ON INTEGRATION OF A STRAPDOWN INERTIAL NAVIGATION SYSTEM WITH MODERN MAGNETIC SENSORS*

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***Abstract*—In this paper, we consider algorithms of integration of strapdown navigation systems with measurements of the geomagnetic field. We present numerical results for these algorithms, obtained with the help of geophysical survey data and characteristics of inertial and magnetic sensors from datasheets. We analyze high-grade navigation systems as well as low-grade ones. In conclusion, we present a table with performance results for all considered variants of integrated navigation systems.**

Keywords – integrated navigation system, strapdown inertial navigation system, magnetic field, magnetic field gradient

I. INTRODUCTION

It is well known that the major drawback of inertial navigation systems (INS) is a dramatic increase of navigation errors with time. Existing approaches to solve this problem use integration of inertial data and data from other systems to correct navigation errors.

One of promising sources of correcting measurements is the anomalous geomagnetic field. Existing sensors are capable of measuring the magnetic field and its gradient with high accuracy and compact enough. State-of-art experimental and commercial magnetic sensors are capable of measuring various parameters of the magnetic field with high accuracy, at high output rates and reliable enough to work in harsh conditions, e.g. on board of an aircraft [1-4].

In this paper, we will give a detailed description of developed integration algorithms for different types of sensors and an algorithm of field model calculation. We present results of numerical simulation demonstrating the effectiveness of the algorithms, as well as precision estimate for navigation solution for described integrated systems.

II. INTEGRATION OF INS WITH MAGNETIC SENSORS

The main idea of our approach is following: we formulate linear INS error equations and linear measurement equations, which describe relations between magnetic measurements and INS navigation errors. Thus, our problem belongs to a class of linear optimal estimation problems, which can be solved with the help of Kalman filtering.

Let us consider described concept in details. At first, we formulate navigation error equations for a navigation system with the wander-azimuth mechanization. In that case, discrete time error equation are:

$$\begin{gathered} X_{k+1} = F_k X_k + q_k, \\ X_k = (\Delta x_k^T, \delta v_k^T, \beta_k^T, \Delta\omega_k^T, \Delta f_k^T)^T, \\ F_k = I_9 + A_k \Delta t + A_k^2 \Delta t^2 / 2, \\ q_k \in N(0, Q_k), \end{gathered} \tag{1}$$

where index k denotes corresponding time, X_k – the state vector, Δx_k – coordinate errors, δv_k – dynamical speed errors, β_k – small rotation angle reflecting attitude errors, Δt – a sampling interval, A_k – the matrix of linear differential error equations, $\Delta\omega_k, \Delta f_k$ – biases of rate gyros and accelerometers, respectively [5].

Next, we should consider linear measurement equations for different types of magnetic sensors, which, generally, will take the form:

$$\begin{gathered} z_k = H_k X_k + r_k, \\ r_k \in N(0, R_k), \end{gathered} \tag{2}$$

where the measurement matrix H_k and the measurement vector z_k depend on type of the geomagnetic field parameter used. Now, given eq. (1), (2), we can write down relations for prognosis and estimation steps of the discrete Kalman filter [6], which solve the aforementioned problem.

III. LINEAR MEASUREMENT EQUATIONS

Let us consider specific measurement equations for different type of magnetic measurements. It can be shown [7, 8] that for integrated navigation systems using an absolute value of the magnetic induction vector (total magnetic field) as a navigation aid, measurement equations will take the form:

$$H_k = ((J_{m_0})_k^{mod}\, O_{1\times 6}),$$
$$z_k = (m_0)_k^{mod} - (m_0)_k^{mes},$$

where $(J_{m_0})_k^{mod}$ – the model gradient of the total magnetic field, $(m_0)_k^{mod,mes}$ – the model and measured total magnetic field, respectively. $O_{M\times N}$ is a zero matrix containing M rows and N columns. When we use the magnetic vector as a navigation aid, these equations will take the form:

$$H_k = ((J_{m_v})_k^{mod}\, O_{3\times 3}\, (m_v)_k^{mod}),$$
$$z_k = (m_v)_k^{mod} - \Omega_k^T (m_v)_k^{mes},$$

where $(J_{m_v})_k^{mod}$ – the model gradient of the magnetic vector, $(m_v)_k^{mod,mes}$ – the model and measured magnetic vectors, Ω_k – the direction cosine matrix. In case of the magnetic gradient vector, we obtain:

$$H_k = ((J_{m_g})_k^{mod}\, O_{3\times 3}\, (m_g)_k^{mod}),$$
$$z_k = (m_g)_k^{mod} - \Omega_k^T (m_g)_k^{mes},$$

where $(J_{m_g})_k^{mod}$ – the model gradient of the magnetic gradient vector, i.e. 2nd derivative of the magnetic field, $(m_g)_k^{mod,mes}$ – the model and measured magnetic gradient vectors. Finally, when the magnetic gradient tensor in use, error equations are:

$$H_k = ((J_{m_t})_k^{mod}\, O_{5\times 3} - K_k),$$
$$z_k = (m_t)_k^{mod} - M_k^T (m_t)_k^{mes},$$

where $(J_{m_t})_k^{mod}$ – the model gradient (2nd derivative) of the magnetic gradient tensor, $(m_t)_k^{mod,mes}$ – the model and measured magnetic gradient tensors (hereinafter we use the m_t vector instead of a full-rank magnetic gradient tensor, given that only five of nine magnetic gradient tensor components are independent, so we can use $m_t = (\Gamma_{11}, \Gamma_{22}, \Gamma_{12}, \Gamma_{13}, \Gamma_{23})^T$ instead of Γ_{ij}) and

$$K = \begin{pmatrix} 0 & -2(m_t)_4 & 2(m_t)_3 \\ 2(m_t)_5 & 0 & -2(m_t)_3 \\ (m_t)_4 & -(m_t)_5 & (m_t)_2 - (m_t)_1 \\ -(m_t)_3 & 2(m_t)_1 + (m_t)_2 & (m_t)_5 \\ -(m_t)_1 + 2(m_t)_2 & (m_t)_3 & -(m_t)_4 \end{pmatrix},$$

M – the transformation matrix of the magnetic gradient tensor.

IV. EARTH'S MAGNETIC FIELD MODELING

During numerical simulations, we calculate the geomagnetic field as sum of three independent components: main field, anomalous field and magnetic time variations. For main field we use the IGRF-12 magnetic model. We model magnetic time variations as a quasi-stationary field, the frequency spectrum of which was based on results on lengthy series of measurements obtained at geomagnetic observatories.

The model of the anomalous magnetic field was based on real-world data. The main difficulty is only total magnetic field measurement are readily available, so information of the geomagnetic field components and their gradients was obtained with the help of an additional processing procedure, which will be described below. First, we applied the discrete two dimensional Fourier transform to measured values of the total magnetic field $u(x, y, z)$, so we obtained the following expansion:

$$u(x, y, z) = \sum_{n_1, n_2} \hat{u}(n_1, n_1, z) \exp\left[i k_x(n_1) x + i k_y(n_2) y \right].$$

where $k_x(n_1) = 2\pi n_1 / L_x$, $k_y(n_2) = 2\pi n_2 / L_y$, and L_x, L_y – grid cell dimensions. In frequency domain, the problem can be solved with the help of linear operators deduced in [9]. For example, for extraction of the vertical magnetic vector component from total magnetic field data, the linear operator to be applied is:

$$\hat{B}_z(n_1, n_2, z) = \frac{i k_x(n_1)}{i(\alpha k_x(n_1) + \beta k_y(n_2)) + \gamma k(n_1, n_2)} \hat{u}(n_1, n_2, z),$$

where $k(n_1, n_2) = \sqrt{k_x^2(n_1) + k_y^2(n_2)}$. The inverse Fourier transform was utilized to obtain final results.

It should be noted, that extraction information of the magnetic field and its gradient from total magnetic field data is ill-posed problem, so Tikhonov regularization [10] with modifications provided by [11, 12] was used. To test reliability of the aforementioned approach before processing real-world data, we applied the extraction procedure to synthetic data. We assumed, that the anomalous magnetic field was generated by the system of many magnetic dipoles beneath of the Earth's surface. The accuracy of extraction was satisfactory (we obtained results similar to ones given in [9]), that allowed us to apply this approach to real-world data as well.

V. NUMERICAL SIMULATION

For purposes of the numerical simulation we assumed that the test object moved with slow varying speed along some trajectory, the orientation of the object changed too. Random and systematic errors of inertial sensors were obtained from datasheets of commercially available inertial measurement units. Fig. 1-4 show results of the geomagnetic filed and its gradient modeling.

We considered low-grade and high-grade INS. Moreover, we considered different navigation aids: precise total magnetic field data with and without compensated field variations; imprecise magnetic vector data; precise magnetic gradient vector data; magnetic gradient tensor data obtained by either imprecise fluxgate sensors or precise SQUID (Superconductive Quantum Interferometer Device). Result of our numerical experiments as RMS errors in coordinates, velocities and rotation angles are summarized in Tab. 1, 2.

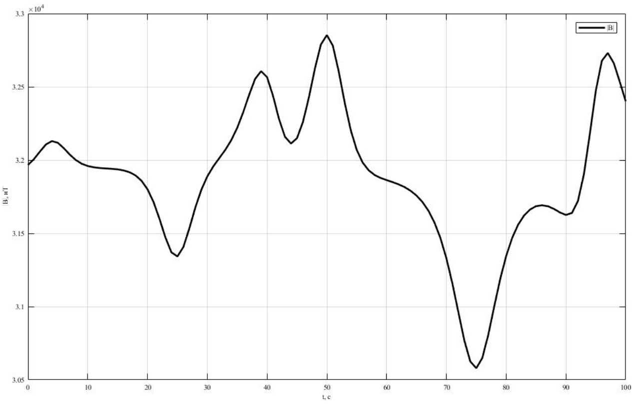

Fig. 1. Total magnetic field

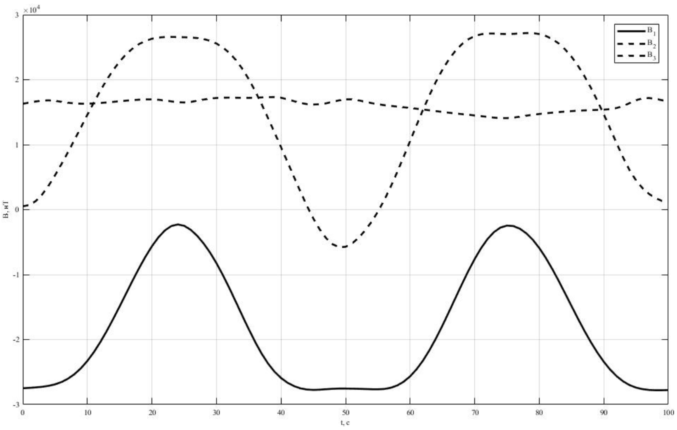

Fig. 2. Vector of the magnetic induction

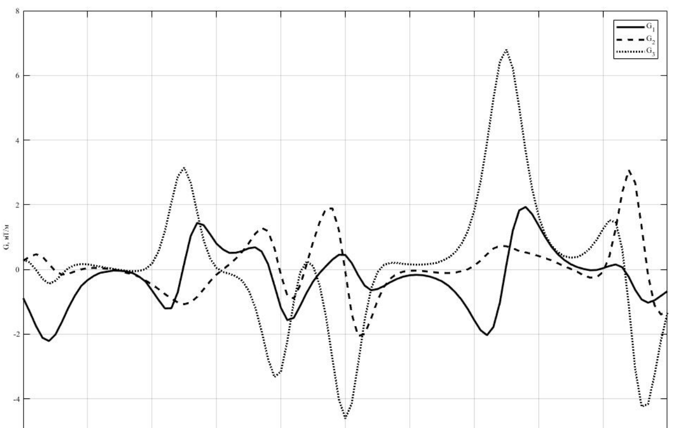

Fig. 3. Magnetic gradient vector

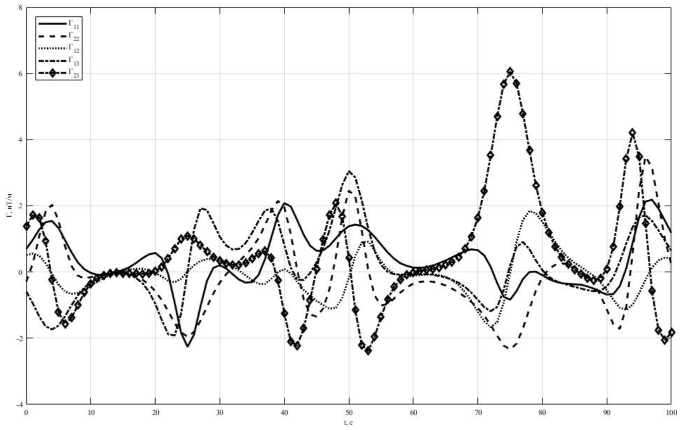

Fig. 4. Magnetic gradient tenso

TABLE I. RESULTS OF NUMERICAL EXPERIMENTS (PART 1)

INS	Navigation aid		
	Total m. field	***Total m. f. (comp. variations)***	***Mag. vector***
low	divergence	(1,93;0,68;2,81)	divergence
high	(4,88;0,22; 0,26)[a]	(0,074;0,022;0,10)	(1,03;0,08;0,03)
INS	**Navigation aid**		
	Gradient vector	***Gradient tensor (fluxgate)***	***Gradient tensor (SQUID)***
low	(3,29;0,58; 0,83)	(27,5;3,50;2,20)	(2,0;0,29;0,48)
high	(0,23;0,04; 0,03)	(11,8;0,59;0,61)	(0,26;0,037;0,03)

[a] navigation errors, rms: coordinates, m, velocities, m/s, rotation angles, degrees

VI. CONCLUSION

In this paper, we considered different navigation aids for a strapdown INS: total magnetic field measurements, magnetic vector measurements, magnetic gradient vector measurements, magnetic gradient tensor measurements. Numerical experiments confirmed the effectiveness of the proposed integration algorithms. From results of these experiments, we conclude that modern SQUID-based magnetic gradiometers are the most promising (in terms of accuracy) navigation aids for high-grade INS and low-grade INS. Moreover, Tab. 2 shows the advantages of field gradient parameters – low-grade INS is corrected by imprecise tensor gradient measurements.

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