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Advanced Programming

Algebraic Data Types

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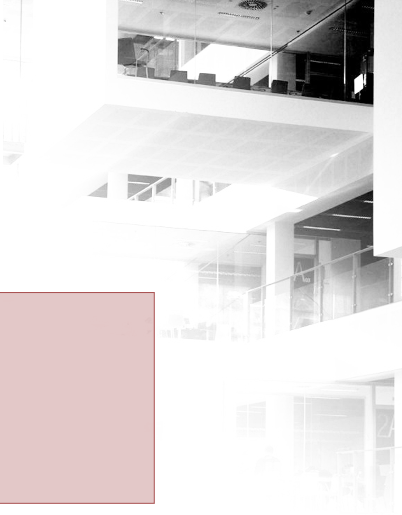
SOFTWARE QUALITY RESEARCH



- Algebraic Data Types
- Functional lists and trees
- Fold functions
- Style guidelines for functional programming in ADPRO

AGENDA

Algebraic Data Types (ADTs)

Def. Algebraic Data Type

A type generated by one or more constructors, each taking zero or more arguments.

The sets of objects generated by each constructor are **summed** (unioned), each constructor can be seen as a representation of a Cartesian **product** (tuple) of its arguments; thus the name **algebraic**.

Example: lists

sealed: extensible in the same file only

`Nothing`: **subtype of any type**

```scala
sealed trait List[+A]
case object Nil extends List[Nothing]
case class Cons[+A] (head: A, tail: List[A]) extends List[A]
```

operations on lists

companion object of `List[+A]`

pattern matching against case constructors

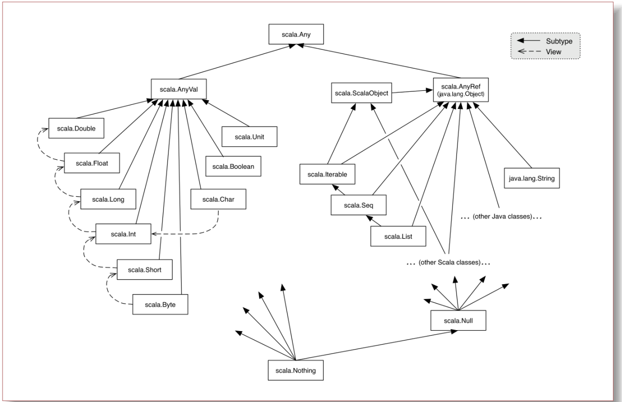
overloading function application for the object

variadic function

```scala
object List {
  def sum(ints: List[Int]): Int =
    ints match { case Nil => 0
                 case Cons (x,xs) => x + sum(xs) }
  def apply[A] (as: A*): List[A] =
    if (as.isEmpty) Nil
    else Cons (as.head, apply(as.tail: _*))
}
```

Lists are covariant

All share the same tail!



For any type `A` we have that

`Nil <:List[Nothing] <:List[A]`

```scala
sealed trait List[+A]
case object Nil extends List[Nothing]
case class Cons[+A] (head: A, tail: List[A]) extends List[A]
```

Poll: How is your recursion?

```scala
def f (a: List[Int]): Int = a match {
  case Nil => 0
  case Cons (h, t) => h + f (t)
}
```

What is `f (List (42, -1, 1, -1, 1, -1))` ?

Function Values

- In functional programing **functions are values**
- Functions can be **passed to other functions**, composed, etc.
- Functions operating on function values are **higher order** (HOFs)

```scala
def map (a: List[Int]) (f: Int => Int): List[Int] =
  a match { case Nil => Nil
            case Cons (h, tail) => Cons (f (h), map (tail) (f)) }
```

A functional (pure) example

```scala
val mixed = List (-1, 2, -3, 4)
map (mixed) (abs _)
```

```scala
map (mixed) ((factorial _) compose (abs _))
```

see method abs as a function value

or type explicitly:

`(abs: Int =>Int)`

An imperative (impure) example

```scala
val mixed = Array (-1, 2, -3, 4)
for (i <- 0 until mixed.length)
  mixed (i) = abs (mixed (i))
```

```scala
val mixed1 = Array (-1, 2, -3, 4)
for (i <- 0 until mixed1.length)
  mixed1 (i) = factorial (abs (mixed1 (i)))
```

Parametric Polymorphism

Monomorphic functions operate on fixed types:

A monomorphic map in Scala

```
def map (a: List[Int]) (f: Int => Int): List[Int] =
  a match { case Nil => Nil
            case Cons (h, tail) => Cons (f (h), map (tail) (f)) }
```

There is nothing specific here regarding Int.

A polymorphic map in Scala

```
def map[A,B] (a: List[A]) (f: A => B): List[B] =
  a match { case Nil => Nil
            case Cons (h, tail) => Cons (f (h), map (tail) (f)) }
```

An example of use:

```
map[Int,String] (mixed_list) {
  (_.toString) compose (factorial _) compose (abs _) }
```

- A **polymorphic** function operates on values of (m)any types
- A polymorphic **type constructor** defines a parameterized family of types
- Don’t confuse with OO-polymorphism AKA "**dynamic dispatch**" (dependent on the inheritance hierarchy)

HOFs in the Standard Library

Methods of class `List[A]`, operate on `this` list, type `A` is bound in the class

`map[B] (f: A =>B): List[B]`
Translate `this` list of As into a list of Bs using `f` to convert the values

`filter (p: A =>Boolean): List[A]`
A sublist of `this` containing elements satisfying predicate p

`flatMap[B] (f: A =>List[B]): List[B]` *type slightly simplified
Apply `f` to elements of `this` and concatenate the produced lists

`take (n: Int): List[A]`
A list of first n elements of `this`.

`takeWhile (p: A =>Boolean): List[A]`
A prefix of `this` containing elements satisfying p

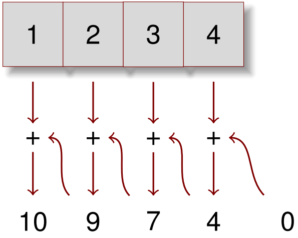
`forall (p: A =>Boolean): Boolean`
True iff p holds for all elements of `this`

`exists (p: A =>Boolean): Boolean`
True iff p holds for at least one element of `this`

More at `https://www.scala-lang.org/api/current/scala/collection/immutable/List.html`

Folds: Functional Loops

Sum of a list



What characterizes folds ?

- An **input list** `l = List(1,2,3,4)`
- An **initial value** `z = 0`
- A **binary operation** `f: (Int,Int) => Int = _ + _`
- An **iteration algorithm**

```
def foldRight[A,B] (f: (A,B) => B) (z: B) (l: List[A]): B =
  l match {
    case Cons (x,xs) => f (x, foldRight (f) (z) (xs))
    case Nil => z
  }
val l1 = List (1,2,3,4,5,6)
val sum     = foldRight[Int,Int] (_+_) (0) (l1)
val product = foldRight[Int,Int] (_*_) (1) (l1)
def map[A,B] (f: A=>B) (l: List[A])=
  foldRight[A,List[B]] ((x, z) => Cons (f (x), z)) (Nil) (l)
```

Many HOFs are special cases of folding

Preferred Programming Style in ADPRO

Always choose the best possible style for an exercise and your abilities

Condemned (fail) →	Forgivable (medium grade*) →	Enlightened (top grade)
variables <		< values
assignments <		< value bindings
return statement <		< expression value
Any/Object type <		< parametric polymorphism
loops <	tail recursion* < simple recursion < folds*	< compose dedicated HOFs
	if conditions < pattern matching*	< use dedicated API
exceptions <		< Option or Either monad

* unless asked for explicitly, or really important for memory use.

Scala: Summary

- **Basics** (objects, modules, functions, expressions, values, variables, operator overloading, infix methods, interpolated strings.)
- **Pure functions** (referential transparency, side effects)
- **Loops and recursion** (tail recursion)
- **Functions as values** (higher-order functions)
- **Parametric polymorphism** (monomorphic functions, dynamic and static dispatch)
- **Standard HOFs** in Scala's library
- **Anonymous functions** (currying, partial function application)
- **Traits** (fat interfaces, multiple inheritance, mixins)
- **Algebraic Data Types** (pattern matching, case classes)
- **Folding**

