

Time Series - Assignment 1

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Question 1

```
require(ggplot2)

## Loading required package: ggplot2

require(dplyr)

## Loading required package: dplyr
##
## Attaching package: 'dplyr'
##
## The following objects are masked from 'package:stats':
##
##   filter, lag
##
## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union

require(zoo)

## Loading required package: zoo
##
## Attaching package: 'zoo'
##
## The following objects are masked from 'package:base':
##
##   as.Date, as.Date.numeric

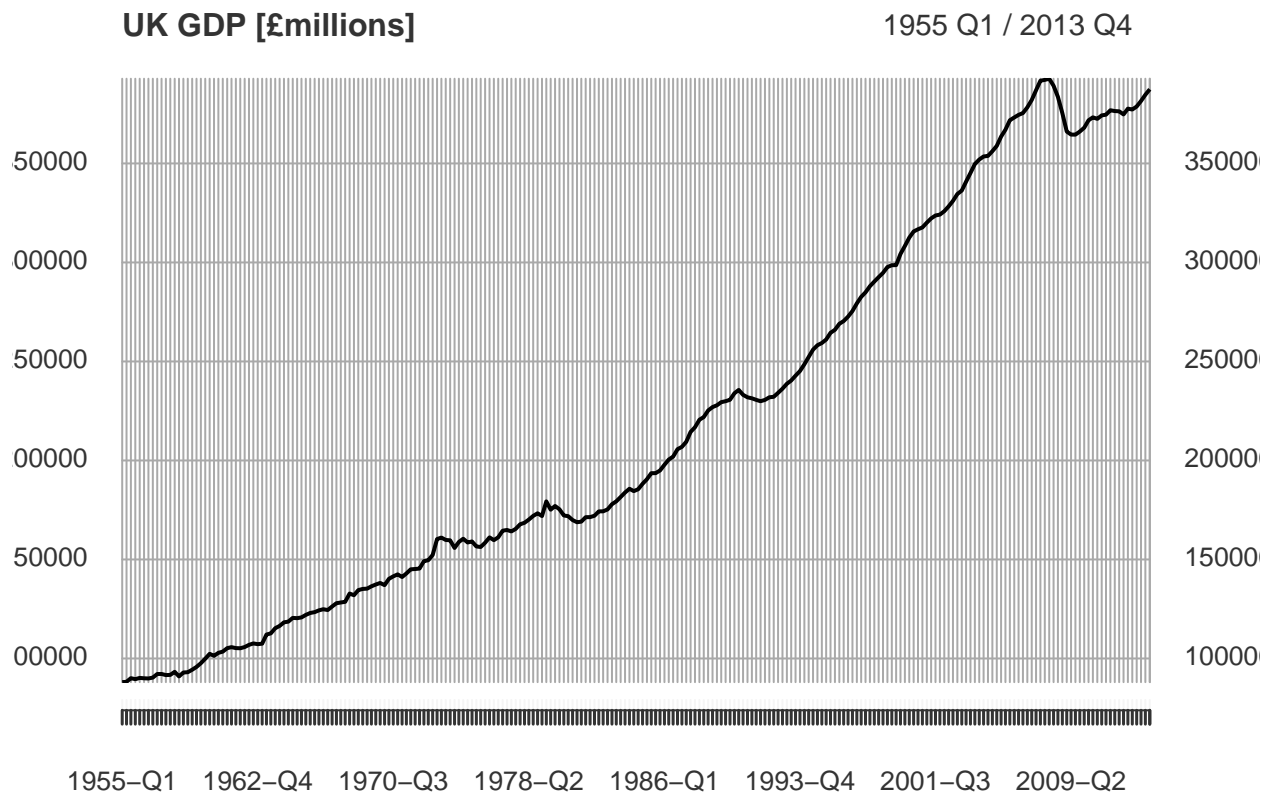
require(xts)

## Loading required package: xts
##
## Attaching package: 'xts'
##
## The following objects are masked from 'package:dplyr':
##
##   first, last

load("UKGDP.RData")

yq <- paste(UKGDP$V1, UKGDP$V2)
df.yq <- cbind(UKGDP, as.yearqtr(yq))
df.xts <- xts(df.yq$V3, order.by = df.yq$`as.yearqtr(yq)`)

plot(df.xts, main = "UK GDP [£millions]")
```



As can be observed on the above plot, the gross domestic product of the UK has an trend that varies according to t (the year). This implies that the UK GDP does not have a constant trend, therefore the expected value of X_t (UK GDP) is not constant. The aforementioned indicates that using an ARMA(p,q) model would not be appropriate.

Question 2

```
ARIMA1.Order <- c(0, 1, 1)
ARIMA2.Order <- c(1, 1, 0)
ARIMA3.Order <- c(1, 1, 1)

ARIMA1 <- arima(x = df.xts, order = ARIMA1.Order)
ARIMA2 <- arima(x = df.xts, order = ARIMA2.Order)
ARIMA3 <- arima(x = df.xts, order = ARIMA3.Order)

AICs <- c(ARIMA1$aic, ARIMA2$aic, ARIMA3$aic)

AIC.Probs <- exp((min(AICs) - AICs)/2)
```

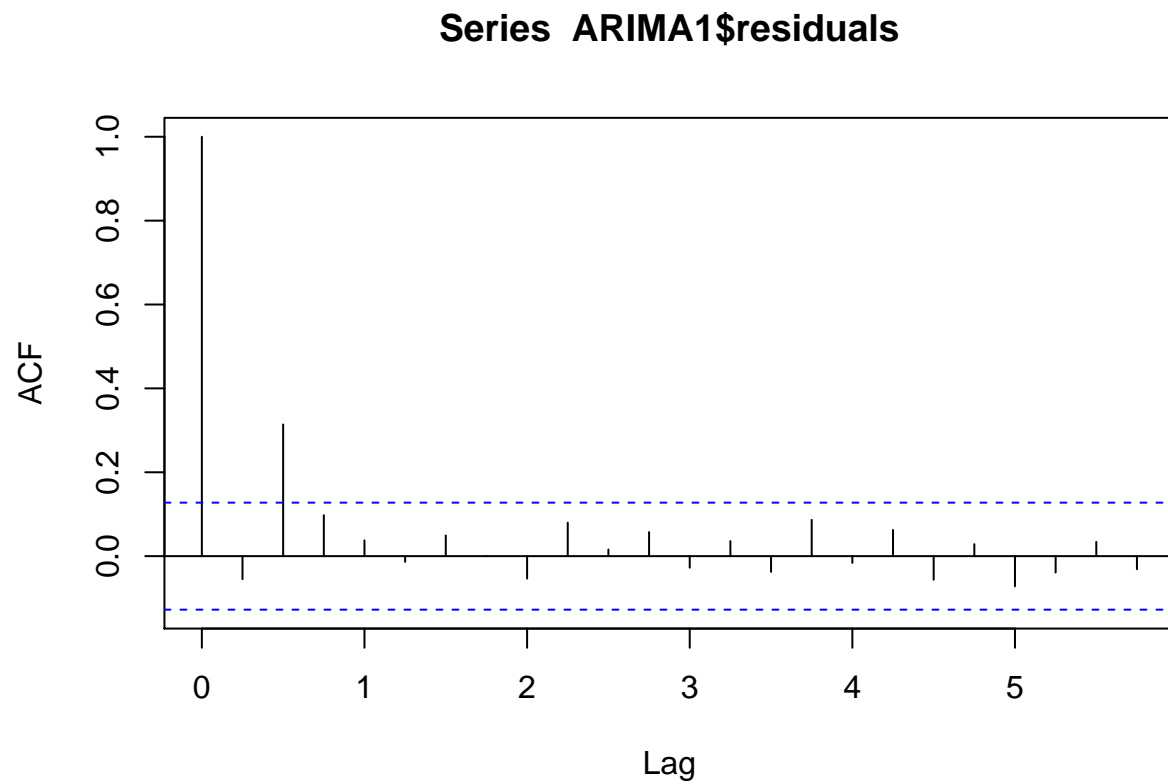
(a) Model selection

The ARIMA(1,1,1) model presents the lowest AIC value and further to that fact, the ARIMA(0,1,1) and ARIMA(1,1,0) models are $2.261 \cdot 10^{-13}$ and $1.707 \cdot 10^{-5}$ times as probable as the ARIMA(1,1,1) model to

minimise the information loss.

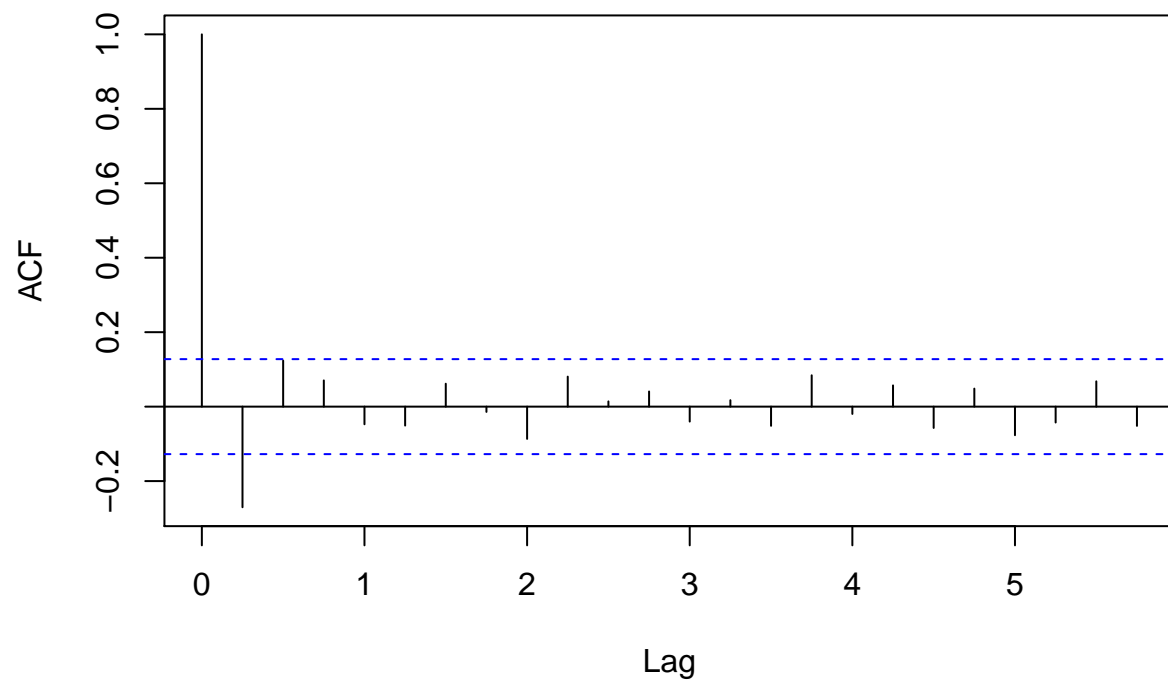
The correlograms of the residuals are depicted in the following plots.

```
# Correlogram of ARIMA(0,1,1) model  
acf(ARIMA1$residuals)
```



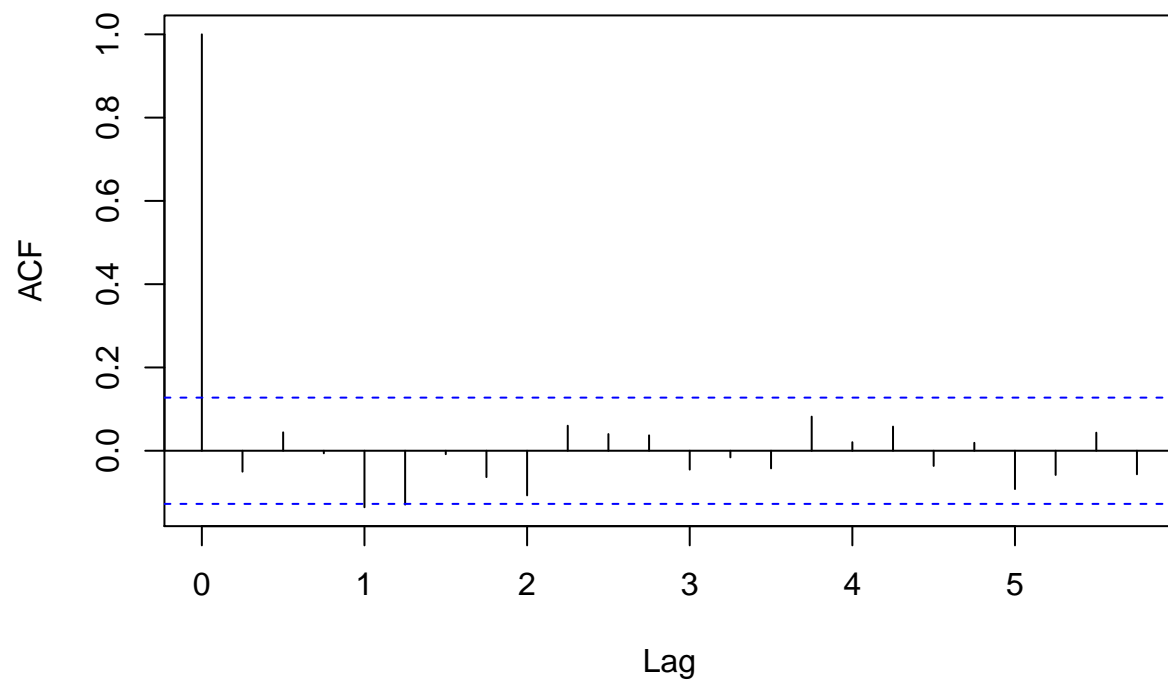
```
# Correlogram of ARIMA(1,1,0) model  
acf(ARIMA2$residuals)
```

Series ARIMA2\$residuals



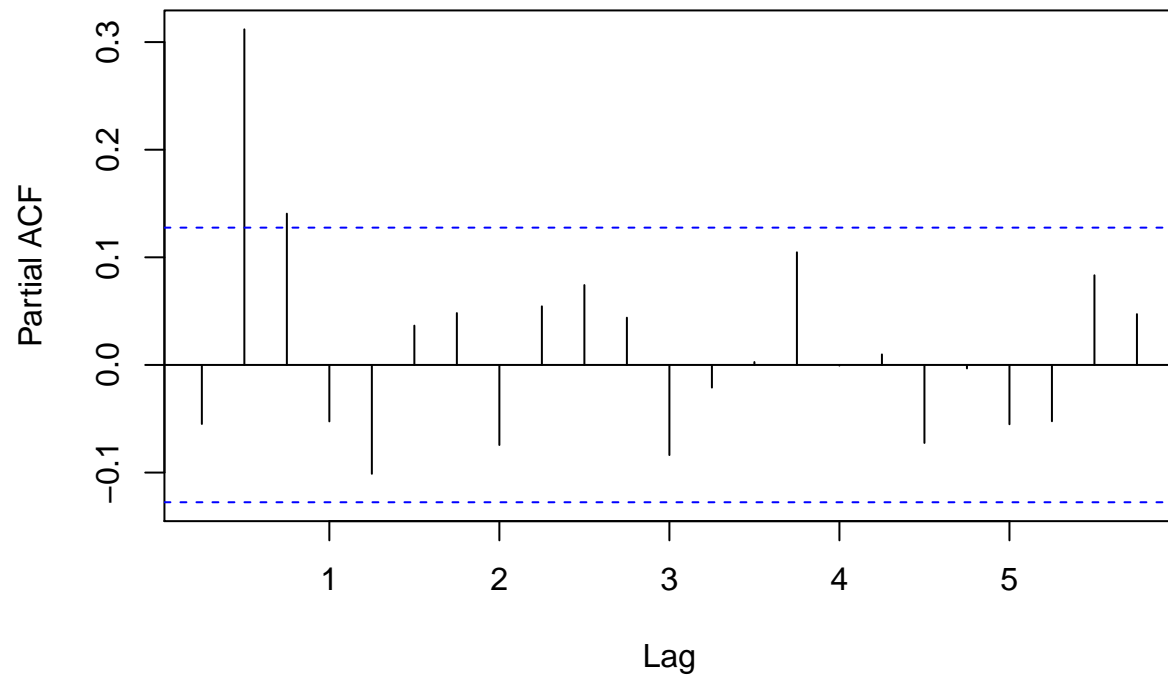
```
# Correlogram of ARIMA(1,1,1) model  
acf(ARIMA3$residuals)
```

Series ARIMA3\$residuals



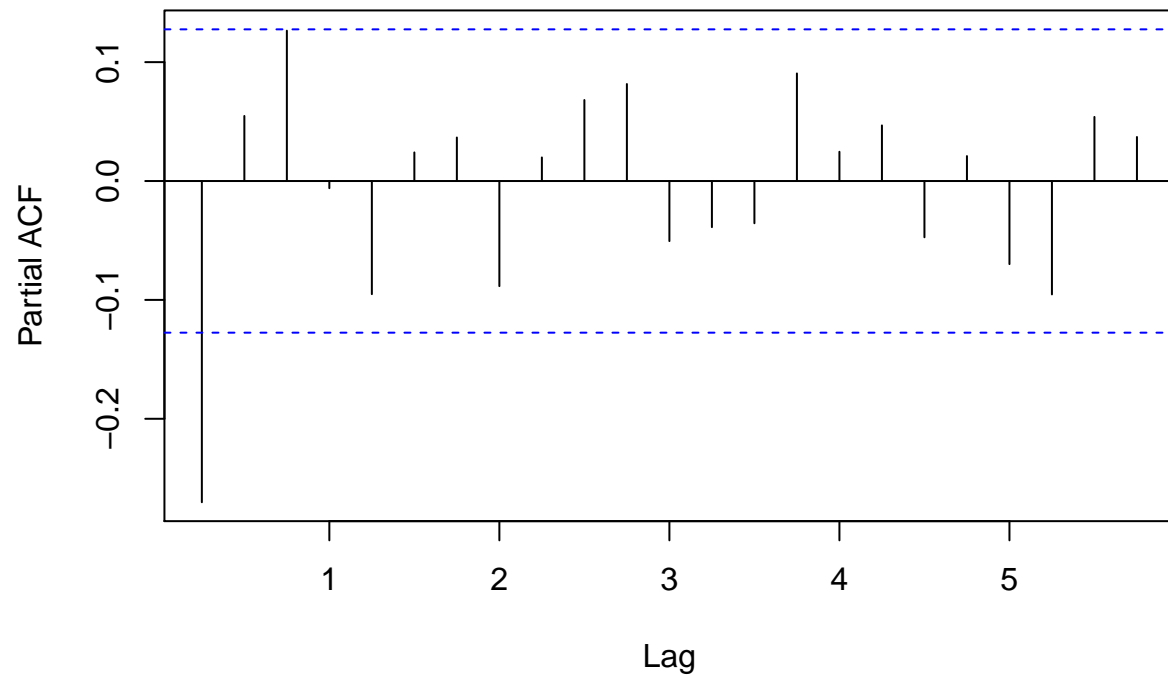
```
# PACF of ARIMA(0,1,1) model  
pacf(ARIMA1$residuals)
```

Series ARIMA1\$residuals



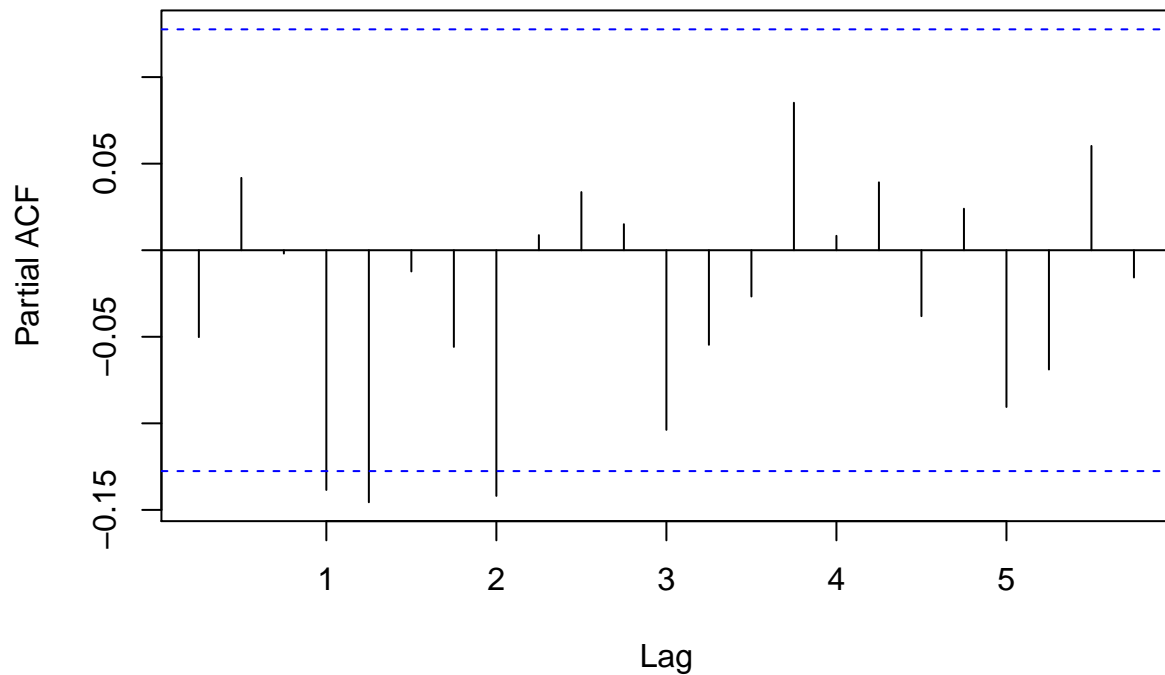
```
# PACF of ARIMA(1,1,0) model  
pacf(ARIMA2$residuals)
```

Series ARIMA2\$residuals



```
# PACF of ARIMA(1,1,1) model  
pacf(ARIMA3$residuals)
```

Series ARIMA3\$residuals



```
#Ljung-Box Test Statistics for lag = 1:50

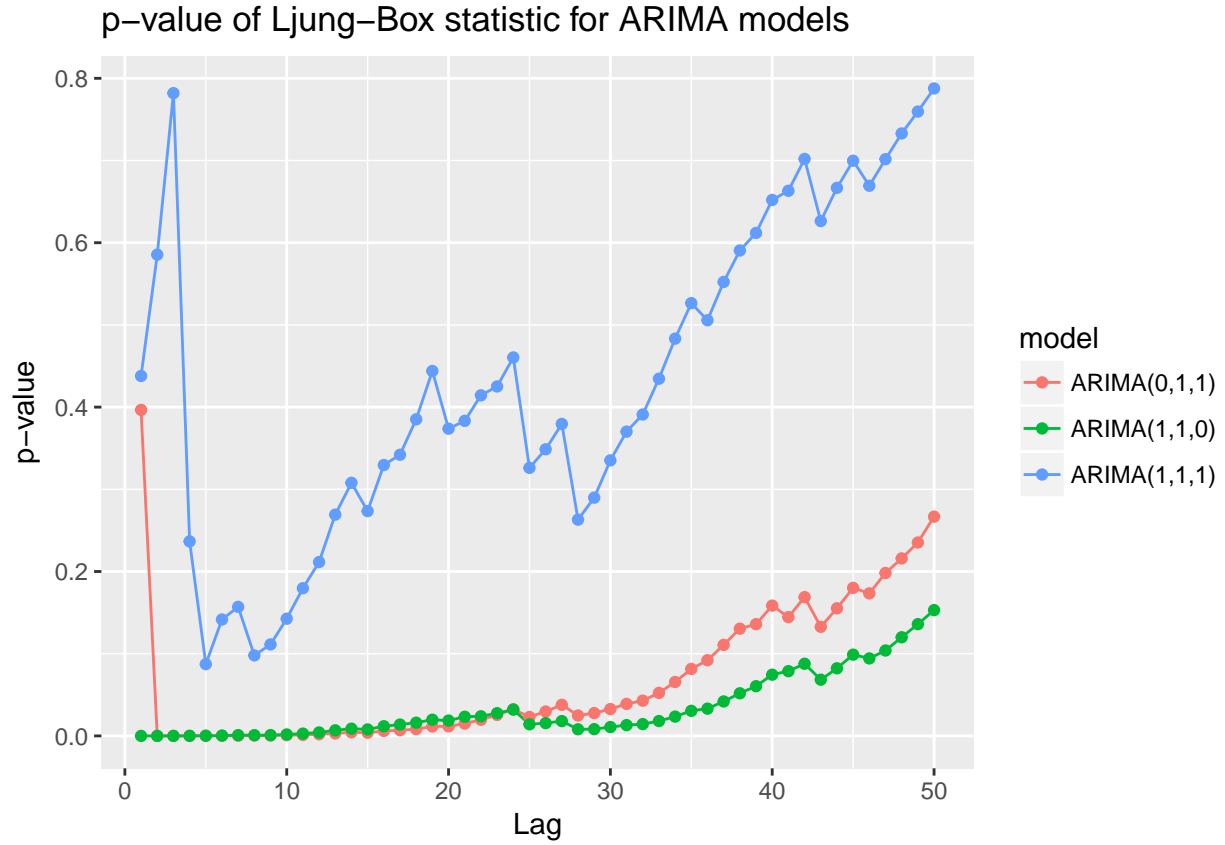
lb <- function(lag, x, type = "Ljung-Box"){
  Box.test(x = x, lag = lag, type = type)$p.value
}

Lags <- 1:50
n.Lags <- length(Lags)

Model.p.values <- data.frame(model = rep(c("ARIMA(0,1,1)",
                                           "ARIMA(1,1,0)",
                                           "ARIMA(1,1,1)"),
                                           each = n.Lags),
                             lag = rep(Lags, times = 3))

Model.p.values$p.value <- c(sapply(Lags, lb, x = ARIMA1$residuals),
                           sapply(Lags, lb, x = ARIMA2$residuals),
                           sapply(Lags, lb, x = ARIMA3$residuals))

ggplot(data = Model.p.values, aes(x = lag, y = p.value, colour = model)) +
  geom_line() +
  geom_point() +
  ggtitle("p-value of Ljung-Box statistic for ARIMA models") +
  xlab("Lag") +
  ylab("p-value")
```

From the above correlograms and partial ACF it is possible to visualize that the residuals are uncorrelated for the ARIMA(1,1,1) model. The same is not seen This indicates that the estimated residuals are white noise.

(b) Chosen model

The fitted ARIMA(1,1,1) model can be written as follows:

$$X_t = \alpha_1 X_{t-1} + Z_t - \theta_1 Z_{t-1}$$