Solutions to Exam in Microwave Engineering (MCC121)

Friday, December 20, 2013

MCC121

Exam in **Microwave Engineering**

Friday, December 20, 2013, 1400 - 18:00, "H-salar"

Teachers: Jan Stake phone: 031 -772 1836
Mark Whale phone: 031 -772 5653

During the exam, teacher will visit around 1430 and 1600.

Examiner: Prof. Jan Stake. Terahertz and millimetre wave lab., Department of microtechnology and nanoscience (MC2), Chalmers University of Technology.

The inspection of the results can be done in my office (D615), MC2-building, Friday, January 17th, 13:00-14:30. The final results will be sent to registrar office on January 20th, 2013.

This is an open book exam. The following is allowed:

- *Calculator (approved by Chalmers)*
- *"Microwave Engineering" by Pozar*
- *Mathematics handbook (Beta)*
- *Smith charts*

To pass this written examination, you need at least 24p out of 60p. Final grade of the course will also include results from assignment 1. That is: 3 (≥28p), 4 (≥42p) and 5 (≥56p).

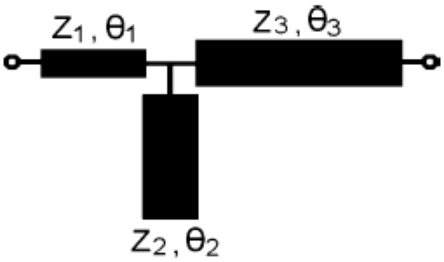
Teamwork is not permitted on this examination. The university academic integrity policy will be strictly enforced. Failure to comply with the academic integrity policy will result in a zero for this examination.

Make sure you have understood the question before you go ahead. Write shortly but make sure your way of thinking is clearly described. It is imperative to clearly explain how the results have been obtained. Solve the problem as far as you can – constructive, creative and valuable approaches are also rewarded. Assume realistic numbers/parameters when needed if data is missing in order to solve the problem.

❖ ❖ ❖

Problem 1

As described in the appended paper, a possible route to reduce the outer dimensions of a Branch-Line Coupler is to replace the quarter wavelength sections with the T-shaped shown below. Prove and derive design equations for the special case when $Z_1=Z_3$ and $\theta_1=\theta_3$. What is the total length of the equivalent circuit, if we chose $Z_1=1.7Z_0$, where Z_0 is the characteristic impedance of the original quarter wave section.



Equivalent T-shaped structure of quarter-wavelength transmission line.

From: Liao, S.-S, et al. (2005). A novel compact-size branch-line coupler. *Microwave and Wireless Components Letters, IEEE*, *15*(9), 588–590. http://dx.doi.org/10.1109/LMWC.2005.855378

(10p)

Solution:

Given: $Z_1=Z_3$ and $\theta=\theta_1=\theta_3$. Set the susceptance presented by the open parallel stub equal to jb. The ABCD matrix for a quarter wave section and the equivalent circuit has to be equivalent at the centre frequency, that is:

$$\begin{bmatrix} 0 & jZ_0 \\ j/Z_0 & 0 \end{bmatrix}_{quarter\ wave} =$$

$$\begin{bmatrix} cos\theta & j\cdot Z_1\cdot sin\theta \\ \frac{j}{Z_1}sin\theta & cos\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ jb & 1 \end{bmatrix}\begin{bmatrix} cos\theta & j\cdot Z_1\cdot sin\theta \\ \frac{j}{Z_1}sin\theta & cos\theta \end{bmatrix}=$$

$$\begin{bmatrix} cos\theta & j\cdot Z_1\cdot sin\theta \\ \frac{j}{Z_1}sin\theta & cos\theta \end{bmatrix}\begin{bmatrix} cos\theta & j\cdot Z_1\cdot sin\theta \\ jbcos\theta+\frac{j}{Z_1}sin\theta & -bZ_1\cdot sin\theta+cos\theta \end{bmatrix}=$$

$$\begin{bmatrix} cos\theta cos\theta - Z_1\cdot sin\theta\left(bcos\theta+\frac{1}{Z_1}sin\theta\right) & j\cdot Z_1\cdot cos\theta sin\theta + j\cdot Z_1\cdot sin\theta(-bZ_1\cdot sin\theta+cos\theta) \\ \frac{j}{Z_1}sin\theta cos\theta + cos\theta\left(jbcos\theta+\frac{j}{Z_1}sin\theta\right) & -sin\theta sin\theta + cos\theta(-bZ_1\cdot sin\theta+cos\theta) \end{bmatrix}$$

$$=\begin{bmatrix} cos\theta cos\theta - sin\theta sin\theta - Z_1\cdot b\cdot sin\theta cos\theta & j\cdot Z_1\cdot(2cos\theta sin\theta - Z_1\cdot b\cdot sin\theta sin\theta) \\ \frac{j}{Z_1}(2sin\theta cos\theta + Z_1\cdot b\cdot sin\theta cos\theta) & cos\theta cos\theta - sin\theta sin\theta - Z_1\cdot b\cdot sin\theta cos\theta \end{bmatrix}$$

$$A = D = 0 = cos\theta cos\theta - sin\theta sin\theta - Z_1\cdot b\cdot sin\theta cos\theta \rightarrow Z_1\cdot b\cdot sin2\theta = 2cos2\theta$$

$$\rightarrow Z_1\cdot b = \frac{2}{tan2\theta}$$

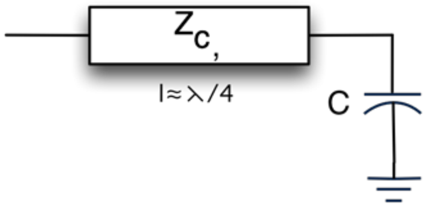
B=j$Z_0 \rightarrow \frac{Z_0}{Z_1} = (2cos\theta sin\theta - Z_1 \cdot b \cdot sin\theta sin\theta) = \{use\ identity\ from\ \ A = 0\} =$
$sin\theta\left(2cos\theta - \frac{cos\theta cos\theta - sin\theta sin\theta}{cos\theta}\right) = tan\theta(cos\theta cos\theta + sin\theta sin\theta) \rightarrow$
$Z_1 = \frac{Z_0}{tan\theta}$ ***(5p points total for correct solution strategy, 3 points for correct calculations)***

So, design equations are: $Z_1 = \frac{Z_0}{tan\theta}$ and $b = \frac{2}{Z_1 tan2\theta}$

For Z_1=1.7Z_0, we have $1.7 = \frac{1}{tan\theta} \rightarrow \theta = 30°$. Hence, the total length is reduced from 90 degrees (quarter wave) to 2θ=**60 degrees** ***(2p correct length, also without derivation)***

Problem 2

An open transmission line with α=β, is equivalent to a series resonant circuit for a frequency when l≈λ/4. However, due to stray fields at the end of the line, which give rise to an equivalent capacitance C, the resonant frequency is shifted. Determine the exact length, l, for resonance. Explain also the outcome of the analytical solution in a Smith Chart.



(10p)

Solution: At the resonant frequency the input impedance, Z_{in}, should be real. For a lossy transmission line of length l and characteristic impedance Z_0, it is given by:
$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} = Z_0 \frac{\frac{Z_L}{Z_0} + \tanh(\gamma l)}{1 + \frac{Z_L}{Z_0}\tanh(\gamma l)}$, where $\frac{Z_L}{Z_0} = \frac{1}{j\omega C Z_0} = -j\tan x = -\tanh(jx)$ with $x = \arctan(\frac{1}{\omega C Z_0})$. This yields now $Z_{in} = Z_0 \frac{\tanh(\gamma l) - \tanh(jx)}{1 - \tanh(\gamma l)\tanh(jx)} = Z_0 \tanh(\gamma l - jx)$. The trigonometric identity is found in Beta p.123 or Pozar Ch. 6 on TL resonators. The imaginary part of Z_{in} is zero for certain lengths when $\beta l - x = 0 + n\pi \rightarrow \beta l = \arctan\left(\frac{1}{\omega C Z_0}\right) + n\pi$. Expressed in wavelengths via $\beta = \frac{2\pi}{\lambda}$ this yields $l = \frac{\lambda}{2\pi}\arctan\left(\frac{1}{\omega C Z_0}\right) + n\frac{\lambda}{2}$. Rewriting emphasizes the deviation from quarter wavelength $\boldsymbol{l = \frac{\lambda}{4} - \arctan(\omega C Z_0) + n\frac{\lambda}{2}}$.
(5p total for correct solution strategy outline, 3p for correct calculation, 2p for SC sketch)

Problem 3

A detector diode exhibits a rather high small signal impedance of ca R_L = 150 ohm at 10GHz. Design a binomial 3-section transformer, using microstrip transmission lines, to match the diode load impedance to 50 ohm. What is the bandwidth of the signal, which can be received and detected by the diode? Theory of small reflections may be applied. Use a 0.5 mm thick alumina substrate (ε_r=9).

(10p)

Solution: The characteristic impedance values are found from Table 5.1 with Z_L/Z_0=150/50=3 and N=3 or calculated from the line towards the load using (5.53). This results in $Z_0 = 50\Omega, \boldsymbol{Z_1 = 57.4\Omega},\ \boldsymbol{Z_2 = 86.6\Omega, Z_3 = 130.7\Omega}$ and $Z_L = 150\Omega$ ***(3p correct impedances)***. The possible relative bandwidth is given by $\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}(\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N})$ with A=0.0625 given by (5.49). Assuming a match $\boldsymbol{\Gamma_m} < 0.05$ gives an acceptable power absorbed in the diode (strict condition 0.25% power reflection) this yields $\boldsymbol{\frac{\Delta f}{f_0} = 0.6 = 60\%}$**.** At f_0=10 GHz this results in $\Delta f = 6\ GHz$. So the match is ok within $\boldsymbol{f_0 \pm 3}$**GHz** ***(4p with motivated, reasonable value)***.

Finally, the widths and lengths of the microstrip lines are calculated for the alumina substrate. First, $Z_1 \to A_1 = 2.33,\ Z_2 \to A_2 = 3.42\ and\ Z_3 \to A_3 = 5.06$ which give the line widths in relation to the conductor spacing ($\frac{W}{d} < 2$) by (3.197) as $\frac{W_1}{d} = 0.79,\ \frac{W_2}{d} = 0.26\ and\ \frac{W_3}{d} = 0.05$. This results in $\boldsymbol{W_1 = 0.4mm, W_2 = 0.13mm}$ **and** $\boldsymbol{W_3 = 0.025mm}$. All the line sections are quarter wavelength, meaning $l = \frac{c}{4f\sqrt{\varepsilon_e}}$ where the effective permittivity is calculated by (3.195) for each width, $\boldsymbol{l_1 = 3.1mm,\ l_2 = 3.2mm}$ **and** $\boldsymbol{l_3 = 3.3mm}$**.**
(3p total for microstrip dimensions)

Problem 4

The first three resonant modes of an air filled rectangular cavity are 6.7 GHz, 12.6 GHz, and 13.0 GHz. Presuming the standard dimension rules for such a cavity (b<a<d), calculate the dimensions of the cavity.

(10p)

Solution:

The dominant resonant mode with b < a < d is TE_{101}, so from (6.40)

$$f_{101} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{d}\right)^2} = 6.7\ GHz$$

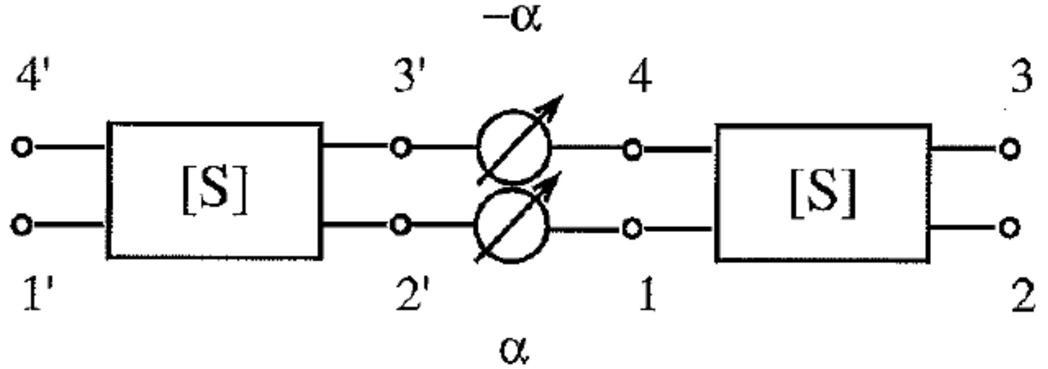
Possible higher order TE-modes: TE_{011}, TE_{102}, TE_{201}, TE_{111}
In addition, the fundamental TM-mode is TM_{110}
Due to dimensional restrictions, $f_{011} < f_{110}$ and $f_{102} < f_{201}$ applies since d > a.
Which modes are second and third are not evident, but depends on the ratios of b/a and a/d. This needs some further exploration.
- TE101, TE102 and TE201 give 3 equations not simultaneously solvable (in a and d only).
- If f_{101}=6.7 GHz and f_{102}=12.6GHz d < a, which is not ok.
- If f_{101}=6.7 GHz and f_{201}=12.6GHz d > a. The third mode should then be f_{011}=13GHz. This yields **b=11.8mm < a=24.3mm < d=57.0mm**. However, f_{102}=8.1GHz is the second mode!
- If f_{101}=6.7GHz, f_{011}=12.6GHz and f_{110}=13GHz, **b=12.6mm < a=28.6mm < d=36.0mm**. However, now f_{102}=9.8GHz and f_{201}=11.3GHz are the second and third modes!
It turns out the problem is *not* solvable with the given resonance frequencies.
(6p total are given for identifying three reasonable lowest order modes)
(2p are further given for calculating dimensions such that b<a<d, see above)
(2p are given for commenting/discussing the inconsistency/insolubility of the problem)

Problem 5

Two variable length 50-ohm lines are connected between two 3dB-couplers, with the scattering matrix:

$$[S] = \begin{bmatrix} 0 & C & jC & 0 \\ C & 0 & 0 & jC \\ jC & 0 & 0 & C \\ 0 & jC & C & 0 \end{bmatrix} \text{ where } C = \frac{1}{\sqrt{2}}$$

The electrical length can be adjusted between -α and α degrees, see figure below. Calculate the signals at port 2 and 3 if port 1 is excited with the signal $1\angle 0°$. The system impedance is 50 ohm. Describe the function of the circuit.



(10p)

<u>Solution:</u> The output of the first coupler is $V_{2'}^- = CV_{1'}^+$ (direct port) and $V_{3'}^- = jCV_{1'}^+$ (coupled port), from the definition of S-parameters in (4.40) and given [S]. After the signal has propagated the interconnecting lines a pure phase shift is added, so the input the second coupler is $V_1^+ = V_{2'}^- e^{j\alpha} = CV_{1'}^+ e^{j\alpha}$ and $V_4^+ = V_{3'}^- e^{-j\alpha} = jCV_{1'}^+ e^{-j\alpha}$. The requested signals are now superposition of signals from the $-\alpha$ and $+\alpha$ lines, via direct port and coupled port, for the output at port 2 and port 3. The result is finally:
$V_2^- = CV_1^+ + jCV_4^+ = C^2(e^{j\alpha} - e^{-j\alpha})V_{1'}^+$ / $V_3^- = CV_4^+ + jCV_1^+ = jC^2(e^{j\alpha} + e^{-j\alpha})V_{1'}^+$.
Euler's formulas, $V_1^+ = 1$ and $C = \frac{1}{\sqrt{2}}$ gives final result $\boldsymbol{V_2^- = j\sin\alpha}$ **and** $\boldsymbol{V_3^- = j\cos\alpha}$**. The circuit is thus an implementation of a variable attenuator via the phase of the lines** $\boldsymbol{\alpha}$**.**
(4p for correct coupler treatment, 4p for correct inclusion of phase, 2p for circuit function)

Problem 6

An air filled rectangular WR-10 waveguide is connected to a 5cm long WR-10 waveguide filled with a dielectric material (ε_r=1.5), which is terminated in a matched load. The load is absorbing 1 Watt, determine the maximum electrical field strength inside the air filled waveguide. Assume lossless case, 94GHz signal and principal wave propagation. WR-10 inside dimensions are 2.54 mm x 1.57 mm.

(10p)

Solution:

All power propagating in the dielectric filled waveguide is absorbed in the load P_{abs} (matched load). The power incident in the air filled waveguide (P_f) is related as $P_{abs}=P_f *(1-|\Gamma^2|)$. The reflection coefficient Γ between the two waveguides is given by the difference in wave impedances for the TE_{10} (principal/fundamental wave propagation) mode by (3.86) $Z_{TE} = \frac{k\eta}{\beta}$ and Table 3.2. Due to the different ε_r $Z_1 = Z_{TE,air} = 485\Omega$ and $Z_2 = Z_{TE,dielectric} = 359\Omega$. The reflection coefficient is then $\Gamma = \frac{Z_2-Z_1}{Z_2+Z_1} = -0.15$ and $P_f = \frac{P_{abs}}{1-|\Gamma|^2} = 1.02W$.

(3p total for correct Γ, 2p for incident power P_f)

The power flow in the TE_{10} mode is given (3.92) by $P_{10} = \frac{\omega\mu a^3|A_{10}|^2 b}{4\pi^2}Re(\beta)$ from which the constant $|A_{10}| = 37$ A/m is calculated. From (3.89b) the maximum field is

$|\mathbf{E}_{\mathbf{y,max}}| = \frac{\omega\mu a}{\pi}|\mathbf{A}_{\mathbf{10}}| = \mathbf{22.3\ kV/m}$ ***(5p total for final calculation of E-field)***.

Good Luck! / JS

Enclosures:

1) Smith Charts
2) Liao, S.-S., Sun, P.-T., Chin, N.-C., & Peng, J.-T. (2005). A novel compact-size branch-line coupler. *Microwave and Wireless Components Letters, IEEE*, *15*(9), 588–590. doi:10.1109/LMWC.2005.855378