DESIGN OF BEAMS II

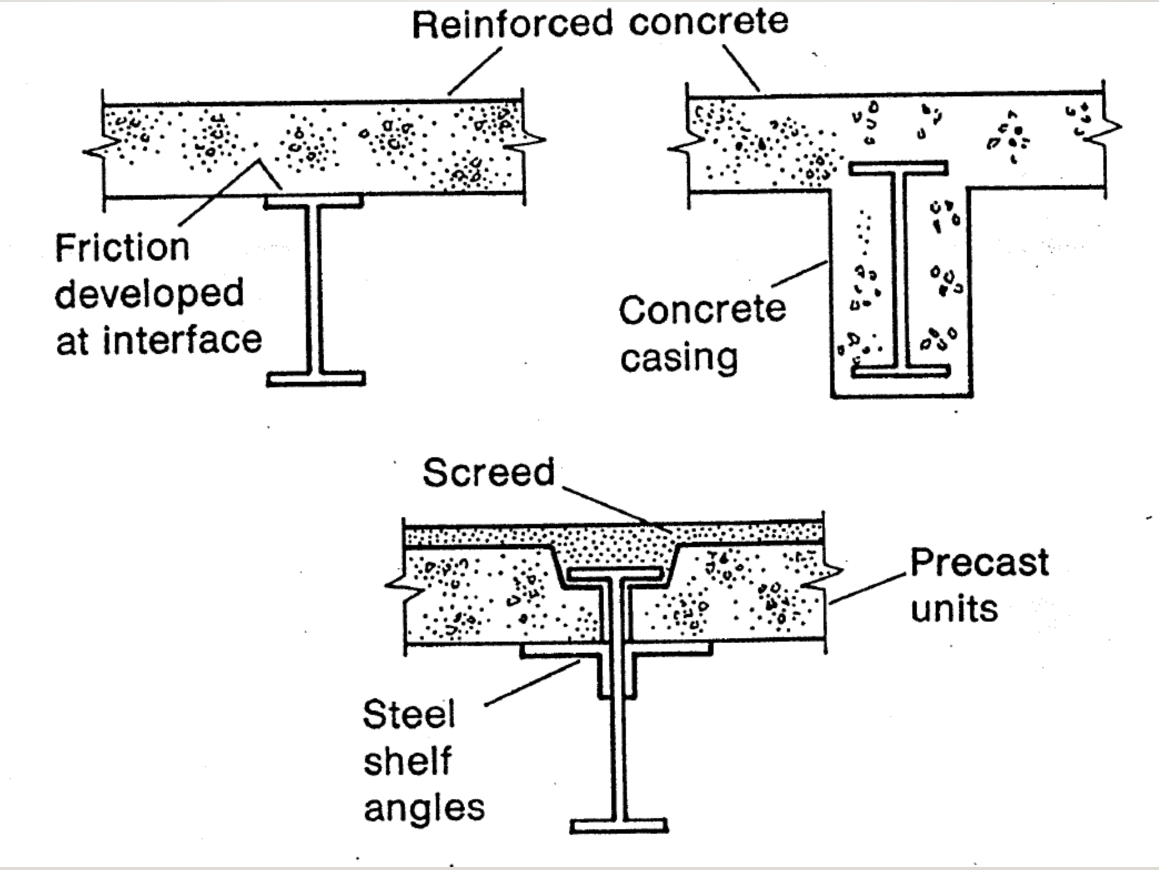
UNRESTRAINED BEAMS

BEAM TYPES

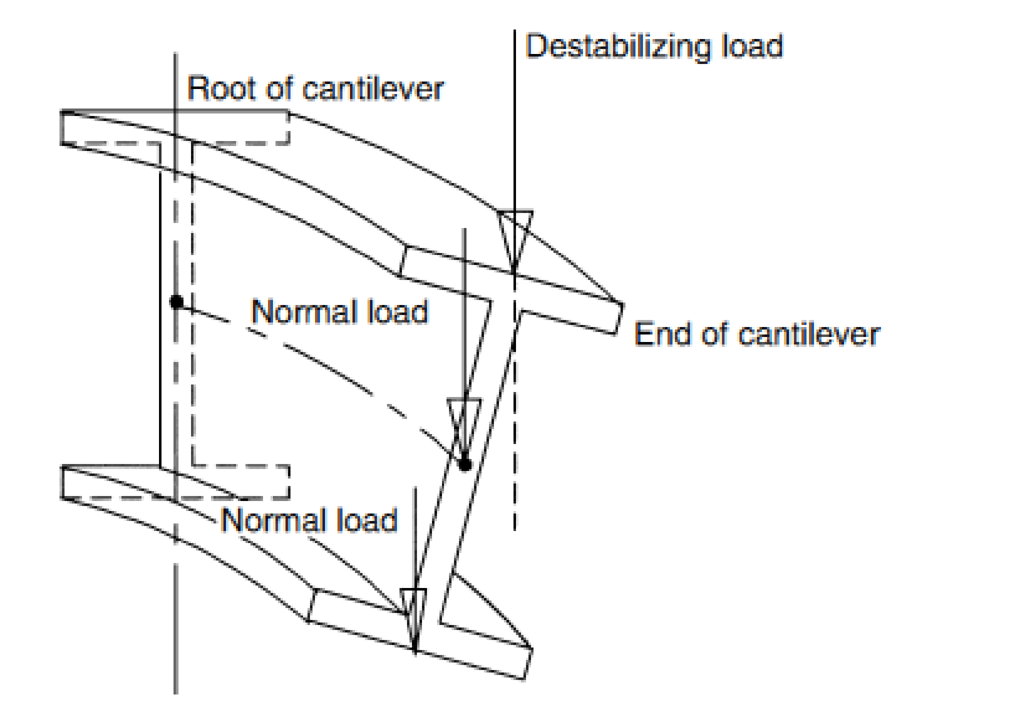
TWO MAIN BEAM TYPES ENCOUNTERED IN STEEL DESIGN:

- **LATERALLY RESTRAINED**
- **LATERALLY UNRESTRAINED**

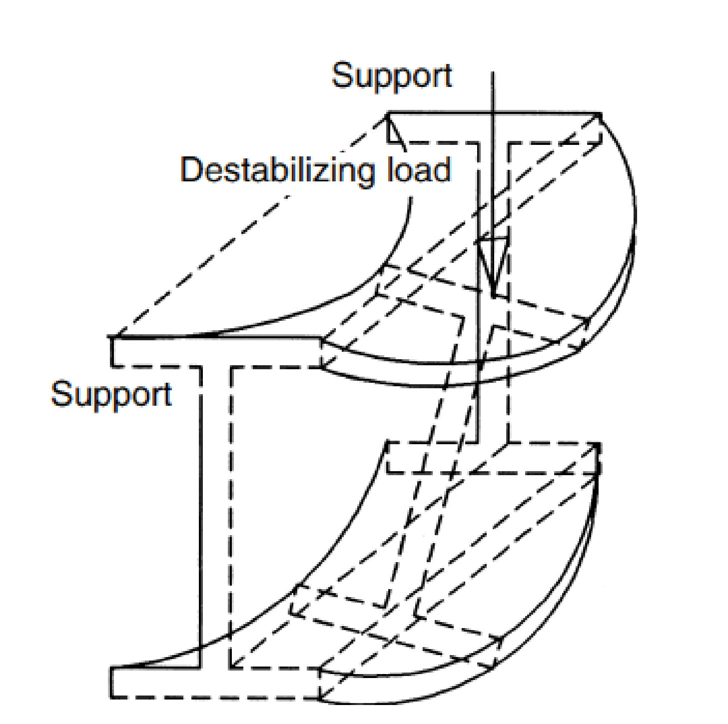
RESTRAINED BEAMS



UNRESTRAINED CANTILEVER BEAM



UNRESTRAINED SIMPLY SUPPORTED BEAM



STRUCTURAL BEHAVIOUR

- **LATERALLY RESTRAINED BEAMS WILL EXPERIENCE IN-PLANE BENDING**
- **LATERALLY UNRESTRAINED BEAM MAY EXPERIENCE BOTH IN-PLANE AND OUT OF PLANE BENDING AS WELL AS TWISTING**

DESIGN CONSIDERATIONS

LATERALLY UNRESTRAINED BEAMS SHOULD BE CHECKED FOR THE FOLLOWING:

- **SHEAR**
- **BENDING**
- **WEB FAILURE**
- **DEFLECTION**
- **LATERAL TORSIONAL BUCKLING**

LATERAL TORSIONAL BUCKLING

FACTORS INFLUENCING LATERAL TORSIONAL BUCKLING:

- **LENGTH OF BEAM**
- **RESTRAINT AT SUPPORTS**
- **LOADING CONDITION**
- **SECTION SHAPE**
- **SHAPE OF BENDING MOMENT DIAGRAM**

ELASTIC CRITICAL MOMENT, M_{cr}

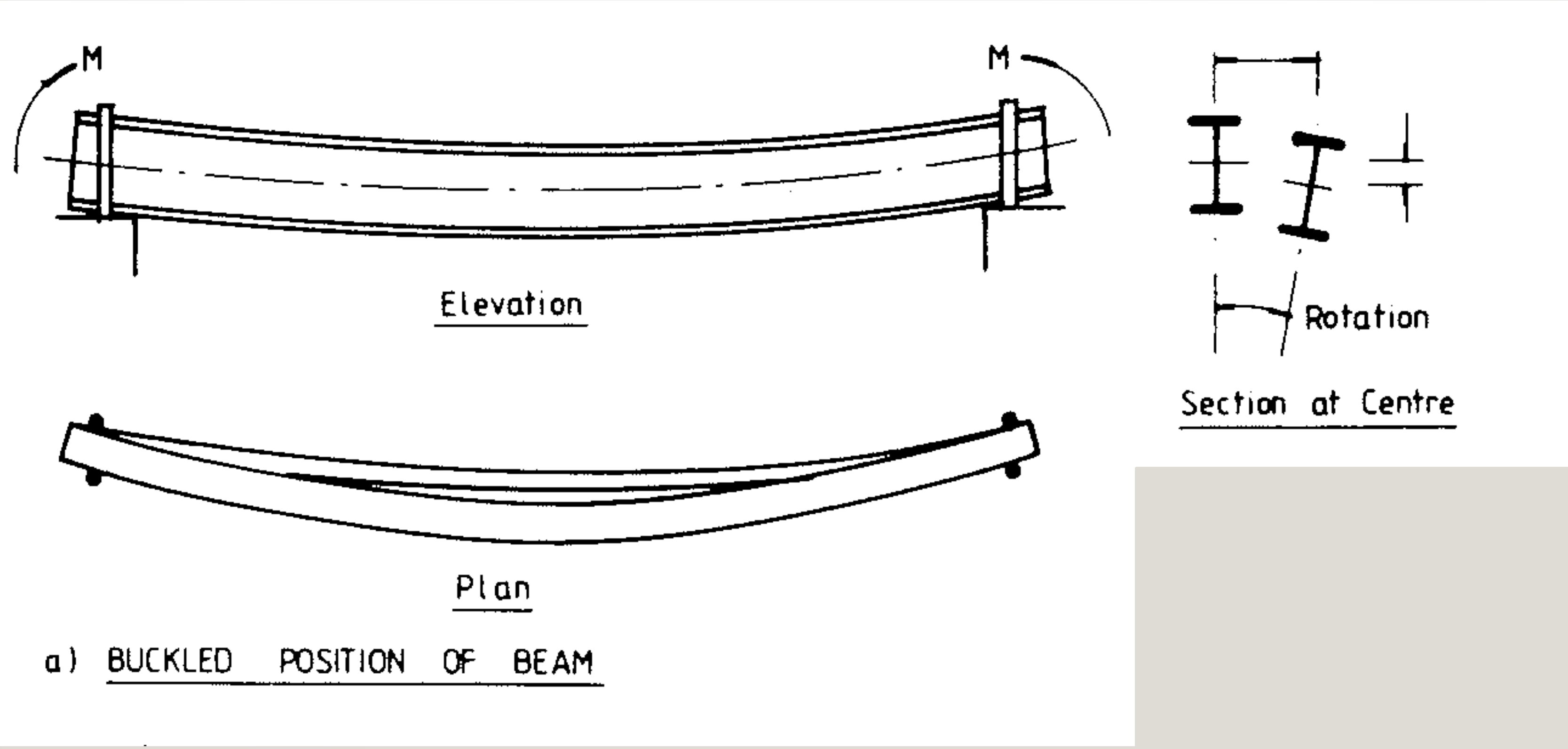
FROM ELASTIC THEORY

$$\boldsymbol{M_{cr} = \frac{\pi^2 EI_z}{L_{cr}^2}\left(\frac{I_w}{I_z} + \frac{L_{cr}^2 GI_t}{\pi^2 EI_z}\right)^{0.5} = \frac{\pi}{L_{cr}}(EI_z GI_t)^{0.5}\left(1 + \frac{\pi^2 EI_w}{L_{cr}^2 GI_t}\right)^{0.5}}$$

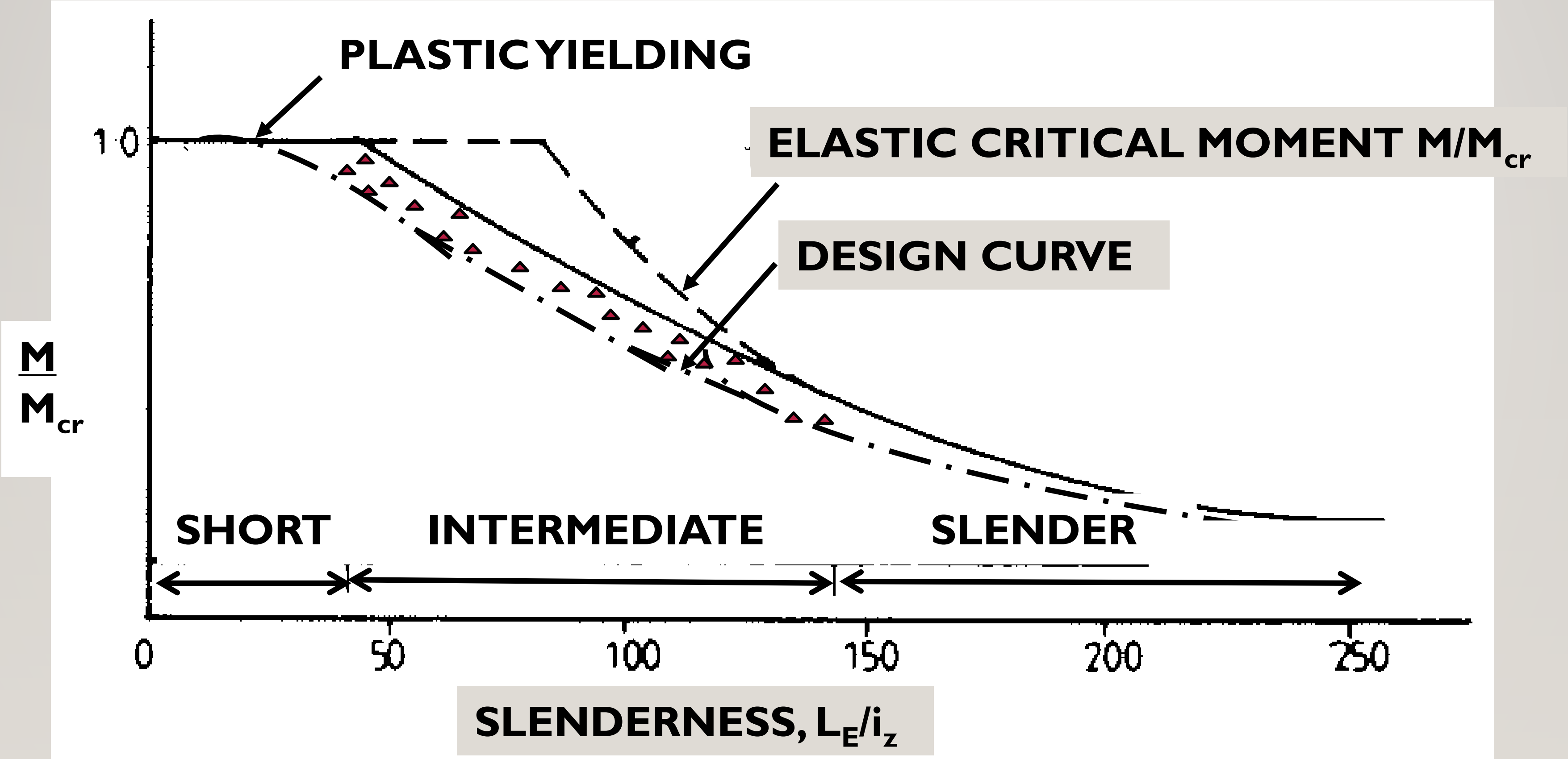
where

E	**YOUNG'S MODULUS**
G	**SHEAR MODULUS**
I_t	**TORSION CONSTANT FOR THE SECTION**
I_w	**WARPING CONSTANT FOR THE SECTION**
L_{cr}	**SPAN**
I_z	**MOMENT OF INERTIA ABOUT Z-Z AXIS**

LATERAL TORSIONAL BUCKLING OF SIMPLY SUPPORTED BEAM



BEHAVIOUR CURVE



EFFECTIVE LENGTH, L_E (TABLE 13, BS5950)

Conditions of restraint at supports	*Loading conditions*	
	Normal	*Destab.*
Comp. flange laterally restrained		
Nominal torsional restraint against rotation about longitudinal axis		
Both flanges fully restrained against rotation on plan	$0.7L_{LT}$	$0.85L_{LT}$
Compression flange fully restrained against rotation on plan	$0.75L_{LT}$	$0.9L_{LT}$
Both flanges partially restrained against rotation on plan	$0.8L_{LT}$	$0.95L_{LT}$
Compression flange partially restrained against rotation on plan	$0.85L_{LT}$	$1.0L$
Both flanges free to rotate on plan	$1.0L_{LT}$	$1.2L_{LT}$
Comp. flange laterally unrestrained		
Both flanges free to rotate on plan		
Partial torsional restraint against rotation about longitudinal axis provided by connection of bottom flange to supports	$1.0L_{LT} + 2D$	$1.2L_{LT} + 2D$
Partial torsional restraint against rotation about longitudinal axis provided only by pressure of bottom flange onto supports	$1.2L_{LT} + 2D$	$1.4L_{LT} + 2D$

DESIGN METHODOLOGY

EC3 DESCRIBES THREE METHODS FOR CHECKING THE LATERAL TORSIONAL BUCKLING OF BEAMS BUT ONLY THE METHODS SET OUT IN CLAUSE 6.3.2.2 (GENERAL CASE) AND CLAUSE 6.3.2.3 (FOR ROLLED SECTIONS AND EQUIVALENT WELDED SECTIONS) WILL BE DISCUSSED.

GENERAL CASE

$$\frac{M_{Ed}}{M_{\mathrm{b},Rd}} \leq 1.0$$

where

M_{Ed} IS THE MAXIMUM MAJOR AXIS MOMENT IN THE SEGMENT

$\mathrm{M}_{\mathrm{b,Rd}}$ IS THE BUCKLING RESISTANCE MOMENT

GENERAL CASE (CONT'D)

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}}$$

where

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1.0$$

in which

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$$

THE IMPERFECTION FACTOR, α_{LT}, IS DETERMINED FROM TABLE 6.3 IN EC3 VIA THE BUCKLING CURVE OBTAINED FROM TABLE 6.4.

TABLES 6.3 AND 6.4

BUCKLING CURVE	a	b	c	d
IMPERFECTION FACTOR, α_{LT}	0.21	0.34	0.49	0.76

TABLE 6.3: IMPERFECTION FACTORS

CROSS-SECTION	LIMITS	BUCKLING CURVE
ROLLED I SECTION	$h/b \leq 2$ $h/b > 2$	a b
WELDED I SECTION	$h/b \leq 2$ $h/b > 2$	c d

TABLE 6.4: BUCKLING CURVES FOR CROSS-SECTIONS

GENERAL CASE (CONT'D)

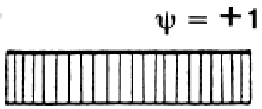
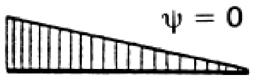
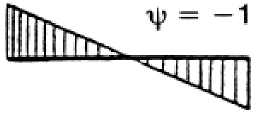
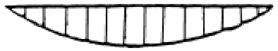
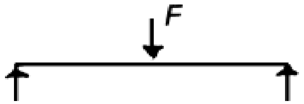
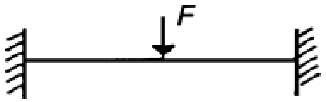
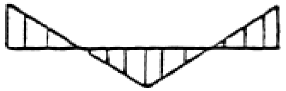
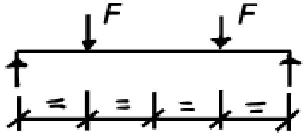
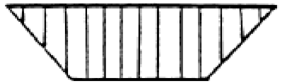
$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

EC3 DOES NOT GIVE ANY GUIDANCE ON HOW TO CALCULATE M_{CR} AND IT IS THEREFORE RECOMMENDED IN NCCI SN003 THAT THE FOLLOWING EQUATION IS USED

$$M_{cr} = C_1 \frac{\pi^2 E I_Z}{L_{cr}^2} \left(\frac{I_w}{I_z} + \frac{L_{cr}^2 G I_t}{\pi^2 E I_z} \right)^{0.5}$$

VALUES OF C_1 MAY BE DETERMINED FROM THE TABLE SHOWN BELOW. IT IS CONSERVATIVE TO ASSUME C_1 = 1.

VALUES OF C_1

Loading and support conditions	*Bending moment diagram*	ψ	C_1
	(a) 	+1	1.000
	(b) 	0	1.879
	(c) 	–1	2.752
	(d) 	–	1.132
	(e) 	–	1.285
	(f) 	–	1.365
	(g) 	–	1.565
	(h) 	–	1.046

NON-DIMENSIONAL SLENDERNESS RATIO

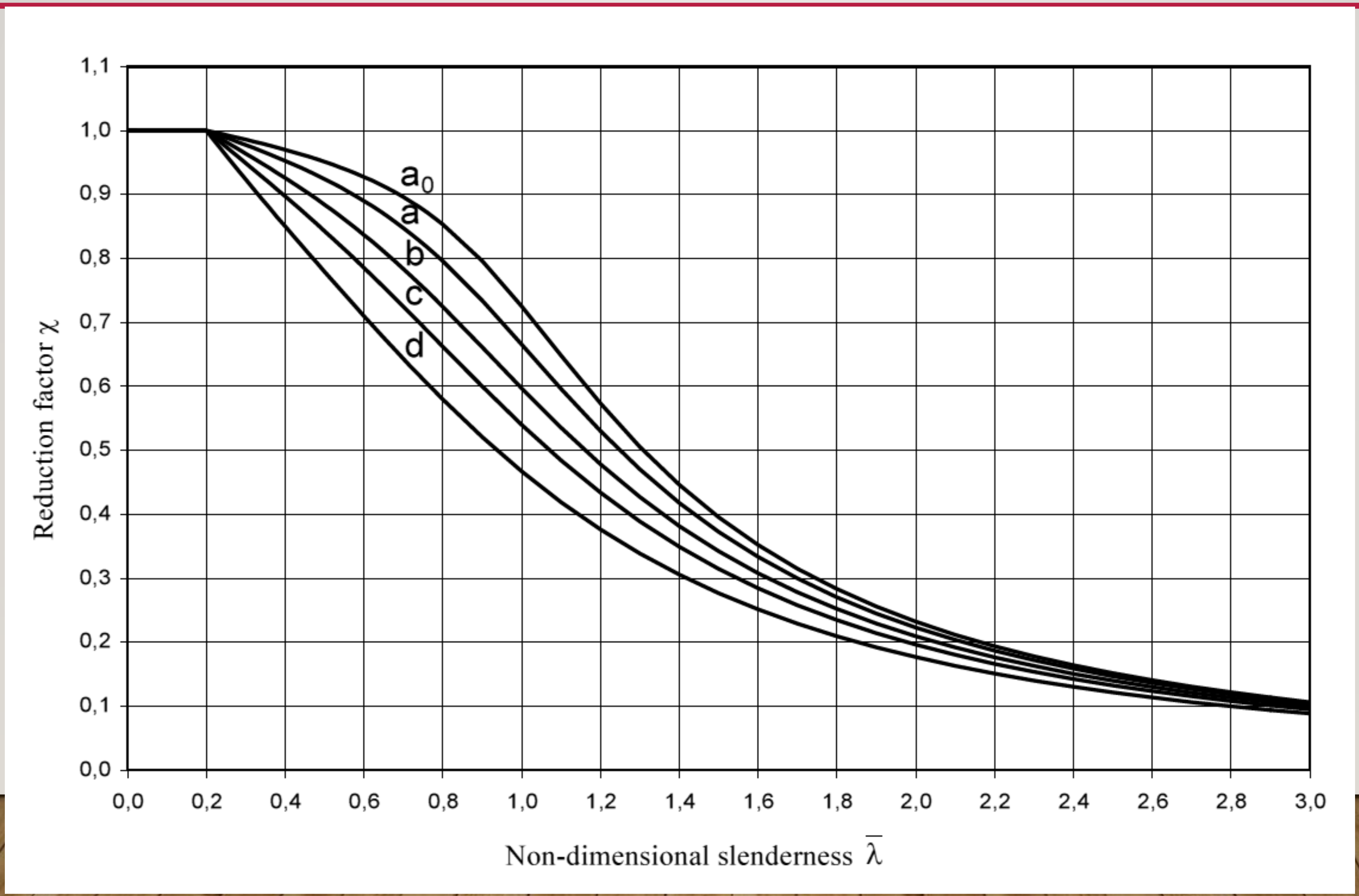
VALUES OF THE REDUCTION FACTOR, χ_{LT}, MAY ALSO BE ESTIMATED FROM FIGURE 6.4, EC3 USING THE NON-DIMENSIONAL SLENDERNESS RATIO, $\bar{\lambda}$, **WHERE**

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{L_{cr}}{i} \times \frac{1}{\lambda_1} \textbf{ FOR CLASS 1,2 AND 3 SECTIONS}$$

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93.9\epsilon$$

$$\varepsilon = \sqrt{\frac{235}{f_y}}$$

BUCKLING CURVES (FIG. 6.4, EC3)



SPECIAL CASE

Here again

$$\boldsymbol{M}_{\mathbf{b},\boldsymbol{Rd}} = \chi_{LT} \boldsymbol{W}_{\boldsymbol{y}} \frac{\boldsymbol{f}_{\boldsymbol{y}}}{\boldsymbol{\gamma}_{\boldsymbol{M1}}}$$

where

$$\chi_{LT} = \frac{\mathbf{1}}{\emptyset_{LT} + \sqrt{\emptyset_{LT}^{2} - \boldsymbol{\beta}\bar{\lambda}_{LT}^{2}}} \leq \mathbf{1.0}\ \boldsymbol{and}\ \leq \frac{\mathbf{1}}{\bar{\lambda}_{LT}}$$

in which

$$\emptyset_{LT} = \mathbf{0.5}\left[\mathbf{1} + \boldsymbol{\alpha}_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{\mathbf{LT,0}}\right) + \boldsymbol{\beta}\bar{\lambda}_{LT}^{2}\right]$$

•

SPECIAL CASE (CONT'D)

THE UK NA HAS SET β = 0.75 AND $\bar{\lambda}_{LT,0} = 0.4$. THE IMPERFECTION FACTOR, α_{LT}, IS DETERMINED FROM TABLE 6.3 IN EC3 VIA THE RECOMMENDATIONS FOR BUCKLING CURVES GIVEN IN TABLE NA.1 (REPRODUCED BELOW), CLAUSE NA.2.17, UK NA.

CROSS-SECTION	LIMITS	BUCKLING CURVE
ROLLED I AND H SECTIONS	h/b ≤ 2 2.0 < h/b ≤ 3.1	b c
WELDED DOUBLY SYMMETRIC SECTIONS	h/b ≤ 2 2.0 < h/b ≤ 3.1	c d

SPECIAL CASE (CONT'D)

EC3 ALSO ALLOWS A MODIFIED VALUE OF χ_{LT} TO BE USED, $\chi_{LT,mod}$, WHICH IS GIVEN BY

$$\chi_{\mathrm{LT},mod} = \frac{\chi_{LT}}{f}$$

where

$$f = 1 - 0.5(1 - k_c)\left[1 - 2.0\left(\bar{\lambda}_{LT} - 0.8\right)^2\right] \leq 1.0$$

in which

$k_c = \frac{1}{\sqrt{C_1}}$ **(NA 2.18)**

COMPARISON BETWEEN GENERAL CURVE AND CURVES FOR ROLLED SECTIONS (I SECTIONS - h/b > 2)