

ACM/ICPC: Competitive Programming Notebook

Lucas Mattioli, Marlon Mendes, Simião Carvalho

Contents

Points	4
Comparing floating point values	4
Lines	4
General equation of a line	4
General equation of a line normalized	4
Point on a line	5
Equal and parallel lines	5
Orthogonal	5
Intersection	5
Angle between lines	6
Distance to point	6
Bisector / Mediatriz	7
Orientation between point and line	7
Line segments	7
Contains point	7
Closest point	8
Intersectin with segment	8
Vectors	9
Angle between vector and X-axis	9
Translation	9
Rotation around origin	9
Rotation around another point	9
Rotation around origin 3D	10
Scale	10
Normalization	10
Dot product	10
Angle between vectors	11
Cross product	11
Circles	11
Definition	11
Perimeter, Area	11
From 2 points	12
From 3 points	12
Intersection between 2 circles	13
Intersection between circle and line	14
Intersection between circle and point	14
Triangles	15
Perimeter	15
Area	15
Side classification	16
By sides	16
By angles	16
Important points	17

Barycenter	17
Incenter	17
Orthocenter	17
Circumcircle	18
Quadrilaterals	19
Area	19
Trapezium	19
Quadrilateral	19
Rectangles	20
From 2 points	20
Intersection between rectangles	20
Polygons	21
Definition	21
UVA 11265	24
Codeforces 1C	26
URI 2202	28
Convex Hull	30
Monotone Chain	30
Example: Codeforces 166B	31

Points

Comparing floating point values

Returns true if double values a and b are equal

```
1  const double EPS { 1e-9 };
2  bool equals(double a, double b)
3  {
4      return fabs(a - b) < EPS;
5  }
```

Listing 1: equals

Lines

General equation of a line

Non-normalized form: $ax + by + c = 0$

```
1  class Line {
2  public:
3      double a;
4      double b;
5      double c;
6
7      Line(double av, double bv, double cv) : a(av), b(bv), c(cv) {}
8
9      Line(const Point& p, const Point& q)
10     {
11         a = p.y - q.y;
12         b = q.x - p.x;
13         c = p.x * q.y - p.y * q.x;
14     }
15 };
```

Listing 2: General equation of a line

General equation of a line normalized

```
1  class Line {
2  public:
3      double a;
4      double b;
5      double c;
6
7      Line(double av, double bv, double cv) : a(av), b(bv), c(cv) {}
8
9      Line(const Point& p, const Point& q)
10     {
11         a = p.y - q.y;
12         b = q.x - p.x;
13         c = p.x * q.y - p.y * q.x;
14
15         auto k = a ? a : b;
16
17         a /= k;
18         b /= k;
```

```
19         c /= k;  
20     }  
21 };
```

Listing 3: General equation of a line

Point on a line

Is the given point located on the given Line?

```
1 template<typename T>  
2 struct Line {  
3     bool contains(const Point<T>& P) const  
4     {  
5         return equals(a*P.x + b*P.y + c, 0);  
6     }  
7 };
```

Listing 4: Point on line

Equal and parallel lines

```
1 template<typename T>  
2 struct Line {  
3     bool operator==(const Line& r) const  
4     {  
5         auto k = a ? a : b;  
6         auto s = r.a ? r.a : r.b;  
7  
8         return equals(a*s, r.a*k) && equals(b*s, r.b*k)  
9             && equals(c*s, r.c*k);  
10    }  
11  
12    bool parallel(const Line& r) const  
13    {  
14        auto det = a*r.b - b*r.a;  
15        return det == 0 and !(*this == r);  
16    }  
17 };
```

Orthogonal

```
1 template<typename T>  
2 struct Line  
3 {  
4     bool orthogonal(const Line& r) const  
5     {  
6         return equals(a * r.a + b * r.b, 0);  
7     }  
8 };
```

Intersection

```
1 const int INF { -1 };  
2 template<typename T>  
3 std::pair<int, Point<T>> intersections(const Line<T>& r, const Line<T>& s)
```

```
4 {  
6     auto det = r.a * s.b - r.b * s.a;  
8     if (equals(det, 0)) // Coincidentes ou paralelas  
9     {  
10        int qtd = (r == s) ? INF : 0;  
11        return std::pair<int, Point<T>>(qtd, Point());  
12    } else // Concorrentes  
13    {  
14        auto x = (-r.c * s.b + s.c * r.b) / det;  
15        auto y = (-s.c * r.a + r.c * s.a) / det;  
16  
17        return std::pair<int, Point<T>>(1, Point<T>(x, y));  
18    }
```

Angle between lines

```
1 template<typename T>  
2 double angle(const Point<T>& P, const Point<T>& Q,  
3             const Point<T>& R, const Point<T>& S)  
4 {  
5     auto ux = P.x - Q.x;  
6     auto uy = P.y - Q.y;  
7  
8     auto vx = R.x - S.x;  
9     auto vy = R.y - S.y;  
10  
11     auto num = ux * vx + uy * vy;  
12     auto den = hypot(ux, uy) * hypot(vx, vy);  
13     return acos(num / den);  
14 }
```

Distance to point

```
1 #include <cmath>  
2 #include <iostream>  
3  
4 template<typename T>  
5 struct Point {  
6     T x, y;  
7 };  
8  
9 template<typename T>  
10 struct Line {  
11     T a, b, c;  
12  
13     double distance(const Point<T>& p) const  
14     {  
15         return fabs(a*p.x + b*p.y + c)/hypot(a, b);  
16     }  
17  
18     Point<T> closest(const Point<T>& p) const  
19     {  
20         auto den = (a*a + b*b);  
21  
22         auto x = (b*(b*p.x - a*p.y) - a*c)/den;  
23         auto y = (a*(-b*p.x + a*p.y) - b*c)/den;
```

```
25         return Point<T> { x, y };
26     }
27 };
28
29 int main()
30 {
31     Point<double> P { 1.0, 4.0 };
32     Line<double> r { 1.0, -1.0, 0 };
33
34     std::cout << "Distance: " << r.distance(P) << '\n';
35
36     auto Q = r.closest(P);
37
38     std::cout << "Closest: Q = (" << Q.x << ", " << Q.y << ")\n";
39
40     return 0;
41 }
```

Bisector / Mediatriz

```
typename<template T>
2 Line<T> perpendicular_bisector(const Point<T>& P, const Point<T>& Q)
3 {
4     auto a = 2*(Q.x - P.x);
5     auto b = 2*(Q.y - P.y);
6     auto c = (P.x * P.x + P.y * P.y) - (Q.x * Q.x + Q.y * Q.y);
7
8     return Line<T>(a, b, c);
9 }
```

Orientation between point and line

```
typedef pair<long long, long long> ii;
2
3 // D = 0: R lies on line PQ
4 // D > 0: R is to the left of line PQ
5 // D < 0: R is to the right of line PQ
6 long long D(const ii &a, const ii &b, const ii &c) {
7     return (a.first * b.second + a.second * c.first + b.first * c.second)
8         - (c.first * b.second + c.second * a.first + b.first * a.second);
9 }
```

Line segments

Contains point

```
1 template<typename T>
2 bool contains(const Point<T>& A, const Point<T>& B, const Point<T>& P)
3 {
4     if (P == A || P == B)
5         return true;
6
7     auto xmin = min(A.x, B.x);
8     auto xmax = max(A.x, B.x);
9     auto ymin = min(A.y, B.y);
10    auto ymax = max(A.y, B.y);
```

```
11 |  
12 |     if (P.x < xmin || P.x > xmax || P.y < ymin || P.y > ymax)  
13 |         return false;  
  
14 |  
15 |     return equals((P.y - A.y)*(B.x - A.x), (P.x - A.x)*(B.y - A.y));  
16 | }
```

Closest point

```
1 | template<typename T>  
2 | struct Segment {  
3 |     Point<T> A, B;  
  
4 |  
5 |     bool contains(const Point<T>& P) const  
6 |     {  
7 |         if (equals(A.x, B.x))  
8 |             return min(A.y, B.y) <= P.y and P.y <= max(A.y, B.y);  
9 |         else  
10 |             return min(A.x, B.x) <= P.x and P.x <= max(A.x, B.x);  
11 |     }  
  
12 |  
13 |     Point<T> closest(const Point<T>& P)  
14 |     {  
15 |         Line<T> r(A, B);  
16 |         auto Q = r.closest(P);  
  
17 |         if (this->contains(Q))  
18 |             return Q;  
  
19 |         auto distA = P.distanceTo(A);  
20 |         auto distB = P.distanceTo(B);  
  
21 |         if (distA <= distB)  
22 |             return A;  
23 |         else  
24 |             return B;  
25 |     }  
26 | }  
27 |  
28 |  
29 | }
```

Intersectin with segment

```
1 | template<typename T>  
2 | class Segment {  
3 | public:  
4 |     Point<T> A, B;  
  
5 |  
6 |     bool intersect(const Segment& s) const  
7 |     {  
8 |         auto d1 = D(A, B, s.A);  
9 |         auto d2 = D(A, B, s.B);  
  
10 |         if ((equals(d1, 0) && contains(s.A)) ||  
11 |             (equals(d2, 0) && contains(s.B)))  
12 |             return true;  
  
13 |         auto d3 = D(s.A, s.B, A);  
14 |         auto d4 = D(s.A, s.B, B);  
  
15 |         if ((equals(d3, 0) && s.contains(A)) ||  
16 |             (equals(d4, 0) && s.contains(B)))  
17 |             return true;  
18 |     }  
19 | }
```



```
        (equals(d4, 0) && s.contains(B)))  
20        return true;  
  
22        return (d1 * d2 < 0) && (d3 * d4 < 0);  
24    }
```

Vectors

Angle between vector and X-axis

Returns an angle in radians in the interval $[-\pi, +\pi]$. A positive angle means in the COUNTER-clockwise direction. A negative angle is measured in the clockwise direction. Note that the atan2 swaped the parameters.

```
1 inline double angle(double x, double y) {  
    return atan2(y, x);  
3 }
```

Listing 5: angle between X-axis and vectorx, y

Translation

```
1 Point translate(const Point& P, double dx, double dy)  
{  
3     return Point { P.x + dx, P.y + dy };  
}
```

Listing 6: Translate point

Rotation around origin

```
Point rotate(const Point& P, double angle)  
2 {  
    auto x = cos(angle) * P.x - sin(angle) * P.y;  
4    auto y = sin(angle) * P.x + cos(angle) * P.y;  
  
6    return Point { x, y };  
}
```

Rotation around another point

```
1 Point rotate(const Point& P, double angle, const Point& C)  
{  
3     auto Q = translate(P, -C.x, -C.y);  
    Q = rotate(Q, angle);  
5     Q = translate(Q, C.x, C.y);  
  
7     return Q;  
}
```

Rotation around origin 3D

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}, \quad R_y = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$
$$R_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scale

```
1 Point scale(double sx, double sy)
2 {
3     return Point(sx * P.x, sy * P.y);
4 }
```

Listing 7: Scale vector by a factor of sx and sy

Normalization

```
1 Vector normalize(const Vector& v)
2 {
3     auto len = v.length();
4     auto u = Vector(v.x / len, v.y / len);
5
6     return u;
7 }
```

Listing 8: Returns a unit vector with the same direction as the given vector

Dot product

$$\langle \vec{u}, \vec{v} \rangle = \vec{u} \cdot \vec{v} = u_x v_x + u_y v_y = |\vec{u}| |\vec{v}| \cos \theta$$

```
1 double dot_product(const Vector& u, const Vector& v)
2 {
3     return u.x * v.x + u.y * v.y;
4 }
```

Angle between vectors

```
double angle(const Vector& u, const Vector& v)
2 {
    auto lu = u.length();
    auto lv = v.length();
    auto prod = dot_product(u, v);
    return acos(prod/(lu * lv));
8 }
```

Cross product

$$u \times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

- $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta$
- where $\vec{i}, \vec{j}, \vec{k}$ are unity vectors on the same direction and orientation as x, y, z , respectively
- the result vector \vec{w} is orthogonal to both \vec{u} and \vec{v}
- it is the area of the parallelogram formed by \vec{u} and \vec{v}

```
Vector cross_product(const Vector& u, const Vector& v)
2 {
    auto x = u.y*v.z - v.y*u.z;
    auto y = u.z*v.x - u.x*v.z;
    auto z = u.x*v.y - u.y*v.x;
    return Vector(x, y, z);
8 }
```

Circles

Definition

```
template<typename T>
2 struct Circle {
    Point<T> C;
    T r;
4 };
};
```

Perimeter, Area

```
template<typename T>
2 struct Circle
{
    double perimeter() const
    {
        return 2.0 * PI * r;
    }
8
    double area() const
10 {
```

```
        return PI * r * r;
12    }

14    double arc(double theta) const
    {
16        return theta * r;
    }

18    double chord(double theta) const
    {
20        return 2 * r * sin(theta/2);
22    }

24    double sector(double theta) const
    {
26        return (theta * r * r)/2;
    }

28    double segment(double a) const
    {
30        auto c = chord(a);
32        auto s = (r + r + c)/2.0;
34        auto T = sqrt(s*(s - r)*(s - r)*(s - c));

        return sector(a) - T;
36    }
38};
```

From 2 points

```
#include <optional>
2
template<typename T>
4 struct Circle {
    static std::optional<Circle>
6    from_2_points_and_r(const Point<T>& P, const Point<T>& Q, T r)
    {
8        double d2 = (P.x - Q.x) * (P.x - Q.x) + (P.y - Q.y) * (P.y - Q.y);
        double det = r * r / d2 - 0.25;

10
        if (det < 0.0)
12            return { };

14        double h = sqrt(det);

16        auto x = (P.x + Q.x) * 0.5 + (P.y - Q.y) * h;
        auto y = (P.y + Q.y) * 0.5 + (Q.x - P.x) * h;

18        return Circle { Point(x, y), r };
20    }
}
```

From 3 points

```
#include <optional>
2
template<typename T>
4 struct Circle {
```

```

6      static std::optional<Circle>
      from_3_points(const Point<T>& P, const Point<T>& Q, const Point<T>& R)
      {
8          auto a = 2*(Q.x - P.x);
          auto b = 2*(Q.y - P.y);
10         auto c = 2*(R.x - P.x);
          auto d = 2*(R.y - P.y);

12         auto det = a*d - b*c;
14         if (equals(det, 0))
             return { };

16         auto k1 = (Q.x*Q.x + Q.y*Q.y) - (P.x*P.x + P.y*P.y);
18         auto k2 = (R.x*R.x + R.y*R.y) - (P.x*P.x + P.y*P.y);

20         auto cx = (k1*d - k2*b)/det;
          auto cy = (a*k2 - c*k1)/det;

22         Point<T> C { cx, cy };
24         auto r = distance(P, C);

26         return Circle<T>(C, r);
      }
28 };

```

Intersection between 2 circles

```

1  #include <variant>
   #include <vector>
3
   const int oo { 2000000000 };
5
   template<typename T> std::variant<int, std::vector<Point<T>>>
7   intersection(const Circle<T>& c1, const Circle<T>& c2)
   {
9       double d = distance(c1.C, c2.C);

11      if (d > c1.r + c2.r or d < fabs(c1.r - c2.r))
          return 0;

13      if (equals(d, 0.0) and equals(c1.r, c2.r))
          return oo;

15      auto a = (c1.r * c1.r - c2.r * c2.r + d * d)/(2 * d);
          auto h = sqrt(c1.r * c1.r - a * a);

17      auto x = c1.C.x + (a/d)*(c2.C.x - c1.C.x);
          auto y = c1.C.y + (a/d)*(c2.C.y - c1.C.y);

19      auto P = Point<T> { x, y };

21      x = P.x + (h/d)*(c2.C.y - c1.C.y);
          y = P.y - (h/d)*(c2.C.x - c1.C.x);

23      auto P1 = Point<T> { x, y };

25      x = P.x - (h/d)*(c2.C.y - c1.C.y);
          y = P.y + (h/d)*(c2.C.x - c1.C.x);

27      auto P2 = Point<T> { x, y };

29      x = P.x + (h/d)*(c2.C.y - c1.C.y);
          y = P.y - (h/d)*(c2.C.x - c1.C.x);

31      auto P2 = Point<T> { x, y };

33

```

```
35     return std::vector<Point<T>> { P1, P2 };  
}
```

Intersection between circle and line

```
template<typename T> std::vector<Point<T>>  
2 intersection(const Circle<T>& c, const Point<T>& P, const Point<T>& Q)  
{  
4     auto a = pow(Q.x - P.x, 2.0) + pow(Q.y - P.y, 2.0);  
    auto b = 2*((Q.x - P.x) * (P.x - c.C.x) + (Q.y - P.y) * (P.y - c.C.y));  
6     auto d = pow(c.C.x, 2.0) + pow(c.C.y, 2.0) + pow(P.x, 2.0)  
        + pow(P.y, 2.0) + 2*(c.C.x * P.x + c.C.y * P.y);  
8     auto D = b * b - 4 * a * d;  
  
10    if (D < 0)  
        return { };  
12    else if (equals(D, 0))  
    {  
14        auto u = -b/(2*a);  
        auto x = P.x + u*(Q.x - P.x);  
16        auto y = P.y + u*(Q.y - P.y);  
        return { Point { x, y } };  
18    }  
  
20    auto u = (-b + sqrt(D))/(2*a);  
  
22    auto x = P.x + u*(Q.x - P.x);  
    auto y = P.y + u*(Q.y - P.y);  
24  
    auto P1 = Point { x, y };  
26  
    u = (-b - sqrt(D))/(2*a);  
28  
    x = P.x + u*(Q.x - P.x);  
30    y = P.y + u*(Q.y - P.y);  
32  
    auto P2 = Point { x, y };  
34  
    return { P1, P2 };  
}
```

Intersection between circle and point

```
1 template<typename T>  
struct Circle {  
3     Point<T> C;  
    T r;  
5  
    enum { IN, ON, OUT } PointPosition;  
7  
    PointPosition position(const Point& P) const  
9    {  
        auto d = dist(P, C);  
11  
        return equals(d, r) ? ON : (d < r ? IN : OUT);  
13    }  
};
```

Triangles

Perimeter

```

1 template<typename T>
2 struct Triangle {
3     Point<T> A, B, C;
4
5     double perimeter() const
6     {
7         auto a = dist(A, B);
8         auto b = dist(B, C);
9         auto c = dist(C, A);
10
11         return a + b + c;
12     }
13 };

```

Area

```

1 // Definição das estruturas Point e Line
2
3 template<typename T>
4 struct Triangle {
5     Point<T> A, B, C;
6
7     double area() const
8     {
9         Line<T> r(A, B);
10
11         auto b = dist(A, B);
12         auto h = r.distance(C);
13
14         return (b * h)/2;
15     }
16 };

```

```

1 // Definição da estrutura Point
2
3 template<typename T>
4 struct Triangle {
5     Point<T> A, B, C;
6
7     double area() const
8     {
9         auto a = dist(A, B);
10        auto b = dist(B, C);
11        auto c = dist(C, A);
12
13        auto s = (a + b + c)/2
14
15        return sqrt(s)*sqrt(s - a)*sqrt(s - b)*sqrt(s - c);
16    }
17 };

```

```

1 // Definição da estrutura Point
2
3 template<typename T>

```

```
struct Triangle {
5     Point<T> A, B, C;

7     double area() const
    {
9         double det = (A.x*B.y + A.y*C.x + B.x*C.y)
                        - (C.x*B.y + C.y*A.x + B.x*A.y);

11        return 0.5 * fabs(det);
13    }
};
```

Side classification

By sides

```
template<typename T>
2 struct Triangle {
    Point<T> A, B, C;

4     enum Sides { EQUILATERAL, ISOSCELES, SCALENE };

6     Sides classification_by_sides() const
    {
8         auto a = dist(A, B);
        auto b = dist(B, C);
        auto c = dist(C, A);

12        if (equals(a, b) and equals(b, c))
            return EQUILATERAL;

14        if (equals(a, b) or equals(a, c) or equals(b, c))
            return ISOSCELES;

16        return SCALENE;
18    }
20 };
```

By angles

```
1 // Defini o da classe Point, da fun o de compara o equals() e
  // da fun o de dist ncia entre pontos dist()
3
4 template<typename T>
5 struct Triangle {
    Point<T> A, B, C;

7     enum Angles { RIGHT, ACUTE, OBTUSE };

9     Angles classification_by_angles() const
    {
11        auto a = dist(A, B);
        auto b = dist(B, C);
        auto c = dist(C, A);

13        auto alpha = acos((a*a - b*b - c*c)/(-2*b*c));
        auto beta = acos((b*b - a*a - c*c)/(-2*a*c));
        auto gamma = acos((c*c - a*a - b*b)/(-2*a*b));
15
17
19    }
```



```
        auto right = PI / 2.0;

21
        if (equals(alpha, right) || equals(beta, right)
23            || equals(gamma, right))
            return RIGHT;

25
        if (alpha > right || beta > right || gamma > right)
27            return OBTUSE;

29        return ACUTE;
    }
31};
```

Important points

Barycenter

```
1 // Definição da estrutura Point

3 template<typename T>
struct Triangle {
5     Point<T> A, B, C;

7     Point<T> barycenter() const
    {
9         auto x = (A.x + B.x + C.x) / 3.0;
        auto y = (A.y + B.y + C.y) / 3.0;

11        return Point<T> { x, y };

13    }
};
```

Incenter

```
template<typename T>
2 struct Triangle {
    Point<T> A, B, C;

4
    // Definição dos métodos area() e perimeter()

6
    double inradius() const
8    {
        return (2 * area()) / perimeter();

10    }

12    Point<T> incenter() const
    {
14        auto P = perimeter();
        auto x = (a*A.x + b*B.x + c*C.x)/P;
16        auto y = (a*A.y + b*B.y + c*C.y)/P;

18        return { x, y };

20    }
};
```

Orthocenter

```

1 #include <iostream>
2
3 using namespace std;
4
5 template<typename T>
6 struct Point {
7     T x, y;
8 };
9
10 template<typename T>
11 struct Line {
12     T a, b, c;
13
14     Line(T av, T bv, T cv) : a(av), b(bv), c(cv) {}
15
16     Line(const Point<T>& P, const Point<T>& Q)
17         : a(P.y - Q.y), b(Q.x - P.x), c(P.x * Q.y - Q.x * P.y)
18     {
19     }
20 };
21
22 template<typename T>
23 struct Triangle {
24     Point<T> A, B, C;
25
26     Point<T> orthocenter() const
27     {
28         Line<T> r(A, B), s(A, C);
29
30         Line<T> u { r.b, -r.a, -(C.x*r.b - C.y*r.a) };
31         Line<T> v { s.b, -s.a, -(B.x*s.b - B.y*s.a) };
32
33         auto det = u.a * v.b - u.b * v.a;
34         auto x = (-u.c * v.b + v.c * u.b) / det;
35         auto y = (-v.c * u.a + u.c * v.a) / det;
36
37         return { x, y };
38     }
39 };
40
41 int main()
42 {
43     Point<double> A { 0, 0 }, B { 3, 6 }, C { 9, 1 };
44     Triangle<double> T { A, B, C };
45
46     auto O = T.orthocenter();
47
48     cout << "(" << O.x << ", " << O.y << ")\n";
49
50     return 0;
51 }

```

Circumcircle

```

1 // Definição da estrutura Point e da função de distância
2 // entre pontos dist()
3
4 template<typename T>
5 struct Triangle {
6     Point<T> A, B, C;

```

```
7 // Definição do método area()
9
11 double circumradius() const
12 {
13     auto a = dist(B, C);
14     auto b = dist(A, C);
15     auto c = dist(A, B);
16
17     return (a * b * c) / (4 * area());
18 }
19
20 Point<T> circumcenter() const
21 {
22     auto D = 2 * (A.x * (B.y - C.y) + B.x * (C.y - A.y) + C.x * (A.y - B.y));
23
24     auto A2 = A.x * A.x + A.y * A.y;
25     auto B2 = B.x * B.x + B.y * B.y;
26     auto C2 = C.x * C.x + C.y * C.y;
27
28     auto x = (A2 * (B.y - C.y) + B2 * (C.y - A.y) + C2 * (A.y - B.y)) / D;
29     auto y = (A2 * (C.x - B.x) + B2 * (A.x - C.x) + C2 * (B.x - A.x)) / D;
30
31     return { x, y };
32 };
```

Quadrilaterals

Area

Trapezium

```
template<typename T>
2 struct Trapezium {
3     T b, B, h;
4
5     T area() const
6     {
7         return (b + B) * h / 2;
8     }
9 };
```

Quadrilateral

```
1 // Definição das estruturas Point e Line
2
3 template<typename T>
4 struct Triangle {
5     Point<T> A, B, C;
6
7     double area() const
8     {
9         Line<T> r(A, B);
10
11         auto b = dist(A, B);
12         auto h = r.distance(C);
13     }
```

```
        return (b * h) / 2;
15    }
};
```

Rectangles

From 2 points

```
// Definição da estrutura Point
2
template<typename T>
4 class Rectangle {
public:
6     Point<T> P, Q;
    T b, h;

8     Rectangle(const Point<T>& p, const Point<T>& q) : P(p), Q(q)
10    {
        b = max(P.x, Q.x) - min(P.x, Q.x);
12        h = max(P.y, Q.y) - min(P.y, Q.y);
    }

14     Rectangle(const T& base, const T& height)
16         : P(0, 0), Q(base, height), b(base), h(height) {}
};
```

Intersection between rectangles

```
1 // Definição da classe Point
template<typename T>
3 struct Rectangle {
    // Membros e construtores
5
6     Rectangle intersection(const Rectangle& r) const
7     {
8         using interval = pair<T, T>;
9
10        auto I = interval(min(P.x, Q.x), max(P.x, Q.x));
11        auto U = interval(min(r.P.x, r.Q.x), max(r.P.x, r.Q.x));
12
13        auto a = max(I.first, U.first);
14        auto b = min(I.second, U.second);
15
16        if (b < a)
17            return { {-1, -1}, {-1, -1} };
18
19        I = interval(min(P.y, Q.y), max(P.y, Q.y));
20        U = interval(min(r.P.y, r.Q.y), max(r.P.y, r.Q.y));
21
22        auto c = max(I.first, U.first);
23        auto d = min(I.second, U.second);
24
25        if (d < c)
26            return { {-1, -1}, {-1, -1} };
27
28        inter = Rectangle(Point(a, c), Point(b, d));
29
30        return { {a, c}, {b, d} };
31    }
};
```

```
};
```

Polygons

Definition

```
1 #include <bits/stdc++.h>
3 using namespace std;
5 template<typename T>
6 struct Point { T x, y; };
7
8 template<typename T>
9 class Polygon {
10 private:
11     vector<Point<T>> vs;
12     int n;
13
14 public:
15     // O par metro deve conter os n v r tices do pol gono
16     Polygon(const vector<Point<T>>& ps) : vs(ps), n(vs.size())
17     {
18         vs.push_back(vs.front());
19     }
20
21 private:
22     T D(const Point<T>& P, const Point<T>& Q, const Point<T>& R) const
23     {
24         return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
25                (R.x * Q.y + R.y * P.x + Q.x * P.y);
26     }
27
28 public:
29     bool convex() const {
30         // Um pol gono deve ter, no minimo, 3 v r tices
31         if (n < 3) return false;
32
33         int P = 0, N = 0, Z = 0;
34
35         for (int i = 0; i < n; ++i) {
36             auto d = D(vs[i], vs[(i + 1) % n], vs[(i + 2) % n]);
37             d ? (d > 0 ? ++P : ++N) : ++Z;
38         }
39
40         return not ((P and N) or (P == 0 and N == 0));
41     }
42
43 private:
44     double distance(const Point<T>&P, const Point<T>& Q)
45     {
46         return hypot(P.x - Q.x, P.y - Q.y);
47     }
48
49 public:
50     double perimeter() const
51     {
52         auto p = 0.0;
53
54         for (int i = 0; i < n; ++i)
```

```

        p += distance(vs[i], vs[i + 1]);
55
    }
57
    return p;
59
double area() const
{
61
    auto a = 0.0;

63
    for (int i = 0; i < n; ++i)
    {
65
        a += vs[i].x * vs[i + 1].y;
        a -= vs[i + 1].x * vs[i].y;
67
    }

69
    return 0.5 * fabs(a);
    }
71
private:
73
    // ngulo APB, em radianos
    double angle(const Point<T>& P, const Point<T>& A, const Point<T>& B)
75
    {
        auto ux = P.x - A.x;
77
        auto uy = P.y - A.y;

79
        auto vx = P.x - B.x;
        auto vy = P.y - B.y;

81
        auto num = ux * vx + uy * vy;
83
        auto den = hypot(ux, uy) * hypot(vx, vy);

85
        // Caso especial: se den == 0, algum dos vetores degenerado: os
        // dois pontos s o iguais. Neste caso, o ngulo n o est definido
87

89
        return acos(num / den);
    }

91
    bool equals(double x, double y)
    {
93
        static const double EPS { 1e-9 };

95
        return fabs(x - y) < EPS;
    }
97
public:
99
    bool contains(const Point<T>& P) const
    {
101
        if (n < 3)
            return false;

103

        auto sum = 0.0;

105
        for (int i = 0; i < n; ++i)
        {
107
            auto d = D(P, vs[i], vs[i + 1]);

109
            // Pontos sobre as arestas ou v r tices s o considerados
            // interiores
            if (equals(d, 0) and AB_contains(P))
111
                return true;

113

            auto a = angle(P, vs[i], vs[i + 1]);
115

```

```

117         sum += d > 0 ? a : -a;
118     }
119
120     static const double PI = acos(-1.0);
121
122     return equals(fabs(sum), 2*PI);
123 }
124
125 private:
126     // Interseção entre a reta AB e o segmento de reta PQ
127     Point<T> intersection(const Point<T>& P, const Point<T>& Q,
128                          const Point<T>& A, const Point<T>& B)
129     {
130         auto a = B.y - A.y;
131         auto b = A.x - B.x;
132         auto c = B.x * A.y - A.x * B.y;
133         auto u = fabs(a * P.x + b * P.y + c);
134         auto v = fabs(a * Q.x + b * Q.y + c);
135
136         // Média ponderada pelas distâncias de P e Q à reta AB
137         return {(P.x * v + Q.x * u)/(u + v), (P.y * v + Q.y * u)/(u + v)};
138     }
139
140 public:
141     // Corta o polígono com a reta r que passa por A e B
142     Polygon cut_polygon(const Point<T>& A, const Point<T>& B) const
143     {
144         vector<Point<T>> points;
145         const double EPS { 1e-9 };
146
147         for (int i = 0; i < n; ++i)
148         {
149             auto d1 = D(A, B, vs[i]);
150             auto d2 = D(A, B, vs[i + 1]);
151
152             // Vertice esquerda da reta
153             if (d1 > -EPS)
154                 points.push_back(vs[i]);
155
156             // A aresta cruza a reta
157             if (d1 * d2 < -EPS)
158                 points.push_back(intersection(vs[i], vs[i + 1], A, B));
159         }
160
161         return Polygon(points);
162     }
163
164     double circumradius() const
165     {
166         auto s = distance(vs[0], vs[1]);
167         const double PI { acos(-1.0) };
168
169         return (s/2.0)*(1.0/sin(PI/n));
170     }
171
172     double apothem() const
173     {
174         auto s = distance(vs[0], vs[1]);
175         const double PI { acos(-1.0) };
176
177         return (s/2.0)*(1.0/tan(PI/n));

```

```

    }
179 };

181 int main()
182 {
183     vector<Point<int>> xs { { 0, 0 }, { 2, 1 }, { 3, 4 }, { 5, 2 }, { 4, 0 } };
185     vector<Point<int>> ys { { 6, 1 }, { 9, 3 }, { 9, 1 }, { 6, 3 } };
186     vector<Point<double>> zs { { 0, 0 }, { 1, 0 }, { 0.5, 0.5 }, { 1, 1 }, { 0, 1 }, {
187         0.5, 0.5 } };

188     Polygon<int> A(xs), B(ys);
189     Polygon<double> C(zs);

191     cout << "A is convex? " << A.convex() << '\n';
192     cout << "B is convex? " << B.convex() << '\n';
193     cout << "C is convex? " << C.convex() << '\n';

195     cout << C.area() << '\n';

197     return 0;
198 }

```

UVA 11265

```

#include <bits/stdc++.h>
2
using namespace std;
4
struct Point {
6     double x, y;

8     Point(double xv = 0, double yv = 0) : x(xv), y(yv) {}

10     double distance(const Point& P) const
11     {
12         return hypot(x - P.x, y - P.y);
13     }

14     bool operator==(const Point& P) const
15     {
16         const double EPS { 1e-6 };
17         return fabs(x - P.x) < EPS and fabs(y - P.y) < EPS;
18     }
19 };

21 struct Polygon {
22     vector<Point> vs;
23     int n;

24     Polygon(const vector<Point>& vs) : vs(vs), n(vs.size())
25     {
26         vs.push_back(vs[0]);
27     }

28     double area() const {
29         double a = 0;

30         for (int i = 0; i < n; ++i)
31         {

```



```

36         a += vs[i].x * vs[i+1].y;
37         a -= vs[i+1].x * vs[i].y;
38     }
39
40     return 0.5 * fabs(a);
41 }
42 };
43
44 Point intersection(const Point& P, const Point& Q,
45                  const Point& A, const Point& B)
46 {
47     auto a = B.y - A.y;
48     auto b = A.x - B.x;
49     auto c = B.x * A.y - A.x * B.y;
50     auto u = fabs(a * P.x + b * P.y + c);
51     auto v = fabs(a * Q.x + b * Q.y + c);
52
53     return Point((P.x*v + Q.x*u)/(u + v), (P.y*v + Q.y*u)/(u + v));
54 }
55
56 double D(const Point& P, const Point& Q, const Point& R)
57 {
58     return (P.x * Q.y + P.y * R.x + Q.x * R.y)
59         - (R.x * Q.y + R.y * P.x + Q.x * P.y);
60 }
61
62 Polygon cut_polygon(const Polygon& P, const Point& A, const Point& B)
63 {
64     vector<Point> points;
65
66     for (int i = 0; i < P.n; ++i)
67     {
68         auto d1 = D(A, B, P.vs[i]);
69         auto d2 = D(A, B, P.vs[i + 1]);
70
71         if (d1 > -EPS)
72             points.push_back(P.vs[i]);
73
74         if (d1 * d2 < -EPS)
75             points.push_back(intersection(P.vs[i], P.vs[i+1], A, B));
76     }
77
78     return Polygon(points);
79 }
80
81 int main() {
82     int N, W, H, x, y, test = 0;
83
84     while (cin >> N >> W >> x >> y) {
85         Polygon p({ Point(0, 0), Point(W, 0), Point(W, H), Point(0, H) });
86         Point F(x, y);
87
88         while (N--) {
89             Point Q, R;
90             cin >> Q.x >> Q.y >> R.x >> R.y;
91
92             if (D(Q, R, F) > 0)
93                 p = cut_polygon(p, Q, R);
94             else
95                 p = cut_polygon(p, R, Q);
96         }

```

```
98         printf("Case #%d: %.3f\n", ++test, p.area());
99     }
100     return 0;
101 }
```

Codeforces 1C

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 const double PI { acos(-1.0) };
6 const int MAX { 110 };
7
8 double angles[MAX];
9
10 struct Point {
11     double x, y;
12
13     double distance(const Point& P) const
14     {
15         return hypot(x - P.x, y - P.y);
16     }
17
18     Point translate(const Point& P) const
19     {
20         return Point { x + P.x, y + P.y };
21     }
22
23     Point rotate(double angle) const
24     {
25         auto xv = x*cos(angle) - y*sin(angle);
26         auto yv = x*sin(angle) + y*cos(angle);
27
28         return Point { xv, yv };
29     }
30
31     bool operator==(const Point& P) const
32     {
33         const double EPS { 1e-5 };
34
35         return fabs(x - P.x) < EPS and fabs(y - P.y) < EPS;
36     }
37 };
38
39 struct Triangle {
40     Point A, B, C;
41
42     double area() const
43     {
44         auto a = A.distance(B);
45         auto b = B.distance(C);
46         auto c = C.distance(A);
47         auto s = (a + b + c) / 2;
48
49         return sqrt(s*(s - a)*(s - b)*(s - c));
50     }
51
52     double circumradius() const
53     {
```

```

55     auto a = A.distance(B);
56     auto b = B.distance(C);
57     auto c = C.distance(A);
58
59     return (a * b * c)/(4 * area());
60 }
61
62 Point circumcenter() const
63 {
64     auto d = 2*(A.x*(B.y - C.y) + B.x*(C.y - A.y) + C.x*(A.y - B.y));
65
66     auto A2 = A.x*A.x + A.y*A.y;
67     auto B2 = B.x*B.x + B.y*B.y;
68     auto C2 = C.x*C.x + C.y*C.y;
69
70     auto x = (A2*(B.y - C.y) + B2*(C.y - A.y) + C2*(A.y - B.y))/d;
71     auto y = (A2*(C.x - B.x) + B2*(A.x - C.x) + C2*(B.x - A.x))/d;
72
73     return Point { x, y };
74 }
75 };
76
77 void precomp()
78 {
79     for (int i = 1; i < MAX; ++i)
80         angles[i] = (2.0*PI)/i;
81 }
82
83 int sides(const Point& P, const Point& Q, const Point& R)
84 {
85     for (int i = 3; i < 100; ++i)
86     {
87         auto angle = angles[i];
88         int match = 0;
89         Point S { P };
90
91         for (int j = 0; j < i; ++j)
92         {
93             if (Q == S)
94                 ++match;
95
96             if (R == S)
97                 ++match;
98
99             S = S.rotate(angle);
100         }
101
102         if (match == 2)
103             return i;
104     }
105
106     return 100;
107 }
108
109 int main()
110 {
111     precomp();
112
113     Point P, Q, R;
114
115     cin >> P.x >> P.y >> Q.x >> Q.y >> R.x >> R.y;

```

```
Triangle t { P, Q, R };

117
    auto r = t.circumradius();
119    auto C = t.circumcenter();

121    P = P.translate(Point { -C.x, -C.y } );
    Q = Q.translate(Point { -C.x, -C.y } );
123    R = R.translate(Point { -C.x, -C.y } );

125    int min_sides = sides(P, Q, R);

127    auto area = (r * r * min_sides*sin(angles[min_sides]))/2.0;

129    cout.precision(6);
    cout << fixed << area << '\n';
131
    return 0;
133 }
```

URI 2202

```
1 #include <bits/stdc++.h>

3 using namespace std;
using ll = long long;

5
struct Point {
7     ll x, y;

9     bool operator<(const Point& P) const
    {
11         return x == P.x ? y < P.y : x < P.x;
    }
13 };

15 struct Line {
    ll a, b, c;

17     Line(const Point& p, const Point& q)
    {
19         a = p.y - q.y;
21         b = q.x - p.x;
        c = p.x * q.y - p.y * q.x;
23     }

25     double distanceTo(const Point& P) const
    {
27         auto num = a*P.x + b*P.y + c;
        auto den = sqrt(a*a + b*b);
29
        return fabs(num/den);
31     }
};

33 ll D(const Point& P, const Point& Q, const Point& R)
35 {
    return (P.x * Q.y + P.y * R.x + Q.x * R.y) -
37         (R.x * Q.y + R.y * P.x + Q.x * P.y);
}

39
```

```

// Andrew monotonic chain
41 vector<Point> convex_hull(vector<Point>& P)
{
43     sort(P.begin(), P.end());

45     vector<Point> L, U;

47     for (auto p : P)
    {
49         while (L.size() >= 2 and D(L[L.size() - 2], L[L.size() - 1], p) < 0)
            L.pop_back();

51         L.push_back(p);

53     }

55     reverse(P.begin(), P.end());

57     for (auto p : P)
    {
59         while (U.size() >= 2 and D(U[U.size() - 2], U[U.size() - 1], p) < 0)
            U.pop_back();

61         U.push_back(p);

63     }

65     L.pop_back();
    U.pop_back();

67     L.reserve(L.size() + U.size());
69     L.insert(L.end(), U.begin(), U.end());

71     return L;
}

73 int main()
75 {
    ios::sync_with_stdio(false);

77     int n, test = 0;

79     while (cin >> n, n)
    {
81         vector<Point> ps(n);

83         for (int i = 0; i < n; ++i)
            cin >> ps[i].x >> ps[i].y;

87         auto ch = convex_hull(ps);
            ch.push_back(ch.front());
89 cout << "convex hull (size = " << ch.size() << ", n = " << n << ") = ";
        for (const auto& v : ch)
91             cout << "(" << v.x << ", " << v.y << ") ";
93 cout << '\n';
        auto min_dist = 1000000000.0;

95         for (int i = 0; i < n; ++i)
        {
97             auto A = ch[i];
            auto B = ch[i + 1];
99             auto max_dist = 0.0;

101             Line r(A, B);

```

```

103 cout << "A = (" << A.x << ", " << A.y << ")\n";
cout << "B = (" << B.x << ", " << B.y << ")\n";
    for (int j = 0; j < n; ++j)
    {
        auto d = r.distanceTo(ch[j]);
107 cout << "dist = " << d << "\n";
cout << "P = (" << ch[j].x << ", " << ch[j].y << ")\n";
109     max_dist = max(max_dist, d);
    }
111 cout << "max dist = " << max_dist << '\n';
    min_dist = min(min_dist, max_dist);
113 cout << "\n";
    }
115
    cout << "Case " << ++test << ": ";
117    cout.precision(2);
    cout << fixed << min_dist << '\n';
119 }

121 return 0;
}

```

Convex Hull

Monotone Chain

```

#include <bits/stdc++.h>
2
using namespace std;
4
typedef long long ll;
6 typedef pair<int, int> ii;

8 ll cross(const ii &O, const ii &A, const ii &B){
    return (A.first - O.first) * (B.second - O.second) - (A.second - O.second) * (B.
        first - O.first);
10 }

12 // Return the points of the convex-hull (CH) of P, in the counter-clockwise order.
// The first point of the return value is the left most (smallest x value) point of the
    CH.
14 // Each edge of the returned CH contains no 3 collinear points.
// There are no duplicated points in the return value unless the input contains
    duplicates already
16 vector<ii> convex_hull(vector<ii> P) {
    int n = P.size(),
18     k=0;
    vector<ii> H(2 * n);
20    sort(P.begin(), P.end());

22    for(int i=0; i<n; ++i){
        while(k >= 2 && cross(H[k-2], H[k-1], P[i]) <= 0) {
24            k--;
        }
        H[k++] = P[i];
26    }

28    for(int i = n - 2, t = k + 1; i >= 0; --i){
        while(k >= t && cross(H[k-2], H[k-1], P[i]) <= 0) {
30            k--;
        }
    }
}

```

```

        k--;
    }
    H[k++] = P[i];
}
H.resize(k - 1); // Avoids duplicating the first and last point of the convex hull.
                // Do H.resize(k) if you want the first and last points of the
convex hull to be the same
return H;
}

inline void print(const vector<ii> &P) {
    cout << "P.size() = " << P.size() << endl;
    cout << "P = ";
    for(auto p : P) {
        cout << "(" << p.first << ", " << p.second << ")", ";
    }
    cout << endl;
}

int main() {
    vector<ii> P({
        {1, 9}, {2, 4}, {3, 12}, {4, 2}, {4, 6}, {5, 7}, {5, 14},
        {6, 5}, {6, 9}, {6, 12}, {7, 1}, {7, 8}, {7, 10}, {8, 15},
        {9, 11}, {10, 4}, {11, 3}, {11, 13}, {12, 6}, {12, 9}, {13, 7},
    });
    sort(P.begin(), P.end());
    print(P);

    vector<ii> ch = convex_hull(P);
    print(ch);

    vector<ii> triangle({{1, 3}, {5, 6}, {7, 4}});
    ch = convex_hull(triangle);
    print(ch);
    return 0;
}

```

Example: Codeforces 166B

```

1 #include <bits/stdc++.h>

3 #define fori(i, ini, lim) for(int i = int(ini); i < int(lim); i++)
#define ford(i, ini, lim) for(int i = int(ini); i >= int(lim); i--)

5 #define debug(x) cout << "> " << #x << " = " << x << endl;
7 #define debug-at(arr, at) cout << "> " << #arr << "[" << at << "]" = " << arr[at] <<
    endl;
#define debug-pair(p) cout << "> " << #p << " = (" << p.first << ", " << p.second << ")
    << endl;

9 using namespace std;

11 typedef long long ll;
13 typedef pair<ll, ll> ii;

15 ll cross(const ii &O, const ii &A, const ii &B){
    return (A.first - O.first) * (B.second - O.second) - (A.second - O.second) * (B.
        first - O.first);
17 }

```

```

19 // Codeforces 166B (codeforces.com/problemset/problem/166/B)
// This variation accepts 3 or more collinear points on the same edge of the returned
   convex hull
21 vector<ii> convex_hull(vector<ii> P) {
   int n = P.size() ,
23     k=0;
   vector<ii> H(2 * n);
25   sort(P.begin() , P.end());

27   for(int i = 0; i < n; ++i) {
       while(k >= 2 && cross(H[k-2], H[k-1], P[i]) < 0) {
29         k--;
       }
31     H[k++] = P[i];
   }

33   for(int i = n - 2, t = k + 1; i >= 0; --i) {
       while(k >= t && cross(H[k-2], H[k-1], P[i]) < 0) {
35         k--;
       }
37     H[k++] = P[i];
39   }
   H.resize(k - 1);
41   return H;
   }
43
45
47 int main() {
   ios_base::sync_with_stdio(false);

49   int n;
   cin >> n;
51   vector<ii> A(n);
   vector<ii> all(n);
53   for(i, 0, n) {
       cin >> A[i].first >> A[i].second;
55       all[i] = A[i];
   }

57   int m;
   cin >> m;
59   vector<ii> B(m);
   for(i, 0, m) {
61       cin >> B[i].first >> B[i].second;
63       all.push_back(B[i]);
   }

65   vector<ii> ch = convex_hull(all);
   sort(ch.begin() , ch.end());
67   sort(A.begin() , A.end());
   cout << (ch == A ? "YES" : "NO") << '\n';
69
   return 0;
71 }

```