Conceptual questions

(Solution)

a) If we redefine the time origin as well as the space origin, we can always bring the equation of a harmonic oscillator back to $x(t) = A\sin(\omega t)$. In that form, the speed and acceleration are respectively $v(t) = A\omega\cos(\omega t)$ and $a(t) = -A\omega^2\sin(\omega t)$. We can then deduce that...

- ...the norm of the velocity is maximal at $\omega t = n\pi$ (where n is an integer), that is, at the equilibrium position $x = 0$.
- ...the acceleration is naught at $\omega t = n\pi$, that is, at the equilibrium position $x = 0$.
- ...the speed is naught at $\omega t = \pi/2 + n\pi$, that is, at the extremal positions $x = \pm A$.
- ...the norm of the acceleration is maximal at $\omega t = \pi/2 + n\pi$, that is, at the extremal positions $x = \pm A$.

b) Yes, it is possible: for the speed to increase, the acceleration must be positive. It is, of course, possible for the acceleration to decrease while remaining positive.

- Let's see the example of an object moving along the x axis, whose acceleration decreases linearly as a function of time $a = a_0(1 - \alpha t)$, $a_0 > 0$, $\alpha > 0$. If we calculate the variation, between times t and $t + \Delta t$, of the acceleration and of the speed, we find :

$$\Delta a = a(t + \Delta t) - a(t) = a_0(1 - \alpha(t + \Delta t)) - a_0(1 - \alpha t) = -a_0\alpha\Delta t < 0$$

$$\Delta v = a_0(1 - \alpha t)\Delta t > 0$$

  Therefore, so long as $\alpha t < 1$, the speed variation Δv is positive even though the acceleration variation Δa is negative.

- For a more day-to-day example, imagine a cyclist on a downwards slope. She is pedalling at the beginning of the slope, but then stops to free-wheel down. When she is pedalling, her acceleration is greater than when she is free-wheeling; nevertheless, even then, her speed will keep increasing.

- In harmonic oscillator motion, as shown in the following diagram for a $\pi/10$ pulsation, there are times, like between 15 and 20 s or between 35 and 40 s, when the acceleration decreases (green curve) while the speed increases (blue curve). This can be expressed analytically when we consider the motion $x(t) = A\sin(\omega t)$, of which the speed is $v(t) = \frac{dx}{dt} = A\omega\cos(\omega t)$, the acceleration is $a(t) = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t)$, and the acceleration variation is $\frac{da}{dt} = -A\omega^3\cos(\omega t)$. The speed variation, then, is opposite in sign to the acceleration variation $\frac{da}{dt}$ if $\pi/2 < \omega < \pi$ or $3\pi/2 < \omega < 2\pi$.