HFSS part 2: Chebyshev transformer

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1 Task 1

This part involved designing a Chebyshev transformer (with a center frequency of 6 GHz for a waveguide with dimensions $a = 80$ mm and $b = 10$ mm) with some predefined properties depending on assigned design id. For me the design id was 6 which means that I used the filter properties in Table 1
The fractional bandwidth presented in Table 1 can be used to calculate θ_m using (1)

$$1 = \frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} \Leftrightarrow \theta_m = \frac{\pi}{4} \tag{1}$$

When θ_m is known the number of sections needed can be calculated using (2)

$$\cos^{-1}\theta_m = \cosh\left[\frac{1}{N}\operatorname{arccosh}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right] \tag{2}$$

Since the resulting value of N from (2) is not an integer it has to be rounded off and used to calculate a new value of θ_m. The result is $N = 3$ and $\theta_m \approx 0.808$.

When the number of sections is known what remains is to calculate the reflection coefficien Γ in each interface and from there calculate what wave impedances are needed in the sections. By matching the total reflection coefficient and its components to the Chebyshev polynom (type I) corresponging

Table 1: Design constraints of the Chebyshev transformer.

Design ID	$\frac{Z_0}{Z_L}$	Fractional bandwidth	Γ_m
6	3.4	1	0.07

to the number of sections using (3)

$$\Gamma(\theta) = 2e^{-j3\theta}\sum_{n}\Big(\Gamma_n\cos(N-2n)\theta\Big) = -\Gamma_m e^{-j3\theta}T_N\Big(\frac{\cos\theta}{\cos\theta_m}\Big) \tag{3}$$

In this case we have $N = 3$ and $T_3(x) = 4x^3 - 3x$ so the reflection coefficients can be gotten by equating cosine terms as seen in (4)

$$\Gamma_0 = \Gamma_3 = -\frac{\Gamma_m}{2\cos^2\theta_m}, \tag{4a}$$

$$\Gamma_1 = \Gamma_2 = -\frac{3}{2}\Gamma_m\Big(\frac{1}{\cos^3\theta_m} - \frac{1}{\cos\theta_m}\Big) \tag{4b}$$

then the characteristic impedances can be acquired from (5)

$$Z_{i+1} = Z_i\frac{1+\Gamma_i}{1-\Gamma_i} \tag{5}$$

We now encounter a potential problem because (4) and (5) can be used to calculate the ratio between Z_L and Z_0, but this can also be calculated since the ratio $\frac{Z_0}{Z_L}$ was given as a design property. As it truns out those expressions do not match so this problem has no solution. Instead we have to find the solution that have properties that are as close to those in Table 1 as possible. This will be done by minimizing (6a) with the condition that (6b) and (6c) must be fulfilled. (6b) comes from the fact that Γ_m must be the same in (4a) and (4b) while (6c) comes from the fact that recursive use of (5) must yield the same ratio between Z_L and Z_0 as the ratio given as a design property.

$$f(\theta_m^{'},\Gamma_0^{'},\Gamma_1^{'}) = \Big(\frac{|\theta_m^{'}-\theta_m|}{\theta_m}\Big)^2 + \Big(\frac{|\Gamma_0^{'}-\Gamma_0|}{\Gamma_0}\Big)^2 + \Big(\frac{|\Gamma_1^{'}-\Gamma_1|}{\Gamma_1}\Big)^2, \tag{6a}$$

$$\Gamma_0^{'} = \frac{\Gamma_1^{'}}{3\sin^3\theta_m}, \tag{6b}$$

$$\frac{Z_L}{Z_0} = \Big(\frac{1+\Gamma_0^{'}}{1-\Gamma_0^{'}}\Big)^2\Big(\frac{1+\Gamma_1^{'}}{1-\Gamma_1^{'}}\Big)^2, \tag{6c}$$

The result from this optimization problem was min $f(\theta_m^{'},\Gamma_0^{'},\Gamma_1^{'}) = 0.31453276$ after the final iteration (and 0.3959806 after one iteration) and the final values can be seen in Table 2 and the theoretical S_{11} plot can be seen in Figure 1

1.1 Part A

One way to create a section of the transformer is to use the same dimensions as the waveguide but fill it with a different dielectric. the relative permittivity

Table 2: Final transformer properties after optimization.

Property	Value
$\frac{Z_0}{Z_L}$	3.4
Γ_m	0.077
$\frac{\Delta f}{f_0}$	0.969
N	3
θ_m	0.810

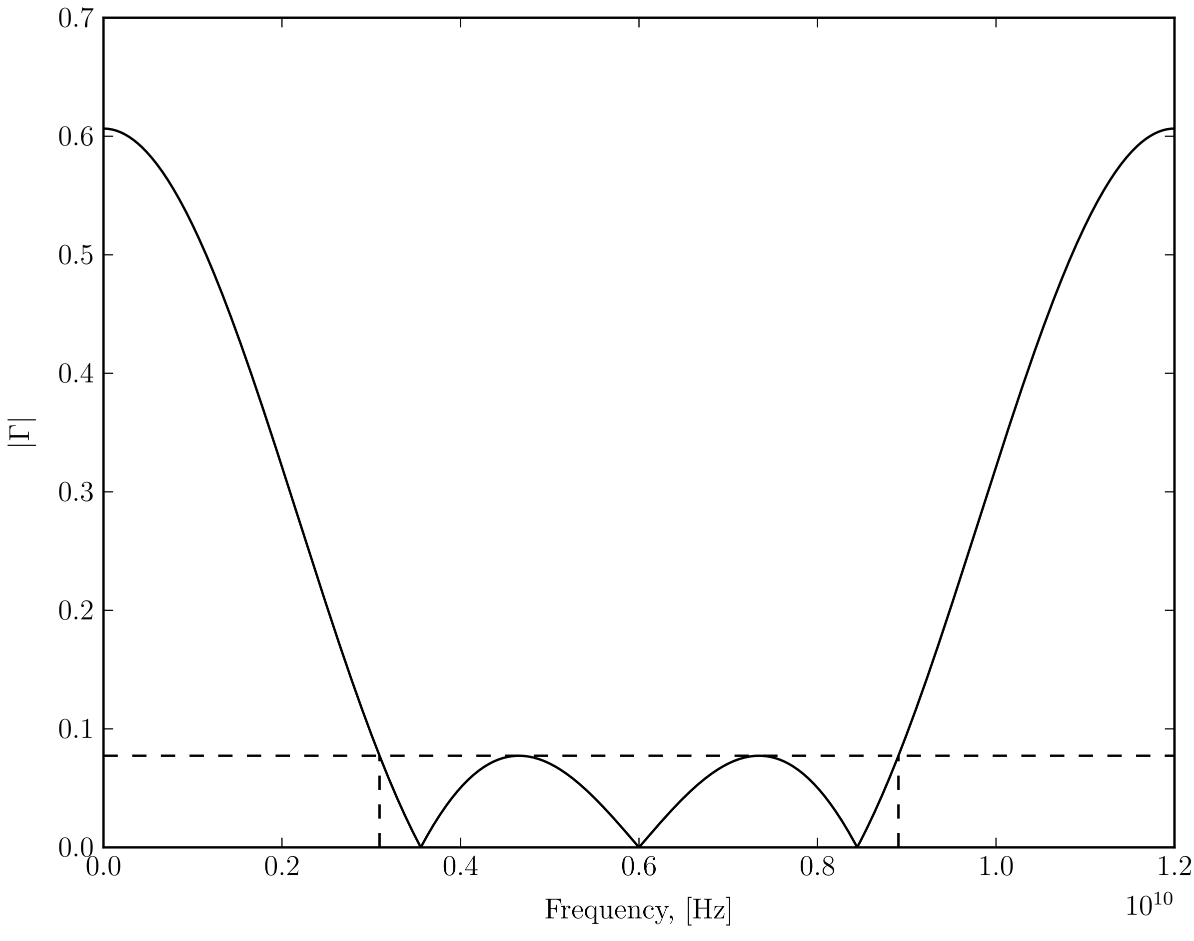

Figure 1: Theoretical S_{11} output for both transformers using the properties in Table 2.

Table 3: Chebyshev transformer properties using variable dielectric.

	ϵ_r	$\frac{\lambda_g}{4}$
Z_0	1.00	13.15 mm
Z_1	1.55	10.38 mm
Z_2	3.17	7.13 mm
Z_3	6.60	4.90 mm
Z_4	10.53	3.87 mm

of the dielectric can be calculated from (7)

$$Z_{\mathrm{TE}} = \frac{k\eta}{\beta} = \frac{\omega\mu}{\sqrt{\omega^2\mu\epsilon - k_c^2}} = \frac{k\eta}{\beta} = \frac{\omega\mu}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}} \tag{7}$$

For a TE_{10}-mode the relative permittivity (assuming the relative permeability is equal to unity) can be calculated from (8)

$$\epsilon_r = \frac{c_0}{\omega}\left[\left(\frac{\omega\mu}{Z_{\mathrm{TE}}}\right)^2 + \left(\frac{\pi}{a}\right)^2\right] \tag{8}$$

The relation between the relative permittivity of the dielectric can be found regardless of the wave impedance of the waveguide but the length of the section (which is $\frac{\lambda_g}{4}$) requires knowlege of the wave impedance of the waveguide. The value of the relative permittivity and section length, assuming the waveguide has $\epsilon_r = 1$, can be found in Table 3.

The simulations results shown in Figure 2 indicates that the theoretical bandwidth ($\Delta f = 6\,\mathrm{GHz}$) was not achieved with this setup, but instead only 5.41 GHz was achieved. One reason for this could be that the transformer in *HFSS* is not ideal.

To improve the results in *HFSS* one could tweak the material parameters or perhaps change the design properties of the transformer.

1.2 Part B

Another way to create a section of the transformer is to vary the height of the sections and use the same dielectric as that of the waveguide. the height of the sections can be calculated from (9)

$$b_i = b_0\frac{Z_i}{Z_0} \tag{9}$$

The value of the section height and section length, assuming the waveguide has $\epsilon_r = 1$, can be found in Table 4 The simulation results shown in Figure 3

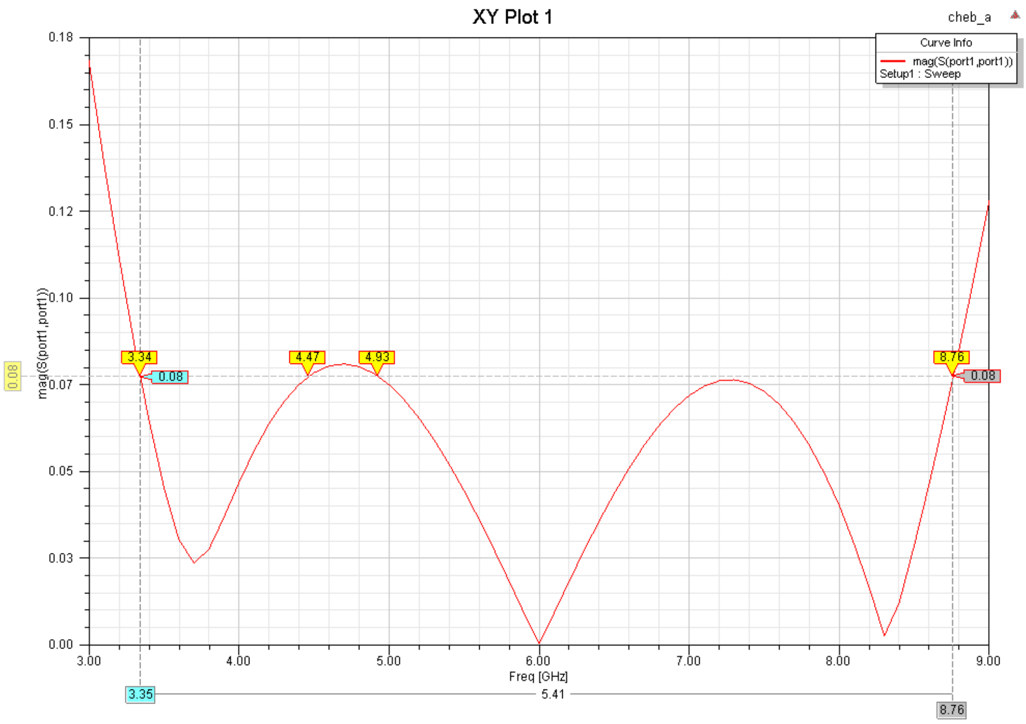

Figure 2: S_{11} plot using a Chebyshev transformer with 3 sections with varying dielectric.

Table 4: Chebyshev transformer properties using variable height.

	b	$\frac{\lambda_g}{4}$
Z_0	10.00 mm	13.50 mm
Z_1	7.89 mm	13.50 mm
Z_2	5.42 mm	13.50 mm
Z_3	3.73 mm	13.50 mm
Z_4	2.94 mm	13.50 mm

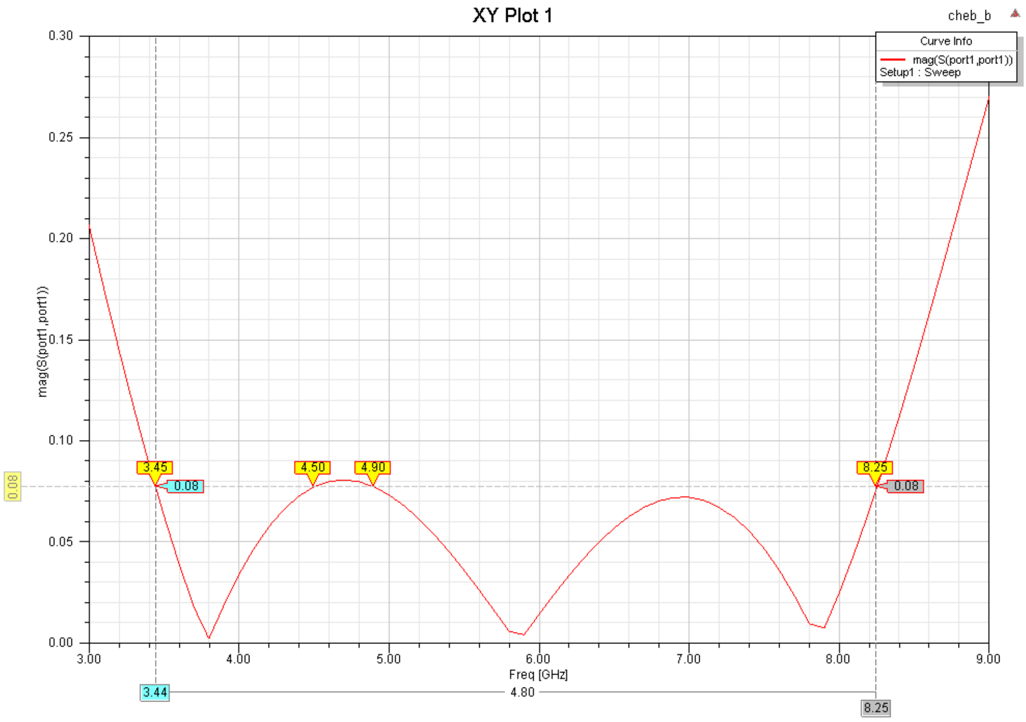

Figure 3: S_{11} plot using a Chebyshev transformer with 3 sections with varying height.

shows that the theoretical bandwidth ($\Delta f = 6\,\text{GHz}$) was not achieved with this setup, but instead only 4.80 GHz was achieved. One reason for this could be that the transformer in *HFSS* is not ideal.

2 Discussion

The transformer in Section 1.1 has better bandwidth than the transformer in Section 1.2 so theoretically a transformer with variable dielectric should be preferred. In reality it might not be as easy to find materials with dielectric that completely match the desired permittivity while the dimensions of the transformer is much easier to modify so for practical reasons the transformer in Section 1.2 should be preferred.