University of Zurich[UZH]

Measuring the electron mass through Compton scattering

Data analysis 2025 – Group project II

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1 Motivation

In this project, you simulate an experiment to determine the electron mass using Compton scattering.

Compton scattering, the inelastic scattering of a photon on a charged particle (usually an electron), is named after Arthur Holly Compton who first described this process in 1923. In the scattering process, some of the energy of the photon is transfered to the recoiling charged particle, i.e. the energy of the photon decreases and its wavelength increases. Compton derived the following relation between the wavelength λ_{in} of the incoming photon, the wavelength λ_{out} of the outgoing photon and the angle θ by which the photon is scattered:

$$\lambda_{\text{out}} - \lambda_{\text{in}} = \frac{h}{m_e c}\,(1 - \cos\theta)\ ,$$

where h is Planck's constant, c is the speed of light and m_e is the mass of the electron. It is easy to rewrite this equation in terms of the energy E_{in} of the incoming photon and the energy E_{out} of the outgoing photon:

$$\frac{E_{\text{out}}}{E_{\text{in}}} = \frac{1}{1 + \frac{E_{\text{in}}}{mc^2}\,(1 - \cos\theta)}\ .$$

Measuring E_{out} as a function of $\cos\theta$ for known E_{in} will allow to determine m_e .

An important cross check in simulation studies is the so-called "pull", defined as

$$\text{pull} = \frac{\text{reconstructed quantity} - \text{generated quantity}}{\text{uncertainty on reconstructed quantity}}\ ,$$

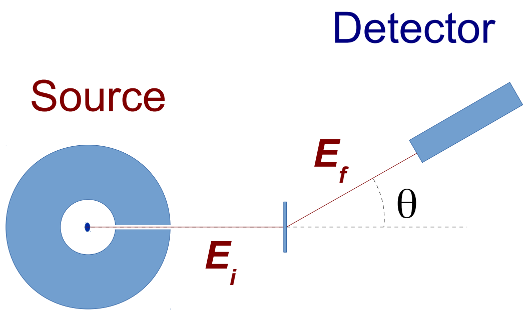

Figure 1: Setup of the experiment.

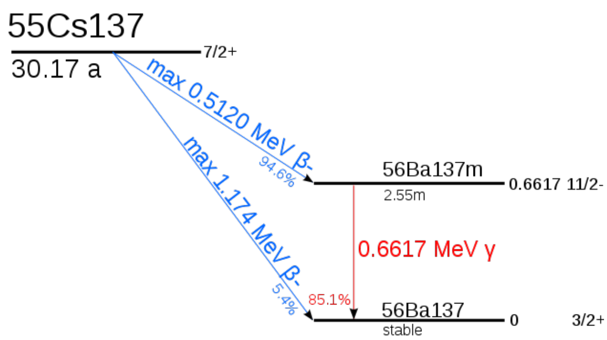

Figure 2: Decay scheme of ^{137}Cs (source: Wikipedia).

where the "uncertainty on reconstructed quantity" is for example that returned from your fit to the data. If the reconstruction is unbiased and the uncertainties on the reconstructed quantities is estimated correctly, the distribution of the pull over many simulated experiments should follow a Gaussian distribution with $\mu = 0$ and $\sigma = 1$. If μ deviates significantly from zero, the measurement is biased. If σ deviates significantly from one, the uncertainties are not estimated correctly.

2 Setup

A sketch of the setup is shown in Figure 1. It consists of a radioactive source emitting photons of a known energy E_{in}, a thin target in which these photons scatter on electrons, and a moveable detector to measure the final energy E_{out} of the photons at various scattering angles θ.

The source contains ^{137}Cs, a radioisotope of Cesium that decays to ^{137}Ba by β decay, i.e. by emitting an electron; with about 95% probability, the ^{137}Ba is produced in an excited state, which decays to the stable ground state by emitting a photon with an energy of 0.6617 MeV. The decay scheme is illustrated in Figure 2. The source is embedded in a protective shielding with only a small hole through which photons can escape. The incoming direction of the photons can be assumed to be known with negligible uncertainty and the scattering angle is given by the position of the detector.

In case you wonder, the electronvolt (eV) is a unit of energy used in nuclear and particle physics: 1 eV corresponds to the amount of energy that is gained by a particle with charge e when it is moved across a potential $\Delta V = 1$ V.

3 Simulate the experiment

First, simulate one measurement:

(a) Using the known mass of the electron and the known incoming energy of the photons, calculate the expected outgoing energy $E_{\text{out}}^{\text{true}}(\theta_i)$ of the photon for $\theta_i = 10, 20, 30 \ldots 80°$. Simulate the finite energy resolution of the photon detector by adding to each $E_{\text{out}}^{\text{true}}(\theta_i)$ a different random value ΔE_i, drawn from a Gaussian distribution with mean $\mu = 0$ and standard deviation $\sigma_E = 0.01$ MeV. This gives you a measured energy $E_{\text{out}}^{\text{reco}}(\theta_i) = E_{\text{out}}^{\text{true}}(\theta_i) + \Delta E_i$ for each θ_i. Plot your data points.

(b) Perform a maximum likelihood fit to your set of simulated data points $E_{\text{out}}^{\text{reco}}(\theta_i)$ to determine the measured electron mass, m_e^{reco}, and its uncertainty. Plot the result of your fit together with your data points.

4 Estimate the uncertainty of the experiment

Now simulate many measurements to check that the uncertainty returned by your fit makes sense:

(a) Repeat the simulation 1'000 times (each time with a new set of ΔE values, always drawn from a Gaussian distribution with mean $\mu = 0$ and standard deviation $\sigma_E = 0.01$ MeV).

(b) Produce a histogram with the 1'000 values of m_e^{reco} that you obtained. Determine the standard deviation of the distribution.

(c) Produce a histogram with the pull distribution for m_e^{reco}. Determine the mean and the standard deviation of the pull distribution.

Finally, check how the uncertainty on the electron mass depends on the energy resolution of your detector:

(a) Repeat the exercise for different values of the assumed energy resolution of your detector, i.e. change the standard deviation of the Gaussian from which you draw your ΔE values, first to $\sigma_E = 0.05$ MeV and then to $\sigma_E = 0.1$ MeV.