# Social Networks & Recommendation Systems

## V. Static random graphs.

Grzegorz Siudem

Warsaw University of Technology

European Funds
Knowledge Education Development

Warsaw University of Technology

European Union
European Social Fund

MSc program in Data Science has been developed
as a part of task 10 of the project
„NERW PW. Science - Education - Development - Cooperation”
co-funded by European Union from European Social Fund.

# Project

# Networks with given hamiltonian

Let us considet the space of every possible graph with $N$ vertices i.e. set $M_N = \mathbb{M}^{N \times N}(\{0,1\})$. We want to define a probability distribution on it.

We maximize entropy:

$$-\sum_{G \in M_N} \mathcal{P}(G) \ln \mathcal{P}(G),$$

Under certain condition $f(\mathcal{P}(G)) = 0$,

Which leads us to Lagrange multipliers

$$\mathcal{L}[\mathcal{P}(G)] = -\sum_{G \in M_N} \mathcal{P}(G) \ln \mathcal{P}(G) + \lambda f(\mathcal{P}(G))$$

It *only* remains to solve this equation

$$\frac{\partial \mathcal{L}}{\partial \mathcal{P}(G)} = 0.$$

# Example - Excercise 1.

# Implementation of $G_{N,p}$ version of ER graph – case study

**Ecercise 2.**

Implement a function that returns the adjacency matrix of one realization of the ER graph with given values of $N$ and $p$. Watch out for the trap!

# Implementation of $G_{N,p}$ version of ER graph – case study

**Ecercise 2.**

Implement a function that returns the adjacency matrix of one realization of the ER graph with given values of $N$ and $p$. Watch out for the trap!

**Excercise 3.**

Draw resulting graph

# Implementation of $G_{N,p}$ version of ER graph – case study

**Ecercise 2.**

Implement a function that returns the adjacency matrix of one realization of the ER graph with given values of $N$ and $p$. Watch out for the trap!

**Excercise 3.**

Draw resulting graph

**Excercise 4.**

Draw histogram of degree distribution.

# Implementation of $G_{N,p}$ version of ER graph – case study

**Ecercise 2.**

Implement a function that returns the adjacency matrix of one realization of the ER graph with given values of $N$ and $p$. Watch out for the trap!

**Excercise 3.**

Draw resulting graph

**Excercise 4.**

Draw histogram of degree distribution.

**Excercise 5.**

What degree of vertex distribution do we expect?

# ER model - comparison of theoretical results and simulations

**Excercise 6.**
Give the *mathematical* justification for the Poisson approximation used.

# ER model - comparison of theoretical results and simulations

**Excercise 6.**

Give the *mathematical* justification for the Poisson approximation used.

**Excercise 7.**

Plot both the simulation results and analytically obtained distributions on one graph. Test appropriate hypotheses.

# ER model - comparison of theoretical results and simulations

**Excercise 6.**

Give the *mathematical* justification for the Poisson approximation used.

**Excercise 7.**

Plot both the simulation results and analytically obtained distributions on one graph. Test appropriate hypotheses.

**Excercises 8.**

Check dependence of the results of the previous excercise for various values of $p$ and $N$.

# ER model - comparison of theoretical results and simulations

**Excercise 6.**

Give the *mathematical* justification for the Poisson approximation used.

**Excercise 7.**

Plot both the simulation results and analytically obtained distributions on one graph. Test appropriate hypotheses.

**Excercises 8.**

Check dependence of the results of the previous excercise for various values of $p$ and $N$.

**Attention!**

Excercises 1-8 in total are worth 1$P$ for the project.

# Non-physicality of ER graphs

Assuming the Poisson approximation, we calculate the variance

$$\mathbb{E}(K) = \sum_{k=0}^{\infty} \frac{k e^{-\langle k \rangle} \langle k \rangle^k}{k!} = \cdots = \langle k \rangle,$$

$$\mathbb{E}(K^2) = \sum_{k=0}^{\infty} \frac{k^2 e^{-\langle k \rangle} \langle k \rangle^k}{k!} = \cdots = \langle k \rangle + \langle k \rangle^2.$$

$$\mathrm{Var}(K) = \mathbb{E}(K^2) - [\mathbb{E}(K)]^2 = \langle k \rangle$$

P5.1 Complete the missing calculations.[0.5P]

# Non-physicality of ER graphs

**Clustering coefficient**

$$\langle C \rangle = p.$$

P5.2 Check the above analytical result by simulation. [1P]

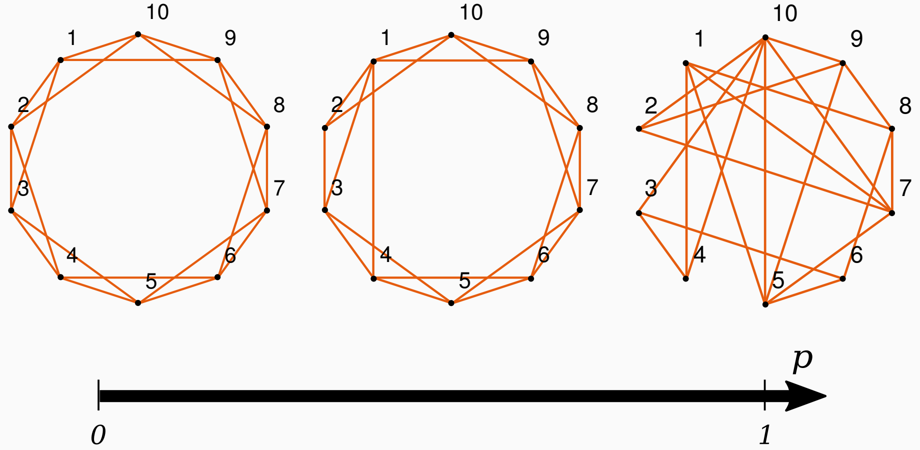
# Stochastic block model

ER model generalization

$$\begin{bmatrix} [p_{11}] & [p_{12}] & \dots & [p_{1N}] \\ [p_{21}] & [p_{22}] & \dots & [p_{2N}] \\ \dots & \dots & \ddots & \dots \\ [p_{N1}] & [p_{N2}] & \dots & [p_{NN}] \end{bmatrix}$$

P5.3 Generate and draw a graph consisting of 4 community each with $N = 20$ nodes and the probability of connection within the community higher than between them. Draw the result. How it depends on the parameter values? [2P]

# Watts-Strogatz model




P5.4 Draw a graph of the averaged coefficient of clustering of the WS network against its parameter $p$. [1.5P]

# Other projects

P5.5 With (or without) Mathematica solve ER model in the case of $G_{N,E}$. [2P]

P5.6 Implement configuration model and test when the procedure converge. [2.5P]

P5.7 Compute partition function and distribution of the network with given hamiltonian for the case with fixed number of edges. [2.5P]

MSc program in Data Science has been developed
as a part of task 10 of the project
„NERW PW. Science - Education - Development - Cooperation”
co-funded by European Union from European Social Fund.