Home Assigment 4

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1 Task 1

The scattering matrix $\mathbf{S}$ of a four port is shown in (1).

$$\mathbf{S} = \begin{pmatrix} 0.2e^{j\frac{\pi}{2}} & 0.5e^{-j\frac{\pi}{4}} & 0.5e^{j\frac{\pi}{4}} & 0 \\ 0.5e^{-j\frac{\pi}{4}} & 0 & 0 & 0.5e^{j\frac{\pi}{4}} \\ 0.5e^{-j\frac{\pi}{4}} & 0 & 0 & 0.5e^{-j\frac{\pi}{4}} \\ 0 & 0.5e^{-j\frac{\pi}{4}} & 0.5e^{-j\frac{\pi}{4}} & 0 \end{pmatrix} \tag{1}$$

1.1 a

The circuit is lossless if $\mathbf{S}$ is unitary. In order for $\mathbf{S}$ to be unitary it has to be equal to its Hermitian conjugate (denoted †) (2), which is not the case for $\mathbf{S}$ as it is defined in (1).

$$\mathbf{S}\mathbf{S}^\dagger = \mathbf{I} \tag{2}$$

1.2 b

The circuit islossless if $\mathbf{S}$ is symmetric (3), which is not the case for $\mathbf{S}$ as it is defined in (1).

$$\mathbf{S} = \mathbf{S}^T \tag{3}$$

1.3 c

The incident and reflected voltage waves are related through the scattering matrix as shown in (4)

$$\vec{V}^- = \mathbf{S}\vec{V}^+ \tag{4}$$

If the system is driven by a voltage wave incident at port 1 while all other ports are matched then

$$V_1^- = \mathbf{S}_{11}V_1^+ + \mathbf{S}_{1j}\overbrace{(1-\delta_{i1})\mathbf{\Gamma}\underbrace{\mathbf{S}\vec{V}^+}_{\vec{V}^-}}^{\vec{V}'^+} = \mathbf{S}_{11}V_1^+ \tag{5}$$

where $\mathbf{\Gamma}_{ij} = \delta_{i1}\delta_{j1}$. Equation (5) implies that the reflection coefficient and return loss is

$$\Gamma = \mathbf{S}_{11} = 0.2e^{j\frac{\pi}{2}}, \tag{6a}$$
$$L_{\text{return}} = 10\log_{10}|\mathbf{S}_{11}|^{-2} = 13.98\,\text{dB} \tag{6b}$$

1.4 c

If the system is driven by a voltage wave incident at port 3 or 4 and measurements are made at port 4 or 3 while all other ports are matched then the outgoing wave at the port is (7a) or (7b)

$$V_4^- = \mathbf{S}_{43}V_3^+, \tag{7a}$$
$$V_3^- = \mathbf{S}_{34}V_4^+ \tag{7b}$$

When the system is driven at port j and the output power is measured at port i then the losses between port i and j for this system can be calculated using (8).

$$L_{ij} = 10\log_{10}\frac{P_{\text{in}}}{P_{\text{out}}} = 10\log_{10}|\mathbf{S}_{ij}|^{-2} \tag{8}$$

When evaluating the losses between port 3 and 4 it is evident that $\mathbf{S}_{34} = \mathbf{S}_{43} = 0.5e^{-j\frac{\pi}{4}}$ so the loss is 6 dB both ways.

1.5 d

If the system is driven by a voltage wave incident at port 1 while port 3 is shorted and ports 2 and 4 are matched

$$V_1^- = \mathbf{S}_{11}V_1^+ + \mathbf{S}_{1j}\overbrace{(1-\delta_{i1})\mathbf{\Gamma}\,\underbrace{\mathbf{S}\vec{V}^+}_{\vec{V}^-}}^{\vec{V}'^+} = (\mathbf{S}_{11} - \mathbf{S}_{13}\mathbf{S}_{31})V_1^+ \tag{9}$$

where $\mathbf{\Gamma}_{ij} = \delta_{i1}\delta_{j1} - \delta_{i3}\delta_{j3}$. Equation (9) implies that the reflection coefficient and return loss is

$$\Gamma = \mathbf{S}_{11} - \mathbf{S}_{13}\mathbf{S}_{31} = 0.2e^{j\frac{\pi}{2}} - 0.5^2 = -0.25 + j0.2, \tag{10a}$$
$$L_{\text{return}} = 10\log_{10}|\mathbf{S}_{11} - \mathbf{S}_{13}\mathbf{S}_{31}|^{-2} = 9.89\,\text{dB}. \tag{10b}$$