Solutions to Exam in Microwave Engineering (MCC121)

Friday, January 16, 2015

MCC121

Exam in **Microwave Engineering**

Friday, January 16, 2015, 1400 - 18:00, "V-salar"

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During the exam, teacher will visit around 1430 and 1600.

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The inspection of the results can be done in my office (D615), MC2-building, Friday, January 30th, 13:00-14:30. The final results will be sent to registrar office on February 2, 2015.

This is an open book exam. The following is allowed:

- *Calculator (approved by Chalmers)*
- *"Microwave Engineering" by Pozar*
- *Mathematics handbook (Beta)*
- *Smith charts*

To pass this written examination, you need at least 24p out of 60p. Final grade of the course will also include results from assignment 1. That is: 3 (≥28p), 4 (≥42p) and 5 (≥56p).

Teamwork is not permitted on this examination. The university academic integrity policy will be strictly enforced. Failure to comply with the academic integrity policy will result in a zero for this examination.

Make sure you have understood the question before you go ahead. Write shortly but make sure your way of thinking is clearly described. It is imperative to clearly explain how the results have been obtained. Solve the problem as far as you can – constructive, creative and valuable approaches are also rewarded. Assume realistic numbers/parameters when needed if data is missing in order to solve the problem.

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Problem 1

For the principal mode (TEM), the maximum power capacity of an air-filled coaxial line occurs when the corresponding characteristic impedance is about 30 Ω, i.e. when the following condition apply *2*ln(b/a)=1* ➔ ($Z_0 = 377/(4\pi)$). But for low loss, show that minimum attenuation due to conductor loss occurs when the characteristic impedance is $Z_0 \approx$ 77 Ω. Derive the corresponding condition for outer radius, b, and inner radius, a.

(Comment: standard 50 Ω line (≈geometrical mean of 30 and 77 Ω) is a good compromise between power capacity and loss)

(10p)

<u>Solution:</u>

Impedance for an air coaxial line is: $Z_o = \eta \frac{ln(b/a)}{2\pi}$

The attenuation due to conductor loss is: $\alpha = \frac{R_s}{2\eta \times ln(b/a)} \left[\frac{1}{a} + \frac{1}{b}\right] =$

It is possible to find the local minima of the above expression, but perhaps a bit easier to find the derivative by inserting the expression for the characteristic impedance:

$$\alpha = \frac{R_s}{4\pi Z_o}\left[\frac{1}{a} + \frac{1}{b}\right] = \frac{R_s}{4\pi Z_o}\frac{1}{b}\left[e^{\frac{2\pi Z_o}{\eta}} + 1\right]$$

Keeping e.g. b constant, find the minimum with respect to characteristic impedance. i.e:

$$\frac{\partial\alpha}{\partial Z_o} = \frac{R_s}{4\pi}\frac{1}{b}\left[\frac{-1}{{Z_o}^2}\left(e^{\frac{2\pi Z_o}{\eta}} + 1\right) + \frac{1}{Z_o}\left(\frac{2\pi}{\eta}e^{\frac{2\pi Z_o}{\eta}}\right)\right] = \frac{R_s}{4\pi Z_o}\frac{1}{b}\left[\frac{2\pi}{\eta}e^{\frac{2\pi Z_o}{\eta}} - \frac{1}{Z_o}\left(e^{\frac{2\pi Z_o}{\eta}} + 1\right)\right] = 0$$

- ⇨ $\frac{2\pi}{\eta}Z_o e^{\frac{2\pi Z_o}{\eta}} = \left(e^{\frac{2\pi Z_o}{\eta}} + 1\right) \rightarrow xe^x = e^x + 1 \; where \; x = \frac{2\pi Z_o}{\eta} \rightarrow e^x(x-1) = 1 \; (solution \; for \; x \approx 1.28)$
- ⇨ $Z_o \approx 1.28\frac{\eta}{2\pi} = 1.28\frac{377}{2\pi} \cong 77\Omega$ Q.E.D.

to find the condition between inner and outer radius: we have $x = \frac{2\pi Z_o}{\eta} = \ln\left(b/a\right) \rightarrow$ $e^x(x-1) = \frac{b}{a}\left(\ln\left(b/a\right) - 1\right) = 1 \rightarrow \ln\left(b/a\right) = 1 + \frac{a}{b} \rightarrow \frac{b}{a} \approx 1/(1.28-1) \cong 3.57$

Answer: $\ln\left(b/a\right) = 1 + \frac{a}{b}$ *, with numerical solution* $b \cong 3.57a$, is the analytical condition for minimum conductor loss in an air filled coaxial line.

Problem 2

A load impedance $Z_L = 300 - j150$ ohm at f = 5GHz is to be matched to a 50-ohm line

a) Design a complete matching network using a single series short-circuited stub tuner

b) Design a complete matching network based on a quarter-wave transformer

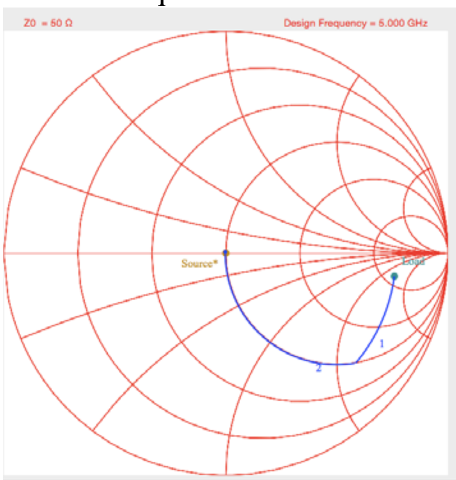
c) Discuss pros and cons of the two matching solutions above.

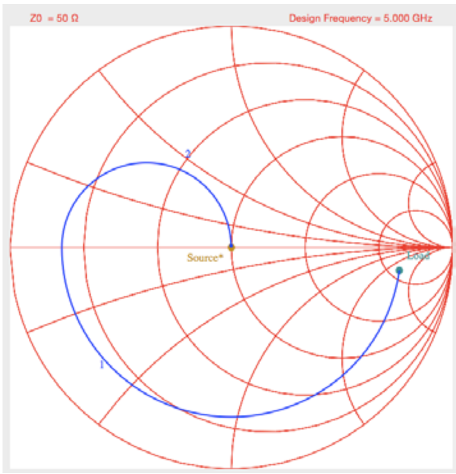
(10p)

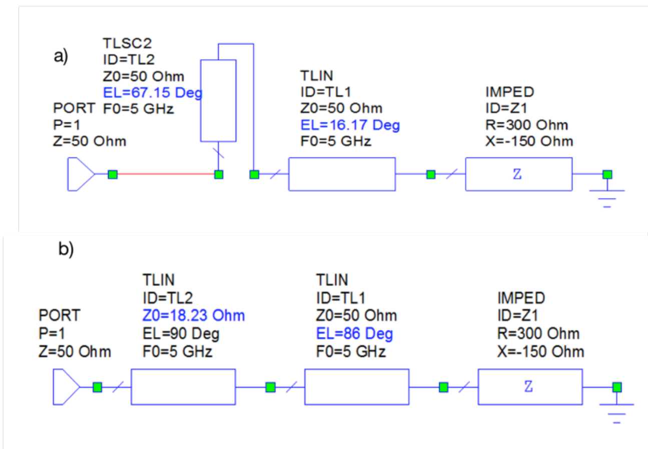
Solution:

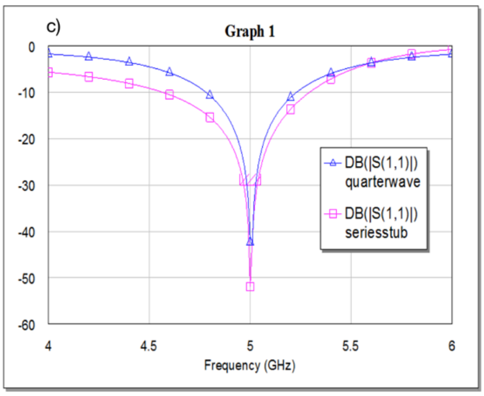
First, plot normalised impedance, i.e. $z_L=(300-j150)/50=6-j3$, in Smith Chart. We can work with the impedance chart for both a and b. a) A short transmission line (0.045λ) will move to r=1 circle (on a circle/clockwise), adding a short-series (inductive stub / 0.187λ) will bring us to origo. b) a transmission line (0.239λ) will bring us to a low real impedance (r=0.14 => R= 7 Ohm), and the quarter wave transformer with impedance $Z_{\lambda/4} = \text{sqrt}(7*50)=18.7$ Ohm will finally transform us to origo.

Answer: see equivalent circuits below.

a) 

b) 



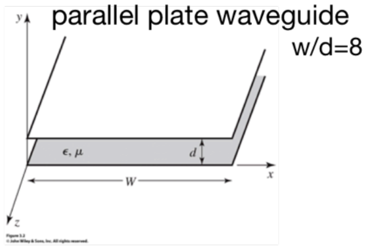


c) There are many practical concerns to be mentioned, such as the difficulty to realise series stubs, tuning possibilities, fabrication tolerances, etc. In this case, the series stub is electrically shorter than the quarter wave based network. This is usually preferred in terms of bandwidth as well as sensitivity to fabrication tolerances. The series stub is a better match over a larger bandwidth (load is set to 300-j150 Ohm) as shown in the plot above.

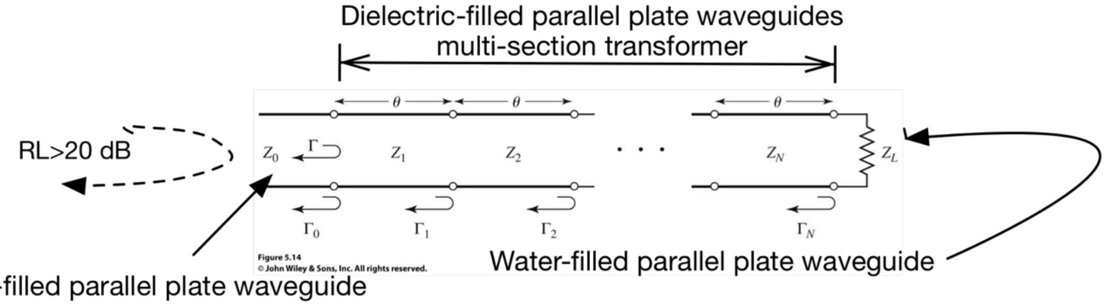
Problem 3

Dr. H. Rodilla is using a parallel plate waveguide (TEM mode), for biomolecular dielectric spectroscopy between 2.0 - 4.5 GHz, and would like to feed the incoming microwave radiation into a section filled with water (containing small amount of biomolecules to be studied). The feeding line is just filled with air and it is important to achieve a return loss better than 20dB, across the whole frequency band. Design a binomial multi-section transformer, using minimum number of dielectric filled parallel plate waveguide sections (see fig b), in order to match the water-filled section to the air-filled feeding line. The width over plate separation is kept constant: *W*/*d*=8 (see fig a). Theory of small reflections may be applied. To avoid onset of higher modes, what is the maximum plate separation, *d*?

a)



b)



(10p)

<u>Solution:</u>

We aim for a RL better than 20 dB, or $|\Gamma|<0.1$, over a bandwidth of $\Delta f = (4.5\text{Gc}-2\text{Gc}) = 2.5\text{GHz}$, and a centre frequency of $f_0=\sqrt{(4.5*2)}=3\text{GHz}$. In this frequency range, the relative permittivity of water is close to 77 (appendix Pozar). First, we need to determine the load impedance of the water-filled line, as well as the characteristic impedance of the air-filled feeding line. Impedance (TEM) of a parallel-plate waveguide: $Z_{TEM}=\eta * W/d$ ->

$Z_L=377(8*\underline{\sqrt{(77)}})=5.4$ Ohm

$Z_o=377/8=47.1$ Ohm. Or ratio $\underline{Z_o/Z_L=\sqrt{(77)}=8.8\approx 9}$

We will also need to calculate the following:

Fractional bandwidth: $\Delta f/f_0=2.5\text{GHz}/3\text{GHz}=83\%$

Wavelength in the air filled parallel plate waveguide: $\lambda_{air}=c/f=10\text{cm}$.

Wavelength in the water filled parallel plate waveguide: $\lambda_{water}=v/f=10/\sqrt{77}\ \text{cm}=1.14\ \text{cm}$.

We need to estimate number of binomial sections, *N*, that can match an impedance ratio of ca 9, over 83% bandwidth ($|\Gamma|\leq 0.1$). This can be calculated from equation (5.55).

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} cos^{-1}\left(\frac{1}{2}\left(\left|\frac{\Gamma(Z_L+Z_o)}{Z_L-Z_o}\right| 2^N\right)^{\frac{1}{N}}\right) = 2 - \frac{4}{\pi} cos^{-1}\left(\left(\left|\frac{\Gamma(Z_L+Z_o)}{Z_L-Z_o}\right|\right)^{\frac{1}{N}}\right) \approx 0.83$$

$$cos^{-1}\left(\left(\left|\frac{0.1(5.4+47.1)}{5.4-47.1}\right|\right)^{\frac{1}{N}}\right) \approx \frac{\pi}{4}(2-0.83)$$

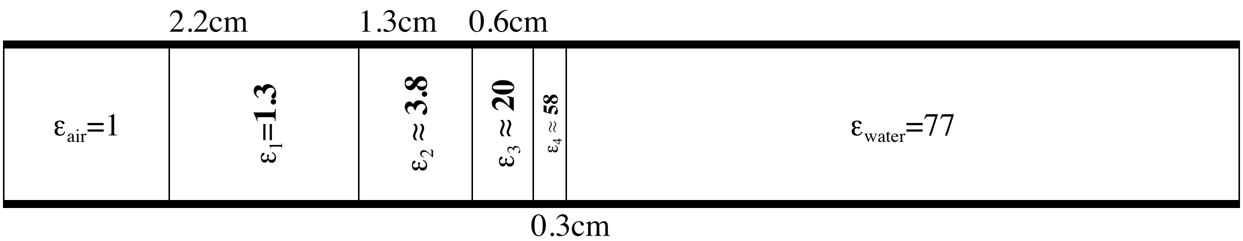
From this expression, we find that ***N=4*** meets the specifications. For simplicity, use tabulated data in table 5.1. Let's make a linear interpolation between Z_o/Z_L =8 and 10, in order to find corresponding impedances (Z_o/Z_L =9) as: $Z_1 \approx 7.8* Z_L$; $Z_2 \approx 4.5* Z_L$; $Z_3 \approx 1.96* Z_L$; $Z_4 \approx 1.15* Z_L$ Ohm.

The maximum distance *d*, will be set by the part containing the high permittivity media (water) and the highest operating frequency (4.5Ghz), which should be less than the onset of higher modes. Next mode can propagate when λ=2d. -> d=λ/2=1.14cm*(3/4.5)*0.5=0.4cm

Answer: The final transformer design, using dielectric filled parallel plate waveguides, is:

No	Z	permittivity	Physical length
Feed	Z_o= 47.1 Ω;	ε_{air}=1	
1	Z_1= 42 Ω	=> ε_1= (47.1/42)^2=**1.3**	$L_1 = \lambda_1/4$=10cm / (4*sqrt(ε_1)) = **2.2 cm**
2	$Z_2 \approx$ 24 Ω	=> $\varepsilon_2 \approx$ **3.8**	$L_2 = \lambda_2/4$= **1.3 cm**
3	$Z_3 \approx$11 Ω	=> $\varepsilon_3 \approx$ **20**	$L_3 = \lambda_3/4$= **0.6 cm**
4	$Z_4 \approx$6.2 Ω	=> $\varepsilon_4 \approx$ **58**	$L_4 = \lambda_4/4$= **0.3 cm**
Load	Z_L=5.4 Ω,	ε_{water}=77	

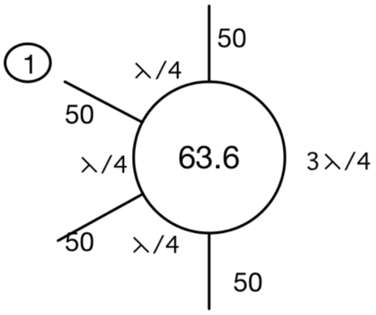
Plate separation, *d*, must be kept less than 4mm.



NB! Not to scale.

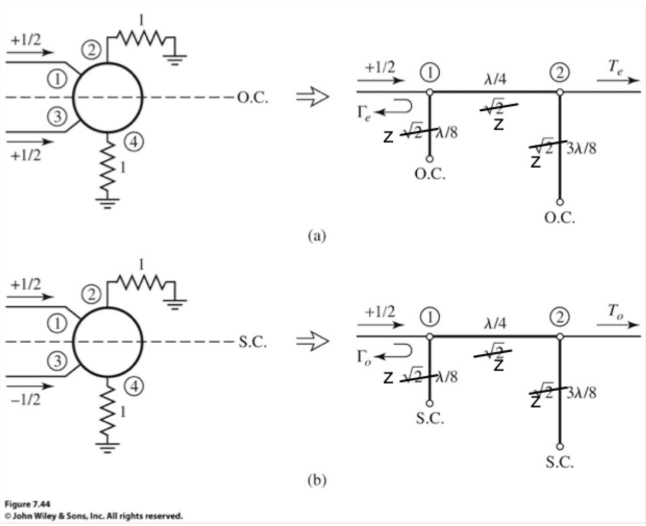
Problem 4

The rat-race coupler below was built and, by mistake, the impedance of the "ring" is 63.6 Ω instead of √2*50 Ω. Determine the performance of the ring hybrid for an input signal at port 1. Discuss the consequences due to different ring-line impedance and its implications for practical use.



(10p)

<u>Solution:</u>
To evaluate the rat-race coupler, we need to find the [S] matrix. Use even and odd mode analysis to find the input match, and coupling to each port.



ABCD representation:

$$\begin{bmatrix} A_e & B_e \\ C_e & D_e \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/_{jz} & 1 \end{bmatrix} \begin{bmatrix} 0 & jz \\ j1/z & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/_{jz} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/_{jz} & 1 \end{bmatrix} \begin{bmatrix} 1 & jz \\ j/_{z} & 0 \end{bmatrix} = \begin{bmatrix} 1 & jz \\ 2j/z & -1 \end{bmatrix}$$

$$\begin{bmatrix} A_o & B_o \\ C_o & D_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/_{jz} & 1 \end{bmatrix} \begin{bmatrix} 0 & jz \\ j1/z & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/_{jz} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/_{jz} & 1 \end{bmatrix} \begin{bmatrix} -1 & jz \\ j/_{z} & 0 \end{bmatrix} = \begin{bmatrix} -1 & jz \\ 2j/z & 1 \end{bmatrix}$$

$$\Gamma_e = \frac{A_e + B_e - C_e - D_e}{A_e + B_e + C_e + D_e} = \frac{2 + j(z - \frac{2}{z})}{j(z + \frac{2}{z})};\ \mathrm{T}_e = \frac{2}{A_e + B_e + C_e + D_e} = \frac{2}{j(z + \frac{2}{z})}$$

$$\Gamma_o = \frac{A_o + B_o - C_o - D_o}{A_o + B_o + C_o + D_o} = \frac{-2 + j(z - \frac{2}{z})}{j(z + \frac{2}{z})};\ \mathrm{T}_o = \frac{2}{A_o + B_o + C_o + D_o} = \frac{2}{j(z + \frac{2}{z})}$$

$\frac{\Gamma_e+\Gamma_o}{2}=\frac{(z-\frac{2}{z})}{(z+\frac{2}{z})}$; $\frac{\Gamma_e-\Gamma_o}{2}=\frac{2}{j(z+\frac{2}{z})}$

$\frac{T_e+T_o}{2}=\frac{2}{j(z+\frac{2}{z})}$; $\frac{T_e-T_o}{2}=0$

With the nominal value of z=√2, we have a perfect input match, and 3-dB coupler. Isolated port (=0) is not affected by the impedance of the "ring".

With a ring-line impedance of z=63.6/50=1.272, we have

$\frac{\Gamma_e+\Gamma_o}{2}=-0.106$ => -19.5dB; $\frac{\Gamma_e-\Gamma_o}{2}=-j0.703$ => -3.06dB

$\frac{T_e+T_o}{2}=-j0.703$ => -3.06dB; $\frac{T_e-T_o}{2}=0$ => "perfect isolation"

Answer: The rat-race will still work fine! Small mismatch loss (RL=19dB), but still equal division between the output ports ports.

Problem 5

A cylindrical cavity is used to determine the permittivity of a gas. The gas causes a resonance shift of 2% for the TM010 mode. What is the dielectric permittivity of the gas? The cavity diameter is 2cm and length 10cm.

(10p)

Solution:

The general resonance condition for the TM010 mode is $f_{010} = \frac{c}{2\pi\sqrt{\epsilon\mu}}\sqrt{\left(\frac{p_{nm}}{a}\right)^2 + (0)^2}$

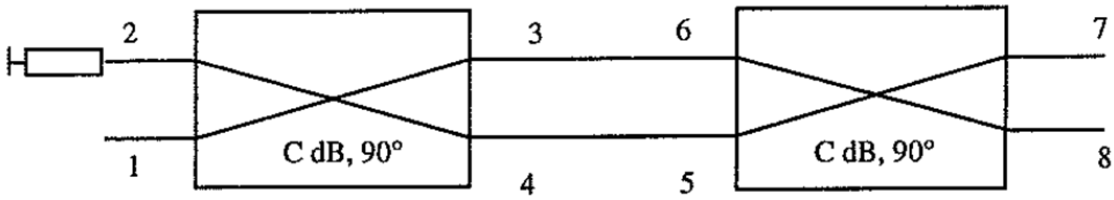
Assume an air-filled cavity. The relative permittivity of the gas is larger than one, so the resonance will shift to a lower frequency when gas is added within the cavity. Hence

$$f_{010}^{gas} = 0.98 f_{010}^{air} = \frac{f_{010}^{air}}{\sqrt{\epsilon_{gas}}} \rightarrow \epsilon_{gas} = \left(\frac{1}{0.98}\right)^2 = 1.04$$

Answer: the relative dielectric permittivity of the gas is 1.04.

Problem 6

A 3-dB hybrid can be realised by two directional couplers in cascade, see below.



Determine C, and design the directional coupler as a coupled transmission line coupler in stripline technology, using quartz substrate (ε_r= 3.78). Impedance level is 50 ohm. Centre frequency is 30GHz. Hint! See attached paper by Cohn for Strip-line design.

(10p)

Solution:

First, we need to find the individual coupling, C, in order to realise a 3-dB hybrid. Start from a symmetrical directional coupler:

$$\begin{matrix} V_4^- = \alpha V_1^+ \\ V_3^- = j\beta V_1^+ \end{matrix} \rightarrow \begin{matrix} V_8^- = \alpha V_4^- + j\beta V_3^- = (\alpha^2 - \beta^2)V_1^+ \\ V_7^- = \alpha V_3^- + j\beta V_4^- = (2\alpha\beta j)V_1^+ \end{matrix}$$

We also have $\alpha^2 + \beta^2 = 1$ (lossless)

$$3dB\ overall\ coupling \rightarrow \frac{1}{\sqrt{2}} = \frac{V_7^-}{V_1^+} = \frac{V_8^-}{V_1^+} = \alpha^2 - \beta^2 = 1 - 2\beta^2 \rightarrow \beta = C = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$
$$= 0.3827\ \ (8.34dB)$$

Hence, next step is to realise a coupled transmission line coupler in stripline technology, with a coupling of C = 8.34dB. Centre frequency is 30 GHz or λ_o=c/f=10 mm -> λ_{quartz}=10 mm / √3.78=5.14mm.

To avoid high order modes, keep ground plane separation b, smaller that λ_{quartz}/2 =2.57mm. In this case, let's design for b=2 mm, which will bring some margin.

Even and odd mode impedances (eq 7.87 in Pozar):

$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}} = 50\sqrt{\frac{1+0.3827}{1-0.3827}} = 75 \text{ Ohm}$$

$$Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}} = 50\sqrt{\frac{1-0.3827}{1+0.3827}} = 33 \text{ Ohm}$$

To use the nomogram for edge coupled striplines in appended paper, calculate $\sqrt{\varepsilon_r}Z_{0e} = 146$ and $\sqrt{\varepsilon_r}Z_{0o} = 64$. Now use the nomogram in fig 5-6 or fig 7.29 in Pozar. From fig 5 (appended paper), one can read w/b=0.4 and from fig 6, s/b=0.05.

Answer: In conclusion, we have b= 2mm, w=0.8mm, s=0.1 mm, for our edge coupled stripline, supported and embedded with quartz substrate, The physical length of the 90 degree coupled section is l= λ_{quartz}/4= 5.14/4=1.29 mm. The coupling factor is C=8.3 dB.

Enclosures: Smith Charts

S. B. Cohn, "Shielded Coupled-Strip Transmission Line," *IRE Transactions on Microwave Theory and Techniques*, vol. 3, no. 5, pp. 29–38, 1955.