Asymptotics of Weighted Reflectable Walks in Weyl Chambers

List of questions:

- (During MRC) Can we analyze A_2^n, A_3^n, and other chambers?
- (June 22) Is there always a clear meaning for "large weights"? How often does "large weights" mean the weights are independently > 1?
- (During MRC) Is there a meaningful way to interpret the generating function trick from Steve and Mark's paper?

$$\frac{(x^2-y)(1-\overline{xy})(x-y^2)}{(1-x)(1-y)(1-xyt(\overline{y}+y\overline{x}+x))} = \\ -\frac{(1-\overline{xy})(x-y^2)(x+1)}{(1-y)(1-xyt(\overline{y}+y\overline{x}+x)} + \frac{(1-\overline{xy})(x-y^2)}{(1-x)(1-xyt(\overline{y}+y\overline{x}+x))}$$

  - On June 16, we looked at coefficients from generating functions on the right. No clear cancellations emerged. The problem is likely difficult. Perhaps code to track collisions more systematically would be helpful.

Meeting Notes

June 22, 2021

- Helen wrote code to find the asymptotics for walks in the positive quadrant with step set $\{\leftarrow, \uparrow, \searrow\}$. The code uses Theorem 10.3.1 of Pemantle and Wilson to evaluate the asymptotics.
  - There may be a faster way of achieving the same thing – but, there is a mysterious factor of ab, and it's not clear if the starting point is (x, y) or (ax, by).
  - Theorem 10.3.1 does simplify massively here, because the matrix has many 0 entries, and the diagonal (except the (1, 1)-entry) is all ± 1's when we plug in the critical point.
- The asymptotics appear to be correct by using recursive code to compute the weighted paths directly.
  - This recursive code is inefficient. Perhaps we can improve it by using Sage packages like `http://www.luschny.de/math/seq/gfun/gfun.html` or `https://gitlab.inria.fr/discretewalks/comb_walks`.
- Next steps include: writing more efficient code in both parts, and looking at one small weight and one large weight.