

### Part 1: IEEE 802.11 MAC (10 points)

An Ad Hoc network using IEEE802.11 has 4 nodes: N1, N2, N3, N4. Assume that SIFS is 1 unit of time, PIFS 2 units of time, DIFS 3 units of time, and slot time is 2 (these value are not the real values but are taken to simplify the packets scheduling).

Assume that at the beginning the channel is idle (no transmission), and that at instant 1, N2 has a packet to be sent to N4. At instant 2, both N1 and N3 have a packet to be sent to N4. Assume that the random number generator (for backoff) will give the following values for N1: 2, 5, ... and for N2: 4, 3, ... and for N3: 1, 4, ... Assume that we don't use RTS/CTS nor fragmentation, and that all data packets have the same length: 6 units of time and that the Ack packet has length 3 units of time. Furthermore the channel Bit Error Rate is assumed to be 0. Show the execution of the DCF mode of IEEE802.11.

At time unit 1 packet is sent from N2 to N4. Following events takes place

1. **Time unit 1:** N2 senses free channel and waits for DIFS time units
2. **Time unit 2:** At time unit 2 both N1 and N3 has a packet to send. Both sense channel is idle and waits for DIFS time unit
3. **Time unit 4:** N2 transmits the packet noting that the channel is free. Also here N1 and N3 selects back off time, which is Slot Time \* Random Number ( $2*2 = 4$  for N1 and  $1*2 = 2$  for N3)
4. **Time unit 10:** At 10 units N2 completes transmitting packet. N4 waits for SIFS time unit to send the ACK
5. **Time unit 11:** N4 sends the ACK
6. **Time unit 14:** N2 receives ACK
7. **Time unit 17:** DIFS time units have passed since the transmission of last packet. So back off counter begins for N1 and N3
8. **Time unit 19:** N3's counter has timed out so it begins transmission. N1 freezes its counter sensing that channel isn't idle
9. **Time unit 25:** N3 completes its transmission. N4 waits SIFS before transmitting ACK
10. **Time unit 26:** N4 transmits ACK
11. **Time unit 29:** N3 receives ACK and N1 resumes it's clock
12. **Time unit 31:** N1's timer times out so N1 begins transmission
13. **Time unit 37:** N4 receives N1's packet and waits DIFS before sending ACK
14. **Time unit 38:** N4 transmits ACK
15. **Time unit 41:** N1 receives ACK

### Text book Problems

1. For radio transmission in free space signal power is reduced in proportion to the square of distance from the source whereas in wire transmission attenuation is a fixed number of Db per kilometre. The following table is used to show the Db reduction relative to some reference for free space radio and uniform wire. Fill the missing numbers to complete the table.

Distance(KM)	Radio(dB)	Wire(dB)
1	-6	-3
2	-12	-6
4	-18	-12
8	-24	-24
16	-30	-48

### Radio Channel

$$\begin{aligned}L_{db} &= 10 \log(P_0/P_r) \\&= 10 \log(4\pi d/\lambda)^2 \\&= 20\log(4\pi d/\lambda) \\&= 20\log(d) + 20\log(4\pi/\lambda)\end{aligned}$$

Putting the value of Distance 1Km in above equation

$$\begin{aligned}6 &= 20\log(1) + 20\log(4\pi/\lambda) \\6 &= 20\log(4\pi/\lambda)\end{aligned}$$

For d=2,4,8,16

$$d = 2$$

$$\begin{aligned}L_{db} &= 20\log(d) + 20\log(4\pi/\lambda) \\&= 20\log(2) + 6 \\&= 6.02 + 6 \\&= 12.02\end{aligned}$$

$$d = 4$$

$$\begin{aligned}L_{db} &= 20\log(d) + 20\log(4\pi/\lambda) \\&= 20\log(4) + 6 \\&= 12.04 + 6 \\&= 18.04\end{aligned}$$

$$d = 8$$

$$\begin{aligned}L_{db} &= 20\log(d) + 20\log(4\pi/\lambda) \\&= 20\log(8) + 6 \\&= 18.06 + 6 \\&= 24.06\end{aligned}$$

$$d = 16$$

$$\begin{aligned} L_{db} &= 20\log(16) + 20\log(4\pi/\lambda) \\ &= 20\log(d) + 6 \\ &= 24.08 + 6 \\ &= 30.08 \end{aligned}$$

### **Wired Channel:**

Attenuation  $\alpha$  (distance)

$$\text{Attenuation} = K * \text{Distance}$$

$$3 = K * 1$$

$$K = 3$$

For  $d = 2, 4, 8, 16$

$$\text{Attenuation} = K * \text{Distance}$$

$$\text{Attenuation} = 3 * \text{Distance}$$

For  $d=2$

$$\text{Attenuation} = 3 * d = 3 * 2$$

$$\text{Attenuation} = 6$$

For  $d=4$

$$\text{Attenuation} = 3 * d = 3 * 4$$

$$\text{Attenuation} = 12$$

for  $d=8$

$$\text{Attenuation} = 3 * d = 3 * 8$$

$$\text{Attenuation} = 24$$

for  $d=16$

$$\text{Attenuation} = 3 * d = 3 * 16$$

$$\text{Attenuation} = 48$$

**Question:**

Suppose a transmitter produces 50Watts of Power.

- Express transmit power in dBm and dBW.
- If a transmitter's power is applied to Unity gain Antenna with 900MHz carrier frequency, what is the free space power in dBm at a free space distance of 100m?
- Repeat (b) for 10Km
- Repeat (C) for antenna gain 2

**Answer:**

$$\begin{aligned} \text{a) } P_{(\text{dBW})} &= 10 \cdot \log_{10}(P_{(\text{W})} / 1\text{W}) \\ &= 10 \cdot \log_{10}(50 / 1\text{W}) \\ &= 16.98 \text{ dBW} \end{aligned}$$

$$\begin{aligned} P_{(\text{dBm})} &= P_{(\text{dBW})} + 30 \\ &= 46.98 \text{ dBm} \end{aligned}$$

- b) According to Friss's transmission equation:

$$\begin{aligned} P_r &= (P_t * G_t * G_r * \lambda^2) / (4\pi * d)^2 \\ P_r &= (50 * 1 * 1 * \lambda^2) / (4\pi * 100)^2 \quad [\lambda = c/f \Rightarrow 3 * 10^8 / 900 * 10^6 \Rightarrow (1/3)] \\ P_r &= (50 * 1 * 1 * (1/3)^2) / (4\pi * 100)^2 \\ P_r &= 3.51 * 10^{-6} \text{ W} \\ P_r &= 10 \cdot \log_{10}(3.51 * 10^{-6} \text{ W} / 1\text{W}) + 30 \\ P_r &= -24 \text{ dBm} \end{aligned}$$

- c) According to Friss's transmission equation:

$$\begin{aligned} P_r &= (P_t * G_t * G_r * \lambda^2) / (4\pi * d)^2 \\ P_r &= (50 * 1 * 1 * \lambda^2) / (4\pi * 10000)^2 \quad [\lambda = c/f \Rightarrow 3 * 10^8 / 900 * 10^6 \Rightarrow (1/3)] \\ P_r &= (50 * 1 * 1 * (1/3)^2) / (4\pi * 10000)^2 \\ P_r &= 3.51 * 10^{-10} \text{ W} \\ P_r &= 10 \cdot \log_{10}(3.51 * 10^{-10} \text{ W} / 1\text{W}) + 30 \\ P_r &= -64.54 \text{ dBm} \end{aligned}$$

- d) According to Friss's transmission equation:

$$\begin{aligned} P_r &= (P_t * G_t * G_r * \lambda^2) / (4\pi * d)^2 \\ P_r &= (50 * 2 * 1 * \lambda^2) / (4\pi * 10000)^2 \quad [\lambda = c/f \Rightarrow 3 * 10^8 / 900 * 10^6 \Rightarrow (1/3)] \\ P_r &= (50 * 2 * 1 * (1/3)^2) / (4\pi * 10000)^2 \\ P_r &= 5.75 * 10^{-11} \text{ W} \\ P_r &= 10 \cdot \log_{10}(5.75 * 10^{-11} \text{ W} / 1\text{W}) + 30 \\ P_r &= -72.403 \text{ dBm} \end{aligned}$$

**Question:**

Instead of assuming free space environment in 6.6 assume an urban area cellular radio scenario. Path loss exponent  $n=3.1$  and a transmitter power of 50W.

- What is the range of path loss exponent for this environment?
- If a transmitter's power is applied to Unity gain Antenna with 900MHz carrier frequency, what is the free space power in dBm at a free space distance of 100m?

**Answer:**

a) *Range of Path loss exponent in urban area cellular radio scenario is 2.7 – 3.5.*

b) According to Friss's transmission equation for Free Space:

$$P_r = (P_t * G_t * G_r * \lambda^2) / (4\pi * d)^2$$

$$P_r = (50 * 1 * 1 * \lambda^2) / (4\pi * 100)^2 \quad [\lambda = c/f \Rightarrow 3*10^8/900*10^6 \Rightarrow (1/3)]$$

$$P_r = (50 * 1 * 1 * (1/3)^2) / (4\pi * 100)^2$$

$$P_r = 3.51 * 10^{-6} \text{ W}$$

$$P_r = 10 \cdot \log_{10}(3.51 * 10^{-6} \text{ W} / 1 \text{ W}) + 30$$

$$P_r = -24 \text{ dBm}$$

### Question:

Determine the height of the Antenna for TV stations that must be able to reach customers 80Kms away. Use Okumura-Hatta Model for rural environment with  $f_c=75\text{Mhz}$  and  $H_r=1.5\text{m}$ . Transmitter power 150Kw and received power must be greater than  $10^{-13}\text{ W}$

### Answer:

$$L_{ru} = 10 \log(150000/10^{-13})$$

$$L_{ru} = 181.760\text{dBW}$$

$$L_{ru} = L_u - 4.78(\log f)^2 - 18.33 \log f - 40.94$$

$$L_u = L_{ru} + 4.78(\log f)^2 + 18.33 \log f + 40.94$$

$$L_u = 663.48917269$$

According to Okumura Hata Model,

$$L_u = 69.55 + 26.16 \log f - 13.82 \log h_b - A(h_m) + (44.9 - 6.55 \log h_b) \log d$$

$$A(h_m) = (1.1 \log f - 0.7)h_m - (1.56 \log f - 0.8)$$

$$A(h_m) = 0.4587555137$$

$$= 663.48917269 = 69.55 + 26.16 \log f - 13.82 \log h_b - 0.4587555137 + (44.9 - 6.55 \log h_b) \log d$$

$$= 663.48917269 = 275.102847137 - 13.82 \log h_b + (44.9 - 6.55 \log h_b) \log d$$

$$= 663.48917269 = 495.251587553 - (13.82 + 32.1152394148) \log h_b$$

$$= 168.237585137 = - (13.82 + 32.1152394148) \log h_b$$

$$= 3.6624950099 = -\log_{10} h_b$$

$$= 0.0002175229 = h_b$$

$$h_b = 0.0002175229 \text{ m}$$

$$\mathbf{h_b=0.0002175229 \text{ m}}$$

### Question

Suppose a car is moving through the suburban environment that has a wireless channel with a coherence time of 10ms and the coherence bandwidth of 600Khz. The bitrate of system used is 50kbps. Characterize the channel.

- a. Is the channel slow or fast fading?
- b. Is the channel flat or frequency selective fading?

Answer:

- a) Bit Rate = 50kbps  
Bit time  $T_b = 1/(50 * 10^3) = 20 \mu s$   
Since,  $T_c > T_b$ , The Channel is Slow Fading
- b)  $B_c = 600 \text{ KHz}$   
Assume Single Bandwidth, So,  
 $B_s = 50 \text{ KHz}$   
Now,  $B_c > 10 * (B_s)$   
Hence,  $B_c \gg B_s$ , So flat fading

## Fading and Doppler Shift

The goal of this exercise is to compute the BER using some simple assumptions. Consider a binary digital communication at bitrate 50bps. The receiver is mobile and is moving toward the transmitter at speed 10m/s and the communication is over 1500MHZ frequency band.

1. What is the value of the frequency of Doppler shift?
2. Consider fadings (due to Doppler shift) of the received signal strength  $R$  below  $0.1 \cdot \rho_{RMS}$ . What is the average fade duration?
3. Assume that a bit is lost whenever the received signal strength  $R$  of any portion of the bit is below  $0.1 \cdot \rho_{RMS}$ . What would the BER of this communication?

$$1. \quad f_m = \frac{v}{\lambda} = \frac{v \times f_c}{c} = \frac{10 \times 1500 \times 10^6}{3 \times 10^8} = 50 \text{ Hz}$$

Doppler Shift = 50Hz

$$2. \quad \text{Threshold Level } (\rho) = \frac{R_{thresh}}{R_{rms}}$$

$$\text{For } \rho \text{ smaller than } 0.1 \cdot \rho_{RMS}, \rho = \frac{0.1 \cdot R_{rms}}{R_{rms}}$$

Hence,  $\rho = 0.1$

$$AFD = \frac{e^{\rho^2} - 1}{\rho f_d \sqrt{2\pi}}.$$

$$T = \frac{0.01005016708}{12.5331413732}$$

$$= 0.00080188731 \text{ s}$$

= 0.8 ms

$$3. \quad N_T = \sqrt{2\pi} \cdot f_m \cdot \rho \cdot e^{-\rho^2}$$

$$= 2.5 \cdot 50 \cdot 0.1 \cdot 0.990049$$

$$= 12.375$$

Since a bit is lost whenever the received signal strength  $R$  of any portion of the bit is below  $0.1 \cdot \rho_{RMS}$ , Number of Bits lost is approximately equal to the number of times level crosses.

Thus, B.E.R =  $12/50 = 0.24$

Bit Error Rate = 0.24



## Network Planning: Constrained Access Points Placement (30 points)

The goal of this problem is to find locations where to place access points (AP) in order to cover an area  $A$ . The constraint is that the location of access points is limited to a set of positions  $L = \{l_1, l_2, \dots, l_n\}$ . The goal is to minimize the deployment cost of the wireless network and therefore the number of access points. The reason behind the location constraint is that wireless network operators have to negotiate with buildings to be able to place an access points (or base station). In a practical setting each location has a cost and allows for some coverage (radius  $R$ ). In this problem we assume that all locations have the same cost and the same coverage.

You will have to find some heuristics/algorithms to select a minimum number of locations from the set  $L$  such that any point in the area  $A$  is within range from at least one access point. Assume that  $A$  is  $2000 \times 2000$  m<sup>2</sup>, and the access points range is 200m.

### [10 points] First Step: Randomly generate $L$ .

In practice the set  $L$  is given. But to be able to evaluate your algorithms you will need some input data. This part is about randomly generating the input data. There are many ways to generate the input data. Here is one that you can use:

1. Start with  $L = \emptyset$ . While the whole area  $A$  is still not covered randomly pick a point that is not covered by any AP and add it to the set  $L$ .
2. At this point all  $A$  is covered. Let  $m$  be the size of  $L$ . Randomly pick  $m$  other locations and add them to  $L$ . This second step is to provide some redundancy in coverage.

### [10 points] Second Step: Heuristics.

1. Heuristic  $H_0$ . One heuristic to find select locations is as follows: start with  $L' = \{ \}$ . As long as there are areas within  $A$  that are not covered by an AP select an AP outside the covered area and add it to  $L'$ . Implement  $H_0$ .
2. Propose two heuristics ( $H_1$ , and  $H_2$ ) for selecting  $L'$  a subset of  $L$  such that every point in the area is covered. The goal of the heuristics is to minimize  $L'$  (cost of network). Implement the heuristics using Python/Java.

**[10 points] Third Step: Comparison.** Through simulation of steps 1 and 2, compare your two heuristics to  $H_0$ , to each other and to  $\text{best}(H_0, H_1, H_2)$ . Plot the performance of each heuristic and  $\text{best}(H_0, H_1, H_2)$  averaged 100 simulation runs. Discuss.

## Heuristics:

### Heuristic 1:

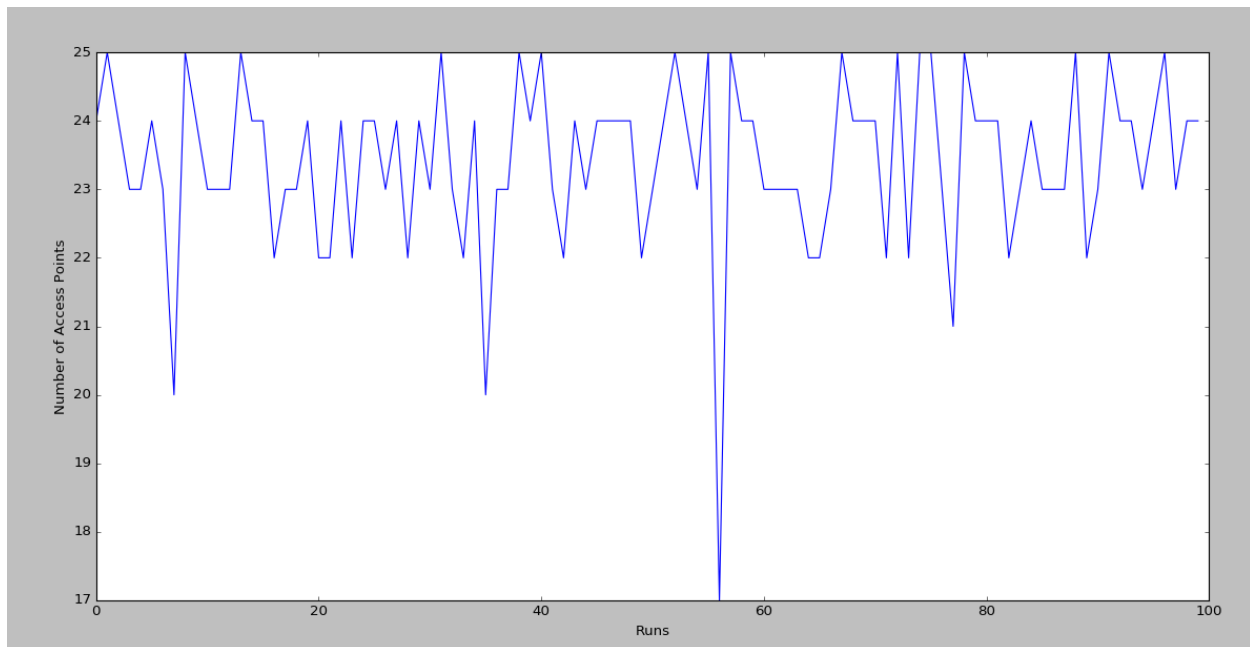
Pseudo Code

```
// Initialize the list
APList = list()
// Initialize the XCoverage and YCoverage variables that keep a track of the coverage
while(currentCoverage != totalCoverage){
//increment Current XCoverage by taking up the next point

XCoverage+=200
// Traverse through the list to increment current Coverage in Y-Direction

}
```

Time complexity of this method is  $O(n^2)$



### Heuristic 2:

Pseudo Code

```
//Sort the list
// Sorted list will decrease the number of times the inner loop runs
// Choose list[0] as the starting point
// Initialize the list
```

```

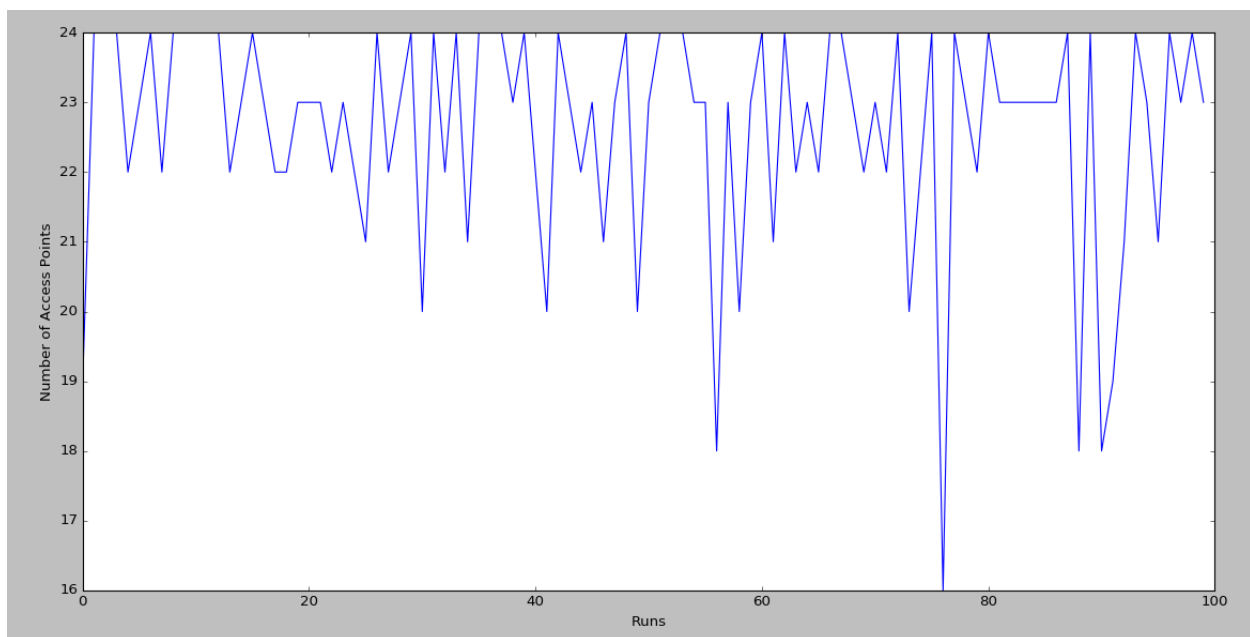
APList = list()
// Initialize the XCoverage and YCoverage variables that keep a track of the coverage

//
while(currentCoverage != totalCoverage){
//increment Current XCoverage by taking up the next point

XCoverage+=200
// Traverse through the list to increment current Coverage in Y-Direction
}

```

Time Complexity of the code is  $O(n \log n + n^2)$



### Heuristic 3:

**Heuristic 3 follows the greedy approach by picking up the first element in the sorted list and finding the farthest element in distance of 400m from it since, that will be least likely to cause significant overlap in the coverage area but still cover the entire area as the coverage radius of AP is 200m**

Pseudo Code

```

//Sort the list

// Sorted list will decrease the number of times the inner loop runs

// Choose list[0] as the starting point
// Initialize the list

```

```

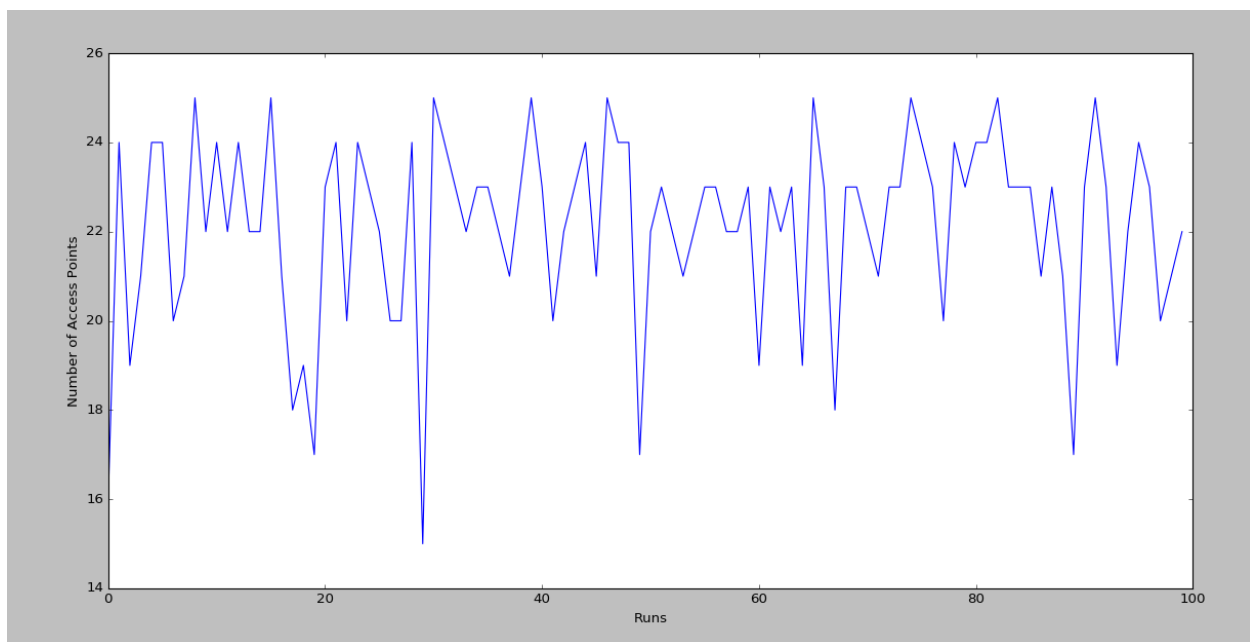
APList = list()
// Initialize the XCoverage and YCoverage variables that keep a track of the coverage

//
while(currentCoverage != totalCoverage){
//increment Current XCoverage by taking up the next point

XCoverage+=400
// Traverse through the list to increment current Coverage in Y-Direction
}

```

Time Complexity of the code is  $O(n \log n + n^2)$



### Comparison:

As it can be seen in the graph, Greedy approach in Heuristic 3 results in best results by decreasing the amount of access points required from Heuristic 2 and Heuristic 1

