Exercise 1. Show that if $\mathbf{A}$ is triangular and unitary, then it is diagonal.

Solution 1.

Exercise 2. Consider Hermitian (self-adjoint) matrices $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{n \times m}$.

a. Prove that all eigenvalues of $\mathbf{A}$ are real.

b. Porve that if $\mathbf{x}_k$ is the kth eigenvector, then eigenvectors with distinct eigenvalues are orthogonal.

c. Prove that the sum of two Hermitian matrices is Hermitian.

d. Prove that the inverse of an invertible Hermitian matrix is Hermitian.

e. Prove that the product of two Hermitian matrices is Hermitian if and only if $\vec{AB} = \vec{BA}$.

Solution 2.

Exercise 3. Consider a Unitary matrix $\mathbf{U} \in \mathbb{C}^{n \times m}$.

a. Prove that the matrix is diagonalizable

b. Prove that the iverse is $\mathbf{U}^{-1} = \mathbf{U}^*$.

c. Prove it is isometric with respect to the L^2 norm.

d. Prove that all eigenvalues have modulus 1.

Solution 3.