Three-dimensional Transport Calculation of Multiple Alpha Modes in Subcritical Systems

Information

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Introduction

Reactivity measurements usually assume flux to be in fundamental mode. However, knowledge of higher modes could be used to remove possible contamination of higher modes that are present and contribute to the flux behavior. Paper introduces computational scheme to calculate moderate dominant alpha-eigenspaces of 3D multigroup S_n transport problems discretized by the finite element method. Focuses on prompt alpha-eigenvalue problem though methods also apply to the delayed alpha-eigenvalue problem.

Method

Transport equation on spatial domain V with scatter and fission terms expanded in spherical harmonic series and angular flux discretized using method of discrete ordinates:

$$\frac{1}{v_g}\frac{\partial \varphi_{d,g}(\mathbf{r},t)}{\partial t} = -\mathbf{\Omega}_d \cdot \nabla \varphi_{d,g}(\mathbf{r},t) - \Sigma_g^t \varphi_{d,g}(\mathbf{r},t) + \sum_{g'}\sum_{l=0}^{L}\Sigma_{g'\to g}^{s,l}\sum_{m=-l}^{l} Y^{l,m}(\mathbf{\Omega}_d)\phi_{g'}^{l,m}(\mathbf{r},t) + \chi_g \sum_{g'} \nu\Sigma_{g'}^{f} Y^{0,0}(\mathbf{\Omega}_d)\phi_{g'}^{0,0}(\mathbf{r},t)$$

where d is index for direction. Flux moments calculated by angular quadrature:

$$\phi_g^{l,m}(\mathbf{r},t) = \sum_d w_d Y^{l,m}(\mathbf{\Omega}_d)\varphi_{d,g}(\mathbf{r},t).$$

Boundary conditions (vacuum, reflected):

$$\varphi_{d,g}(\mathbf{r},t) = 0, \quad \mathbf{r} \in \partial V_B, \quad \mathbf{\Omega}_d \cdot \mathbf{n} < 0$$

$$\varphi_{d,g}(\mathbf{r},t) = \varphi_{d',g}(\mathbf{r},t), \quad \mathbf{r} \in \partial V_R, \quad \mathbf{\Omega}_d \cdot \mathbf{n} < 0$$

Substitute exponential time dependency of the flux into the NTE:

$$\varphi_{d,g}(\mathbf{r},t) = \varphi_{d,g}(\mathbf{r})e^{\alpha t}$$

to obtain alpha-eigenvalue equation.

Spatial Discretization

Discontinuous finite element method. Domain divided into spatial elements $V \equiv \cup_e V_e$ where $V_{e1} \cap V_{e2} \equiv , \forall e_1 \neq e_2$ where e is the element index. Each element is bounded by ∂V_e where the element boundary consists of $\partial V_e \equiv \cup_j V_e^j, \partial V_e^{j_1} \cap \partial V_e^{j_2} \equiv , \forall j_i \neq j_2$ faces.

Sobolev space $\mathbf{S}^{V,P}$ introduced to describe flux in group g and direction d. Consists of discontinuous polynomial functions of order P at each element:

$$\mathbf{S}^{V,P} = \{f \in L_2(V) : f|_{\forall V_e} \in \mathbb{P}_P\}$$

Review your finite elements because this is rough....and I'm not sure where it's going....