FUNDAMENTALS OF

PHOTONICS

Numerical tutorial 2 - Resonator

Outline

- **Gaussian beam**
  - Mathematical formula
  - Beam properties

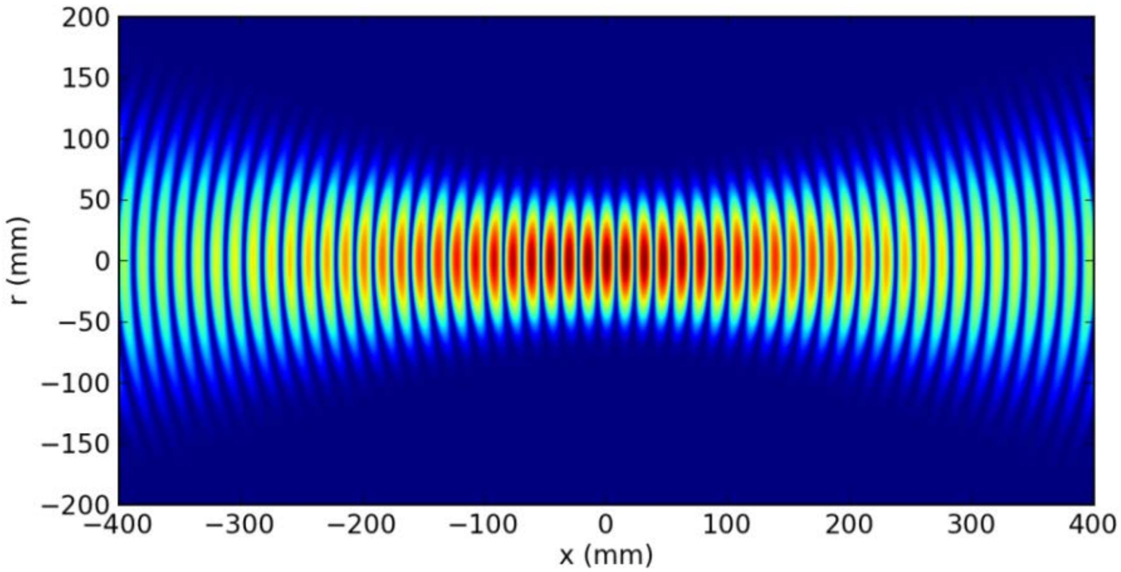
- **Spherical mirror resonator**
  - Gaussian beam solution

- **Unfolded resonator model (HA2)**

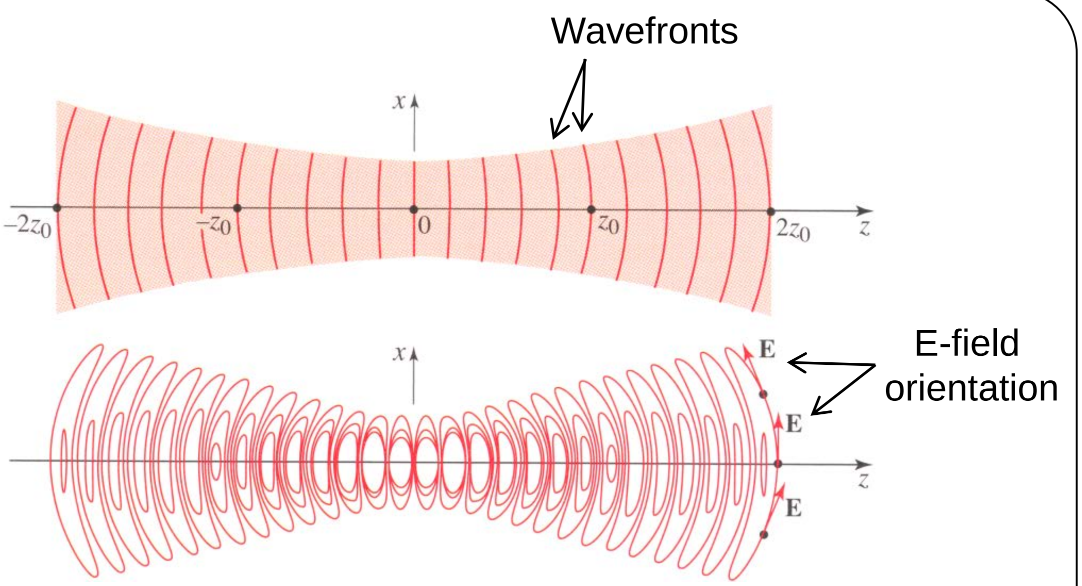
Gaussian beam

- Beam ↔ spatially localized wave with low divergence
- **Solution to paraxial Helmholtz equation** (wave equation for monochromatic and paraxial wave)
- **Lowest divergence of all beams** for given waist diameter (laser light under ideal conditions)

Simulated Gaussian beam, (instantaneous intensity, i.e. $2\cdot(u(\boldsymbol{r},t=0))^2 = 2\cdot(Re\{U(\boldsymbol{r},t=0)\})^2$



Mathematical formula



$$\mathbf{E}(r) = \left(-\hat{x} + \frac{x}{z + jz_0}\hat{z} \right) U(\mathbf{r})$$

Electric field complex amplitude vector

Scalar complex amplitude

- **Course book sign convention:**

$$U(\mathbf{r}) \sim \frac{1}{z + jz_0} \exp\left(-jk \frac{\overbrace{x^2 + y^2}^{= r^2}}{2(z + jz_0)} \right) \exp(-jkz)$$

z_0 = **Rayleigh range**

All beam properties *(except max intensity)* ***are given by z_0 and λ!***

- **Jörgen's sign convention:**

$$U(\mathbf{r}) \sim \frac{1}{z + jz_0} \exp\left(jk \frac{x^2 + y^2}{2(z + jz_0)} \right) \exp(jkz)$$

$$\boxed{k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0 / n} = \frac{\omega}{c_0 / n} = \frac{\omega}{c}}$$

Properties

- **Rewriting complex amplitude** (course book sign convention):

$$U(\mathbf{r}) \sim \frac{W_0}{W(z)} \exp\left(-\frac{\overbrace{x^2+y^2}^{=r^2}}{W^2(z)}\right) \exp\left(-jkz + j\zeta(z) - jk\frac{\overbrace{x^2+y^2}^{=r^2}}{2R(z)}\right)$$

$$W(z) = W_0\sqrt{\left(1+(z/z_0)^2\right)} \qquad R(z) = z\left(1+(z_0/z)^2\right)$$

$$\zeta(z) = \tan^{-1}(z/z_0) \qquad W_0 = \sqrt{\lambda z_0/\pi}$$

Beam radius

Wavefront **radius of curvature**

Waist radius ($2\cdot W_0$=spot size)

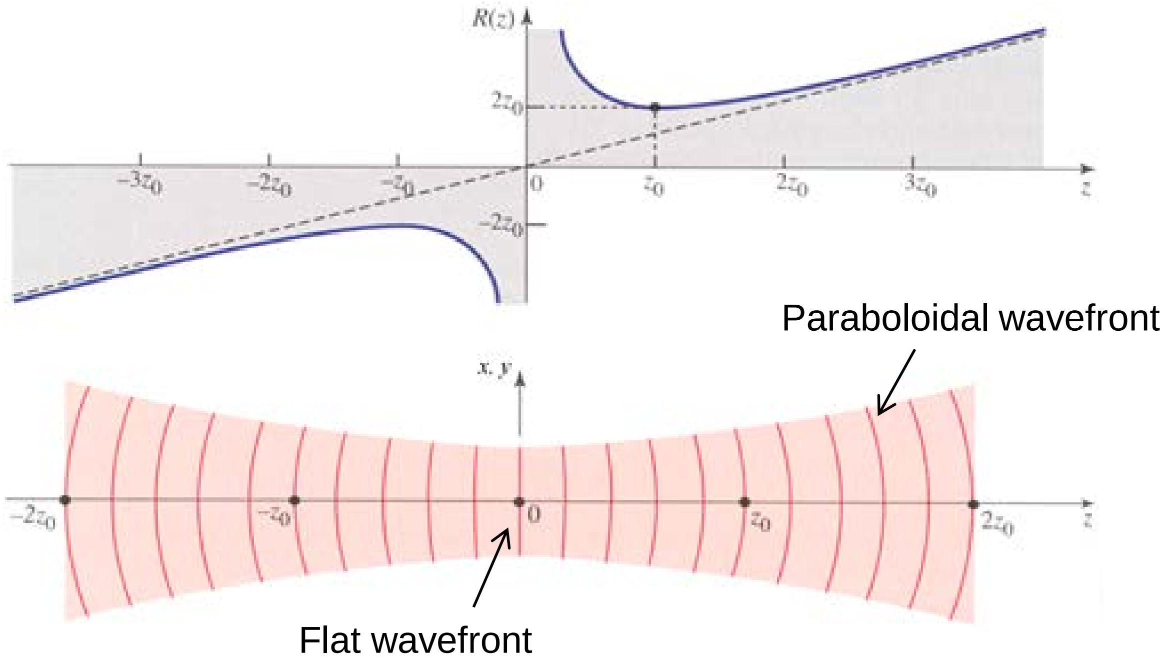
- **Beam waist** is located at z=0, where the beam radius is at minimum

- **Intensity distribution** in radial direction = the **Gaussian function** (hence the name…)

- **Power:** $P = \frac{1}{2} I_0 \pi W_0^2$ where I_0 is the max intensity, located in (x=0,y=0,z=0)

- **Divergence angle:** $\theta_0 \approx \frac{W_0}{z_0} = \frac{\lambda}{\pi W_0}$

- **Wavefronts:**

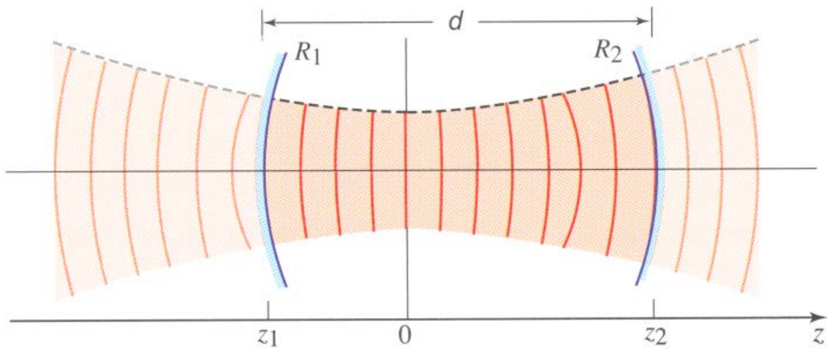


Starting with a field $E_1(x,y)$~$\exp(-r^2/W_0^2)$ in the TSM
→ the Gaussian beam will emerge!

Spherical mirror resonator

- **Gaussian beam** has paraboloidal wavefronts → **can perfectly adapt to the curvature of the spherical mirrors** to retrace itself (for minimal losses):

z=0 at beam center



R_1 = **mirror 1 radius of curvature**
R_2 = mirror 2 radius of curvature

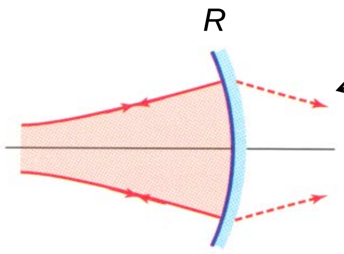
Course book sign convention:
- Concave mirror $R < 0$
- Beam $R > 0$ for $z > 0$, $R < 0$ for $z < 0$

$$\left.\begin{aligned} z_2 &= z_1 + d \\ R_1 &= z_1 + z_0^2/z_1 \\ -R_2 &= z_2 + z_0^2/z_2 \end{aligned}\right\} \Rightarrow \begin{array}{l} \left.\begin{array}{c} z_1 = -d\left(R_2+d\right)/\left(R_2+R_1+2d\right) \\ z_2 = z_1 + d \end{array}\right] \longrightarrow \text{Waist position} \\ z_0^2 = -d\left(R_1+d\right)\left(R_2+d\right)\left(R_2+R_1+d\right)/\left(R_2+R_1+2d\right)^2 \longrightarrow \text{Waist radius} \end{array}$$

Starting with an initial field $E_1(x,y)$~$\exp(-r^2/W_0^2)$ in the TSM
→ the above Gaussian beam solution will emerge after some roundtrips
(assuming finite diameter of mirrors…) ***!***

Unfolded resonator model (HA2)

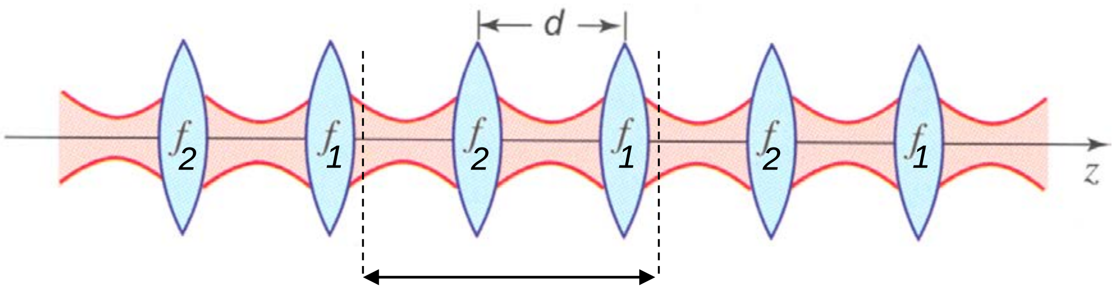
- **A spherical mirror has the same wavefront bending effect as a lens** of focal length $f = -R/2$ (course book sign convention), where R is the mirror radius of curvature:



Dashed curve = effect of replacing mirror by lens with $f = -R/2$

Mirror amplitude reflectance	**Lens amplitude transmittance**
$r(x,y) \sim \exp\left(-jk\frac{x^2+y^2}{R}\right)$	$t(x,y) \sim \exp\left(jk\frac{x^2+y^2}{2f}\right)$

- **The resonator can thus be modelled as a successive propagation of the beam through lenses:**



$f_1 = -R_1/2$
$f_2 = -R_2/2$

HA2: use TSM to propagate between the lenses,
and lens amplitude transmittance function to propagate through lens!