Computational Modeling
Project 1

Maroua Ouahbi
Constructor University

INTRODUCTION

The project simulates the falling of an object under the influence of earth's gravitational field, with the equation:

$$F = G\frac{mM}{(R+h)^2} \tag{1}$$

where F is the gravitational force, G the gravitational constant ($G = 6.67430\times10^{-11}$N m^2kg^{-2}), m the mass of the object, M mass of earth ($M = 5.972\times10^{24}\ kg$), R the radius of earth (R =6.371km).

The simulation recreates how the scenario would play out in real life conditions. Which means that along with the gravitational force acting on the object there's also a drag force that is being accounted for. The formula used for the drag force being:

$$F_{drag} = kv^2 \tag{2}$$

where k is the damping coefficient.
As the height increases the force of the gravity decreases considering the influence of the gravitational field decreases the further away the object is. All this behavior has been analyzed through the python program.

METHODS

- Euler Method: At each time step we update the velocity and position of the body using gravitational acceleration and drag force to compute the motion. where e.g. the height dependent gravity g(h)=g0(R/R+h)² with g0 as gravitational acceleration on sea level andRRis the radius of earth. The drag force is used as a negative acceleration term.
- Euler-Richardson Method: This method introduces improved accuracy over the Euler method by using this intermediate velocity and position values to find better approximations of the motion at every time step. This is a technique used to simulate free fall with just gravity and no drag force.

CODE VALIDATION

To check if the value of gravitational acceleration is accurate, the following parameters were set: Mass (kg) = 1 and the Damping coefficient (kg/s) = 1. This ensures that the calculated value matches Earth's gravitational force g0. Additionally, to verify if the acceleration is correct, the Damping coefficient (kg/s) is set to 0, eliminating friction. Under these conditions, the acceleration should be equal to the value of g, Earth's gravitational acceleration, which it was.

RESULTS

Euler Method: Simulating a body of mass $m=1.5$ kg, falling from a height of 10,000 m with a drag coefficient of $k=2\times10-4$ kg/m, the height and velocity were plotted over time. The results showed a rapid initial increase in velocity, and also it gets closer to the terminal velocity due to the drag force.

Euler-Richardson Method: This method was tested for constant gravitational acceleration $g=9.81$ m/s^2 and without drag. Compared to the Euler method, the Euler-Richardson method required fewer iterations to give stable results, which makes it a lot more accurate.

CONCLUSION

After using both the Euler and Euler-Richardson methods, they have both effectively simulated the behavior of a falling body while following physics rules. The Euler-Richardson method allows the use of the larger time steps and allows stability when the drag is not in the picture. Verification of gravitational acceleration and acceleration without drag confirmed the correctness of the code. And the simulation outcomes are consistent with theoretical predictions.