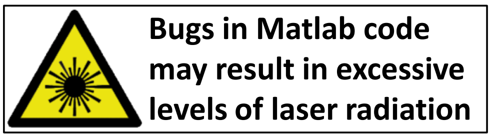
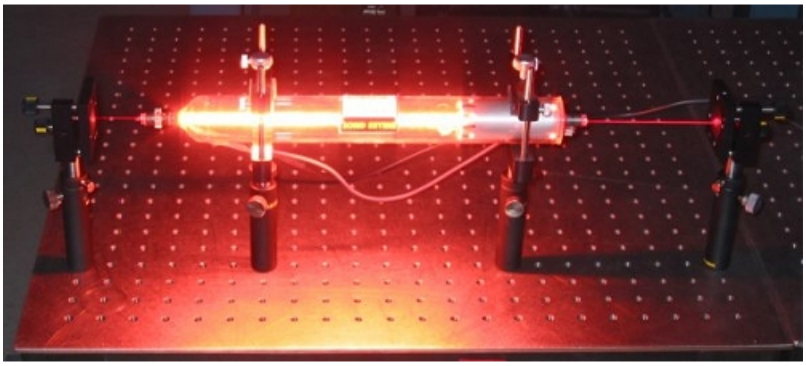




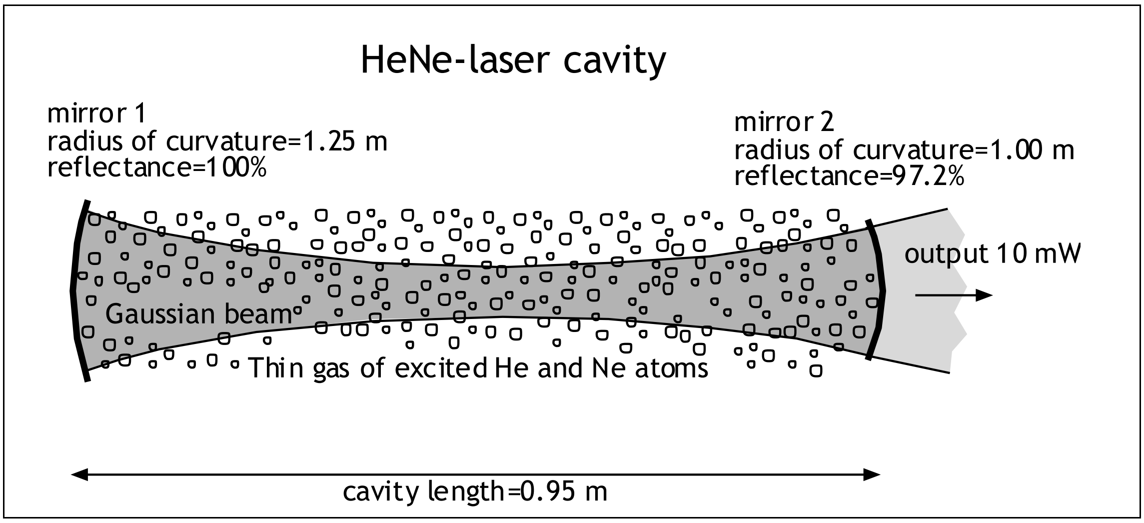
Home assignment 2

Build a laser cavity!

The photo shows a Helium-Neon laser (a "HeNe-laser" if you are in the laser business) with the classic red emission wavelength of 633 nm. It is one of the energy transitions in the Neon atoms that is responsible for the emission of this light, but the excited Helium atoms are also important since they deliver their excess energy to the Neon atoms when they collide (and thus excite the Neon atoms). The particular laser cavity (resonator) shown below, which you now will simulate in Matlab, you will further study experimentally in the course *Laser engineering* in the next study period.
In the laser cavity the light bounces back and forth between the two mirrors. Finally, the field has adapted to the cavity (the cavity mirrors) so that it repeats itself after each double pass (roundtrip). For a given cavity there are only certain spatial distributions of the optical field that manage to do so, they are called the modes of the cavity. The "simplest" mode, the so called fundamental mode, is a Gaussian beam. The beam parameters (the size and position of the beam waist) are determined by the boundary conditions: at the mirrors the beam must reflect back "into itself", i.e., the local propagation direction of the light must be normal to the mirror surface. This is equivalent to saying that a wavefront coincides with the mirror surface.

1. In the course book, Chapter 10.2 B, this case is treated analytically in the section "The Gaussian beam is a mode of the spherical-mirror resonator". If you use the expressions stated there, *where would the waist be positioned*, and *how large is the beam diameter at the left mirror* (mirror 1) in this laser cavity? Since the mirrors have different curvatures, the waist is not located halfway between them (so the picture below is misleading when it comes to the spatial distribution of the Gaussian beam).

Note: The book uses a strange convention that a focusing (i.e. concave) mirror has a negative radius of curvature. Both mirrors in our setup are focusing so in the formulas in the book (but not in the simulations) you need to use -1.25 m and -1.0 m as the radii of curvature for the left and right mirrors, respectively. **(End of 1)**



Now you will demonstrate – using TSM – how the beam automatically adjusts towards the fundamental mode by repeated reflections in the cavity mirrors. Start at the left mirror and propagate toward the right. Assume that the starting field at the left mirror is a Gaussian field with planar wavefront and $1/e^2$-intensity radius of ω=200µm. That this $E_l(x, y)$ does **not** represent a cavity mode is obvious since it does not match the boundary condition – the planar wavefront of the starting field does not coincide with the curved mirror there. Suitable sampling distances could be 10µm at the left mirror and 20µm at the right mirror. (The mirrors have different curvatures, which cause the beam waist to be closer to the left mirror. At the right mirror the light has diverged to a larger diameter and a larger sampling distance is needed to have a larger numerical window.)
At every reflection at either of the mirrors the phase of the field is changed, since a spherical mirror is a phase modulating optical component, acting like a lens with a focal length that is half the radius of curvature – and now we use the standard convention that focusing mirrors like the ones in the cavity have *positive* radii of curvature. Moreover, we assume that the mirrors, just as in real life, have a finite diameter so that light that falls outside the mirror is lost. *It is by these losses, which are generally quite small for one roundtrip, that the fundamental mode is selected.* If you think the method converges too slowly, test to make the mirrors smaller to increase the losses. Just make sure the mirrors are large enough so that the optical field maintains approximately a Gaussian intensity profile.

2. Determine the ω-value of the Gaussian beam every time you come back to the left mirror. Plot this value as a function of the number of roundtrips in your simulation. Does this value converge? Compare with the analytic result in assignment **1**. Comments? Which result is more correct?

(End of 2)

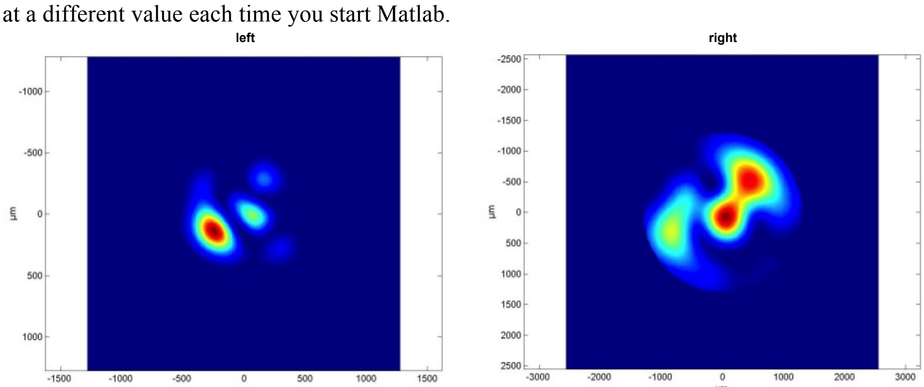
Hint: A very simple way to determine the ω-value of a sampled two-dimensional Gaussian function, in this case the intensity ($I = |E|^2$) distribution at the left mirror, is to make use of the integral formula

$$\iint_{-\infty}^{\infty} \text{Gaussian function } dxdy = \frac{I_{max}\pi\omega^2}{2}$$

where I_{max} is the maximum value of the Gaussian function and ω is the $1/e^2$-radius of the Gaussian function. The double integral is calculated in Matlab with sum(sum(...)), but don´t forget to also multiply by the "area of one sampling position"!

3. Is it possible for an even more incorrect and random starting field to converge toward the fundamental mode? Test by creating a starting field that is a sum of 30 Gaussian functions that each has the parameters $R_l \approx \infty$ and ω_l =200µm, but differ by a random phase-offset (i.e., each Gaussian function is multiplied by the random scalar phase-changing constant **exp(1i*2*pi*rand)**) as well as a random center position which deviates from the origin with up to 300 µm (this is most simply done by replacing **xmat** with **xmat-xoffset** where **xoffset=300e-6*(rand-0.5)*2**, and analogously for **ymat**). See one example in the figures below. **(End of 3)**

Remark: Write **rand('state',sum(100*clock))** on the first line of your program if you want the random number generator to start at a different value each time you start Matlab.



Intensity of the starting field at the left mirror (left) and intensity of the field after the first propagation to the right mirror (and cut-off by the finite diameter of the right mirror).

4. In a classic laboratory exercise in the course *Laser engineering* a hair (typical diameter of 50μm) glued to a frame is inserted into the cavity. For simplicity, assume that the hair is inserted immediately before the right mirror and that it absorbs all the light that falls onto it, see the figure below. Does this "disturbed" cavity have a mode? What name would you like to give this mode? Generate your starting field as in **3.** **(End of 4)**

Hint: To create the transmission function of the hair plane in this – and particularly the following – tasks, I personally find it easier mentally to first create the "inverted" plane where the hair is "one" and the surroundings are "zero":

```
inverted_t_hairplane= abs(umat)< hair_diameter/2;
t_hairplane= 1 - inverted_t_hairplane;
```

Remark 1: Since this optical field extends somewhat further away from the optical axis than the fundamental mode you may find it necessary to increase the diameter of the mirrors to avoid "cutting off" the field too abruptly at each mirror reflection, which may otherwise lead to disturbances in the field. In any case the optical field will appear somewhat "noisier" than before because of the disturbing influence of the hair.

Remark 2: There is a convention for the naming of the modes, where a mode is called the (m,p)-mode, where the integer m is the number of intensity zeros along the x-axis (or u-axis), not counting infinity, and the integer p is the number of intensity zeros along the y-axis. The fundamental mode, which is a Gaussian, is thus also called the (0,0)-mode.

Remark 3: Just as in an experimental setup, these modes are more difficult to obtain than the fundamental mode also numerically, and generally require a larger number of iterations or even completely new trials with a new starting field.

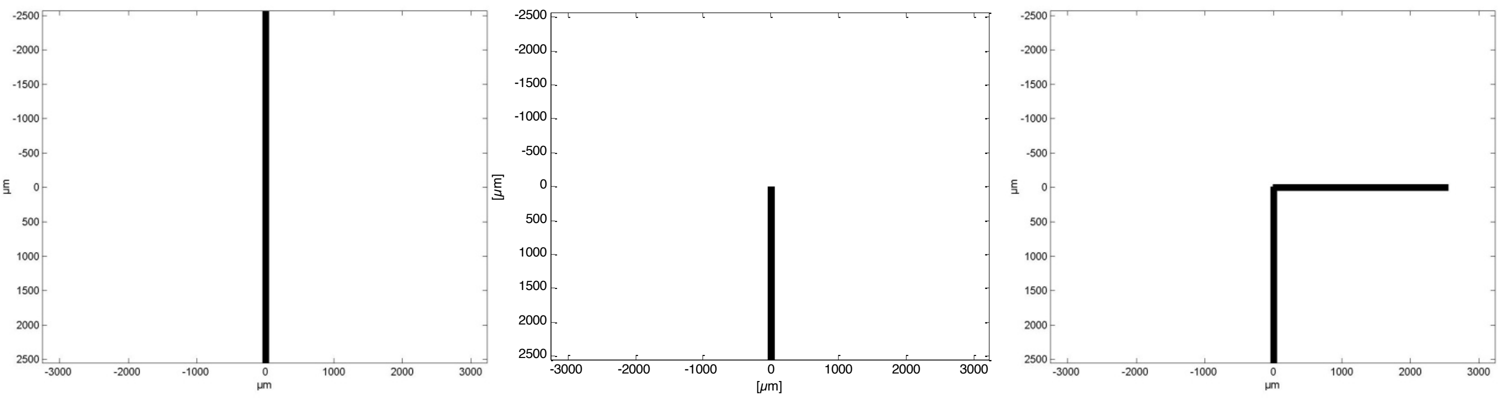
5. Repeat assignment **4**, but only insert the hair halfway, such that one of its ends reaches the origin, see the figure. (This is tricky to do in practice, but we have Matlab!) What happens? **(End of 5)**

Hint: Use Matlab´s logical operator & to create the transmission function

```
inverted_t_hairplane= (abs(umat)< hair_diameter/2) & (vmat>0);
t_hairplane= 1 - inverted_t_hairplane;
```

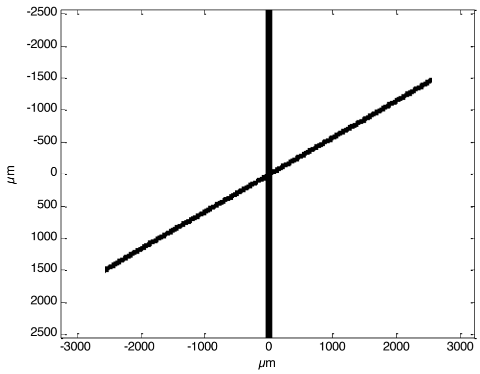
6. Repeat assignment **4**, but instead insert *two* hairs that both start at the origin and make an angle of 90° with respect to each other, as in the figure. Does this cavity possess a mode, and if so, what would you call it? **(End of 6)**

Hint: Create an inverted "horizontal hair" in the same way you created an inverted vertical hair in task 5. Then "add" them using the logical operator 'or', which is written as | in Matlab. (If you can´t find the character you can write 'help relop' in Matlab´s command window, and get a list of all logical operators, and copy-paste the desired one into your code). Finally, you invert everything back, as before.



The transmission function of the "hair plane" in assignments 4, 5, and 6, respectively (white=1, black=0).

PS. At the follow-up we will also demonstrate inserting two hairs through the origin, where one is vertical, and the other one makes an angle 60° with respect to the first. Such a mount exists in the laboratory exercise in *Laser engineering*, as two hairs that are glued onto a frame. Unfortunately, it is very rare that the laser wants to lase when these are inserted into the cavity - as far as I know it is many years since this mode was last observed, and then probably only for a second or so until someone in the room happened to breathe a little carelessly.



The lab frame: two crossed hairs.

<u>Strictly</u> resonance: Is there still something our laser cavity simulation doesn´t tell us about the optical field in the laser cavity?

7. The cavity resonance condition of a laser cavity "dictates" that the field must repeat itself after one round trip (except for some attenuation due to outcoupling losses). Our Matlab simulation is a method to enforce the field repetition. However, in one respect the field repetition condition is *not* strictly fulfilled in the solutions we obtain with our method, i.e., it is still possible that the field before and after a roundtrip are mathematically different.

a) In what way can they differ (again neglecting the attenuation due to cavity loss)?
(End of 7a)

b) Confirm that the field you received in assignment **2** (the fundamental mode) indeed *does not repeat itself* in this respect! **(End of 7b)**

c) What physical simulation parameter would you adjust (very slightly) in your simulation to get perfect repetition in a cavity with a fixed geometry (such adjustment also occurs in a real laser if the field does not repeat itself anymore, for instance after a sudden change of the length of the cavity)? **(End of 7c)**