

# Computing the Alpha-Eigenvalue Using Nonlinear Solvers

## Information

Authors: Erin D. Fichtl and James S. Warsa

Organization: Los Alamos National Laboratory, Computational Physics Group

## Introduction

$\alpha$ -eigenvalue formulation of the multigroup (MG) neutron transport equation (NTE) formed by assuming exponential time-behavior of flux

$$\psi_{g,n}(\mathbf{r}, t) = e^{\alpha t} \psi_{g,n}^{\alpha}(\mathbf{r})$$

Plugging in the flux to the MG NTE:

$$\left( \frac{\alpha}{v_g} + \hat{\Omega}_n \cdot \nabla + \sigma_{t,g}(\mathbf{r}) \right) \psi_{g,n}^{\alpha} = \sum_{g'} \sigma_{s,g' \rightarrow g}(\mathbf{r}) \phi_{g'}^{\alpha} + \sum_{g'} \bar{\nu} \sigma_{f,g' \rightarrow g}(\mathbf{r}) \phi_{g'}^{\alpha}$$

Dominant mode corresponds to algebraically largest eigenvalue.

## Methods

### k-eigenvalue method

Traditionally computed using multiple k-eigenvalue solutions.

Limitation: If  $\alpha$  is negative enough, effective total cross section becomes negative in some groups:

$$\tilde{\sigma}_{t,g} = \left( \frac{\alpha}{v_g} + \sigma_{t,g} \right) < 0.$$

Negative XS cause instabilities in transport sweeps.

### Reformulation as Standard Eigenvalue Problem

Rewrite discretized form of  $\alpha$ -NTE:

$$(\alpha \mathbf{L}_{\alpha} + \mathbf{L}) \psi^{\alpha} = \mathbf{M}(\mathbf{S} + \mathbf{F}) \mathbf{D} \psi^{\alpha}$$

as

$$\alpha \psi^{\alpha} = \mathbf{L}_{\alpha}^{-1} [\mathbf{M}(\mathbf{S} + \mathbf{F}) \mathbf{D} - \mathbf{L}] \psi^{\alpha}$$

and use standard eigensolvers for subcritical systems ( $\alpha < 0$ ) for which dominant  $\alpha$  is smallest in magnitude. For supercritical systems, need correction  $\frac{\beta}{v} \psi^{\alpha}$  that is subtracted from both sides:

$$\alpha \psi^{\alpha} - \frac{\beta}{v} \psi^{\alpha} = \mathbf{L}_{\alpha}^{-1} [\mathbf{M}(\mathbf{S} + \mathbf{F}) \mathbf{D} - \mathbf{L}] \psi^{\alpha} - \frac{\beta}{v} \psi^{\alpha}$$

Effective eigenvalues,  $(\alpha - \beta)$ , are all negative though method requires prior knowledge of eigenvalue to determine correction term. Also problematic since codes usually apply  $\mathbf{L}^{-1}$  instead of  $\mathbf{L}$  directly.

### Recast into Nonlinear Iteration Problem

Eigenvalue problems can be thought of nonlinear problems since they involve product of two unknowns, the eigenvalue and eigenvector.

For  $\alpha$ -NTE, unknown is vector  $\mathbf{x} = \begin{bmatrix} \mathbf{y} \\ \alpha \end{bmatrix}$

Fixed-point iteration:

$$\begin{aligned} \mathbf{y}^{\ell+1} &= \mathbf{P}(\alpha^{(\ell)})\mathbf{y}^{\ell} \\ \alpha^{\ell+1} &= \alpha^{(\ell)} + a(\mathbf{x}^{\ell}) \end{aligned}$$

Can be rewritten as  $\mathbf{x}^{(\ell+1)} = \mathbf{x}^{(\ell)} - \mathcal{F}(\mathbf{x}^{(\ell)})$  where  $\mathcal{F}$  is a nonlinear residual given by

$$\mathcal{F}(\mathbf{x}) = \begin{bmatrix} \mathbf{y} - \mathbf{P}(\alpha)\mathbf{y} \\ -a(\mathbf{x}) \end{bmatrix}$$

$\mathbf{y}$  represents either  $\phi^\alpha$  or  $\psi^\alpha$  depending on the formulation and  $a$  is the eigenvalue update.