

a02_2

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```
[1]: import numpy as np
import matplotlib as plt

%load_ext autoreload
%autoreload 2

from a02_helper import *
from a02_functions import sigma, logsigma, l, dl, gd, sgdepoche, sgd, optimize, l
    ↪ classify, predict
```

```
[2]: %matplotlib inline
```

1 2. Maximum Likelihood Estimation

1.1 Helper functions

Define the logistic function in `a02_functions.py`. Make sure it operates on both scalars and vectors. Then test it below.

```
[3]: # this should give:
# [0.5, array([0.26894142, 0.5, 0.73105858])]
[sigma(0), sigma(np.array([-1, 0, 1]))]
```

```
[3]: [0.5, array([0.26894142, 0.5, 0.73105858])]
```

Define the logarithm of the logistic function in `a02_functions.py`. Make sure it operates on both scalars and vectors.

```
[4]: # this should give:
# [-0.69314718055994529, array([-1.31326169, -0.69314718, -0.31326169])]
[logsigma(0), logsigma(np.array([-1, 0, 1]))]
```

```
[4]: [-0.6931471805599453, array([-1.31326169, -0.69314718, -0.31326169])]
```

1.2 2b Log-likelihood and gradient

Complete the function for the log-likelihood of the logistic regression model in `a02_functions.py` and test it below.

```
[5]: # this should give:
# -47066.641667825766
l(y, Xz, np.linspace(-5, 5, D))
```

```
[5]: -47066.64166782577
```

Now fill out the function that returns the gradient of the log-likelihood of the logistic regression model in `a02_functions.py`, and test it below.

```
[6]: # this should give:
# array([ 551.33985842,  143.84116318,  841.83373606,  156.87237578,
#         802.61217579,  795.96202907,  920.69045803,  621.96516752,
#         659.18724769,  470.81259805,  771.32406968,  352.40325626,
#         455.66972482,  234.36600888,  562.45454038,  864.83981264,
#         787.19723703,  649.48042176,  902.6478154 ,  544.00539886,
#        1174.78638035,  120.3598967 ,  839.61141672,  633.30453444,
#        -706.66815087, -630.2039816 , -569.3451386 , -527.50996698,
#        -359.53701083, -476.64334832, -411.60620464, -375.11950586,
#        -345.37195689, -376.22044258, -407.31761977, -456.23251936,
#        -596.86960184, -107.97072355, -394.82170044, -229.18125598,
#        -288.46356547, -362.13402385, -450.87896465, -277.03932676,
#        -414.99293368, -452.28771693, -167.54649092, -270.9043748 ,
#        -252.20140951, -357.72497343, -259.12468742,  418.35938483,
#         604.54173228,   43.10390907,  152.24258478,  378.16731033,
#         416.12032881])
dl(y, Xz, np.linspace(-5, 5, D))
```

```
[6]: array([ 551.33985842,  143.84116318,  841.83373606,  156.87237578,
            802.61217579,  795.96202907,  920.69045803,  621.96516752,
            659.18724769,  470.81259805,  771.32406968,  352.40325626,
            455.66972482,  234.36600888,  562.45454038,  864.83981264,
            787.19723703,  649.48042176,  902.6478154 ,  544.00539886,
           1174.78638035,  120.3598967 ,  839.61141672,  633.30453444,
           -706.66815087, -630.2039816 , -569.3451386 , -527.50996698,
           -359.53701083, -476.64334832, -411.60620464, -375.11950586,
           -345.37195689, -376.22044258, -407.31761977, -456.23251936,
           -596.86960184, -107.97072355, -394.82170044, -229.18125598,
           -288.46356547, -362.13402385, -450.87896465, -277.03932676,
           -414.99293368, -452.28771693, -167.54649092, -270.9043748 ,
           -252.20140951, -357.72497343, -259.12468742,  418.35938483,
            604.54173228,   43.10390907,  152.24258478,  378.16731033,
            416.12032881])
```

1.3 2c Gradient descent

In `a02_functions.py`, define the objective and update function for one gradient-descent epoch for fitting an MLE estimate of logistic regression with gradient descent (should return a tuple of two functions; see `optimize` in `a02_functions.py`). Then test it below.

```
[7]: # this should give
# [47066.641667825766,
# array([ 4.13777838e+01, -1.56745627e+01,  5.75882538e+01,
#         1.14225143e+01,  5.54249703e+01,  5.99229049e+01,
#         7.11220141e+01,  4.84761728e+01,  5.78067289e+01,
#         4.54794720e+01,  7.14638492e+01,  1.51369386e+01,
#         3.36375739e+01,  2.15061217e+01,  5.78014255e+01,
#         6.72743066e+01,  7.00829312e+01,  5.29328088e+01,
#         6.16042473e+01,  5.50018510e+01,  8.94624817e+01,
#         2.74784480e+01,  8.51763599e+01,  5.60363965e+01,
#        -2.55865589e+01, -1.53788213e+01, -4.67015412e+01,
#        -2.50356570e+00, -3.85357592e+00, -2.21819155e+00,
#         3.32098671e+00,  3.86933390e+00, -2.00309898e+01,
#         3.84684492e+00, -2.19847927e-01, -1.29775457e+00,
#        -1.28374302e+01, -2.78303173e+00, -5.61671182e+00,
#         1.73657121e+01, -6.81197570e+00, -1.20249002e+01,
#         2.65789491e+00, -1.39557852e+01, -2.01135653e+01,
#        -2.72134051e+01, -9.45952961e-01, -1.02239111e+01,
#         1.52794293e-04, -5.18938123e-01, -3.19717561e+00,
#         4.62953437e+01,  7.87893022e+01,  1.88618651e+01,
#         2.85195027e+01,  5.04698358e+01,  6.41240689e+01]])
f, update = gd(y, Xz)
[f(np.linspace(-5, 5, D)), update(np.linspace(-5, -5, D), 0.1)]
```

(3065, 57)

3065

```
[7]: [47066.64166782577,
array([ 4.13777838e+01, -1.56745627e+01,  5.75882538e+01,  1.14225143e+01,
        5.54249703e+01,  5.99229049e+01,  7.11220141e+01,  4.84761728e+01,
        5.78067289e+01,  4.54794720e+01,  7.14638492e+01,  1.51369386e+01,
        3.36375739e+01,  2.15061217e+01,  5.78014255e+01,  6.72743066e+01,
        7.00829312e+01,  5.29328088e+01,  6.16042473e+01,  5.50018510e+01,
        8.94624817e+01,  2.74784480e+01,  8.51763599e+01,  5.60363965e+01,
       -2.55865589e+01, -1.53788213e+01, -4.67015412e+01, -2.50356570e+00,
       -3.85357592e+00, -2.21819155e+00,  3.32098671e+00,  3.86933390e+00,
       -2.00309898e+01,  3.84684492e+00, -2.19847927e-01, -1.29775457e+00,
       -1.28374302e+01, -2.78303173e+00, -5.61671182e+00,  1.73657121e+01,
       -6.81197570e+00, -1.20249002e+01,  2.65789491e+00, -1.39557852e+01,
       -2.01135653e+01, -2.72134051e+01, -9.45952961e-01, -1.02239111e+01,
        1.52794293e-04, -5.18938123e-01, -3.19717561e+00,  4.62953437e+01,
        7.87893022e+01,  1.88618651e+01,  2.85195027e+01,  5.04698358e+01,
        6.41240689e+01]])]
```

```
[13]: # Now you can run gradient descent!
np.random.seed(0)
w0 = np.random.normal(size=D)
wz_gd, vz_gd, ez_gd = optimize(gd(y, Xz), w0, nepochs=500)
```

(3065, 57)

Epoch	0: f=	6636.208, eps=0.010000000
3065		
Epoch	1: f=	4216.957, eps=0.010500000
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Epoch	2: f=	2657.519, eps=0.011025000
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Epoch	3: f=	1926.135, eps=0.011576250
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Epoch	4: f=	1449.495, eps=0.012155063
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Epoch	5: f=	1207.529, eps=0.012762816
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Epoch	6: f=	1052.489, eps=0.013400956
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Epoch	7: f=	957.275, eps=0.014071004
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Epoch	8: f=	899.610, eps=0.014774554
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Epoch	9: f=	882.904, eps=0.015513282
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Epoch	10: f=	1017.083, eps=0.007756641
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Epoch	11: f=	840.760, eps=0.008144473
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Epoch	12: f=	805.649, eps=0.008551697
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Epoch	13: f=	822.108, eps=0.004275848
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Epoch	14: f=	746.377, eps=0.004489641
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Epoch	15: f=	735.803, eps=0.004714123
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Epoch	16: f=	729.780, eps=0.004949829
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Epoch	17: f=	724.467, eps=0.005197320
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Epoch	18: f=	719.408, eps=0.005457186
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Epoch	19: f=	714.564, eps=0.005730046
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Epoch	20: f=	709.932, eps=0.006016548
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Epoch	21: f=	705.514, eps=0.006317375
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Epoch	22: f=	701.321, eps=0.006633244
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Epoch	23: f=	697.373, eps=0.006964906

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Epoch 24: f= 693.728, eps=0.007313152
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Epoch 25: f= 690.591, eps=0.007678809
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Epoch 26: f= 688.614, eps=0.008062750
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Epoch 27: f= 688.607, eps=0.008465887
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Epoch 28: f= 690.854, eps=0.004232944
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Epoch 29: f= 679.967, eps=0.004444591
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Epoch 30: f= 678.649, eps=0.004666820
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Epoch 31: f= 677.447, eps=0.004900161
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Epoch 32: f= 676.292, eps=0.005145169
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Epoch 33: f= 675.182, eps=0.005402428
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Epoch 34: f= 674.120, eps=0.005672549
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Epoch 35: f= 673.114, eps=0.005956177
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Epoch 36: f= 672.177, eps=0.006253986
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Epoch 37: f= 671.334, eps=0.006566685
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Epoch 38: f= 670.656, eps=0.006895019
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Epoch 39: f= 670.397, eps=0.007239770
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Epoch 40: f= 671.342, eps=0.003619885
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Epoch 41: f= 668.932, eps=0.003800879
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Epoch 42: f= 668.378, eps=0.003990923
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Epoch 43: f= 668.027, eps=0.004190469
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Epoch 44: f= 667.720, eps=0.004399993
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Epoch 45: f= 667.433, eps=0.004619993
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Epoch 46: f= 667.159, eps=0.004850992
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Epoch 47: f= 666.897, eps=0.005093542

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Epoch 48: f= 666.650, eps=0.005348219
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Epoch 49: f= 666.417, eps=0.005615630
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Epoch 50: f= 666.201, eps=0.005896411
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Epoch 51: f= 666.008, eps=0.006191232
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Epoch 52: f= 665.858, eps=0.006500794
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Epoch 53: f= 665.812, eps=0.006825833
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Epoch 54: f= 666.068, eps=0.003412917
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Epoch 55: f= 665.424, eps=0.003583562
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Epoch 56: f= 665.290, eps=0.003762741
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Epoch 57: f= 665.204, eps=0.003950878
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Epoch 58: f= 665.128, eps=0.004148421
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Epoch 59: f= 665.054, eps=0.004355843
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Epoch 60: f= 664.982, eps=0.004573635
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Epoch 61: f= 664.911, eps=0.004802316
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Epoch 62: f= 664.842, eps=0.005042432
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Epoch 63: f= 664.773, eps=0.005294554
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Epoch 64: f= 664.707, eps=0.005559282
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Epoch 65: f= 664.641, eps=0.005837246
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Epoch 66: f= 664.578, eps=0.006129108
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Epoch 67: f= 664.518, eps=0.006435563
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Epoch 68: f= 664.467, eps=0.006757341
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Epoch 69: f= 664.446, eps=0.007095208
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Epoch 70: f= 664.544, eps=0.003547604
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Epoch 71: f= 664.339, eps=0.003724984

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Epoch 72: f= 664.278, eps=0.003911234
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Epoch 73: f= 664.239, eps=0.004106795
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Epoch 74: f= 664.206, eps=0.004312135
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Epoch 75: f= 664.173, eps=0.004527742
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Epoch 76: f= 664.139, eps=0.004754129
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Epoch 77: f= 664.106, eps=0.004991835
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Epoch 78: f= 664.072, eps=0.005241427
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Epoch 79: f= 664.037, eps=0.005503499
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Epoch 80: f= 664.002, eps=0.005778674
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Epoch 81: f= 663.967, eps=0.006067607
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Epoch 82: f= 663.936, eps=0.006370988
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Epoch 83: f= 663.918, eps=0.006689537
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Epoch 84: f= 663.948, eps=0.003344768
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Epoch 85: f= 663.839, eps=0.003512007
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Epoch 86: f= 663.807, eps=0.003687607
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Epoch 87: f= 663.783, eps=0.003871988
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Epoch 88: f= 663.760, eps=0.004065587
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Epoch 89: f= 663.737, eps=0.004268866
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Epoch 90: f= 663.713, eps=0.004482310
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Epoch 91: f= 663.688, eps=0.004706425
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Epoch 92: f= 663.661, eps=0.004941746
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Epoch 93: f= 663.634, eps=0.005188834
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Epoch 94: f= 663.606, eps=0.005448275
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Epoch 95: f= 663.576, eps=0.005720689

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Epoch 96: f= 663.546, eps=0.006006724
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Epoch 97: f= 663.514, eps=0.006307060
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Epoch 98: f= 663.482, eps=0.006622413
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Epoch 99: f= 663.451, eps=0.006953533
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Epoch 100: f= 663.427, eps=0.007301210
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Epoch 101: f= 663.442, eps=0.003650605
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Epoch 102: f= 663.371, eps=0.003833135
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Epoch 103: f= 663.340, eps=0.004024792
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Epoch 104: f= 663.316, eps=0.004226032
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Epoch 105: f= 663.294, eps=0.004437333
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Epoch 106: f= 663.271, eps=0.004659200
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Epoch 107: f= 663.248, eps=0.004892160
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Epoch 108: f= 663.223, eps=0.005136768
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Epoch 109: f= 663.198, eps=0.005393606
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Epoch 110: f= 663.172, eps=0.005663287
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Epoch 111: f= 663.146, eps=0.005946451
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Epoch 112: f= 663.121, eps=0.006243773
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Epoch 113: f= 663.102, eps=0.006555962
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Epoch 114: f= 663.108, eps=0.003277981
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Epoch 115: f= 663.042, eps=0.003441880
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Epoch 116: f= 663.019, eps=0.003613974
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Epoch 117: f= 663.001, eps=0.003794673
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Epoch 118: f= 662.982, eps=0.003984406
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Epoch 119: f= 662.963, eps=0.004183627

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Epoch 120: f= 662.943, eps=0.004392808
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Epoch 121: f= 662.922, eps=0.004612449
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Epoch 122: f= 662.900, eps=0.004843071
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Epoch 123: f= 662.877, eps=0.005085225
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Epoch 124: f= 662.853, eps=0.005339486
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Epoch 125: f= 662.828, eps=0.005606460
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Epoch 126: f= 662.802, eps=0.005886783
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Epoch 127: f= 662.774, eps=0.006181122
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Epoch 128: f= 662.745, eps=0.006490178
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Epoch 129: f= 662.715, eps=0.006814687
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Epoch 130: f= 662.685, eps=0.007155422
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Epoch 131: f= 662.659, eps=0.007513193
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Epoch 132: f= 662.656, eps=0.007888852
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Epoch 133: f= 662.786, eps=0.003944426
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Epoch 134: f= 662.631, eps=0.004141647
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Epoch 135: f= 662.578, eps=0.004348730
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Epoch 136: f= 662.545, eps=0.004566166
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Epoch 137: f= 662.519, eps=0.004794475
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Epoch 138: f= 662.497, eps=0.005034198
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Epoch 139: f= 662.477, eps=0.005285908
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Epoch 140: f= 662.462, eps=0.005550204
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Epoch 141: f= 662.457, eps=0.005827714
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Epoch 142: f= 662.476, eps=0.002913857
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Epoch 143: f= 662.373, eps=0.003059550

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Epoch 144: f= 662.355, eps=0.003212527
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Epoch 145: f= 662.340, eps=0.003373154
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Epoch 146: f= 662.325, eps=0.003541811
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Epoch 147: f= 662.310, eps=0.003718902
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Epoch 148: f= 662.293, eps=0.003904847
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Epoch 149: f= 662.276, eps=0.004100089
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Epoch 150: f= 662.257, eps=0.004305094
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Epoch 151: f= 662.238, eps=0.004520348
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Epoch 152: f= 662.218, eps=0.004746366
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Epoch 153: f= 662.197, eps=0.004983684
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Epoch 154: f= 662.175, eps=0.005232868
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Epoch 155: f= 662.152, eps=0.005494512
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Epoch 156: f= 662.128, eps=0.005769237
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Epoch 157: f= 662.103, eps=0.006057699
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Epoch 158: f= 662.076, eps=0.006360584
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Epoch 159: f= 662.048, eps=0.006678613
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Epoch 160: f= 662.019, eps=0.007012544
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Epoch 161: f= 661.989, eps=0.007363171
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Epoch 162: f= 661.957, eps=0.007731330
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Epoch 163: f= 661.924, eps=0.008117896
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Epoch 164: f= 661.890, eps=0.008523791
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Epoch 165: f= 661.859, eps=0.008949981
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Epoch 166: f= 661.868, eps=0.004474990
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Epoch 167: f= 661.834, eps=0.004698740

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Epoch 168: f= 661.809, eps=0.004933677
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Epoch 169: f= 661.791, eps=0.005180361
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Epoch 170: f= 661.780, eps=0.005439379
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Epoch 171: f= 661.784, eps=0.002719689
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Epoch 172: f= 661.698, eps=0.002855674
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Epoch 173: f= 661.685, eps=0.002998458
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Epoch 174: f= 661.672, eps=0.003148380
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Epoch 175: f= 661.659, eps=0.003305799
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Epoch 176: f= 661.645, eps=0.003471089
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Epoch 177: f= 661.630, eps=0.003644644
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Epoch 178: f= 661.615, eps=0.003826876
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Epoch 179: f= 661.599, eps=0.004018220
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Epoch 180: f= 661.582, eps=0.004219131
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Epoch 181: f= 661.564, eps=0.004430087
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Epoch 182: f= 661.546, eps=0.004651592
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Epoch 183: f= 661.526, eps=0.004884171
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Epoch 184: f= 661.506, eps=0.005128380
3065
Epoch 185: f= 661.485, eps=0.005384799
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Epoch 186: f= 661.462, eps=0.005654039
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Epoch 187: f= 661.439, eps=0.005936741
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Epoch 188: f= 661.414, eps=0.006233578
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Epoch 189: f= 661.388, eps=0.006545257
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Epoch 190: f= 661.361, eps=0.006872520
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Epoch 191: f= 661.333, eps=0.007216146

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Epoch 192: f= 661.303, eps=0.007576953
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Epoch 193: f= 661.272, eps=0.007955801
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Epoch 194: f= 661.240, eps=0.008353591
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Epoch 195: f= 661.206, eps=0.008771270
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Epoch 196: f= 661.170, eps=0.009209834
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Epoch 197: f= 661.133, eps=0.009670325
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Epoch 198: f= 661.097, eps=0.010153842
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Epoch 199: f= 661.093, eps=0.010661534
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Epoch 200: f= 661.463, eps=0.005330767
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Epoch 201: f= 661.555, eps=0.002665383
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Epoch 202: f= 660.978, eps=0.002798653
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Epoch 203: f= 660.966, eps=0.002938585
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Epoch 204: f= 660.955, eps=0.003085514
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Epoch 205: f= 660.942, eps=0.003239790
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Epoch 206: f= 660.929, eps=0.003401780
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Epoch 207: f= 660.916, eps=0.003571869
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Epoch 208: f= 660.902, eps=0.003750462
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Epoch 209: f= 660.887, eps=0.003937985
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Epoch 210: f= 660.871, eps=0.004134885
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Epoch 211: f= 660.855, eps=0.004341629
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Epoch 213: f= 660.819, eps=0.004786646
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Epoch 215: f= 660.781, eps=0.005277277

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Epoch 216: f= 660.760, eps=0.005541141
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Epoch 218: f= 660.715, eps=0.006109108
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Epoch 219: f= 660.691, eps=0.006414563
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Epoch 220: f= 660.666, eps=0.006735291
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Epoch 221: f= 660.640, eps=0.007072056
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Epoch 222: f= 660.612, eps=0.007425659
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Epoch 223: f= 660.583, eps=0.007796941
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Epoch 224: f= 660.553, eps=0.008186788
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Epoch 225: f= 660.521, eps=0.008596128
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Epoch 226: f= 660.488, eps=0.009025934
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Epoch 227: f= 660.453, eps=0.009477231
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Epoch 228: f= 660.417, eps=0.009951093
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Epoch 229: f= 660.379, eps=0.010448647
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Epoch 230: f= 660.344, eps=0.010971080
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Epoch 231: f= 660.362, eps=0.005485540
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Epoch 232: f= 660.377, eps=0.002742770
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Epoch 233: f= 660.267, eps=0.002879908
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Epoch 234: f= 660.254, eps=0.003023904
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Epoch 236: f= 660.231, eps=0.003333854
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Epoch 238: f= 660.205, eps=0.003675574
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Epoch 239: f= 660.191, eps=0.003859353

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 Epoch 242: f= 660.145, eps=0.004467683
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 Epoch 244: f= 660.111, eps=0.004925621
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 Epoch 245: f= 660.092, eps=0.005171902
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 Epoch 246: f= 660.073, eps=0.005430497
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 Epoch 247: f= 660.052, eps=0.005702022
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 Epoch 248: f= 660.031, eps=0.005987123
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 Epoch 249: f= 660.009, eps=0.006286479
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 Epoch 251: f= 659.961, eps=0.006930843
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 Epoch 252: f= 659.935, eps=0.007277385
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 Epoch 253: f= 659.908, eps=0.007641254
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 Epoch 254: f= 659.880, eps=0.008023317
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 Epoch 255: f= 659.850, eps=0.008424483
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 Epoch 260: f= 659.888, eps=0.005120006
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Epoch 269: f= 659.572, eps=0.003782290
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Epoch 270: f= 659.558, eps=0.003971405
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Epoch 271: f= 659.543, eps=0.004169975
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Epoch 272: f= 659.528, eps=0.004378474
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Epoch 273: f= 659.513, eps=0.004597397
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Epoch 274: f= 659.496, eps=0.004827267
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Epoch 275: f= 659.479, eps=0.005068631
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Epoch 276: f= 659.460, eps=0.005322062
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Epoch 278: f= 659.421, eps=0.005867574
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Epoch 279: f= 659.400, eps=0.006160952
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Epoch 280: f= 659.378, eps=0.006469000
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Epoch 281: f= 659.355, eps=0.006792450
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Epoch 282: f= 659.331, eps=0.007132072
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Epoch 283: f= 659.305, eps=0.007488676
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Epoch 284: f= 659.279, eps=0.007863110
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Epoch 285: f= 659.251, eps=0.008256265
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Epoch 286: f= 659.222, eps=0.008669078
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Epoch 287: f= 659.191, eps=0.009102532

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Epoch 288: f= 659.159, eps=0.009557659
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Epoch 289: f= 659.125, eps=0.010035542
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Epoch 290: f= 659.090, eps=0.010537319
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Epoch 291: f= 659.053, eps=0.011064185
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Epoch 292: f= 659.016, eps=0.011617394
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Epoch 293: f= 658.992, eps=0.012198264
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Epoch 294: f= 659.226, eps=0.006099132
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Epoch 295: f= 659.526, eps=0.003049566
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Epoch 296: f= 658.916, eps=0.003202044
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Epoch 297: f= 658.891, eps=0.003362147
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Epoch 298: f= 658.878, eps=0.003530254
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Epoch 299: f= 658.865, eps=0.003706767
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Epoch 300: f= 658.852, eps=0.003892105
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Epoch 301: f= 658.839, eps=0.004086710
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Epoch 302: f= 658.825, eps=0.004291046
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Epoch 303: f= 658.810, eps=0.004505598
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Epoch 304: f= 658.795, eps=0.004730878
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Epoch 305: f= 658.778, eps=0.004967422
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Epoch 306: f= 658.761, eps=0.005215793
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Epoch 307: f= 658.743, eps=0.005476582
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Epoch 308: f= 658.725, eps=0.005750412
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Epoch 309: f= 658.705, eps=0.006037932
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Epoch 310: f= 658.684, eps=0.006339829
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Epoch 311: f= 658.663, eps=0.006656820

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Epoch 312: f= 658.640, eps=0.006989661
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Epoch 313: f= 658.617, eps=0.007339144
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Epoch 314: f= 658.593, eps=0.007706101
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Epoch 315: f= 658.573, eps=0.008091406
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Epoch 316: f= 658.582, eps=0.004045703
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Epoch 317: f= 658.544, eps=0.004247988
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Epoch 318: f= 658.521, eps=0.004460388
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Epoch 319: f= 658.503, eps=0.004683407
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Epoch 320: f= 658.486, eps=0.004917578
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Epoch 321: f= 658.470, eps=0.005163456
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Epoch 322: f= 658.455, eps=0.005421629
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Epoch 323: f= 658.443, eps=0.005692711
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Epoch 324: f= 658.436, eps=0.005977346
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Epoch 325: f= 658.450, eps=0.002988673
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Epoch 326: f= 658.381, eps=0.003138107
3065
Epoch 327: f= 658.368, eps=0.003295012
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Epoch 328: f= 658.356, eps=0.003459763
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Epoch 329: f= 658.345, eps=0.003632751
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Epoch 330: f= 658.333, eps=0.003814388
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Epoch 331: f= 658.320, eps=0.004005108
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Epoch 332: f= 658.307, eps=0.004205363
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Epoch 333: f= 658.293, eps=0.004415631
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Epoch 334: f= 658.278, eps=0.004636413
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Epoch 335: f= 658.263, eps=0.004868234

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Epoch 336: f= 658.247, eps=0.005111645
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Epoch 337: f= 658.230, eps=0.005367228
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Epoch 338: f= 658.212, eps=0.005635589
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Epoch 339: f= 658.193, eps=0.005917368
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Epoch 340: f= 658.174, eps=0.006213237
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Epoch 341: f= 658.153, eps=0.006523899
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Epoch 342: f= 658.132, eps=0.006850094
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Epoch 343: f= 658.109, eps=0.007192598
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Epoch 344: f= 658.086, eps=0.007552228
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Epoch 345: f= 658.061, eps=0.007929840
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Epoch 346: f= 658.036, eps=0.008326332
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Epoch 347: f= 658.017, eps=0.008742648
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Epoch 348: f= 658.040, eps=0.004371324
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Epoch 349: f= 658.004, eps=0.004589890
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Epoch 350: f= 657.981, eps=0.004819385
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Epoch 351: f= 657.965, eps=0.005060354
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Epoch 352: f= 657.954, eps=0.005313372
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Epoch 353: f= 657.953, eps=0.005579040
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Epoch 354: f= 657.969, eps=0.002789520
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Epoch 355: f= 657.876, eps=0.002928996
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Epoch 356: f= 657.864, eps=0.003075446
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Epoch 357: f= 657.854, eps=0.003229218
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Epoch 358: f= 657.844, eps=0.003390679
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Epoch 359: f= 657.833, eps=0.003560213

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Epoch 360: f= 657.821, eps=0.003738224
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Epoch 361: f= 657.809, eps=0.003925135
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Epoch 362: f= 657.797, eps=0.004121392
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Epoch 363: f= 657.783, eps=0.004327461
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Epoch 364: f= 657.770, eps=0.004543834
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Epoch 365: f= 657.755, eps=0.004771026
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Epoch 366: f= 657.740, eps=0.005009577
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Epoch 367: f= 657.724, eps=0.005260056
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Epoch 368: f= 657.707, eps=0.005523059
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Epoch 369: f= 657.689, eps=0.005799212
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Epoch 370: f= 657.671, eps=0.006089173
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Epoch 371: f= 657.651, eps=0.006393631
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Epoch 372: f= 657.631, eps=0.006713313
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Epoch 373: f= 657.609, eps=0.007048978
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Epoch 374: f= 657.587, eps=0.007401427
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Epoch 375: f= 657.564, eps=0.007771499
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Epoch 376: f= 657.539, eps=0.008160074
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Epoch 377: f= 657.513, eps=0.008568077
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Epoch 378: f= 657.486, eps=0.008996481
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Epoch 379: f= 657.460, eps=0.009446305
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Epoch 380: f= 657.445, eps=0.009918621
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Epoch 381: f= 657.554, eps=0.004959310
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Epoch 382: f= 657.540, eps=0.005207276
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Epoch 383: f= 657.567, eps=0.002603638

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 Epoch 384: f= 657.357, eps=0.002733820
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 Epoch 385: f= 657.348, eps=0.002870511
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 Epoch 386: f= 657.339, eps=0.003014036
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 Epoch 387: f= 657.330, eps=0.003164738
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 Epoch 388: f= 657.320, eps=0.003322975
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 Epoch 389: f= 657.310, eps=0.003489124
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 Epoch 390: f= 657.299, eps=0.003663580
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 Epoch 391: f= 657.287, eps=0.003846759
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 Epoch 392: f= 657.275, eps=0.004039097
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 Epoch 393: f= 657.263, eps=0.004241052
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 Epoch 394: f= 657.250, eps=0.004453104
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 Epoch 395: f= 657.236, eps=0.004675760
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 Epoch 396: f= 657.221, eps=0.004909548
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 Epoch 397: f= 657.206, eps=0.005155025
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 Epoch 398: f= 657.190, eps=0.005412776
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 Epoch 399: f= 657.173, eps=0.005683415
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 Epoch 400: f= 657.156, eps=0.005967586
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 Epoch 401: f= 657.138, eps=0.006265965
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 Epoch 402: f= 657.118, eps=0.006579263
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 Epoch 403: f= 657.098, eps=0.006908226
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 Epoch 404: f= 657.077, eps=0.007253638
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 Epoch 405: f= 657.054, eps=0.007616320
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 Epoch 406: f= 657.031, eps=0.007997136
 3065
 Epoch 407: f= 657.007, eps=0.008396992

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 Epoch 408: f= 656.981, eps=0.008816842
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 Epoch 409: f= 656.954, eps=0.009257684
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 Epoch 410: f= 656.926, eps=0.009720568
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 Epoch 411: f= 656.896, eps=0.010206597
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 Epoch 412: f= 656.866, eps=0.010716927
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 Epoch 413: f= 656.838, eps=0.011252773
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 Epoch 414: f= 656.871, eps=0.005626387
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 Epoch 415: f= 656.908, eps=0.002813193
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 Epoch 416: f= 656.776, eps=0.002953853
 3065
 Epoch 417: f= 656.765, eps=0.003101546
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 Epoch 418: f= 656.755, eps=0.003256623
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 Epoch 419: f= 656.745, eps=0.003419454
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 Epoch 420: f= 656.735, eps=0.003590427
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 Epoch 421: f= 656.724, eps=0.003769948
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 Epoch 422: f= 656.713, eps=0.003958445
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 Epoch 423: f= 656.701, eps=0.004156368
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 Epoch 424: f= 656.689, eps=0.004364186
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 Epoch 425: f= 656.676, eps=0.004582395
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 Epoch 426: f= 656.662, eps=0.004811515
 3065
 Epoch 427: f= 656.648, eps=0.005052091
 3065
 Epoch 428: f= 656.632, eps=0.005304695
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 Epoch 429: f= 656.617, eps=0.005569930
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 Epoch 430: f= 656.600, eps=0.005848427
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 Epoch 431: f= 656.583, eps=0.006140848

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Epoch 432: f= 656.564, eps=0.006447890
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Epoch 433: f= 656.545, eps=0.006770285
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Epoch 434: f= 656.525, eps=0.007108799
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Epoch 435: f= 656.504, eps=0.007464239
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Epoch 436: f= 656.482, eps=0.007837451
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Epoch 437: f= 656.459, eps=0.008229324
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Epoch 438: f= 656.435, eps=0.008640790
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Epoch 439: f= 656.410, eps=0.009072829
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Epoch 440: f= 656.388, eps=0.009526471
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Epoch 441: f= 656.406, eps=0.004763235
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Epoch 442: f= 656.387, eps=0.005001397
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Epoch 443: f= 656.379, eps=0.005251467
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Epoch 444: f= 656.381, eps=0.002625734
3065
Epoch 445: f= 656.303, eps=0.002757020
3065
Epoch 446: f= 656.295, eps=0.002894871
3065
Epoch 447: f= 656.286, eps=0.003039615
3065
Epoch 448: f= 656.277, eps=0.003191596
3065
Epoch 449: f= 656.268, eps=0.003351175
3065
Epoch 450: f= 656.258, eps=0.003518734
3065
Epoch 451: f= 656.248, eps=0.003694671
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Epoch 452: f= 656.237, eps=0.003879404
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Epoch 453: f= 656.226, eps=0.004073375
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Epoch 454: f= 656.214, eps=0.004277043
3065
Epoch 455: f= 656.202, eps=0.004490895

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Epoch 456: f= 656.189, eps=0.004715440
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Epoch 457: f= 656.175, eps=0.004951212
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Epoch 458: f= 656.161, eps=0.005198773
3065
Epoch 459: f= 656.145, eps=0.005458711
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Epoch 460: f= 656.130, eps=0.005731647
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Epoch 461: f= 656.113, eps=0.006018229
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Epoch 462: f= 656.096, eps=0.006319141
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Epoch 463: f= 656.077, eps=0.006635098
3065
Epoch 464: f= 656.058, eps=0.006966853
3065
Epoch 465: f= 656.038, eps=0.007315195
3065
Epoch 466: f= 656.017, eps=0.007680955
3065
Epoch 467: f= 655.995, eps=0.008065003
3065
Epoch 468: f= 655.972, eps=0.008468253
3065
Epoch 469: f= 655.948, eps=0.008891666
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Epoch 470: f= 655.923, eps=0.009336249
3065
Epoch 471: f= 655.896, eps=0.009803061
3065
Epoch 472: f= 655.868, eps=0.010293215
3065
Epoch 473: f= 655.841, eps=0.010807875
3065
Epoch 474: f= 655.835, eps=0.011348269
3065
Epoch 475: f= 656.135, eps=0.005674135
3065
Epoch 476: f= 656.301, eps=0.002837067
3065
Epoch 477: f= 655.760, eps=0.002978921
3065
Epoch 478: f= 655.744, eps=0.003127867
3065
Epoch 479: f= 655.735, eps=0.003284260

```

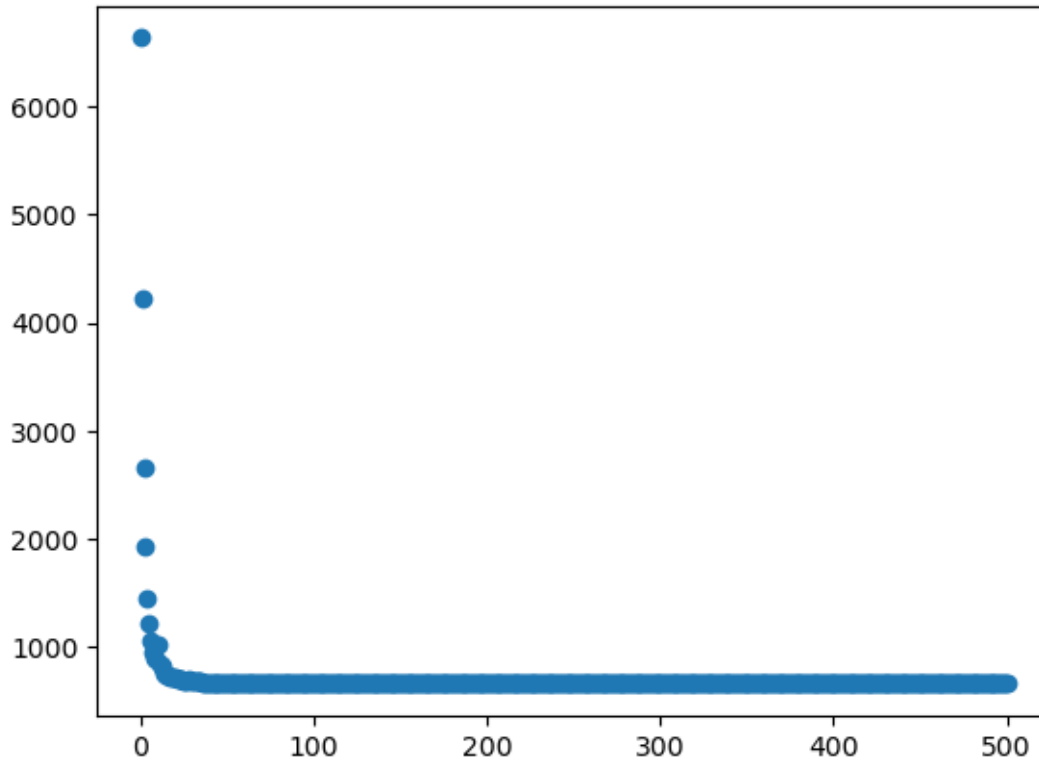
3065
Epoch 480: f= 655.725, eps=0.003448473
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Epoch 481: f= 655.716, eps=0.003620897
3065
Epoch 482: f= 655.705, eps=0.003801941
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Epoch 483: f= 655.695, eps=0.003992039
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Epoch 484: f= 655.684, eps=0.004191640
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Epoch 485: f= 655.672, eps=0.004401222
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Epoch 486: f= 655.659, eps=0.004621284
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Epoch 487: f= 655.646, eps=0.004852348
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Epoch 488: f= 655.633, eps=0.005094965
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Epoch 489: f= 655.619, eps=0.005349713
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Epoch 490: f= 655.604, eps=0.005617199
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Epoch 491: f= 655.588, eps=0.005898059
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Epoch 492: f= 655.571, eps=0.006192962
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Epoch 493: f= 655.554, eps=0.006502610
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Epoch 494: f= 655.536, eps=0.006827741
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Epoch 495: f= 655.517, eps=0.007169128
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Epoch 496: f= 655.497, eps=0.007527584
3065
Epoch 497: f= 655.476, eps=0.007903963
3065
Epoch 498: f= 655.454, eps=0.008299161
3065
Epoch 499: f= 655.432, eps=0.008714119
3065
Result after 500 epochs: f=655.4134964699433

```

```

[14]: # look at how gradient descent made progress
plt.scatter(np.arange(0, 501), vz_gd, label='GD')
plt.show()

```

1.4 2d Stochastic gradient descent

Implement stochastic gradient descent in `a02_functions.py`. Then test it with the given results below.

```
[15]: # when you run this multiple times, with 50% probability you should get the
# following result (there is one other result which is very close):
# array([ -3.43689655e+02,  -1.71161311e+02,  -5.71093536e+02,
#         -5.16478220e+01,   4.66294348e+02,  -3.71589878e+02,
#         5.21493183e+02,   1.25699230e+03,   8.33804130e+02,
#         5.63185399e+02,   1.32761302e+03,  -2.64104011e+02,
#         7.10693307e+02,  -1.75497331e+02,  -1.94174427e+02,
#         1.11641507e+02,  -3.30817509e+02,  -3.46754913e+02,
#         8.48722111e+02,  -1.89136304e+02,  -4.25693844e+02,
#        -1.23084189e+02,  -2.95894797e+02,  -2.35789333e+02,
#        -3.38695243e+02,  -3.05642830e+02,  -2.28975383e+02,
#        -2.38075137e+02,  -1.66702530e+02,  -2.27341599e+02,
#        -1.77575620e+02,  -1.49093855e+02,  -1.70028859e+02,
#        -1.50243833e+02,  -1.82986008e+02,  -2.41143708e+02,
#        -3.31047159e+02,  -5.79991185e+01,  -1.98477863e+02,
#        -1.91264948e+02,  -1.17371919e+02,  -1.66953779e+02,
#        -2.01472565e+02,  -1.23330949e+02,  -3.00857740e+02,
```

```
#      -1.95853348e+02,  -7.44868073e+01,  -1.11172370e+02,
#      -1.57618226e+02,  -1.25729512e+00,  -1.45536466e+02,
#      -1.43362438e+02,  -3.00429708e+02,  -9.84391082e+01,
#      -4.54152047e+01,  -5.26492232e+01,  -1.45175427e+02])
sgdepoch(y[1:3], Xz[1:3, :], np.linspace(-5, 5, D), 1000)
```

```
[15]: array([-3.43689655e+02, -1.71161311e+02, -5.71093536e+02, -5.16478220e+01,
 4.66294348e+02, -3.71589878e+02,  5.21493183e+02,  1.25699230e+03,
 8.33804130e+02,  5.63185399e+02,  1.32761302e+03, -2.64104011e+02,
 7.10693307e+02, -1.75497331e+02, -1.94174427e+02,  1.11641507e+02,
-3.30817509e+02, -3.46754913e+02,  8.48722111e+02, -1.89136304e+02,
-4.25693844e+02, -1.23084189e+02, -2.95894797e+02, -2.35789333e+02,
-3.38695243e+02, -3.05642830e+02, -2.28975383e+02, -2.38075137e+02,
-1.66702530e+02, -2.27341599e+02, -1.77575620e+02, -1.49093855e+02,
-1.70028859e+02, -1.50243833e+02, -1.82986008e+02, -2.41143708e+02,
-3.31047159e+02, -5.79991185e+01, -1.98477863e+02, -1.91264948e+02,
-1.17371919e+02, -1.66953779e+02, -2.01472565e+02, -1.23330949e+02,
-3.00857740e+02, -1.95853348e+02, -7.44868073e+01, -1.11172370e+02,
-1.57618226e+02, -1.25729512e+00, -1.45536466e+02, -1.43362438e+02,
-3.00429708e+02, -9.84391082e+01, -4.54152047e+01, -5.26492232e+01,
-1.45175427e+02])
```

```
[16]: # with 50% probability, you should get:
# [40.864973045695081,
#      array([-3.43689655e+02, -1.71161311e+02, -5.71093536e+02,
#      -5.16478220e+01,  4.66294348e+02, -3.71589878e+02,
#      5.21493183e+02,  1.25699230e+03,  8.33804130e+02,
#      5.63185399e+02,  1.32761302e+03, -2.64104011e+02,
#      7.10693307e+02, -1.75497331e+02, -1.94174427e+02,
#      1.11641507e+02, -3.30817509e+02, -3.46754913e+02,
#      8.48722111e+02, -1.89136304e+02, -4.25693844e+02,
#      -1.23084189e+02, -2.95894797e+02, -2.35789333e+02,
#      -3.38695243e+02, -3.05642830e+02, -2.28975383e+02,
#      -2.38075137e+02, -1.66702530e+02, -2.27341599e+02,
#      -1.77575620e+02, -1.49093855e+02, -1.70028859e+02,
#      -1.50243833e+02, -1.82986008e+02, -2.41143708e+02,
#      -3.31047159e+02, -5.79991185e+01, -1.98477863e+02,
#      -1.91264948e+02, -1.17371919e+02, -1.66953779e+02,
#      -2.01472565e+02, -1.23330949e+02, -3.00857740e+02,
#      -1.95853348e+02, -7.44868073e+01, -1.11172370e+02,
#      -1.57618226e+02, -1.25729512e+00, -1.45536466e+02,
#      -1.43362438e+02, -3.00429708e+02, -9.84391082e+01,
#      -4.54152047e+01, -5.26492232e+01, -1.45175427e+02])]
f, update = sgd(y[1:3], Xz[1:3, :])
[f(np.linspace(-5, 5, D)), update(np.linspace(-5, 5, D), 1000)]
```

```
[16]: [40.86497304569509,
       array([-3.43689655e+02, -1.71161311e+02, -5.71093536e+02, -5.16478220e+01,
              4.66294348e+02, -3.71589878e+02,  5.21493183e+02,  1.25699230e+03,
              8.33804130e+02,  5.63185399e+02,  1.32761302e+03, -2.64104011e+02,
              7.10693307e+02, -1.75497331e+02, -1.94174427e+02,  1.11641507e+02,
              -3.30817509e+02, -3.46754913e+02,  8.48722111e+02, -1.89136304e+02,
              -4.25693844e+02, -1.23084189e+02, -2.95894797e+02, -2.35789333e+02,
              -3.38695243e+02, -3.05642830e+02, -2.28975383e+02, -2.38075137e+02,
              -1.66702530e+02, -2.27341599e+02, -1.77575620e+02, -1.49093855e+02,
              -1.70028859e+02, -1.50243833e+02, -1.82986008e+02, -2.41143708e+02,
              -3.31047159e+02, -5.79991185e+01, -1.98477863e+02, -1.91264948e+02,
              -1.17371919e+02, -1.66953779e+02, -2.01472565e+02, -1.23330949e+02,
              -3.00857740e+02, -1.95853348e+02, -7.44868073e+01, -1.11172370e+02,
              -1.57618226e+02, -1.25729512e+00, -1.45536466e+02, -1.43362438e+02,
              -3.00429708e+02, -9.84391082e+01, -4.54152047e+01, -5.26492232e+01,
              -1.45175427e+02]])]
```

```
[17]: # you can run stochastic gradient descent!
      wz_sgd, vz_sgd, ez_sgd = optimize(sgd(y, Xz), w0, nepochs=500)
```

```
Epoch 0: f= 6636.208, eps=0.010000000
Epoch 1: f= 958.654, eps=0.010500000
Epoch 2: f= 786.651, eps=0.011025000
Epoch 3: f= 738.739, eps=0.011576250
Epoch 4: f= 718.166, eps=0.012155063
Epoch 5: f= 709.413, eps=0.012762816
Epoch 6: f= 696.048, eps=0.013400956
Epoch 7: f= 701.674, eps=0.006700478
Epoch 8: f= 686.406, eps=0.007035502
Epoch 9: f= 683.692, eps=0.007387277
Epoch 10: f= 684.500, eps=0.003693639
Epoch 11: f= 679.969, eps=0.003878321
Epoch 12: f= 679.230, eps=0.004072237
Epoch 13: f= 678.181, eps=0.004275848
Epoch 14: f= 677.570, eps=0.004489641
Epoch 15: f= 676.763, eps=0.004714123
Epoch 16: f= 675.966, eps=0.004949829
Epoch 17: f= 676.625, eps=0.002474914
Epoch 18: f= 675.185, eps=0.002598660
Epoch 19: f= 674.531, eps=0.002728593
Epoch 20: f= 674.095, eps=0.002865023
Epoch 21: f= 673.692, eps=0.003008274
Epoch 22: f= 673.359, eps=0.003158688
Epoch 23: f= 673.075, eps=0.003316622
Epoch 24: f= 672.914, eps=0.003482453
Epoch 25: f= 672.549, eps=0.003656576
Epoch 26: f= 672.208, eps=0.003839405
Epoch 27: f= 672.128, eps=0.004031375
```

Epoch	28:	f=	671.943,	eps=0.004232944
Epoch	29:	f=	671.364,	eps=0.004444591
Epoch	30:	f=	671.437,	eps=0.002222295
Epoch	31:	f=	670.780,	eps=0.002333410
Epoch	32:	f=	670.619,	eps=0.002450081
Epoch	33:	f=	670.405,	eps=0.002572585
Epoch	34:	f=	670.230,	eps=0.002701214
Epoch	35:	f=	670.065,	eps=0.002836275
Epoch	36:	f=	669.883,	eps=0.002978088
Epoch	37:	f=	669.804,	eps=0.003126993
Epoch	38:	f=	669.708,	eps=0.003283342
Epoch	39:	f=	669.592,	eps=0.003447510
Epoch	40:	f=	669.531,	eps=0.003619885
Epoch	41:	f=	669.408,	eps=0.003800879
Epoch	42:	f=	669.209,	eps=0.003990923
Epoch	43:	f=	669.264,	eps=0.001995462
Epoch	44:	f=	668.761,	eps=0.002095235
Epoch	45:	f=	668.598,	eps=0.002199996
Epoch	46:	f=	668.608,	eps=0.001099998
Epoch	47:	f=	668.502,	eps=0.001154998
Epoch	48:	f=	668.400,	eps=0.001212748
Epoch	49:	f=	668.324,	eps=0.001273385
Epoch	50:	f=	668.278,	eps=0.001337055
Epoch	51:	f=	668.235,	eps=0.001403907
Epoch	52:	f=	668.189,	eps=0.001474103
Epoch	53:	f=	668.134,	eps=0.001547808
Epoch	54:	f=	668.080,	eps=0.001625198
Epoch	55:	f=	668.035,	eps=0.001706458
Epoch	56:	f=	667.966,	eps=0.001791781
Epoch	57:	f=	667.927,	eps=0.001881370
Epoch	58:	f=	667.878,	eps=0.001975439
Epoch	59:	f=	667.813,	eps=0.002074211
Epoch	60:	f=	667.757,	eps=0.002177921
Epoch	61:	f=	667.722,	eps=0.002286817
Epoch	62:	f=	667.667,	eps=0.002401158
Epoch	63:	f=	667.613,	eps=0.002521216
Epoch	64:	f=	667.515,	eps=0.002647277
Epoch	65:	f=	667.483,	eps=0.002779641
Epoch	66:	f=	667.370,	eps=0.002918623
Epoch	67:	f=	667.319,	eps=0.003064554
Epoch	68:	f=	667.369,	eps=0.001532277
Epoch	69:	f=	667.264,	eps=0.001608891
Epoch	70:	f=	667.200,	eps=0.001689335
Epoch	71:	f=	667.172,	eps=0.001773802
Epoch	72:	f=	667.112,	eps=0.001862492
Epoch	73:	f=	667.006,	eps=0.001955617
Epoch	74:	f=	667.045,	eps=0.000977808
Epoch	75:	f=	666.973,	eps=0.001026699

Epoch	76:	f=	666.911,	eps=0.001078034
Epoch	77:	f=	666.884,	eps=0.001131935
Epoch	78:	f=	666.844,	eps=0.001188532
Epoch	79:	f=	666.823,	eps=0.001247959
Epoch	80:	f=	666.794,	eps=0.001310357
Epoch	81:	f=	666.766,	eps=0.001375875
Epoch	82:	f=	666.745,	eps=0.001444668
Epoch	83:	f=	666.708,	eps=0.001516902
Epoch	84:	f=	666.707,	eps=0.001592747
Epoch	85:	f=	666.649,	eps=0.001672384
Epoch	86:	f=	666.632,	eps=0.001756003
Epoch	87:	f=	666.610,	eps=0.001843804
Epoch	88:	f=	666.579,	eps=0.001935994
Epoch	89:	f=	666.532,	eps=0.002032793
Epoch	90:	f=	666.482,	eps=0.002134433
Epoch	91:	f=	666.461,	eps=0.002241155
Epoch	92:	f=	666.440,	eps=0.002353213
Epoch	93:	f=	666.365,	eps=0.002470873
Epoch	94:	f=	666.433,	eps=0.001235437
Epoch	95:	f=	666.342,	eps=0.001297208
Epoch	96:	f=	666.294,	eps=0.001362069
Epoch	97:	f=	666.259,	eps=0.001430172
Epoch	98:	f=	666.240,	eps=0.001501681
Epoch	99:	f=	666.209,	eps=0.001576765
Epoch	100:	f=	666.185,	eps=0.001655603
Epoch	101:	f=	666.153,	eps=0.001738383
Epoch	102:	f=	666.153,	eps=0.001825303
Epoch	103:	f=	666.166,	eps=0.000912651
Epoch	104:	f=	666.137,	eps=0.000958284
Epoch	105:	f=	666.104,	eps=0.001006198
Epoch	106:	f=	666.091,	eps=0.001056508
Epoch	107:	f=	666.047,	eps=0.001109333
Epoch	108:	f=	666.025,	eps=0.001164800
Epoch	109:	f=	666.006,	eps=0.001223040
Epoch	110:	f=	665.992,	eps=0.001284192
Epoch	111:	f=	665.967,	eps=0.001348402
Epoch	112:	f=	665.966,	eps=0.001415822
Epoch	113:	f=	665.960,	eps=0.001486613
Epoch	114:	f=	665.916,	eps=0.001560943
Epoch	115:	f=	665.935,	eps=0.000780472
Epoch	116:	f=	665.913,	eps=0.000819495
Epoch	117:	f=	665.879,	eps=0.000860470
Epoch	118:	f=	665.865,	eps=0.000903494
Epoch	119:	f=	665.855,	eps=0.000948668
Epoch	120:	f=	665.834,	eps=0.000996102
Epoch	121:	f=	665.824,	eps=0.001045907
Epoch	122:	f=	665.811,	eps=0.001098202
Epoch	123:	f=	665.786,	eps=0.001153112

Epoch 124: f= 665.777, eps=0.001210768
Epoch 125: f= 665.778, eps=0.000605384
Epoch 126: f= 665.761, eps=0.000635653
Epoch 127: f= 665.745, eps=0.000667436
Epoch 128: f= 665.737, eps=0.000700808
Epoch 129: f= 665.725, eps=0.000735848
Epoch 130: f= 665.716, eps=0.000772640
Epoch 131: f= 665.704, eps=0.000811272
Epoch 132: f= 665.700, eps=0.000851836
Epoch 133: f= 665.687, eps=0.000894428
Epoch 134: f= 665.676, eps=0.000939149
Epoch 135: f= 665.662, eps=0.000986107
Epoch 136: f= 665.651, eps=0.001035412
Epoch 137: f= 665.640, eps=0.001087182
Epoch 138: f= 665.630, eps=0.001141542
Epoch 139: f= 665.614, eps=0.001198619
Epoch 140: f= 665.602, eps=0.001258550
Epoch 141: f= 665.586, eps=0.001321477
Epoch 142: f= 665.573, eps=0.001387551
Epoch 143: f= 665.570, eps=0.001456928
Epoch 144: f= 665.538, eps=0.001529775
Epoch 145: f= 665.519, eps=0.001606264
Epoch 146: f= 665.536, eps=0.000803132
Epoch 147: f= 665.500, eps=0.000843288
Epoch 148: f= 665.486, eps=0.000885453
Epoch 149: f= 665.471, eps=0.000929725
Epoch 150: f= 665.461, eps=0.000976212
Epoch 151: f= 665.451, eps=0.001025022
Epoch 152: f= 665.439, eps=0.001076273
Epoch 153: f= 665.432, eps=0.001130087
Epoch 154: f= 665.418, eps=0.001186591
Epoch 155: f= 665.420, eps=0.000593296
Epoch 156: f= 665.403, eps=0.000622961
Epoch 157: f= 665.393, eps=0.000654109
Epoch 158: f= 665.383, eps=0.000686814
Epoch 159: f= 665.376, eps=0.000721155
Epoch 160: f= 665.366, eps=0.000757212
Epoch 161: f= 665.357, eps=0.000795073
Epoch 162: f= 665.350, eps=0.000834827
Epoch 163: f= 665.344, eps=0.000876568
Epoch 164: f= 665.333, eps=0.000920396
Epoch 165: f= 665.325, eps=0.000966416
Epoch 166: f= 665.316, eps=0.001014737
Epoch 167: f= 665.313, eps=0.001065474
Epoch 168: f= 665.296, eps=0.001118748
Epoch 169: f= 665.280, eps=0.001174685
Epoch 170: f= 665.267, eps=0.001233419
Epoch 171: f= 665.265, eps=0.001295090

Epoch 172: f= 665.251, eps=0.001359845
Epoch 173: f= 665.235, eps=0.001427837
Epoch 174: f= 665.250, eps=0.000713918
Epoch 175: f= 665.225, eps=0.000749614
Epoch 176: f= 665.214, eps=0.000787095
Epoch 177: f= 665.202, eps=0.000826450
Epoch 178: f= 665.192, eps=0.000867772
Epoch 179: f= 665.190, eps=0.000911161
Epoch 180: f= 665.176, eps=0.000956719
Epoch 181: f= 665.166, eps=0.001004555
Epoch 182: f= 665.151, eps=0.001054783
Epoch 183: f= 665.146, eps=0.001107522
Epoch 184: f= 665.138, eps=0.001162898
Epoch 185: f= 665.121, eps=0.001221043
Epoch 186: f= 665.106, eps=0.001282095
Epoch 187: f= 665.099, eps=0.001346200
Epoch 188: f= 665.096, eps=0.001413510
Epoch 189: f= 665.087, eps=0.001484185
Epoch 190: f= 665.082, eps=0.001558394
Epoch 191: f= 665.062, eps=0.001636314
Epoch 192: f= 665.038, eps=0.001718130
Epoch 193: f= 665.037, eps=0.001804036
Epoch 194: f= 665.042, eps=0.000902018
Epoch 195: f= 665.013, eps=0.000947119
Epoch 196: f= 664.992, eps=0.000994475
Epoch 197: f= 664.982, eps=0.001044199
Epoch 198: f= 664.975, eps=0.001096409
Epoch 199: f= 664.970, eps=0.001151229
Epoch 200: f= 664.960, eps=0.001208791
Epoch 201: f= 664.939, eps=0.001269230
Epoch 202: f= 664.944, eps=0.000634615
Epoch 203: f= 664.928, eps=0.000666346
Epoch 204: f= 664.916, eps=0.000699663
Epoch 205: f= 664.908, eps=0.000734646
Epoch 206: f= 664.900, eps=0.000771379
Epoch 207: f= 664.891, eps=0.000809948
Epoch 208: f= 664.886, eps=0.000850445
Epoch 209: f= 664.876, eps=0.000892967
Epoch 210: f= 664.870, eps=0.000937616
Epoch 211: f= 664.863, eps=0.000984496
Epoch 212: f= 664.856, eps=0.001033721
Epoch 213: f= 664.847, eps=0.001085407
Epoch 214: f= 664.842, eps=0.001139678
Epoch 215: f= 664.840, eps=0.001196661
Epoch 216: f= 664.823, eps=0.001256494
Epoch 217: f= 664.812, eps=0.001319319
Epoch 218: f= 664.828, eps=0.000659660
Epoch 219: f= 664.812, eps=0.000692643

Epoch 220: f= 664.796, eps=0.000727275
Epoch 221: f= 664.785, eps=0.000763638
Epoch 222: f= 664.779, eps=0.000801820
Epoch 223: f= 664.769, eps=0.000841911
Epoch 224: f= 664.763, eps=0.000884007
Epoch 225: f= 664.757, eps=0.000928207
Epoch 226: f= 664.747, eps=0.000974618
Epoch 227: f= 664.740, eps=0.001023349
Epoch 228: f= 664.730, eps=0.001074516
Epoch 229: f= 664.721, eps=0.001128242
Epoch 230: f= 664.713, eps=0.001184654
Epoch 231: f= 664.707, eps=0.001243887
Epoch 232: f= 664.706, eps=0.001306081
Epoch 233: f= 664.685, eps=0.001371385
Epoch 234: f= 664.690, eps=0.000685692
Epoch 235: f= 664.680, eps=0.000719977
Epoch 236: f= 664.675, eps=0.000755976
Epoch 237: f= 664.666, eps=0.000793775
Epoch 238: f= 664.659, eps=0.000833463
Epoch 239: f= 664.653, eps=0.000875137
Epoch 240: f= 664.636, eps=0.000918893
Epoch 241: f= 664.634, eps=0.000964838
Epoch 242: f= 664.624, eps=0.001013080
Epoch 243: f= 664.614, eps=0.001063734
Epoch 244: f= 664.608, eps=0.001116921
Epoch 245: f= 664.598, eps=0.001172767
Epoch 246: f= 664.592, eps=0.001231405
Epoch 247: f= 664.588, eps=0.001292975
Epoch 248: f= 664.581, eps=0.001357624
Epoch 249: f= 664.579, eps=0.001425505
Epoch 250: f= 664.566, eps=0.001496781
Epoch 251: f= 664.579, eps=0.000748390
Epoch 252: f= 664.553, eps=0.000785810
Epoch 253: f= 664.543, eps=0.000825100
Epoch 254: f= 664.524, eps=0.000866355
Epoch 255: f= 664.517, eps=0.000909673
Epoch 256: f= 664.511, eps=0.000955157
Epoch 257: f= 664.504, eps=0.001002915
Epoch 258: f= 664.495, eps=0.001053060
Epoch 259: f= 664.488, eps=0.001105713
Epoch 260: f= 664.479, eps=0.001160999
Epoch 261: f= 664.480, eps=0.000580500
Epoch 262: f= 664.474, eps=0.000609525
Epoch 263: f= 664.467, eps=0.000640001
Epoch 264: f= 664.459, eps=0.000672001
Epoch 265: f= 664.453, eps=0.000705601
Epoch 266: f= 664.448, eps=0.000740881
Epoch 267: f= 664.441, eps=0.000777925

Epoch 268: f= 664.436, eps=0.000816821
Epoch 269: f= 664.430, eps=0.000857662
Epoch 270: f= 664.428, eps=0.000900545
Epoch 271: f= 664.417, eps=0.000945573
Epoch 272: f= 664.413, eps=0.000992851
Epoch 273: f= 664.404, eps=0.001042494
Epoch 274: f= 664.399, eps=0.001094618
Epoch 275: f= 664.393, eps=0.001149349
Epoch 276: f= 664.391, eps=0.001206817
Epoch 277: f= 664.388, eps=0.001267158
Epoch 278: f= 664.377, eps=0.001330516
Epoch 279: f= 664.378, eps=0.000665258
Epoch 280: f= 664.370, eps=0.000698521
Epoch 281: f= 664.359, eps=0.000733447
Epoch 282: f= 664.351, eps=0.000770119
Epoch 283: f= 664.349, eps=0.000808625
Epoch 284: f= 664.343, eps=0.000849056
Epoch 285: f= 664.332, eps=0.000891509
Epoch 286: f= 664.326, eps=0.000936084
Epoch 287: f= 664.318, eps=0.000982889
Epoch 288: f= 664.314, eps=0.001032033
Epoch 289: f= 664.310, eps=0.001083635
Epoch 290: f= 664.297, eps=0.001137817
Epoch 291: f= 664.292, eps=0.001194707
Epoch 292: f= 664.287, eps=0.001254443
Epoch 293: f= 664.275, eps=0.001317165
Epoch 294: f= 664.261, eps=0.001383023
Epoch 295: f= 664.263, eps=0.000691512
Epoch 296: f= 664.250, eps=0.000726087
Epoch 297: f= 664.244, eps=0.000762391
Epoch 298: f= 664.237, eps=0.000800511
Epoch 299: f= 664.233, eps=0.000840537
Epoch 300: f= 664.226, eps=0.000882563
Epoch 301: f= 664.219, eps=0.000926692
Epoch 302: f= 664.214, eps=0.000973026
Epoch 303: f= 664.211, eps=0.001021678
Epoch 304: f= 664.198, eps=0.001072761
Epoch 305: f= 664.193, eps=0.001126399
Epoch 306: f= 664.189, eps=0.001182719
Epoch 307: f= 664.187, eps=0.001241855
Epoch 308: f= 664.198, eps=0.000620928
Epoch 309: f= 664.177, eps=0.000651974
Epoch 310: f= 664.168, eps=0.000684573
Epoch 311: f= 664.162, eps=0.000718801
Epoch 312: f= 664.156, eps=0.000754742
Epoch 313: f= 664.147, eps=0.000792479
Epoch 314: f= 664.142, eps=0.000832103
Epoch 315: f= 664.137, eps=0.000873708

Epoch 316: f= 664.131, eps=0.000917393
Epoch 317: f= 664.127, eps=0.000963263
Epoch 318: f= 664.121, eps=0.001011426
Epoch 319: f= 664.115, eps=0.001061997
Epoch 320: f= 664.108, eps=0.001115097
Epoch 321: f= 664.116, eps=0.000557548
Epoch 322: f= 664.105, eps=0.000585426
Epoch 323: f= 664.096, eps=0.000614697
Epoch 324: f= 664.088, eps=0.000645432
Epoch 325: f= 664.083, eps=0.000677704
Epoch 326: f= 664.078, eps=0.000711589
Epoch 327: f= 664.071, eps=0.000747168
Epoch 328: f= 664.069, eps=0.000784527
Epoch 329: f= 664.064, eps=0.000823753
Epoch 330: f= 664.057, eps=0.000864941
Epoch 331: f= 664.050, eps=0.000908188
Epoch 332: f= 664.048, eps=0.000953597
Epoch 333: f= 664.044, eps=0.001001277
Epoch 334: f= 664.043, eps=0.001051341
Epoch 335: f= 664.028, eps=0.001103908
Epoch 336: f= 664.029, eps=0.000551954
Epoch 337: f= 664.022, eps=0.000579552
Epoch 338: f= 664.016, eps=0.000608529
Epoch 339: f= 664.014, eps=0.000638956
Epoch 340: f= 664.006, eps=0.000670903
Epoch 341: f= 664.003, eps=0.000704449
Epoch 342: f= 664.003, eps=0.000739671
Epoch 343: f= 664.000, eps=0.000776655
Epoch 344: f= 663.994, eps=0.000815487
Epoch 345: f= 663.984, eps=0.000856262
Epoch 346: f= 663.979, eps=0.000899075
Epoch 347: f= 663.980, eps=0.000449537
Epoch 348: f= 663.976, eps=0.000472014
Epoch 349: f= 663.971, eps=0.000495615
Epoch 350: f= 663.967, eps=0.000520396
Epoch 351: f= 663.963, eps=0.000546416
Epoch 352: f= 663.960, eps=0.000573736
Epoch 353: f= 663.953, eps=0.000602423
Epoch 354: f= 663.947, eps=0.000632544
Epoch 355: f= 663.943, eps=0.000664171
Epoch 356: f= 663.939, eps=0.000697380
Epoch 357: f= 663.934, eps=0.000732249
Epoch 358: f= 663.931, eps=0.000768861
Epoch 359: f= 663.928, eps=0.000807305
Epoch 360: f= 663.924, eps=0.000847670
Epoch 361: f= 663.918, eps=0.000890053
Epoch 362: f= 663.918, eps=0.000445027
Epoch 363: f= 663.913, eps=0.000467278

Epoch 364: f= 663.910, eps=0.000490642
Epoch 365: f= 663.907, eps=0.000515174
Epoch 366: f= 663.901, eps=0.000540933
Epoch 367: f= 663.897, eps=0.000567979
Epoch 368: f= 663.893, eps=0.000596378
Epoch 369: f= 663.890, eps=0.000626197
Epoch 370: f= 663.886, eps=0.000657507
Epoch 371: f= 663.882, eps=0.000690382
Epoch 372: f= 663.877, eps=0.000724902
Epoch 373: f= 663.877, eps=0.000761147
Epoch 374: f= 663.874, eps=0.000799204
Epoch 375: f= 663.869, eps=0.000839164
Epoch 376: f= 663.861, eps=0.000881122
Epoch 377: f= 663.855, eps=0.000925178
Epoch 378: f= 663.850, eps=0.000971437
Epoch 379: f= 663.843, eps=0.001020009
Epoch 380: f= 663.841, eps=0.001071010
Epoch 381: f= 663.841, eps=0.001124560
Epoch 382: f= 663.837, eps=0.001180788
Epoch 383: f= 663.832, eps=0.001239828
Epoch 384: f= 663.818, eps=0.001301819
Epoch 385: f= 663.807, eps=0.001366910
Epoch 386: f= 663.813, eps=0.000683455
Epoch 387: f= 663.800, eps=0.000717628
Epoch 388: f= 663.794, eps=0.000753509
Epoch 389: f= 663.788, eps=0.000791185
Epoch 390: f= 663.781, eps=0.000830744
Epoch 391: f= 663.778, eps=0.000872281
Epoch 392: f= 663.777, eps=0.000915895
Epoch 393: f= 663.766, eps=0.000961690
Epoch 394: f= 663.762, eps=0.001009774
Epoch 395: f= 663.757, eps=0.001060263
Epoch 396: f= 663.747, eps=0.001113276
Epoch 397: f= 663.745, eps=0.001168940
Epoch 398: f= 663.740, eps=0.001227387
Epoch 399: f= 663.758, eps=0.000613693
Epoch 400: f= 663.745, eps=0.000644378
Epoch 401: f= 663.735, eps=0.000676597
Epoch 402: f= 663.729, eps=0.000710427
Epoch 403: f= 663.730, eps=0.000355213
Epoch 404: f= 663.723, eps=0.000372974
Epoch 405: f= 663.718, eps=0.000391623
Epoch 406: f= 663.712, eps=0.000411204
Epoch 407: f= 663.709, eps=0.000431764
Epoch 408: f= 663.704, eps=0.000453352
Epoch 409: f= 663.701, eps=0.000476020
Epoch 410: f= 663.698, eps=0.000499821
Epoch 411: f= 663.694, eps=0.000524812

Epoch 412: f= 663.691, eps=0.000551053
Epoch 413: f= 663.686, eps=0.000578605
Epoch 414: f= 663.683, eps=0.000607536
Epoch 415: f= 663.678, eps=0.000637912
Epoch 416: f= 663.675, eps=0.000669808
Epoch 417: f= 663.669, eps=0.000703298
Epoch 418: f= 663.666, eps=0.000738463
Epoch 419: f= 663.660, eps=0.000775386
Epoch 420: f= 663.656, eps=0.000814156
Epoch 421: f= 663.651, eps=0.000854863
Epoch 422: f= 663.645, eps=0.000897607
Epoch 423: f= 663.640, eps=0.000942487
Epoch 424: f= 663.634, eps=0.000989611
Epoch 425: f= 663.632, eps=0.001039092
Epoch 426: f= 663.632, eps=0.000519546
Epoch 427: f= 663.626, eps=0.000545523
Epoch 428: f= 663.622, eps=0.000572799
Epoch 429: f= 663.615, eps=0.000601439
Epoch 430: f= 663.611, eps=0.000631511
Epoch 431: f= 663.607, eps=0.000663087
Epoch 432: f= 663.604, eps=0.000696241
Epoch 433: f= 663.600, eps=0.000731053
Epoch 434: f= 663.598, eps=0.000767606
Epoch 435: f= 663.592, eps=0.000805986
Epoch 436: f= 663.588, eps=0.000846286
Epoch 437: f= 663.582, eps=0.000888600
Epoch 438: f= 663.579, eps=0.000933030
Epoch 439: f= 663.574, eps=0.000979681
Epoch 440: f= 663.575, eps=0.000489841
Epoch 441: f= 663.570, eps=0.000514333
Epoch 442: f= 663.567, eps=0.000540049
Epoch 443: f= 663.561, eps=0.000567052
Epoch 444: f= 663.556, eps=0.000595404
Epoch 445: f= 663.553, eps=0.000625175
Epoch 446: f= 663.549, eps=0.000656433
Epoch 447: f= 663.547, eps=0.000689255
Epoch 448: f= 663.542, eps=0.000723718
Epoch 449: f= 663.537, eps=0.000759904
Epoch 450: f= 663.532, eps=0.000797899
Epoch 451: f= 663.529, eps=0.000837794
Epoch 452: f= 663.525, eps=0.000879684
Epoch 453: f= 663.523, eps=0.000923668
Epoch 454: f= 663.517, eps=0.000969851
Epoch 455: f= 663.510, eps=0.001018344
Epoch 456: f= 663.514, eps=0.000509172
Epoch 457: f= 663.510, eps=0.000534630
Epoch 458: f= 663.505, eps=0.000561362
Epoch 459: f= 663.499, eps=0.000589430

```
Epoch 460: f= 663.496, eps=0.000618902
Epoch 461: f= 663.490, eps=0.000649847
Epoch 462: f= 663.489, eps=0.000682339
Epoch 463: f= 663.486, eps=0.000716456
Epoch 464: f= 663.479, eps=0.000752279
Epoch 465: f= 663.476, eps=0.000789893
Epoch 466: f= 663.468, eps=0.000829387
Epoch 467: f= 663.463, eps=0.000870857
Epoch 468: f= 663.460, eps=0.000914399
Epoch 469: f= 663.456, eps=0.000960119
Epoch 470: f= 663.455, eps=0.001008125
Epoch 471: f= 663.446, eps=0.001058532
Epoch 472: f= 663.438, eps=0.001111458
Epoch 473: f= 663.432, eps=0.001167031
Epoch 474: f= 663.428, eps=0.001225383
Epoch 475: f= 663.418, eps=0.001286652
Epoch 476: f= 663.417, eps=0.001350984
Epoch 477: f= 663.423, eps=0.000675492
Epoch 478: f= 663.418, eps=0.000709267
Epoch 479: f= 663.409, eps=0.000744730
Epoch 480: f= 663.403, eps=0.000781967
Epoch 481: f= 663.396, eps=0.000821065
Epoch 482: f= 663.393, eps=0.000862118
Epoch 483: f= 663.388, eps=0.000905224
Epoch 484: f= 663.384, eps=0.000950485
Epoch 485: f= 663.377, eps=0.000998010
Epoch 486: f= 663.363, eps=0.001047910
Epoch 487: f= 663.362, eps=0.001100306
Epoch 488: f= 663.356, eps=0.001155321
Epoch 489: f= 663.358, eps=0.000577660
Epoch 490: f= 663.351, eps=0.000606543
Epoch 491: f= 663.343, eps=0.000636871
Epoch 492: f= 663.342, eps=0.000668714
Epoch 493: f= 663.339, eps=0.000702150
Epoch 494: f= 663.336, eps=0.000737257
Epoch 495: f= 663.333, eps=0.000774120
Epoch 496: f= 663.322, eps=0.000812826
Epoch 497: f= 663.317, eps=0.000853468
Epoch 498: f= 663.314, eps=0.000896141
Epoch 499: f= 663.311, eps=0.000940948
Result after 500 epochs: f=663.3023109198814
```

1.5 2e Compare GD and SGD

Explore the behavior of both methods, given the parameters provided to you!

```
[18]: # reproducibility
      np.random.seed(21)
```


[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

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[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

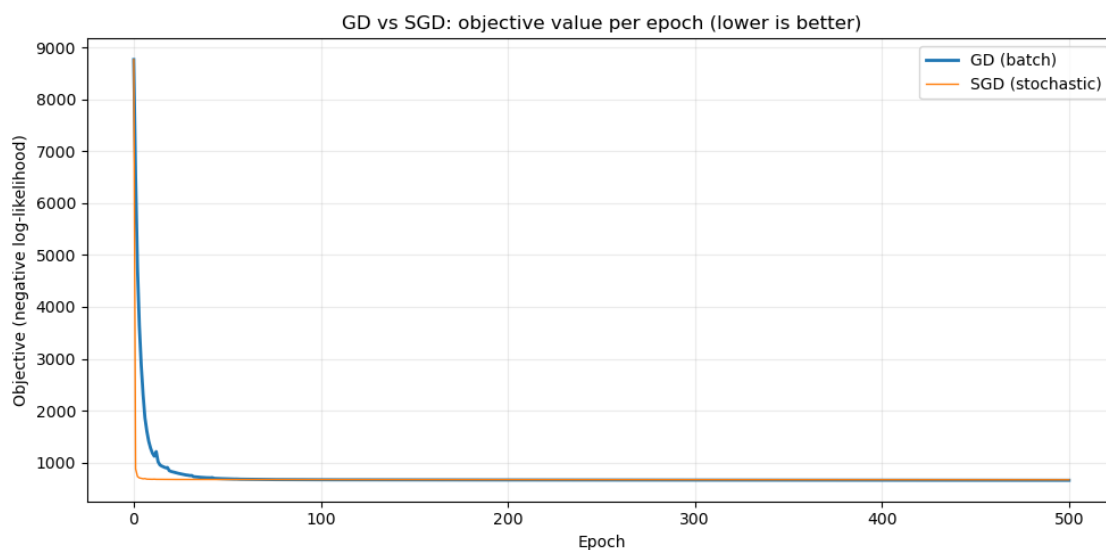
[illegible]

[illegible]

[illegible]

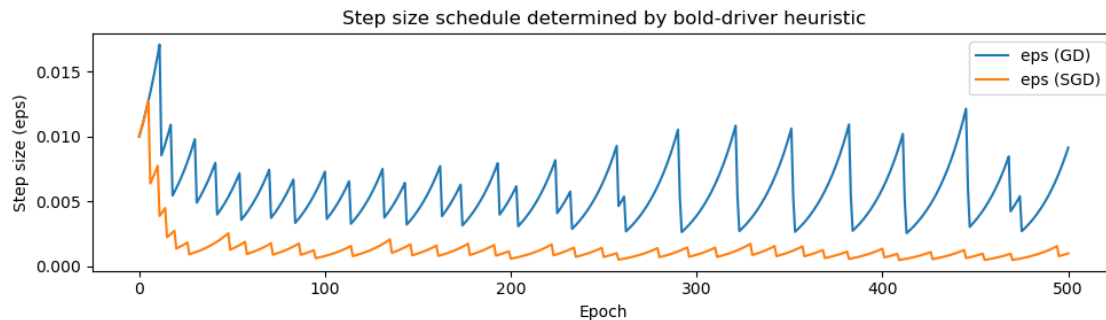
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```
[26]: # objective (optimize minimizes the provided objective f; here f =  $\mathcal{L}$ 
      ↪ -log-likelihood)
plt.figure(figsize=(10,5))
plt.plot(vals_gd, label="GD (batch)", linewidth=2)
plt.plot(vals_sgd, label="SGD (stochastic)", linewidth=1)
plt.xlabel("Epoch")
plt.ylabel("Objective (negative log-likelihood)")
plt.title("GD vs SGD: objective value per epoch (lower is better)")
plt.legend()
plt.grid(alpha=0.25)
plt.tight_layout()
plt.savefig('images/convergence_rate.png', dpi=300, bbox_inches='tight')
plt.show()
```



```
[25]: # step sizes chosen by bold-driver heuristic
plt.figure(figsize=(10,3))
plt.plot(eps_gd, label="eps (GD)")
```

```
plt.plot(eps_sgd, label="eps (SGD)")
plt.xlabel("Epoch")
plt.ylabel("Step size (eps)")
plt.title("Step size schedule determined by bold-driver heuristic")
plt.legend()
plt.tight_layout()
plt.savefig('images/learning_rate.png', dpi=300, bbox_inches='tight')
plt.show()
```



```
[22]: # Compute final accuracies
train_pred_gd = classify(Xz, w_gd)
test_pred_gd  = classify(Xtestz, w_gd)
train_acc_gd  = np.mean(train_pred_gd == y)
test_acc_gd   = np.mean(test_pred_gd == ytest)

train_pred_sgd = classify(Xz, w_sgd)
test_pred_sgd  = classify(Xtestz, w_sgd)
train_acc_sgd  = np.mean(train_pred_sgd == y)
test_acc_sgd   = np.mean(test_pred_sgd == ytest)
```

```
[23]: print("Final results after 500 epochs")
print("GD:  objective = {:.3f}, train acc = {:.4f}, test acc = {:.4f}".
      ↪format(vals_gd[-1], train_acc_gd, test_acc_gd))
print("SGD: objective = {:.3f}, train acc = {:.4f}, test acc = {:.4f}".
      ↪format(vals_sgd[-1], train_acc_sgd, test_acc_sgd))
```

Final results after 500 epochs

GD: objective = 657.277, train acc = 0.9214, test acc = 0.9186

SGD: objective = 667.409, train acc = 0.9214, test acc = 0.9186

```
[ ]:
```