

ACM/ICPC: Competitive Programming Notebook

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Points

Comparing floating point values

Returns true if double values a and b are equal

```

1  const double EPS { 1e-9 };
2  bool equals(double a, double b)
3  {
4      return fabs(a - b) < EPS;
5  }

```

Listing 1: equals

Lines

General equation of a line

Non-normalized form: $ax + by + c = 0$

```

1  class Line {
2  public:
3      double a;
4      double b;
5      double c;
6
7      Line(double av, double bv, double cv) : a(av), b(bv), c(cv) {}
8
9      Line(const Point& p, const Point& q)
10     {
11         a = p.y - q.y;
12         b = q.x - p.x;
13         c = p.x * q.y - p.y * q.x;
14     }
15 };

```

Listing 2: General equation of a line

General equation of a line normalized

```

1  class Line {
2  public:
3      double a;
4      double b;
5      double c;
6
7      Line(double av, double bv, double cv) : a(av), b(bv), c(cv) {}
8
9      Line(const Point& p, const Point& q)
10     {
11         a = p.y - q.y;
12         b = q.x - p.x;
13         c = p.x * q.y - p.y * q.x;
14
15         auto k = a ? a : b;
16
17         a /= k;
18         b /= k;
19     }
20 };

```

```
19         c /= k;  
20     }  
21 };
```

Listing 3: General equation of a line

Point on a line

Is the given point located on the given Line?

```
1 template<typename T>  
2 struct Line {  
3     bool contains(const Point<T>& P) const  
4     {  
5         return equals(a*P.x + b*P.y + c, 0);  
6     }  
7 };
```

Listing 4: Point on line

Equal and parallel lines

```
1 template<typename T>  
2 struct Line {  
3     bool operator==(const Line& r) const  
4     {  
5         auto k = a ? a : b;  
6         auto s = r.a ? r.a : r.b;  
7  
8         return equals(a*s, r.a*k) && equals(b*s, r.b*k)  
9             && equals(c*s, r.c*k);  
10    }  
11  
12    bool parallel(const Line& r) const  
13    {  
14        auto det = a*r.b - b*r.a;  
15        return det == 0 and !(*this == r);  
16    }  
17 };
```

Orthogonal

```
1 template<typename T>  
2 struct Line  
3 {  
4     bool orthogonal(const Line& r) const  
5     {  
6         return equals(a * r.a + b * r.b, 0);  
7     }  
8 };
```

Intersection

```
1 const int INF { -1 };  
2 template<typename T>  
3 std::pair<int, Point<T>> intersections(const Line<T>& r, const Line<T>& s)
```

```
4 {  
6     auto det = r.a * s.b - r.b * s.a;  
8     if (equals(det, 0)) // Coincidentes ou paralelas  
9     {  
10        int qtd = (r == s) ? INF : 0;  
11        return std::pair<int, Point<T>>(qtd, Point());  
12    } else // Concorrentes  
13    {  
14        auto x = (-r.c * s.b + s.c * r.b) / det;  
15        auto y = (-s.c * r.a + r.c * s.a) / det;  
16  
17        return std::pair<int, Point<T>>(1, Point<T>(x, y));  
18    }
```

Angle between lines

```
1 template<typename T>  
2 double angle(const Point<T>& P, const Point<T>& Q,  
3             const Point<T>& R, const Point<T>& S)  
4 {  
5     auto ux = P.x - Q.x;  
6     auto uy = P.y - Q.y;  
7  
8     auto vx = R.x - S.x;  
9     auto vy = R.y - S.y;  
10  
11     auto num = ux * vx + uy * vy;  
12     auto den = hypot(ux, uy) * hypot(vx, vy);  
13     return acos(num / den);  
14 }
```

Distance to point

```
1 #include <cmath>  
2 #include <iostream>  
3  
4 template<typename T>  
5 struct Point {  
6     T x, y;  
7 };  
8  
9 template<typename T>  
10 struct Line {  
11     T a, b, c;  
12  
13     double distance(const Point<T>& p) const  
14     {  
15         return fabs(a*p.x + b*p.y + c)/hypot(a, b);  
16     }  
17  
18     Point<T> closest(const Point<T>& p) const  
19     {  
20         auto den = (a*a + b*b);  
21  
22         auto x = (b*(b*p.x - a*p.y) - a*c)/den;  
23         auto y = (a*(-b*p.x + a*p.y) - b*c)/den;
```

```
25         return Point<T> { x, y };
26     }
27 };
28
29 int main()
30 {
31     Point<double> P { 1.0, 4.0 };
32     Line<double> r { 1.0, -1.0, 0 };
33
34     std::cout << "Distance: " << r.distance(P) << '\n';
35
36     auto Q = r.closest(P);
37
38     std::cout << "Closest: Q = (" << Q.x << ", " << Q.y << ")\n";
39
40     return 0;
41 }
```

Bisector / Mediatriz

```
typename<template T>
2 Line<T> perpendicular_bisector(const Point<T>& P, const Point<T>& Q)
3 {
4     auto a = 2*(Q.x - P.x);
5     auto b = 2*(Q.y - P.y);
6     auto c = (P.x * P.x + P.y * P.y) - (Q.x * Q.x + Q.y * Q.y);
7
8     return Line<T>(a, b, c);
9 }
```

Orientation between point and line

```
typedef pair<long long, long long> ii;
2
3 // D = 0: R lies on line PQ
4 // D > 0: R is to the left of line PQ
5 // D < 0: R is to the right of line PQ
6 long long D(const ii &a, const ii &b, const ii &c) {
7     return (a.first * b.second + a.second * c.first + b.first * c.second)
8           - (c.first * b.second + c.second * a.first + b.first * a.second);
9 }
```

Line segments

Contains point

```
1 template<typename T>
2 bool contains(const Point<T>& A, const Point<T>& B, const Point<T>& P)
3 {
4     if (P == A || P == B)
5         return true;
6
7     auto xmin = min(A.x, B.x);
8     auto xmax = max(A.x, B.x);
9     auto ymin = min(A.y, B.y);
10    auto ymax = max(A.y, B.y);
```

```
11 |  
12 |     if (P.x < xmin || P.x > xmax || P.y < ymin || P.y > ymax)  
13 |         return false;  
  
14 |  
15 |     return equals((P.y - A.y)*(B.x - A.x), (P.x - A.x)*(B.y - A.y));  
16 | }
```

Closest point

```
1 | template<typename T>  
2 | struct Segment {  
3 |     Point<T> A, B;  
  
4 |  
5 |     bool contains(const Point<T>& P) const  
6 |     {  
7 |         if (equals(A.x, B.x))  
8 |             return min(A.y, B.y) <= P.y and P.y <= max(A.y, B.y);  
9 |         else  
10 |             return min(A.x, B.x) <= P.x and P.x <= max(A.x, B.x);  
11 |     }  
  
12 |  
13 |     Point<T> closest(const Point<T>& P)  
14 |     {  
15 |         Line<T> r(A, B);  
16 |         auto Q = r.closest(P);  
  
17 |         if (this->contains(Q))  
18 |             return Q;  
  
19 |         auto distA = P.distanceTo(A);  
20 |         auto distB = P.distanceTo(B);  
  
21 |         if (distA <= distB)  
22 |             return A;  
23 |         else  
24 |             return B;  
25 |     }  
26 | }  
27 |  
28 |  
29 | }
```

Intersectin with segment

```
1 | template<typename T>  
2 | class Segment {  
3 | public:  
4 |     Point<T> A, B;  
  
5 |  
6 |     bool intersect(const Segment& s) const  
7 |     {  
8 |         auto d1 = D(A, B, s.A);  
9 |         auto d2 = D(A, B, s.B);  
  
10 |         if ((equals(d1, 0) && contains(s.A)) ||  
11 |             (equals(d2, 0) && contains(s.B)))  
12 |             return true;  
  
13 |         auto d3 = D(s.A, s.B, A);  
14 |         auto d4 = D(s.A, s.B, B);  
  
15 |         if ((equals(d3, 0) && s.contains(A)) ||  
16 |             (equals(d4, 0) && s.contains(B)))  
17 |             return true;  
18 |     }  
19 | }
```



```
        (equals(d4, 0) && s.contains(B)))
20        return true;
22        return (d1 * d2 < 0) && (d3 * d4 < 0);
24    }
```

Vectors

Angle between vector and X-axis

Returns an angle in radians in the interval $[-\pi, +\pi]$. A positive angle means in the COUNTER-clockwise direction. A negative angle is measured in the clockwise direction. Note that the atan2 swaped the parameters.

```
1 inline double angle(double x, double y) {
    return atan2(y, x);
3 }
```

Listing 5: angle between X-axis and vectorx, y

Translation

```
1 Point translate(const Point& P, double dx, double dy)
{
3     return Point { P.x + dx, P.y + dy };
}
```

Listing 6: Translate point

Rotation around origin

```
Point rotate(const Point& P, double angle)
2 {
    auto x = cos(angle) * P.x - sin(angle) * P.y;
    auto y = sin(angle) * P.x + cos(angle) * P.y;
4
    return Point { x, y };
6
}
```

Rotation around another point

```
1 Point rotate(const Point& P, double angle, const Point& C)
{
3     auto Q = translate(P, -C.x, -C.y);
    Q = rotate(Q, angle);
    Q = translate(Q, C.x, C.y);
5
7     return Q;
}
```

Rotation around origin 3D

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}, \quad R_y = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$
$$R_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scale

```
1 Point scale(double sx, double sy)
2 {
3     return Point(sx * P.x, sy * P.y);
4 }
```

Listing 7: Scale vector by a factor of sx and sy

Normalization

```
1 Vector normalize(const Vector& v)
2 {
3     auto len = v.length();
4     auto u = Vector(v.x / len, v.y / len);
5
6     return u;
7 }
```

Listing 8: Returns a unit vector with the same direction as the given vector

Dot product

$$\langle \vec{u}, \vec{v} \rangle = \vec{u} \cdot \vec{v} = u_x v_x + u_y v_y = |\vec{u}| |\vec{v}| \cos \theta$$

```
1 double dot_product(const Vector& u, const Vector& v)
2 {
3     return u.x * v.x + u.y * v.y;
4 }
```

Angle between vectors

```
double angle(const Vector& u, const Vector& v)
2 {
    auto lu = u.length();
    auto lv = v.length();
    auto prod = dot_product(u, v);
    return acos(prod/(lu * lv));
8 }
```

Cross product

$$u \times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

- $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta$
- where $\vec{i}, \vec{j}, \vec{k}$ are unity vectors on the same direction and orientation as x, y, z , respectively
- the result vector \vec{w} is orthogonal to both \vec{u} and \vec{v}
- it is the area of the parallelogram formed by \vec{u} and \vec{v}

```
Vector cross_product(const Vector& u, const Vector& v)
2 {
    auto x = u.y*v.z - v.y*u.z;
    auto y = u.z*v.x - u.x*v.z;
    auto z = u.x*v.y - u.y*v.x;
    return Vector(x, y, z);
8 }
```

Circles

Definition

```
template<typename T>
2 struct Circle {
    Point<T> C;
    T r;
4 };
};
```

Perimeter, Area

```
template<typename T>
2 struct Circle
{
    double perimeter() const
    {
        return 2.0 * PI * r;
    }
8
    double area() const
10 {
```

```

    return PI * r * r;
12 }

14 double arc(double theta) const
15 {
16     return theta * r;
17 }

18 double chord(double theta) const
19 {
20     return 2 * r * sin(theta/2);
21 }
22 }

24 double sector(double theta) const
25 {
26     return (theta * r * r)/2;
27 }

28 double segment(double a) const
29 {
30     auto c = chord(a);
31     auto s = (r + r + c)/2.0;
32     auto T = sqrt(s*(s - r)*(s - r)*(s - c));
33
34     return sector(a) - T;
35 }
36 }
37 };
38

```

From 2 points

```

#include <optional>
2
template<typename T>
4 struct Circle {
    static std::optional<Circle>
6     from_2_points_and_r(const Point<T>& P, const Point<T>& Q, T r)
    {
8         double d2 = (P.x - Q.x) * (P.x - Q.x) + (P.y - Q.y) * (P.y - Q.y);
9         double det = r * r / d2 - 0.25;
10
11         if (det < 0.0)
12             return { };
13
14         double h = sqrt(det);
15
16         auto x = (P.x + Q.x) * 0.5 + (P.y - Q.y) * h;
17         auto y = (P.y + Q.y) * 0.5 + (Q.x - P.x) * h;
18
19         return Circle { Point(x, y), r };
20     }
21 }

```

From 3 points

```

#include <optional>
2
template<typename T>
4 struct Circle {

```

```

static std::optional<Circle>
6 from_3_points(const Point<T>& P, const Point<T>& Q, const Point<T>& R)
{
8     auto a = 2*(Q.x - P.x);
    auto b = 2*(Q.y - P.y);
10    auto c = 2*(R.x - P.x);
    auto d = 2*(R.y - P.y);

12    auto det = a*d - b*c;
14    if (equals(det, 0))
        return { };

16    auto k1 = (Q.x*Q.x + Q.y*Q.y) - (P.x*P.x + P.y*P.y);
18    auto k2 = (R.x*R.x + R.y*R.y) - (P.x*P.x + P.y*P.y);

20    auto cx = (k1*d - k2*b)/det;
    auto cy = (a*k2 - c*k1)/det;

22    Point<T> C { cx, cy };
24    auto r = distance(P, C);

26    return Circle<T>(C, r);
28 }
};

```

Intersection between 2 circles

```

1 #include <variant>
#include <vector>

3 const int oo { 2000000000 };

5 template<typename T> std::variant<int, std::vector<Point<T>>>
7 intersection(const Circle<T>& c1, const Circle<T>& c2)
{
9     double d = distance(c1.C, c2.C);

11    if (d > c1.r + c2.r or d < fabs(c1.r - c2.r))
        return 0;

13    if (equals(d, 0.0) and equals(c1.r, c2.r))
        return oo;

15    auto a = (c1.r * c1.r - c2.r * c2.r + d * d)/(2 * d);
    auto h = sqrt(c1.r * c1.r - a * a);

17    auto x = c1.C.x + (a/d)*(c2.C.x - c1.C.x);
    auto y = c1.C.y + (a/d)*(c2.C.y - c1.C.y);

19    auto P = Point<T> { x, y };

21    x = P.x + (h/d)*(c2.C.y - c1.C.y);
    y = P.y - (h/d)*(c2.C.x - c1.C.x);

23    auto P1 = Point<T> { x, y };

25    x = P.x - (h/d)*(c2.C.y - c1.C.y);
    y = P.y + (h/d)*(c2.C.x - c1.C.x);

27    auto P2 = Point<T> { x, y };

29    x = P.x - (h/d)*(c2.C.y - c1.C.y);
    y = P.y + (h/d)*(c2.C.x - c1.C.x);

31    auto P2 = Point<T> { x, y };

33

```

```
35     return std::vector<Point<T>> { P1, P2 };  
}
```

Intersection between circle and line

```
template<typename T> std::vector<Point<T>>  
2 intersection(const Circle<T>& c, const Point<T>& P, const Point<T>& Q)  
{  
4     auto a = pow(Q.x - P.x, 2.0) + pow(Q.y - P.y, 2.0);  
    auto b = 2*((Q.x - P.x) * (P.x - c.C.x) + (Q.y - P.y) * (P.y - c.C.y));  
6     auto d = pow(c.C.x, 2.0) + pow(c.C.y, 2.0) + pow(P.x, 2.0)  
        + pow(P.y, 2.0) + 2*(c.C.x * P.x + c.C.y * P.y);  
8     auto D = b * b - 4 * a * d;  
  
10    if (D < 0)  
        return { };  
12    else if (equals(D, 0))  
    {  
14        auto u = -b/(2*a);  
        auto x = P.x + u*(Q.x - P.x);  
16        auto y = P.y + u*(Q.y - P.y);  
        return { Point { x, y } };  
18    }  
  
20    auto u = (-b + sqrt(D))/(2*a);  
  
22    auto x = P.x + u*(Q.x - P.x);  
    auto y = P.y + u*(Q.y - P.y);  
24  
    auto P1 = Point { x, y };  
26  
    u = (-b - sqrt(D))/(2*a);  
28  
    x = P.x + u*(Q.x - P.x);  
30    y = P.y + u*(Q.y - P.y);  
32  
    auto P2 = Point { x, y };  
34  
    return { P1, P2 };  
}
```

Intersection between circle and point

```
1 template<typename T>  
struct Circle {  
3     Point<T> C;  
    T r;  
5  
    enum { IN, ON, OUT } PointPosition;  
7  
    PointPosition position(const Point& P) const  
9    {  
        auto d = dist(P, C);  
11  
        return equals(d, r) ? ON : (d < r ? IN : OUT);  
13    }  
};
```

Triangles

Perimeter

```
1 template<typename T>
2 struct Triangle {
3     Point<T> A, B, C;
4
5     double perimeter() const
6     {
7         auto a = dist(A, B);
8         auto b = dist(B, C);
9         auto c = dist(C, A);
10
11         return a + b + c;
12     }
13 };
```

Area

```
1 // Definição das estruturas Point e Line
2
3 template<typename T>
4 struct Triangle {
5     Point<T> A, B, C;
6
7     double area() const
8     {
9         Line<T> r(A, B);
10
11         auto b = dist(A, B);
12         auto h = r.distance(C);
13
14         return (b * h)/2;
15     }
16 };
```

```
1 // Definição da estrutura Point
2
3 template<typename T>
4 struct Triangle {
5     Point<T> A, B, C;
6
7     double area() const
8     {
9         auto a = dist(A, B);
10        auto b = dist(B, C);
11        auto c = dist(C, A);
12
13        auto s = (a + b + c)/2
14
15        return sqrt(s)*sqrt(s - a)*sqrt(s - b)*sqrt(s - c);
16    }
17 };
```

```
1 // Definição da estrutura Point
2
3 template<typename T>
```

```
struct Triangle {  
5     Point<T> A, B, C;  
  
7     double area() const  
    {  
9         double det = (A.x*B.y + A.y*C.x + B.x*C.y)  
                - (C.x*B.y + C.y*A.x + B.x*A.y);  
11  
        return 0.5 * fabs(det);  
13    }  
};
```

Side classification

By sides

```
template<typename T>  
2 struct Triangle {  
    Point<T> A, B, C;  
  
4     enum Sides { EQUILATERAL, ISOSCELES, SCALENE };  
  
6     Sides classification_by_sides() const  
8     {  
        auto a = dist(A, B);  
10       auto b = dist(B, C);  
        auto c = dist(C, A);  
  
12       if (equals(a, b) and equals(b, c))  
14         return EQUILATERAL;  
  
16       if (equals(a, b) or equals(a, c) or equals(b, c))  
18         return ISOSCELES;  
  
        return SCALENE;  
20    }  
};
```

By angles

```
1 // Defini o da classe Point, da fun o de compara o equals() e  
  // da fun o de dist ncia entre pontos dist()  
3  
template<typename T>  
5 struct Triangle {  
    Point<T> A, B, C;  
  
7     enum Angles { RIGHT, ACUTE, OBTUSE };  
  
9     Angles classification_by_angles() const  
11    {  
        auto a = dist(A, B);  
13       auto b = dist(B, C);  
        auto c = dist(C, A);  
  
15       auto alpha = acos((a*a - b*b - c*c)/(-2*b*c));  
17       auto beta = acos((b*b - a*a - c*c)/(-2*a*c));  
        auto gamma = acos((c*c - a*a - b*b)/(-2*a*b));  
19    }
```



```
        auto right = PI / 2.0;

21
        if (equals(alpha, right) || equals(beta, right)
23            || equals(gamma, right))
            return RIGHT;

25
        if (alpha > right || beta > right || gamma > right)
27            return OBTUSE;

29        return ACUTE;
    }
31};
```

Important points

Barycenter

```
1 // Definição da estrutura Point

3 template<typename T>
struct Triangle {
5     Point<T> A, B, C;

7     Point<T> barycenter() const
    {
9         auto x = (A.x + B.x + C.x) / 3.0;
        auto y = (A.y + B.y + C.y) / 3.0;

11        return Point<T> { x, y };

13    }
};
```

Incenter

```
template<typename T>
2 struct Triangle {
    Point<T> A, B, C;

4
    // Definição dos métodos area() e perimeter()

6
    double inradius() const
8    {
        return (2 * area()) / perimeter();

10    }

12    Point<T> incenter() const
    {
14        auto P = perimeter();
        auto x = (a*A.x + b*B.x + c*C.x)/P;
16        auto y = (a*A.y + b*B.y + c*C.y)/P;

18        return { x, y };

20    }
};
```

Orthocenter

```

1 #include <iostream>
2
3 using namespace std;
4
5 template<typename T>
6 struct Point {
7     T x, y;
8 };
9
10 template<typename T>
11 struct Line {
12     T a, b, c;
13
14     Line(T av, T bv, T cv) : a(av), b(bv), c(cv) {}
15
16     Line(const Point<T>& P, const Point<T>& Q)
17         : a(P.y - Q.y), b(Q.x - P.x), c(P.x * Q.y - Q.x * P.y)
18     {
19     }
20 };
21
22 template<typename T>
23 struct Triangle {
24     Point<T> A, B, C;
25
26     Point<T> orthocenter() const
27     {
28         Line<T> r(A, B), s(A, C);
29
30         Line<T> u { r.b, -r.a, -(C.x*r.b - C.y*r.a) };
31         Line<T> v { s.b, -s.a, -(B.x*s.b - B.y*s.a) };
32
33         auto det = u.a * v.b - u.b * v.a;
34         auto x = (-u.c * v.b + v.c * u.b) / det;
35         auto y = (-v.c * u.a + u.c * v.a) / det;
36
37         return { x, y };
38     }
39 };
40
41 int main()
42 {
43     Point<double> A { 0, 0 }, B { 3, 6 }, C { 9, 1 };
44     Triangle<double> T { A, B, C };
45
46     auto O = T.orthocenter();
47
48     cout << "(" << O.x << ", " << O.y << ")\n";
49
50     return 0;
51 }

```

Circumcircle

```

1 // Definição da estrutura Point e da função de distância
2 // entre pontos dist()
3
4 template<typename T>
5 struct Triangle {
6     Point<T> A, B, C;

```

```
7 // Definição do método area()
9
11 double circumradius() const
12 {
13     auto a = dist(B, C);
14     auto b = dist(A, C);
15     auto c = dist(A, B);
16
17     return (a * b * c) / (4 * area());
18 }
19
20 Point<T> circumcenter() const
21 {
22     auto D = 2 * (A.x * (B.y - C.y) + B.x * (C.y - A.y) + C.x * (A.y - B.y));
23
24     auto A2 = A.x * A.x + A.y * A.y;
25     auto B2 = B.x * B.x + B.y * B.y;
26     auto C2 = C.x * C.x + C.y * C.y;
27
28     auto x = (A2 * (B.y - C.y) + B2 * (C.y - A.y) + C2 * (A.y - B.y)) / D;
29     auto y = (A2 * (C.x - B.x) + B2 * (A.x - C.x) + C2 * (B.x - A.x)) / D;
30
31     return { x, y };
32 };
```

Quadrilaterals

Area

Trapezium

```
template<typename T>
2 struct Trapezium {
3     T b, B, h;
4
5     T area() const
6     {
7         return (b + B) * h / 2;
8     }
9 };
```

Quadrilateral

```
1 // Definição das estruturas Point e Line
2
3 template<typename T>
4 struct Triangle {
5     Point<T> A, B, C;
6
7     double area() const
8     {
9         Line<T> r(A, B);
10
11         auto b = dist(A, B);
12         auto h = r.distance(C);
13     }
```

```
        return (b * h) / 2;
15    }
};
```

Rectangles

From 2 points

```
// Definição da estrutura Point
2
template<typename T>
4 class Rectangle {
public:
6     Point<T> P, Q;
    T b, h;

8     Rectangle(const Point<T>& p, const Point<T>& q) : P(p), Q(q)
10    {
        b = max(P.x, Q.x) - min(P.x, Q.x);
12        h = max(P.y, Q.y) - min(P.y, Q.y);
    }

14    Rectangle(const T& base, const T& height)
16        : P(0, 0), Q(base, height), b(base), h(height) {}
};
```

Intersection between rectangles

```
1 // Definição da classe Point
template<typename T>
3 struct Rectangle {
    // Membros e construtores
5
    Rectangle intersection(const Rectangle& r) const
7    {
        using interval = pair<T, T>;
9
        auto I = interval(min(P.x, Q.x), max(P.x, Q.x));
11       auto U = interval(min(r.P.x, r.Q.x), max(r.P.x, r.Q.x));

13       auto a = max(I.first, U.first);
        auto b = min(I.second, U.second);
15
17       if (b < a)
            return { {-1, -1}, {-1, -1} };

19       I = interval(min(P.y, Q.y), max(P.y, Q.y));
        U = interval(min(r.P.y, r.Q.y), max(r.P.y, r.Q.y));
21
        auto c = max(I.first, U.first);
23       auto d = min(I.second, U.second);

25       if (d < c)
            return { {-1, -1}, {-1, -1} };
27
        inter = Rectangle(Point(a, c), Point(b, d));
29
        return { {a, c}, {b, d} };
31    }
```

```
};
```

Polygons

Definition

```
1 #include <bits/stdc++.h>
3 using namespace std;
5 template<typename T>
6 struct Point { T x, y; };
7
8 template<typename T>
9 class Polygon {
10 private:
11     vector<Point<T>> vs;
12     int n;
13
14 public:
15     // O par metro deve conter os n v r tices do pol gono
16     Polygon(const vector<Point<T>>& ps) : vs(ps), n(vs.size())
17     {
18         vs.push_back(vs.front());
19     }
20
21 private:
22     T D(const Point<T>& P, const Point<T>& Q, const Point<T>& R) const
23     {
24         return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
25             (R.x * Q.y + R.y * P.x + Q.x * P.y);
26     }
27
28 public:
29     bool convex() const {
30         // Um pol gono deve ter, no minimo, 3 v r tices
31         if (n < 3) return false;
32
33         int P = 0, N = 0, Z = 0;
34
35         for (int i = 0; i < n; ++i) {
36             auto d = D(vs[i], vs[(i + 1) % n], vs[(i + 2) % n]);
37             d ? (d > 0 ? ++P : ++N) : ++Z;
38         }
39
40         return not ((P and N) or (P == 0 and N == 0));
41     }
42
43 private:
44     double distance(const Point<T>&P, const Point<T>& Q)
45     {
46         return hypot(P.x - Q.x, P.y - Q.y);
47     }
48
49 public:
50     double perimeter() const
51     {
52         auto p = 0.0;
53
54         for (int i = 0; i < n; ++i)
```

```

        p += distance(vs[i], vs[i + 1]);
55
    }
57
    return p;
59
double area() const
{
61
    auto a = 0.0;

63
    for (int i = 0; i < n; ++i)
    {
65
        a += vs[i].x * vs[i + 1].y;
        a -= vs[i + 1].x * vs[i].y;
67
    }

69
    return 0.5 * fabs(a);
    }
71
private:
73
    // ngulo APB, em radianos
    double angle(const Point<T>& P, const Point<T>& A, const Point<T>& B)
75
    {
        auto ux = P.x - A.x;
77
        auto uy = P.y - A.y;

79
        auto vx = P.x - B.x;
        auto vy = P.y - B.y;

81
        auto num = ux * vx + uy * vy;
83
        auto den = hypot(ux, uy) * hypot(vx, vy);

85
        // Caso especial: se den == 0, algum dos vetores degenerado: os
        // dois pontos s o iguais. Neste caso, o ngulo n o est definido
87

89
        return acos(num / den);
    }

91
    bool equals(double x, double y)
    {
93
        static const double EPS { 1e-9 };

95
        return fabs(x - y) < EPS;
    }
97
public:
99
    bool contains(const Point<T>& P) const
    {
101
        if (n < 3)
            return false;

103

        auto sum = 0.0;

105

        for (int i = 0; i < n; ++i)
        {
107
            auto d = D(P, vs[i], vs[i + 1]);

109

            // Pontos sobre as arestas ou v r tices s o considerados
            // interiores
111
            if (equals(d, 0) and AB_contains(P))
                return true;
113

            auto a = angle(P, vs[i], vs[i + 1]);
115

```

```

117         sum += d > 0 ? a : -a;
118     }
119
120     static const double PI = acos(-1.0);
121
122     return equals(fabs(sum), 2*PI);
123 }
124
125 private:
126     // Interse o entre a reta AB e o segmento de reta PQ
127     Point<T> intersection(const Point<T>& P, const Point<T>& Q,
128                          const Point<T>& A, const Point<T>& B)
129     {
130         auto a = B.y - A.y;
131         auto b = A.x - B.x;
132         auto c = B.x * A.y - A.x * B.y;
133         auto u = fabs(a * P.x + b * P.y + c);
134         auto v = fabs(a * Q.x + b * Q.y + c);
135
136         // M dia ponderada pelas dist ncias de P e Q at a reta AB
137         return {(P.x * v + Q.x * u)/(u + v), (P.y * v + Q.y * u)/(u + v)};
138     }
139
140 public:
141     // Corta o pol gono com a reta r que passa por A e B
142     Polygon cut_polygon(const Point<T>& A, const Point<T>& B) const
143     {
144         vector<Point<T>> points;
145         const double EPS { 1e-9 };
146
147         for (int i = 0; i < n; ++i)
148         {
149             auto d1 = D(A, B, vs[i]);
150             auto d2 = D(A, B, vs[i + 1]);
151
152             // V rtice esquerda da reta
153             if (d1 > -EPS)
154                 points.push_back(vs[i]);
155
156             // A aresta cruza a reta
157             if (d1 * d2 < -EPS)
158                 points.push_back(intersection(vs[i], vs[i + 1], A, B));
159         }
160
161         return Polygon(points);
162     }
163
164     double circumradius() const
165     {
166         auto s = distance(vs[0], vs[1]);
167         const double PI { acos(-1.0) };
168
169         return (s/2.0)*(1.0/sin(PI/n));
170     }
171
172     double apothem() const
173     {
174         auto s = distance(vs[0], vs[1]);
175         const double PI { acos(-1.0) };
176
177         return (s/2.0)*(1.0/tan(PI/n));

```

```

    }
179 };

181 int main()
182 {
183     vector<Point<int>> xs { { 0, 0 }, { 2, 1 }, { 3, 4 }, { 5, 2 }, { 4, 0 } };
185     vector<Point<int>> ys { { 6, 1 }, { 9, 3 }, { 9, 1 }, { 6, 3 } };
187     vector<Point<double>> zs { { 0, 0 }, { 1, 0 }, { 0.5, 0.5 }, { 1, 1 }, { 0, 1 }, {
188         0.5, 0.5 } };

189     Polygon<int> A(xs), B(ys);
190     Polygon<double> C(zs);

191     cout << "A is convex? " << A.convex() << '\n';
192     cout << "B is convex? " << B.convex() << '\n';
193     cout << "C is convex? " << C.convex() << '\n';

195     cout << C.area() << '\n';

197     return 0;
198 }

```

UVA 11265

```

#include <bits/stdc++.h>
2
using namespace std;
4
struct Point {
6     double x, y;

8     Point(double xv = 0, double yv = 0) : x(xv), y(yv) {}

10     double distance(const Point& P) const
11     {
12         return hypot(x - P.x, y - P.y);
13     }

14     bool operator==(const Point& P) const
15     {
16         const double EPS { 1e-6 };
17         return fabs(x - P.x) < EPS and fabs(y - P.y) < EPS;
18     }
19 };

21 struct Polygon {
22     vector<Point> vs;
23     int n;

24     Polygon(const vector<Point>& vs) : vs(vs), n(vs.size())
25     {
26         vs.push_back(vs[0]);
27     }

28     double area() const {
29         double a = 0;

30         for (int i = 0; i < n; ++i)
31         {

```



```

36         a += vs[i].x * vs[i+1].y;
37         a -= vs[i+1].x * vs[i].y;
38     }
39
40     return 0.5 * fabs(a);
41 }
42 };
43
44 Point intersection(const Point& P, const Point& Q,
45                  const Point& A, const Point& B)
46 {
47     auto a = B.y - A.y;
48     auto b = A.x - B.x;
49     auto c = B.x * A.y - A.x * B.y;
50     auto u = fabs(a * P.x + b * P.y + c);
51     auto v = fabs(a * Q.x + b * Q.y + c);
52
53     return Point((P.x*v + Q.x*u)/(u + v), (P.y*v + Q.y*u)/(u + v));
54 }
55
56 double D(const Point& P, const Point& Q, const Point& R)
57 {
58     return (P.x * Q.y + P.y * R.x + Q.x * R.y)
59         - (R.x * Q.y + R.y * P.x + Q.x * P.y);
60 }
61
62 Polygon cut_polygon(const Polygon& P, const Point& A, const Point& B)
63 {
64     vector<Point> points;
65
66     for (int i = 0; i < P.n; ++i)
67     {
68         auto d1 = D(A, B, P.vs[i]);
69         auto d2 = D(A, B, P.vs[i + 1]);
70
71         if (d1 > -EPS)
72             points.push_back(P.vs[i]);
73
74         if (d1 * d2 < -EPS)
75             points.push_back(intersection(P.vs[i], P.vs[i+1], A, B));
76     }
77
78     return Polygon(points);
79 }
80
81 int main() {
82     int N, W, H, x, y, test = 0;
83
84     while (cin >> N >> W >> x >> y) {
85         Polygon p({ Point(0, 0), Point(W, 0), Point(W, H), Point(0, H) });
86         Point F(x, y);
87
88         while (N--) {
89             Point Q, R;
90             cin >> Q.x >> Q.y >> R.x >> R.y;
91
92             if (D(Q, R, F) > 0)
93                 p = cut_polygon(p, Q, R);
94             else
95                 p = cut_polygon(p, R, Q);
96         }

```

```
98         printf("Case #%d: %.3f\n", ++test, p.area());
99     }
100     return 0;
101 }
```

Codeforces 1C

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 const double PI { acos(-1.0) };
6 const int MAX { 110 };
7
8 double angles[MAX];
9
10 struct Point {
11     double x, y;
12
13     double distance(const Point& P) const
14     {
15         return hypot(x - P.x, y - P.y);
16     }
17
18     Point translate(const Point& P) const
19     {
20         return Point { x + P.x, y + P.y };
21     }
22
23     Point rotate(double angle) const
24     {
25         auto xv = x*cos(angle) - y*sin(angle);
26         auto yv = x*sin(angle) + y*cos(angle);
27
28         return Point { xv, yv };
29     }
30
31     bool operator==(const Point& P) const
32     {
33         const double EPS { 1e-5 };
34
35         return fabs(x - P.x) < EPS and fabs(y - P.y) < EPS;
36     }
37 };
38
39 struct Triangle {
40     Point A, B, C;
41
42     double area() const
43     {
44         auto a = A.distance(B);
45         auto b = B.distance(C);
46         auto c = C.distance(A);
47         auto s = (a + b + c) / 2;
48
49         return sqrt(s*(s - a)*(s - b)*(s - c));
50     }
51
52     double circumradius() const
53     {
```

```

55     auto a = A.distance(B);
56     auto b = B.distance(C);
57     auto c = C.distance(A);
58
59     return (a * b * c)/(4 * area());
60 }
61
62 Point circumcenter() const
63 {
64     auto d = 2*(A.x*(B.y - C.y) + B.x*(C.y - A.y) + C.x*(A.y - B.y));
65
66     auto A2 = A.x*A.x + A.y*A.y;
67     auto B2 = B.x*B.x + B.y*B.y;
68     auto C2 = C.x*C.x + C.y*C.y;
69
70     auto x = (A2*(B.y - C.y) + B2*(C.y - A.y) + C2*(A.y - B.y))/d;
71     auto y = (A2*(C.x - B.x) + B2*(A.x - C.x) + C2*(B.x - A.x))/d;
72
73     return Point { x, y };
74 }
75 };
76
77 void precomp()
78 {
79     for (int i = 1; i < MAX; ++i)
80         angles[i] = (2.0*PI)/i;
81 }
82
83 int sides(const Point& P, const Point& Q, const Point& R)
84 {
85     for (int i = 3; i < 100; ++i)
86     {
87         auto angle = angles[i];
88         int match = 0;
89         Point S { P };
90
91         for (int j = 0; j < i; ++j)
92         {
93             if (Q == S)
94                 ++match;
95
96             if (R == S)
97                 ++match;
98
99             S = S.rotate(angle);
100         }
101
102         if (match == 2)
103             return i;
104     }
105
106     return 100;
107 }
108
109 int main()
110 {
111     precomp();
112
113     Point P, Q, R;
114
115     cin >> P.x >> P.y >> Q.x >> Q.y >> R.x >> R.y;

```

```
Triangle t { P, Q, R };

117
    auto r = t.circumradius();
119    auto C = t.circumcenter();

121    P = P.translate(Point { -C.x, -C.y } );
    Q = Q.translate(Point { -C.x, -C.y } );
123    R = R.translate(Point { -C.x, -C.y } );

125    int min_sides = sides(P, Q, R);

127    auto area = (r * r * min_sides*sin(angles[min_sides]))/2.0;

129    cout.precision(6);
    cout << fixed << area << '\n';
131
    return 0;
133 }
```

URI 2202

```
1 #include <bits/stdc++.h>

3 using namespace std;
using ll = long long;

5
struct Point {
7     ll x, y;

9     bool operator<(const Point& P) const
    {
11         return x == P.x ? y < P.y : x < P.x;
    }
13 };

15 struct Line {
    ll a, b, c;

17     Line(const Point& p, const Point& q)
    {
19         a = p.y - q.y;
21         b = q.x - p.x;
        c = p.x * q.y - p.y * q.x;
23     }

25     double distanceTo(const Point& P) const
    {
27         auto num = a*P.x + b*P.y + c;
        auto den = sqrt(a*a + b*b);
29
        return fabs(num/den);
31     }
};

33 ll D(const Point& P, const Point& Q, const Point& R)
35 {
    return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
37         (R.x * Q.y + R.y * P.x + Q.x * P.y);
}

39
```

```

// Andrew monotonic chain
41 vector<Point> convex_hull(vector<Point>& P)
{
43     sort(P.begin(), P.end());

45     vector<Point> L, U;

47     for (auto p : P)
    {
49         while (L.size() >= 2 and D(L[L.size() - 2], L[L.size() - 1], p) < 0)
            L.pop_back();

51         L.push_back(p);

53     }

55     reverse(P.begin(), P.end());

57     for (auto p : P)
    {
59         while (U.size() >= 2 and D(U[U.size() - 2], U[U.size() - 1], p) < 0)
            U.pop_back();

61         U.push_back(p);

63     }

65     L.pop_back();
    U.pop_back();

67     L.reserve(L.size() + U.size());
    L.insert(L.end(), U.begin(), U.end());

71     return L;
}

73 int main()
75 {
    ios::sync_with_stdio(false);

77     int n, test = 0;

79     while (cin >> n, n)
    {
81         vector<Point> ps(n);

83         for (int i = 0; i < n; ++i)
            cin >> ps[i].x >> ps[i].y;

87         auto ch = convex_hull(ps);
        ch.push_back(ch.front());
89 cout << "convex hull (size = " << ch.size() << ", n = " << n << ") = ";
        for (const auto& v : ch)
91             cout << "(" << v.x << ", " << v.y << ") ";
        cout << '\n';
93         auto min_dist = 1000000000.0;

95         for (int i = 0; i < n; ++i)
        {
97             auto A = ch[i];
            auto B = ch[i + 1];
99             auto max_dist = 0.0;

101             Line r(A, B);

```

```
103 cout << "A = (" << A.x << ", " << A.y << ")\n";
    cout << "B = (" << B.x << ", " << B.y << ")\n";
        for (int j = 0; j < n; ++j)
        {
            auto d = r.distanceTo(ch[j]);
107 cout << "dist = " << d << "\n";
            cout << "P = (" << ch[j].x << ", " << ch[j].y << ")\n";
109             max_dist = max(max_dist, d);
        }
111 cout << "max dist = " << max_dist << '\n';
            min_dist = min(min_dist, max_dist);
113 cout << "\n";
        }
115
            cout << "Case " << ++test << ": ";
117            cout.precision(2);
            cout << fixed << min_dist << '\n';
119        }

121    return 0;
}
```

Convex Hull

Graham

```
#include <bits/stdc++.h>
2
using namespace std;
4
const double EPS { 1e-6 };
6
template<typename T>
8 bool equals(T a, T b)
{
10     if (std::is_floating_point<T>::value)
        return fabs(a - b) < EPS;
12     else
        return a == b;
14 }

16 template<typename T>
struct Point
18 {
    T x, y;
20
    double distance(const Point& P) const
22     {
        return hypot(x - P.x, y - P.y);
24     }
};

26
template<typename T>
28 T D(const Point<T>& P, const Point<T>& Q, const Point<T>& R)
{
30     return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
        (R.x * Q.y + R.y * P.x + Q.x * P.y);
32 }
```

```

34 template<typename T>
35 class GrahamScan
36 {
37 private:
38     static Point<T> pivot(vector<Point<T>>& P)
39     {
40         size_t idx = 0;
41
42         for (size_t i = 1; i < P.size(); ++i)
43             if (P[i].y < P[idx].y or
44                 (equals(P[i].y, P[idx].y) and P[i].x > P[idx].x))
45                 idx = i;
46
47         swap(P[0], P[idx]);
48
49         return P[0];
50     }
51
52     static void sort_by_angle(vector<Point<T>>& P)
53     {
54         auto P0 = pivot(P);
55
56         sort(P.begin() + 1, P.end(),
57             [&](const Point<T>& A, const Point<T>& B) {
58                 // pontos colineares: escolhe-se o mais prximo do piv
59                 if (equals(D(P0, A), D(P0, B)))
60                     return A.distance(P0) < B.distance(P0);
61
62                 auto alfa = atan2(A.y - P0.y, A.x - P0.x);
63                 auto beta = atan2(B.y - P0.y, B.x - P0.x);
64
65                 return alfa < beta;
66             });
67     }
68 }
69
70 public:
71     static vector<Point<T>> convex_hull(const vector<Point<T>>& points)
72     {
73         vector<Point<T>> P(points);
74         auto N = P.size();
75
76         // Corner case: com 3 v r tices ou menos, P o pr prio convex hull
77         if (N <= 3)
78             return P;
79
80         sort_by_angle(P);
81
82         vector<Point<T>> ch;
83         ch.push_back(P[N - 1]);
84         ch.push_back(P[0]);
85         ch.push_back(P[1]);
86
87         size_t i = 2;
88
89         while (i < N)
90         {
91             auto j = ch.size() - 1;
92
93             if (D(ch[j - 1], ch[j], P[i]) > 0)
94                 ch.push_back(P[i++]);
95             else

```

```

96         ch.pop_back();
97     }
98     // O envoltório é um caminho fechado: o primeiro ponto é igual
100    // ao último
101    return ch;
102 }
103 };
104
105 int main()
106 {
107     vector<Point<int>> P { { 0, 0 }, { 5, 3 }, { 8, -2 }, { 4, 4 }, { 2, 1 }, { 2, 5 },
108                          { 3, -1 },
109                          { 7, 2 }, { 5, 0 }, { 0, 4 }, { 1, -1 }, { 7, -2 }, { 6, 4 }, { 6, 0 }, { 1, 3 }
110                      };
111
112     auto ch = GrahamScan<int>::convex_hull(P);
113
114     for (size_t i = 0; i < ch.size(); ++i)
115         cout << i + 1 << ": (" << ch[i].x << ", " << ch[i].y << ")\n";
116
117     return 0;
118 }
```