$$p(t_n) = f(Y_n, t_n), \cdots p(t_{n-r+1}) = f(Y_{n-r+1}, t_{n-r+1}) \tag{11.70}$$

and to calculate the approximation

$$Y_{n+1} - Y_n \approx \int_{t_n}^{t_{n+1}} p(t)dt \tag{11.71}$$

which is generally a linear combination of $f_n \cdots f_{n-r+1}$. For example, the Adams–Bashforth formulas of order 2, 3, 4 are

$$\begin{aligned}
Y_{n+1} - Y_n &= \frac{\Delta t}{2}(3 f_n - f_{n-1}) + O(\Delta t^3) \\
Y_{n+1} - Y_n &= \frac{\Delta t}{12}(23 f_n - 16 f_{n-1} + 5 f_{m-2}) + O(\Delta t^4) \\
Y_{n+1} - Y_n &= \frac{\Delta t}{24}(55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3}) + O(\Delta t^5).
\end{aligned} \tag{11.72}$$

11.10.2 Implicit Multistep Methods

The implicit Adams–Moulton method also uses the yet not known value y_{n+1} to obtain the polynomial (Fig. 11.9)

$$p(t_{n+1}) = f_{n+1}, \ldots, p(t_{n-r+2}) = f_{n-r+2}. \tag{11.73}$$

The corresponding Adams–Moulton formulas of order 2–4 are

$$\begin{aligned}
Y_{n+1} - Y_n &= \frac{\Delta t}{2}(f_{n+1} + f_n) + O(\Delta t^3) \\
Y_{n+1} - Y_n &= \frac{\Delta t}{12}(5 f_{n+1} + 8 f_n - f_{n-1}) + O(\Delta t^4) \\
Y_{n+1} - Y_n &= \frac{\Delta t}{24}(9 f_{n+1} + 19 f_n - 5 f_{n-1} + f_{n-2}) + O(\Delta t^5).
\end{aligned} \tag{11.74}$$

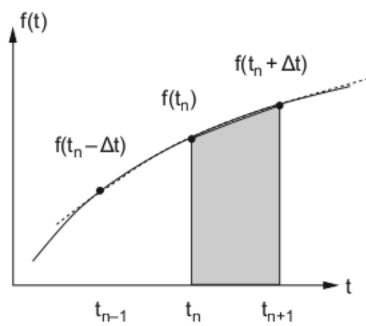

Fig. 11.9 Adams–Moulton method