CEGE0009: Structural Analysis and Design

Course Examination – 2018/2019

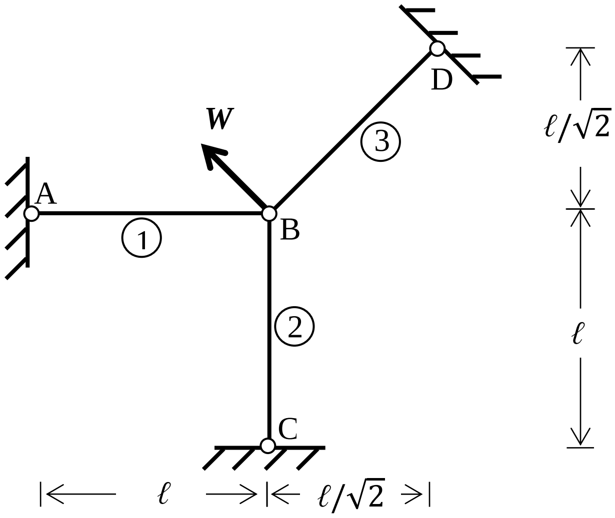
Time allowed: 3 hours

- **This paper has two sections: A and B**
- **Answer ALL questions**
- **Answer questions in sections A and B on separate answer books**
- **Data sheets for each section are provided at the end of the paper**
- **Candidates can use their own unmarked copies of Eurocode 2: Part 1-1**

<u>Section A</u> – Answer all questions from this section (Total = 50 marks)

Question 1

The structure shown in Figure Q1 is a pin-jointed truss. It is made of three bars of axial stiffnesses as indicated in the table inserted in Figure Q1. It is loaded at B by a force ***W*** perpendicular to BD as shown in the diagram. The truss is stress-free before the application of the load and all members have their nominal length unless otherwise stated.



Bar	Axial Stiffness
1	AE/2
2	AE
3	AE/2

Figure Q1

(a) Calculate the bar forces throughout the truss and the horizontal deflection at *B* caused by ***W***.

[20 marks]

(b) The load ***W*** is now removed. Calculate the new bar forces if the unloaded lengths of all three bars are 1% shorter than their nominal values.

[5 marks]

Question 2

(a) The frame shown in Figure Q2a is pinned at *A* and *D*, moment-resisting at *B* and *C*, and loaded by a point moment M as indicated in the diagram. The frame is made of members of constant bending stiffness *EI* throughout. Find numerically the reactions at A and D. Draw the bending moment diagram throughout the frame indicating actual values at *A*, *B*, *C*, *D*. Draw a consistent deflected shape.

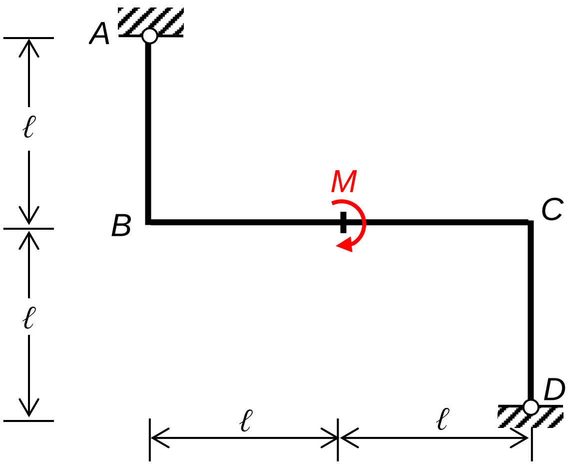


Figure Q2a

Note: the table of standard cases of beam deflections is provided in appendix A.

[20 marks]

(b) Using qualitative analysis **only**, draw the free body diagram, the bending moment diagram and a consistent deflected shape for the frame shown in Figure Q2b.

[5 marks]

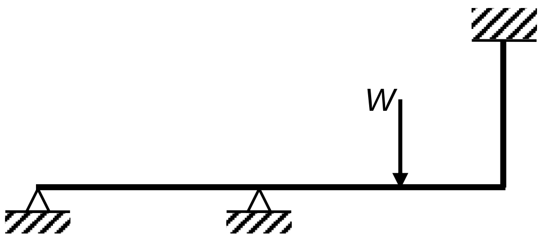


Figure Q2b

<u>Section B</u> – Answer all questions from this section (Total = 50 marks)

Question 3

(a) Distinguish between serviceability and ultimate limit states and give two examples of each limit state in the context of reinforced concrete design.

[6 marks]

(b) The simply-supported beam with cantilevering ends shown in Figure Q3 supports characteristic uniformly distributed permanent and variable loads of g_k and q_k respectively. Sketch the loading arrangements which could be used to estimate values of the design

(i) sagging moment
(ii) hogging moment
(iii) shear force in the beam.

[6 marks]

(c) A beam 300mm wide with an effective depth of 500mm is subject to an ultimate design moment of 350 kNm and ultimate design shear force of 300 kN. Assuming the beam is made of class C30/37 concrete and the characteristic strength of the reinforcement is 500 N/mm^2

(i) calculate the number of 25 mm diameter bars required in bending
(ii) design the shear reinforcement
(ii) draw a labelled sketch of the strain distribution indicating values of the depth of the neutral axis, strain in the concrete and strain in the steel reinforcement at failure.

[13 marks]

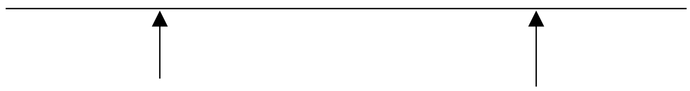


Figure Q3

Question 4

(a) Explain with the aid of diagrams what you understand by the term effective height/length of a column and describe how its value is determined in concrete design.

[8 marks]

(b) Figure Q4 shows a cross-section of a non-slender, braced 400mm square column made of class C30/37 concrete and grade 500 steel.

(i) Estimate the axial load capacity of the column and comment on why a lower value would be used in practice.

[5 marks]

(ii) Design the links and draw a sketch of the arrangement. The nominal cover to all reinforcement may be assumed to be 40mm.

[4 marks]

(iii) Assuming the column is subjected to a moment of 192 kNm about the y-y axis determine with the aid of the column design chart in Appendix B the axial load capacity of the column.

[8 marks]

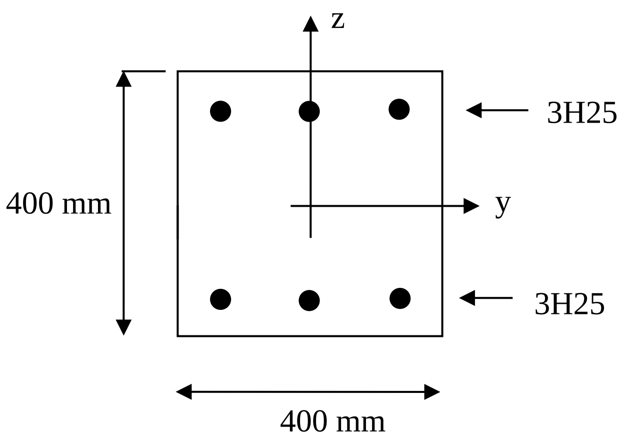


Figure Q4

APPPENDIX FOR SECTION A

1. **TABLE OF STANDARD CASES OF BEAM DEFLECTION**
2. **TABLE OF STANDARD VIRTUAL WORKS INTEGRALS**

TABLE OF STANDARD CASES OF BEAM DEFLECTION

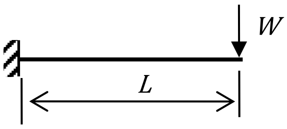
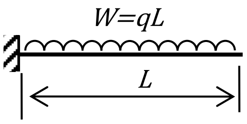
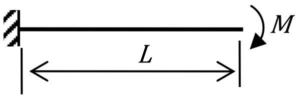
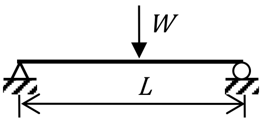
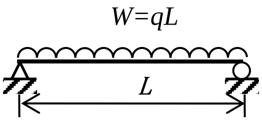
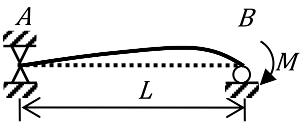
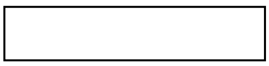
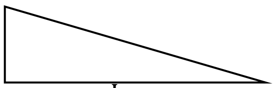
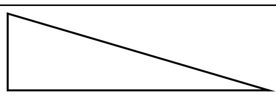
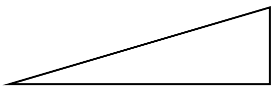
	Tip or central deflection	End Rotation
	$\frac{WL^3}{3EI}$	$\frac{WL^2}{2EI}$
	$\frac{WL^3}{8EI}$	$\frac{WL^2}{6EI}$
	$\frac{ML^2}{2EI}$	$\frac{ML}{EI}$
	$\frac{WL^3}{48EI}$	$\frac{WL^2}{16EI}$
	$\frac{5WL^3}{384EI}$	$\frac{WL^2}{24EI}$
	$\frac{ML^2}{16EI}$ @centre	$\theta_A = \frac{\theta_B}{2}; \quad \theta_B = \frac{ML}{3EI}$

TABLE OF STANDARD VIRTUAL WORK INTEGRALS

M(x)	m(x)	$\int M(x)m(x)dx$
M_0  L	m_0  L	$M_0 m_0 L$
M_0  L	m_0  L	$\frac{1}{2} M_0 m_0 L$
M_0  L	m_0  L	$\frac{1}{3} M_0 m_0 L$
M_0  L	 m_0 L	$\frac{1}{6} M_0 m_0 L$

APPENDIX FOR SECTION B
DESIGN DATA

1. Bar area tables
2. Design formulae for rectangular beams
3. Column design charts

1. BAR AREAS TABLES

(i) Cross-sectional areas of groups of bars (mm^2)

Bar size (mm)	Number of bars									
	1	2	3	4	5	6	7	8	9	10
6	28.3	56.6	84.9	113	142	170	198	226	255	283
8	50.3	101	151	201	252	302	352	402	453	503
10	78.5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	566	679	792	905	1020	1130
16	201	402	603	804	1010	1210	1410	1610	1810	2010
20	314	628	943	1260	1570	1890	2200	2510	2830	3140
25	491	982	1470	1960	2450	2950	3440	3930	4420	4910
32	804	1610	2410	3220	4020	4830	5630	6430	7240	8040
40	1260	2510	3770	5030	6280	7540	8800	10100	11300	12600

(ii) Shear Reinforcement

	Spacing of links (mm)										
Diameter (mm)	*85*	*90*	*100*	*125*	*150*	*175*	*200*	*225*	*250*	*275*	*300*
8	1.183	1.118	1.006	0.805	0.671	0.575	0.503	0.447	0.402	0.336	0.335
10	1.847	1.744	1.57	1.256	1.047	0.897	0.785	0.698	0.628	0.571	0.523
12	2.659	2.511	2.26	1.808	1.507	1.291	1.13	1.004	0.904	0.822	0.753
16	4.729	4.467	4.02	3.216	2.68	2.297	2.01	1.787	1.608	1.462	1.34

A_{sv}/s_v for varying stirrup diameters and spacing

(iii) Cross-sectional area per metre width for various bar spacing (mm^2)

	Spacing of bars (mm)								
Diameter (mm)	*50*	*75*	*100*	*125*	*150*	*175*	*200*	*250*	*300*
6	566	377	283	226	189	162	142	113	94.3
8	1010	671	503	402	335	287	252	201	168
10	1570	1050	785	628	523	449	393	314	262
12	2260	1510	1130	905	754	646	566	452	377
16	4020	2680	2010	1610	1340	1150	1010	804	670
20	6280	4190	3140	2510	2090	1800	1570	1260	1050
25	9820	6550	4910	3930	3270	2810	2450	1960	1640
32	16100	10700	8040	6430	5360	4600	4020	3220	2680
40	25100	16800	12600	10100	8380	7180	6280	5030	4190

2. DESIGN FORMULAE FOR RECTANGULAR BEAMS

2.1 Bending

$M_{Rd} = 0.167 f_{ck} b d^2$

(i) $M \le M_{Rd}$
$z = d[0.5 + \sqrt{(0.25 - 3K/3.4)}]$
where $K = \dfrac{M}{f_{ck} b d^2}$
$x = (d - z)/0.4$
$$A_{s1} = \frac{M}{0.87 f_{yk} z}$$

(ii) $M > M_{Rd}$
$z = d[0.5 + \sqrt{(0.25 - 3K'/3.4)}]$
$K' = 0.167$
$x = (d - z)/0.4$

The area of compression reinforcement, A_{s2}, is given by:
$$A_{s2} = \frac{M - M_{Rd}}{0.87 f_{yk} (d - d_2)}$$

The area of tension reinforcement, A_{s1}, is given by:
$$A_{s1} = \frac{M_{Rd}}{0.87 f_{yk} z} + A_{s2}$$

If d_2/x exceeds 0.38 the compression stress will be less than $0.87 f_y$ i.e. yield stress

where
d_2 is the depth of the compression steel from the compression face

3. Column Design Chart

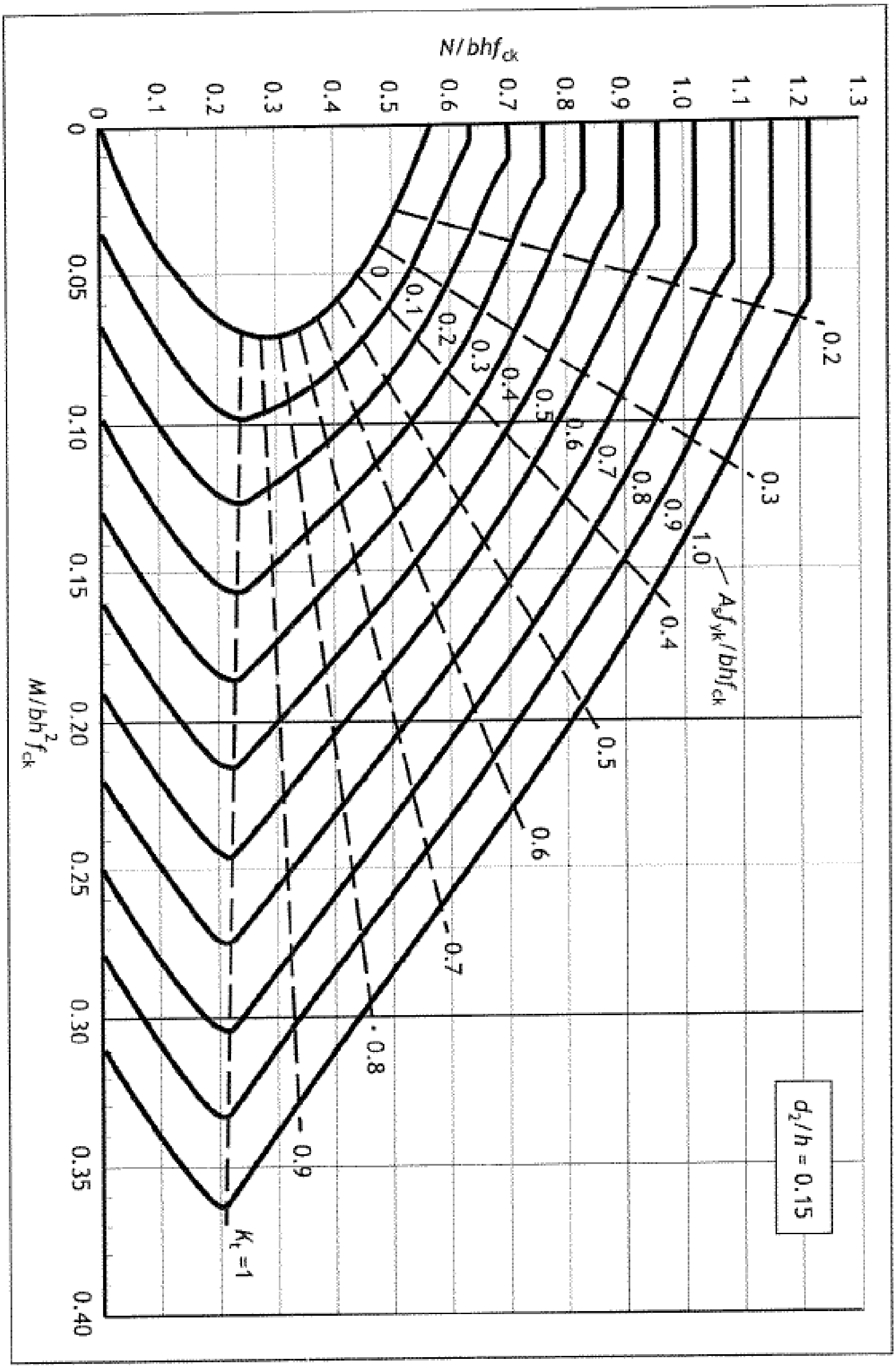


Figure 15.5(c)
Rectangular columns $d2/h$ = 0.15 (indicative)

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