WHEN IS A PUISEUX MONOID ATOMIC?

Scott T. Chapman, Felix Gotti, **Marly Gotti**

Department of Mathematics
Sam Houston State University,Huntsville, TX 77341
scott.chapman@shsu.edu

Department of Mathematics
UC Berkeley, Berkeley, CA 94720
felixgotti@berkeley.edu

Department of Mathematics
University of Florida, Gainesville, FL 32611
marlycormar@ufl.edu

Abstract: A Puiseux monoid is an additive submonoid of the nonnegative cone of $\mathbb{Q}$. If M is a Puiseux monoid, then the question of when each nonunit element of M can be written as a sum of irreducible elements (or is *atomic*) is surprisingly difficult. For instance, although various techniques have been developed over the past few years to identify subclasses of Puiseux monoids which are atomic, no general characterization of such monoids is known. Here we discuss some of the most relevant aspects related to the atomicity of Puiseux monoids. We provide characterizations of when M is finitely generated, factorial, half-factorial, other-half-factorial, Prüfer, seminormal, root-closed, and completely integrally closed. In addition to the atomic property, precise characterizations are also not known for when M satisfies the ACCP, is a BF-monoid, or is an FF-monoid; in each of these four cases, we construct classes of Puiseux monoids satisfying these properties.

Keywords: Puiseux monoids, atomicity, factorization theory, Prüfer monoids, numerical monoids, ACCP, BF-monoids, FF-monoids.

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