CEGE0009: Structural Analysis and Design

BEng/MEng/

Course Examination – 2019/2020

Time allowed: 3 hours

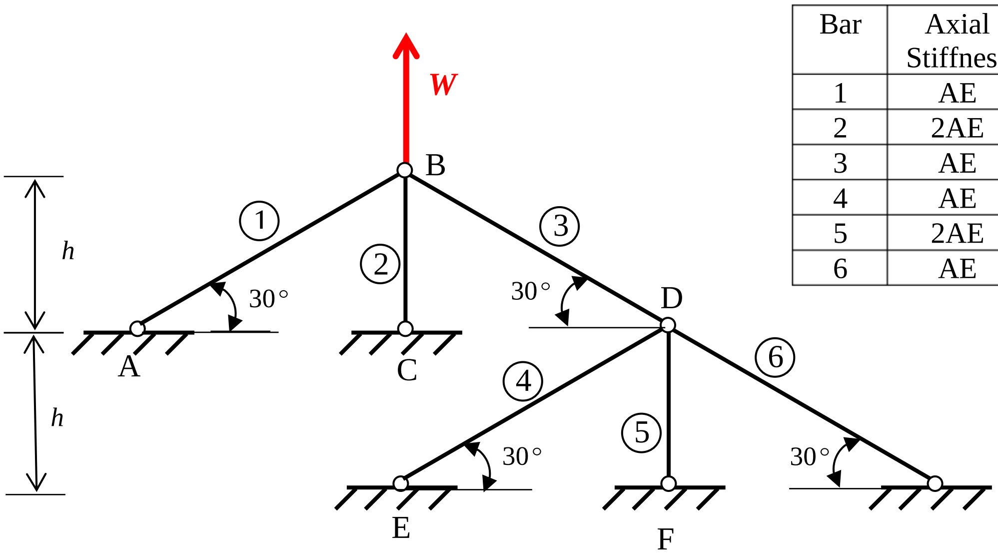
- **This paper has two sections: A and B**
- **Answer ALL questions**

PAGE LIMIT: answer each question on no more than four sides of an A4 page.

<u>Section A</u> – Answer all questions from this section (Total = 50 marks)

Question 1

The structure shown in Figure Q1 is a pin-jointed truss. It is made of bars of axial stiffnesses as indicated in the table inserted in Figure Q1. The truss is loaded at B by a vertical force ***W*** as shown in the diagram. The truss is stress-free before the application of the load and all members have their nominal length.



Bar	Axial Stiffness
1	AE
2	2AE
3	AE
4	AE
5	2AE
6	AE

Figure Q1

(a) Calculate the bar forces throughout the truss caused by ***W***.

[20marks]

(b) The load W is now removed. Without providing actual numerical answers, describe with the help of matrix notation the step by step procedure you would follow to calculate the bar forces if support F settles vertically downward by $h/100$.

[5 marks]

Question 2

(a) The frame shown in Figure Q2a is made of a vertical column AB of bending stiffness *EI* rigidly connected to a horizontal beam BC of bending stiffness 2*EI* loaded uniformly along its span by *q* and an inclined member CD of bending stiffness *EI* pinned at both ends C and D.

Calculate the bending moment, and the axial and shear forces throughout the frame. Draw a diagram for each internal force, indicating their values at *A*, *B*, C and *D*. Draw a consistent deflected shape. Calculate the horizontal deflection of B.

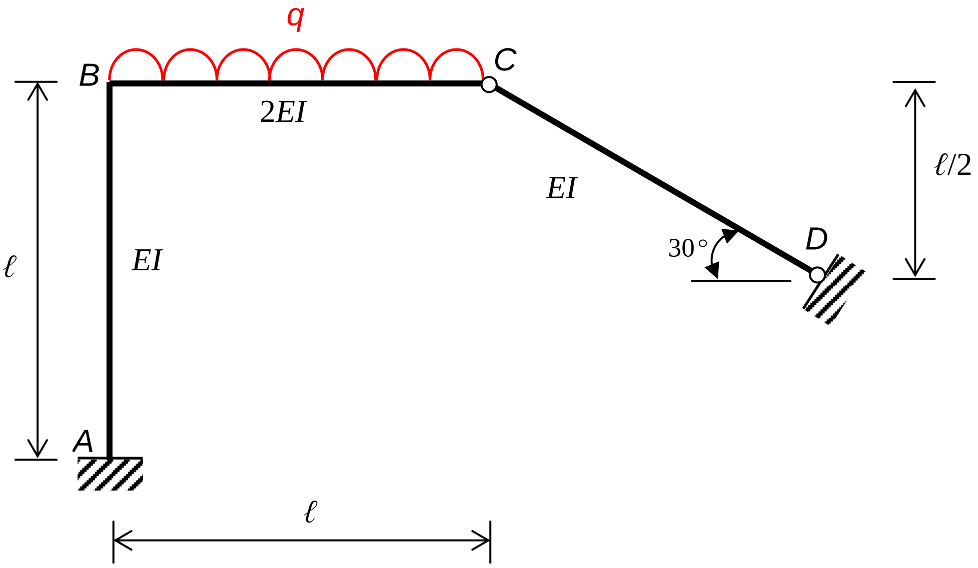


Figure Q2a

Note: Tables of standard cases of beam deflection and virtual work integrals are provided in appendix A.

[20 marks]

(b) Describe qualitatively, including diagrams as appropriate, how the bending moment from Question 2a changes if

- (i) The bending stiffness of beam BC is much smaller than that of the other two members
- (ii) The bending stiffness of beam BC is much larger than that of the other two members

[5 marks]

<u>Section B</u> – Answer all questions from this section (Total = 50 marks)

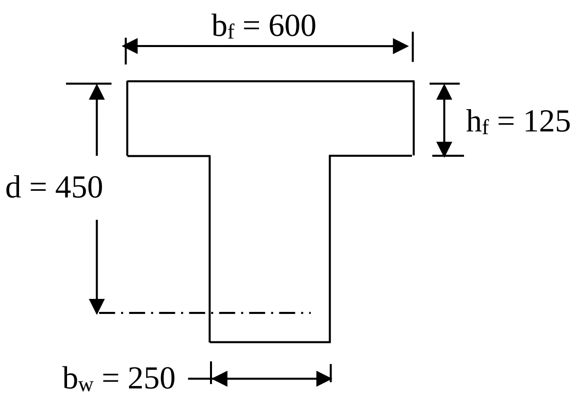
Question 3

(a) Discuss the significance of the assumption "plane sections remain plane" in the design of reinforced concrete sections for bending. List the other assumptions. **[5 marks]**

(b) FIG. Q3 shows a T-beam which is made of class C25/30 concrete. The beam is 6m long and simply supported at its ends. Assuming it supports characteristic permanent (including self-weight) and variable loads of 30 kN/m and 20 kN/m respectively

- (i) Design the longitudinal reinforcement assuming the use of 20mm diameter bars
- (ii) Design the shear reinforcement
- (iii) Check deflection
- (iv) Draw a labelled sketch of the reinforcement arrangement in cross-section and elevation

(N.B. Bar Area tables are appended to this paper)



All dimensions in mm

FIG. Q3

[20 marks]

Question 4

(a) For the braced frame shown in FIG. Q4(a) draw sketches of the various load combinations necessary for the design of

(i) Beam EFG

(ii) Column FH.

Assume the floors and roof support uniformly distributed characteristic permanent and variable loads of g_k and q_k respectively. **[6 marks]**

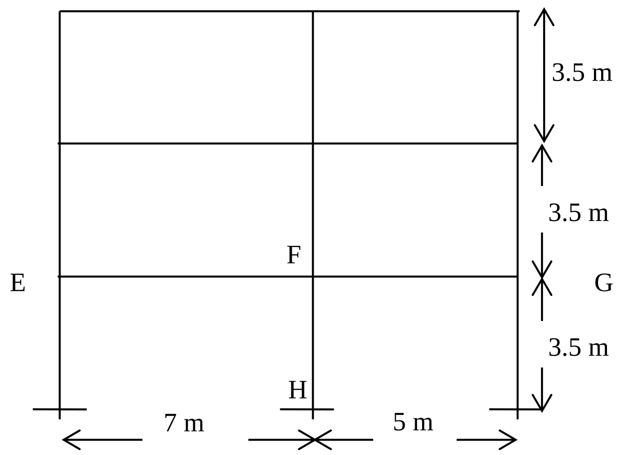


FIG. Q4a

(b) The column section shown in FIG. Q4(b) is made of class C30/37 concrete and reinforced with six 20 mm diameter, grade 500 bars. Assuming a rectangular stress block for concrete determine

(i) The maximum axial load capacity of the column section **[5 marks]**

(ii) The axial load and moment capacity of the section assuming the neutral axis occurs 235mm below the apex. **[10 marks]**

(c) Design the transverse steel reinforcement and produce a sketch of the arrangement. **[4 marks]**

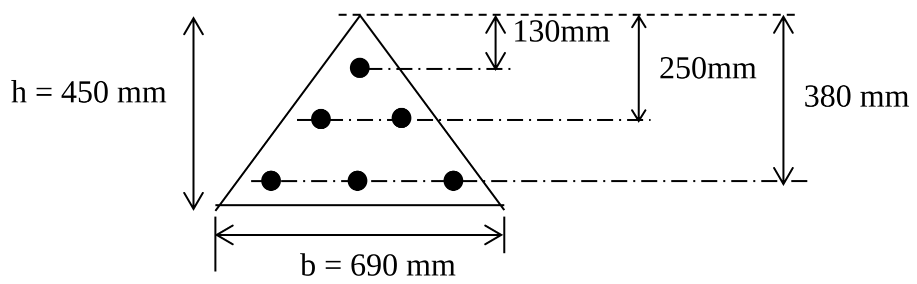


FIG. Q4(b)

APPPENDIX FOR SECTION A

1. **TABLE OF STANDARD CASES OF BEAM DEFLECTION**
2. **TABLE OF STANDARD VIRTUAL WORKS INTEGRALS**

TABLE OF STANDARD CASES OF BEAM DEFLECTION

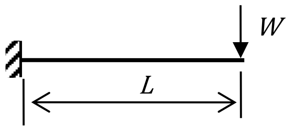
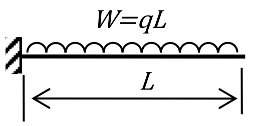
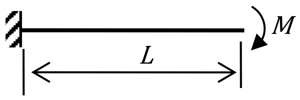
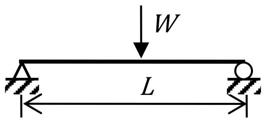
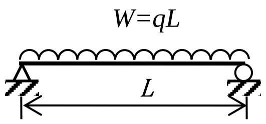
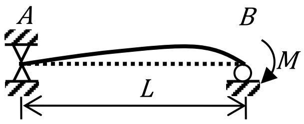
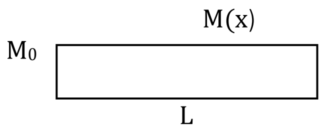
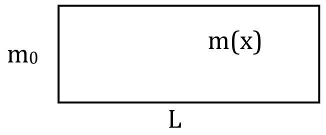
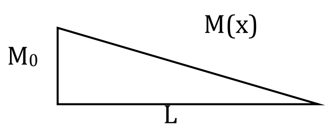
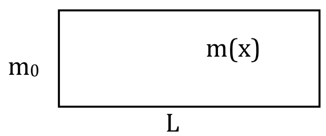
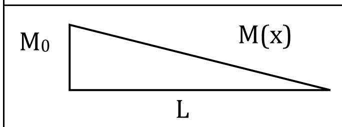
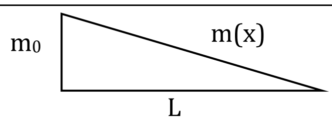
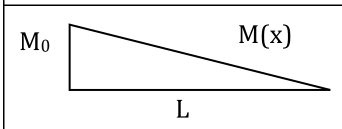
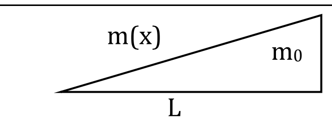
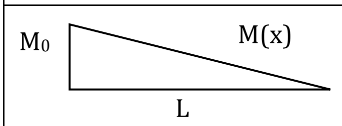
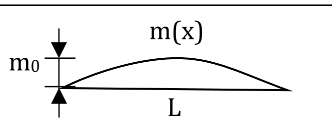
	Tip or central deflection	End Rotation
	$\dfrac{WL^3}{3EI}$	$\dfrac{WL^2}{2EI}$
	$\dfrac{WL^3}{8EI}$	$\dfrac{WL^2}{6EI}$
	$\dfrac{ML^2}{2EI}$	$\dfrac{ML}{EI}$
	$\dfrac{WL^3}{48EI}$	$\dfrac{WL^2}{16EI}$
	$\dfrac{5WL^3}{384EI}$	$\dfrac{WL^2}{24EI}$
	$\dfrac{ML^2}{16EI}$ @centre	$\theta_A = \dfrac{\theta_B}{2}; \quad \theta_B = \dfrac{ML}{3EI}$

TABLE OF STANDARD VIRTUAL WORK INTEGRALS

		$\int M(x)m(x)dx = M_0 m_0 L$
		$\int M(x)m(x)dx = \frac{1}{2} M_0 m_0 L$
		$\int M(x)m(x)dx = \frac{1}{3} M_0 m_0 L$
		$\int M(x)m(x)dx = \frac{1}{6} M_0 m_0 L$
		$\int M(x)m(x)dx = \frac{1}{3} M_0 m_0 L$

APPENDIX FOR SECTION B
DESIGN DATA

1. Bar area tables
2. Design formulae for rectangular beams

1. BAR AREAS TABLES

(i) Cross-sectional areas of groups of bars (mm^2)

Bar size (mm)	Number of bars									
	1	2	3	4	5	6	7	8	9	10
6	28.3	56.6	84.9	113	142	170	198	226	255	283
8	50.3	101	151	201	252	302	352	402	453	503
10	78.5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	566	679	792	905	1020	1130
16	201	402	603	804	1010	1210	1410	1610	1810	2010
20	314	628	943	1260	1570	1890	2200	2510	2830	3140
25	491	982	1470	1960	2450	2950	3440	3930	4420	4910
32	804	1610	2410	3220	4020	4830	5630	6430	7240	8040
40	1260	2510	3770	5030	6280	7540	8800	10100	11300	12600

(ii) Shear Reinforcement

	Spacing of links (mm)										
Diameter (mm)	*85*	*90*	*100*	*125*	*150*	*175*	*200*	*225*	*250*	*275*	*300*
8	1.183	1.118	1.006	0.805	0.671	0.575	0.503	0.447	0.402	0.336	0.335
10	1.847	1.744	1.57	1.256	1.047	0.897	0.785	0.698	0.628	0.571	0.523
12	2.659	2.511	2.26	1.808	1.507	1.291	1.13	1.004	0.904	0.822	0.753
16	4.729	4.467	4.02	3.216	2.68	2.297	2.01	1.787	1.608	1.462	1.34

A_{SV}/s_V for varying stirrup diameters and spacing

(iii) Cross-sectional area per metre width for various bar spacing (mm^2)

	Spacing of bars (mm)								
Diameter (mm)	*50*	*75*	*100*	*125*	*150*	*175*	*200*	*250*	*300*
6	566	377	283	226	189	162	142	113	94.3
8	1010	671	503	402	335	287	252	201	168
10	1570	1050	785	628	523	449	393	314	262
12	2260	1510	1130	905	754	646	566	452	377
16	4020	2680	2010	1610	1340	1150	1010	804	670
20	6280	4190	3140	2510	2090	1800	1570	1260	1050
25	9820	6550	4910	3930	3270	2810	2450	1960	1640
32	16100	10700	8040	6430	5360	4600	4020	3220	2680
40	25100	16800	12600	10100	8380	7180	6280	5030	4190

2. DESIGN FORMULAE FOR RECTANGULAR BEAMS

2.1 Bending

$M_{Rd} = 0.167 f_{ck} b d^2$

(i) $M \leq M_{Rd}$
$z = d[0.5 + \sqrt{(0.25 - 3K/3.4)}]$
where $K = \dfrac{M}{f_{ck} b d^2}$
$x = (d - z)/0.4$
$$A_{s1} = \frac{M}{0.87 f_{yk} z}$$

(ii) $M > M_{Rd}$
$z = d[0.5 + \sqrt{(0.25 - 3K'/3.4)}]$
$K' = 0.167$
$x = (d - z)/0.4$

The area of compression reinforcement, A_{s2}, is given by:
$$A_{s2} = \frac{M - M_{Rd}}{0.87 f_{yk} (d - d_2)}$$

The area of tension reinforcement, A_{s1}, is given by:
$$A_{s1} = \frac{M_{Rd}}{0.87 f_{yk} z} + A_{s2}$$

If d_2/x exceeds 0.38 the compression stress will be less than $0.87 f_y$ i.e. yield stress

where d_2 is the depth of the compression steel from the compression face

2.2 Shear

The concrete strut capacity, $V_{Rd,max}$, is given by:

$V_{Rd,max} = b_w z \nu f_{cd} / (\cot\theta + \tan\theta)$

where

$z \approx 0.9d$

$f_{cd} = \alpha_{cc} f_{ck}/\gamma_m = 0.85 f_{ck}/1.5$ (for $f_{ck} \leq 50$ N/mm^2)

$\nu = 0.6(1 - f_{ck}/250)$ for $f_{ck} \leq 50$ N/mm^2

θ is the angle between the concrete strut and the axis of the beam

$$\theta = 0.5\sin^{-1}\frac{(V_{Rd,\max}/b_w d)}{0.153 f_{ck}(1 - f_{ck}/250)}$$

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cot\theta$$

where

- $V_{Rd,s}$ is the shear resistance of the member governed by 'failure' of the stirrups
- A_{sw} is the cross-sectional area of the shear reinforcement
- s is the spacing of shear reinforcement
- f_{ywd} is the design yield strength of the shear reinforcement.

See Eurocode 2 for other formulae relevant to shear design

END OF PAPER