Kernel Methods - Practical Session № 1

Linear Algebra Recap

Exercise 1

Let $A \in \mathbb{R}^{m \times n}$ be a real matrix. For each function f, **specify its codomain and compute** ∇f

(a) $f : \begin{cases} \mathbb{R} \to ? \\ x \to xA \end{cases}$

(b) f: $\begin{cases} \mathbb{R}^n \to ? \\ \mathrm{x} \to A\mathrm{x} \end{cases}$

(c) f: $\begin{cases} \mathbb{R}^{m \times n} \to ? \\ X \to Tr(A^T X) \end{cases}$

Exercise 2

Let $X \in \mathbb{R}^{n \times p}$, $\lambda > 0$.
Prove that $M = X^T X + \lambda I_p$ is invertible.
Hint: Prove that the eigenvalues of M are larger than λ

Linear Regression

Using notations from the slides:

- $Y = (y_1, ..., y_n)^T \in \mathbb{R}^n$ the vector of outcomes

- $X = (\mathrm{x}_1, ..., \mathrm{x}_n)^T \in \mathbb{R}^{n \times p}$ the matrix (n rows=samples, p columns=features)

- $\beta \in \mathbb{R}^p$ the linear model's parameters

Exercise 3

(a) State the loss function for ordinary least squares (OLS)

(b) Formulate OLS as a minimization problem

(c) Compute the solution $\hat{\beta}^{OLS}$

(d) What happens if $X^T X$ is singular (not invertible)?

Exercise 4 - Bias and Variance of OLS

slide 16

Exercise 5 - Optimality of OLS

slide 17

Ridge Regression

Exercise 6 - Bias and Variance

slide 33

Exercise 7 - Performance

slide 34

Ridge Logistic Regression

Exercise 8

slide 46