Handin 6

Marcus Malmquist, marmalm, 941022

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1 Task 1

The periodic structure used in this task can be seen in Figure 1 and the properties of then structure can be seen in Table 1. Sine no information about the microstrip was provided, it is assumed that $\epsilon_r = 10$.

The physical length, d, of each section can be calculated using (1)

$$\begin{cases} kd = \theta \\ k = \dfrac{\omega}{c_0}\epsilon_r \end{cases} \Rightarrow d = \frac{c_0}{\omega\epsilon_r}\theta \tag{1}$$

In order to relate k and β it is convenient to use the $ABCD$-matrix which, for a cascaded circuit with transmission lines and impedances in series, can be calculated using (2). The $ABCD$-matrix in (2) is reciprocal since then determinant for each of the matrices in the right hand side in (2) is 1.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\frac{\theta}{2} & jZ_0\sin\frac{\theta}{2} \\ j\frac{1}{Z_0}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} 1 & -j\frac{1}{\omega C} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & jZ_0\sin\frac{\theta}{2} \\ j\frac{1}{Z_0}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \tag{2}$$

It is possible to relate ω and k using (1) once d is known so (2) will be a function of k. Using then hint that $\cosh(\gamma d) = \frac{A+D}{2}$ and the asumption that

f_0	2 GHz
Z_0	50 Ω
C	500 fF
$\theta\|_{f=f_0}$	$\frac{8}{9}\pi$
ϵ_r	10 (assumed)

Table 1: The properties of the periodic structure seen in Figure 1.

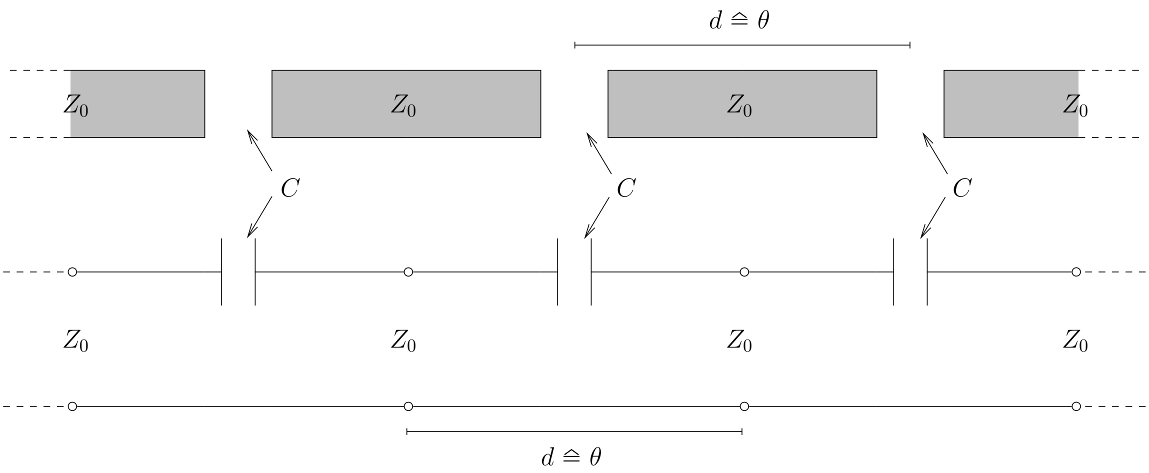

Figure 1: Then periodic structure layout both as microstrip and equivalent circuit.

$\gamma = j\beta$, we can solve (3) which relates β and k. The solution in the region $(-\pi < \beta d < \pi)$ can be seen in Figure 2.

$$\cos(\beta d) = \frac{A + D}{2} \tag{3}$$

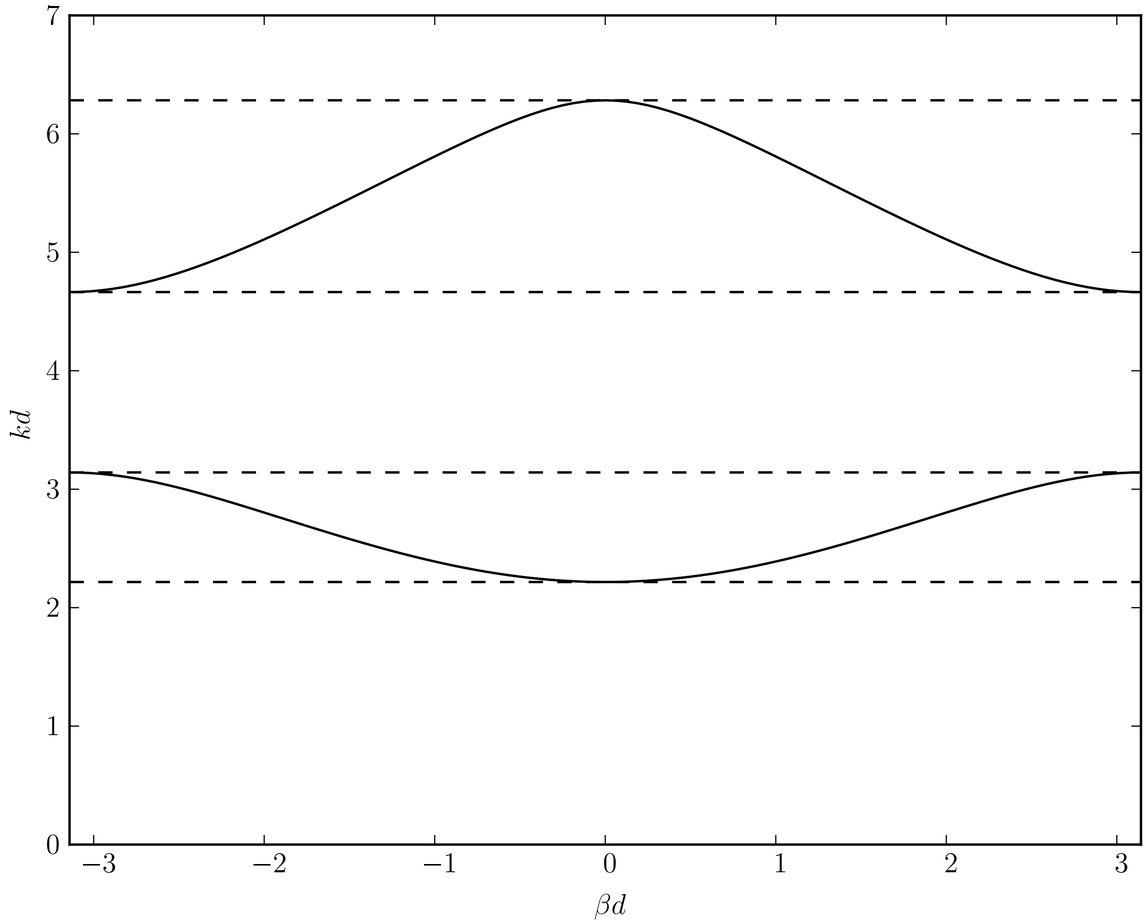

Figure 2: k-β diagram for the structure in Figure 1. The dashed lines are upper and lower bounds of the passbands.