ORF525 - Problem Set 1

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Q.1

1. The conversion is necessary because otherwise we would have an unwanted order relation. For three similar houses A, B, C in zipcodes 98001, 98002, 98003, a linear model would be forced to affect a price for the house A that lies between the price for house A and C, which is a bug and not a future of the data itself.

2.

Q.2

$$||Y-\theta||_2^2 + 4\tau^2||\theta||_0 = \sum (y_i-\theta_i)^2 + 4\tau^2 1_{\theta_i \neq 0} = \sum_i f(\theta_i)$$

Where $f : \theta \to (y-\theta)^2 + 4\tau^2 1_{\theta \neq 0}$, eg

$$f(\theta) = \begin{cases} y^2 & \text{if } \theta = 0 \\ (y-\theta)^2 + 4\tau^2 & \text{if } \theta \neq 0 \end{cases}$$

The problem is linearly separable, we can minimize on each variable θ_i independetly:

- If $|y| > 2\tau$, then $y^2 \geq 4\tau^2$ and $(y-\theta)^2 + 4\tau^2 \geq 4\tau^2 = f(y)$.
- If $|y| \leq 2\tau$, then $f(0) = y^2 \leq 4\tau^2 \leq y^2 + (y-\theta) = f(\theta) \forall \theta \neq 0$.

So $\arg\min ||Y-\theta||_2^2 + 4\tau^2 = \hat{\theta}^{\text{hard}}$.

$$||Y-\theta||_2^2 + 4\tau^2||\theta||_1 = \sum (y_i-\theta_i)^2 + 4\tau^2|\theta_1| = \sum_i g(\theta_i)$$

Where $g : \theta \to (y-\theta)^2 + 4\tau^2|\theta|$, eg

$$g(\theta) = \begin{cases} y^2 & \text{if } \theta = 0 \\ (y-\theta)^2 + 4\tau^2|theta| & \text{if } \theta \neq 0 \end{cases}$$

The problem is linearly separable, we can minimize on each variable θ_i independetly:

- If $|y| > 2\tau$, then $y^2 \geq 4\tau^2$ and $(y-\theta)^2 + 4\tau^2|theta| \geq 4\tau^2 = g()$.

So $\arg\min ||Y-\theta||_2^2 + 4\tau^2 = \hat{\theta}^{\text{hard}}$.