## Multi-Agent Reinforcement Learning

Stefano V. Albrecht, Filippos Christianos, Lukas Schäfer

Multi-Agent Deep Reinforcement Learning – Part 2

#### The MARL Book

This lecture is based on

Multi-Agent Reinforcement Learning: Foundations and Modern Approaches

by Stefano V. Albrecht, Filippos Christianos and Lukas Schäfer

MIT Press, 2024

Download book, slides, and code at: www.marl-book.com



#### Lecture Outline

- Agent modeling with deep learning
- Parameter and experience sharing
- Policy self-play in zero-sum games
- Population-based training

# Agent Modeling with Deep Learning

## Agents Modeling – Motivation

In MARL, agents need to consider the policies of other agents to coordinate their actions.

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Approaches presented so far account for the action selection of other agents through:

- Distribution of training data is dependent on the policies of all agents
- Training centralized critics conditioned on the actions of other agents

Can we provide agents with more **explicit** information about the policies of other agents so they can learn to coordinate better, e.g. by learning best-response policies?

#### Recap: Agent Modeling

#### Reminder

In Chapter 6, we have seen approaches that model other agents' policies:

- Learn models of other agents to predict their actions
- Compute optimal action (best-response) against agent models



S. Albrecht, P. Stone. Autonomous Agents Modelling Other Agents: A Comprehensive Survey and Open Problems. *Artificial Intelligence*, 2018

## Recap: Tabular Agent Modeling

In Chapter 6, we modeled other agents' policies as stationary distributions by maintaining tables of action frequencies for each state

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#### Problem

Similar to tabular value functions, **tabular agent models** are limited due to their inability to generalise across states.

#### Solution

Use **deep learning** to learn generalisable agent models!

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- Agents learn value functions conditioned on the joint action:  $Q_i(s, a)$
- Using the value function and agent models, agent *i* can compute its expected action values under the current models of other agents:

$$AV_i(s, a_i) = \sum_{a_{-i} \in A_{-i}} Q_i(s, \langle a_i, a_{-i} \rangle) \prod_{j \in I \setminus \{i\}} \hat{\pi}_j(a_j \mid s)$$

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• Use AV<sub>i</sub> to select optimal actions and as learning update targets

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- Agent i can train its model for agent j by minimizing the cross-entropy loss between the predicted policy  $\hat{\pi}^i_j$  and the observed actions of agent j:

$$\mathcal{L}(\phi_j^i) = \mathbb{E}_{a_j^t \sim \pi_j(h_j^t)} \Big[ -\log \hat{\pi}_j^i(a_j^t \mid h_i^t; \phi_j^i) \Big]$$

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• Then, agent *i* can compute expected action values:

$$AV(h_i, a_i; \theta_i) = \sum_{a_{-i} \in A_{-i}} Q(h_i, \langle a_i, a_{-i} \rangle; \theta_i) \prod_{j \neq i} \hat{\pi}^i_j (a_j \mid h_i; \phi^i_j)$$

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#### Solution

Approximate AV with only K joint actions sampled from the agent models:

$$AV(h_i, a_i; \theta_i) = \frac{1}{K} \sum_{k=1}^K Q(h_i, \langle a_i, a_{-i}^k \rangle; \theta_i) \Big|_{a_j^k \sim \hat{\pi}_j^i(\cdot | h_i)}$$

#### Problem

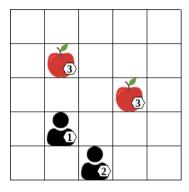
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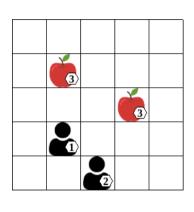
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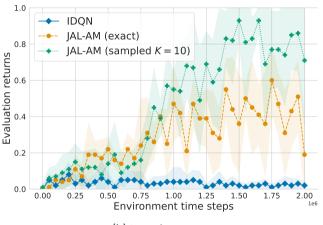
To optimise the centralized joint-action-value function of agent *i*, we then minimize the following loss over batches of experiences sampled from a replay buffer:



(a) Environment



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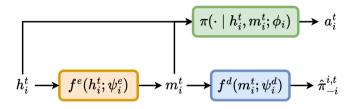
(b) Learning curve

- JAL-AM combines agent models and centralized value functions to compute best-response policies.
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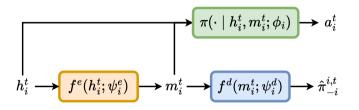
#### Problem

To condition policies (and value functions) of agents on the policies of other agents, we need **compact** representations of the policies of other agents. How can we learn such representations?



Agent i trains encoder-decoder architecture with

- Encoder  $f^e$  with parameters  $\psi_i^e$ : given observation history  $h_i^t$  of agent i, output compact representation  $m_i^t$  of the policies of other agents
- Decoder  $f^d$  with parameters  $\psi_i^d$ : given compact representation  $m_i^t$ , predict the policies  $\hat{\pi}_{-i}^{i,t}$  of other agents



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Then, agent i can condition its policy on the compact representations  $m_i^t$ .

The encoder and decoder are jointly trained to minimize the cross-entropy loss for the predicted action probabilities and true actions of all other agents:

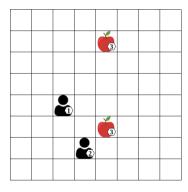
$$\mathcal{L}(\psi_i^e, \psi_i^d) = \sum_{j \neq i} -\log \hat{\pi}_j^{i,t}(a_j^t) \quad ext{with} \quad \hat{\pi}_j^{i,t} = f^d \left(f^e(h_i^t; \psi_i^e); \psi_i^d\right)_j$$

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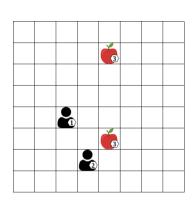
Encoder-decoder agent models can be integrated into MARL algorithm by conditioning value functions and policies on the obtained policy representations.

## Compact Agent Policy Representations in LBF

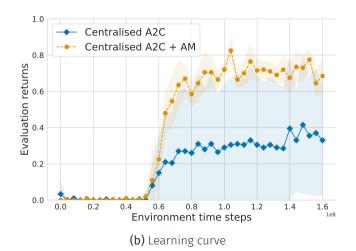


(a) Environment

## Compact Agent Policy Representations in LBF

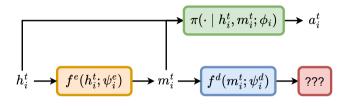


(a) Environment



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#### Reconstruction Targets to Learn Compact Representations



#### Note

We used the ground truth actions as information to encode by using them as targets for the decoder. We could also use

- Observations try to capture information that other agents have access to
- Rewards try to predict the objectives that other agents optimise for
- ..

Parameter and Experience Sharing

#### Parameter and Experience Sharing – Motivation

#### Problem

Training agents with MARL is difficult for environments with many agents due to the increased number of parameters to train  $\Rightarrow$  unstable or slow training!

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Training agents with MARL is difficult for environments with many agents due to the increased number of parameters to train  $\Rightarrow$  unstable or slow training!

#### Solution

Two approaches to improve the efficiency of training many agents:

- Parameter sharing: agents share their network parameters with each other
- Experience sharing: agents share experiences with each other

# **Environments with Homogeneous Agents**

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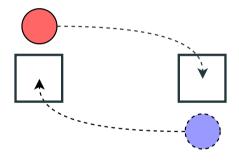
$$\pi_1^* = \ldots = \pi_n^*$$

• Weakly homogeneous agents: Agents can be permuted and their expected returns remain the same under the permutation  $\sigma: I \mapsto I$ :

$$U_i(\pi) = U_{\sigma(i)}\left(\langle \pi_{\sigma(1)}, \pi_{\sigma(2)}, \dots, \pi_{\sigma(n)} \rangle\right), \ \forall i \in I$$

# Environments with Homogeneous Agents – Examples

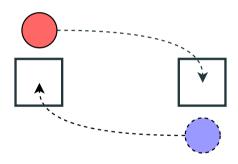
## Weakly homogeneous agents:



Agents need to learn similar policies.

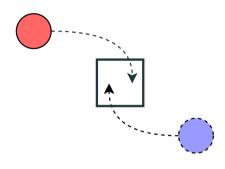
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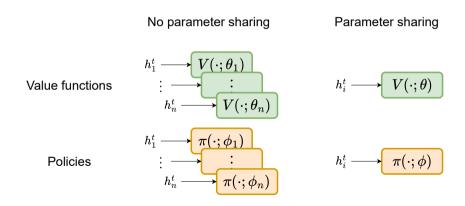
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Sharing network parameters across agents is common practice to make MARL training more efficient. Share parameters across value functions, policies, or both.



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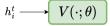
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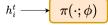
Parameter sharing has two primary benefits:

- Scalability: the number of parameters remains constant independent of the number of agents → less computational cost
- Efficiency: shared parameters are updated using the experiences of all agents → more training data for the shared parameters

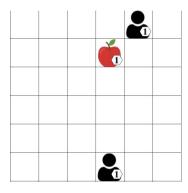
The downside is that (naive) parameter sharing assumes strongly homogeneous agents.

Parameter sharing



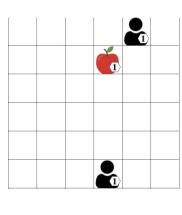


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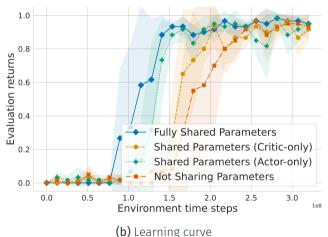


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# Parameter Sharing in LBF



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(b) Learning curv

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**Assumption**: Agents are (at least) weakly homogeneous, i.e. they need to learn similar policies.

#### Note

The experiences of agent j is **off-policy** data for agent  $i \to \text{experience}$  sharing needs to use off-policy MARL algorithms or correct for the differences in data distributions.

# Deep Q-Networks with Shared Experience Replay

We can extend IDQN with experience sharing by following the steps below:

- ullet Collect the experience of all agents in a shared replay buffer  $\mathcal{D}_{ ext{shared}}$
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#### Note

DQN is an off-policy algorithm so it is theoretically sound to use the experience of other agents that have different policies.

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policy loss on own data

$$\mathcal{L}(\phi_i) = -\left(r_i^t + \gamma V(h_i^{t+1}; \theta_i) - V(h_i^t; \theta_i)\right) \log \pi(a_i^t \mid h_i^t; \phi_i)$$

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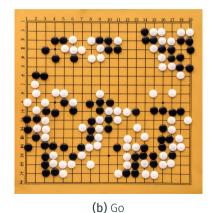
Hyperparameter  $\lambda$  determines weighting for the loss over the experience of other agents. The same IS weight correction can be applied to the critic loss.

Policy Self-Play in Zero-Sum Games

Next we will take a closer look at (turn-based) zero-sum board games such as chess, shogi, or Go.



(a) Chess



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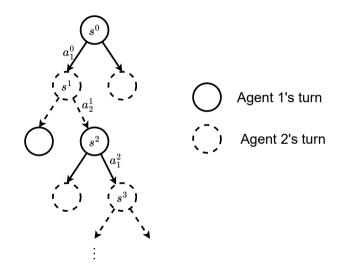
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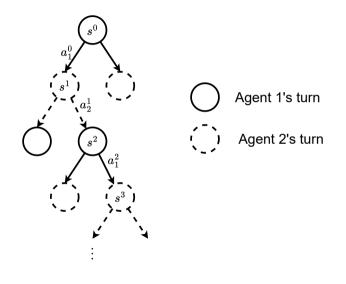
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Fortunately, we can exploit the structure of these games to develop effective algorithms.

## Tree Search for Zero-Sum Games



#### Tree Search for Zero-Sum Games

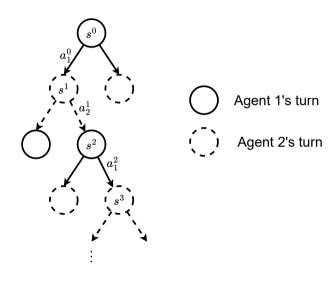


We can view turn-based zerosum games as trees where

- nodes represent game states
- edges represent actions
- leaves represent terminal states

in each node either agent 1 or agent 2 makes a move.

#### Tree Search for Zero-Sum Games



#### Problem

The tree can grow very large depending on its

- Depth: number of time steps until terminal states
- Breadth: number of actions available in each state
- $\rightarrow$  makes search computationally expensive

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- Simulation: simulate a game from the current state until a leaf node is reached
- Expansion: expand the tree by adding a new node for the reached state if it does not exist yet
- Backpropagation: update the estimated values of the nodes visited during the selection step

#### Monte Carlo Tree Search - Simulation

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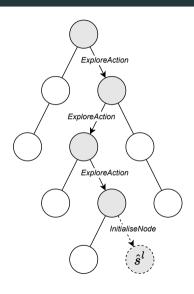
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To sample actions, MCTS commonly uses  $\epsilon$ -greedy policies with respect to action-value function Q, or the upper confidence bound (UCB) policy:

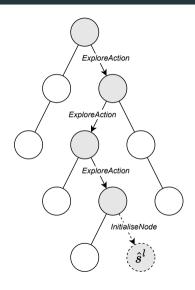
$$\hat{a}^{\tau} = \begin{cases} \hat{a} & \text{if } N(\hat{s}^{\tau}, \hat{a}) = 0\\ \arg\max_{\hat{a} \in A} \left( Q(\hat{s}^{\tau}, \hat{a}) + \sqrt{\frac{2 \ln N(\hat{s}^{\tau})}{N(\hat{s}^{\tau}, \hat{a})}} \right) & \text{otherwise} \end{cases}$$

## Monte Carlo Tree Search – Expansion



If leaf node is reached  $\rightarrow$  expand the search tree by adding a new node for the reached state  $\hat{s}^l$ 

## Monte Carlo Tree Search – Expansion

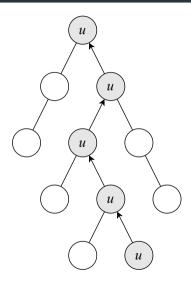


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The new node is initialized with

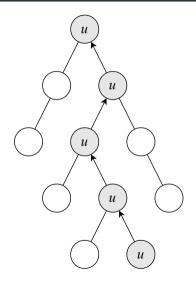
- $N(\hat{s}^l, \hat{a}) = 0$  for all actions  $\hat{a}$
- An initial value estimate Q(\$\hat{s}^l\$, \$\hat{a}\$) for all actions
   \$\hat{a}\$, e.g. from a learned value function, heuristic,
   or random samples of outcomes.

## Monte Carlo Tree Search – Backpropagation



Once a value estimate u for the leaf node is obtained  $\rightarrow$  backpropagate rewards and value estimates starting from the leaf node up to the root node.

## Monte Carlo Tree Search - Backpropagation

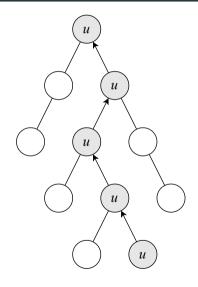


Once a value estimate u for the leaf node is obtained  $\rightarrow$  backpropagate rewards and value estimates starting from the leaf node up to the root node.

For each visited state-action pair  $(\hat{s}^{\tau}, \hat{a}^{\tau})$ , we increment the visitation count and update the value:

$$Q(\hat{\mathbf{s}}^{\tau}, \hat{a}^{\tau}) \leftarrow Q(\hat{\mathbf{s}}^{\tau}, \hat{a}^{\tau}) + \frac{1}{N(\hat{\mathbf{s}}^{\tau}, \hat{a}^{\tau})} \left[ u - Q(\hat{\mathbf{s}}^{\tau}, \hat{a}^{\tau}) \right]$$

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(Any RL TD update rule can be used to update the value estimates.)

## Monte Carlo Tree Search – Action Selection

Simulation, expansion, and backpropagation steps are repeated at every time step to iteratively grow the search tree and obtain better value estimates.

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Simulation, expansion, and backpropagation steps are repeated at every time step to iteratively grow the search tree and obtain better value estimates.

Following k simulations form the current state  $s^t$ , the best action is selected. This process can be done by choosing the action with:

- highest value estimate:  $BestAction(s^t) = arg \max_{\hat{a} \in A} Q(s^t, \hat{a})$
- highest visitation count:  $BestAction(s^t) = arg \max_{\hat{a} \in A} N(s^t, \hat{a})$

## Monte Carlo Tree Search - Pseudocode

#### Algorithm Monte Carlo tree search (MCTS) for MDPs

```
1: Repeat for every episode:
 2: for t = 0, 1, 2, 3, ... do
           Observe current state st
           for k simulations do
               \tau \leftarrow t
                \hat{s}^{\tau} \leftarrow s^t
                                                                                                        ▶ Perform simulation
                 while \hat{s}^{\tau} is non-terminal and \hat{s}^{\tau}-node exists in tree do
                       \hat{a}^{\tau} \leftarrow ExploreAction(\hat{s}^{\tau})
 g.
                       \hat{\mathbf{s}}^{\tau+1} \sim \mathcal{T}(\cdot \mid \hat{\mathbf{s}}^{\tau}, \hat{a}^{\tau})
 g.
                       \hat{\mathbf{r}}^{\tau} \leftarrow \mathcal{R}(\hat{\mathbf{s}}^{\tau}, \hat{\mathbf{a}}^{\tau}, \hat{\mathbf{s}}^{\tau+1})
10:
                     \tau \leftarrow \tau + 1
11:
                 if \hat{s}^{\tau}-node does not exist in tree then
12:
13:
                       InitializeNode(\hat{s}^{\tau})
                                                                                                                     ▷ Expand tree
                 while \tau > t do
14.
                                                                                                                ▶ Backpropagate
                      \tau \leftarrow \tau - 1
15:
                       Update(Q, \hat{s}^{\tau}, \hat{a}^{\tau})
16:
            Select action a^t for state s^t.
17:
               \pi^t \leftarrow BestAction(s^t)
18:
               a^t \sim \pi^t
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MCTS assumes known transition function  $\mathcal{T}$  and reward function  $\mathcal{R}$ .

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MCTS assumes known transition function  $\mathcal{T}$  and reward function  $\mathcal{R}$ .

Can learn estimates of these functions from data to simulate possible outcomes of the game.

## Self-Play Monte Carlo Tree Search

In zero-sum games with symmetrical roles and egocentric observations, agents can use the same policy to control both players

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In zero-sum games with symmetrical roles and egocentric observations, agents can use the same policy to control both players  $\rightarrow$  learn a policy in self-play



(a) Agent 1 perspective



(b) Agent 2 perspective

AlphaZero: combine self-play MCTS with deep learning to learn value estimates and policies → reached superhuman performance in Go, chess, and shogi!



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Learn functions with parameters  $\theta$  conditioned on state s:

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For each episode, a triplet  $(s, \pi, z)$  of data is stored where

- s are the states
- $\pi$  are policy distributions computed by BestAction
- z is the game outcome (+1 for win, -1 for loss, 0 for draw)



The network is randomly initialized and trained using sampled batches of data to minimise the following combined loss:

$$\mathcal{L}(\theta) = \mathcal{L}_{\text{value}} + \mathcal{L}_{\text{policy}} + c ||\theta||^{2}$$

$$\mathcal{L}_{\text{value}} = \mathbb{E}_{(s,\pi,z)\sim\mathcal{D}} \Big[ (V(s;\theta) - u)^{2} \Big]$$

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For exploration, AlphaZero combines a UCB policy with the learned policy:

$$\hat{a}^{\tau} = \begin{cases} \hat{a} & \text{if } N(\hat{s}^{\tau}, \hat{a}) = 0\\ \arg\max_{\hat{a} \in A} \left( Q(\hat{s}^{\tau}, \hat{a}) + C(\hat{s}^{\tau}) P(\hat{s}^{\tau}, \hat{a}) \frac{\sqrt{N(\hat{s}^{\tau})}}{1 + N(\hat{s}^{\tau}, \hat{a})} \right) & \text{otherwise} \end{cases}$$

with the additional exploration rate  $C(\hat{s}^{\tau})$ .

# Population-Based Training

## Population-Based Training – Self-Play for General-Sum Games

#### Problem

With MCTS, we focused on policy self-play in **two-agent zero-sum** games. Can we extend the idea of self-play to **general-sum** games with **more than two agents**?

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Population-based training is a generalisation of self-play to general-sum games:

- Maintain a **population of policies** representing possible strategies of the agent
- Evolve populations so they become more effective against the populations of other agents
- We denote the population of policies for agent i at generation k as  $\Pi_i^k$ .

## Population-Based Training – Overview

First, initialize a population of policies for each agent (e.g. random policies).

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## Policy Space Response Oracles (PSRO)

Policy space response oracles (PSRO) is a population-based MARL algorithm with the following steps:

- Initialize populations of random policies for each agent
- 2. Construct a **meta-game**  $M^k$  at generation k from the populations of all agents as a (non-repeated) normal-form game with
  - Actions: policies of agent populations, i.e.
     A<sub>i</sub> = Π<sup>k</sup><sub>i</sub>
  - Rewards: returns of joint policies  $\langle \pi_1, \dots, \pi_n \rangle$ , i.e.  $\mathcal{R}_i(\pi_1, \dots, \pi_n) = U_i(\pi_1, \dots, \pi_n)$ .

$M^k$	$\pi_2^{(1)}$	$\pi_2^{(2)}$	•••	$\pi_2^{(k)}$
$\pi_1^{(1)}$	0,1	1, 2	• • •	0,3
$\pi_1^{(2)}$	2,1	0, 1	• • •	1,1
:			•	
$\pi_1^{(k)}$	5, 1	0,1		4,3

# Policy Space Response Oracles – Construct Meta-Game

$M^k$	$\pi_2^{(1)}$	$\pi_2^{(2)}$	•••	$\pi_2^{(k)}$
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÷			•	
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## Problem

Need to compute the expected returns of each agent for any joint policy  $\langle \pi_1, \dots, \pi_n \rangle$  in the meta-game. How can we do this?

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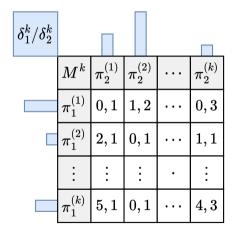
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#### Solution

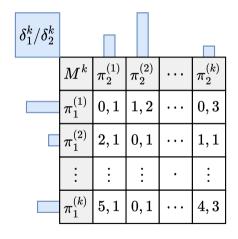
Compute average returns of each agent over multiple episodes of the underlying game with respective joint policy  $\rightarrow$  converges to expected returns in the limit.

## Policy Space Response Oracles – Solve Meta-Game



Compute a solution to the meta-game  $M^k$  following some solution concept, e.g. a Nash equilibrium.

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Compute a solution to the meta-game  $M^k$  following some solution concept, e.g. a Nash equilibrium.

Solution to the meta-game: distributions  $\delta_i^k$  over policies in the population of agent i (for each agent  $i \in I$ ).

# Policy Space Response Oracles – Add Best-Response Policies

$M^k$	$\pi_2^{(1)}$	$\pi_2^{(2)}$	•••	$\pi_2^{(k)}$	$\pi_2'$
$\pi_1^{(1)}$	0, 1	1, 2	:	0,3	?
$\pi_1^{(2)}$	2,1	0,1	•••	1,1	?
:	:		•		?
$\pi_1^{(k)}$	5, 1	0,1	•••	4,3	?
$\pi_1'$	?	?	?	?	?

Each agent i determines an effective **oracle policy**  $\pi'_i$  against the solution distribution of the other agents and adds this policy to its population:

$$\Pi_i^{k+1} = \Pi_i^k \cup \{\pi_i'\}$$

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For example, agent i might determine its best-response policy

$$\pi_i' \in rg \max_{\pi_i} \mathbb{E}_{\pi_{-i} \sim \delta_{-i}^k} [U_i(\langle \pi_i, \pi_{-i} \rangle)]$$

by training a policy  $\pi_i^\prime$  using RL against sampled policies of the other agents.

## Policy Space Response Oracles – Pseudocode

## Algorithm Policy space response oracles (PSRO)

```
1: Initialize populations \Pi_i^1 for all i \in I (e.g., random policies)
2: for each generation k = 1, 2, 3, \dots do
       Construct meta-game M^k from current populations \{\Pi_i^k\}_{i\in I}
3.
       Use meta-solver on M^k to obtain distributions \{\delta_i^k\}_{i\in I}
       for each agent i \in I do
                                                                  ▶ Train best-response policies
5:
           for each episode e = 1, 2, 3, \dots do
6:
                Sample policies for other agents \pi_{-i} \sim \delta_{-i}^{k}
                Use single-agent RL to train \pi'_i wrt. \pi_{-i} in underlying game G
8:
           Grow population \Pi_i^{k+1} \leftarrow \Pi_i^k \cup \{\pi_i'\}
9:
```

# Policy Space Response Oracles – Pseudocode

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- 5: **for** each agent  $i \in I$  **do**  $\triangleright$  Train best-response policies
- 6: **for** each episode e = 1, 2, 3, ... **do**
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- 8: Use single-agent RL to train  $\pi_i'$  wrt.  $\pi_{-i}$  in underlying game G
- 9: Grow population  $\Pi_i^{k+1} \leftarrow \Pi_i^k \cup \{\pi_i'\}$

Repeat this process until policy populations converge (new best-response policies are already in the respective populations), or for a fixed number of generations.

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## Policy Space Response Oracles in Rock-Paper-Scissors

If PSRO computes exact Nash equilibria solutions to the meta-game, and computes exact best-response policies, then the population distributions of PSRO converge to the Nash equilibrium of the underlying game.

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For example for rock-paper-scissors for two agents, with initial populations deterministically choosing rock and paper, respectively:

k	$\Pi_1^k$	$\Pi_2^k$	$\delta_1^k$	$\delta_2^k$	$\pi_1'$	$\pi_2'$
1	<u>R</u>	<u>P</u>	1	1	S	Р
2	R, <u>S</u>	Р	(0,1)	1	S	R
3	R,S	<u>R</u> ,P	$(\frac{2}{3}, \frac{1}{3})$	$(\frac{2}{3}, \frac{1}{3})$	Р	R/P
4	R, <u>P</u> ,S	R,P	$(0,\frac{2}{3},\frac{1}{3})$	$(\frac{1}{3}, \frac{2}{3})$	R	S
5	R,P,S	R,P, <u>S</u>	$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$	$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$	R/P/S	R/P/S

StarCraft II is a real-time strategy game for two or more players in which players have to collect resources, build infrastructure and armies to defeat their opponents.



Figure: StarCraft II game, image source: https://deepmind.google/discover/blog/alphastar-mastering-the-real-time-strategy-game-starcraft-ii/.

StarCraft II is a real-time strategy game for two or more players in which players have to collect resources, build infrastructure and armies to defeat their opponents.

StarCraft II is challenging due to

- Sparse rewards: players only receive a terminal reward at the end of the game
- Large action space: players choose between many actions constituting of a type (e.g., build, move, attack), which unit should execute the action, and the target of the action
- Long horizon: games can last for thousands of time steps

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- Long horizon: games can last for thousands of time steps
- Partial observability: players only observe a limited view of the game state
- **Diversity of strategies:** players choose between three available races offering many units and possible strategies

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Agents of each type are added to the league whenever they become effective (measured by win rates) against their respective opponents.

During population-based training, AlphaStar computes distributions over the policies in the league to train any policy  $\pi'_i$  against using prioritized fictitious self-play (PFSP):

$$\delta_i^k(\pi_i) \propto f(\Pr[\pi_i' \text{ wins against } \pi_i])$$

which is proportional to the probability of the agent winning against the policy  $\pi_i$ .

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→ reached "GrandMaster" level in StarCraft II (top 0.2% of ranked human players).

#### Summary

#### We covered:

- Agent modelling with deep learning
- Parameter and experience sharing
- Policy self-play in zero-sum games
- Population-based training