- $X_0 = 0$
- $X_t - X_s \sim \mathcal{N}(0, t-s)$

  Proof. fix $t_n \to t, s_n \to s$, t_n, s_n are dyadic. By construction: $X_{t_n} - X_{x_n} \sim \mathcal{N}(0, t_n - s_n)$ $X_{t_n} - X_{x_n} \overset{as}{\to} X_t - X_s$ $\mathcal{N}(0, t_n - s_n) \to \mathcal{N}(0, t-s)$ □

- $X_t - X_s \perp \{X_r : r \leq s\} \forall t \geq s$

  Proof. $s = k'2^{-n}, t = k2^{-n}$ Then $X_t - X_s \perp \sigma \underbrace{\{X_r, r \leq s, r = k2^{-n}\}}_{\mathcal{F}_n}$

  Let Z bdd $\sigma\{X_r, r \leq s\}$-meas, Y bdd $\sigma\{X_t - X_s\}$-meas.
  Let's show $E[YZ] = E[Y]E[Z]$.
  Let $Z^{(n)} \to Z, Y^{(n)} \to Y$.
  We know that $E[Y^{(n)}Z^{(n)}] = E[Y^{(n)}]E[Z^{(n)}]$ We conclude by DCT.
  Example of such $Z^{(n)}$: $Z^{(n)} = E[Z|\mathcal{F}_n]$
  Martingal convergence theorem: $Z^{(n)} \to E[Z|\sigma(\underbrace{\cup_n \mathcal{F}_n}_{\mathcal{F}'})]$

  $$\mathcal{F}' = \sigma\{X_r : r \leq s, r \text{ dyadic }\} \overset{?}{=} \sigma\{X_r : r \leq s\}$$

  - $\subseteq$ trivial.
  - $\supseteq$:It is enough to show that X_r is $\mathcal{F}'$-measurable. This follows by continuity.

  □