# a02_3

October 24, 2025

## 1 3 Prediction

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[5]:
```
import numpy as np
import matplotlib.pyplot as plt
import sklearn

%load_ext autoreload
%autoreload 2

from a02_helper import *
from a02_functions import gd, predict, classify, optimize

import pandas as pd
from sklearn import metrics
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

[6]:
```
%matplotlib inline
```

[7]:
```
# Fitted model
w0 = np.random.normal(size=D)
wz_gd, vz_gd, ez_gd = optimize(gd(y, Xz), w0, nepochs=500)
```

```
(3065, 57)
Epoch   0: f=  8661.469, eps=0.010000000
3065
Epoch   1: f=  4767.607, eps=0.010500000
3065
Epoch   2: f=  3573.970, eps=0.011025000
3065
Epoch   3: f=  2711.887, eps=0.011576250
3065
Epoch   4: f=  2069.651, eps=0.012155063
3065
Epoch   5: f=  1604.959, eps=0.012762816
3065
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Epoch   6: f=  1362.409, eps=0.013400956
3065
Epoch   7: f=  1187.560, eps=0.014071004
3065
Epoch   8: f=  1076.212, eps=0.014774554
3065
Epoch   9: f=   991.532, eps=0.015513282
3065
Epoch  10: f=   962.508, eps=0.016288946
3065
Epoch  11: f=  1069.003, eps=0.008144473
3065
Epoch  12: f=   868.784, eps=0.008551697
3065
Epoch  13: f=   832.754, eps=0.008979282
3065
Epoch  14: f=   824.886, eps=0.009428246
3065
Epoch  15: f=   826.584, eps=0.004714123
3065
Epoch  16: f=   786.579, eps=0.004949829
3065
Epoch  17: f=   777.356, eps=0.005197320
3065
Epoch  18: f=   771.012, eps=0.005457186
3065
Epoch  19: f=   764.957, eps=0.005730046
3065
Epoch  20: f=   759.038, eps=0.006016548
3065
Epoch  21: f=   753.311, eps=0.006317375
3065
Epoch  22: f=   747.813, eps=0.006633244
3065
Epoch  23: f=   742.745, eps=0.006964906
3065
Epoch  24: f=   738.253, eps=0.007313152
3065
Epoch  25: f=   734.823, eps=0.007678809
3065
Epoch  26: f=   731.837, eps=0.008062750
3065
Epoch  27: f=   728.983, eps=0.008465887
3065
Epoch  28: f=   727.762, eps=0.008889182
3065
Epoch  29: f=   724.563, eps=0.009333641
3065
```

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Epoch  30: f=   728.125, eps=0.004666820
3065
Epoch  31: f=   707.823, eps=0.004900161
3065
Epoch  32: f=   705.561, eps=0.005145169
3065
Epoch  33: f=   703.524, eps=0.005402428
3065
Epoch  34: f=   701.642, eps=0.005672549
3065
Epoch  35: f=   699.836, eps=0.005956177
3065
Epoch  36: f=   698.243, eps=0.006253986
3065
Epoch  37: f=   696.927, eps=0.006566685
3065
Epoch  38: f=   696.261, eps=0.006895019
3065
Epoch  39: f=   696.344, eps=0.003447510
3065
Epoch  40: f=   691.109, eps=0.003619885
3065
Epoch  41: f=   690.115, eps=0.003800879
3065
Epoch  42: f=   689.207, eps=0.003990923
3065
Epoch  43: f=   688.297, eps=0.004190469
3065
Epoch  44: f=   687.379, eps=0.004399993
3065
Epoch  45: f=   686.455, eps=0.004619993
3065
Epoch  46: f=   685.525, eps=0.004850992
3065
Epoch  47: f=   684.593, eps=0.005093542
3065
Epoch  48: f=   683.661, eps=0.005348219
3065
Epoch  49: f=   682.731, eps=0.005615630
3065
Epoch  50: f=   681.806, eps=0.005896411
3065
Epoch  51: f=   680.890, eps=0.006191232
3065
Epoch  52: f=   679.985, eps=0.006500794
3065
Epoch  53: f=   679.098, eps=0.006825833
3065
```

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Epoch  54: f=   678.236, eps=0.007167125
3065
Epoch  55: f=   677.426, eps=0.007525481
3065
Epoch  56: f=   676.785, eps=0.007901755
3065
Epoch  57: f=   676.841, eps=0.003950878
3065
Epoch  58: f=   675.592, eps=0.004148421
3065
Epoch  59: f=   675.102, eps=0.004355843
3065
Epoch  60: f=   674.688, eps=0.004573635
3065
Epoch  61: f=   674.301, eps=0.004802316
3065
Epoch  62: f=   673.926, eps=0.005042432
3065
Epoch  63: f=   673.564, eps=0.005294554
3065
Epoch  64: f=   673.218, eps=0.005559282
3065
Epoch  65: f=   672.905, eps=0.005837246
3065
Epoch  66: f=   672.659, eps=0.006129108
3065
Epoch  67: f=   672.583, eps=0.006435563
3065
Epoch  68: f=   672.835, eps=0.003217782
3065
Epoch  69: f=   671.664, eps=0.003378671
3065
Epoch  70: f=   671.467, eps=0.003547604
3065
Epoch  71: f=   671.310, eps=0.003724984
3065
Epoch  72: f=   671.160, eps=0.003911234
3065
Epoch  73: f=   671.011, eps=0.004106795
3065
Epoch  74: f=   670.865, eps=0.004312135
3065
Epoch  75: f=   670.719, eps=0.004527742
3065
Epoch  76: f=   670.575, eps=0.004754129
3065
Epoch  77: f=   670.432, eps=0.004991835
3065
```

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Epoch  78: f=   670.290, eps=0.005241427
3065
Epoch  79: f=   670.148, eps=0.005503499
3065
Epoch  80: f=   670.007, eps=0.005778674
3065
Epoch  81: f=   669.867, eps=0.006067607
3065
Epoch  82: f=   669.728, eps=0.006370988
3065
Epoch  83: f=   669.590, eps=0.006689537
3065
Epoch  84: f=   669.456, eps=0.007024014
3065
Epoch  85: f=   669.332, eps=0.007375214
3065
Epoch  86: f=   669.251, eps=0.007743975
3065
Epoch  87: f=   669.385, eps=0.003871988
3065
Epoch  88: f=   669.076, eps=0.004065587
3065
Epoch  89: f=   668.960, eps=0.004268866
3065
Epoch  90: f=   668.874, eps=0.004482310
3065
Epoch  91: f=   668.799, eps=0.004706425
3065
Epoch  92: f=   668.727, eps=0.004941746
3065
Epoch  93: f=   668.658, eps=0.005188834
3065
Epoch  94: f=   668.592, eps=0.005448275
3065
Epoch  95: f=   668.533, eps=0.005720689
3065
Epoch  96: f=   668.491, eps=0.006006724
3065
Epoch  97: f=   668.501, eps=0.003003362
3065
Epoch  98: f=   668.312, eps=0.003153530
3065
Epoch  99: f=   668.267, eps=0.003311206
3065
Epoch 100: f=   668.227, eps=0.003476767
3065
Epoch 101: f=   668.186, eps=0.003650605
3065
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Epoch 102: f=   668.143, eps=0.003833135
3065
Epoch 103: f=   668.099, eps=0.004024792
3065
Epoch 104: f=   668.054, eps=0.004226032
3065
Epoch 105: f=   668.007, eps=0.004437333
3065
Epoch 106: f=   667.959, eps=0.004659200
3065
Epoch 107: f=   667.910, eps=0.004892160
3065
Epoch 108: f=   667.858, eps=0.005136768
3065
Epoch 109: f=   667.806, eps=0.005393606
3065
Epoch 110: f=   667.751, eps=0.005663287
3065
Epoch 111: f=   667.695, eps=0.005946451
3065
Epoch 112: f=   667.638, eps=0.006243773
3065
Epoch 113: f=   667.579, eps=0.006555962
3065
Epoch 114: f=   667.518, eps=0.006883760
3065
Epoch 115: f=   667.455, eps=0.007227948
3065
Epoch 116: f=   667.391, eps=0.007589346
3065
Epoch 117: f=   667.325, eps=0.007968813
3065
Epoch 118: f=   667.263, eps=0.008367254
3065
Epoch 119: f=   667.227, eps=0.008785616
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Epoch 120: f=   667.396, eps=0.004392808
3065
Epoch 121: f=   667.254, eps=0.004612449
3065
Epoch 122: f=   667.172, eps=0.004843071
3065
Epoch 123: f=   667.133, eps=0.005085225
3065
Epoch 124: f=   667.111, eps=0.005339486
3065
Epoch 125: f=   667.132, eps=0.002669743
3065
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Epoch 126: f=   666.902, eps=0.002803230
3065
Epoch 127: f=   666.879, eps=0.002943392
3065
Epoch 128: f=   666.857, eps=0.003090561
3065
Epoch 129: f=   666.834, eps=0.003245089
3065
Epoch 130: f=   666.809, eps=0.003407344
3065
Epoch 131: f=   666.784, eps=0.003577711
3065
Epoch 132: f=   666.757, eps=0.003756596
3065
Epoch 133: f=   666.729, eps=0.003944426
3065
Epoch 134: f=   666.700, eps=0.004141647
3065
Epoch 135: f=   666.670, eps=0.004348730
3065
Epoch 136: f=   666.639, eps=0.004566166
3065
Epoch 137: f=   666.606, eps=0.004794475
3065
Epoch 138: f=   666.572, eps=0.005034198
3065
Epoch 139: f=   666.536, eps=0.005285908
3065
Epoch 140: f=   666.499, eps=0.005550204
3065
Epoch 141: f=   666.460, eps=0.005827714
3065
Epoch 142: f=   666.420, eps=0.006119100
3065
Epoch 143: f=   666.378, eps=0.006425055
3065
Epoch 144: f=   666.334, eps=0.006746307
3065
Epoch 145: f=   666.288, eps=0.007083623
3065
Epoch 146: f=   666.241, eps=0.007437804
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Epoch 147: f=   666.191, eps=0.007809694
3065
Epoch 148: f=   666.140, eps=0.008200179
3065
Epoch 149: f=   666.086, eps=0.008610188
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Epoch 150: f=   666.031, eps=0.009040697
3065
Epoch 151: f=   665.973, eps=0.009492732
3065
Epoch 152: f=   665.913, eps=0.009967368
3065
Epoch 153: f=   665.855, eps=0.010465737
3065
Epoch 154: f=   665.844, eps=0.010989024
3065
Epoch 155: f=   666.432, eps=0.005494512
3065
Epoch 156: f=   666.509, eps=0.002747256
3065
Epoch 157: f=   665.670, eps=0.002884619
3065
Epoch 158: f=   665.648, eps=0.003028850
3065
Epoch 159: f=   665.629, eps=0.003180292
3065
Epoch 160: f=   665.610, eps=0.003339307
3065
Epoch 161: f=   665.590, eps=0.003506272
3065
Epoch 162: f=   665.569, eps=0.003681586
3065
Epoch 163: f=   665.546, eps=0.003865665
3065
Epoch 164: f=   665.523, eps=0.004058948
3065
Epoch 165: f=   665.499, eps=0.004261896
3065
Epoch 166: f=   665.473, eps=0.004474990
3065
Epoch 167: f=   665.447, eps=0.004698740
3065
Epoch 168: f=   665.419, eps=0.004933677
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Epoch 169: f=   665.389, eps=0.005180361
3065
Epoch 170: f=   665.359, eps=0.005439379
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Epoch 171: f=   665.327, eps=0.005711348
3065
Epoch 172: f=   665.293, eps=0.005996915
3065
Epoch 173: f=   665.258, eps=0.006296761
3065
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Epoch 174: f=   665.222, eps=0.006611599
3065
Epoch 175: f=   665.183, eps=0.006942179
3065
Epoch 176: f=   665.143, eps=0.007289288
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Epoch 177: f=   665.102, eps=0.007653752
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Epoch 178: f=   665.058, eps=0.008036440
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Epoch 179: f=   665.012, eps=0.008438262
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Epoch 180: f=   664.964, eps=0.008860175
3065
Epoch 181: f=   664.915, eps=0.009303184
3065
Epoch 182: f=   664.870, eps=0.009768343
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Epoch 183: f=   664.881, eps=0.004884171
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Epoch 184: f=   664.854, eps=0.005128380
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Epoch 185: f=   664.845, eps=0.005384799
3065
Epoch 186: f=   664.855, eps=0.002692399
3065
Epoch 187: f=   664.709, eps=0.002827019
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Epoch 188: f=   664.692, eps=0.002968370
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Epoch 189: f=   664.676, eps=0.003116789
3065
Epoch 190: f=   664.659, eps=0.003272628
3065
Epoch 191: f=   664.641, eps=0.003436260
3065
Epoch 192: f=   664.622, eps=0.003608073
3065
Epoch 193: f=   664.602, eps=0.003788476
3065
Epoch 194: f=   664.581, eps=0.003977900
3065
Epoch 195: f=   664.560, eps=0.004176795
3065
Epoch 196: f=   664.537, eps=0.004385635
3065
Epoch 197: f=   664.513, eps=0.004604917
3065
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Epoch 198: f=   664.489, eps=0.004835163
3065
Epoch 199: f=   664.462, eps=0.005076921
3065
Epoch 200: f=   664.435, eps=0.005330767
3065
Epoch 201: f=   664.407, eps=0.005597305
3065
Epoch 202: f=   664.377, eps=0.005877170
3065
Epoch 203: f=   664.345, eps=0.006171029
3065
Epoch 204: f=   664.312, eps=0.006479580
3065
Epoch 205: f=   664.278, eps=0.006803559
3065
Epoch 206: f=   664.242, eps=0.007143737
3065
Epoch 207: f=   664.204, eps=0.007500924
3065
Epoch 208: f=   664.165, eps=0.007875970
3065
Epoch 209: f=   664.124, eps=0.008269769
3065
Epoch 210: f=   664.080, eps=0.008683257
3065
Epoch 211: f=   664.035, eps=0.009117420
3065
Epoch 212: f=   663.988, eps=0.009573291
3065
Epoch 213: f=   663.941, eps=0.010051956
3065
Epoch 214: f=   663.912, eps=0.010554554
3065
Epoch 215: f=   664.108, eps=0.005277277
3065
Epoch 216: f=   664.138, eps=0.002638638
3065
Epoch 217: f=   663.795, eps=0.002770570
3065
Epoch 218: f=   663.779, eps=0.002909099
3065
Epoch 219: f=   663.764, eps=0.003054554
3065
Epoch 220: f=   663.748, eps=0.003207282
3065
Epoch 221: f=   663.732, eps=0.003367646
3065
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Epoch 222: f=   663.715, eps=0.003536028
3065
Epoch 223: f=   663.697, eps=0.003712829
3065
Epoch 224: f=   663.679, eps=0.003898471
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Epoch 225: f=   663.659, eps=0.004093394
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Epoch 226: f=   663.638, eps=0.004298064
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Epoch 227: f=   663.617, eps=0.004512967
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Epoch 228: f=   663.594, eps=0.004738616
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Epoch 229: f=   663.571, eps=0.004975546
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Epoch 230: f=   663.546, eps=0.005224324
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Epoch 231: f=   663.520, eps=0.005485540
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Epoch 232: f=   663.493, eps=0.005759817
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Epoch 233: f=   663.464, eps=0.006047808
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Epoch 234: f=   663.434, eps=0.006350198
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Epoch 235: f=   663.403, eps=0.006667708
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Epoch 236: f=   663.370, eps=0.007001093
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Epoch 237: f=   663.336, eps=0.007351148
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Epoch 238: f=   663.300, eps=0.007718705
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Epoch 239: f=   663.262, eps=0.008104641
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Epoch 240: f=   663.223, eps=0.008509873
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Epoch 241: f=   663.182, eps=0.008935366
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Epoch 242: f=   663.138, eps=0.009382135
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Epoch 243: f=   663.093, eps=0.009851241
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Epoch 244: f=   663.048, eps=0.010343803
3065
Epoch 245: f=   663.016, eps=0.010860994
3065
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Epoch 246: f=   663.173, eps=0.005430497
3065
Epoch 247: f=   663.253, eps=0.002715248
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Epoch 248: f=   662.908, eps=0.002851011
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Epoch 249: f=   662.893, eps=0.002993561
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Epoch 250: f=   662.879, eps=0.003143239
3065
Epoch 251: f=   662.864, eps=0.003300401
3065
Epoch 252: f=   662.848, eps=0.003465421
3065
Epoch 253: f=   662.832, eps=0.003638693
3065
Epoch 254: f=   662.815, eps=0.003820627
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Epoch 255: f=   662.797, eps=0.004011659
3065
Epoch 256: f=   662.778, eps=0.004212241
3065
Epoch 257: f=   662.759, eps=0.004422854
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Epoch 258: f=   662.738, eps=0.004643996
3065
Epoch 259: f=   662.716, eps=0.004876196
3065
Epoch 260: f=   662.694, eps=0.005120006
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Epoch 261: f=   662.670, eps=0.005376006
3065
Epoch 262: f=   662.645, eps=0.005644806
3065
Epoch 263: f=   662.619, eps=0.005927047
3065
Epoch 264: f=   662.591, eps=0.006223399
3065
Epoch 265: f=   662.563, eps=0.006534569
3065
Epoch 266: f=   662.533, eps=0.006861297
3065
Epoch 267: f=   662.501, eps=0.007204362
3065
Epoch 268: f=   662.468, eps=0.007564580
3065
Epoch 269: f=   662.434, eps=0.007942809
3065
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Epoch 270: f=   662.397, eps=0.008339950
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Epoch 271: f=   662.359, eps=0.008756947
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Epoch 272: f=   662.320, eps=0.009194795
3065
Epoch 273: f=   662.279, eps=0.009654535
3065
Epoch 274: f=   662.241, eps=0.010137261
3065
Epoch 275: f=   662.248, eps=0.005068631
3065
Epoch 276: f=   662.235, eps=0.005322062
3065
Epoch 277: f=   662.241, eps=0.002661031
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Epoch 278: f=   662.132, eps=0.002794083
3065
Epoch 279: f=   662.119, eps=0.002933787
3065
Epoch 280: f=   662.106, eps=0.003080476
3065
Epoch 281: f=   662.092, eps=0.003234500
3065
Epoch 282: f=   662.078, eps=0.003396225
3065
Epoch 283: f=   662.063, eps=0.003566036
3065
Epoch 284: f=   662.047, eps=0.003744338
3065
Epoch 285: f=   662.030, eps=0.003931555
3065
Epoch 286: f=   662.013, eps=0.004128133
3065
Epoch 287: f=   661.995, eps=0.004334539
3065
Epoch 288: f=   661.976, eps=0.004551266
3065
Epoch 289: f=   661.956, eps=0.004778830
3065
Epoch 290: f=   661.935, eps=0.005017771
3065
Epoch 291: f=   661.913, eps=0.005268660
3065
Epoch 292: f=   661.890, eps=0.005532093
3065
Epoch 293: f=   661.865, eps=0.005808697
3065
```

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Epoch 294: f=   661.840, eps=0.006099132
3065
Epoch 295: f=   661.814, eps=0.006404089
3065
Epoch 296: f=   661.786, eps=0.006724293
3065
Epoch 297: f=   661.757, eps=0.007060508
3065
Epoch 298: f=   661.726, eps=0.007413533
3065
Epoch 299: f=   661.694, eps=0.007784210
3065
Epoch 300: f=   661.661, eps=0.008173420
3065
Epoch 301: f=   661.625, eps=0.008582091
3065
Epoch 302: f=   661.589, eps=0.009011196
3065
Epoch 303: f=   661.550, eps=0.009461756
3065
Epoch 304: f=   661.510, eps=0.009934843
3065
Epoch 305: f=   661.470, eps=0.010431585
3065
Epoch 306: f=   661.445, eps=0.010953165
3065
Epoch 307: f=   661.639, eps=0.005476582
3065
Epoch 308: f=   661.719, eps=0.002738291
3065
Epoch 309: f=   661.347, eps=0.002875206
3065
Epoch 310: f=   661.331, eps=0.003018966
3065
Epoch 311: f=   661.318, eps=0.003169914
3065
Epoch 312: f=   661.305, eps=0.003328410
3065
Epoch 313: f=   661.291, eps=0.003494831
3065
Epoch 314: f=   661.277, eps=0.003669572
3065
Epoch 315: f=   661.261, eps=0.003853051
3065
Epoch 316: f=   661.245, eps=0.004045703
3065
Epoch 317: f=   661.228, eps=0.004247988
3065
```

```
Epoch 318: f=   661.211, eps=0.004460388
3065
Epoch 319: f=   661.192, eps=0.004683407
3065
Epoch 320: f=   661.173, eps=0.004917578
3065
Epoch 321: f=   661.152, eps=0.005163456
3065
Epoch 322: f=   661.131, eps=0.005421629
3065
Epoch 323: f=   661.109, eps=0.005692711
3065
Epoch 324: f=   661.085, eps=0.005977346
3065
Epoch 325: f=   661.061, eps=0.006276214
3065
Epoch 326: f=   661.035, eps=0.006590024
3065
Epoch 327: f=   661.008, eps=0.006919525
3065
Epoch 328: f=   660.980, eps=0.007265502
3065
Epoch 329: f=   660.950, eps=0.007628777
3065
Epoch 330: f=   660.919, eps=0.008010216
3065
Epoch 331: f=   660.886, eps=0.008410726
3065
Epoch 332: f=   660.852, eps=0.008831263
3065
Epoch 333: f=   660.817, eps=0.009272826
3065
Epoch 334: f=   660.784, eps=0.009736467
3065
Epoch 335: f=   660.782, eps=0.010223291
3065
Epoch 336: f=   661.122, eps=0.005111645
3065
Epoch 337: f=   661.137, eps=0.002555823
3065
Epoch 338: f=   660.669, eps=0.002683614
3065
Epoch 339: f=   660.658, eps=0.002817794
3065
Epoch 340: f=   660.647, eps=0.002958684
3065
Epoch 341: f=   660.635, eps=0.003106618
3065
```

```
Epoch 342: f=   660.622, eps=0.003261949
3065
Epoch 343: f=   660.609, eps=0.003425047
3065
Epoch 344: f=   660.596, eps=0.003596299
3065
Epoch 345: f=   660.582, eps=0.003776114
3065
Epoch 346: f=   660.567, eps=0.003964920
3065
Epoch 347: f=   660.551, eps=0.004163166
3065
Epoch 348: f=   660.534, eps=0.004371324
3065
Epoch 349: f=   660.517, eps=0.004589890
3065
Epoch 350: f=   660.499, eps=0.004819385
3065
Epoch 351: f=   660.480, eps=0.005060354
3065
Epoch 352: f=   660.460, eps=0.005313372
3065
Epoch 353: f=   660.439, eps=0.005579040
3065
Epoch 354: f=   660.417, eps=0.005857992
3065
Epoch 355: f=   660.394, eps=0.006150892
3065
Epoch 356: f=   660.370, eps=0.006458437
3065
Epoch 357: f=   660.345, eps=0.006781358
3065
Epoch 358: f=   660.319, eps=0.007120426
3065
Epoch 359: f=   660.291, eps=0.007476448
3065
Epoch 360: f=   660.262, eps=0.007850270
3065
Epoch 361: f=   660.232, eps=0.008242783
3065
Epoch 362: f=   660.200, eps=0.008654923
3065
Epoch 363: f=   660.166, eps=0.009087669
3065
Epoch 364: f=   660.131, eps=0.009542052
3065
Epoch 365: f=   660.095, eps=0.010019155
3065
```

```
Epoch 366: f=   660.057, eps=0.010520113
3065
Epoch 367: f=   660.017, eps=0.011046118
3065
Epoch 368: f=   659.976, eps=0.011598424
3065
Epoch 369: f=   659.949, eps=0.012178345
3065
Epoch 370: f=   660.175, eps=0.006089173
3065
Epoch 371: f=   660.438, eps=0.003044586
3065
Epoch 372: f=   659.875, eps=0.003196816
3065
Epoch 373: f=   659.840, eps=0.003356656
3065
Epoch 374: f=   659.826, eps=0.003524489
3065
Epoch 375: f=   659.812, eps=0.003700714
3065
Epoch 376: f=   659.798, eps=0.003885749
3065
Epoch 377: f=   659.784, eps=0.004080037
3065
Epoch 378: f=   659.768, eps=0.004284039
3065
Epoch 379: f=   659.752, eps=0.004498241
3065
Epoch 380: f=   659.736, eps=0.004723153
3065
Epoch 381: f=   659.718, eps=0.004959310
3065
Epoch 382: f=   659.699, eps=0.005207276
3065
Epoch 383: f=   659.680, eps=0.005467640
3065
Epoch 384: f=   659.660, eps=0.005741022
3065
Epoch 385: f=   659.639, eps=0.006028073
3065
Epoch 386: f=   659.616, eps=0.006329476
3065
Epoch 387: f=   659.593, eps=0.006645950
3065
Epoch 388: f=   659.568, eps=0.006978248
3065
Epoch 389: f=   659.543, eps=0.007327160
3065
```

```
Epoch 390: f=   659.518, eps=0.007693518
3065
Epoch 391: f=   659.498, eps=0.008078194
3065
Epoch 392: f=   659.518, eps=0.004039097
3065
Epoch 393: f=   659.469, eps=0.004241052
3065
Epoch 394: f=   659.442, eps=0.004453104
3065
Epoch 395: f=   659.422, eps=0.004675760
3065
Epoch 396: f=   659.404, eps=0.004909548
3065
Epoch 397: f=   659.386, eps=0.005155025
3065
Epoch 398: f=   659.370, eps=0.005412776
3065
Epoch 399: f=   659.358, eps=0.005683415
3065
Epoch 400: f=   659.354, eps=0.005967586
3065
Epoch 401: f=   659.373, eps=0.002983793
3065
Epoch 402: f=   659.289, eps=0.003132983
3065
Epoch 403: f=   659.274, eps=0.003289632
3065
Epoch 404: f=   659.262, eps=0.003454113
3065
Epoch 405: f=   659.249, eps=0.003626819
3065
Epoch 406: f=   659.236, eps=0.003808160
3065
Epoch 407: f=   659.222, eps=0.003998568
3065
Epoch 408: f=   659.208, eps=0.004198496
3065
Epoch 409: f=   659.193, eps=0.004408421
3065
Epoch 410: f=   659.177, eps=0.004628842
3065
Epoch 411: f=   659.161, eps=0.004860284
3065
Epoch 412: f=   659.143, eps=0.005103298
3065
Epoch 413: f=   659.125, eps=0.005358463
3065
```

```
Epoch 414: f=   659.106, eps=0.005626387
3065
Epoch 415: f=   659.086, eps=0.005907706
3065
Epoch 416: f=   659.065, eps=0.006203091
3065
Epoch 417: f=   659.043, eps=0.006513246
3065
Epoch 418: f=   659.019, eps=0.006838908
3065
Epoch 419: f=   658.995, eps=0.007180853
3065
Epoch 420: f=   658.970, eps=0.007539896
3065
Epoch 421: f=   658.943, eps=0.007916891
3065
Epoch 422: f=   658.917, eps=0.008312735
3065
Epoch 423: f=   658.897, eps=0.008728372
3065
Epoch 424: f=   658.930, eps=0.004364186
3065
Epoch 425: f=   658.886, eps=0.004582395
3065
Epoch 426: f=   658.861, eps=0.004811515
3065
Epoch 427: f=   658.843, eps=0.005052091
3065
Epoch 428: f=   658.832, eps=0.005304695
3065
Epoch 429: f=   658.830, eps=0.005569930
3065
Epoch 430: f=   658.852, eps=0.002784965
3065
Epoch 431: f=   658.743, eps=0.002924213
3065
Epoch 432: f=   658.732, eps=0.003070424
3065
Epoch 433: f=   658.721, eps=0.003223945
3065
Epoch 434: f=   658.710, eps=0.003385142
3065
Epoch 435: f=   658.698, eps=0.003554400
3065
Epoch 436: f=   658.686, eps=0.003732120
3065
Epoch 437: f=   658.673, eps=0.003918726
3065
```

```
Epoch 438: f=   658.659, eps=0.004114662
3065
Epoch 439: f=   658.645, eps=0.004320395
3065
Epoch 440: f=   658.630, eps=0.004536415
3065
Epoch 441: f=   658.614, eps=0.004763235
3065
Epoch 442: f=   658.598, eps=0.005001397
3065
Epoch 443: f=   658.581, eps=0.005251467
3065
Epoch 444: f=   658.563, eps=0.005514040
3065
Epoch 445: f=   658.544, eps=0.005789742
3065
Epoch 446: f=   658.524, eps=0.006079230
3065
Epoch 447: f=   658.503, eps=0.006383191
3065
Epoch 448: f=   658.481, eps=0.006702351
3065
Epoch 449: f=   658.458, eps=0.007037468
3065
Epoch 450: f=   658.434, eps=0.007389342
3065
Epoch 451: f=   658.409, eps=0.007758809
3065
Epoch 452: f=   658.383, eps=0.008146749
3065
Epoch 453: f=   658.355, eps=0.008554086
3065
Epoch 454: f=   658.326, eps=0.008981791
3065
Epoch 455: f=   658.297, eps=0.009430880
3065
Epoch 456: f=   658.277, eps=0.009902424
3065
Epoch 457: f=   658.352, eps=0.004951212
3065
Epoch 458: f=   658.344, eps=0.005198773
3065
Epoch 459: f=   658.356, eps=0.002599386
3065
Epoch 460: f=   658.188, eps=0.002729356
3065
Epoch 461: f=   658.179, eps=0.002865824
3065
```

```
Epoch 462: f=   658.169, eps=0.003009115
3065
Epoch 463: f=   658.159, eps=0.003159570
3065
Epoch 464: f=   658.148, eps=0.003317549
3065
Epoch 465: f=   658.137, eps=0.003483426
3065
Epoch 466: f=   658.126, eps=0.003657598
3065
Epoch 467: f=   658.113, eps=0.003840478
3065
Epoch 468: f=   658.101, eps=0.004032501
3065
Epoch 469: f=   658.087, eps=0.004234127
3065
Epoch 470: f=   658.073, eps=0.004445833
3065
Epoch 471: f=   658.058, eps=0.004668125
3065
Epoch 472: f=   658.043, eps=0.004901531
3065
Epoch 473: f=   658.027, eps=0.005146607
3065
Epoch 474: f=   658.010, eps=0.005403938
3065
Epoch 475: f=   657.992, eps=0.005674135
3065
Epoch 476: f=   657.973, eps=0.005957841
3065
Epoch 477: f=   657.953, eps=0.006255733
3065
Epoch 478: f=   657.933, eps=0.006568520
3065
Epoch 479: f=   657.911, eps=0.006896946
3065
Epoch 480: f=   657.888, eps=0.007241793
3065
Epoch 481: f=   657.865, eps=0.007603883
3065
Epoch 482: f=   657.840, eps=0.007984077
3065
Epoch 483: f=   657.814, eps=0.008383281
3065
Epoch 484: f=   657.786, eps=0.008802445
3065
Epoch 485: f=   657.758, eps=0.009242567
3065
```

```
Epoch 486: f=   657.727, eps=0.009704696
3065
Epoch 487: f=   657.696, eps=0.010189930
3065
Epoch 488: f=   657.664, eps=0.010699427
3065
Epoch 489: f=   657.638, eps=0.011234398
3065
Epoch 490: f=   657.725, eps=0.005617199
3065
Epoch 491: f=   657.790, eps=0.002808600
3065
Epoch 492: f=   657.570, eps=0.002949030
3065
Epoch 493: f=   657.556, eps=0.003096481
3065
Epoch 494: f=   657.546, eps=0.003251305
3065
Epoch 495: f=   657.535, eps=0.003413870
3065
Epoch 496: f=   657.524, eps=0.003584564
3065
Epoch 497: f=   657.513, eps=0.003763792
3065
Epoch 498: f=   657.501, eps=0.003951982
3065
Epoch 499: f=   657.488, eps=0.004149581
3065
Result after 500 epochs: f=657.4748688984262
```

In `a02_functions.py`, complete the `predict` and `classify` methods for the predicted spam probability and predicted class label, respectively. Use them to explore your previously fitted model.

[8]:
```
# Exploration example: confusion matrix
yhat = predict(Xtestz, wz_gd)
ypred = classify(Xtestz, wz_gd)
print(sklearn.metrics.confusion_matrix(ytest, ypred))  # true x predicted
```

```
[[887  54]
 [ 71 524]]
```

[9]:
```
# Exploration example: classification report
print(sklearn.metrics.classification_report(ytest, ypred))
```

```
              precision    recall  f1-score   support

           0       0.93      0.94      0.93       941
           1       0.91      0.88      0.89       595
```

```
    accuracy                           0.92      1536
   macro avg       0.92      0.91      0.91      1536
weighted avg       0.92      0.92      0.92      1536
```

[10]:
```
# Exploration Example: precision-recall curve (with annotated thresholds)
nextplot()
precision, recall, thresholds = sklearn.metrics.precision_recall_curve(ytest,␣
 ↪yhat)
plt.plot(recall, precision)
for x in np.linspace(0, 1, 10, endpoint=False):
    index = int(x * (precision.size - 1))
    plt.text(recall[index], precision[index], "{:3.2f}".
 ↪format(thresholds[index]))
plt.xlabel("Recall")
plt.ylabel("Precision")
```

[10]:
```
Text(0, 0.5, 'Precision')
```

[ ]:

[ ]:

[11]:
```
# Explore which features are considered important
# YOUR CODE HERE
```

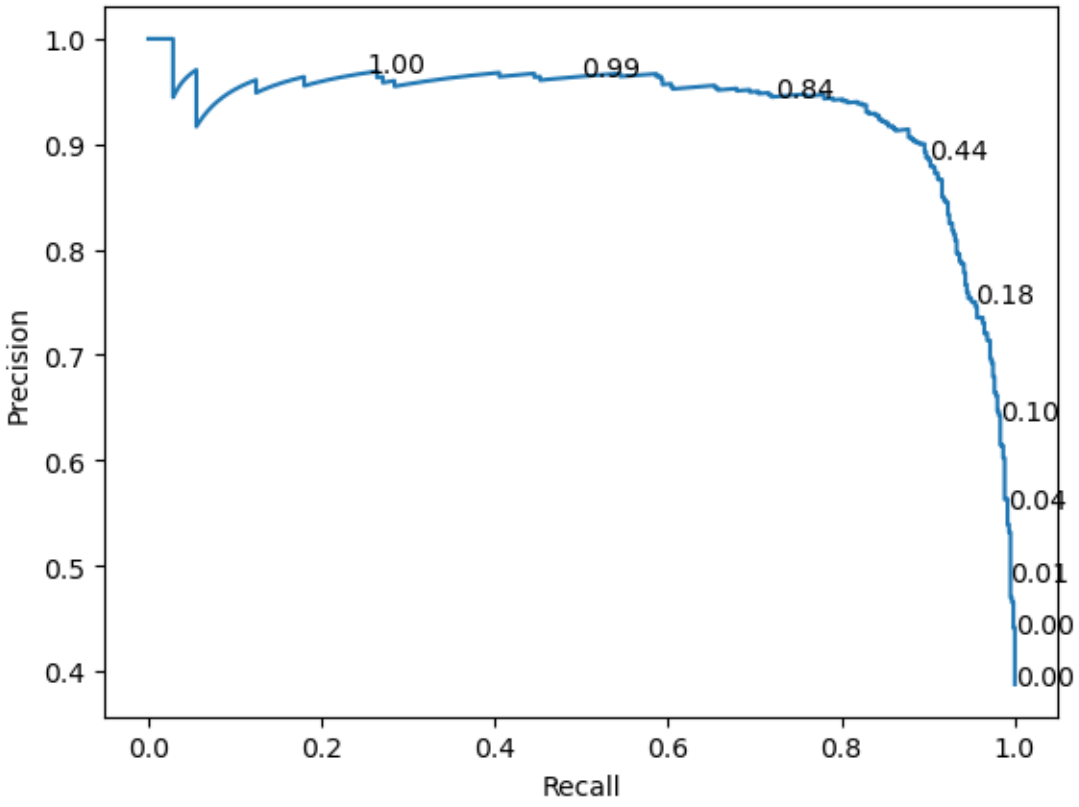
[12]:
```
w = wz_gd.copy()  # already available in your notebook
k = 15  # how many top features to display
```

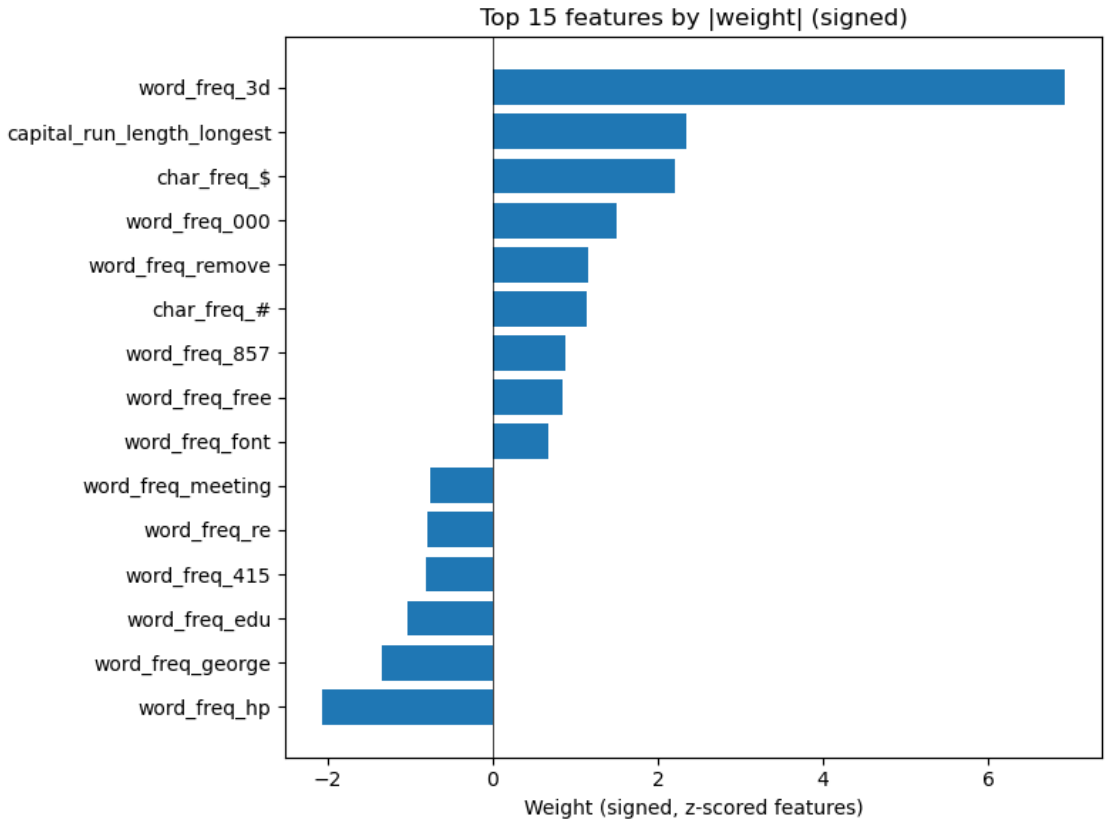
[13]:
```
# ranking by absolute weight
idx_by_abs = np.argsort(np.abs(w))[::-1]
top_idx = idx_by_abs[:k]
top_feats = features[top_idx]
top_weights = w[top_idx]

print("Top {:d} features by absolute weight:".format(k))
for i, (fname, weight) in enumerate(zip(top_feats, top_weights), 1):
    sign = "pos" if weight > 0 else "neg"
    print(f"{i:2d}. {fname:30s} weight = {weight: .4f} ({sign})")
```

```
Top 15 features by absolute weight:
 1. word_freq_3d                   weight =  6.9292 (pos)
 2. capital_run_length_longest     weight =  2.3463 (pos)
 3. char_freq_$                    weight =  2.2042 (pos)
 4. word_freq_hp                   weight = -2.0603 (neg)
 5. word_freq_000                  weight =  1.4965 (pos)
 6. word_freq_george               weight = -1.3474 (neg)
 7. word_freq_remove               weight =  1.1600 (pos)
```

```
 8. char_freq_#                  weight =  1.1469 (pos)
 9. word_freq_edu                weight = -1.0369 (neg)
10. word_freq_857                weight =  0.8728 (pos)
11. word_freq_free               weight =  0.8445 (pos)
12. word_freq_415                weight = -0.8011 (neg)
13. word_freq_re                 weight = -0.7968 (neg)
14. word_freq_meeting            weight = -0.7558 (neg)
15. word_freq_font               weight =  0.6721 (pos)
```

[14]:
```
# bar plot of top features (signed)
plt.figure(figsize=(8, 6))
# plot sorted by weight (so negative at bottom, positive at top)
order = np.argsort(top_weights)
ypos = np.arange(len(top_weights))
plt.barh(ypos, top_weights[order], align='center')
plt.yticks(ypos, top_feats[order])
plt.xlabel("Weight (signed, z-scored features)")
plt.title(f"Top {k} features by |weight| (signed)")
plt.axvline(0, color='k', linewidth=0.5)
plt.tight_layout()
plt.show()
```






[15]:
```
# extra diagnostics: correlation with label and mean feature values by class
top_df = []
for j in top_idx:
    featname = features[j]
    wj = w[j]
    # correlation between feature and binary label (point-biserial = Pearson␣
 ↪with binary)
    corr = np.corrcoef(Xz[:, j], y)[0, 1]
    mean_spam = Xz[y == 1, j].mean()
    mean_ham  = Xz[y == 0, j].mean()
    top_df.append((featname, wj, corr, mean_spam, mean_ham))

df_top = pd.DataFrame(top_df, columns=["feature", "weight", "corr_with_y",␣
 ↪"mean_spam", "mean_ham"])
# sort by absolute weight for display
df_top["abs_weight"] = np.abs(df_top["weight"])
df_top = df_top.sort_values("abs_weight", ascending=False).
 ↪drop(columns="abs_weight")
pd.set_option("display.float_format", "{: .4f}".format)
print("\nDiagnostics for top features (z-scored):")
```

```
display(df_top)
```

```
Diagnostics for top features (z-scored):
```

| | feature | weight | corr_with_y | mean_spam | mean_ham |
|---|---|---|---|---|---|
| 0 | word_freq_3d | 6.9292 | 0.0576 | 0.0709 | -0.0468 |
| 1 | capital_run_length_longest | 2.3463 | 0.1928 | 0.2375 | -0.1566 |
| 2 | char_freq_$ | 2.2042 | 0.3271 | 0.4028 | -0.2656 |
| 3 | word_freq_hp | -2.0603 | -0.2646 | -0.3258 | 0.2149 |
| 4 | word_freq_000 | 1.4965 | 0.3369 | 0.4149 | -0.2736 |
| 5 | word_freq_george | -1.3474 | -0.1855 | -0.2284 | 0.1506 |
| 6 | word_freq_remove | 1.1600 | 0.3421 | 0.4212 | -0.2778 |
| 7 | char_freq_# | 1.1469 | 0.0658 | 0.0811 | -0.0535 |
| 8 | word_freq_edu | -1.0369 | -0.1519 | -0.1871 | 0.1234 |
| 9 | word_freq_857 | 0.8728 | -0.1206 | -0.1485 | 0.0979 |
| 10 | word_freq_free | 0.8445 | 0.3381 | 0.4164 | -0.2746 |
| 11 | word_freq_415 | -0.8011 | -0.1206 | -0.1485 | 0.0979 |
| 12 | word_freq_re | -0.7968 | -0.1402 | -0.1726 | 0.1139 |
| 13 | word_freq_meeting | -0.7558 | -0.1350 | -0.1663 | 0.1096 |
| 14 | word_freq_font | 0.6721 | 0.1021 | 0.1257 | -0.0829 |

[16]:
```
# interpretation helpers (short textual summary)
pos_feats = df_top[df_top.weight > 0].feature.tolist()
neg_feats = df_top[df_top.weight < 0].feature.tolist()
print("Features that increase spam probability (positive weights):", pos_feats[:
 ↪8])
print("Features that decrease spam probability (negative weights):", neg_feats[:
 ↪8])
```

```
Features that increase spam probability (positive weights): ['word_freq_3d',
'capital_run_length_longest', 'char_freq_$', 'word_freq_000',
'word_freq_remove', 'char_freq_#', 'word_freq_857', 'word_freq_free']
Features that decrease spam probability (negative weights): ['word_freq_hp',
'word_freq_george', 'word_freq_edu', 'word_freq_415', 'word_freq_re',
'word_freq_meeting']
```