Solutions to Exam in Microwave Engineering (MCC121)

Friday, December 21, 2012

MCC121

Exam in **Microwave Engineering**

Friday, December 21, 2012, 1400 - 18:00, “V-salar”

Teachers: Jan Stake phone: 031 -772 1836
Vincent Desmaris phone: 031 -772 1846
During the exam, teacher will visit around 1430 and 1600.

Examiner: Prof. Jan Stake. Terahertz and millimetre wave lab., Department of microtechnology and nanoscience (MC2), Chalmers University of Technology.

This is an open book exam. The following is allowed:

- *Calculator (approved by Chalmers)*
- *”Microwave Engineering” by Pozar*
- *Mathematics handbook (Beta)*
- *Smith charts*

To pass this written examination, you need at least 24p out of 60p. Final grade of the course will also include results from assignment 1. That is: 3 (≥28p), 4 (≥42p) and 5 (≥56p).

Teamwork is not permitted on this examination. The university academic integrity policy will be strictly enforced. Failure to comply with the academic integrity policy will result in a zero for this examination.

Make sure you have understood the question before you go ahead. Write shortly but make sure your way of thinking is clearly described. It is imperative to clearly explain how the results have been obtained. Solve the problem as far as you can – constructive, creative and valuable approaches are also rewarded. Assume realistic numbers/parameters when needed if data is missing in order to solve the problem.

❖ ❖ ❖

Problem 1

Prove that a short ($l < \lambda/12$) low impedance transmission line ($Z_0 < Z_L/3$) can be approximated as a shunt capacitor. Determine the value of this capacitor in terms of the line parameters.

(10p)

<u>Solution</u>

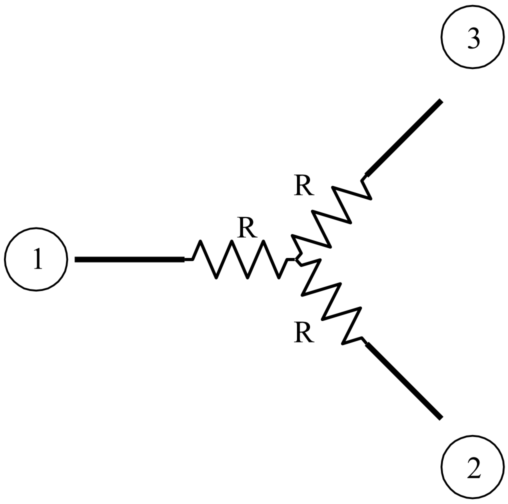
Alt 1) Study the ABCD matrix for a short transmission line, $\begin{bmatrix} cos(\beta l) & jZ_o sin(\beta l) \\ jY_o sin(\beta l) & cos(\beta l) \end{bmatrix} \approx \begin{bmatrix} 1 & jZ_o \beta l \\ jY_o \beta l & 1 \end{bmatrix}$. For low impedance line (high admittance), the ABCD matrix becomes identical to the matrix representation for a shunt capacitor. I.e. $\begin{bmatrix} 1 & 0 \\ jY_o \beta l & 1 \end{bmatrix} \cong \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}$ so $C = \frac{Y_o \beta l}{\omega}$.

Alt 2) Study the input impedance for a short low impedance transmission line, loaded with ZL. $Z = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)} \approx Z_o \frac{Z_L + jZ_o \beta l}{Z_o + jZ_L \beta l} \approx Z_o \frac{Z_L}{Z_o + jZ_L \beta l} = \frac{Z_L}{1 + jZ_L \frac{\beta l}{Z_o}}$. The last expression corresponds to the impedance of a capacitor, C, in parallel to a load, Z_L. So $C = \frac{\beta l}{Z_o \omega} = \frac{Y_o \beta l}{\omega}$

Answer: $C = \frac{Y_o \beta l}{\omega}$.

Problem 2

a. Design a three-port resistive power divider for an equal power split. The impedance level is 75Ω.
b. Calculate also the signal in port 3 if port 2 is mismatched and connected to 100Ω load. Assume other ports matched.
c. Has this leakage signal a significant influence on the performance of the power divider?



(10p)

<u>Solution</u>

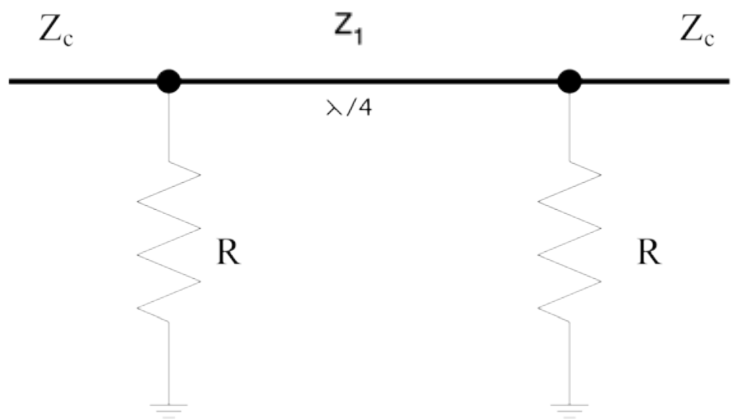
a) For equal split, and matched -> R = Zo/3. So R =25 ohm.

b) $[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ for the equal power split divider. For matched ports: $V_3^- = V_1^+ \frac{1}{2}$. A 100 ohm termination in port 2 will result in a reflection coefficient of $\Gamma = \frac{100-75}{100+75} = \frac{1}{7}$. So additional signal at port 3 due to a mismatch at port 2: $\delta V_3^- = V_1^+ \frac{1}{2}\frac{1}{7}\frac{1}{2} = V_1^+ \frac{1}{28}$. Hence, the total signal at port 3: $V_{3tot}^- = V_3^- + \delta V_3^- = V_1^+ \frac{1}{2}(1 + \frac{1}{14}) = V_1^+ \frac{15}{28}$

c) The leak signal will be, $20\log\left(\frac{1/2}{1/28}\right) = 22.9dB$, below the main signal, so in many cases negligible.

Problem 3

The circuit below can be used as a simple attenuator. Just by soldering two shunt resistors, separated by a quarter wavelength, it is possible to make a quick-fix-attenuator without major modifications in a planar microstrip circuit.



Determine a condition for Z_1 and R so the attenuator is matched. Calculate R and Z_1 to obtain 2dB attenuation. What's the return loss if you choose Z_1=Z_c ?   (Z_c=50Ω)

(10p)

Solution

ABCD representation:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix} \begin{bmatrix} 0 & jZ_1 \\ jY_1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix} \begin{bmatrix} jZ_1/R & jZ_1 \\ jY_1 & 0 \end{bmatrix} = \begin{bmatrix} jZ_1/R & jZ_1 \\ jZ_1/R^2 + jY_1 & jZ_1/R \end{bmatrix}$$

(Reciprocal and AD-BC=1)

S11=0 => $A + B/Z_c - CZ_c - D = 0$, hence $jZ_1/Z_c - Z_c(jZ_1/R^2 + jY_1) = jZ_1(1/Z_c - Z_c/R^2 - Z_c/Z_1^2) = 0$ so $\boxed{1/Z_1^2 = 1/Z_c^2 - 1/R^2 \rightarrow Z_1 = Z_c\sqrt{\frac{R^2}{R^2 - Z_c^2}}}$.

For 2-dB attenuation, we need an expression for S21 in order to find correct value for R:

$$S21 = \frac{2(AD - BC)}{A + B/Z_c + CZ_c + D} = \frac{2}{jZ_1/R + jZ_1/Z_c + (jZ_1/R^2 + jY_1)Z_c + jZ_1/R}$$

$$= \frac{2}{j} \frac{1}{2Z_1/R + Z_1/Z_c + Z_1Z_c/R^2 + Z_c/Z_1} = \frac{2}{jZ_1} \frac{1}{2/R + 1/Z_c + Z_c/R^2 + Z_c/Z_1^2}$$

$$= \{insert\ condition\ for\ S11 = 0\} = \frac{2\sqrt{R^2 - Z_c^2}}{jRZ_c} \frac{1}{2/R + 1/Z_c + Z_c(1/Z_c^2)}$$

$$= \frac{\sqrt{R^2 - Z_c^2}}{jRZ_c} \frac{1}{1/R + 1/Z_c} = \frac{\sqrt{R^2 - Z_c^2}}{j} \frac{1}{R + Z_c} = -j\frac{\sqrt{R - Z_c}}{\sqrt{R + Z_c}}$$

2dB-> $|S21| = 10^{-0.1} = k = \left|\frac{\sqrt{R-Z_c}}{\sqrt{R+Z_c}}\right| \rightarrow R = Z_c\frac{1+k^2}{1-k^2} = 50\frac{1+10^{-0.2}}{1-10^{-0.2}} = 221\Omega$

and Z1=51 ohm. Very close to Zc so lets etstimate return loss if we stick to same characteristic impedance. I.e. RL for Z1=Zc=50.

We can use our ABCD representation above to find S11, or use the following approach to calculate the input impedance presented by the two-port.
The quarter wave transformer will transform the impedance of a shunt connection between Zc and R, which after transformation is shunted with R.

ZL=Zc//R which after the quarter wave transformer (Z1=Zc) become:

$\acute{Z} = \frac{Z_c^2}{ZL} = \frac{Z_c^2}{RZ_c}(R + Z_c) = \frac{Z_c}{R}(R + Z_c)$ in shunt with R

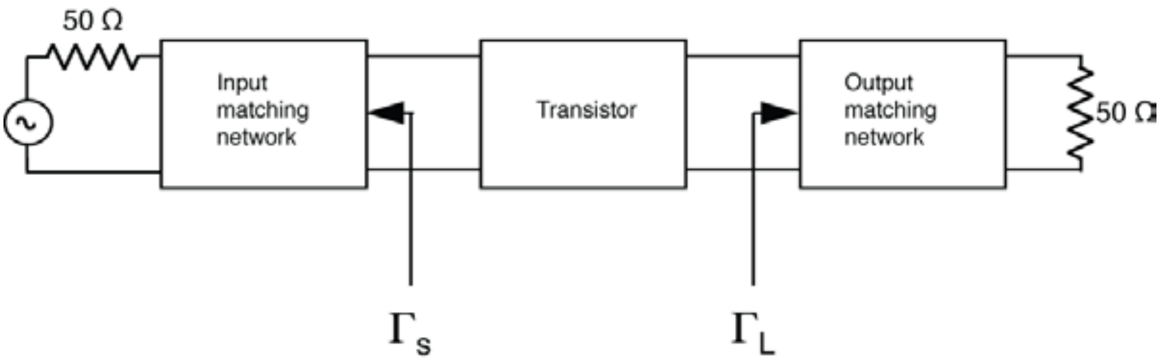
$$1/Z_{in} = \frac{R}{Z_c(R + Z_c)} + \frac{1}{R} = \frac{1}{Z_c}\left(\frac{R}{(R + Z_c)} + \frac{Z_c}{R}\right) \to Z_{in} = 48ohm$$

$|\Gamma| = \left|\frac{48-50}{48+50}\right| = 0.02$ Hence, return loss is about RL=34dB.

Answer: R=221 ohm and Z1= 51ohm for 2dB attenuation (matched case). For Z1=50, we will have a low return loss of 34dB.

Problem 4

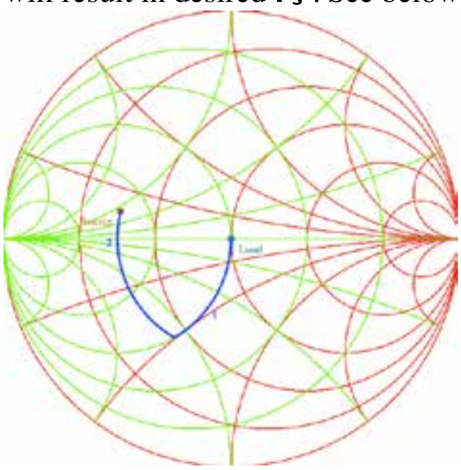
A transistor requires a source reflection coefficient of Γ_S=0.5/<u>166</u>°for proving minimum noise figure in an amplifier circuit. Design the input matching circuit using distributed transmission lines, so embedding impedance corresponding to Γ_S is presented to the transistor as shown below. The amplifier will be used in a Z_c = 50 ohm system impedance environment.

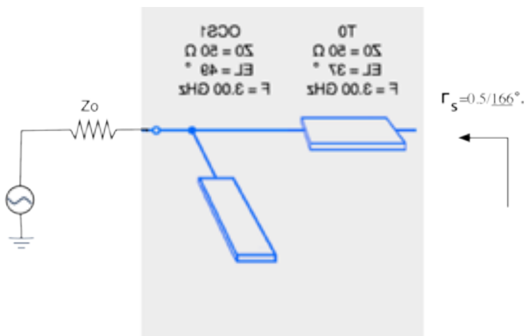


(10p)

<u>Solution</u>

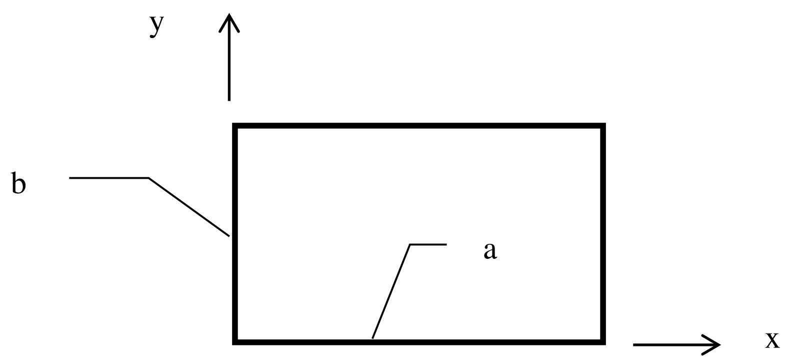
Mark desired source reflection coefficient in a SC. Using, distributed components, we will transform from origin (match) to Γ_S=0.5/<u>166</u>°. See moves in SC below.
Answer: An open stub with an electrical length of 0.136**λ** followed by a line with length 0.1**λ** will result in desired Γ_S . See below.





Problem 5

Consider a rectangular waveguide:



Select the ratio a/b to obtain the largest possible frequency range at the lowest possible attenuation for the dominant mode.

(10p)

<u>Solution</u>

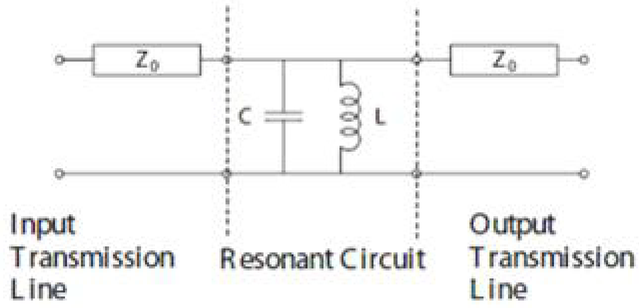
The dominant mode, TE10, in a rectangular hollow waveguide has a cut off frequency, which depends on the width, *a*. Corresponding to half wavelength equal to *a*. Next similar mode, TE20, exhibit a cut off when the wavelength is equal to *a*. Hence, twice the cut off frequency for TE10. The metallic loss (attenuation) depends on both width and height, and for all low order modes, a large waveguide is always preferred. (See fig 3.8 in Pozar or analytic expressions). Consequently, for a fixed width, *a*, determining the dominant mode, we would prefer a large height waveguide. However, corresponding TE01 mode might be excited if the height, *b*, is equal to half a wavelength. Since TE20 cannot be avoided, the maximum frequency span is a factor of two. So a natural design, which at the same time minimizes loss and provides the largest frequency band for the dominant TE10 mode, is when TE01 is designed to have the same cut off frequency as the TE20 mode. **Hence, *b=a/2***. (This gives largest possible bandwidth for dominant mode and at the same time lowest loss.)

Problem 6

Show that near resonance for the circuit below, the following approximations can be made:

$$|S_{21}|^2 = \frac{1}{1+Q^2\left(1-\left(\frac{w}{w_0}\right)^2\right)^2} \quad \text{or} \quad |S_{21}|^2 = \frac{1}{1+Q^2\left(1-\left(\frac{w_0}{w}\right)^2\right)^2}$$

Where Q is the loaded Q-factor and w_0 is the angular frequency of resonance.



What is S_{21} at resonance?

(10p)

Solution

Start by defining: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C + \frac{1}{j\omega L} & 1 \end{bmatrix}$ (Reciprocal and AD-BC=1)

$$S21 = \frac{2(AD-BC)}{A + B/Z_c + CZ_c + D} = \frac{2}{2 + Z_c\left(j\omega C + \frac{1}{j\omega L}\right)} = \frac{1}{1 + \frac{1}{2}Z_c j\omega C\left(1 - \frac{1}{\omega^2 LC}\right)}$$

$$= \left\{ near\ resonance\ , we\ have \quad \begin{matrix} \omega_o = \frac{1}{\sqrt{LC}} \\ Q = \frac{1}{2}Z_c\omega_o C = \frac{Z_c}{2\omega_o L} \end{matrix} \right\} = \frac{1}{1 + jQ\left(1 - \frac{\omega_o^2}{\omega^2}\right)} = \{or\}$$

$$= \frac{1}{1 + \frac{Q}{j}\left(1 - \frac{\omega^2}{\omega_o^2}\right)}$$

$|S21|^2 = \frac{1}{1+Q^2\left(1-\frac{\omega_o^2}{\omega^2}\right)^2} = \frac{1}{1+Q^2\left(1-\frac{\omega^2}{\omega_o^2}\right)^2}$ V.S.V.

Answer: S21 is equal to 1 at resonance.