Algorithms

# Neutron Transport and CMFD methods

## Neutron Transport Equation

The discussion starts with the multi-group steady state neutron transport equations shown in Eq. (1), where the solution can be obtained by Method of Characteristics (MOC), Discrete Ordinate method (SN), etc.

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|  |  | (1) |

Where the parameters are defined as:

: group *g* angular flux;

: group *g* scalar flux ;

: fission spectrum for group *g*;

: scattering cross-section from group *g’* to group *g*;

: total cross-section for group *g*;

: averaged neutron emitted per fission reaction;

 : fission cross-section for group *g*;

: steady state eigenvalue.

## Coarse Mesh Finite Difference Method

As discussed above, the direct solve of the neutron transport equation is extremely slow, and it is usually accelerated by the CMFD method. The CMFD method starts with the neutron balance equation in Eq. (2), where the *m* denotes the coarse mesh node index, *s* means surface of nodes m, and  and are the volume of node *m* and surface of node *m* respectively.

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|  |  | (2) |

The cross sections and flux are homogenized as in Eq. (3), and the superscript *i* denotes the fine mesh cell in each CMFD coarse cell.

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|  |  | (3) |

The essential idea of the CMFD method is to preserve the net neutron current at all surfaces by adding aterm shown in Eq. (4), where the subscript *m+1/2* denotes the surface between node *m* and node *m+1*.

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|  |  | (4) |

Where term is expresses as in Eq. (5).

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|  |  | (5) |

After each high order transport solve, the coupling coefficientsis calculated based on Eq. (6), where the superscript *high* denotes the result from the high order transport solution.

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|  |  | (6) |

The correlation from Eq. (4) is then put back into Eq. (2) and a matrix form of eigenvalue problem can be solved with the CMFD flux and eigenvalue as unknown.

For a given CMFD system, ,and  are given.

## Group Homogenization

The total number of groups are collapse to few groups. The few group flux is the sum of the multi-group flux rather than the average of multi-group flux.

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The D-tilt term is then defined as:

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| --- | --- | --- |
|  |  | (8) |

Actually, the definition of D-tilt is arbitrary since the current is corrected by the D-hat term.

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One important thing is that the higher order current is actually the CMFD calculated current, rather than the current from the MOC level.

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Prolongation:

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## Space homogenization

The spatial homogenization is very similar to the MOC/CMFD homogenization.

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|  |  | (7) |

The D-tilt term is then defined as:

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

Actually, the definition of D-tilt is arbitrary since the current is corrected by the D-hat term.

|  |  |  |
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|  |  | (9) |

One important thing is that the higher order current is actually the CMFD calculated current, rather than the current from the MOC level. The subscript s denotes the all the fine mesh surface in that coarse surface M+1/2.

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|  |  | (10) |

Prolongation:

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