Home Assigment Week 2

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1 Task 1

The parallel-plate waveguide shown in Figure 1 consists of two infinitely wide perfect conductors separated by a distance d and is partially filled with a dielectric, that has a width W and relative permittivity $\epsilon_r = 3$, and vacuum everywhere else. The relative permeability is assumed to be $\mu_r = 1$ and the electric field strength is assumed to be $E_y^0 = |E_y| = 1\,\mathrm{GVm^{-1}}$.

1.1 a

The lowest order TE-mode have the phasor field components seen in (1).

$$\bar{H}_z(x,y) = \begin{cases} A\sin(K_d x) & , |x| \leq \dfrac{W}{2} \\ Be^{-K_a|x|} & , |x| \geq \dfrac{W}{2} \end{cases} \tag{1}$$

The electric and magnetic field components for a TE-mode can be acquired from (2).

$$\bar{H}_x = -\frac{\gamma}{h^2}\frac{\partial \bar{H}_z}{\partial x}, \tag{2a}$$

$$\bar{H}_y = -\frac{\gamma}{h^2}\frac{\partial \bar{H}_z}{\partial y}, \tag{2b}$$

$$\bar{E}_x = -\frac{j\omega\mu}{h^2}\frac{\partial \bar{H}_z}{\partial y}, \tag{2c}$$

$$\bar{E}_y = \frac{j\omega\mu}{h^2}\frac{\partial \bar{H}_z}{\partial x}, \tag{2d}$$

where

$$\gamma = \alpha + j\beta, \tag{3a}$$

$$h^2 = \gamma^2 + \omega^2\mu\epsilon = \begin{cases} h_d^2 = K_d^2 & , |x| \leq \dfrac{W}{2} \\ h_a^2 = K_a^2 & , |x| \geq \dfrac{W}{2} \end{cases}, \tag{3b}$$

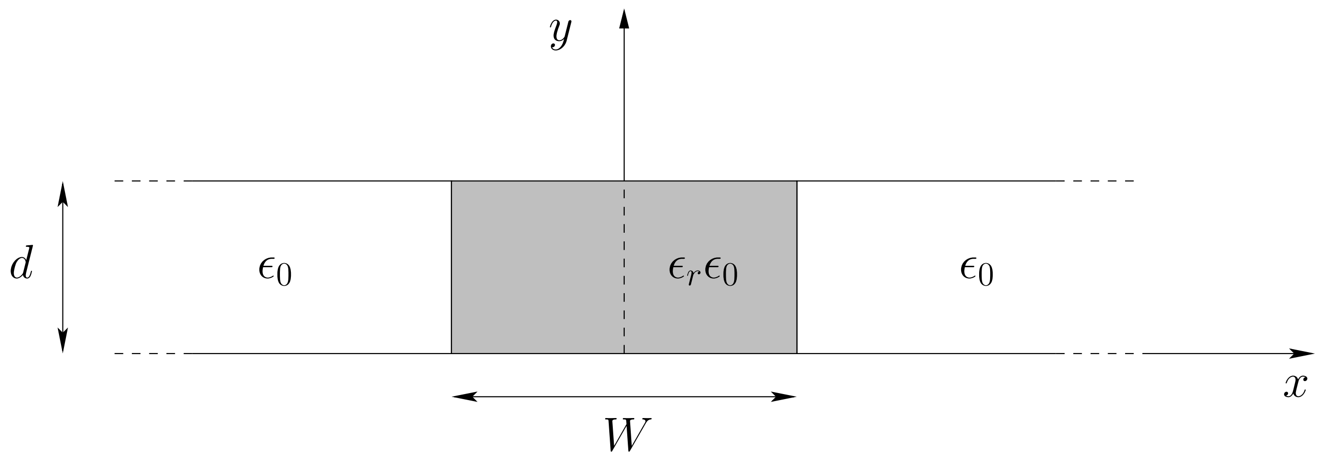

Figure 1: The figure depicts a parallel-plate waveguide consisting of two infinitely wide perfect conductors separated by a distance d and is partially filled with a dielectric slab that has a width W and vacuum everywhere else.

We can conclude that $\bar{H}_z$ has no y-components and we get

$$\bar{H}_x = \begin{cases} \frac{\gamma}{h^2} AK_d \cos(K_d x) & , |x| \leq \frac{W}{2} \\ \frac{\gamma}{h^2} \operatorname{sgn}(x) BK_a e^{-K_a|x|} & , |x| \geq \frac{W}{2} \end{cases}, \tag{4a}$$

$$\bar{H}_y = 0, \forall x, \tag{4b}$$

$$\bar{E}_x = 0, \forall x, \tag{4c}$$

$$\bar{E}_y = \begin{cases} \frac{j\omega\mu}{h^2} AK_d \cos(K_d x) & , |x| \leq \frac{W}{2} \\ \frac{j\omega\mu}{h^2} \operatorname{sgn}(x) BK_a e^{-K_a|x|} & , |x| \geq \frac{W}{2} \end{cases} \tag{4d}$$

Knowing that the the the boundary condition for the H-field at the interface between vacuum and the dielectric is continous (since there are no free currents), we can relate $\bar{H}_z$ in vacuum to $\bar{H}_z$ in the dielectric since $H_t = H_z$.

$$A \sin(K_d \frac{W}{2}) = Be^{-K_a \frac{W}{2}} \Leftrightarrow B = Ae^{K_a \frac{W}{2}} \sin(K_d \frac{W}{2}) \tag{5}$$

A boundary condition is the E_x and E_y needs to vanish at $y = 0$ and $y = d$ but this is already fulfilled since $E_x \equiv 0$ and $E_y \equiv 0$. Another boundary condition is that the normal component of the magetic field and the tangential component of the electric field is continous across an interface, which along with (5) gives

$$\begin{aligned} K_d \cos(K_d \tfrac{W}{2}) &= \operatorname{sgn}(x) K_a e^{K_a \frac{W}{2}} \sin(K_d \tfrac{W}{2}) e^{-K_a \frac{W}{2}} \\ \Leftrightarrow \frac{K_d}{K_a} &= \tan(K_d \tfrac{W}{2}) \end{aligned} \tag{6}$$

Since the electric field strength between the two plates is $1\,\mathrm{GVm}^{-1}$ the amplitude of $\bar{H}_z$ can be related to E_y^0.

$$\frac{j\omega\mu}{h^2} A K_d = E_y^0 \Leftrightarrow A = -j\frac{h^2}{\omega\mu}\frac{E_y^0}{K_d} \tag{7}$$

This makes the solution for an arbitrary wavelength

$$\bar{H}_z = \begin{cases} -j\dfrac{K_d}{\omega\mu} E_y^0 \sin(K_d x) & , |x| \le \dfrac{W}{2} \\ -j\dfrac{K_d}{\omega\mu} E_y^0 \sin(K_d \dfrac{W}{2}) e^{-K_a(|x| - \frac{W}{2})} & , |x| \ge \dfrac{W}{2} \end{cases}, \tag{8a}$$

$$\bar{H}_x = \begin{cases} -j\dfrac{\gamma}{\omega\mu} E_y^0 \cos(K_d x) & , |x| \le \dfrac{W}{2} \\ -j \operatorname{sgn}(x) \dfrac{\gamma}{\omega\mu} E_y^0 \cos(K_d \dfrac{W}{2}) e^{-K_a(|x| - \frac{W}{2})} & , |x| \ge \dfrac{W}{2} \end{cases}, \tag{8b}$$

$$\bar{H}_y = 0, \forall x, \tag{8c}$$

$$\bar{E}_x = 0, \forall x, \tag{8d}$$

$$\bar{E}_y = \begin{cases} E_y^0 \cos(K_d x) & , |x| \le \dfrac{W}{2} \\ \operatorname{sgn}(x) E_y^0 \cos(K_d \dfrac{W}{2}) e^{-K_a(|x| - \frac{W}{2})} & , |x| \ge \dfrac{W}{2} \end{cases} \tag{8e}$$

Using (3b) and (6) we can calculated K_d and K_a numerically. The result for the lowest order TE-mode can be seen graphically in Figure 2 and was calculated to $K_d = 1690$ which gives $K_a = 890$.

1.2 b

In the calculations made to plot the intensity for the relevant components a frequency above the cuf-off frequency was used. The chosen frequency was 50 GHz and the width of the dielectric was 5 mm

1.3 c

The cut-off frequency can be found from (9) using appropriate values for the dielectric. The cut-off frequency is frequency dependent and is about 46.8 GHz for lower frequencies ($f \ll 10^{10}$Hz) and 34.6 GHz for higher frequencies ($f \gg$

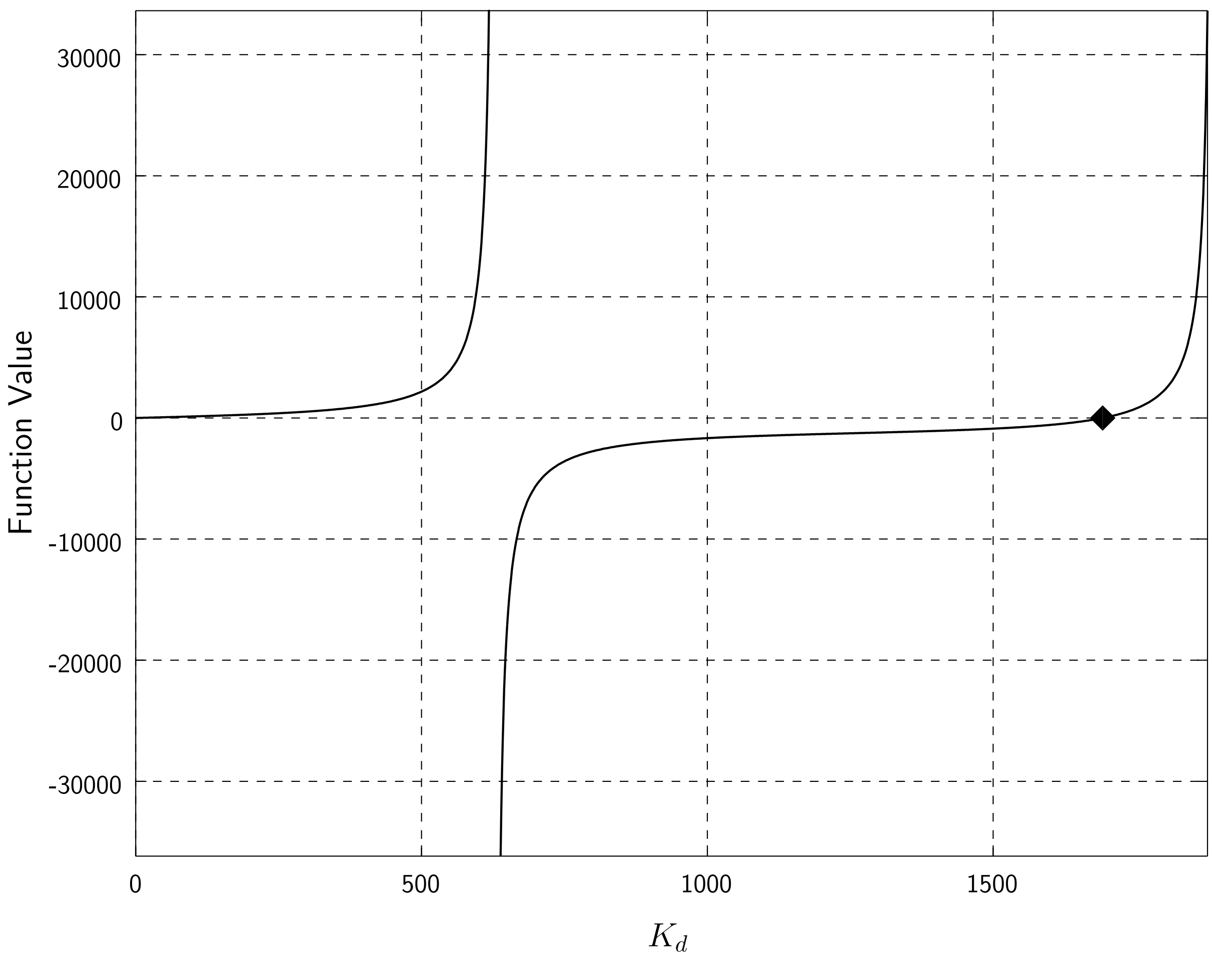

Figure 2: The figure depicts the numerical solution for K_d at 30 GHz.

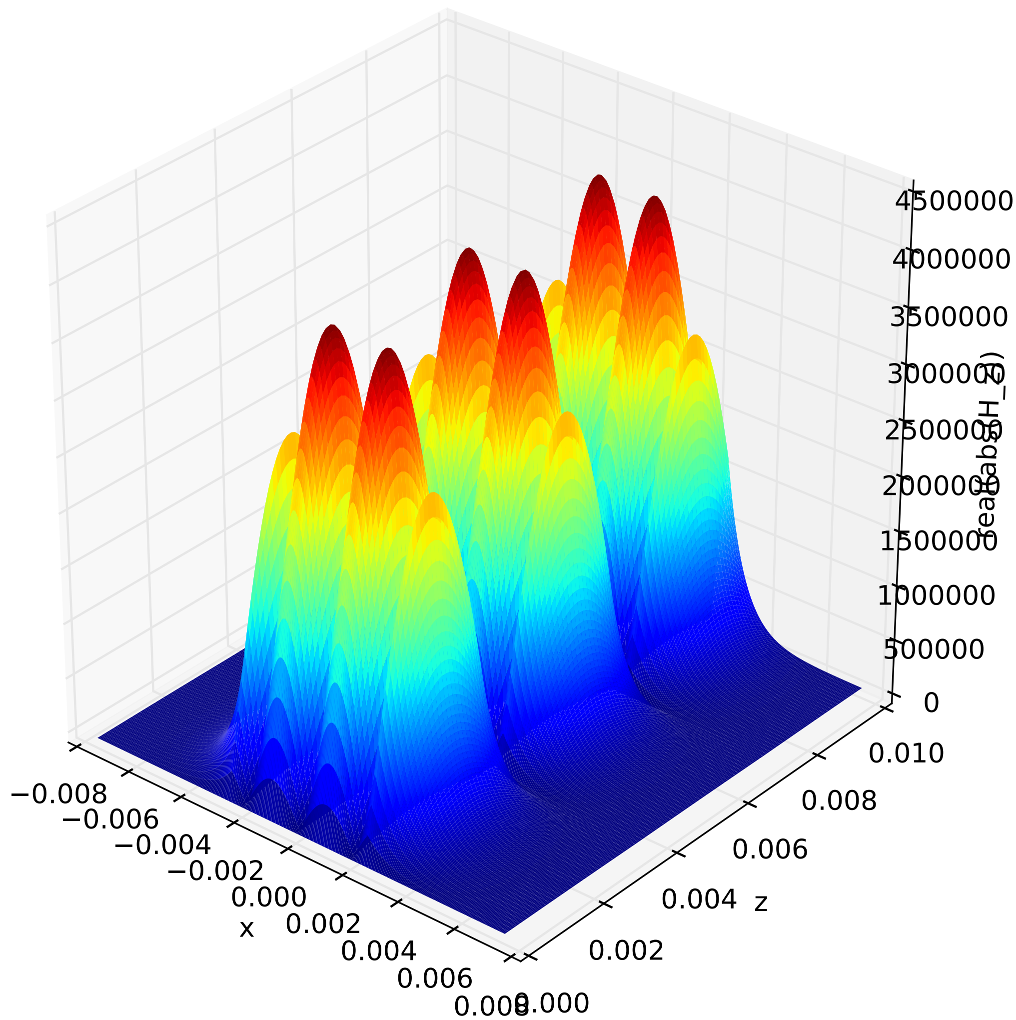

Figure 3: The figure depicts $|\Re(H_z(x,z,t))|$ at 50 GHz when the dielctric has a width of 5 mm.

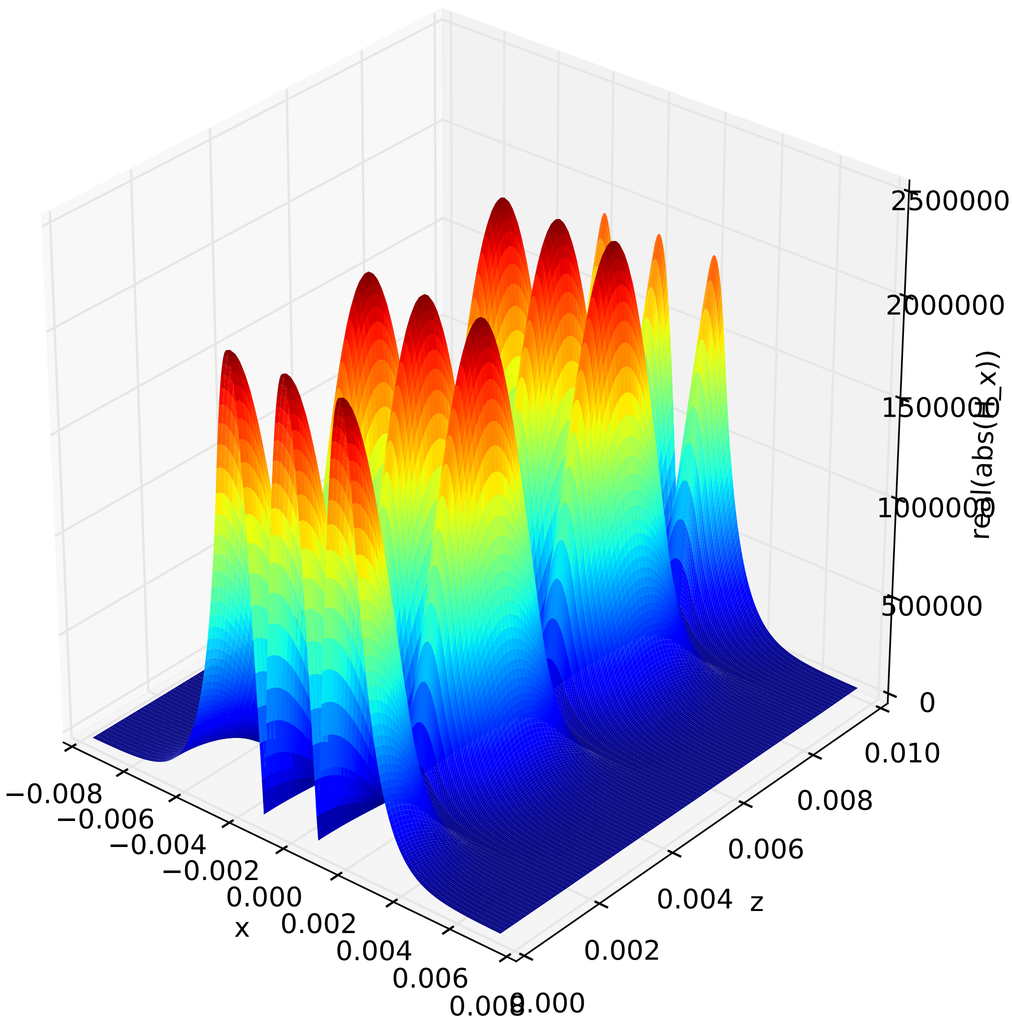

Figure 4: The figure depicts $|\Re(H_x(x,z,t))|$ at 50 GHz when the dielctric has a width of 5 mm.

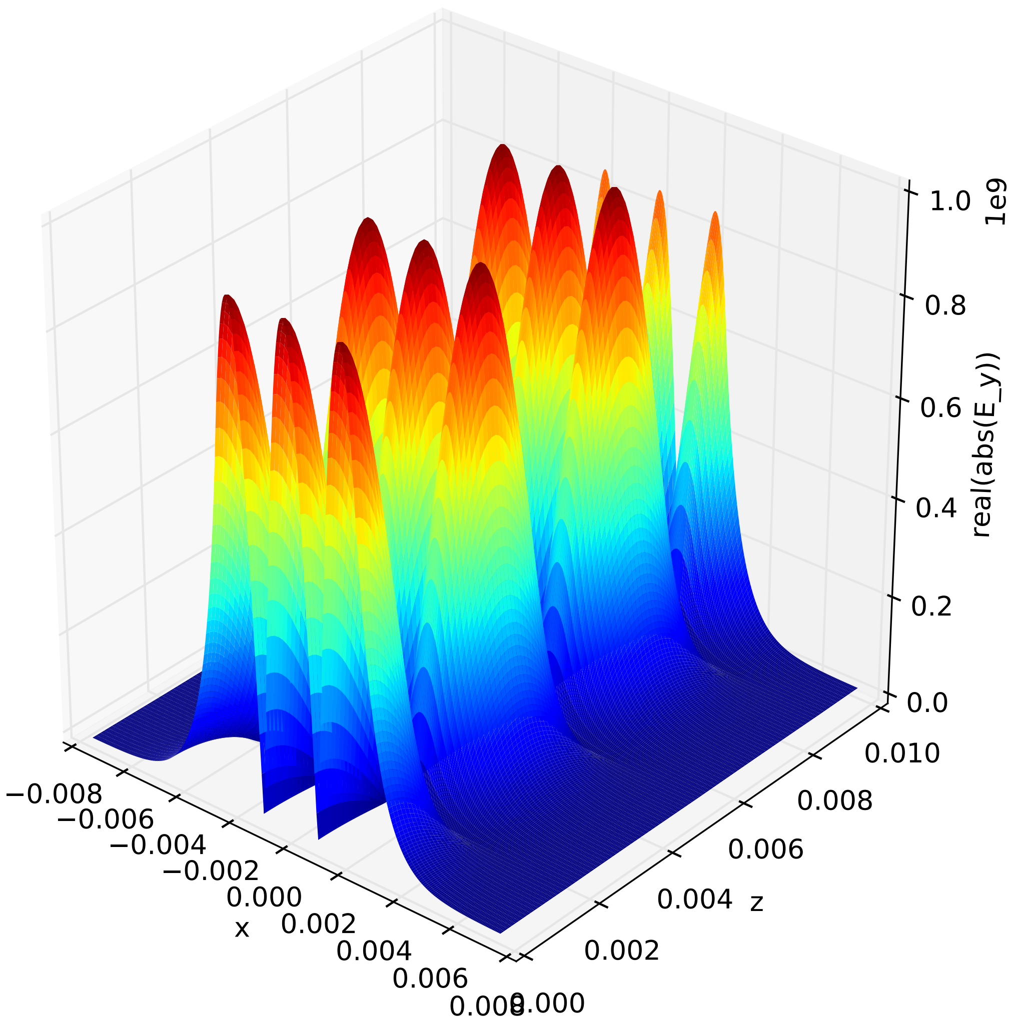

Figure 5: The figure depicts $|\Re(E_y(x,z,t))|$ at 50 GHz when the dielctric has a width of 5 mm.

10^{10}Hz). For the frequency at which the waveguide is designed for the cut-off frequency is 46.6 GHz. Howerver, the wave starts to propagate at about 44.4 GHz which should be the “true” cut-off frequency.

$$f_c = \frac{h}{2\pi\sqrt{\mu\epsilon}} \tag{9}$$

1.4 d

At $f = 30$ GHz the wave is evanescent since this frequency is below the cut-off frequency so it is not possible to create a 90° section.

1.5 e

1.6 f

Magnetic walls are located where $\hat{n} \times \bar{H} = 0$. While there are no complete magnetic walls there are “walls” where the current is several orders of magnitude lower than the peak values, which can bre seen in Figure 6.

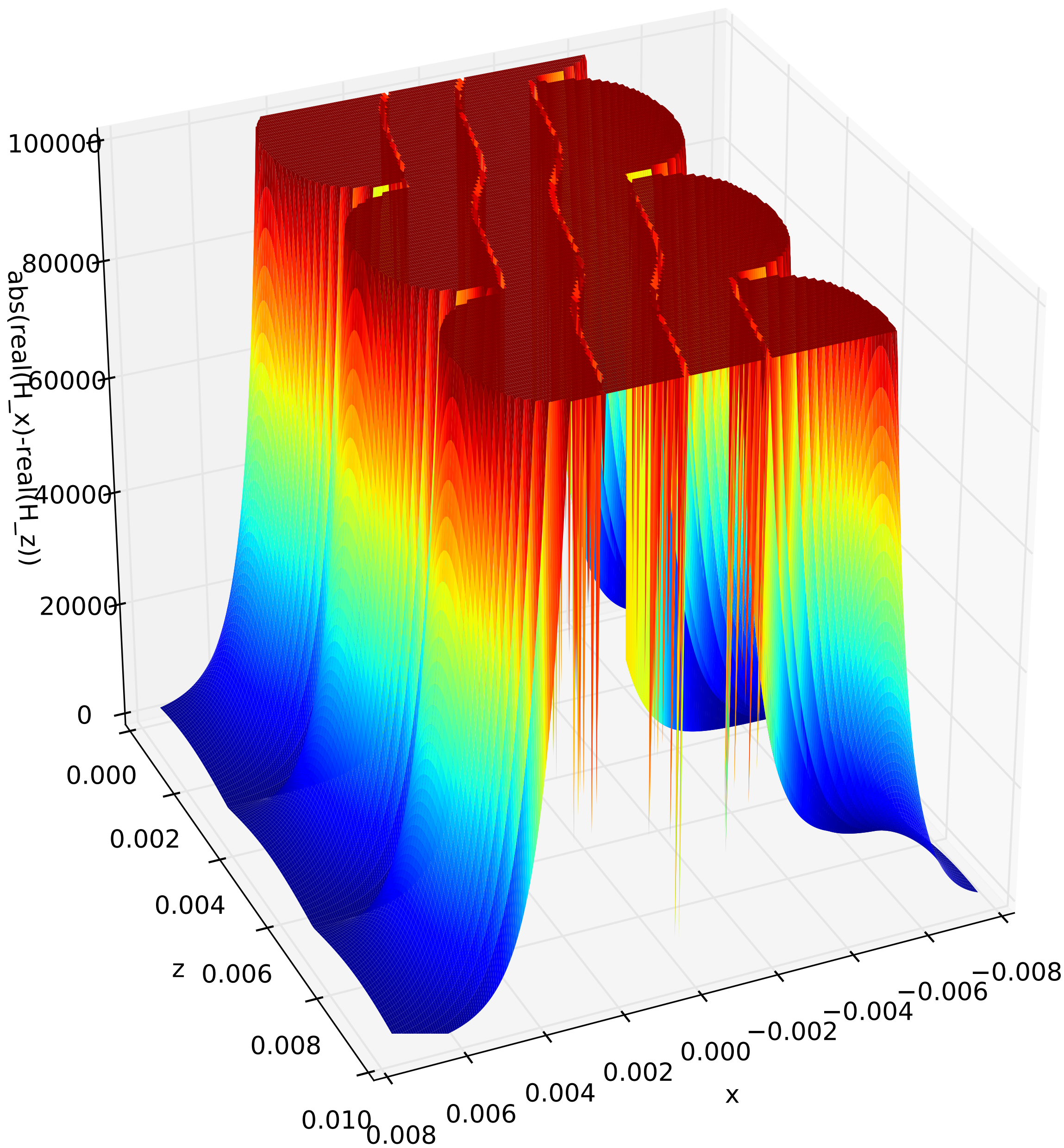

Figure 6: The figure depicts the magnitude of the current at 50 gigaHz when the dielctric has a width of 5 mm. There are parts where the current is several orders of magnitude lower than the peak values.