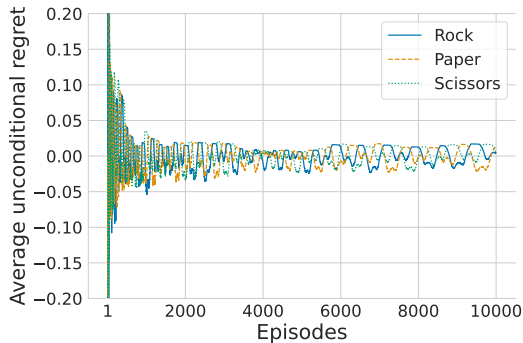


(a) Policies  $\pi_i^z$

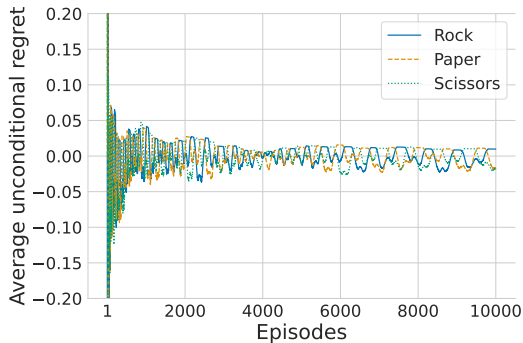


(b) Empirical distributions  $\bar{\pi}_i^z$

# Unconditional Regret Matching in Rock-Paper-Scissors



(a) Agent 1



(b) Agent 2

# Summary

## We covered:

- Value iteration for stochastic games
- Joint-action value learning algorithms
  - JAL-GT: temporal-difference learning with game theory solution concepts
  - JAL-AM: temporal-difference learning with agent models and best responses
- Learning policies by optimising policy parameters with gradient ascent
- Learning policies by minimising notions of regret

## Next we'll cover:

- Deep learning