# ORF524 - Final Exam

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**Problem 1**

1. True
2. False
3. False
4. True (If By optimzer we mean minimizer, otherwise it is the intersection of the feasible directions with the ascent directions that should be empty, and in that case the answer is False)
5. 
6. True

**Problem 2**

1. 
2. Let $\lambda \in (0,1)$, $c_1, c_2, c, b_1, b_2, b \in \mathbb{R}^n$ so that $c = \lambda c_1 + (1-\lambda)c_2, b = \lambda b_1 + (1-\lambda)b_2$
   - Convexity in $c$: For two sets $A, B$, if $a, b \in A, B$, then $\lambda a \leq \lambda \sup A$, $a + b \leq \sup A + \sup B$, so that $\sup(A+B) \leq \sup A + \sup B$ and $\sup \lambda A \leq \lambda \sup A$ $V(b, \lambda c_1 + (1-\lambda)c_2) = \max_{Ax \leq b, x \geq 0} \lambda c_1^T x + (1-\lambda)c_2^T x \leq \max_{Ax \leq b, x \geq 0} \lambda c_1^T x + \max_{Ax \leq b, x \geq 0} (1-\lambda) c_2^T x \leq \lambda \max_{Ax \leq b, x \geq 0} c_1^T x + (1-\lambda) \max_{Ax \leq b, x \geq 0} c_2^T x = \lambda V(b, c_1) + (1-\lambda) V(b, c_2)$
   - Concavity $b$: Let $x_i$ be a feasible solution to $\max_{Ax_i \leq b_i, x_i \geq 0} c^T x_i$ for $i = 1, 2$. Then $x = \lambda x_1 + (1-\lambda)x_2$ is a feasible solution to $\max_{Ax \leq b, x \geq 0} c^T x$, and we have that $\lambda c^T x_1 + (1-\lambda)c^T x_2 = c^T x$ Which means that $\max_{Ax_i \leq b_i, x_i \geq 0, i=1,2} \lambda c^T x_1 + (1-\lambda)c^T x_2 \leq \max_{Ax \leq b, x \geq 0} c^T x$ Since the max on the left invloves two independent variables, it can be distributed so that we have:

   $$\lambda V(b_1, c) + (1-\lambda) V(b_2, c) \leq V(b, c)$$

3. Take $f(x) = 1_{x>0}$, the $\text{epi} f = (-\infty, 0] \times [0, \infty) \cup [0, \infty) \times [1, \infty)$, which is close as the union of two closed sets, but $f$ is not continuous.
4. 
5. $f(n, -n) \to_{n\infty} 0$, but $||(n, -n)|| \to_n \infty$