Computing the Alpha-Eigenvalue Using Nonlinear Solvers

Information

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Introduction

α-eigenvalue formulation of the multigroup (MG) neutron transport equation (NTE) formed by assuming exponential time-behavior of flux

$$\psi_{g,n}(\mathbf{r},t) = e^{\alpha t}\psi^{\alpha}_{g,n}(\mathbf{r})$$

Plugging in the flux to the MG NTE:

$$\left(\frac{\alpha}{v_g} + \hat{\Omega}_n \cdot \nabla + \sigma_{t,g}(\mathbf{r})\right)\psi^{\alpha}_{g,n} = \sum_{g'} \sigma_{s,g'\to g}(\mathbf{r})\phi^{\alpha}_{g'} + \sum_{g'} \bar{\nu}\sigma_{f,g'\to g}(\mathbf{r})\phi^{\alpha}_{g'}$$

Dominant mode corresponds to algebraically largest eigenvalue.

Methods

k-eigenvalue method

Traditionally computed using multiple k-eigenvalue solutions.
Limitation: If α is negative enough, effective total cross section becomes negative in some groups:

$$\tilde{\sigma}_{t,g} = \left(\frac{\alpha}{v_g} + \sigma_{t,g}\right) < 0.$$

Negative XS cause instabilities in transport sweeps.

Reformulation as Standard Eigenvalue Problem

Rewrite discretized form of α-NTE:

$$(\alpha\mathbf{L}_{\alpha} + \mathbf{L})\psi^{\alpha} = \mathbf{M}(\mathbf{S}+\mathbf{F})\mathbf{D}\psi^{\alpha}$$

as

$$\alpha\psi^{\alpha} = \mathbf{L}_{\alpha}^{-1}[\mathbf{M}(\mathbf{S}+\mathbf{F})\mathbf{D} - \mathbf{L}]\psi^{\alpha}$$

and use standard eigensolvers for subcritical systems ($\alpha < 0$) for which dominant α is smallest in magnitude. For supercritical systems, need correction $\frac{\beta}{v}\psi^{\alpha}$ that is subtracted from both sides:

$$\alpha\psi^{\alpha} - \frac{\beta}{v}\psi^{\alpha} = \mathbf{L}_{\alpha}^{-1}[\mathbf{M}(\mathbf{S}+\mathbf{F})\mathbf{D} - \mathbf{L}]\psi^{\alpha} - \frac{\beta}{v}\psi^{\alpha}$$

Effective eigenvalues, $(\alpha - \beta)$, are all negative though method requires prior knowledge of eigenvalue to determine correction term. Also problematic since codes usually apply $\mathbf{L}^{-1}$ instead of $\mathbf{L}$ directly.

Recast into Nonlinear Iteration Problem

Eigenvalue problems can be thought of nonlinear problems since they involve product of two unknowns, the eigenvalue and eigenvector.

For α-NTE, unknown is vector $\mathbf{x} = \begin{bmatrix} \mathbf{y} \\ \alpha \end{bmatrix}$

Fixed-point iteration:

$$\mathbf{y}^{\ell+1} = \mathbf{P}(\alpha^{(\ell)})\mathbf{y}^{\ell}$$
$$\alpha^{\ell+1} = \alpha^{(\ell)} + a(\mathbf{x}^{\ell})$$

Can be rewritten as $\mathbf{x}^{(\ell+1)} = \mathbf{x}^{(\ell)} - \mathcal{F}(\mathbf{x}^{(\ell)})$ where $\mathcal{F}$ is a nonlinear residual given by

$$\mathcal{F}(\mathbf{x}) = \begin{bmatrix} \mathbf{y} - \mathbf{P}(\alpha)\mathbf{y} \\ -a(\mathbf{x}) \end{bmatrix}$$

$\mathbf{y}$ represents either ϕ^{α} or ψ^{α} depending on the formulation and a is the eigenvalue update.