FUNDAMENTALS OF

PHOTONICS

Numerical tutorial 3 - Imaging

Outline

- **Coherence**
  - Partially coherent light (coherence time, length, area)
  - Effect on interference in space and time

- **Ordinary light sources**
  - Incoherent approximation
  - Modelling wave propagation

- **Imaging in ordinary light**
  - Point spread function

- **Galilean telescope (HA3)**
  - Simulation method
  - Numerical windows

Introduction

Electromagnetic radiation

Coherent

Predictable wave = wave from ”ideal” laser

- Characteristic: **single frequency with ordered phase in time and space** (e.g. plane, spherical wave)
- In microwaves: always assumed
- In photonics: always assumed except
- **Numerical propagation techniques:** **HFM, TSM, FDTD, …**

Incoherent

Chaotic wave ≈ wave from (objects illuminated by) **ordinary light sources**

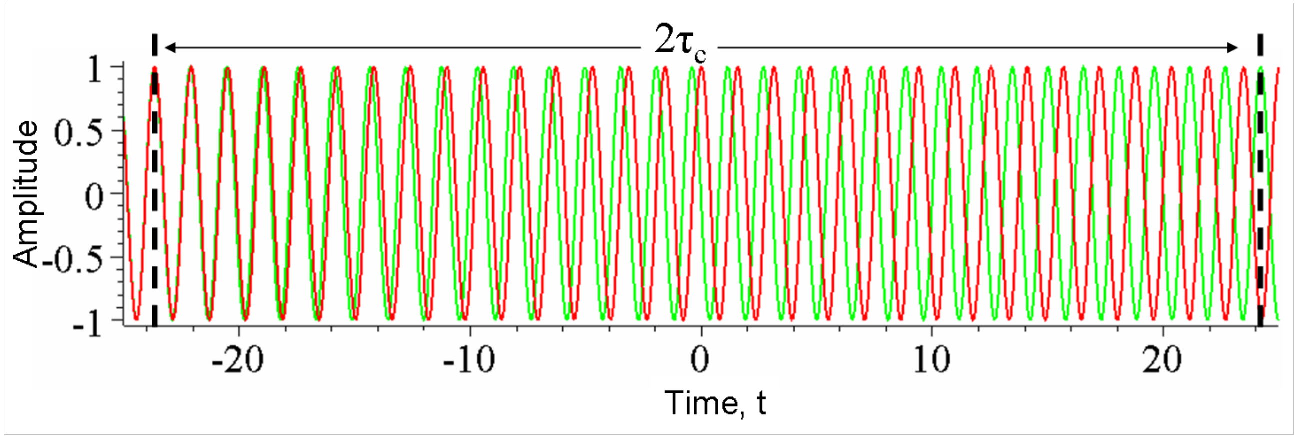
- Characteristic: **broad spectrum with random phase in time and space**
- Case of imaging and lighting applications
- **Numerical propagation techniques:** **None directly !**

Partially coherent light (Statistical Optics)

- **Temporal coherence**

  Time interval within which phase of wave is predictable (wave auto-correlation ↔ phase correlation of a point in wave relative to itself, over time)

  - **Coherence time**, $\tau_c = 1/\Delta\nu$ where $\Delta\nu$ = spectral width of source
  - **Coherence length** (along propagation direction), $L_c = c \cdot \tau_c$



Wave can only interfere with itself delayed with a time < τ_c

↔

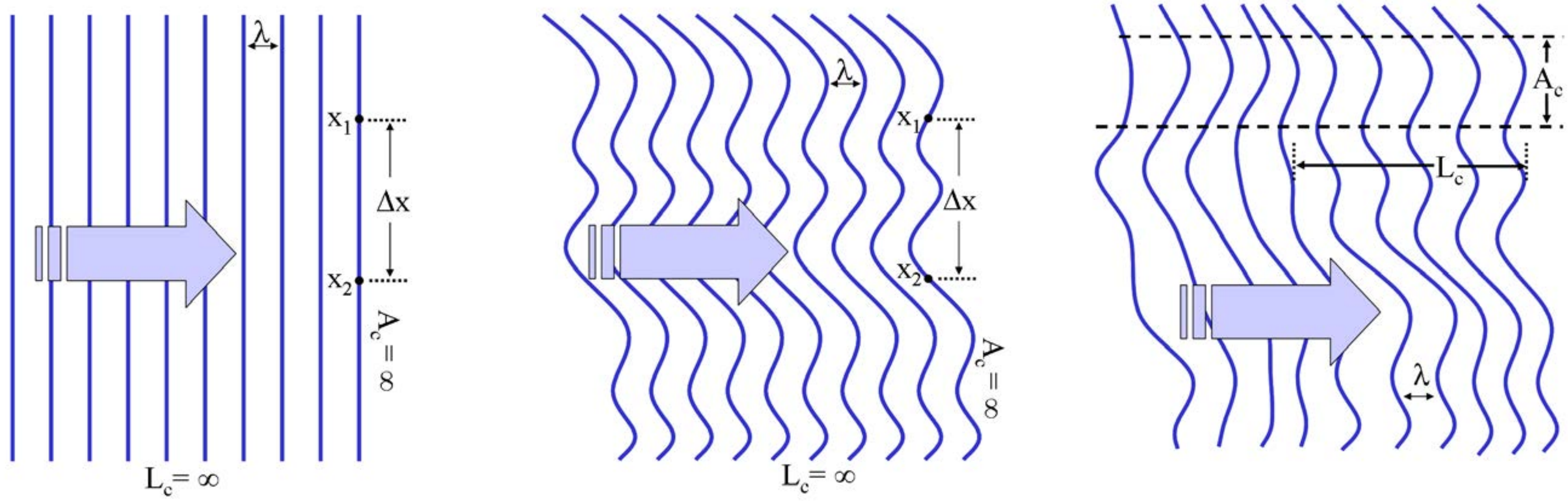
Wave can only interfere with itself propagated a distance < L_c

Source	$\Delta\nu_c$ (Hz)	$\tau_c = 1/\Delta\nu_c$	$l_c = c\tau_c$
Filtered sunlight (λ_o = 0.4–0.8 μm)	3.75×10^{14}	2.67 fs	800 nm
Light-emitting diode (λ_o = 1 μm, $\Delta\lambda_o$ = 50 nm)	1.5×10^{13}	67 fs	20 μm
Low-pressure sodium lamp	5×10^{11}	2 ps	600 μm
Multimode He–Ne laser (λ_o = 633 nm)	1.5×10^{9}	0.67 ns	20 cm
Single-mode He–Ne laser (λ_o = 633 nm)	1×10^{6}	1 μs	300 m

- **Spatial coherence**

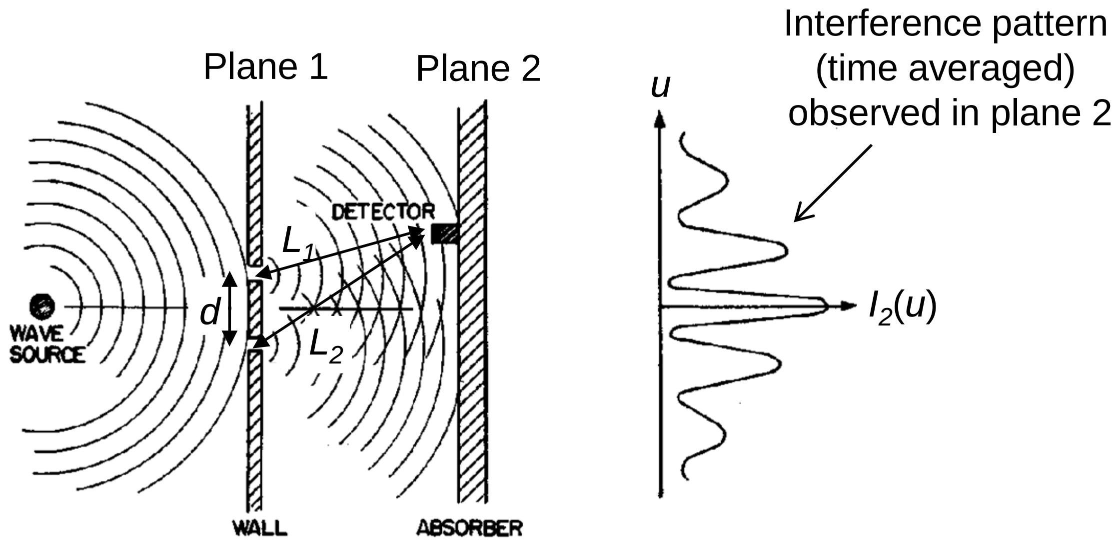
Space interval within which phase of wave is predictable (wave cross-correlation ↔ phase correlation of two points in wave, over time)

- **Coherence area** (normal to propagation direction), A_c, having a coherence **diameter** d_c
- Coherence area changes along free-space propagation (in contrast to coherence time/length), as wave diverges/converges



Interference patterns can only be produced from area A_c

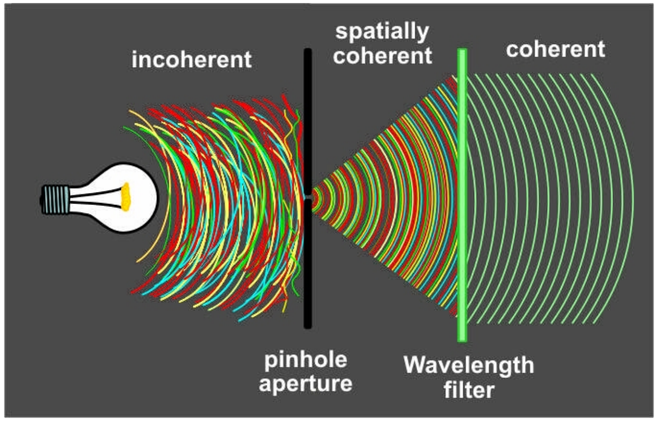
- **Double-slit experiment**



- **Interference pattern appear in plane 2 <u>if:</u>**

1. Slit separation $d<d_c$, where d_c is coherence diameter of source wave at plane 1
2. Optical path difference L_2-L_1<L_c, where L_c is coherence length of source wave

- Coherence of incident wave at plane 1 can be improved:



Alternative to using pinhole aperture is to increase distance between wave source and plane 1

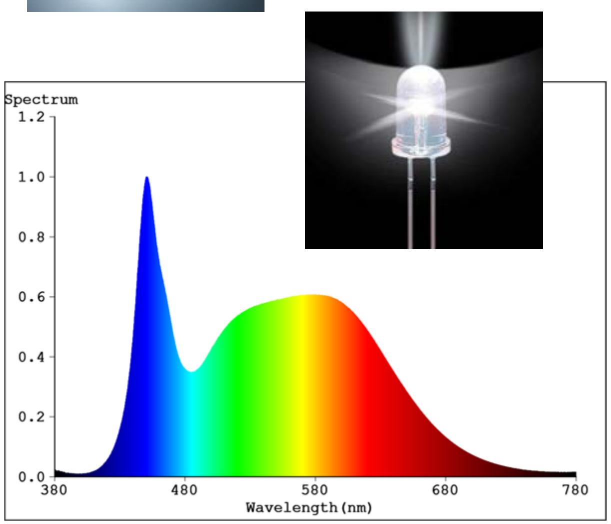
Ordinary light sources

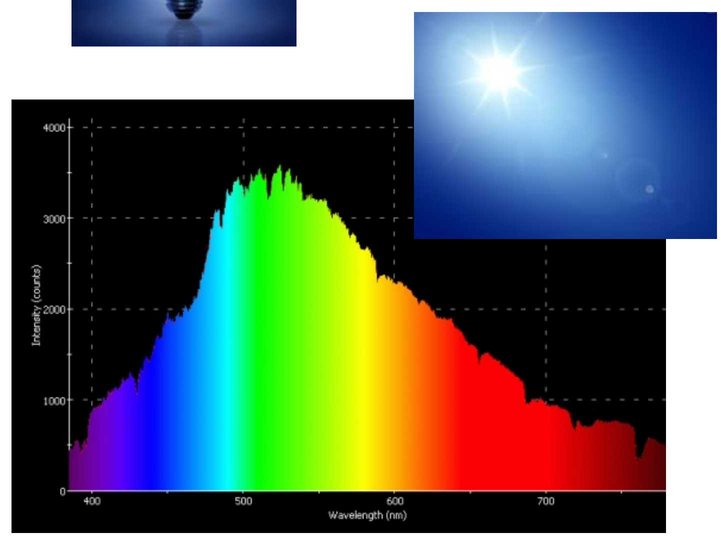
- Ordinary light sources for illumination, such as LED, fluorescent tube, incandescent bulb, and the sun are approximately incoherent:

  1. White light emission ↔ broad frequency extent → **coherence time/length ≈ 0**

  2. Physical size ↔ spatial extent → **coherence diameter ≈ 0** (close to source)

Thus, there will be no time averaged interference effect in space!





- **How to model wave propagation?**

  1. Consider one frequency of source at a time

  2. Consider a point on source surface (or object surface illuminated by source) as a point source, i.e. oscillating dipole, generating a coherent spherical wave with frequency given by 1.

  3. Propagate this spherical wave using e.g. HFM or TSM through optical system

  4. Repeat 1-3. for all frequencies***** and points on source surface

  5. <u>Sum intensities</u> (not E-fields!) to get total intensity, resulting from all propagated spherical waves in 4. This is equivalent to that point sources on source surface are uncorrelated and no interference effects occur

- **Alternative ”natural” model (not used in HA3):**

  Same as above, but in 2. generate a random starting phase for spherical wave. In 5. E-fields can now be conventionally summed to get total intensity.

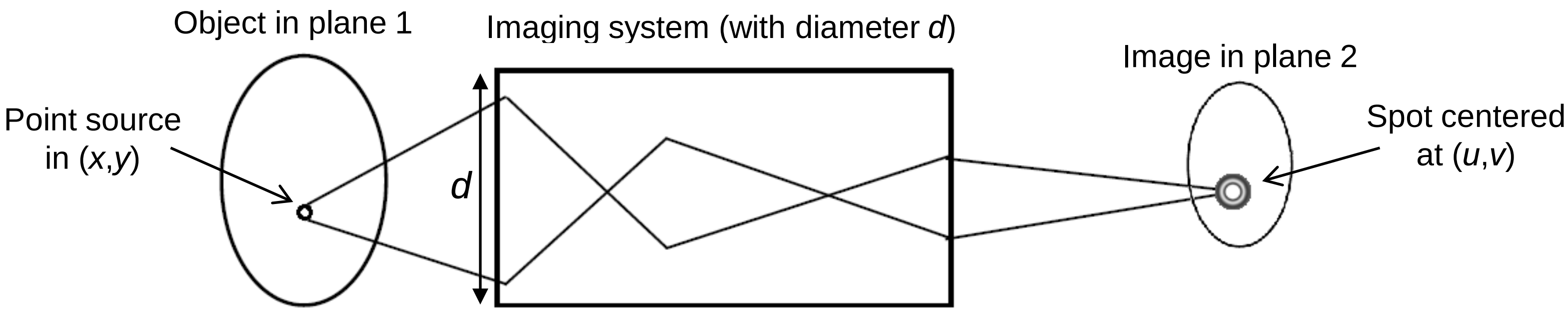
  However, to get a correct statistical average points 1-5 must be redone a large number of times
  → **inefficient numerically method!**

*Often only one ”typical” wavelength is studied (as in HA3)

Imaging

Definitions:

- Plane 1 ≡ **object plane** ↔ **object illuminated by ordinary light**, i.e. from an incoherent light source
- Optical system ≡ imaging system
- Plane 2 ≡ **image plane** ↔ **contains a ”blurred” and scaled replica of input plane**



Function:

Coherent light emitted from point source in plan 1, and captured by optical system, ends up in spot with diameter ~λ/d, centered at unique point in plane 2.

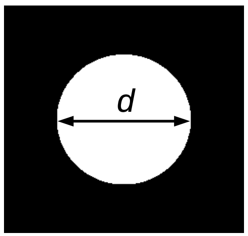
- Spot centered at (u=0,v=0) ↔ **impulse respons function** $h(u,v)$, but here referred to as **point spread function**
- Linear system + incoherent summation → image intensity (considering just one source frequency, $\nu=c/\lambda$):

$$I_2(u,v) \sim |E_1(x,y)|^2 \otimes |h(u,v)|^2 = I_1(x,y) \otimes |h(u,v)|^2$$

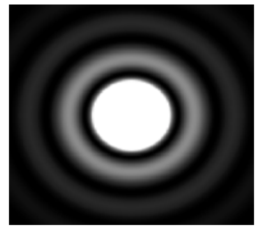
Point spread function (PSF)

- **Finite diameter of PSF results from diffraction** of coherent wave from point source with circular aperture of imaging system (e.g. shutter aperture of a camera lens)

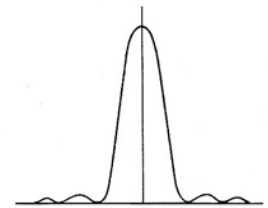
Aperture



Airy pattern



2.44λL/d



2.44λL/d

1st-order Bessel function

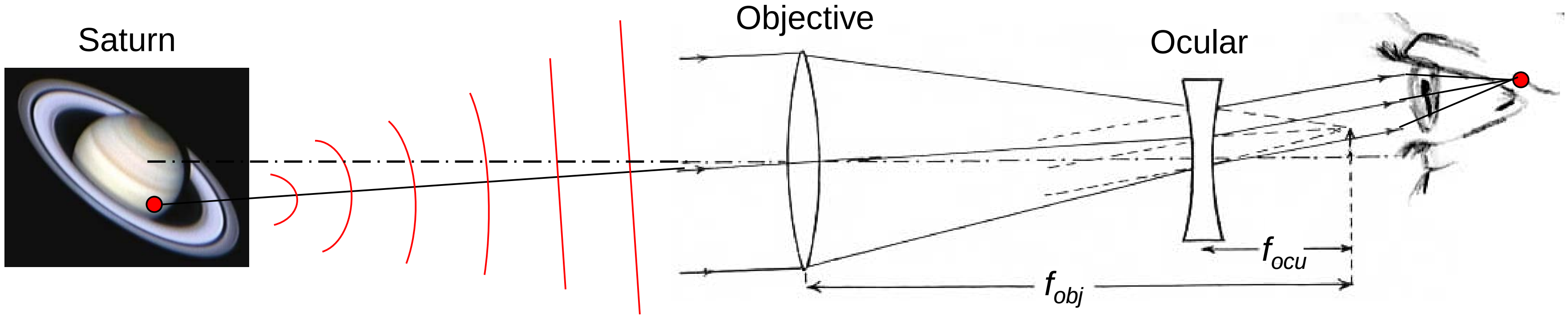
$$I(u,v) \propto \left(\frac{2J_1(\pi d r/\lambda L)}{(\pi d r/\lambda L)} \right)^2,$$

$$r = \sqrt{u^2 + v^2}$$

- Larger aperture → increased image resolution (but trade-off with depth of focus…)

Pictures taken by the Hubble telescope before and after improving PSF:

Galilean telescope (HA3)



- **Light coming from a distant point source has approximately planar wavefronts**, i.e. parallel incoming rays:

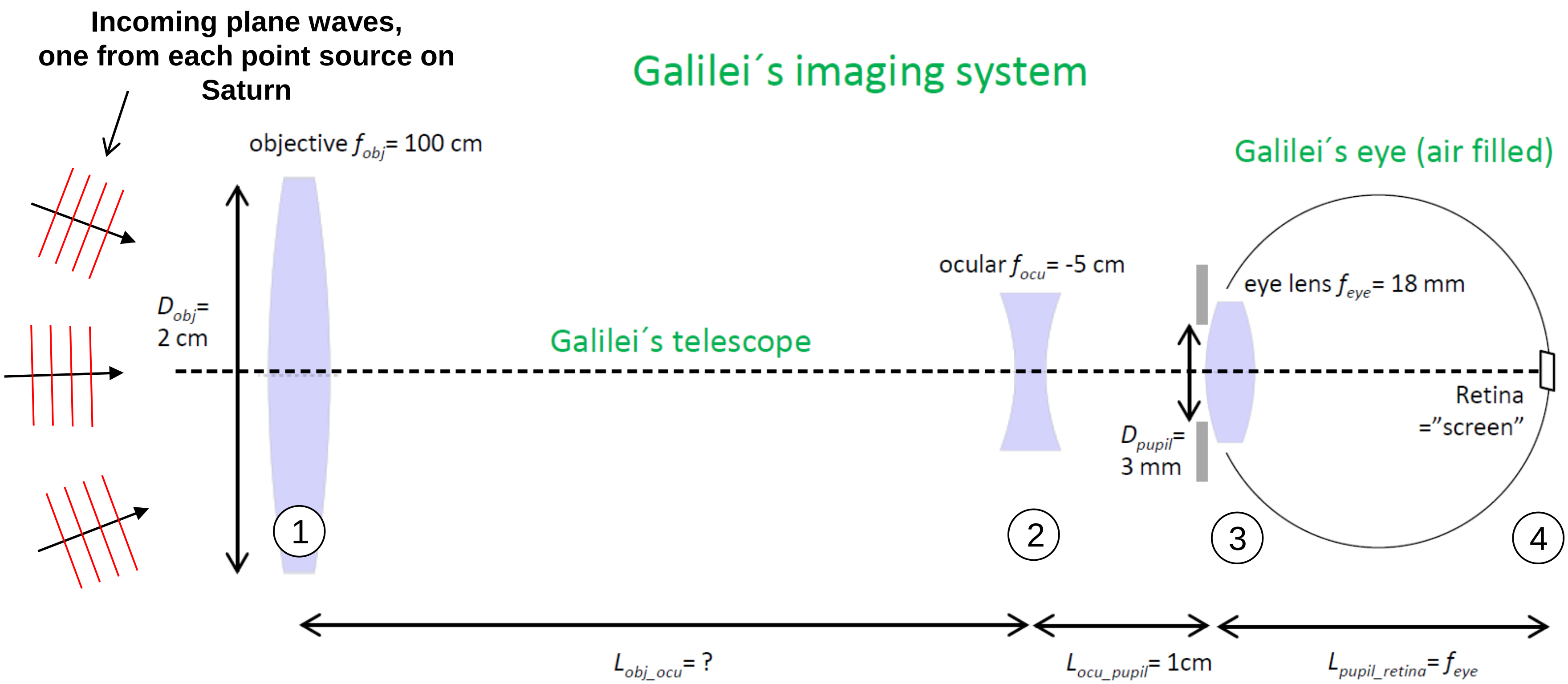
  HFM for a distant point source:

$$E_1(x,y) = \delta(x,y), \quad L \to \infty \quad \Rightarrow \quad E_2(u,v) \sim \exp\left(-jk\frac{ux+vy}{L}\right) \quad \Leftrightarrow$$

Plane wave arriving at an angle (*x*/*L*,*y*/*L*)!

- A convex objective lens is used to capture parallel incoming rays, and a concave ocular lens is used to redirect them towards observer’s eye
- Lens in eye itself brings parallel incoming rays to a focus on observer’s retina

Galilei´s imaging system



Propagation	**1**	**1→2**	**2**	**2→3**	**3**	**3→4**	**4**
Method	Objective aperture + lens transmission function	TSM	Lens transmission function	TSM	Pupil aperture + lens transmission function	TSM	
Numerical window	2 x 2 cm		2 x 2 mm		2 x 2 mm		0.2 x 0.2 mm