Solutions to exam in Microwave Engineering

Friday, January 15, 2016, 14:00 - 18:00

in "H-salar", Chalmers University of Technology.

During the exam, teacher will visit around 1430 and 1600.
Contact info: Jan Stake (1836), Michael Andersson (1875)

Examiner: Prof. Jan Stake, Terahertz and millimetre wave laboratory, Department of microtechnology and nanoscience (MC2), Chalmers University of Technology.

The inspection of the results can be done in my office (D615), MC2-building, February 5th.

This is an open book exam. The following is allowed:

- *Calculator (approved by Chalmers)*
- *"Microwave Engineering" by David M. Pozar*
- *Mathematics handbook (Beta)*
- *Smith charts*

To pass this written examination, you need at least 24p out of 60p. Final grade of the exam is: 3 (≥24p), 4 (≥36p) and 5 (≥48p). At the main written examination, bonus points gained from assignments (≤11p) can raise the final grade of the course. That is: 4 (≥42p) and 5 (≥56p).

Teamwork is not permitted on this examination. The university academic integrity policy will be strictly enforced. Failure to comply with the academic integrity policy will result in a zero for this examination.

Make sure you have understood the question before you go ahead. Write shortly but make sure your way of thinking is clearly described. It is imperative to clearly explain how the results have been obtained. Solve the problem as far as you can – constructive, creative and valuable approaches are also rewarded. Assume realistic numbers/parameters when needed if data is missing in order to solve the problem.

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Problem 1

An air filled rectangular waveguide (WR-90), in which the TE_{10} mode propagates, is left open at the output flange so part of the guided microwave energy radiates into free space (Z_o=377 Ohm). If you neglect any influence from discontinuities,

a. calculate the return loss at the junction between the waveguide and open space;
b. calculate the standing wave ratio.
c. A dielectric slab can be inserted at end of the waveguide in order to improve the mismatch. Find the physical length and dielectric permittivity of a quarter-wave (dielectric) transformer.

Frequency: 10GHz; Waveguide dimensions (*a* x *b*): 23mm x 10mm.

(10p)

Solution:

Dimensions: a = 23mm cm; b = 10mm;

f = 10e9 Hz; $\varepsilon = 8.854e - 12\ s^4A^2/(m^3kg^1)$; $\mu = 1.257e - 6\ m\ kg\ /(s^2A^2)$; c = 3e8 m/s;

$\rho = 377\ \Omega$; $Z0 = 377\ \Omega$

a.

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k_c = \frac{\pi}{a}$$

$$\beta = \sqrt{k^2 - kc^2}$$

$$\eta_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

Wave impedance in waveguide

$$ZTE = \frac{k\,\eta}{\beta}$$

$$\Gamma = \frac{Z_0 - ZTE}{Z_0 + ZTE}$$

Solution:

$$RL = -20\log(|\Gamma|) = 17\ dB$$

b. **Solution:**

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 1.32$$

c. Wave impedance for quarter wavelength dielectric slab has to full fill

$$ZTE_1 = \sqrt{ZTE\ Z_0}$$

$$ZTE_1 = \frac{k_1\eta_1}{\beta_1} = \frac{k_0\sqrt{\varepsilon_r}\eta_0}{\beta_1\sqrt{\varepsilon_r}} = \frac{k_0\ \eta_0}{\beta_1} \ \rightarrow\ \beta_1 = \frac{k_0\ \eta_0}{ZTE_1}$$

$$\beta_1 = \sqrt{{k_1}^2 - {k_c}^2}\ \rightarrow k_1 = \sqrt{{\beta_1}^2 + {k_c}^2} = \sqrt{(\frac{k_0\ \eta_0}{ZTE_1})^2 + {k_c}^2}$$

$$with\ k_1 = w\sqrt{\mu\ \varepsilon\ \varepsilon_r} = k_0\sqrt{\varepsilon_r}\ \rightarrow\ \varepsilon_r = \frac{{k_1}^2}{{k_0}^2} = 1.18$$

$$l = \frac{\lambda_g}{4}\ \ with\ \lambda_g = \frac{2\pi}{\beta_1} \rightarrow\ l = 9\ mm$$

Solution: ε_r = 1.2; $l = 8.6\ mm$

Problem 2

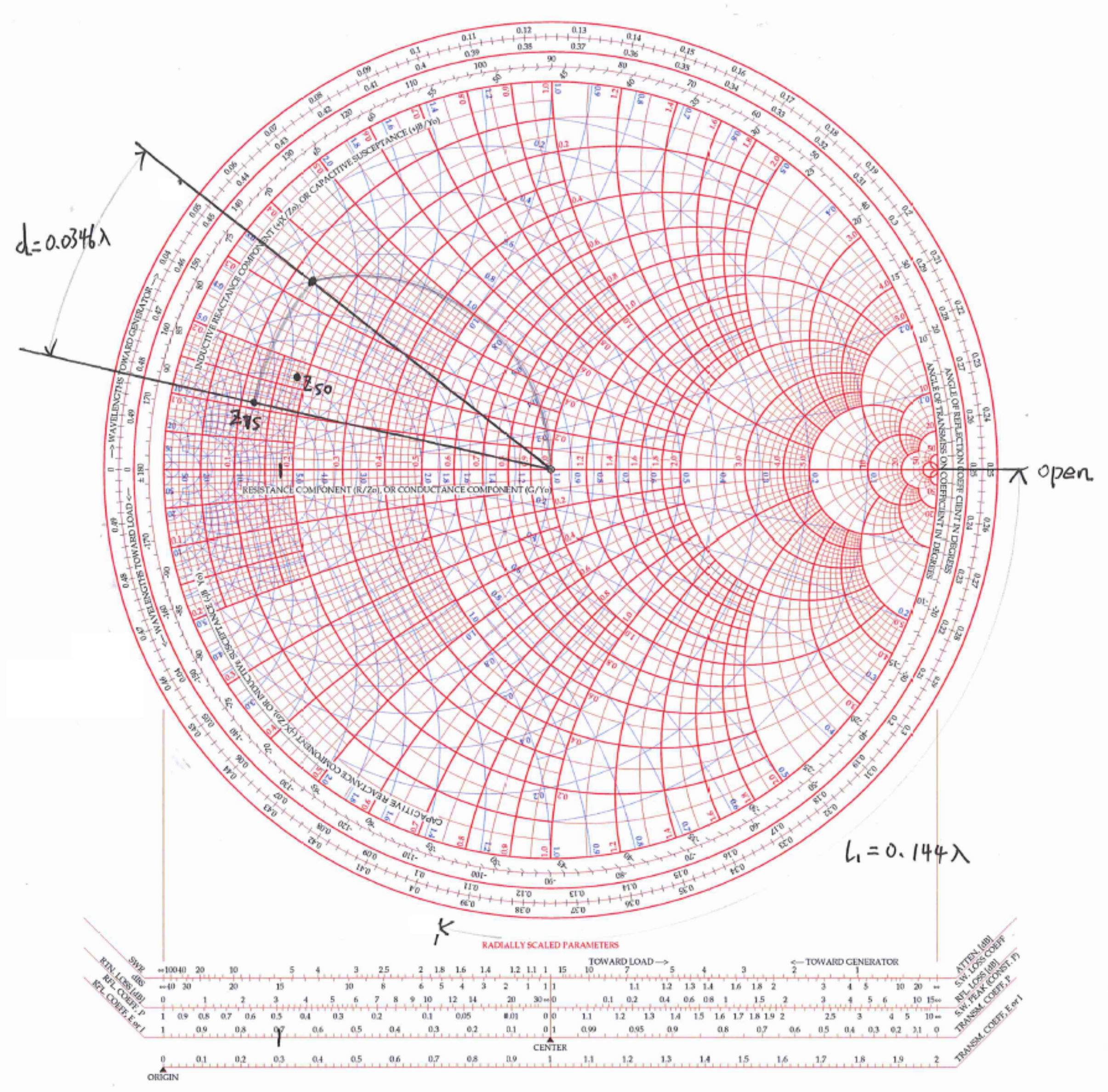
The reflection coefficient, $\Gamma = 0.7\underline{/160^\circ}$ at 5 GHz, of a home build antenna was measured using a vector network analyser, calibrated in a 50-ohm line environment. The antenna should be connected to a receiver, with <u>75-ohm</u> coaxial connector.

a) Design a matching network using a two open-circuit stubs in parallel (balanced stubs). Show the equivalent electrical circuit.

b) Show the physical realisation in microstrip technology, with a strip-to-ground-plane spacing of 1 mm, a dielectric constant of 2.2, loss tangent of 0.003, and a 50 µm copper conductor thickness.

(10p)

<u>Solution:</u>



a)
Find z_{L50} by plotting Γ in the Smith Chart. z_{L50} = 0.19 + j*0.18. Renormalize with Z_0 = 50 Ω.
Z_L = z_{L50} *50 Ω = 9.5 + j*9 . Normalize to 75 Ω : z_{L75} = Z_L/75 = 0.13+j*0.12. Plot z_{L75} in Smith chart. Rotate to g = 1 circle by adding a transmission line of length d = 0.034$\boldsymbol{\lambda}$, which corresponds to βd = 13°. Y = -j2.5 at this point. If we want to match using a double balanced stub (the stubs have the same length) we need to add Y = Y_{stub}+Y_{stub}= j2.5 -> Y_{stub} = j*1.25. Find the length of the stub from open to y = j*1.25 -> l = 0.14$\boldsymbol{\lambda}$ corresponds to βl = 50°.

b)
Use Eq. on p. 149. $Z_0 = 75\ \Omega;\ \epsilon_r = 2.2;\ d = 1\ mm;\ tan\delta = 0.003; T = 50\ \mu m$

$$A = \frac{Z_0}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}\left(0.23 + \frac{0.11}{\epsilon_r}\right)$$

Assume W/d < 2 and use Eq (3.197)

$$\frac{W}{d} = \frac{2e^A}{e^{2A}-2} \rightarrow w = 0.16\ cm$$

Calculate effective dielectric constant

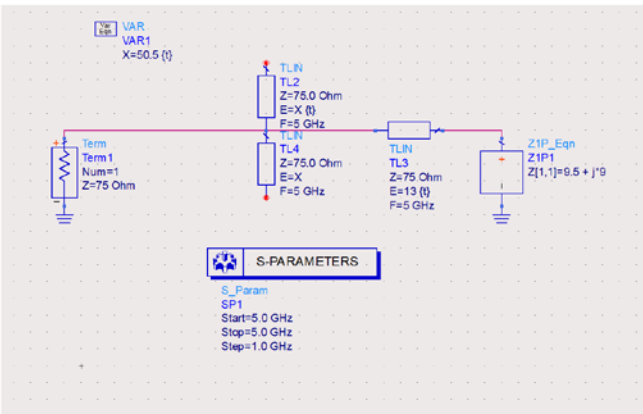
$$\epsilon_e = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\frac{1}{\sqrt{1+12d/w}} = 1.81 \rightarrow \boldsymbol{\lambda} = \frac{\boldsymbol{c}}{\boldsymbol{f}\sqrt{\epsilon_e}} = 4.5\ cm$$

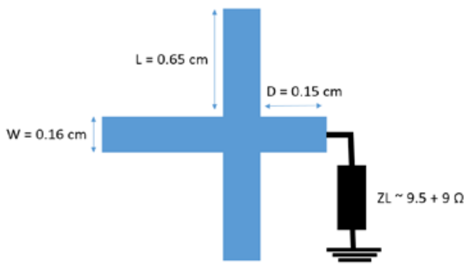
Length first transmission line $d = 0.034\boldsymbol{\lambda} = 0.15\ cm$

Length of stubs $l =\ 0.14\boldsymbol{\lambda} =\ 0.65\ cm$

Solution: $w = 0.16\ cm;\ d = 0.15\ cm;\ l =\ 0.65\ cm$

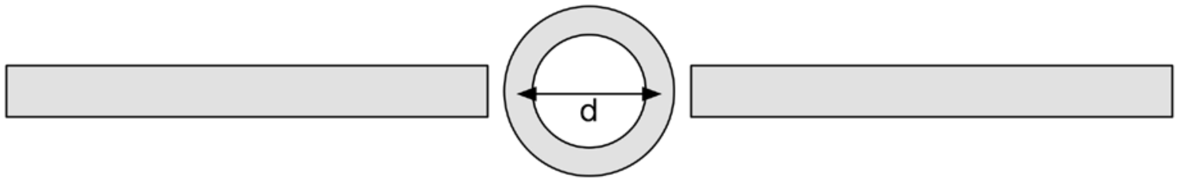
Physical realisation cross section: compare with Figure 3.26 (a). Equivalent circuit and Top view:





Problem 3

Design a ring-resonator, i.e. find the diameter of the ring shown below, in order to exhibit a fundamental resonance at 9 GHz. Transmission lines are realised in strip-line technology, with ε_r =2.5, loss tangent of 0.003. Calculate the unloaded Q-value of the resonator, assuming the conductor loss is negligible.



(10p)

<u>Solution:</u>

Resonance occurs when the length (circumference) of the ring resonator is equal to an integer number of guided wavelengths:

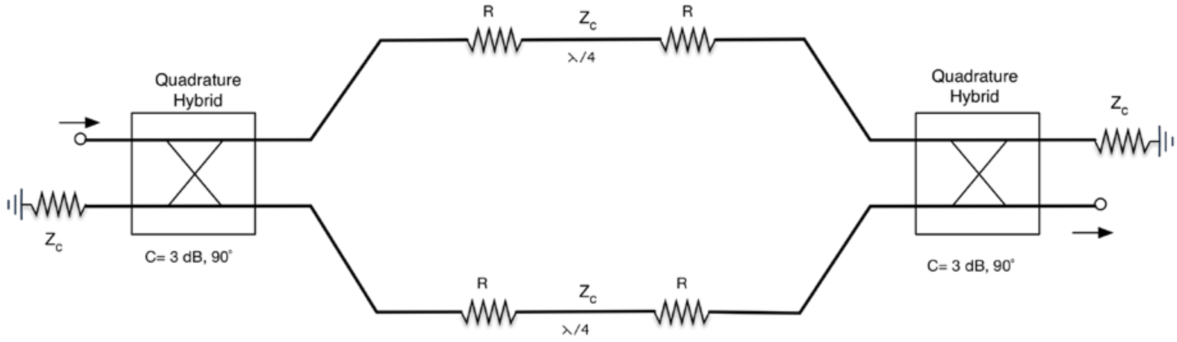
$d\pi = \mathrm{n}\lambda \rightarrow d = \frac{\lambda}{\pi} = \frac{c}{\pi f \sqrt{\epsilon_r}} \approx 6.7\ mm.$

The unloaded Q-value (TEM wave) : $Q = \frac{\beta}{2\alpha} = \frac{\beta}{2\alpha_d} = \frac{2\mathrm{k}}{2\cdot\mathrm{k}\cdot\tan(\delta)} = \frac{1}{\tan(\delta)} \approx 330$

Answer: The unloaded Q is 330 and the diameter of the ring should be 7mm for a resonance at 9GHz

Problem 4

What is the signal attenuation for the circuit below, and calculate also input matching, i.e. return loss. Characteristic impedance is Z_c= 50 Ohm and R = 340 Ohm.



(10p)

Solution:

This is a balanced circuit. Input signal is divided in equal parts, but with a 90-degree phase difference, and then recombined at the output. Thanks to the 90 degree hybrids, any mismatch will cancel at the input port and terminated in the load. For the attenuation, it is enough to calculate S21 for the individual attenuator as:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & jZ_c \\ j/Z_c & 0 \end{bmatrix} \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & jZ_c \\ j/Z_c & jR/Z_c \end{bmatrix} = \begin{bmatrix} jR/Z_c & jZ_c + jR^2/Z_c \\ j/Z_c & jR/Z_c \end{bmatrix}$$

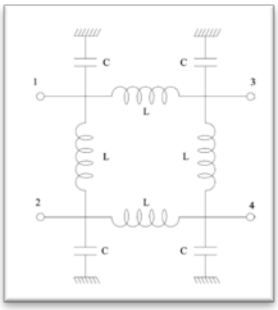
$$S_{21} = \frac{2}{A + B/Z_c + CZ_c + D} = \frac{2}{2A + B/Z_c + CZ_c} = \frac{2}{2\,jR/Z_c + 1 + jR^2/Z_c^2 + j}$$

$$= \frac{2}{1 + j(1 + 2\frac{340}{50} + \left(\frac{340}{50}\right)^2)} \cong \frac{2}{1 + 60.84j} \rightarrow |S_{21}| = 0.0329 \ \rightarrow -30dB$$

Answer: The attenuation is 30dB and no reflections at the input (perfect input match).

Problem 5

Derive the complete S-matrix for the following four-port, with an equal reference/system impedance, Z_c, at all ports.



(10p)

<u>Solution:</u>

Using the even/odd method, the four port can be reduced to two 2-port (even four 1 ports). The circuit is reciprocal and lossless. Using symmetry, we have two PI-networks as:

Even mode (C-L-C PI network)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{even} = \begin{bmatrix} 1-\omega^2 LC & j\omega L \\ 2j\omega C - j\omega^3 LC^2 & 1-\omega^2 LC \end{bmatrix}$$

Odd mode (C//0.5L – L - C//0.5L PI network):

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{odd} = \begin{bmatrix} 3-\omega^2 LC & j\omega L \\ 6j\omega C + \dfrac{8}{j\omega L} - j\omega^3 LC^2 & 3-\omega^2 LC \end{bmatrix}$$

$$T_e = \frac{2}{A + B/Z_c + CZ_c + D} = \frac{2}{2(1-\omega^2 LC) + j\omega L/Z_c + (2j\omega C - j\omega^3 LC^2)Z_c}$$

$$\Gamma_e = \frac{A + B/Z_c - CZ_c - D}{A + B/Z_c + CZ_c + D} = \frac{j\omega L/Z_c - (2j\omega C - j\omega^3 LC^2)Z_c}{2(1-\omega^2 LC) + j\omega L/Z_c + (2j\omega C - j\omega^3 LC^2)Z_c}$$

$$T_o = \frac{2}{2(3-\omega^2 LC) + j\omega L/Z_c + \left(6j\omega C + \frac{8}{j\omega L} - j\omega^3 LC^2\right)Z_c}$$

$$\Gamma_o = \frac{j\omega L/Z_c - \left(6j\omega C + \frac{8}{j\omega L} - j\omega^3 LC^2\right)Z_c}{2(3-\omega^2 LC) + j\omega L/Z_c + \left(6j\omega C + \frac{8}{j\omega L} - j\omega^3 LC^2\right)Z_c}$$

Answer: From above, the complete S-matrix elements are:

$$S_{11} = S_{22} = S_{33} = S_{44} = \frac{\Gamma_e + \Gamma_o}{2} = \cdots$$

$$S_{12} = S_{21} = S_{43} = S_{34} = \frac{\Gamma_e - \Gamma_o}{2} = \cdots$$

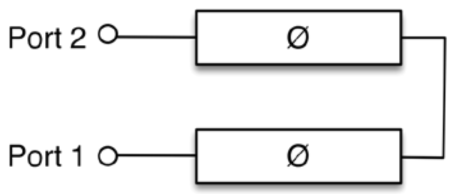
$$S_{31} = S_{13} = S_{42} = S_{24} = \frac{T_e + T_o}{2} = \cdots$$

$$S_{41} = S_{14} = S_{32} = S_{23} = \frac{T_e - T_o}{2} = \cdots$$

Check: Low frequency limit: S31=0.5, and S41=0.5, and S11=-0.5, and S12=0.5 OK! (L short / C open)
High frequency limit: S11=-1, and S12=0, and S31=S41=0 OK! (C short / L open)

Problem 6

A so-called C-section, which consists of a coupled line directional coupler having two adjacent ports connected, is shown below. These types of networks are useful for phase equalizers and phase shifters (e.g. Shiffman differential phase shifter) and as a replacement of a quarter wave transformer for dual-band matching applications.
Design a dual-band impedance transformer utilizing a single *C*-section between Z_G = 50 ohm and Z_L = 75 ohm, providing a simultaneous impedance match at 2 and 5 GHz. See enclosed article [1]. Show the electrical equivalent circuit as well as the physical layout of a microstrip circuit on a substrate with ε_r =10 (see design chart in Pozar Fig 7.30).



[1] J. E. Page and J. Esteban, "Dual-Band Matching Properties of the C-Section All-Pass Network," *IEEE Trans. Microw. Theory. Tech.*, vol. 61, no. 2, pp. 827–832, Feb. 2013. http://dx.doi.org/10.1109/TMTT.2012.2231876

(10p)

Solution:
Start by looking at the design graph Fig 4 in the attached paper, with the centre frequency ratio 5GHz/2GHz = 2.5 which results in a coupling factor of ca c=13dB. Furthermore, we combine (5) and (8) to calculate the even and odd mode impedances of the C-section, coupled transmission lines

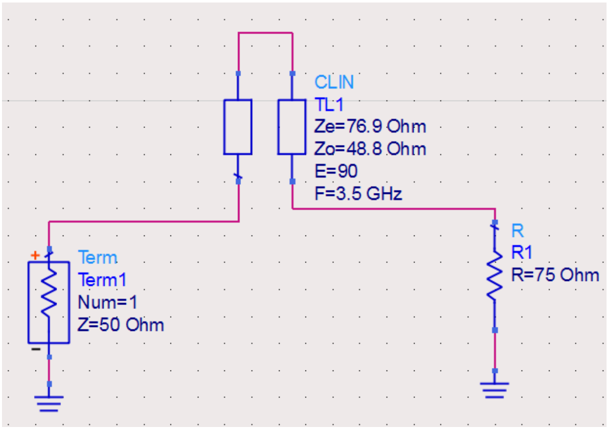
$$r = \sqrt{\frac{Z_o}{Z_e}} = \sqrt{\frac{1 - 10^{-c/20}}{1 + 10^{-c/20}}} = 0.7963$$

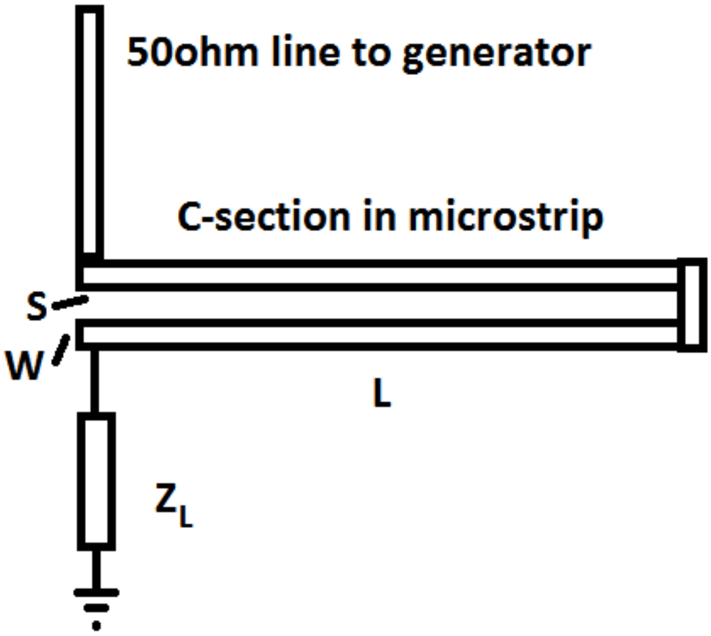
$$Z_e Z_o = Z_g Z_L$$

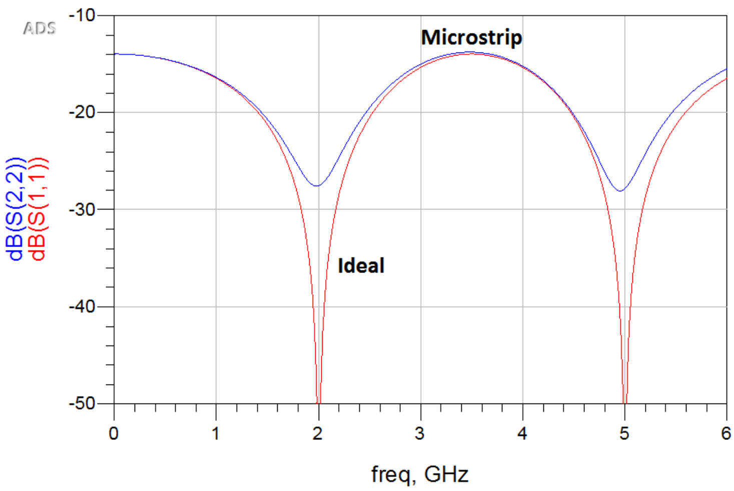
$$\Rightarrow Z_e = \sqrt{\frac{Z_g Z_L}{r^2}} = 76.9\Omega \; and \; Z_o = \frac{Z_g Z_L}{Z_e} = 48.8\Omega.$$

Now, we use the design chart in Pozar Fig 7.30 which results in $W/d \approx 0.55$ and $S/d \approx 0.6$, for the line widths and separation, respectively, normalized to the substrate thickness *d*. Since *d* is not given assume for example $d = 1mm \ll \lambda$ (thin enough to avoid parasitic mode propagation). This means that the coupled microstrip line has parameters $W = 0.55mm$ and $S = 0.6mm$. Finally, from (11), the length of the coupled line section should be 90 degrees at the average of the frequencies, 3.5GHz. It is appr. the same as $l = \lambda/4 = \frac{c}{4\sqrt{\varepsilon_{eff}}} \approx 11.8mm$ as for an isolated microstrip line.

Answer: Even and odd impedances are 77 and 49 ohm respectively. The coupled line section should be 90 degrees long at 3.5 GHz, in order to simultaneously match the load at 3 and 5 GHz. For a realisation in microstrip technology, the dimensions are W/d=0.5 and S/d =0.6, where for example d=1 mm is a suitable choice. Below are given the equivalent circuit and physical realisation. The S-parameters of the ideal C-section and a (tuned) microstrip version are also shown.


CLIN
TL1
Ze=76.9 Ohm
Zo=48.8 Ohm
E=90
F=3.5 GHz
Term
Term1
Num=1
Z=50 Ohm
R
R1
R=75 Ohm


50ohm line to generator
C-section in microstrip
S
W
L
Z_L


ADS
Microstrip
Ideal
dB(S(2,2))
dB(S(1,1))
-10
-20
-30
-40
-50
0
1
2
3
4
5
6
freq, GHz