Exercise 1. Consider the inverted pendulum dynamics

$$u'' + (\delta + \epsilon \cos \omega t) \sin u = 0.$$

(a) Perform a Floquet analysis (computationally) of the pendulum with continous forceing $\cos \omega t$

(b) Evaluate for what values of δ, ϵ, and ω the pendulum is stabilized.

Solution 1. a) We perform a Floquet analysis computationally by simulating the ODE above with two sets of intial conditions

$$u_1(0) = 0, u_1'(0) = 1$$
$$u_2(0) = 1, u_2'(0) = 0.$$

We can then compute the Floquet discriminant as

$$\Gamma = u_1(T) + u_2'(T),$$

where $T = 2\pi/\omega$. Notice that since we can vary the parameters in the ODE, Γ is can be viewed as a function of $(\delta, \epsilon, \omega)$. We also recreate figure 36 from the class notes using the given equation. That is, we visualize Γ as a function of ω for two regimes of δ and ϵ which are $\delta > \epsilon$ and $\delta < \epsilon$. This is shown in Figure 1. We observe that for large values of ω, Γ approaches 2 in both regimes. Suggesting that the pendulum is stabilized under high frequency forcing.
b) We also generate heatmaps for Γ for fixed ω and varying δ and ϵ to see for which values the solution is stable i.e.

$$|\Gamma(\delta, \epsilon, \omega)| < 2.$$

These heatmaps can be seen in Figure 2. We find that for large enoough of ω nearly all solutions are stable regardless of the choice of δ, ϵ.

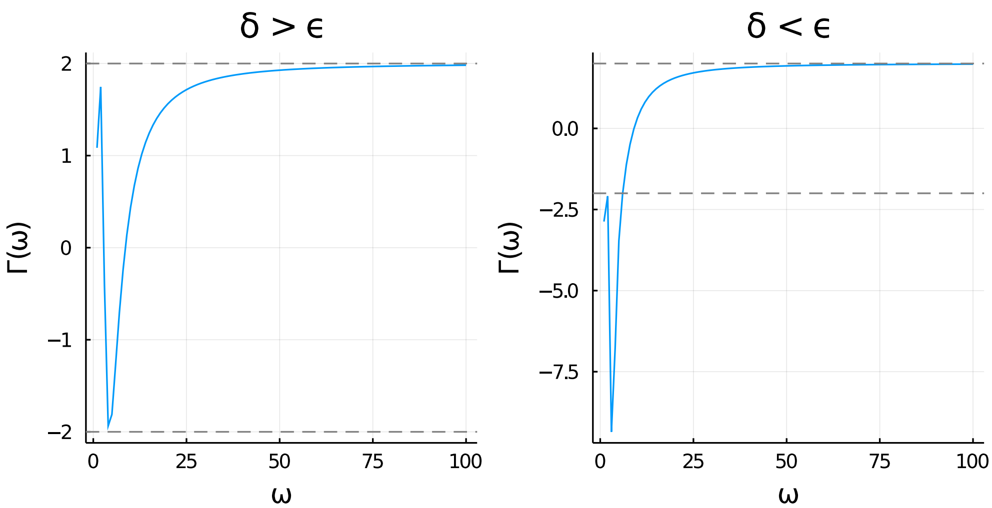

Figure 1: We visualize the Floquet discriminant as a function of ω in two parameter regimes.

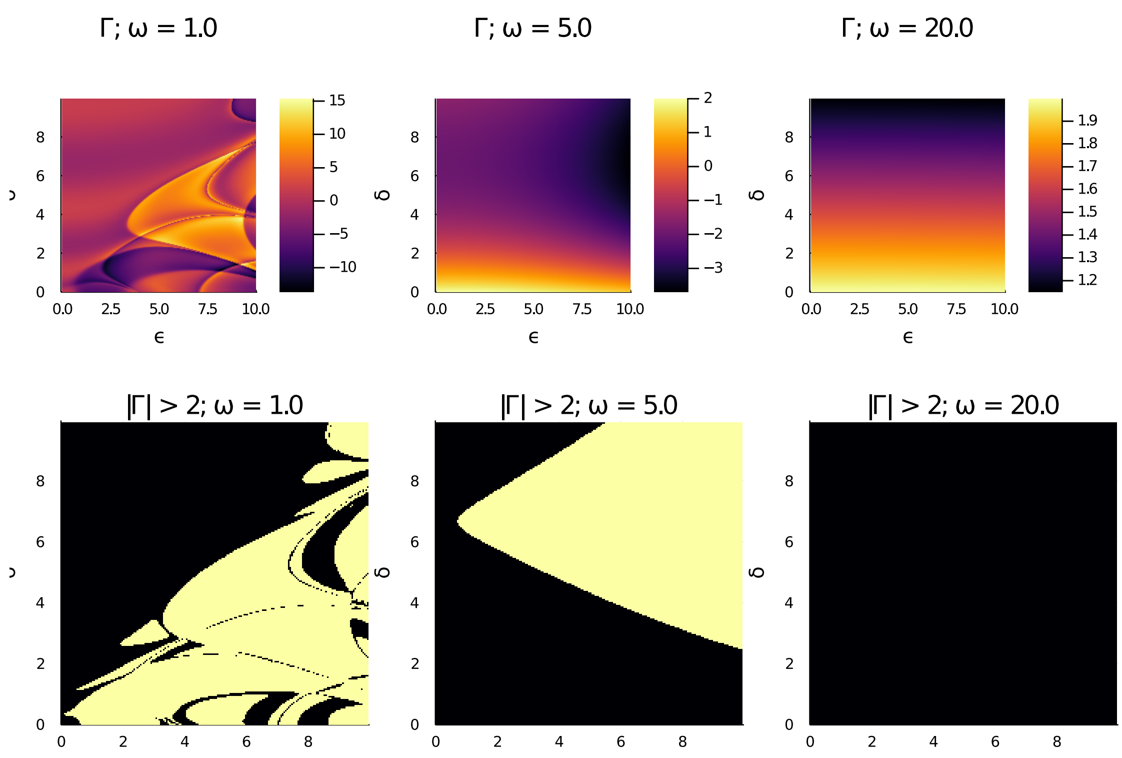

Figure 2: Visualizing the Floquet discriminant for $(\delta, \epsilon) \in [0, 10] \times [0, 10]$. We see for larger values of ω the pendulum is stablized.