Homework 4-NE 255

Information

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1 Problem 1

1.1 (a)

For $\mu > 0$, we sweep along the space from left to right using the following equation:

$$\psi_{in} = \left(1 + \frac{\sigma_i \Delta_i}{2|\mu_n|}\right)^{-1} \left(\psi_{i-1/2,n} + \frac{\Delta_i q_{in}}{2|\mu_n|}\right)$$

where we determine $\psi_{i+1/2,n}$ using the relation

$$\psi_{i+1/2,n} = 2\psi_{in} - \psi_{i-1/2,n}.$$

We sweep from left to right and need to specify the value of $\psi_{1/2,n}$ to begin the sweep.

1.2 (b)

For $\mu < 0$, we sweep along the space from right to left using the following equation:

$$\psi_{in} = \left(1 + \frac{\sigma_i \Delta_i}{2|\mu_n|}\right)^{-1} \left(\psi_{i+1/2,n} + \frac{\Delta_i q_{in}}{2|\mu_n|}\right)$$

where we determine $\psi_{i-1/2,n}$ using the relation

$$\psi_{i-1/2,n} = 2\psi_{in} - \psi_{i+1/2,n}.$$

We sweep right to left and need to specify the value of $\psi_{I+1/2,n}$ to begin the sweep where I is the node on the right boundary.

1.3 (c)

For a reflecting boundary condition on the right boundary, we begin our sweep on the left boundary and sweep across the domain until we reach the right boundary. When we reach the boundary, we use the positive angle (n) angular flux value as the starting value for the negative angle (N+1-n) sweep:

$$\psi_{I+1/2,N+1-n} = \psi_{I+1/2,n}.$$

1.4 (d)

To generate flux moments, you need to store the scalar flux during the sweep.

2 Problem 2

2.1 (a)

The expression for the flux at some location x' in terms of the flux at location x is

$$\psi_a(x') = \psi_a(x) \exp\left[-\frac{\Sigma_t(x'-x)}{\mu_a}\right], \quad x' > x.$$

2.2 (b)

Imposing a Cartesian grid with mesh index i and mesh spacing $\Delta_i = x_{i+1/2} - x_{i-1/2}$, we obtain the expression for $\psi_{a,i+1/2}$

$$\psi_{a,i+1/2} = e^{-2h}\psi_{a,i-1/2},$$

where $h \equiv \frac{\Sigma_t \Delta_i}{2|\mu_a|}$.

2.3 (c)

To obtain another expression for $\psi_{a,i+1/2}$ in terms of $\psi_{a,i-1/2}$ and h, we use the two following relationships

$$\mu_n \left[\psi_{a,i+1/2} - \psi_{a,i-1/2}\right] + \Sigma_i \Delta_i \psi_{in} = 0$$

and

$$\psi_{in} = \frac{1}{2}\left(\psi_{a,i+1/2} + \psi_{a,i-1/2}\right).$$

We eliminate ψ_{in} in the first equation and obtain the following expression

$$\psi_{a,i+1/2}\left(\mu_a + \frac{\Sigma_t\Delta_i}{2}\right) = \psi_{a,i-1/2}\left(\mu_a - \frac{\Sigma_t\Delta_i}{2}\right).$$

Solving for $\psi_{a,i+1/2}$ and using our previous definition of h, we obtain

$$\psi_{a,i+1/2} = \psi_{a,i-1/2}\left(\frac{1-h}{1+h}\right).$$

2.4 (d)

We expand the expression in Part (b) in a power series

$$\psi_{a,i+1/2} = e^{-2h}\psi_{a,i-1/2} = \psi_{a,i-1/2}\left(1 - 2h + 2h^2 - O(h^3)\right).$$

We also expand the expression in Part (c) in a power series to obtain

$$\psi_{a,i+1/2} = \psi_{a,i-1/2}\left(\frac{1-h}{1+h}\right) = \psi_{a,i-1/2}\left(1 - 2h + 2h^2 - O(h^3)\right).$$

Comparing the expressions, we see that they are equivalent through order h^2. This implies that the relationship is at least $O(h^2)$ accurate.

2.5 (e)

Looking at the expression

$$\psi_{a,i+1/2} = \psi_{a,i-1/2}\left(\frac{1-h}{1+h}\right)$$

we wish to avoid negative fluxes. To avoid negative flux values, the following must be true

$$\left(\frac{1-h}{1+h}\right) > 0 \implies (1-h) > 0 \therefore h < 1.$$

For the smallest μ_a and a specific Σ_t, the mesh spacing must assure that the following is true

$$\Sigma_t \Delta_i < 2|\mu_a|.$$

Therefore, the mesh spacing must be a factor of $2|\mu_a|$ less than the mean free path in order to assure there will be no negative flux values.

3 Problem 3

A 1D discrete ordinates code was written and can be seen at the end of this report. A sourceless, nonscattering problem was solved and the scalar flux plotted for various mesh spacing sizes. As seen in Figure 3.1, as we increase the mesh spacing, we can no longer guarantee that we will have positive scalar flux values. Using the expression from before with $\Sigma_t = 1$ and $|\mu_a| = 0.1$, we obtain the following condition

$$\Sigma_t \Delta_i < 2|\mu_a| \rightarrow \Delta_i < 0.2.$$

We see that for $\Delta_i = 0.2$ and $\Delta_i = 0.4$, the mesh spacing no longer guarantees positive fluxes. In the case of mesh spacing of 0.4, we begin to see negative flux values as expected from our previous expression.

3.1 (a)

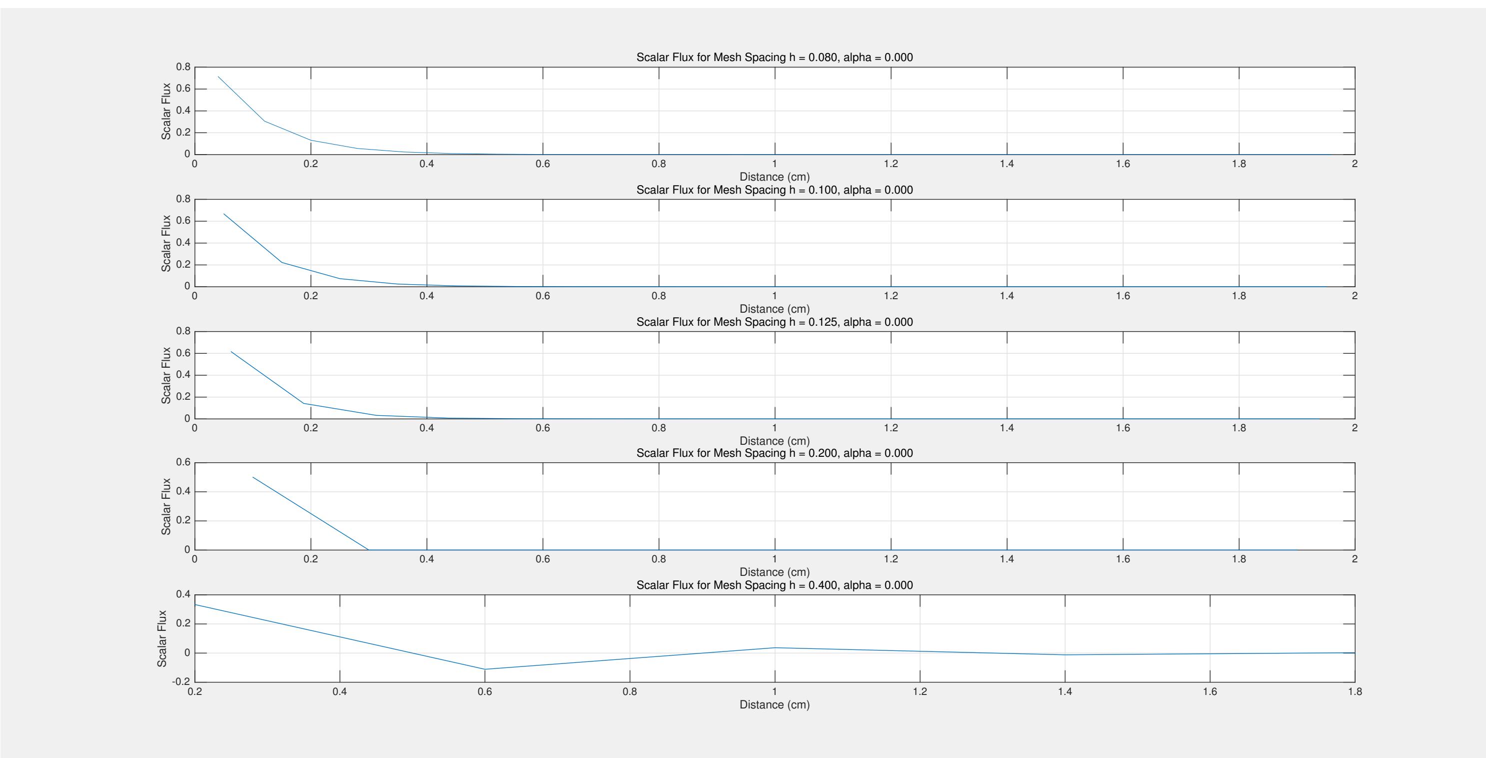

Figure 3.1: Scalar Flux for Various Mesh Size ($\alpha = 0$)

3.2 (b)

By using different α values, we can improve the solution by weighing incoming or outgoing angular flux values differently. However, as we increase mesh spacing, we once again run into negative flux values once we violate our condition. By weighting the incoming/outgoing fluxes differently, we can make better estimates for the flux in a cell when we know something about the problem. In this particular case, the left to right angular flux is the major part of the scalar flux and we can weight accordingly.

3.3 (c)

We add a source and vary the problem parameters as follows: $\alpha = [-0.5, 0, 0.5], \mu = \pm[0.2, 0.7], \Sigma_t = 1.0, \Sigma_s = 0.5, q_e(x) = 1.0$. We plot the solution in Figures 3.7, 3.8, and 3.9. For different α, we change the inflection of the solution. This is expected as we change the weighting of the incoming and outgoing angular fluxes.

3.4 (d)

Increasing the scattering cross section to $\Sigma_s = 0.9$ increases the scalar flux in the system since the probability of absorption decreases substantially. With increased scattering of neutron and the reflective boundary condition on the right, we require a larger number of iterations to converge the scalar flux. The plot of the scalar flux can be seen in Figure 3.10.

3.5 Problem 3 MATLAB Functions and Scripts

```
function [xi,scalar_flux] = OneDDiscreteOrdinates(mu,wi,h,alpha,L,sigt,sigs,qex)

xii = 0:h:L;
xi = h/2:h:L;

N = length(xi);

half_angular_flux = zeros(length(mu),length(xii));
angular_flux = zeros(length(mu),length(xi));

for iter = 1:1000

    scalar_flux_old = Calculate_ScalarFlux(xi,wi,angular_flux);

    for j = 1:length(mu)

        if ( mu(j) > 0 )

            %Set boundary condition on angular flux
            half_angular_flux(j,1) = 2.0;

            for k = 1:length(xi)

                qscat = 0;

                for l = 1:length(wi)

                    qscat = qscat + (1/2)*sigs*wi(l)*angular_flux(l,k);

                end

                q = qscat + qex;

                angular_flux(j,k) = (2*abs(mu(j))/(1+alpha) + sigt*h)^(-1)*(h*q + ...
                    abs(mu(j))*half_angular_flux(j,k)*(1 + (1-alpha)/(1+alpha)));

                half_angular_flux(j,k+1) = (2/(1+alpha))*angular_flux(j,k) - ...
                    ((1-alpha)/(1+alpha))*half_angular_flux(j,k);

            end
```

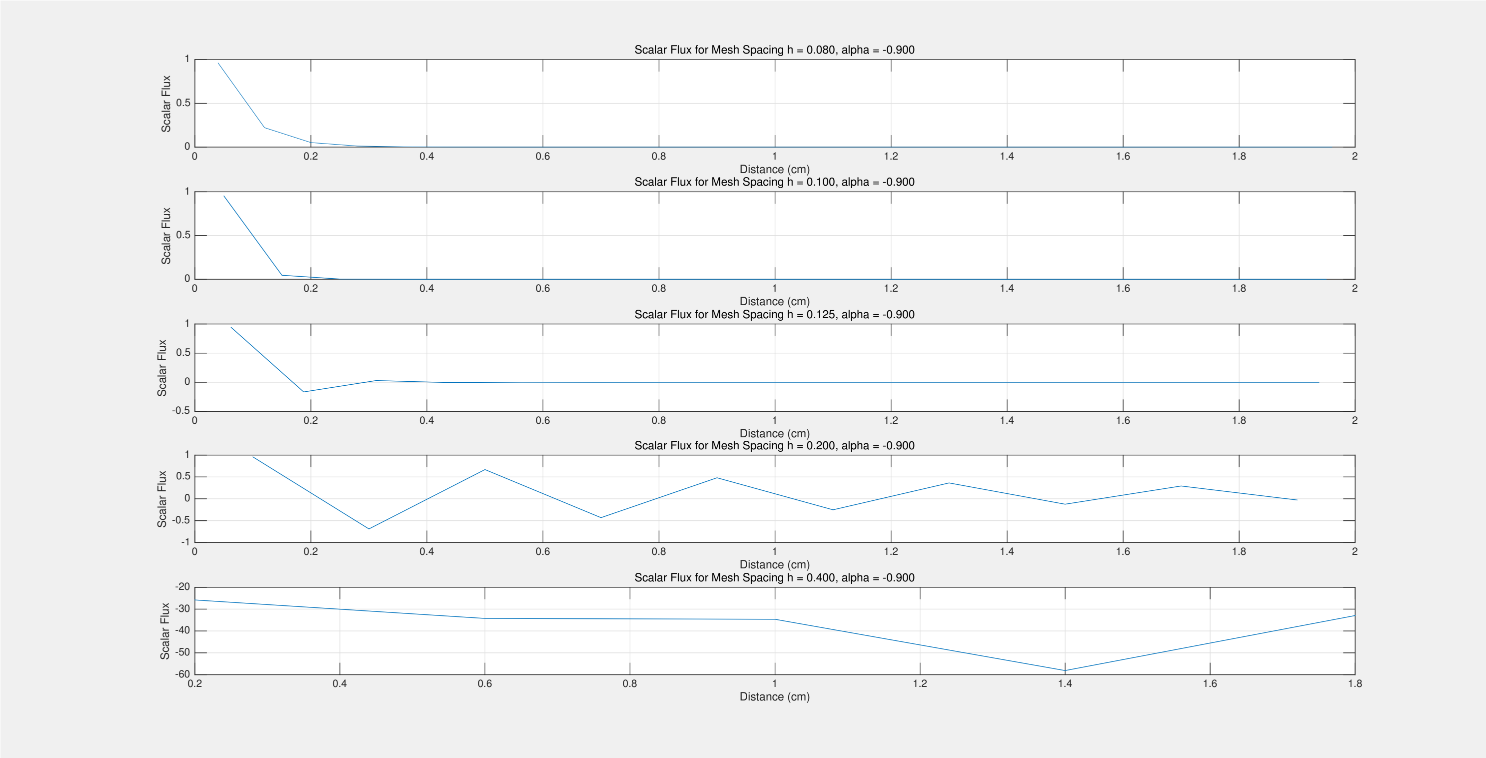

Figure 3.2: Scalar Flux for Various Mesh Size ($\alpha = -0.9$)

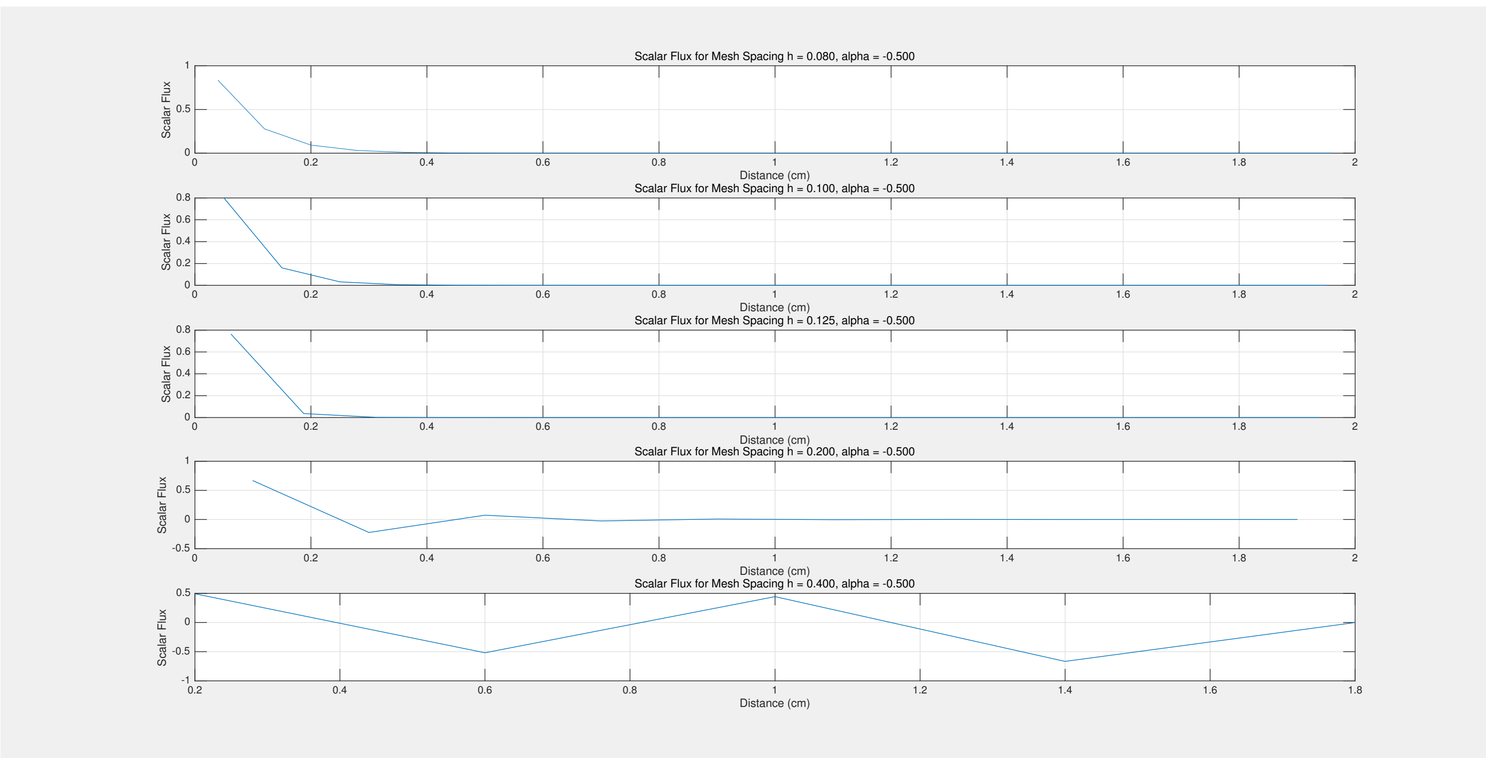

Figure 3.3: Scalar Flux for Various Mesh Size ($\alpha = -0.5$)

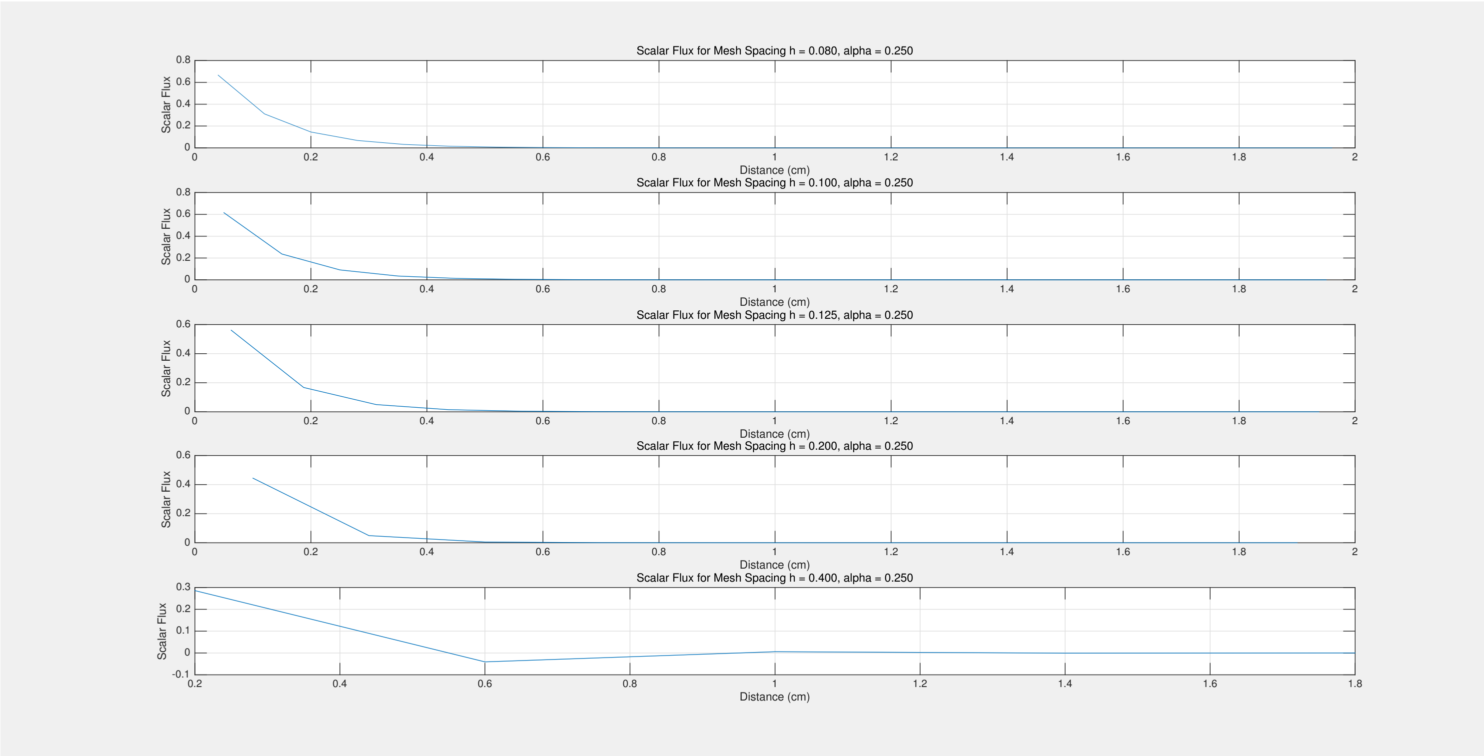

Figure 3.4: Scalar Flux for Various Mesh Size ($\alpha = 0.25$)

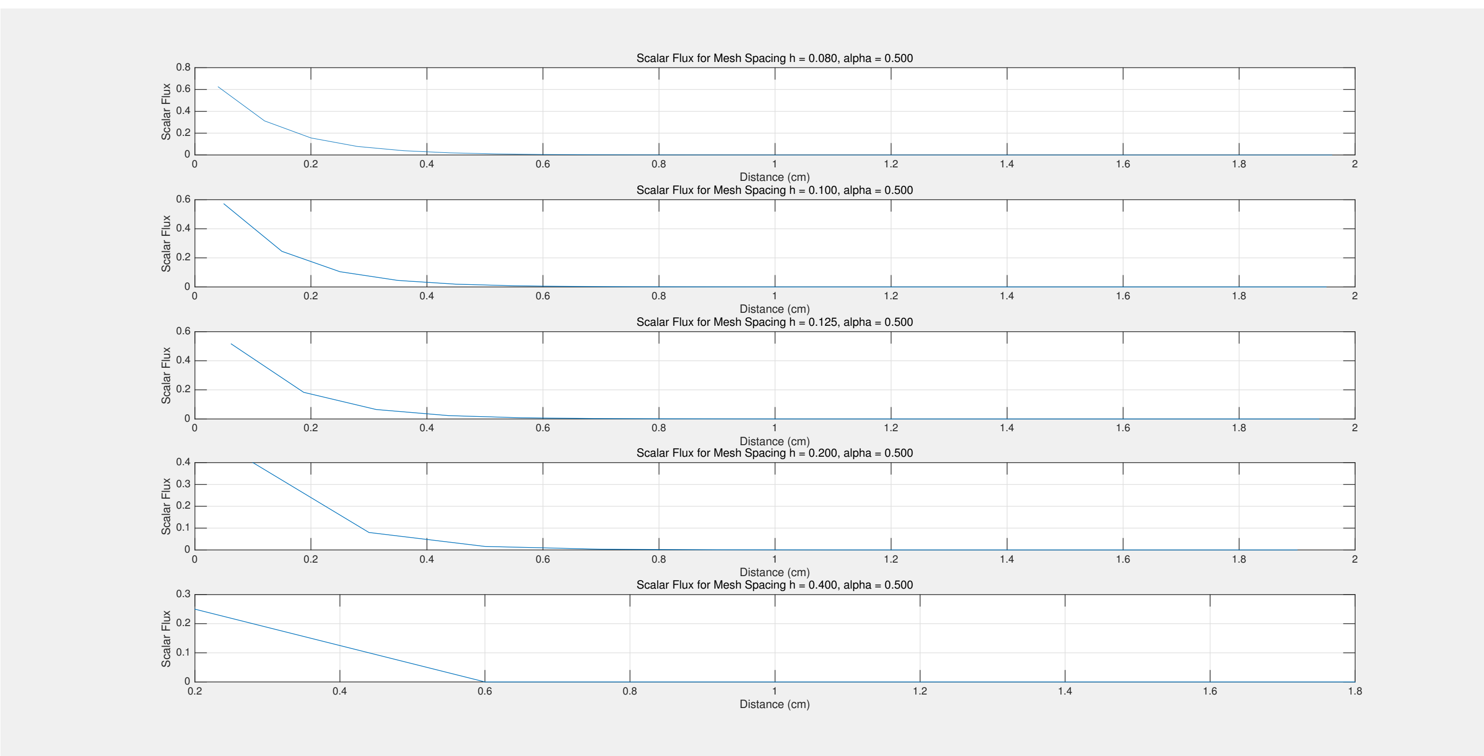

Figure 3.5: Scalar Flux for Various Mesh Size ($\alpha = 0.50$)

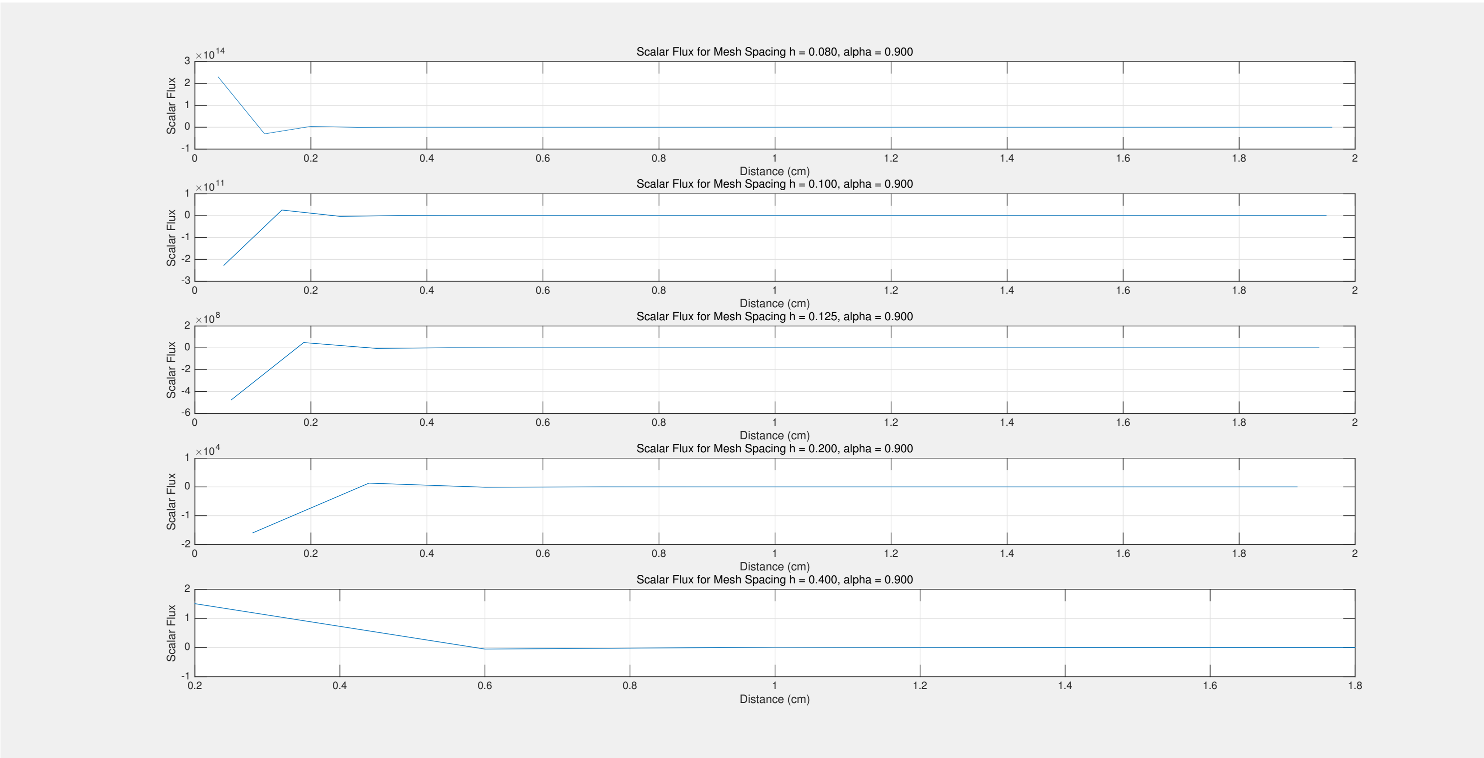

Figure 3.6: Scalar Flux for Various Mesh Size ($\alpha = 0.90$)

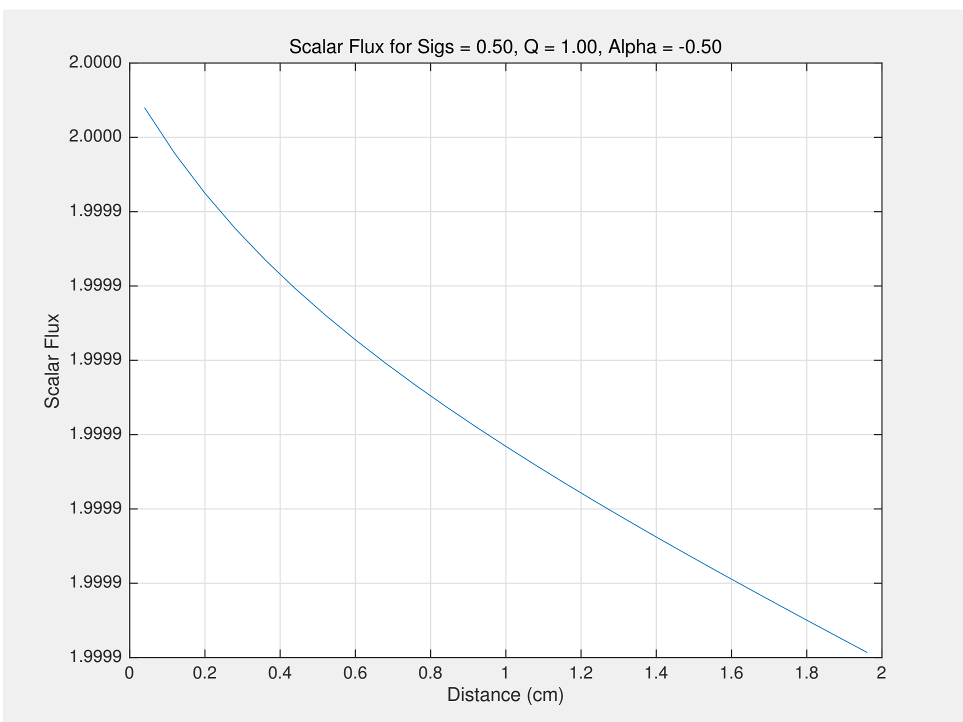

Figure 3.7: Scalar Flux ($\alpha = -0.50$)

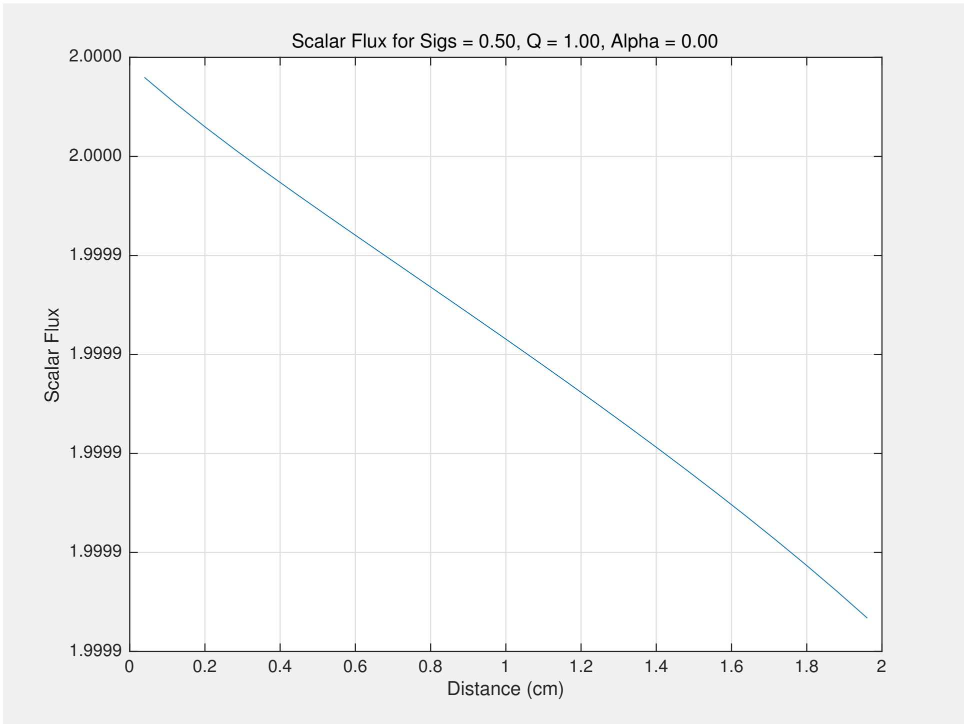

Figure 3.8: Scalar Flux ($\alpha = 0.00$)

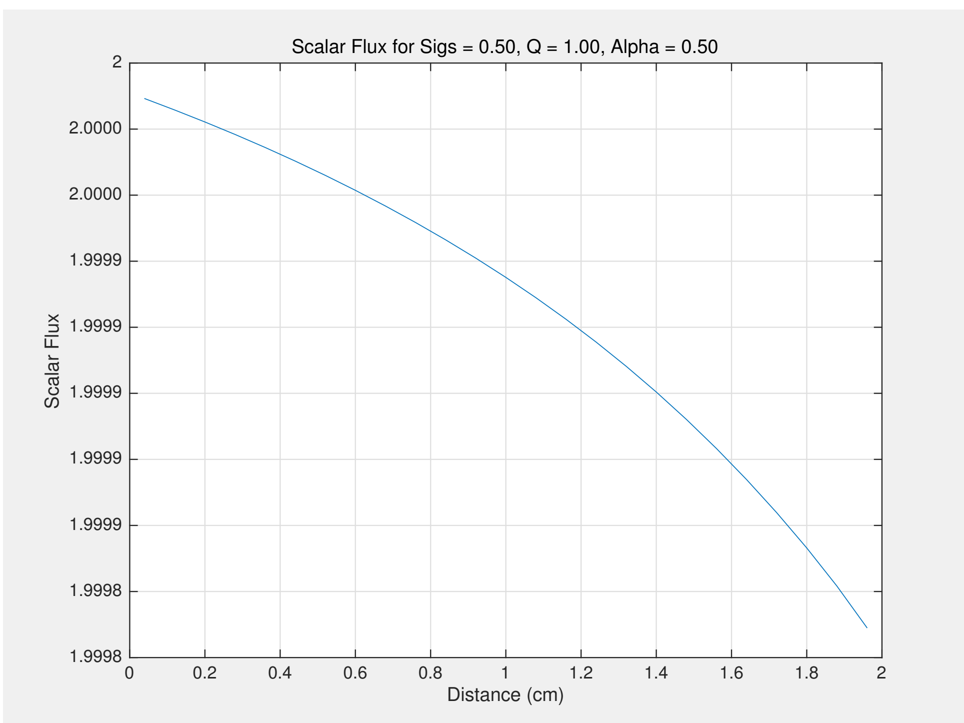

Figure 3.9: Scalar Flux ($\alpha = 0.50$)

```

        elseif ( mu(j) < 0 )

            %Set boundary condition on angular flux
            half_angular_flux(j,N+1) = half_angular_flux(length(wi)+1-j,N+1);

            for k = length(xi):-1:1

                qscat = 0;

                for l = 1:length(wi)

                    qscat = qscat + (1/2)*sigs*wi(l)*angular_flux(l,k);

                end

                q = qscat + qex;

                angular_flux(j,k) = (2*abs(mu(j))/(1+alpha) + sigt*h)^(-1)*(h*q + ...
                    abs(mu(j))*half_angular_flux(j,k+1)*(1 + (1-alpha)/(1+alpha)));

                half_angular_flux(j,k) = (2/(1-alpha))*angular_flux(j,k) - ...
                    ((1+alpha)/(1-alpha))*half_angular_flux(j,k+1);

            end

        end

    end

    scalar_flux = Calculate_ScalarFlux(xi,wi,angular_flux);

    norm_flux = sqrt(sum((scalar_flux - scalar_flux_old).^2))

    if ( norm_flux < 1e-3)

        break

    end

end

return
```

```
%Problem 3
%NE 255
%1D Discrete Ordinates Code

clc, clear, clf

L = 2.0;

alpha = [-0.5 0 0.5];
mu = [0.7 0.2 -0.2 -0.7];
wi = [0.5 0.5 0.5 0.5];
sigt = 1.0;
sigs = 0.9;
qex = 1.0;

h = 0.08;

for i = 1:length(alpha)

    [xi,scalar_flux] = OneDDiscreteOrdinates(mu,wi,h,alpha(i),L,sigt,sigs,qex);
    plot(xi,scalar_flux)
    grid on
```

```
    xlabel('Distance (cm)');
    ylabel('Scalar Flux');
    titl = sprintf('Scalar Flux for Sigs = %.2f, Q = %.2f, Alpha = %.2f',sigs,qex,alpha(i));
    title(titl);
    fid = sprintf('scattering_alpha%i',i);
    export_fig(fid,'-pdf','-nocrop')

end
```

4 Problem 4

Starting from the following general system of equations

$$\frac{\mu_a}{h_i}(\psi^g_{a,i+1/2} - \psi^g_{a,i-1/2}) + \Sigma^g_{t,i}\psi^g_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{gg'}_{s,i}(a' \to a)\psi^{g'}_{a,i} + \frac{\chi_g}{2}\sum_{g'=1}^{G} \nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^g_i$$

we assume the following: neutrons only downscatter from fast groups (1 and 2) to thermal groups (3, 4, 5), thermal groups can upscatter into other thermal groups and downscatter, and there is an external and fission source present. We write down five coupled equations for a five group problem:

- Group 1

$$\frac{\mu_a}{h_i}(\psi^1_{a,i+1/2} - \psi^1_{a,i-1/2}) + \Sigma^1_{t,i}\psi^1_{a,i} = 2\pi \sum_{a=1}^{N} w_a \Sigma^1_{s,i}\psi^{g'}_{a,i} + \frac{\chi_1}{2}\sum_{g'=1}^{5} \nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^1_i$$

- Group 2

$$\frac{\mu_a}{h_i}(\psi^2_{a,i+1/2} - \psi^2_{a,i-1/2}) + \Sigma^2_{t,i}\psi^2_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{2} \Sigma^{2g'}_{s,i}(a' \to a)\psi^{g'}_{a,i} + \frac{\chi_2}{2}\sum_{g'=1}^{5} \nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^2_i$$

- Group 3

$$\frac{\mu_a}{h_i}(\psi^3_{a,i+1/2} - \psi^3_{a,i-1/2}) + \Sigma^3_{t,i}\psi^3_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{5} \Sigma^{3g'}_{s,i}(a' \to a)\psi^{g'}_{a,i} + \frac{\chi_3}{2}\sum_{g'=1}^{5} \nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^3_i$$

- Group 4

$$\frac{\mu_a}{h_i}(\psi^4_{a,i+1/2} - \psi^4_{a,i-1/2}) + \Sigma^4_{t,i}\psi^4_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{5} \Sigma^{4g'}_{s,i}(a' \to a)\psi^{g'}_{a,i} + \frac{\chi_4}{2}\sum_{g'=1}^{5} \nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^4_i$$

- Group 5

$$\frac{\mu_a}{h_i}(\psi^5_{a,i+1/2} - \psi^5_{a,i-1/2}) + \Sigma^5_{t,i}\psi^5_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{5} \Sigma^{5g'}_{s,i}(a' \to a)\psi^{g'}_{a,i} + \frac{\chi_5}{2}\sum_{g'=1}^{5} \nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^5_i$$

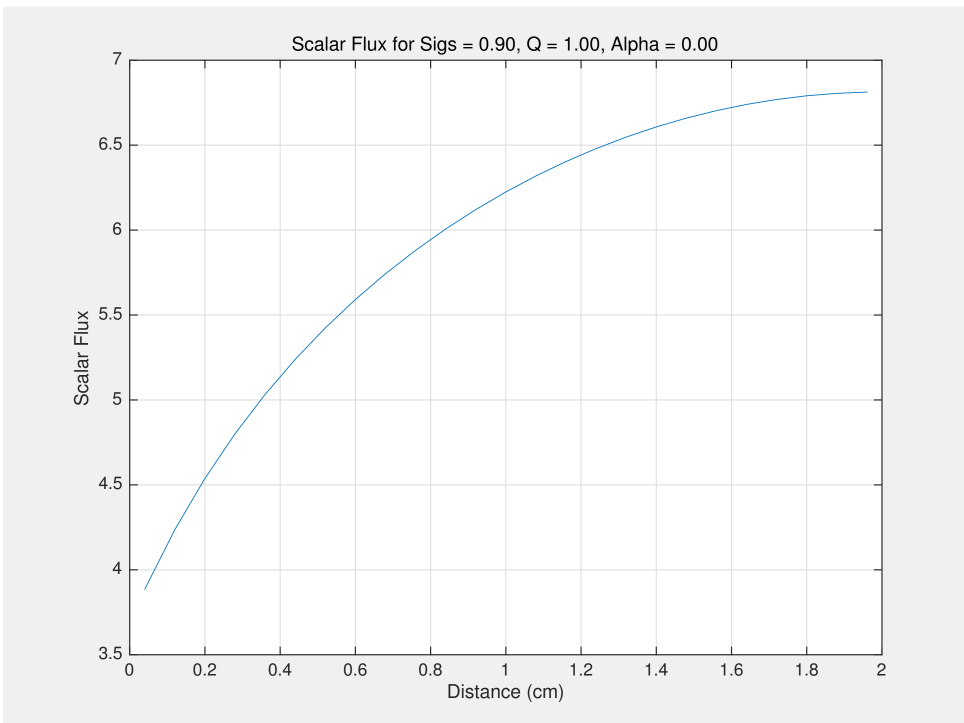

Figure 3.10: Scalar Flux for $\Sigma_s = 0.9$