Computation of Fundamental Time-eigenvalue of the Neutron Transport Equation

Information

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Methods

Modified α-k Power Iteration Method

$$\hat{\Omega}\cdot\nabla(\phi_\alpha-\phi_k)+\Sigma_t(\phi_\alpha-\phi_k)+\frac{\alpha}{v}\phi_\alpha=\int_0^\infty dE'\int_{4\pi}d\Omega'\Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\cdot\hat{\Omega})(\phi_\alpha(\vec{r},E',\hat{\Omega}')-\phi_k(\vec{r},E',\hat{\Omega}'))+ \frac{\chi(E)}{4\pi}\int_0^\infty dE'\nu_p\Sigma_f\int_{4\pi}d\hat{\Omega}'(\phi_\alpha(\vec{r},E',\hat{\Omega}')-\frac{\phi_k(\vec{r},E',\hat{\Omega}')}{k}$$

Assume $\phi_\alpha \approx \phi_k$. The previous expression simplifies to

$$\alpha \approx \frac{1}{\phi_k/v}\frac{k-1}{k}\frac{\chi(E)}{4\pi}\int_0^\infty dE'\nu_p\Sigma_f\int_{4\pi}d\Omega'\phi_k(\vec{r},E',\hat{\Omega}')$$

α can be estimated at any point where $\Sigma_f \neq 0$.

Using the following relationship:

$$\int_0^\infty dE\int_{4\pi}d\Omega\frac{\chi(E)}{4\pi}=1$$

we can approximate α as

$$\alpha \approx \frac{k-1}{k}\frac{\int_0^\infty dE'\nu_p\Sigma_f\int_{4\pi}d\hat{\Omega}'\phi_k(\vec{r},E',\hat{\Omega}')}{\int_0^\infty\int_{4\pi}d\hat{\Omega}\frac{\phi_k}{v}}$$

If problem has symmetry, better to calculate α at central point if there is fissionable material at that point.
Better estimate by integrating over everywhere where there is fissionable material:

$$\alpha \approx \frac{k-1}{k}\frac{\int_{\Sigma_f\neq 0}dV\int_0^\infty dE'\nu_p\Sigma_f\int_{4\pi}d\hat{\Omega}'\phi_k(\vec{r},E',\hat{\Omega}')}{\int_{\Sigma_f\neq 0}dV\int_0^\infty\int_{4\pi}d\hat{\Omega}\frac{\phi_k}{v}}$$

Subcritical Problem $\alpha < 0$

Method is unstable when $\alpha < 0$ (i.e. when the system is subcritical). α-eigenvalue equation rewritten with an arbitrary parameter η that is greater than zero.

$$\hat{\Omega} \cdot \phi_\alpha + \left(\Sigma_t - \eta\frac{\alpha}{v}\right)\phi_\alpha = \int_0^\infty dE' \int_{4\pi} d\hat{\Omega}' \Sigma_s(\vec{r}, E' \rightarrow E, \hat{\Omega}' \cdot \hat{\Omega})\phi_\alpha(\vec{r}, E', \hat{\Omega}') +$$
$$\frac{1}{k}\frac{\chi(E)}{4\pi}\int_0^\infty dE' \nu_p \Sigma_f(\vec{r}, E') \int_{4\pi} d\hat{\Omega}' \phi_\alpha(\vec{r}, E', \hat{\Omega}' - (1+\eta)\frac{\alpha}{v}\phi_\alpha(\vec{r}, E', \hat{\Omega}')$$

Default is usually $\eta = 1$.

Modified Iterative Method

Iterative Method Algorithm:

1. Solve k-effective TE with $\alpha = 0$ for k_0 and ϕ_0.
2. Estimate the value $\alpha_{n-1/2}$ from α expressions seen above.
3. Let $\alpha_n = \alpha_{n-1} + \alpha_{n-1/2}$ and solve α-NTE to find new k_n and ϕ_n.
4. Repeat steps 2 and 3 until $k_n = 1$. α_n will be desired eigenvalue.