Lazy Streams

The main file to work in this week is located in `src/main/scala/adpro/Stream.scala`. You need to scroll down in the file to find Exercise 1.

The test suite and the exercise template assume that you are using Scala standard library immutable lists (not the book lists). The test suite is weaker this week. We will develop a proper test suite in a later homework. For some exercises, the tests are missing, because it was impossible to write them without revealing the interface that you need to design.

Hand in: `src/main/scala/adpro/Stream.scala`

Exercise 1. Define functions `from` and `to` that generate streams of natural numbers above (and below) a given natural number `n`. The function `from` should create a stream producing all numbers larger than n, starting from n, increasing. The function `to` should create a stream producing all numbers smaller than n, starting from n, decreasing. In the source file, this exercise is in the very bottom, in the companion object of `Stream`.

```
def from (n: Int): Stream[Int]
def to (n: Int): Stream[Int]
```

Use `from` to create a value `naturals: Stream[Int]` representing all natural numbers in order.

Exercise 2. Write a function to convert a `Stream` to a `List`, which will force its evaluation and let you look at it in the REPL. You can convert to the regular `List` type in the standard library. You can place this and other functions that operate on a `Stream` inside the `Stream` trait. Use pattern matching.

```
def toList: List[A]
```

Test this function using the factory of streams to build finite streams and converting the to lists (to see whether they yield expected lists). Then create a few finite streams of integers using `to (n)` from the previous exercise, and convert them to lists.[1] You will need to import the `Stream` definition as in: `import adpro.Stream`.

Exercise 3. Write the function `take(n)` for returning the first `n` elements of a Stream, and `drop(n)` for skipping the first `n` elements of a `Stream`. Use pattern matching.

```
def take (n: Int): Stream[A]
def drop (n: Int): Stream[A]
```

For fluency, try the following test case in REPL (should terminate with no memory exceptions and very fast). Why does it terminate without exception? Answer this question as a comment in the Scala file under the same exercise number.

```
naturals.take (1000000000).drop (41).take (10).toList
```

Exercise 4. Write the function `takeWhile (p)` for returning all starting elements of a `Stream` that match the given predicate `p`. Use pattern matching to implement it.

```
def takeWhile(p: A => Boolean): Stream[A]
```

Test your implementation on the following test case:

[1] Exercise 5.1 [Chiusano, Bjarnason 2014]

`naturals.takeWhile { _ < 1000000000 }.drop (100).take (50).toList`

It should terminate very fast, with no exceptions thrown. Why?[2]

Exercise 5. Implement `forAll (p)` that checks that all elements in `this Stream` satisfy a given predicate. Terminate the traversal as soon as it encounters a non-matching value. Use recursion and pattern matching.

```
def forAll(p: A => Boolean): Boolean
```

We use the following test case for `forAll`: `naturals.forAll (_ < 0)`

If we used this one, it would be crashing: `naturals.forAll (_ >=0)`. Explain why.

Recall that `exists` has already been implemented before (in the book). Both `forAll` and `exists` are a bit strange for infinite streams; you should not use them unless you know the result; but once you know the result there is no need to use them. They are fine to use on finite streams. Why?[3]

Exercise 6. Use `foldRight` to implement `takeWhile`. Reuse the test case from Exercise 4.[4]

Exercise 7. Implement `headOption` using `foldRight`.

Exercise 8. Implement the following functions. The task involves designing their types. Implement map, filter, append, and flatMap using foldRight. The append method should be non-strict in its argument.[5]

1. `map (f)`, using an analogous signature to the one from lists
   Test case: `naturals.map (_*2).drop (30).take (50).toList`
2. `filter (p)`
   Test case: `naturals.drop (42).filter (_%2 ==0).take (30).toList`
3. `append (that)`
   This one requires sorting out the variance of type parameters carefully. You may find it easier to implement it as a function in the companion object first.
   Test case: `naturals.append (naturals)` (useless, but should not crash)
   Test case: `naturals.take(10).append(naturals).take(20).toList`
4. `flatMap`
   Test case: `naturals.flatMap (to _).take (100).toList`
   Test case: `naturals.flatMap (x =>from (x)).take (100).toList`

There are no automatic tests for this exercise—because the types are not already present in the template file, the test suite would be failing to compile, if we wrote them. This would make the test suite useless for the earlier exercises. Test in the REPL instead.

Exercise 9. The book presents the following implementation for find:

```
def find (p: A => Boolean): Option[A]= this.filter (p).headOption
```

Explain why this implementation is suitable (efficient) for streams and would not be optimal for lists.

[2] Exercise 5.3 [Chiusano, Bjarnason 2014]
[3] Exercise 5.4 [Chiusano, Bjarnason 2014]
[4] Exercise 5.5 [Chiusano, Bjarnason 2014]
[5] Exercise 5.7 [Chiusano, Bjarnason 2014]

Exercise 10. Compute a lazy stream of Fibonacci numbers `fibs`: 0, 1, 1, 2, 3, 5, 8, and so on. It can be done with functions available so far. Test it in REPL by translating a finite prefix of `fibs` to `List`, and a finite prefix of some infinite suffix.[6] Again, no ready-made tests, because the types are not prescribed (you need to write the type yourself).

Exercise 11. Write a more general stream-building function called `unfold`. It takes an initial state, and a function for producing both the next state and the next value in the generated stream.

```
def unfold[A, S] (z: S) (f: S => Option[(A, S)]): Stream[A]
```

If you solve it *without* using pattern matching, then you obtain a particularly concise solution, that combines aspects of this and last week's material.

You can test this function in REPL by unfolding the stream of natural numbers and checking whether its finite prefix is equal to the corresponding prefix of `naturals`.[7]

The exercise is placed in the companion object of `Stream`, in the bottom of `Stream.scala`.

Exercise 12. Write `fib` and `from` in terms of `unfold`. Use these test cases in REPL:

```
from (1).take (1000000000).drop (41).take (10).toList ==
  from1 (1).take (1000000000).drop (41).take (10).toList
```

and `fibs1.take (100).toList ==fibs.take (100).toList`,

where identifiers suffixed with 1 refer to the new versions of the functions.[8]

Exercise 13. Use unfold to implement `map`, `take`, `takeWhile`, and `zipWith`.[9]

Note that there is a choice whether the operation used by `zipWith` is strict or not. The lazy (by-name) is more general as it allows using efficiently functions that ignore the first (or the second) operand if the other one is a special case (so if you zip with || or &&).

Some of the test cases for REPL listed above can be used here again. This is a good test case for `zipWith` in REPL:

```
naturals
  .zipWith[Int, Int] (_+_) (naturals)
  .take (2000000000)
  .take (20)
  .toList
```

What should be the result of this?

```
naturals
  .map { _%2==0 }
  .zipWith[Boolean,Boolean] (_||_) (naturals.map { _ % 2 == 1 })
  .take(10)
  .toList
```

Convince yourself what the results of these test cases should be before you run the code.

[6] Exercise 5.10 [Chiusano, Bjarnason 2014]
[7] Exercise 5.11 [Chiusano, Bjarnason 2014]
[8] Exercise 5.12 [Chiusano, Bjarnason 2014]
[9] Exercise 5.13 [Chiusano, Bjarnason 2014]