Hoja 6 de problemas y prácticas con R

Estadística Computacional I. Grado en Estadística

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Ejercicio 1

1. Responder a los siguientes apartados:
   i) Generar secuencias de tamaño 500 según los generadores de Mersenne-Twister, Congruencia lineal, “Knuth-TAOCP-2002” y una secuencia determinista.
   ii) Dibujar gráficos de líneas e histogramas para las cuatro secuencias.
   iii) Aplicar contrastes de aleatoriedad sobre las cuatro secuencias.

Solución

Apartado i)

```
library(randtoolbox)
```

```
## Loading required package: rngWELL
```

```
## This is randtoolbox. For an overview, type 'help("randtoolbox")'.
```

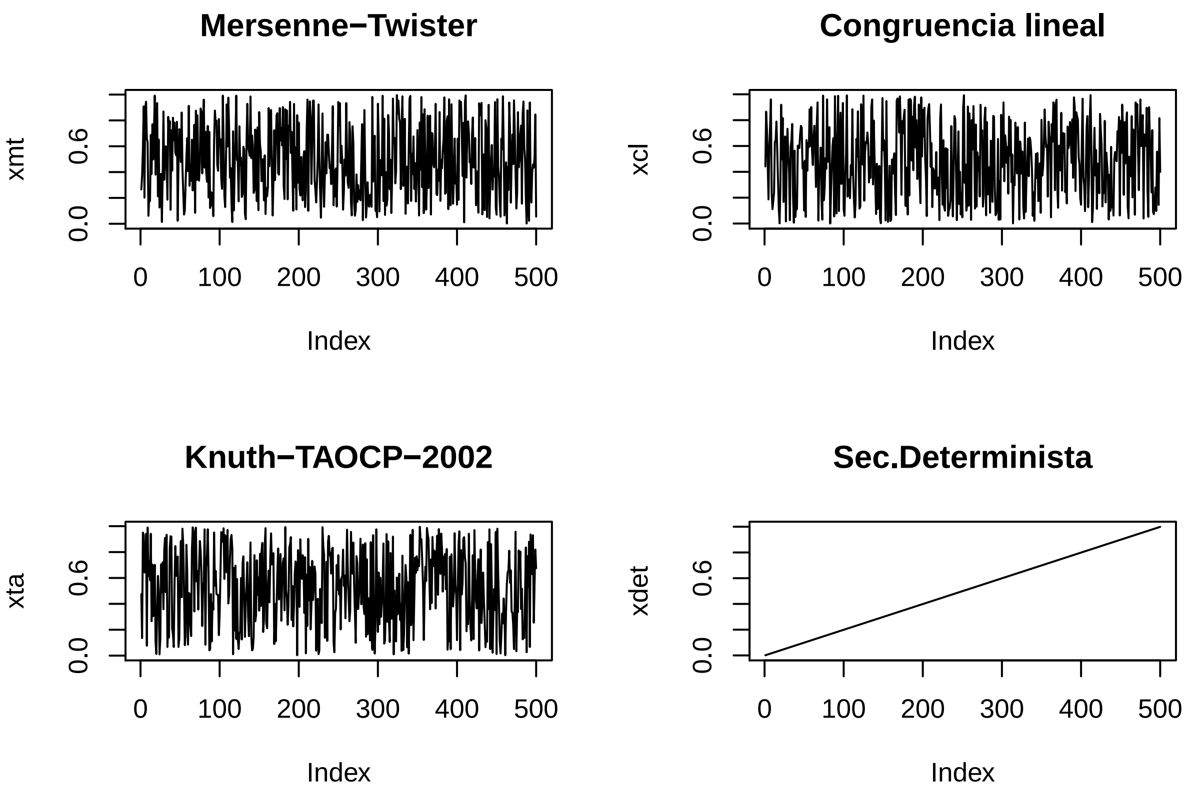
```
#Generador de congruencia lineal y algunos contrastes
library(tseries)   #Para contrastes de independencia
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

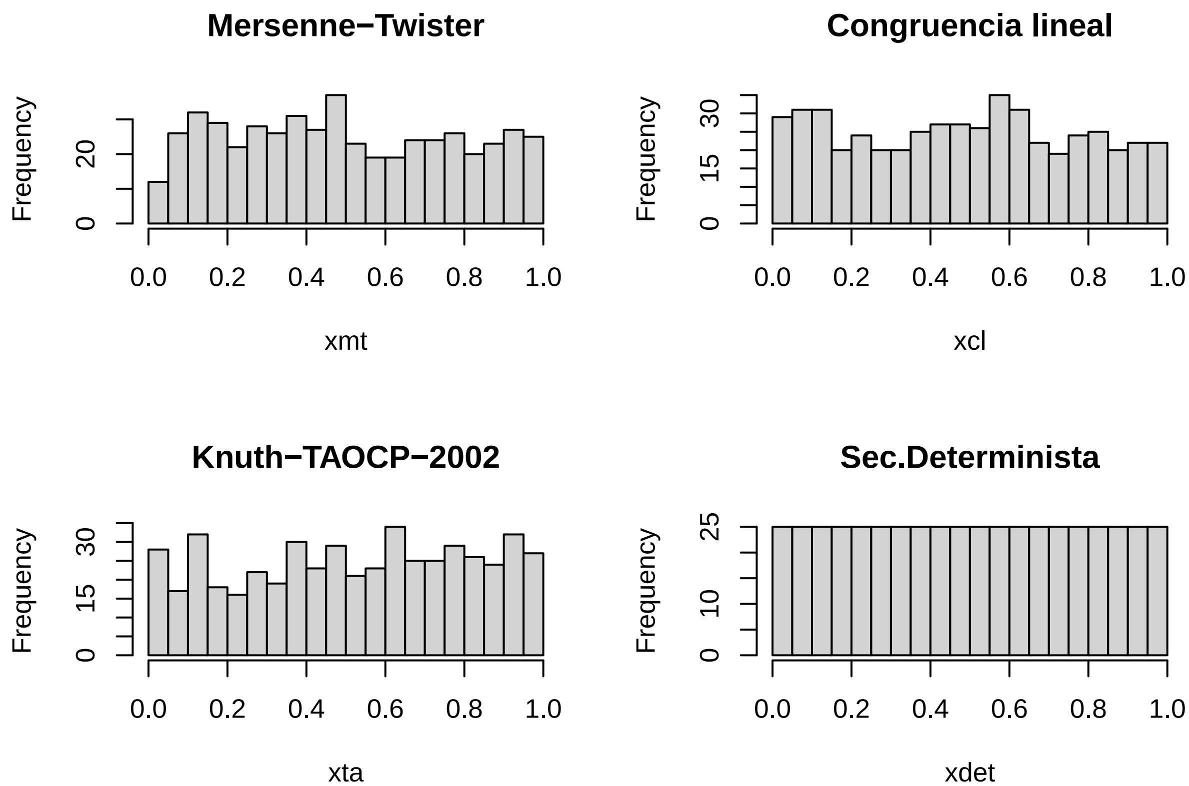
```
n<-500
set.seed(1)
xmt<- runif(n)       #Mersenne-Twister
xcl<- congruRand(n)  #Generador de congruencia lineal
xta<-knuthTAOCP(n)   #Knuth-TAOCP-2002
xdet<- ((1:n)-0.5)/n
```

Apartado ii)

```
#####    Completar
par(mfrow=c(2,2))
plot(xmt,type="l",main="Mersenne-Twister")
plot(xcl,type="l",main="Congruencia lineal")
plot(xta,type="l",main="Knuth-TAOCP-2002")
plot(xdet,type="l",main="Sec.Determinista")
```




```
par(mfrow=c(1,1))

par(mfrow=c(2,2))
hist(xmt,br=20,main="Mersenne-Twister")
hist(xcl,br=20,main="Congruencia lineal")
hist(xta,br=20,main="Knuth-TAOCP-2002")
hist(xdet,br=20,main="Sec.Determinista")
```




```
par(mfrow=c(1,1))
```

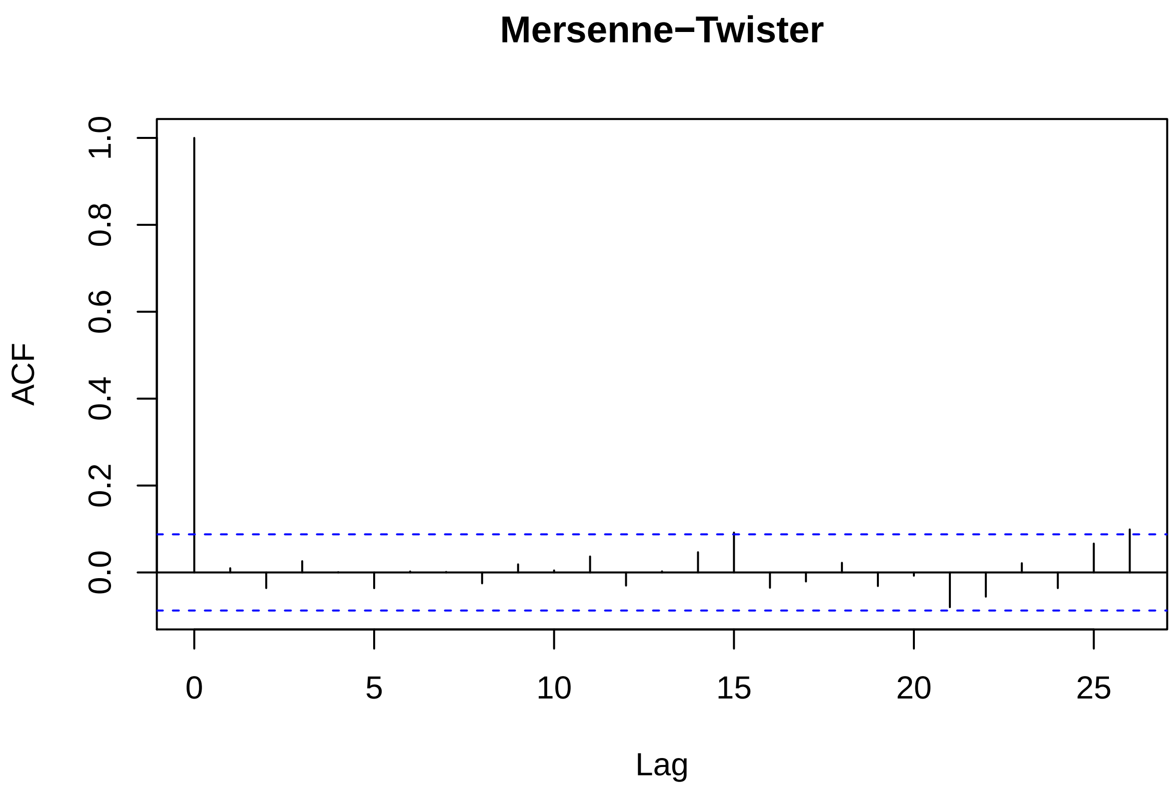
Apartado iii)

```
#Algunos contrastes
contrastes_aleato<-function(x,titulo)
{
 print(titulo)
 ks.test(x, "punif")
 gap.test(x)       #Por defecto, l=0, u=0.5
 order.test(x,d=5)  #d puede ser 2,3,4,5, pero n debe ser múltiplo de d
 freq.test(x, 1:4)#Por defecto, secuencia 1:15
 serial.test(x,d=5)  #n debe ser múltiplo de d (t=2)
 poker.test(x)
 print(runs.test(factor(x>0.5)))
 acf(x,main=titulo)
}
```

```
contrastes_aleato(xmt,"Mersenne-Twister")
```

```
## [1] "Mersenne-Twister"
##
##            Gap test
##
## chisq stat = 16, df = 9, p-value = 0.064
##
##        (sample size : 500)
##
## length    observed freq       theoretical freq
```

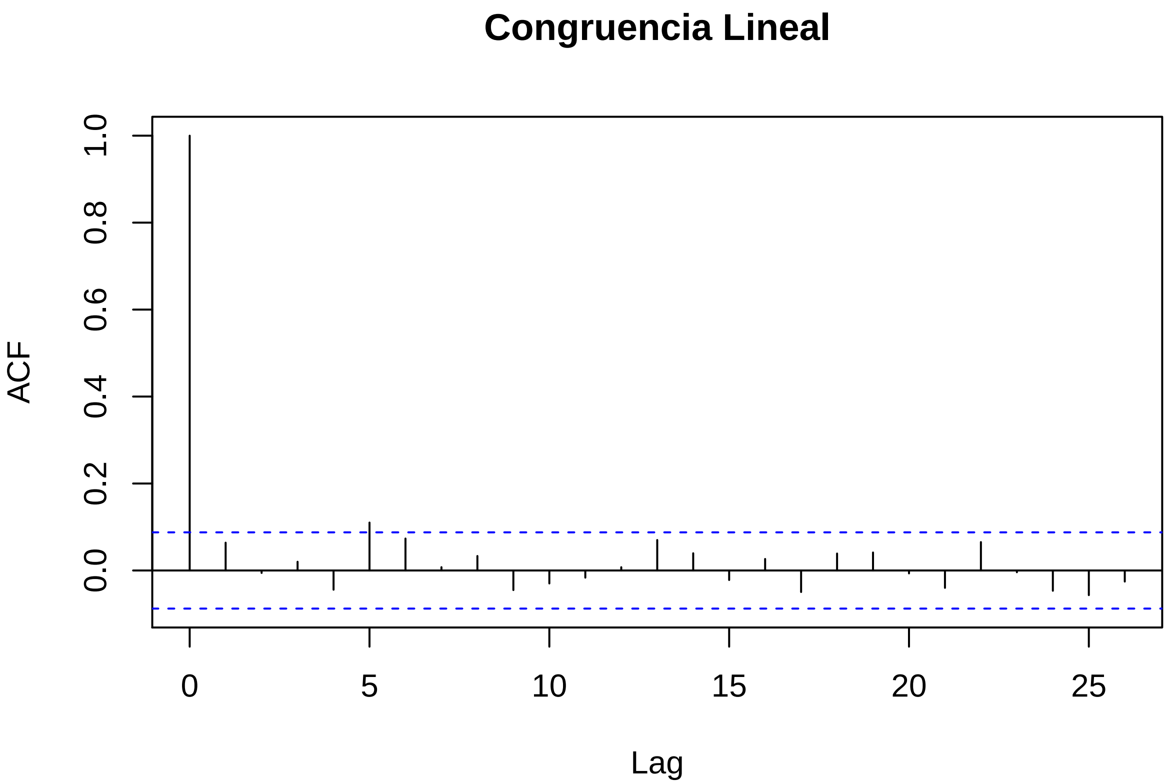
```
## 1             48              62
## 2             37              31
## 3             16              16
## 4             11              7.8
## 5             5           3.9
## 6             2           2
## 7             0           0.98
## 8             0           0.49
## 9             1           0.24
## 10            1           0.12
##
##           Order test
##
## chisq stat = 121, df = 119, p-value = 0.44
##
##       (sample size : 500)
##
##  observed number  0 1 0 0 2 1 1 1 0 1 3 2 2 0 0 2 2 3 0 1 1 0 0 0 0 3 0 0 1 2 1 1 0 0 1 2 0 0 0 1 1
##  expected number  0.83
##
##           Frequency test
##
## chisq stat = 6.9, df = 3, p-value = 0.075
##
##       (sample size : 500)
##
##  observed number  121 149 109 121
##  expected number  125
##
##           Serial test
##
## chisq stat = 33, df = 24, p-value = 0.1
##
##       (sample size : 500)
##
##  observed number  9 7 7 5 9 12 8 11 12 16 9 11 17 5 14 19 12 7 9 7 13 10 8 8 5
##  expected number  10
##
##           Poker test
##
## chisq stat = 3.3, df = 4, p-value = 0.5
##
##       (sample size : 500)
##
##  observed number  0 6 56 35 3
##  expected number  0.16 9.6 48 38 3.8
##
##  Runs Test
##
## data:  factor(x > 0.5)
## Standard Normal = -0.8, p-value = 0.4
## alternative hypothesis: two.sided
```




```
contrastes_aleato(xcl,"Congruencia Lineal")
```

```
## [1] "Congruencia Lineal"
##
##          Gap test
##
## chisq stat = 12, df = 9, p-value = 0.22
##
##      (sample size : 500)
##
## length   observed freq        theoretical freq
## 1             60               62
## 2             22               31
## 3             15               16
## 4             13               7.8
## 5             6           3.9
## 6             1           2
## 7             0           0.98
## 8             1           0.49
## 9             1           0.24
## 10            0           0.12
##
##          Order test
##
## chisq stat = 111, df = 119, p-value = 0.68
##
##      (sample size : 500)
##
##  observed number  2 3 3 0 1 0 0 1 1 1 0 2 0 0 0 0 2 0 0 0 0 0 0 2 1 0 0 0 0 2 1 1 2 3 1 0 0 0 1 1 0
```

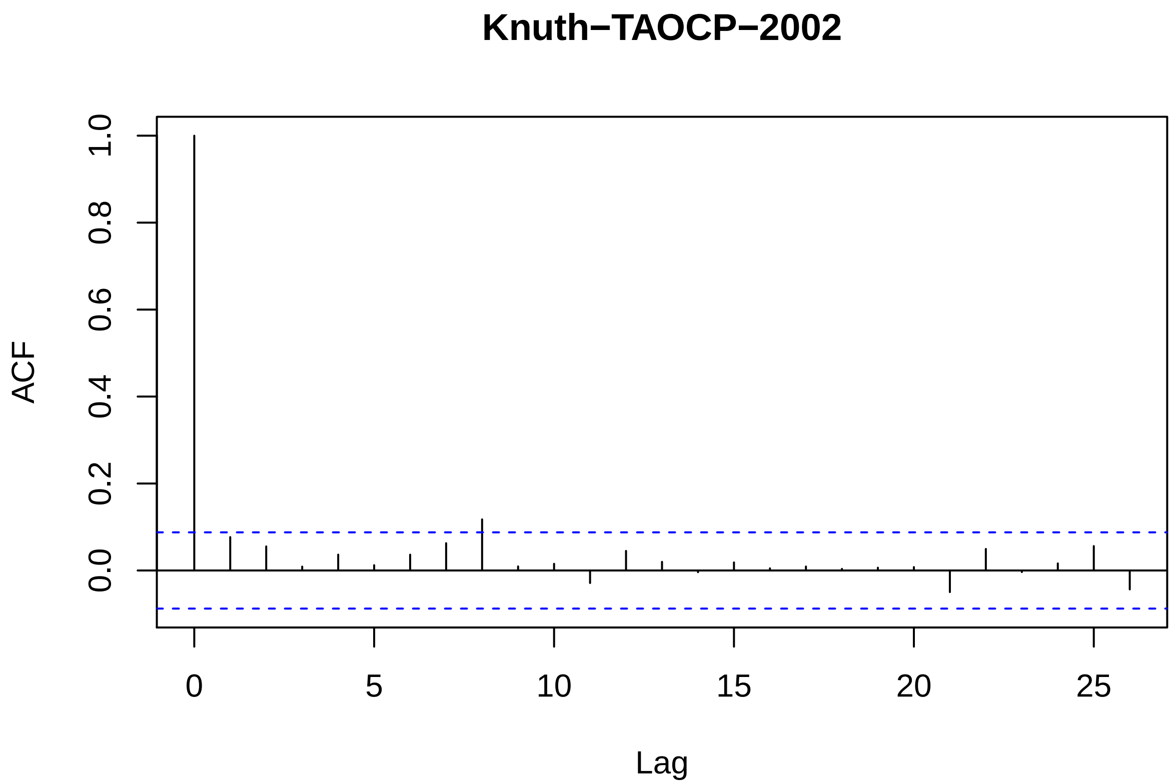
```
##  expected number  0.83
##
##           Frequency test
##
## chisq stat = 2.8, df = 3, p-value = 0.43
##
##       (sample size : 500)
##
##  observed number  135 119 133 113
##  expected number  125
##
##           Serial test
##
## chisq stat = 23, df = 24, p-value = 0.5
##
##       (sample size : 500)
##
##  observed number  11 17 10 11 11 5 10 8 11 7 17 7 13 13 11 8 9 13 7 6 10 5 10 11 9
##  expected number  10
##
##           Poker test
##
## chisq stat = 6.9, df = 4, p-value = 0.14
##
##       (sample size : 500)
##
##  observed number  0 12 58 26 4
##  expected number  0.16 9.6 48 38 3.8
##
##  Runs Test
##
## data:  factor(x > 0.5)
## Standard Normal = -1, p-value = 0.2
## alternative hypothesis: two.sided
```




```
contrastes_aleato(xta,"Knuth-TAOCP-2002")
```

```
## [1] "Knuth-TAOCP-2002"
##
##           Gap test
##
## chisq stat = 9.6, df = 9, p-value = 0.38
##
##       (sample size : 500)
##
## length    observed freq       theoretical freq
## 1             56               62
## 2             32               31
## 3             15               16
## 4             7            7.8
## 5             5            3.9
## 6             1            2
## 7             0            0.98
## 8             0            0.49
## 9             0            0.24
## 10            1            0.12
##
##           Order test
##
## chisq stat = 118, df = 119, p-value = 0.5
##
##       (sample size : 500)
##
##  observed number  1 3 1 0 0 2 0 1 1 0 0 1 1 1 1 0 2 0 0 1 0 1 1 1 1 0 1 0 0 1 0 0 1 0 2 1 0 2 1 0 2
```

```
##  expected number  0.83
##
##           Frequency test
##
## chisq stat = 3, df = 3, p-value = 0.39
##
##       (sample size : 500)
##
##  observed number  111 123 128 138
##  expected number  125
##
##           Serial test
##
## chisq stat = 19, df = 24, p-value = 0.77
##
##       (sample size : 500)
##
##  observed number  9 7 10 9 12 9 9 6 5 11 7 8 11 11 8 14 11 16 14 8 9 12 8 11 15
##  expected number  10
##
##           Poker test
##
## chisq stat = 4.4, df = 4, p-value = 0.36
##
##       (sample size : 500)
##
##  observed number  0 10 52 38 0
##  expected number  0.16 9.6 48 38 3.8
##
##  Runs Test
##
## data:  factor(x > 0.5)
## Standard Normal = -1, p-value = 0.2
## alternative hypothesis: two.sided
```




```r
contrastes_aleato(xdet,"Determinista")
```

```
## [1] "Determinista"
##
##            Gap test
##
## chisq stat = 1.4e+73, df = 249, p-value = 0
##
##       (sample size : 500)
##
## length   observed freq       theoretical freq
## 1             0           62
## 2             0           31
## 3             0           16
## 4             0           7.8
## 5             0           3.9
## 6             0           2
## 7             0           0.98
## 8             0           0.49
## 9             0           0.24
## 10            0           0.12
## 11            0           0.061
## 12            0           0.031
## 13            0           0.015
## 14            0           0.0076
## 15            0           0.0038
## 16            0           0.0019
## 17            0           0.00095
## 18            0           0.00048
```

```
## 19             0             0.00024
## 20             0             0.00012
## 21             0             6e-05
## 22             0             3e-05
## 23             0             1.5e-05
## 24             0             7.5e-06
## 25             0             3.7e-06
## 26             0             1.9e-06
## 27             0             9.3e-07
## 28             0             4.7e-07
## 29             0             2.3e-07
## 30             0             1.2e-07
## 31             0             5.8e-08
## 32             0             2.9e-08
## 33             0             1.5e-08
## 34             0             7.3e-09
## 35             0             3.6e-09
## 36             0             1.8e-09
## 37             0             9.1e-10
## 38             0             4.5e-10
## 39             0             2.3e-10
## 40             0             1.1e-10
## 41             0             5.7e-11
## 42             0             2.8e-11
## 43             0             1.4e-11
## 44             0             7.1e-12
## 45             0             3.6e-12
## 46             0             1.8e-12
## 47             0             8.9e-13
## 48             0             4.4e-13
## 49             0             2.2e-13
## 50             0             1.1e-13
## 51             0             5.6e-14
## 52             0             2.8e-14
## 53             0             1.4e-14
## 54             0             6.9e-15
## 55             0             3.5e-15
## 56             0             1.7e-15
## 57             0             8.7e-16
## 58             0             4.3e-16
## 59             0             2.2e-16
## 60             0             1.1e-16
## 61             0             5.4e-17
## 62             0             2.7e-17
## 63             0             1.4e-17
## 64             0             6.8e-18
## 65             0             3.4e-18
## 66             0             1.7e-18
## 67             0             8.5e-19
## 68             0             4.2e-19
## 69             0             2.1e-19
## 70             0             1.1e-19
## 71             0             5.3e-20
## 72             0             2.6e-20
```

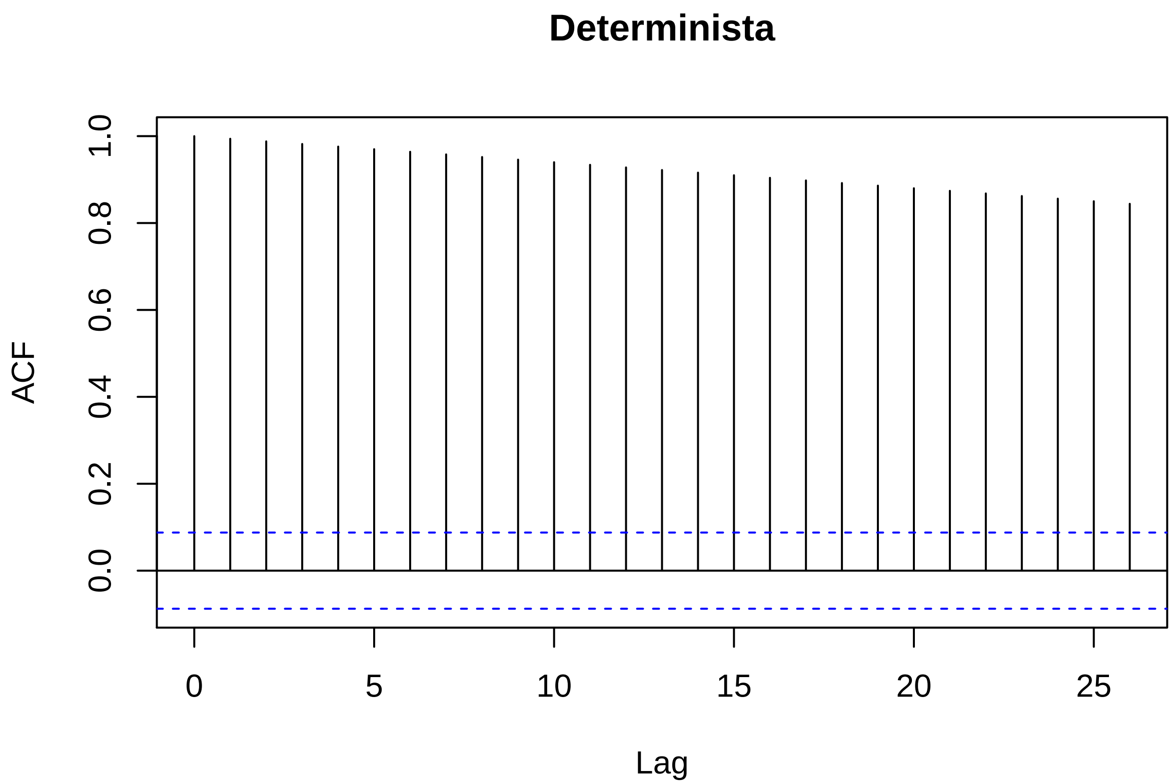
```
## 73              0              1.3e-20
## 74              0              6.6e-21
## 75              0              3.3e-21
## 76              0              1.7e-21
## 77              0              8.3e-22
## 78              0              4.1e-22
## 79              0              2.1e-22
## 80              0              1e-22
## 81              0              5.2e-23
## 82              0              2.6e-23
## 83              0              1.3e-23
## 84              0              6.5e-24
## 85              0              3.2e-24
## 86              0              1.6e-24
## 87              0              8.1e-25
## 88              0              4e-25
## 89              0              2e-25
## 90              0              1e-25
## 91              0              5e-26
## 92              0              2.5e-26
## 93              0              1.3e-26
## 94              0              6.3e-27
## 95              0              3.2e-27
## 96              0              1.6e-27
## 97              0              7.9e-28
## 98              0              3.9e-28
## 99              0              2e-28
## 100             0              9.9e-29
## 101             0              4.9e-29
## 102             0              2.5e-29
## 103             0              1.2e-29
## 104             0              6.2e-30
## 105             0              3.1e-30
## 106             0              1.5e-30
## 107             0              7.7e-31
## 108             0              3.9e-31
## 109             0              1.9e-31
## 110             0              9.6e-32
## 111             0              4.8e-32
## 112             0              2.4e-32
## 113             0              1.2e-32
## 114             0              6e-33
## 115             0              3e-33
## 116             0              1.5e-33
## 117             0              7.5e-34
## 118             0              3.8e-34
## 119             0              1.9e-34
## 120             0              9.4e-35
## 121             0              4.7e-35
## 122             0              2.4e-35
## 123             0              1.2e-35
## 124             0              5.9e-36
## 125             0              2.9e-36
## 126             0              1.5e-36
```

```
## 127           0           7.3e-37
## 128           0           3.7e-37
## 129           0           1.8e-37
## 130           0           9.2e-38
## 131           0           4.6e-38
## 132           0           2.3e-38
## 133           0           1.1e-38
## 134           0           5.7e-39
## 135           0           2.9e-39
## 136           0           1.4e-39
## 137           0           7.2e-40
## 138           0           3.6e-40
## 139           0           1.8e-40
## 140           0           9e-41
## 141           0           4.5e-41
## 142           0           2.2e-41
## 143           0           1.1e-41
## 144           0           5.6e-42
## 145           0           2.8e-42
## 146           0           1.4e-42
## 147           0           7e-43
## 148           0           3.5e-43
## 149           0           1.8e-43
## 150           0           8.8e-44
## 151           0           4.4e-44
## 152           0           2.2e-44
## 153           0           1.1e-44
## 154           0           5.5e-45
## 155           0           2.7e-45
## 156           0           1.4e-45
## 157           0           6.8e-46
## 158           0           3.4e-46
## 159           0           1.7e-46
## 160           0           8.6e-47
## 161           0           4.3e-47
## 162           0           2.1e-47
## 163           0           1.1e-47
## 164           0           5.3e-48
## 165           0           2.7e-48
## 166           0           1.3e-48
## 167           0           6.7e-49
## 168           0           3.3e-49
## 169           0           1.7e-49
## 170           0           8.4e-50
## 171           0           4.2e-50
## 172           0           2.1e-50
## 173           0           1e-50
## 174           0           5.2e-51
## 175           0           2.6e-51
## 176           0           1.3e-51
## 177           0           6.5e-52
## 178           0           3.3e-52
## 179           0           1.6e-52
## 180           0           8.2e-53
```

```
## 181            0             4.1e-53
## 182            0             2e-53
## 183            0             1e-53
## 184            0             5.1e-54
## 185            0             2.5e-54
## 186            0             1.3e-54
## 187            0             6.4e-55
## 188            0             3.2e-55
## 189            0             1.6e-55
## 190            0             8e-56
## 191            0             4e-56
## 192            0             2e-56
## 193            0             1e-56
## 194            0             5e-57
## 195            0             2.5e-57
## 196            0             1.2e-57
## 197            0             6.2e-58
## 198            0             3.1e-58
## 199            0             1.6e-58
## 200            0             7.8e-59
## 201            0             3.9e-59
## 202            0             1.9e-59
## 203            0             9.7e-60
## 204            0             4.9e-60
## 205            0             2.4e-60
## 206            0             1.2e-60
## 207            0             6.1e-61
## 208            0             3e-61
## 209            0             1.5e-61
## 210            0             7.6e-62
## 211            0             3.8e-62
## 212            0             1.9e-62
## 213            0             9.5e-63
## 214            0             4.7e-63
## 215            0             2.4e-63
## 216            0             1.2e-63
## 217            0             5.9e-64
## 218            0             3e-64
## 219            0             1.5e-64
## 220            0             7.4e-65
## 221            0             3.7e-65
## 222            0             1.9e-65
## 223            0             9.3e-66
## 224            0             4.6e-66
## 225            0             2.3e-66
## 226            0             1.2e-66
## 227            0             5.8e-67
## 228            0             2.9e-67
## 229            0             1.4e-67
## 230            0             7.2e-68
## 231            0             3.6e-68
## 232            0             1.8e-68
## 233            0             9.1e-69
## 234            0             4.5e-69
```

```
## 235           0           2.3e-69
## 236           0           1.1e-69
## 237           0           5.7e-70
## 238           0           2.8e-70
## 239           0           1.4e-70
## 240           0           7.1e-71
## 241           0           3.5e-71
## 242           0           1.8e-71
## 243           0           8.8e-72
## 244           0           4.4e-72
## 245           0           2.2e-72
## 246           0           1.1e-72
## 247           0           5.5e-73
## 248           0           2.8e-73
## 249           0           1.4e-73
## 250           1           6.9e-74
##
##           Order test
##
## chisq stat = 11900, df = 119, p-value = 0
##
##       (sample size : 500)
##
##  observed number  100 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##  expected number  0.83
##
##           Frequency test
##
## chisq stat = 0, df = 3, p-value = 1
##
##       (sample size : 500)
##
##  observed number  125 125 125 125
##  expected number  125
##
##           Serial test
##
## chisq stat = 1000, df = 24, p-value = 8.9e-196
##
##       (sample size : 500)
##
##  observed number  0 0 50 50 0 0 0 0 50 50 0 0 0 0 50 0 0 0 0 0 0 0 0 0 0
##  expected number  10
##
##           Poker test
##
## chisq stat = 2504, df = 4, p-value = 0
##
##       (sample size : 500)
##
##  observed number  0 0 0 0 100
##  expected number  0.16 9.6 48 38 3.8
##
##  Runs Test
```

```
##
## data:  factor(x > 0.5)
## Standard Normal = -22, p-value <2e-16
## alternative hypothesis: two.sided
```




```
#Los siguientes tests trabajan con generadores,
#no con una secuencia
coll.test(runif,2^7,2^10)
```

```
##
##           Collision test
##
## chisq stat = 0.008, df = 1, p-value = 0.93
##
## Poisson approximation (sample number : 1000 / sample size : 128 / cell number : 1e+06)
##
##  collision        observed        expected
##  number           count       count
##      0            992             992
##      1            8           7.8
```

```
#m=2^10 secuencias de tamaño n=2^7
#por defecto tdim=2
coll.test(congruRand,2^7,2^10)
```

```
##
##           Collision test
##
## chisq stat = 0.074, df = 1, p-value = 0.79
##
## Poisson approximation (sample number : 1000 / sample size : 128 / cell number : 1e+06)
```

```
##
##   collision        observed        expected
##   number           count       count
##       0            993             992
##       1            7           7.8
```

```
coll.test(knuthTAOCP,2^7,2^10)
```

```
##
##           Collision test
##
## chisq stat = 1.8, df = 1, p-value = 0.18
##
## Poisson approximation (sample number : 1000 / sample size : 128 / cell number : 1e+06)
##
##   collision        observed        expected
##   number           count       count
##       0            996             992
##       1            4           7.8
```

Ejercicio 2

2. Comparar empíricamente los rendimientos de los siguientes generadores de números aleatorios en (0,1), Mersenne-Twister, Congruencia Lineal, K2nuth-TAOCP-2002,2 además de un generador de números determinista, según los resultados del test de Kolmogorov-Smirnov y el test de huecos. Utilizar 1000 muestras de tamaño 100 y analizar de forma numérica y gráfica los resultados.

Solución

```
library(randtoolbox)
M<-1000
n<-100
pvaloresKS<- matrix(NA,M,4)
pvaloresGap<- matrix(NA,M,4)
colnames(pvaloresKS)<-  c("Mersenne-Twister","Congruencia Lineal",
"Knuth-TAOCP-2002","Determinista")
colnames(pvaloresGap)<-  c("Mersenne-Twister","Congruencia Lineal",
"Knuth-TAOCP-2002","Determinista")

for (i in 1:M)
{
  if (i%%50==0) {
     cat("Muestra ",i,"de ",M,"\n")
  }
 xmt<- runif(n)       #Mersenne-Twister
 xcl<- congruRand(n)  #Generador de congruencia lineal
 xta<-knuthTAOCP(n)   #Knuth-TAOCP-2002
 xdet<- ((1:n)-0.5)/n   #Secuencia determinista
 pvaloresKS[i,]<- c(ks.test(xmt, "punif")$p.value,
                    ks.test(xcl, "punif")$p.value,
                    ks.test(xta, "punif")$p.value,
                    ks.test(xdet, "punif")$p.value)
 pvaloresGap[i,]<- c(gap.test(xmt,echo=FALSE)$p.value,
                     gap.test(xcl,echo=FALSE)$p.value,
```

```
                    gap.test(xta,echo=FALSE)$p.value,
                    gap.test(xdet,echo=FALSE)$p.value)

}
```

```
## Muestra  50 de  1000
## Muestra  100 de  1000
## Muestra  150 de  1000
## Muestra  200 de  1000
## Muestra  250 de  1000
## Muestra  300 de  1000
## Muestra  350 de  1000
## Muestra  400 de  1000
## Muestra  450 de  1000
## Muestra  500 de  1000
## Muestra  550 de  1000
## Muestra  600 de  1000
## Muestra  650 de  1000
## Muestra  700 de  1000
## Muestra  750 de  1000
## Muestra  800 de  1000
## Muestra  850 de  1000
## Muestra  900 de  1000
## Muestra  950 de  1000
## Muestra  1000 de  1000
```

```
head(pvaloresKS,10)
```

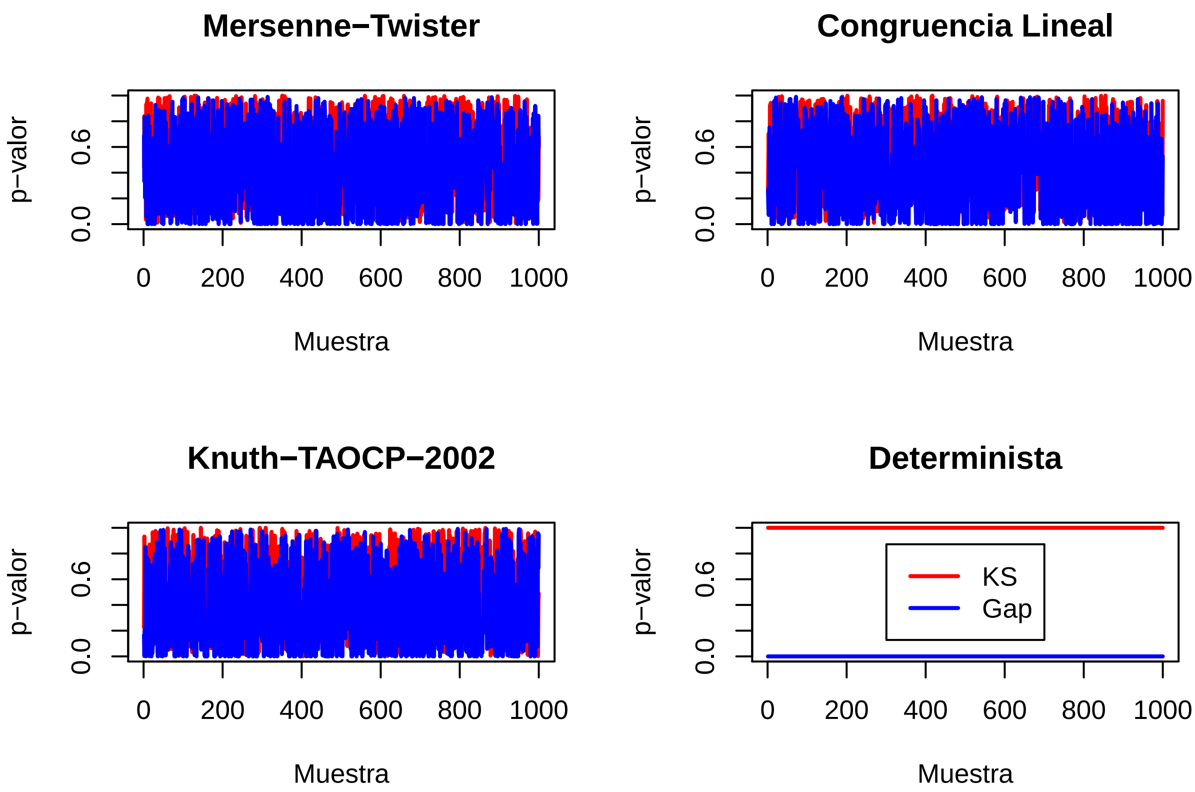
```
##       Mersenne-Twister Congruencia Lineal Knuth-TAOCP-2002 Determinista
##  [1,]            0.442              0.254           0.2229            1
##  [2,]            0.840              0.510           0.9322            1
##  [3,]            0.519              0.706           0.6388            1
##  [4,]            0.715              0.241           0.1957            1
##  [5,]            0.466              0.381           0.6399            1
##  [6,]            0.035              0.925           0.0746            1
##  [7,]            0.933              0.300           0.5846            1
##  [8,]            0.530              0.943           0.5261            1
##  [9,]            0.954              0.510           0.7942            1
## [10,]            0.976              0.079           0.0055            1
```

```
head(pvaloresGap,10)
```

```
##       Mersenne-Twister Congruencia Lineal Knuth-TAOCP-2002 Determinista
##  [1,]          6.9e-01            2.7e-01          0.16588            0
##  [2,]          3.3e-01            1.2e-01          0.00259            0
##  [3,]          8.3e-01            7.1e-02          0.20470            0
##  [4,]          2.0e-01            2.5e-01          0.04058            0
##  [5,]          7.4e-01            7.5e-01          0.85194            0
##  [6,]          6.0e-01            4.7e-01          0.00084            0
##  [7,]          5.8e-02            3.2e-01          0.40500            0
##  [8,]          3.6e-01            6.5e-01          0.77236            0
##  [9,]          5.0e-01            4.3e-01          0.55869            0
## [10,]          4.1e-06            1.1e-63          0.10567            0
```

```
#Ejercicio: dibujar en una sola pantalla los M p-valores KS y Gap para los
#cuatro generadores
```

```
par(mfrow=c(2,2))
for (i in 1:4)
{
 plot(pvaloresKS[,i],type="l",
      main=colnames(pvaloresKS)[i],
      xlab="Muestra",ylab="p-valor",
 col="red",lwd=2,ylim=c(0,1))
 lines(pvaloresGap[,i], col="blue",lwd=2)
}
legend("center",col=c("red","blue"),lty=1,
       lwd=2,legend=c("KS","Gap"))
```



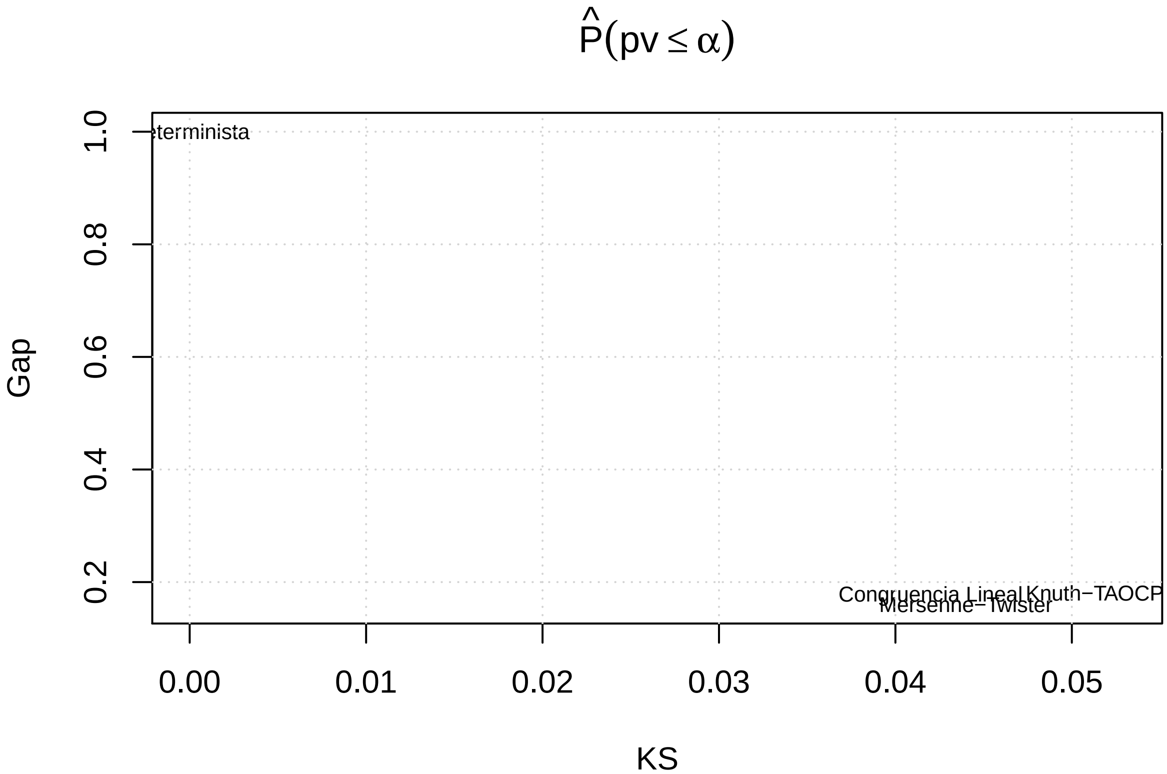

```
par(mfrow=c(1,1))
```

```
#Estimar P[p<=alfa]
alfa<- 0.05
rendim<- matrix(NA,4,2)
colnames(rendim)<- c("KS","Gap")
rownames(rendim)<- c("Mersenne-Twister","Congruencia Lineal",
"Knuth-TAOCP-2002","Determinista")
for (i in 1:4)
 rendim[i,]<-c(mean(pvaloresKS[,i]<=alfa),
              mean(pvaloresGap[,i]<=alfa)) #Ejercicio: completar
rendim
```

```
##                       KS  Gap
## Mersenne-Twister   0.044 0.16
## Congruencia Lineal 0.042 0.17
## Knuth-TAOCP-2002   0.053 0.18
```

```
## Determinista        0.000 1.00
```

```
plot(rendim,type="n",
     main=expression(hat(P)(pv<=alpha)))
text(rendim,label=rownames(rendim),cex=0.7)
grid()
```




```
#Cuántas veces es mayor un pv que el otro
for (i in 1:4)
 cat(colnames(pvaloresKS)[i],mean(pvaloresKS[,i]>pvaloresGap[,i]),"\n")
```

```
## Mersenne-Twister 0.59
## Congruencia Lineal 0.62
## Knuth-TAOCP-2002 0.65
## Determinista 1
```

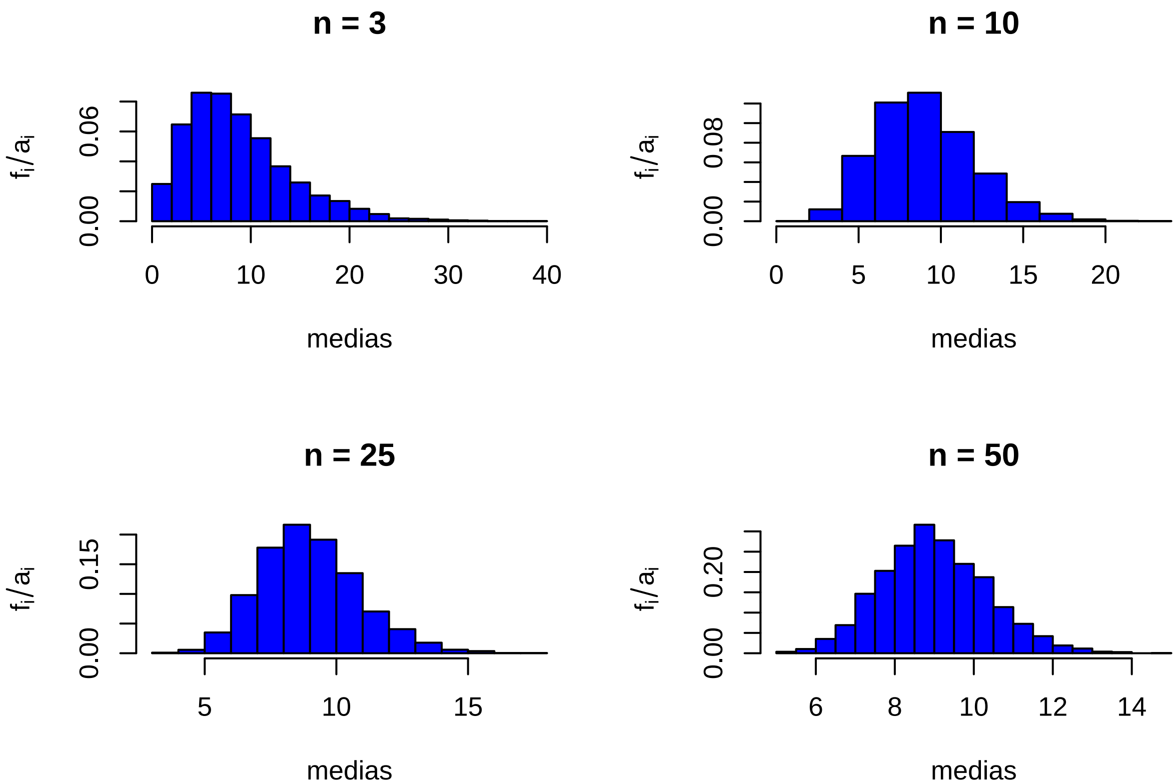
Ejercicio 3

3. Ilustrar el Teorema Central del Límite con muestras de una ley Geométrica con parámetro 0.1 para tamaños muestrales 3, 10, 25, 50 y número de muestras 5000.

Solución

```
M<-5000 #Núm. de muestras
p<-0.1
vn<-c(3,10,25,50)      #Tamaños de cada muestra
paranti<- par(no.readonly=T)
par(mfrow=c(2,2))
for (n in vn) #Ejercicio: completar
```

```
{  x<- matrix(rgeom(M*n,p),nrow=M,ncol=n)
   medias<- rowMeans(x)
   hist(medias,freq=FALSE, col="blue",
        main=paste("n =",n),
   ylab=expression(f[i] /a[i]))
}
```




```
par(paranti)
```

Ejercicio 4

4. Ilustrar el concepto de intervalo de confianza mediante 100 muestras de tamaño 10 de una ley N(0,1), siendo la media poblacional el parámetro de interés.

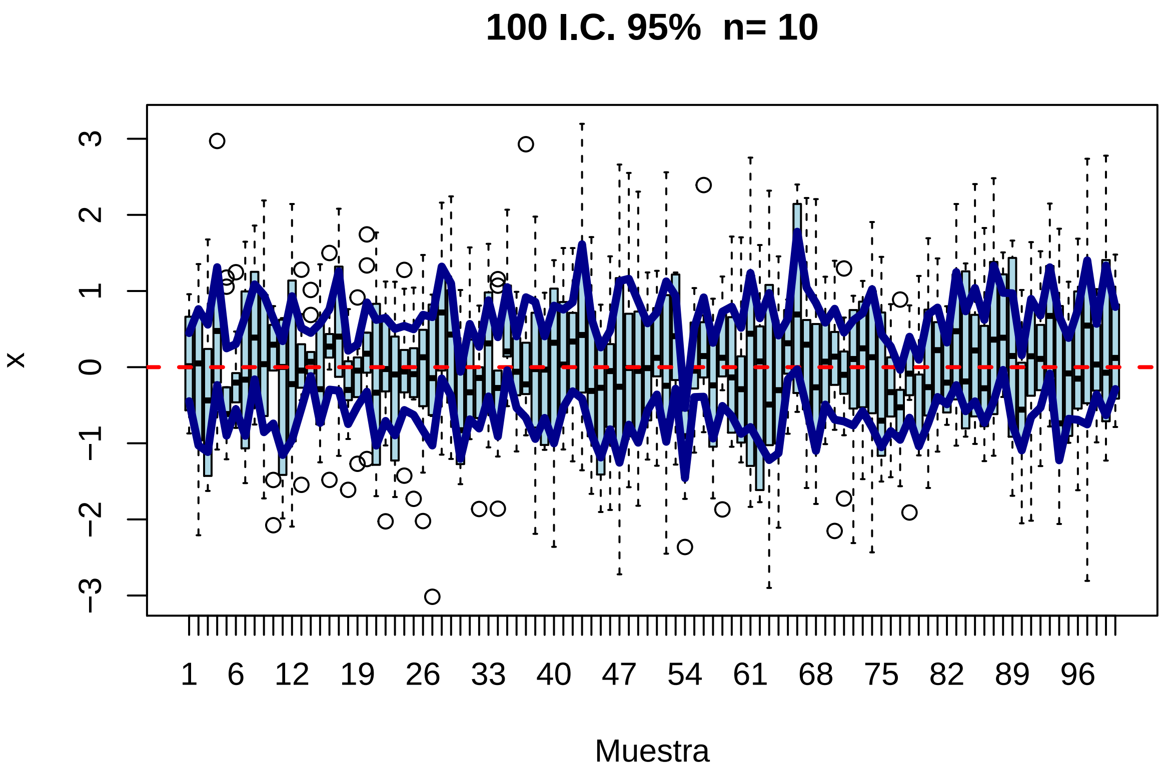
Solución

```
n<-10      #Tamaño muestral
M<-100     #M muestras
alfa<-0.05
media<-0
desvtip<-1
muestras<-matrix(rnorm(M*n,mean=media,sd=desvtip),
                 nrow=M,ncol=n,byrow=TRUE)
```

```
pv<-array(0,M)  #M pvalores
exinf<-numeric(M)
exsup<-numeric(M)
```

```r
for (i in 1:M)
{      #Ejercicio: completar
 test<-t.test(muestras[i,],conf.level=1-alfa)
 exinf[i]<-test$conf.int[1]
 exsup[i]<-test$conf.int[2]
}
```

```r
plot(gl(M,n),as.vector(t(muestras)),main=paste("100 I.C. 95% ","n=",n),
xlab="Muestra", ylab="x",col="lightblue")
abline(h=media,col="red",lwd=2,lty=2)
lines(exinf,col="darkblue",lwd=4)
lines(exsup,col="darkblue",lwd=4)
```




```r
cat("Cobertura observada =",
    100* mean((exinf<= media) & (exsup>=media)),
    "% \n")
```

```
## Cobertura observada = 97 %
```

```r
cat("Longitud media =",
    mean(exsup-exinf),"\n")
```

```
## Longitud media = 1.4
```

Ejercicio 5

5. Escribir y probar una función R para generar muestras de una Weibull:

$$f(t) = \lambda\alpha(\lambda\, t)^{\alpha-1}\, e^{-(\lambda t)^\alpha}, \quad F(t) = 1 - e^{-(\lambda t)^\alpha},\ \lambda > 0, \alpha > 0$$

   i) Generar muestras de tamaño 200 para las configuraciones ($\alpha = 0.5, \lambda = 1$), ($\alpha = 1, \lambda = 1$), ($\alpha = 2, \lambda = 1$) y ($\alpha = 2, \lambda = 3$); α es el parámetro de forma, λ el de escala.

ii) Dibujar los histogramas.
iii) Representar las funciones de distribución empírica y superponer las funciones de distribución teóricas.
iv) Realizar contrastes de bondad de ajuste mediante la librería fitdistrplus de R.

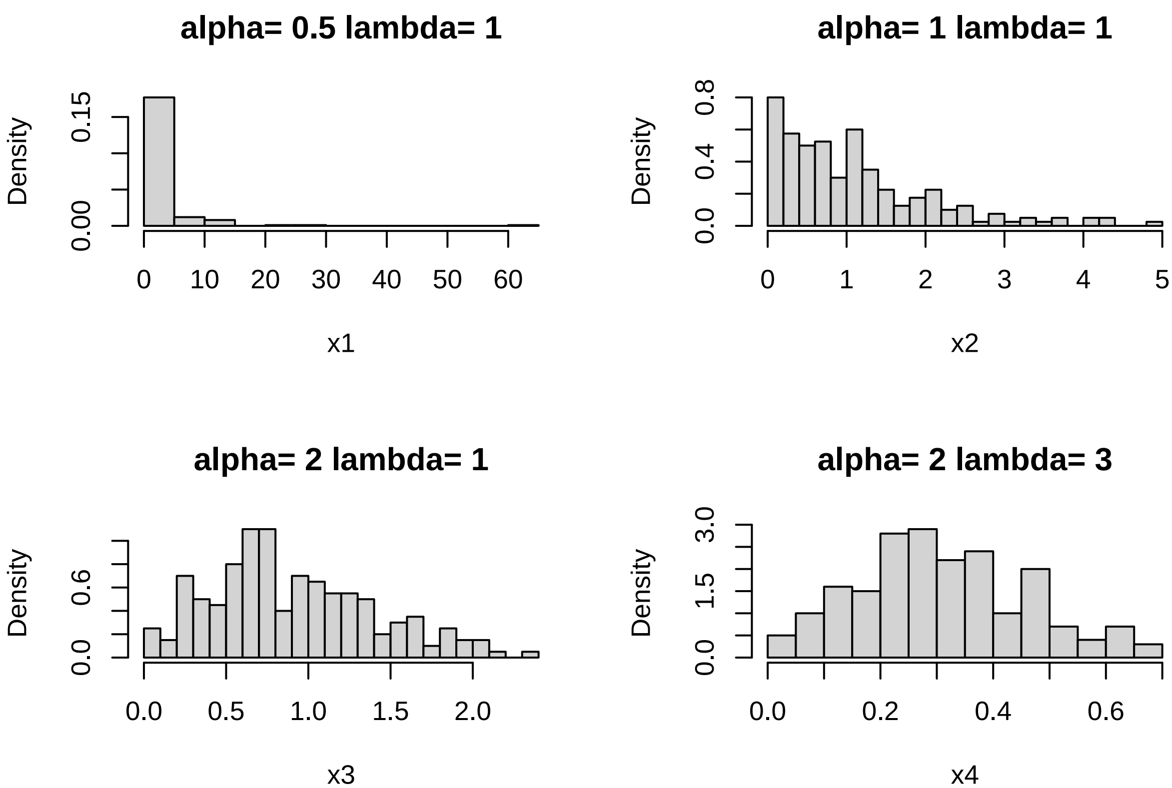
Solución

Apartado (i)

```
generaweib<- function(n,alfa,landa)
{
  U<- runif(n)
  ((-log(1-U))^(1/alfa))/landa   #Ejercicio: completar
}
#i)
n<-200
set.seed(129871)
x1<- generaweib(n,0.5,1)
x2<- generaweib(n,1,1)
x3<- generaweib(n,2,1)
x4<- generaweib(n,2,3)
```

Apartado (ii)

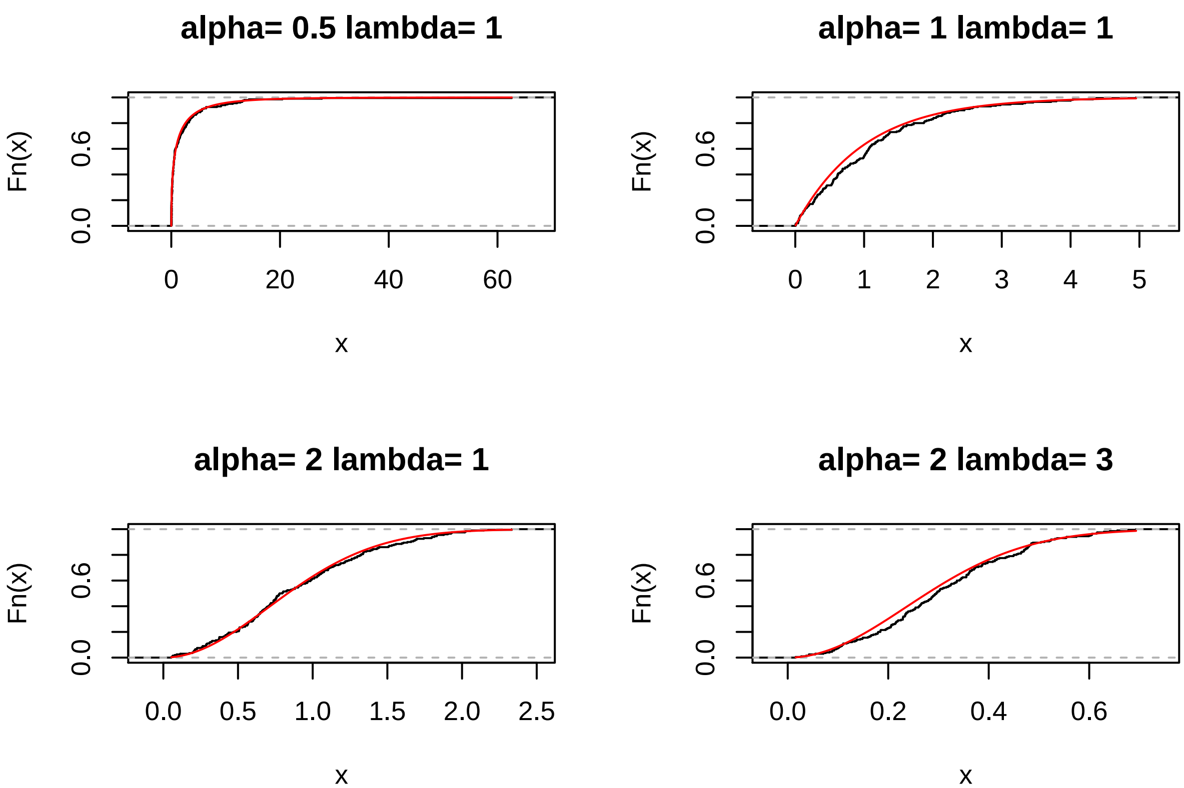
```
tit1<-paste("alpha=",0.5,"lambda=",1)
tit2<-paste("alpha=",1,"lambda=",1)
tit3<-paste("alpha=",2,"lambda=",1)
tit4<-paste("alpha=",2,"lambda=",3)

par(mfrow=c(2,2))
hist(x1,main=tit1,br=20,prob=TRUE)
hist(x2,main=tit2,br=20,prob=TRUE)
hist(x3,main=tit3,br=20,prob=TRUE)
hist(x4,main=tit4,br=20,prob=TRUE)
```




```
par(mfrow=c(1,1))
```

Apartado (iii)

```
par(mfrow=c(2,2))
plot(ecdf(x1),main=tit1,do.points=FALSE,verticals=TRUE)
curve(pweibull(x,0.5,1),min(x1),max(x1),1000,add=TRUE,col="red")
plot(ecdf(x2),main=tit2,do.points=FALSE,verticals=TRUE)
curve(pweibull(x,1,1),min(x2),max(x2),1000,add=TRUE,col="red")
plot(ecdf(x3),main=tit3,do.points=FALSE,verticals=TRUE)
curve(pweibull(x,2,1),min(x3),max(x3),1000,add=TRUE,col="red")
plot(ecdf(x4),main=tit4,do.points=FALSE,verticals=TRUE)
curve(pweibull(x,2,1/3),min(x4),max(x4),1000,add=TRUE,col="red")
```




```
#R usa 1/landa
par(mfrow=c(1,1))
```

Apartado (iv)

```
#iv) se deben estimar los parámetros por
# máxima verosimilitud:
#install.packages("fitdistrplus")
library(fitdistrplus)
```

```
## Loading required package: MASS

## Loading required package: survival
```

```
print(mv1<-mledist(x1,"weibull"))
```

```
## $estimate
## shape scale
##  0.49  1.07
##
## $convergence
## [1] 0
##
## $value
## [1] 236
##
## $hessian
##       shape scale
## shape  1551   -79
## scale   -79    41
```

```
##
## $optim.function
## [1] "optim"
##
## $optim.method
## [1] "Nelder-Mead"
##
## $fix.arg
## NULL
##
## $fix.arg.fun
## NULL
##
## $weights
## NULL
##
## $counts
## function gradient
##       43       NA
##
## $optim.message
## NULL
##
## $loglik
## [1] -236
```

```
ks.test(x1,"pweibull",mv1$estimate[1],
        1/mv1$estimate[2]) #R usa 1/landa
```

```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  x1
## D = 0.05, p-value = 0.8
## alternative hypothesis: two-sided
```

```
#otra opción es ajustar los datos mediante
#fitdist y luego aplicar gofstat:
#Aquí se muestra el p-valor del
#test chi-cuadrado de bondad de ajuste
```

```
fitweib1 <- fitdist(x1, "weibull")
summary(fitweib1)
```

```
## Fitting of the distribution ' weibull ' by maximum likelihood
## Parameters :
##       estimate Std. Error
## shape     0.49      0.027
## scale     1.07      0.165
## Loglikelihood:  -236   AIC:  476   BIC:  483
## Correlation matrix:
##       shape scale
## shape  1.00  0.31
## scale  0.31  1.00
```

```r
gofstat(fitweib1)$chisqpvalue
```

```
## [1] 0.37
```

```r
fitweib2 <- fitdist(x2, "weibull")
summary(fitweib2)
```

```
## Fitting of the distribution ' weibull ' by maximum likelihood
## Parameters :
##       estimate Std. Error
## shape      1.1      0.060
## scale      1.1      0.078
## Loglikelihood:  -218   AIC:  440   BIC:  447
## Correlation matrix:
##       shape scale
## shape  1.00  0.31
## scale  0.31  1.00
```

```r
gofstat(fitweib2)$chisqpvalue
```

```
## [1] 0.25
```

```r
fitweib3 <- fitdist(x3, "weibull")
summary(fitweib3)
```

```
## Fitting of the distribution ' weibull ' by maximum likelihood
## Parameters :
##       estimate Std. Error
## shape      1.9       0.11
## scale      1.0       0.04
## Loglikelihood:  -131   AIC:  267   BIC:  273
## Correlation matrix:
##       shape scale
## shape  1.00  0.31
## scale  0.31  1.00
```

```r
gofstat(fitweib3)$chisqpvalue
```

```
## [1] 0.32
```

```r
fitweib4 <- fitdist(x4, "weibull")
summary(fitweib4)
```

```
## Fitting of the distribution ' weibull ' by maximum likelihood
## Parameters :
##       estimate Std. Error
## shape     2.20      0.124
## scale     0.35      0.012
## Loglikelihood:  103   AIC:  -202   BIC:  -195
## Correlation matrix:
##       shape scale
## shape  1.00  0.31
## scale  0.31  1.00
```

```r
gofstat(fitweib4)$chisqpvalue
```

```
## [1] 0.43
```

Ejercicio 6

6. Leer el fichero datos en "Pesos.RData", y a continuación:

   i) Estimar la densidad por el método del núcleo.

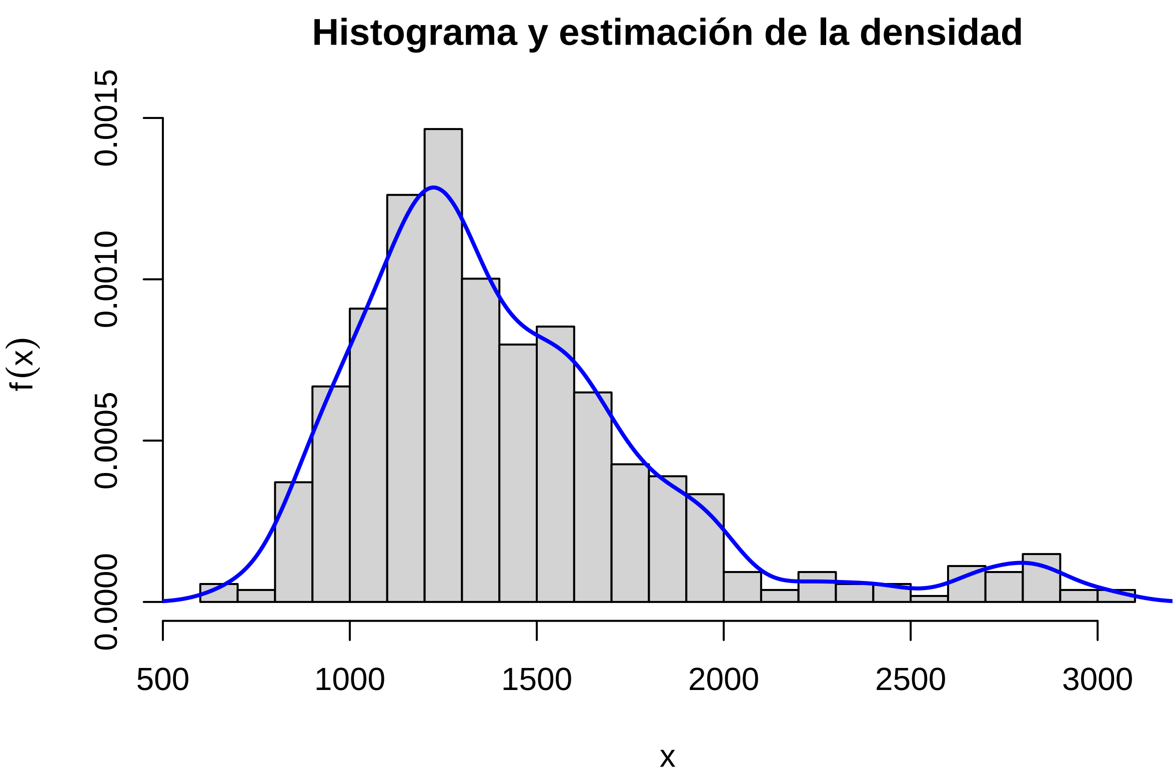
   ii) Escribir una función para generar valores según dicha densidad estimada.

   iii) Comparar las distribuciones de una muestra generada de tamaño 200 y el conjunto de datos original.

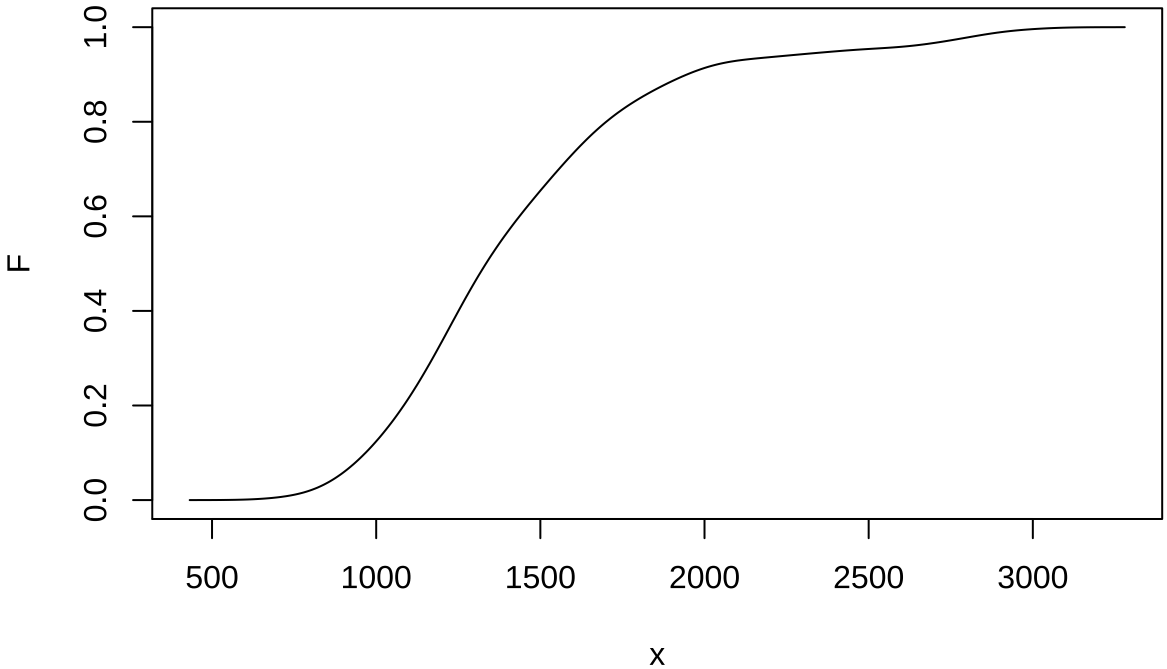
Solución

Apartado (i)

```
load("datos/Pesos.RData")
hist(datos,br=30,prob=TRUE,
     main="Histograma y estimación de la densidad",
     ylab = expression(hat(f)(x)),xlab="x")
lines(density(datos,bw="SJ"),col="blue",lwd=2)
```




```
estimnuc<- density(datos,bw="SJ",n=5000)
#Para tener más puntos
distrib<-data.frame(x=estimnuc$x,
                    F=cumsum(estimnuc$y)/sum(estimnuc$y))
plot(distrib,type="l")
```



Apartado (ii)

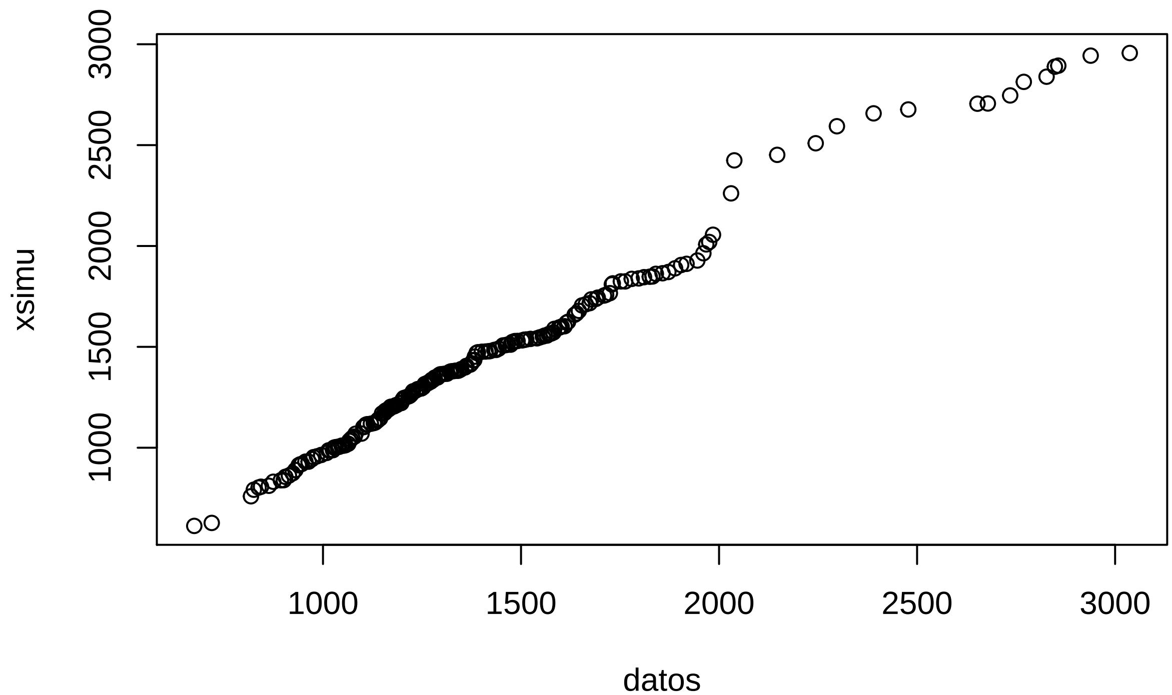
```
generax<- function(n,distrib)
{
 U<-runif(n)
 sapply(U, function(u) min(distrib$x[distrib$F>=u]) )
}

generax(10,distrib)
```

```
##  [1] 1284 1688 1524 1264 1053  870 1369 1310 1476 1257
```

```
xsimu<- generax(200,distrib)
```

Apartado (iii)

```
qqplot(datos,xsimu)
```




```
ks.test(datos,xsimu)
```

```
## Warning in ks.test(datos, xsimu): p-value will be approximate in the presence of
## ties
```

```
##
##  Two-sample Kolmogorov-Smirnov test
##
## data:  datos and xsimu
## D = 0.08, p-value = 0.3
## alternative hypothesis: two-sided
```

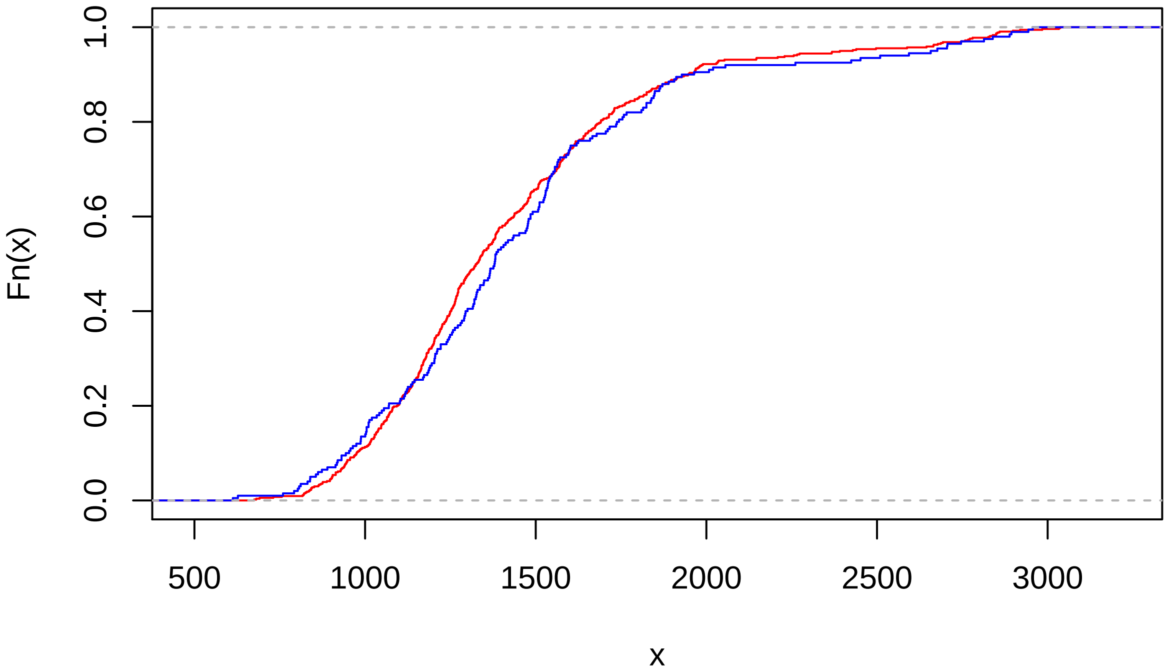
```
summary(datos)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     675    1144    1327    1428    1612    3037
```

```
summary(xsimu)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     613    1144    1379    1457    1607    2956
```

```
plot(ecdf(datos),main="",do.points=FALSE,
     verticals=TRUE,col="red")
lines(ecdf(xsimu),do.points=FALSE,
      verticals=TRUE,col="blue")
```



Ejercicio 7

7. Diseñar una función para generar realizaciones de una ley Geométrica simulando el proceso de conteo del número de fracasos antes del primer éxito en la repetición de ensayos Bernouilli. Probar la función y analizar los resultados.

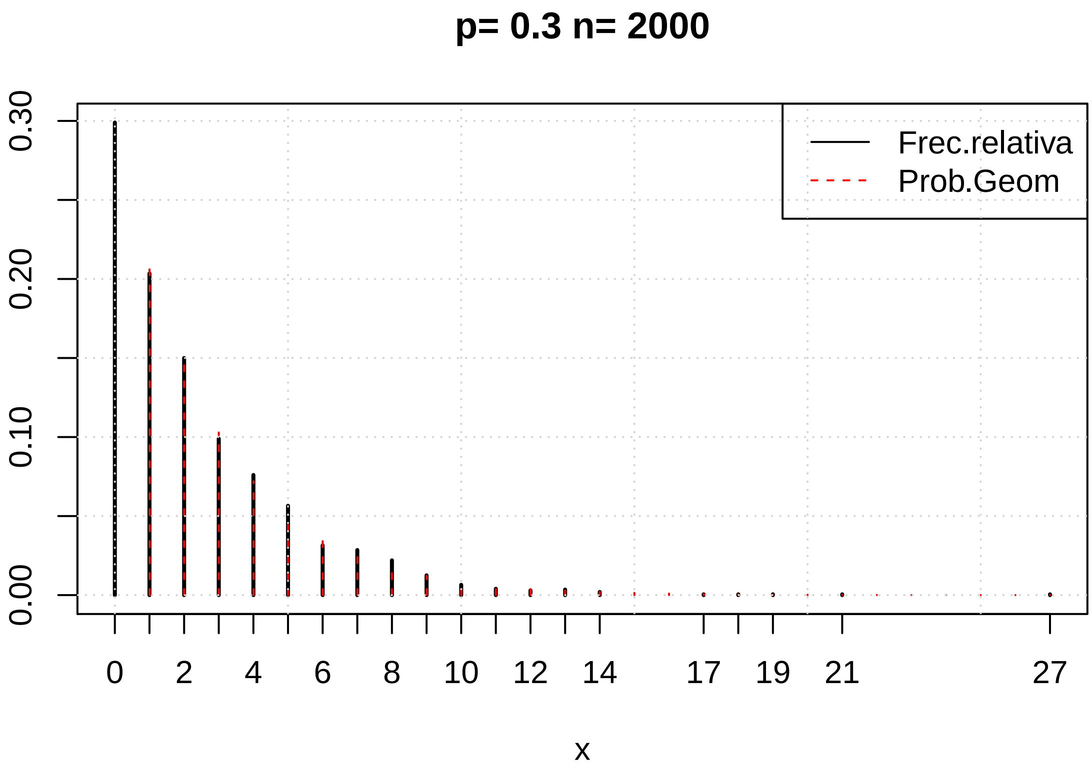
Solución

```
#para generar muestra de tamaño n de Ge(p)
Geom<- function(n,p)
{
 #Inicializaciones
 X<- integer(n)
 #algoritmo
 for (i in 1:n)
 {
 s<- -1
 repeat       #Completar
 {
 s<- s+1
 U<- runif(1)
 if (U <= p) break
}
 X[i]<- s
}
 X
}
```

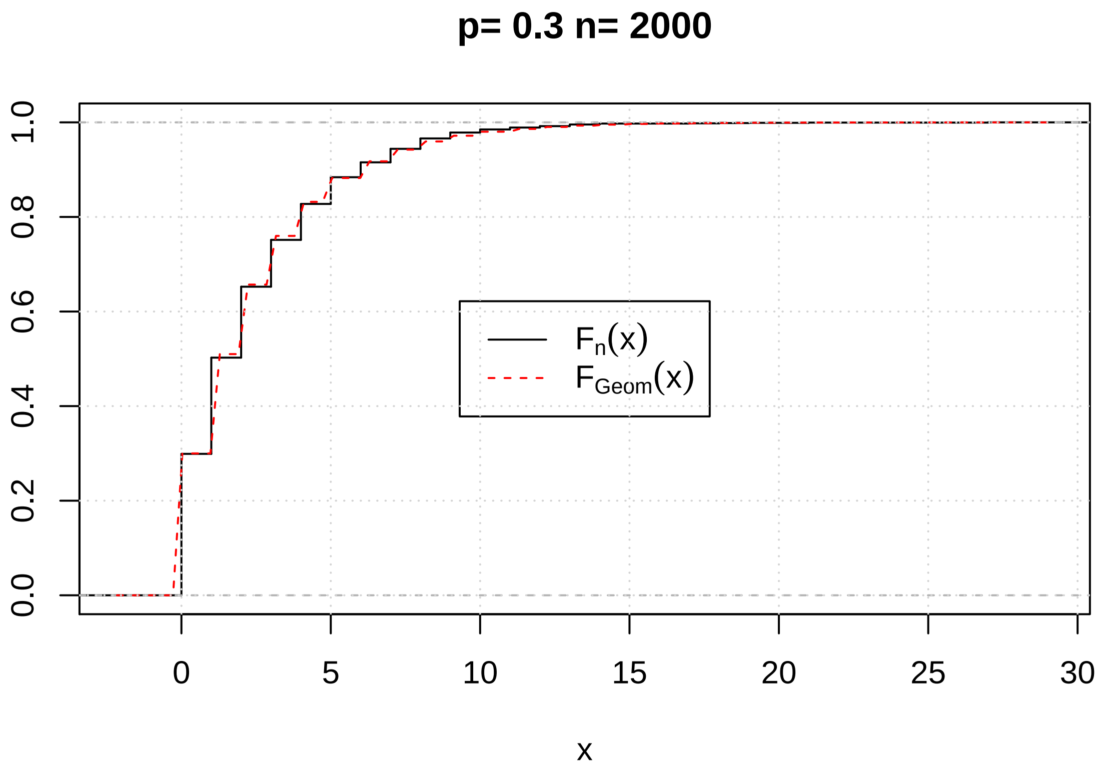
```
p<- 0.3
n<- 2000
set.seed(12345)
```

```r
x<- Geom(n,p)
tabla<-table(x)
```

```r
vx<- 1:max(x)
plot(prop.table(tabla),type="h",
     ylab="",main=paste("p=",p,"n=",n))
lines(vx,dgeom(vx,p),type="h",col="red",lty=2)
legend("topright",lty=1:2,col=c("black","red"),
       legend=c("Frec.relativa","Prob.Geom"))
grid()
```




```r
plot(ecdf(x),do.points=FALSE,verticals=TRUE,
     ylab="",main=paste("p=",p,"n=",n))
curve(pgeom(x,p),add=TRUE,col="red",lty=2)
legend("center",lty=1:2,col=c("black","red"),
       legend=expression(F[n](x),F[Geom](x)))
grid()
```



p= 0.3 n= 2000
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
x
$F_n(x)$
$F_{Geom}(x)$