The Spectrum of the Multigroup Neutron Transport Operator for Bounded Spatial Domains

Information

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Introduction

If $\sigma[A]$, the spectrum of A, contains points in the finite part of the complex plane, then they are discrete and situated in the left half plane, finite in any strip given by $\beta_1 \leq \text{Re } \lambda \leq \beta_2$, and the generalized eigenspaces are all finite-dimensional. Defining the "leading" eigenvalue of A to be one which is real, which is not less than the real part of any other eigenvalue, which is "simple," and has a nonnegative eigenfuction $\psi(\mathbf{x}, \mathbf{\Omega})$ such that $\int \psi(\mathbf{x}, \mathbf{\Omega}) d^2\Omega$ is everywhere positive on the domain D.

Definition 0.1. Multiplicity: Let λ_i be an eigenvalue of a matrix $A \in \mathbb{R}^{n \times n}$. The *algebraic multiplicity* $\mu_A(\lambda_i)$ of λ_i is its multiplicity as the root of the characteristic polynomial.

Definition 0.2. Simple Eigenvalue: If $\mu_A(\lambda_i) = 1$, then λ_i is said to be a simple eigenvalue of A.

Main Results

- $\sigma[A]$ is nonempty if there exists at least one energy group with self-scattering in some nonempty sphere $|\mathbf{x} - \mathbf{x_0}| < r_0$ in D.
- A leading eigenvalue of A exists iff $\sigma[A]$ is nonempty.
- A dominant eigenvalue of A exists if the first condition is met and if any neutron, occurring at any point in D and in any energy group, has a positive probability of being scattered into any other energy group.

Requirements of Cross Sections, Material Boundaries, and Fluxes

Cross sections are required to be piecewise continuous and boundaries of physical system and material subregions to be continuous. Fluxes are allowed to be *square-integrable.* If cross sections and initial condition are piecewise continuous, flux expected to be piecewise continuous as well (not proven).

Definition 0.3. Square-integrable Function: A real- or complex-valued function is square-integrable if it satisfies the following:

$$\int_{-\infty}^{\infty} |f(x)|^2 dx < \infty$$

1 Summary of Results

Theorem 1. For at least one energy group g_i, let there exist an open set $R \subset D$ such that $\sigma_{g_i g_i}(x, \mu) > 0$ for $x \in R$ and $\mu \in [-1, 1]$. Then $\sigma[A]$ contains at least one point.

Theorem 2. If $\sigma[A]$ is nonempty, then it contains a lead eigenvalue.

Theorem 3. Let Theorem 1 hold, then $\sigma[A]$ possesses a leading eigenvalue λ_0 if the following condition holds: For each point $\mathbf{x}* \in D$ and any two energy groups g_1^*, g_2^*, there exists a sequence of integers

$$g_1, g_2, \ldots, g_n,\ 1 \leq g_i \leq G,$$
$$g_1 = g_1^*,\ g_n = g_2^*,$$

such that

$$\sigma_{g,g_i}(\mathbf{x}*, \mu) > 0$$

and

$$\max_{\substack{\mathbf{x} \in D \\ -1 \leq \mu \leq 1}} \sigma_{g,g_i}(x, \mu) > 0,\ 2 \leq i \leq n-1.$$

If these conditions hold, then the eigenfunction $\psi_{\lambda_0}(\mathbf{x}, \mathbf{\Omega})$ is the only nonnegative eigenfunction of A.