

# 1 3D Tools

## TikZ Library `3dtools`

```
\usetikzlibrary{3dtools} %  $\LaTeX$  and plain  $\TeX$ 
\usetikzlibrary[3dtools] % Con $\TeX$ t
```

This library provides additional tools to create 3d-like pictures. It is a collection of reasonably working tools, which however is not streamlined, and may be subject to substantial changes if the library ever happens to get further developed.

TikZ has the `3d` and `tpp` libraries which deal with the projections of three-dimensional drawings. In addition there exist excellent packages like `tikz-3dplot`. The purpose of this library is to provide some means to manipulate the coordinates. It supports linear combinations of vectors, vector and scalar products.

**Note:** Hopefully this library is only temporary and its contents will be absorbed in slightly extended versions of the `3d` and `calc` libraries. The cleanest way will be to record a screen depth when “saving” a coordinate with TikZ. However, this would require changes at the level of `tikz.code.tex` and can only be done consistently if the maintainer(s) of TikZ support this. Note also that it is quite conceivable that the viewers will in the future will be able to achieve 3d ordering, so, in a way, recording the screen depth (see the `screendepth` function below) will become almost mandatory at a given point.

**Filing a bug report or placing a feature request.** This library is currently hosted under <https://github.com/marmotghost/tikz-3dtools>. The author is also active at the *noncommercial* Q & A site <https://topanswers.xyz/tex>, which offers, apart from the possibility of asking questions and browsing through the posts a chat in which one can discuss problems.

**Why is this library not on CTAN?** First of all, ideally this library is only temporary. Secondly, the author does not have a law degree, and finds it very hard to select the appropriate license for submitting the library to CTAN. The library is written as a fun project, everyone is free to use it, and trying to understand all the legal stuff that seems to be required to perform a successful submission to CTAN is not the kind of fun the author is after. This is not to say that CTAN does not have a point in requiring all these data.

## 1.1 Coordinate computations

The `3dtools` library has some options and styles for coordinate computations.

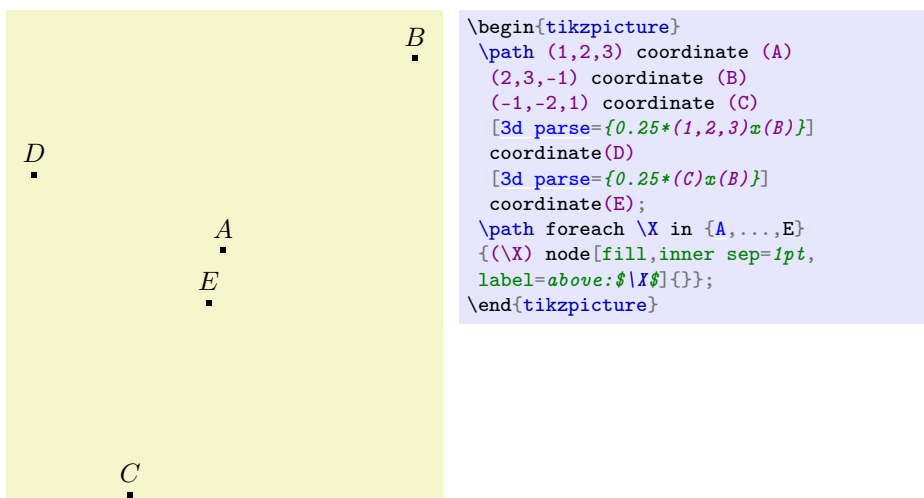
`/tikz/3d parse` (no value)

Parses an expression and inserts the result in form of a coordinate.

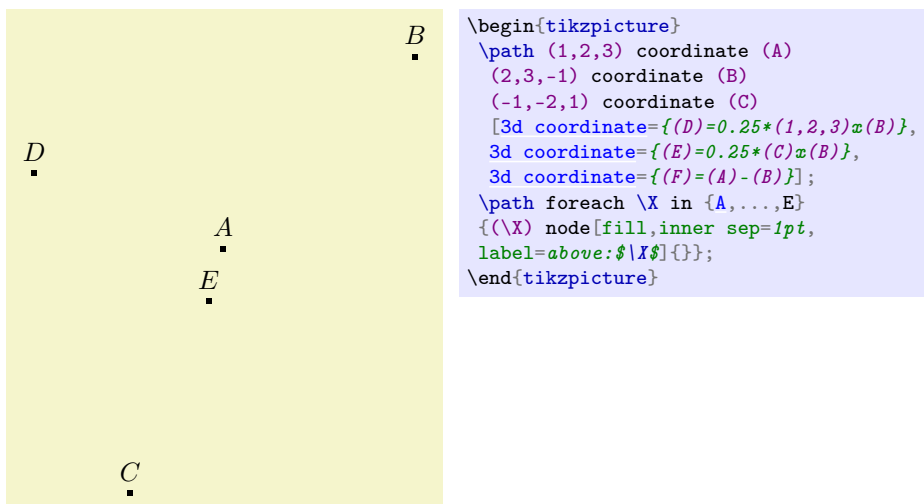
`/tikz/3d coordinate` (no value)

Allow one to define a 3d coordinate from other coordinates.

Both keys support both symbolic and explicit coordinates.



Notice that, as of now, only the syntax `\path (1,2,3) coordinate (A);` works, i.e. `\coordinate (A) at (1,2,3);` does *not* work, but leads to error messages.



The actual parsings are done by the function `\pgfmaththdparse` that allows one to parse 3d expressions. The supported vector operations are + (addition +), - (subtraction -), \* (multiplication of the vector by a scalar), x (vector product  $\times$ ) and o (scalar product).

`\pgfmaththdparse{<x>}`

Parses 3d expressions.

`TDx("vector")`

Yields the  $x$ -component of a 3d expression.

`TDy("vector")`

Yields the  $y$ -component of a 3d expression.

`TDz("vector")`

Yields the  $z$ -component of a 3d expression.

`screendepth("vector")`

Yields distance a coordinate is above (positive) or below (negative) the screen. The values are only really meaningful if the user has installed some reasonable view. The larger the screen depth of a point is, the closer is the point to the observer. The function reconstructs the 3-bein<sup>1</sup> from lengths like `\textbackslash pgf@xy` and so on, so the function is independent of the tool that is employed to install a view (cf. section 1.2). The screen depth is crucial to decide which objects are in front of other objects.

In order to pretty-print the result one may want to use `\pgfmathprintvector`, and use the math function `TD` for parsing.

`\pgfmathprintvector{\langle x \rangle}`

Pretty-prints vectors.

$$0.2 \vec{A} - 0.3 \vec{B} + 0.6 \vec{C} = (-1, -1.7, 1.5)$$

```
\pgfmathparse{TD("0.2*(A)
-0.3*(B)+0.6*(C)")}%
$0.2\,\vec A-0.3\,\vec B+0.6\,\vec C
=(\pgfmathprintvector\pgfmathresult)$
```

The alert reader may wonder why this works, i.e. how would TikZ “know” what the coordinates  $A$ ,  $B$  and  $C$  are. It works because the coordinates in TikZ are global, so they get remembered from the above example.

**Warning.** The expressions that are used in the coordinates will only be evaluated when they are retrieved. So, if you use, say, random numbers, you will get each time a *different* result.

$$\vec{R} = (0.99, 0.67, 0.29)$$

$$\vec{R} = (0.8, 0.78, 0.94)$$

```
\begin{tikzpicture}
\path[overlay] (rnd,rnd,rnd)
coordinate (R);
\node at (0,1)
{\pgfmathparse{TD("(R)")}%
$\vec R=(\pgfmathprintvector\pgfmathresult)$};
\node at (0,0)
{\pgfmathparse{TD("(R)")}%
$\vec R=(\pgfmathprintvector\pgfmathresult)$};
\end{tikzpicture}
```

Its main usage is to strip off unnecessary zeros, which emerge since the internal computations are largely done with the `fp` library.

$$(1, 0, 0)^T \times (0, 1, 0)^T = (0, 0, 1)^T$$

```
\pgfmathparse{TD("(1,0,0)x(0,1,0)")}%
$(1,0,0)^T\times(0,1,0)^T=
(\pgfmathprintvector\pgfmathresult)^T$
```

$$\vec{A} \cdot \vec{B} = 5$$

```
\pgfmathparse{TD("(A)o(B)")}%
$\vec A\cdot\vec B=
\pgfmathprintnumber\pgfmathresult$
```

<sup>1</sup>A 3-bein or dreibein is a German word that stands for a local frame, its literal translation is something like three-leg. Like the term eigenvector this is a foreign word that, to the best of my knowledge, has no commonly used English counterpart.

Notice that, as of now, the only purpose of brackets  $(\dots)$  is to delimit vectors. Further, the addition  $+$  and subtraction  $-$  have a *higher* precedence than vector products  $\times$  and scalar products  $\circ$ . That is,  $(A)+(B)\circ(C)$  gets interpreted as  $(\vec{A} + \vec{B}) \cdot \vec{C}$ , and  $(A)+(B)\times(C)$  as  $(\vec{A} + \vec{B}) \times \vec{C}$ .

$$(\vec{A} + \vec{B}) \cdot \vec{C} = -11$$

```
\pgfmathparse{TD("(A)+(B)\circ(C)")}%
$(\vec{A}+\vec{B})\cdot\vec{C}=
\pgfmathprintnumber\pgfmathresult$
```

$$(\vec{A} + \vec{B}) \times \vec{C} = (9, -5, -1)$$

```
\pgfmathparse{TD("(A)+(B)\times(C)")}%
$(\vec{A}+\vec{B})\times\vec{C}=
(\pgfmathprintvector\pgfmathresult)$
```

Moreover, any expression can only have either one  $\circ$  or one  $\times$ , or none of these. Expressions with more of these can be accidentally right.

`axisangles("vector")`

Yields the the rotation angles that transforms the vector in the  $z$ -axis. Since an axis has a residual rotation symmetry, namely the rotation around this axis, only two angles are required, and thus returned. In the conventions of section 1.2, these are the angles  $\phi$  and  $\psi$ . It corresponds to the macro `\tdplotgetpolarcoords{\marg{x}}{\marg{y}}{\marg{z}}` from the `tikz-3dplot` package.

$$\angle(\vec{A}) = -116.57, -36.7$$

```
\pgfmathparse{axisangles("(A)")}%
$\sphericalangle(\vec{A})=
\pgfmathprintvector\pgfmathresult$
```

$$\angle(\vec{A}) = -116.57, -36.7$$

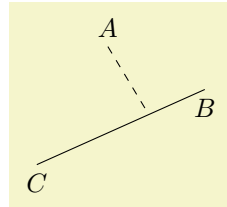
```
\pgfmathparse{axisangles("(A)")}%
$\sphericalangle(\vec{A})=
\pgfmathprintvector\pgfmathresult$
```

`/tikz/3d/projection of`

(no value)

Allows one to compute the projection of a point on a line.

The following code example illustrates the usage. It also makes use of the `install view` key, which we describe in section 1.2.



```
\begin{tikzpicture}[3d/install view={%
phi=110,psi=0,theta=60}]
\draw
(2,1,2) coordinate[label=above:{$A$}] (A)
(1,2,1) coordinate[label=below:{$B$}] (B) --
(2,0,0) coordinate[label=below:{$C$}] (C);
\path[3d/projection of={($A$ on ($B$)--($C$))}]
coordinate (P);
\draw[dashed] (A) -- (P);
\end{tikzpicture}
```

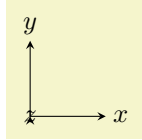
## 1.2 Orthonormal projections

This library can be used together with the `tikz-3dplot` package and/or the `tpp` library. It also has its own means to install orthonormal projections. Orthonormal projections emerge from subjecting 3-dimensional vectors to orthogonal transformations and projecting them to 2 dimensions. They are not to be confused with the perspective projections, which are more realistic and supported by the `tpp` library. Orthonormal projections may be thought of a limit of perspective projections at large distances, where large means that the distance of the observer is much larger than the dimensions of the objects that get depicted.

`/tikz/3d/install view` (no value)

Installs a 3d orthonormal projection.

The initial projection is such that  $x$  is right and  $y$  is up, as if we had no third direction.



```
\begin{tikzpicture}[3d/install view]
\draw[-stealth] (0,0,0) -- (1,0,0)
  node[pos=1.2] {$x$};
\draw[-stealth] (0,0,0) -- (0,1,0)
  node[pos=1.2] {$y$};
\draw[-stealth] (0,0,0) -- (0,0,1)
  node[pos=1.2] {$z$};
\end{tikzpicture}
```

The 3d-like pictures emerge by rotating the view. The conventions for the parametrization of the orthogonal rotations in terms of three rotation angles  $\phi$ ,  $\psi$  and  $\theta$  are

$$O(\phi, \psi, \theta) = \begin{pmatrix} c_\phi c_\psi & s_\phi c_\psi & -s_\psi \\ c_\phi s_\psi s_\theta - s_\phi c_\theta & s_\phi s_\psi s_\theta + c_\phi c_\theta & c_\psi s_\theta \\ c_\phi s_\psi c_\theta + s_\phi s_\theta & s_\phi s_\psi c_\theta - c_\phi s_\theta & c_\psi c_\theta \end{pmatrix}. \quad (1)$$

Here,  $c_\phi := \cos \phi$ ,  $s_\phi := \sin \phi$  and so on.

`/tikz/3d/phi` (initially 0)

3d rotation angle.

`/tikz/3d/psi` (initially 0)

3d rotation angle.

`/tikz/3d/theta` (initially 0)

3d rotation angle.

The rotation angles can be used to define the view. The conventions are chosen in such a way that they resemble those of the `tikz-3dplot` package, which gets widely used. This matrix can be written as

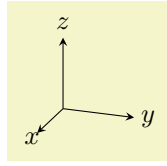
$$O(\phi, \psi, \theta) = R_x(\theta) \cdot R_y(\psi) \cdot R_z(\phi),$$

where

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}, \quad R_y(\psi) = \begin{pmatrix} \cos(\psi) & 0 & -\sin(\psi) \\ 0 & 1 & 0 \\ \sin(\psi) & 0 & \cos(\psi) \end{pmatrix},$$

$$R_z(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix}$$

are rotations about the  $x$ ,  $y$  and  $z$  axis, respectively. For  $\psi = 0$ ,  $O(\phi, 0, \theta) = R^d(\theta_d, \phi_d)$  from the `tikz-3dplot` package. Note, however, that there seems to be an inconsistency in equation (2.1) of that package.<sup>2</sup>



```
\begin{tikzpicture}[3d/install view={phi=110,psi=0,theta=70}]
\draw[-stealth] (0,0,0) -- (1,0,0)
node[pos=1.2] {$x$};
\draw[-stealth] (0,0,0) -- (0,1,0)
node[pos=1.2] {$y$};
\draw[-stealth] (0,0,0) -- (0,0,1)
node[pos=1.2] {$z$};
\end{tikzpicture}
```

### 1.3 Visibility considerations

When drawing objects in 3d, some parts are “visible”, i.e. in the foreground, while other parts are hidden, i.e. in the background. In what follows, we discuss some basic means that allow one to determine whether some stretch is visible or hidden. Very often this amounts to find critical values of some parameter(s) (such as angles) at which some curve changes from visible to hidden, or vice versa. One of the major players in this game is the normal on the screen, which is given by the last row of the orthogonal matrix  $O$  of (1). For  $\psi = 0$ , this normal is also the normal in the so-called “main” coordinates of `tikz-3dplot` with the identifications  $\theta = \texttt{\tdplotmaintheta}$  and  $\phi = \texttt{\tdplotmainphi}$ .

Here is an example. Suppose we wish to draw a latitude circle on a sphere for  $\psi = 0$ . Since the system has a rotational symmetry around the  $z$  axis, we can set  $\phi = 0$  as well. The normal on the screen then becomes

$$n_{\text{screen}} = (0, -\sin(\theta), \cos(\theta))^T.$$

The latitude circle can be parametrized as

$$\gamma_{\text{lat}}(\beta) = (r \cos(\beta), r \sin(\beta), h)^T,$$

where  $r = R \cos(\lambda)$  and  $h = R \sin(\lambda)$  with  $R$  being the radius of the sphere and  $\lambda$  the latitude angle of the latitude circle. The critical angles  $\beta_{\text{crit}}$  at which the curve transitions from hidden to visible are then given by the solutions of

$$n_{\text{screen}} \cdot \gamma_{\text{lat}}(\beta_{\text{crit}}) = -r \sin(\beta) \sin(\theta) + h \cos(\theta) \stackrel{!}{=} 0.$$

Since  $-1 \leq \sin(\beta) \leq 1$ , this equation only has solutions if

$$\left| \frac{h}{r} \cot(\theta) \right| \leq 1. \quad (2)$$

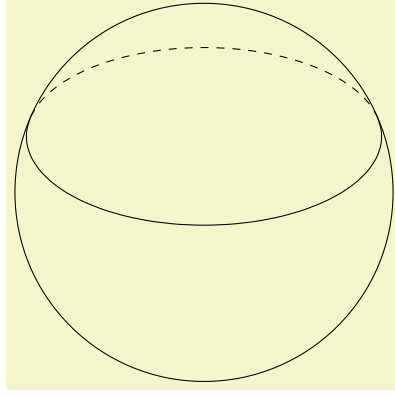
---

<sup>2</sup>I do not know how to contact the author.

The solutions are then given by

$$\beta_{\text{crit}}^{(1)} = \arcsin\left(\frac{h}{r} \cot(\theta)\right) \quad \text{and} \quad \beta_{\text{crit}}^{(2)} = 180 - \beta_{\text{crit}}^{(1)},$$

and  $\beta_{\text{crit}}^{(1)} = \beta_{\text{crit}}^{(2)}$  if the inequality (2) is saturated. The following code example illustrates this. Notice that it could be made more concise with the 3d library.



```
\begin{tikzpicture}[declare function={%
R=2.5;lambda=20;theta=60;
r=R*cos(lambda);h=R*sin(lambda);
betacrit=asin(h/r*cot(theta));}]
\draw (0,0) circle[radius=R];
\begin{scope}[smooth,
3d/install view={phi=0,psi=0,theta=theta}]
\draw[dashed]
plot[variable=\t,
domain=betacrit:180-betacrit]
({r*cos(\t)},{r*sin(\t)},h);
\draw
plot[variable=\t,
domain=betacrit:-180-betacrit]
({r*cos(\t)},{r*sin(\t)},h);
\end{scope}
\end{tikzpicture}
```

Another type of visibility considerations concerns the contours of shapes. To be specific, consider a surface parametrized by

$$f(u, v) = \begin{pmatrix} u \\ R(u) \cos(v) \\ R(u) \sin(v) \end{pmatrix} \quad (3)$$

with some function  $R(u)$ . What is the contour of an orthonormal projection of the surface on the screen? It is the boundary between the hidden and visible parts. What distinguishes the visible from the hidden parts? It is the projection of the normal of the surface on the screen,  $F(u, v) := \vec{n}_S(u, v) \cdot \vec{n}_{\text{screen}}$ . The hidden and visible stretches are separated by the zeros of this projection. In the case of our surface (3),

$$\vec{n}_S(u, v) = \vec{\nabla}_u f(u, v) \times \vec{\nabla}_v f(u, v) = R(u) \begin{pmatrix} -R'(u) \\ \cos(v) \\ \sin(v) \end{pmatrix}, \quad (4)$$

and for  $\psi = 0$

$$n_{\text{screen}} = \begin{pmatrix} \sin(\theta) \sin(\phi) \\ -\sin(\theta) \cos(\phi) \\ \cos(\theta) \end{pmatrix}. \quad (5)$$

So we need to find the zeros of

$$\begin{aligned} F(u, v) &= -\cos(\theta) \sin(v) + \sin(\theta) \cos(\phi) \cos(v) + R'(u) \sin(\theta) \sin(\phi) \\ &=: a \sin(v) + b \cos(v) + c. \end{aligned} \quad (6)$$

This equation can be solved for  $x := \sin(v)$  using  $\cos(v) = \pm\sqrt{1 - \sin^2(v)} = \pm\sqrt{1 - x^2}$ . It is clear that, since  $\sin(v)$  is bounded (and so is  $\cos(v)$ ), this equation

does not always have a solution. In particular, if  $R'(u)$  is too large, there might not be a solution. This just means that for a given  $u$  all points are hidden or visible. This happens e.g. when you look at a cone from the top or bottom. A more advanced code could deal with these cases, the one presented below does not. Note also that these are local visibility conditions, a seemingly visible stretch can always be covered by some other stretch that is a finite distance away in  $u$  direction.

However, for “reasonable” configurations, there are two solutions of the quadratic equation, as they should, they correspond to the upper and lower contour in figure 1. They are a bit unilluminating and given by

$$[\sin(v)]_{1,2} = -\frac{a\sqrt{a^2 + b^2 - c^2} \pm bc}{a^2 + b^2}, \quad (7a)$$

$$[\cos(v)]_{1,2} = -\frac{b\sqrt{a^2 + b^2 - c^2} \mp ac}{a^2 + b^2}, \quad (7b)$$

where  $a$ ,  $b$  and  $c$  are implicitly defined in (6). Hence, for “reasonable” configurations, it is very easy to plot the contour of the surface with  $\text{\LaTeX}$ .

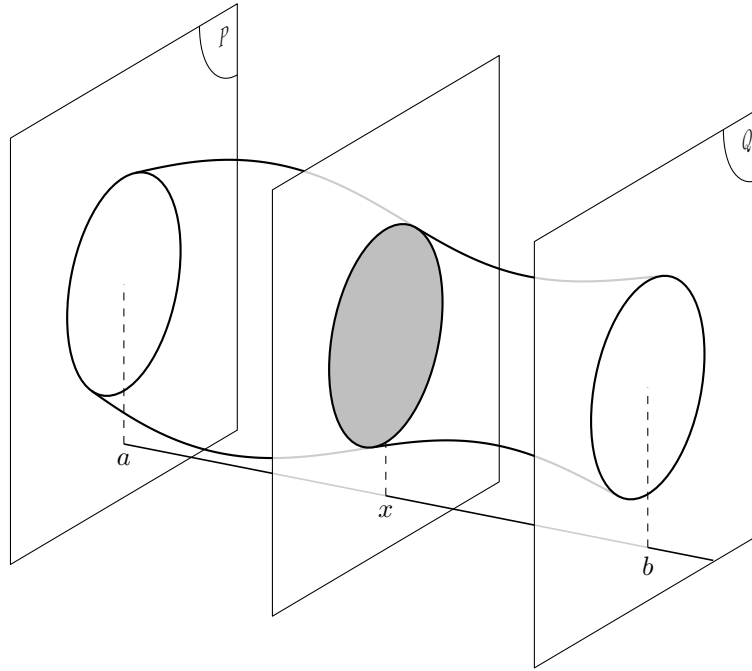


Figure 1: Example of a screen projection of a surface of revolution. Taken from [topanswers.xyz](http://topanswers.xyz).

## 1.4 Predefined pics

`/tikz/pics/3d circle through 3 points=<options>` (no default, initially empty)

Draws a circle through 3 points in 3 dimensions. If the three coordinates are close to linearly dependent, the circle will not be drawn.



`/tikz/3d circle through 3 points/A` (initially (1,0,0))

First coordinate. Can be either symbolic or explicit. Symbolic coordinates need to be defined via `\path (x,y,z) coordinate (name);`.

`/tikz/3d circle through 3 points/B` (initially (0,1,0))

Second coordinate, like above.

`/tikz/3d circle through 3 points/C` (initially (0,0,1))

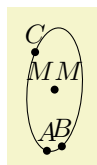
Third coordinate, like above.

`/tikz/3d circle through 3 points/center name` (initially M)

Name of the center coordinate that will be derived.

`/tikz/3d circle through 3 points/auxiliary coordinate prefix`  
(initially tmp)

In TikZ the coordinates are global. The code for the circle is more comprehensible if named coordinates are introduced. Their names will begin with this prefix. Changing the prefix will allow users to avoid overwriting existing coordinates.



```
\begin{tikzpicture}[3d/install view={phi=30,psi=0,theta=70}]
\foreach \X in {A,B,C}
{\pgfmathsetmacro{\myx}{3*(rnd-1/2)}
\pgfmathsetmacro{\myy}{3*(rnd-1/2)}
\pgfmathsetmacro{\myz}{3*(rnd-1/2)}
\path (\myx,\myy,\myz) coordinate (\X);}
\path pic{3d circle through 3 points={%
A={A},B={B},C={C},center name=MM}};
\foreach \X in {A,B,C,MM}
{\fill (\X) circle[radius=1.5pt]
node[above]{\X}};
\end{tikzpicture}
```

`/tikz/pics/3d incircle=<options>` (no default, initially empty)

Inscribes a circle in a triangle in 3 dimensions.

`/tikz/3d incircle/A` (initially (1,0,0))

First coordinate. Can be either symbolic or explicit.

`/tikz/3d incircle/B` (initially (0,1,0))

Second coordinate, like above.

`/tikz/3d incircle/C` (initially (0,0,1))

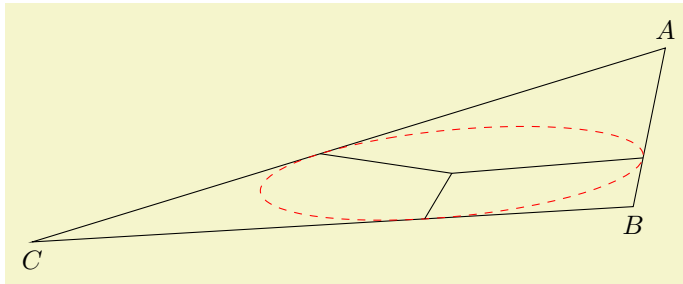
Third coordinate, like above.

`/tikz/3d incircle/center name` (initially I)

Name of the center coordinate that will be derived.

`/tikz/3d incircle/auxiliary coordinate prefix` (initially tmp)

In TikZ the coordinates are global. The code for the circle is more comprehensible if named coordinates are introduced. Their names will begin with this prefix. Changing the prefix will allow users to avoid overwriting existing coordinates.



```
\begin{tikzpicture}[3d/install view={phi=110,psi=0,theta=70}]
\draw
(8,5,5) coordinate[label=above:{$A$}] (A) --
(1,2,0) coordinate[label=below:{$B$}] (B) --
(5,-5,0) coordinate[label=below:{$C$}] (C) -- cycle;
\path pic[red,dashed]{3d incircle={%
A={A},B={B},C={C},center name=I}};
\draw (I) -- (tmppa) (I) -- (tmppb) (I) -- (tmppc);
\end{tikzpicture}
```

`/tikz/pics/ycylinder` (initially empty)

A cylinder in the  $y$ -direction. This pic requires the `calc` library. As of now it does only work for `psi=0`.

`/tikz/pics/3d/r` (initially 1)

Key for radii, e.g. of cylinders.

`/tikz/pics/3d/h` (initially 1)

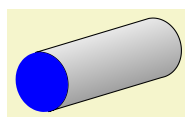
Key for heights, e.g. of cylinders.

`/tikz/pics/3d/mantle` (initially draw)

Style for cylinder mantle. If no fill option is specified, it will be shaded.

`/tikz/pics/3d/top` (initially draw)

Style for cylinder top.



```
\begin{tikzpicture}[3d/install view={phi=30,psi=0,theta=80}]
\pic{ycylinder={r=0.4,h=3,
top/.style={fill=blue}}};
\end{tikzpicture}
```

To do:

- transform to plane given by three non-degenerate coordinates
- transform to plane given by normal and one point
- maybe layering/visibility

## 1.5 3D-like decorations

`/tikz/decorations/3d complete coil`

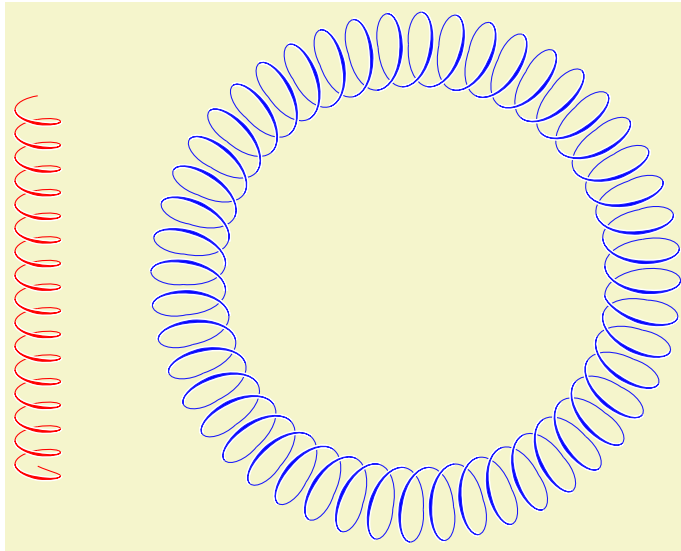
(no value)

3d-like coil where the front is thicker than the back.

`/tikz/decorations/3d coil closed`

(no value)

Indicates that the coil is closed.



```
\begin{tikzpicture}
\draw[decoration={3d coil color=red,aspect=0.35, segment length=3.1mm,
amplitude=3mm,3d complete coil},
decorate] (0,1) -- (0,6);
\draw[decoration={3d coil color=blue,3d coil opacity=0.9,aspect=0.5,
segment length={2*pi*3cm/50}, amplitude=5mm,3d complete coil,
3d coil closed},
decorate] (5,3.5) circle[radius=3cm];
\end{tikzpicture}
```