



# Predicting the viscosity of multi-walled carbon nanotubes/water nanofluid by developing an optimal artificial neural network based on experimental data<sup>☆</sup>

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## ARTICLE INFO

Available online 13 July 2016

### Keywords:

MWCNTs/water nanofluid  
Relative viscosity  
Optimal artificial neural network  
Margin of deviation

## ABSTRACT

Regarding the viscosity of the fluids which is an imperative parameter for calculating the required pumping power and convective heat transfer, based on experimental data, an optimal artificial neural network was designed to predict the relative viscosity of multi-walled carbon nanotubes/water nanofluid. Solid volume fraction and temperature were used as input variables and relative viscosity was employed as output variable. Accurate and efficient artificial neural network was obtained by changing the number of neurons in the hidden layer. The dataset was divided into training and test sets which contained 80 and 20% of data points respectively. The results obtained from the optimal artificial neural network exhibited a maximum deviation margin of 0.28%. Eventually, the ANN outputs were compared with results obtained from the previous empirical correlation and experimental data. It was found that the optimal artificial neural network model is more accurate compared to the previous empirical correlation.

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## 1. Introduction

Water is a common fluid for heat transfer applications such as heat exchangers, solar collectors, nuclear power industries, automobile radiators and so on. Water is inexpensive and non-toxic. The advantages of using water as a heat transfer fluid include high specific heat capacity, density, and availability. Despite all these advantages, this fluid has a low thermal conductivity. In order to improve the thermal conductivity of water, some researchers have proposed the dispersion of nanoparticles in liquids, which are called nanofluids [1–5]. Since dispersing the nanoparticles can lead to enhance the thermal conductivity of fluids, many researchers have focused on this topic [6–10]. These studies revealed that the thermal conductivity of nanofluids is higher than common fluids.

However, the viscosity of the fluids which is an imperative parameter for calculating the required pumping power and convective heat transfer is also affected by the dispersing nanoparticles in the base fluid. In this regard, many researchers have examined the viscosity of nanofluids, and reported that the viscosity of nanofluids is a function of temperature, concentration, size and shape of nanoparticles [11–19].

However, several experiments are needed to determine the viscosity of nanofluids at various temperatures and nanoparticles concentrations. For eschewing the costs of the experiments, which are expensive and time-consuming, artificial intelligence based on artificial neural network (ANN) model can be applied for prediction of the viscosity of nanofluids. Neural network unit processes datasets in the same way that the human brain does. ANN is made up of countless interconnected processing components which called neurons to work in parallel paths achieving a proper solution. An artificial neural network is implemented using an experimental data set, optimized network parameters, and appropriate training algorithms. Industrial applications usually provide large datasets which is an advantage of training an ANN model [20]. In fact, ANN model is an effective and applicable method for forecasting the relation between dependent and independent parameters which mathematical formulation is unavailable [21] or existed relationship has low precision [20,22]. Because of numerous applications of ANNs in various fields, many researchers applied different types of them to predict the thermophysical properties of nanofluids. Mention may be made of the research studies of Hojjat et al. [23], Longo et al. [24], Ariana et al. [25] and Hemmat Esfe et al. [26–30]. However, there are few studies for the viscosity of nanofluids employing soft computing methods. In this regard, Mehrabi et al. [31] recognized a model for predicting the effective viscosity of  $\text{Al}_2\text{O}_3$ ,  $\text{CuO}$ ,  $\text{TiO}_2$  and  $\text{SiO}_2$  water-based nanofluids by an FCM-ANFIS using experimental data. They

<sup>☆</sup> Communicated by W.J. Minkowycz.

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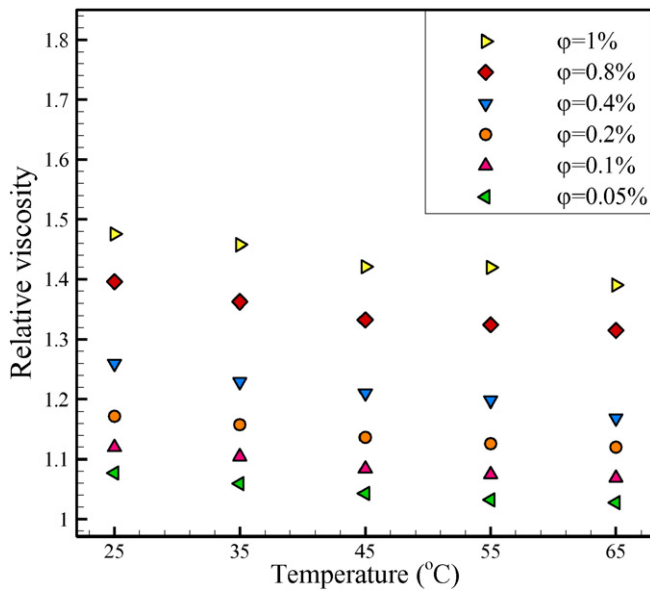


Fig. 1. Relative viscosity versus temperature for various solid volume fractions.

assumed the size of the nanoparticles, nanoparticle concentration and temperature as design parameters. Their comparisons showed that the predicted results agreed with the experimental data. Karimi et al. [32] designed an optimal ANN model based on genetic algorithm to predict the viscosity of nanofluids. They used the temperature and nanoparticle concentration as input variables. Their results revealed that optimal ANN model was in good agreement with experimental data. Lately, Hemmat Esfe et al. [33] designed an optimal ANN to predict both the thermal conductivity and dynamic viscosity of ferromagnetic. They used experimental data which include diameter of particles, temperature and concentration of nanoparticles as input variables. Their model outputs indicated that the ANN was able to predict the experimental data.

The study of the rheological behavior of the viscosity of nanofluids seemed necessary due to their numerous applications which include calculating the pumping power and convective heat transfer. On the other hand, artificial neural network models can be applied for

prediction of the viscosity of nanofluids with less cost and time. Moreover, literature survey indicates that there is no reported work about the ANN modeling of viscosity of MWCNTs/water nanofluid. Hence, in the present study, for the first time, an optimal artificial neural network has been modeled to predict the relative viscosity of this nanofluid using a set of experimental data.

## 2. Methodology

### 2.1. Experimental data for the analysis

Thirty experimental data for the relative viscosity of multi-walled carbon nanotubes MWCNTs/water nanofluids were used for developing models. The experimental data were present in a solid volume fraction ( $\phi$ ) ranging from 0 to 1% and a temperature ( $T$ ) ranging from 25 to 65 °C presented by Hemmat Esfe et al. [18] and shown in Fig. 1.

### 2.2. Development of artificial neural network model

Recently, since linear computing is unable to predict the nonlinear systems, the various ANN models has been used widely for determining relations between process parameters because of the significant advantages of high precision, low cost and time. As previously mentioned, the base of ANN is the human brain, which is a parallel processing network to determine the complex nonlinear relationships between independent and dependent variables [34].

There are three layers in an ANN model which called input, hidden, and output layers. Each layer consisted of some processing components, called neurons, as mentioned before. The ANN architecture gives the connections between the layers and neurons. The role of the hidden layers is to adjust input dataset to the new dataset of outputs. At first, specific weights considering randomly are assigned to inputs. This step is called as a training step of ANN. In the training step, the extracted errors from experiments and predicted values are returned to the beginning of networks where it makes all corrections to weights. This procedure is continued until the desired outcome was extracted. The number of neurons in the input and output layers is related to the number of input and output parameters. In this study, the transfer function of hidden layer was considered as Tan-Sigmoid, while Log-sigmoid

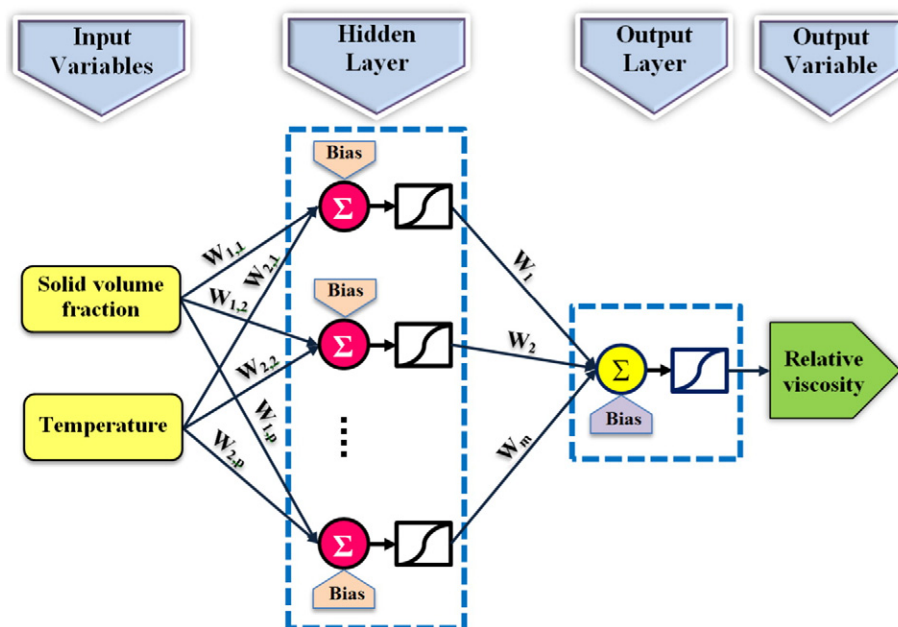


Fig. 2. Structure of the feed forward neural network ( $W_{sp}$  is the weight of sth elements in input vector to pth neurons and  $W_m$  is the weight of mth neurons to output).

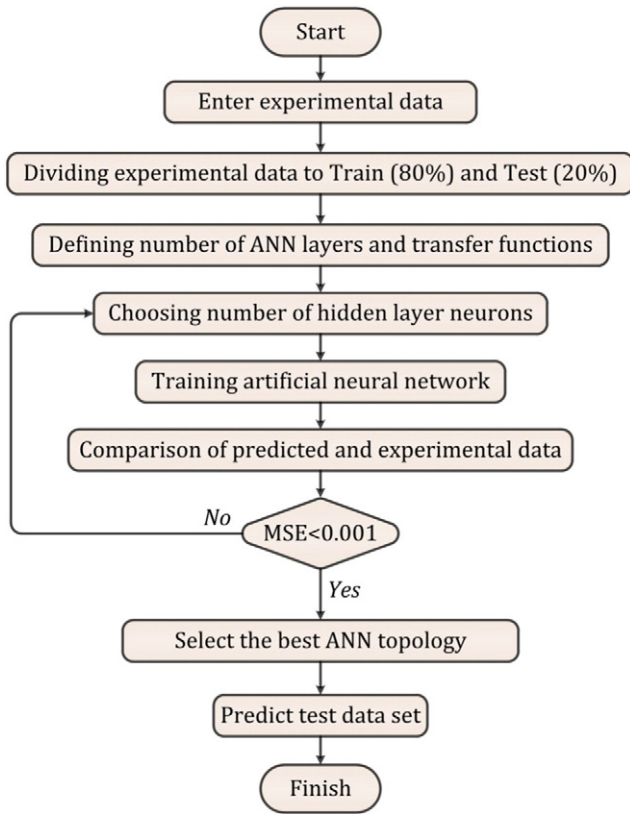


Fig. 3. Flowchart of the suggested algorithm to achieve the optimal neural network.

transfer function was used for the output layer. These transfer functions are more popular than others such as pureline and step functions [35].

Since the Levenberg–Marquardt generally leads to better outputs [36,37], the feed forward Levenberg–Marquardt has been applied as the training algorithm. In this study, network has been used as the training algorithm. The structure of the feed forward neural network is presented in Fig. 2. Temperature ( $T$ ) and solid volume fraction ( $\phi$ ) were employed as input parameters and the relative viscosity of the nanofluid ( $\mu_r$ ) was used as outputs variable. Thirty experimental data, aforementioned in Section 2.1, were employed to feed the model. The

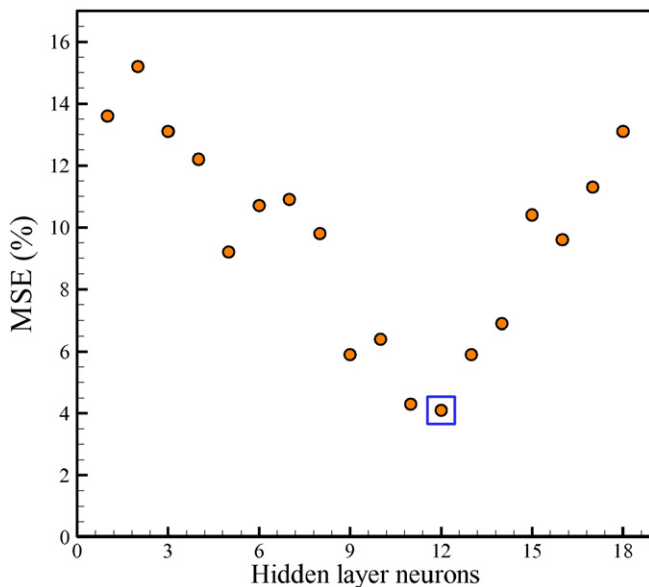


Fig. 4. MSE based on variation of neurons in hidden layer.

Table 1

Comparison between the ANN outputs and experimental data for the test data set.

No.	Input variables		Relative viscosity of the nanofluid		
	$\phi$ (%)	Temperature ( $^{\circ}\text{C}$ )	Experimental	ANN model	APE (%)
1	0.8	25	1.3956	1.4106	1.075
2	0.2	35	1.1577	1.1680	0.890
3	0.05	45	1.0426	1.0530	1.001
4	0.1	55	1.0746	1.0832	0.798
5	0.8	55	1.3245	1.3360	0.866
6	0.8	65	1.3151	1.3260	0.832
MAPE (%)					0.910

dataset was divided to training and test sets which contained 80 and 20% of data points respectively. All data values have been to be scaled into the range of  $(-1, 1)$  using Eq. (1).

$$X_{\text{norm}} = 2 \times \frac{X - X_{\min}}{X_{\max} - X_{\min}} - 1 \quad (1)$$

The criteria for selecting an appropriate network are mean square error (MSE) and avoiding over-fitting. The MSE can be expressed as follows:

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (Y_{\text{pre}} - Y_{\text{ex}})^2 \quad (2)$$

where  $Y_{\text{ex}}$  is the experimental output,  $Y_{\text{pre}}$  is the network output and  $N$  is the number of data points.

Optimized number of neurons in hidden layer based on a variety of different structures is obtained by trial and error [38]. Fig. 3 shows the flowchart of the suggested algorithm to achieve the optimal neural network.

### 3. Results and discussion

As shown in Fig. 3, to attain an accurate and efficient artificial neural network, the number of neurons in the hidden layers has been changed. Then, the optimal network has been selected by considering the minimum MSE and avoiding over-fitting. In this regard, the MSEs of different structures by changing neurons of the hidden layer are illustrated in

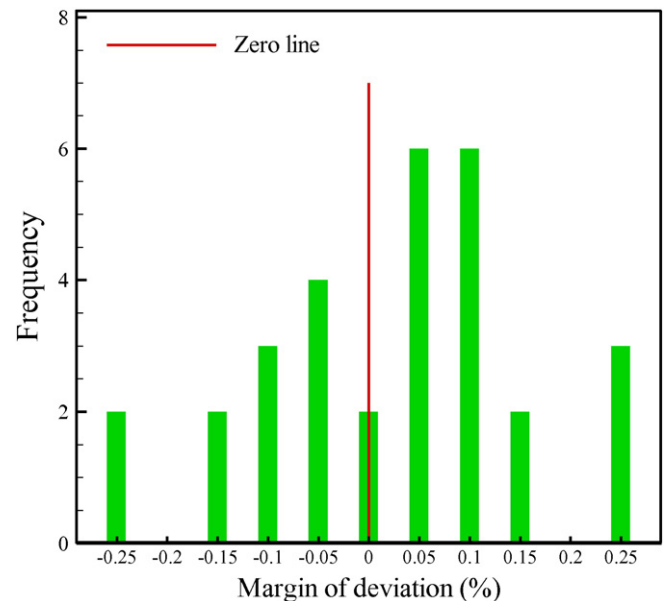


Fig. 5. Histogram of deviation margin for optimal ANN.

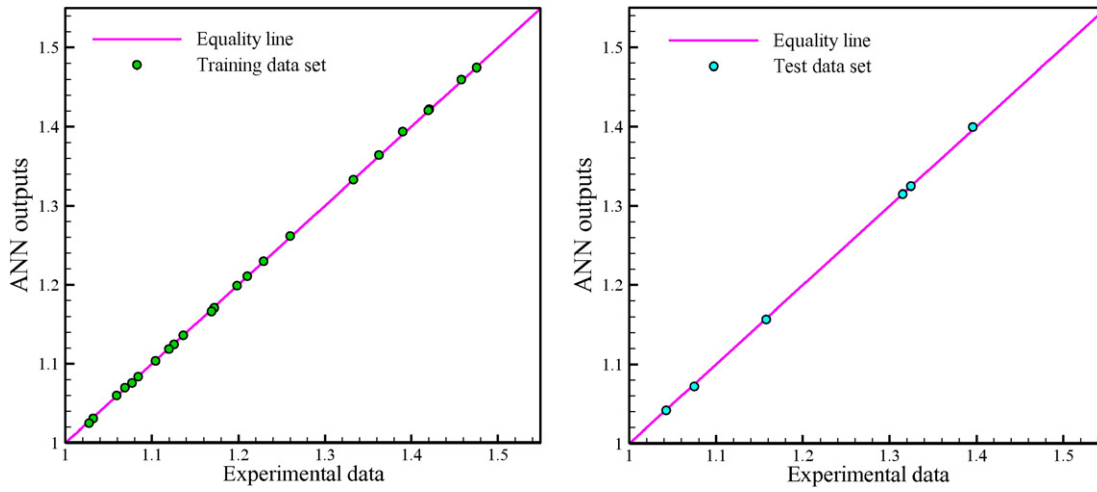


Fig. 6. Optimal ANN outputs versus experimental data for training and test data sets.

Fig. 4. It can be seen that an ANN model with 12 neurons in hidden layers has a minimum value of MSE.

The test data set was chosen from the experimental data points randomly in order to examine the accuracy of neural network model. It is notable that test data set was not employed during the training step. The mean absolute percentage error (MAPE) of relative viscosity of the nanofluid for test data set is 0.910 as presented in Table 1. This value demonstrates the high accuracy and efficiency of the presented ANN model.

The accuracy of optimal ANN can be evaluated by deviation analysis of the relative viscosity. The deviation between ANN outputs and experimental data is calculated as follows:

$$\text{Margin of deviation} = \left[ \frac{\left( \frac{\mu_{nf}}{\mu_{bf}} \right)_{Exp} - \left( \frac{\mu_{nf}}{\mu_{bf}} \right)_{ANN}}{\left( \frac{\mu_{nf}}{\mu_{bf}} \right)_{Exp}} \right] \times 100 \quad (3)$$

The histogram of deviation margin for the optimal ANN is presented in Fig. 5. It can be observed that deviation distribution is concentrated around zero, which means that the optimal ANN has a suitable accuracy.

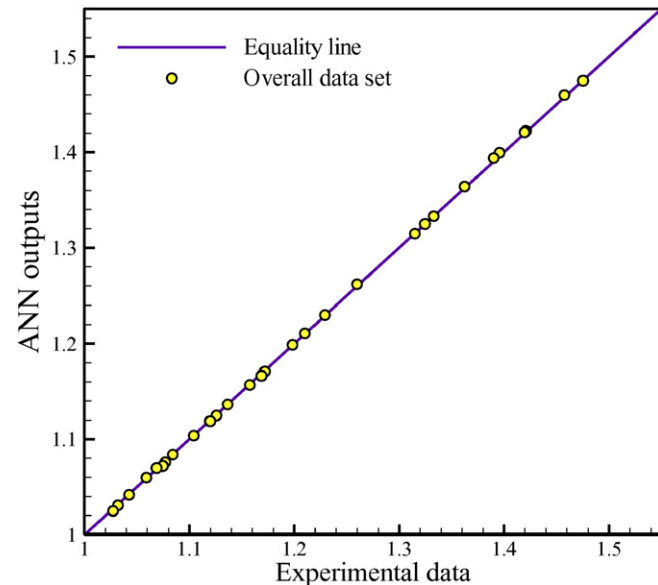


Fig. 7. Optimal ANN outputs versus experimental data for training and test data sets.

Fig. 6 shows the optimal ANN outputs versus experimental relative viscosity for training and test data sets. The excellent fits have been observed on the bisector line for both data sets, which show a well-trained neural network. A comparison between optimal ANN outputs and experimental data for the overall data set is presented in Fig. 7. As can be seen, most points are on the bisector line.

The following empirical correlation is suggested by Hemmat Esfe et al. [18] for determination of the relative viscosity of MWCNTs/water nanofluid (Eq. (4)). They proposed this correlation for volume fractions up to 1%, and the temperatures ranging from 25 °C to 55 °C.

$$\frac{\mu_{nf}}{\mu_{bf}} = 38.158\varphi - 0.0017357T + 1.1296 \quad (4)$$

To demonstrate the ability of ANN to model the experimental data, the experimental relative viscosity, the data obtained from the empirical correlation (Eq. (4)), and the optimal ANN outputs are established in Fig. 8. This figure shows that the experimental results and ANN outputs are in excellent agreement, while the empirical correlation has a high deviation from experimental results. In fact, the ANN

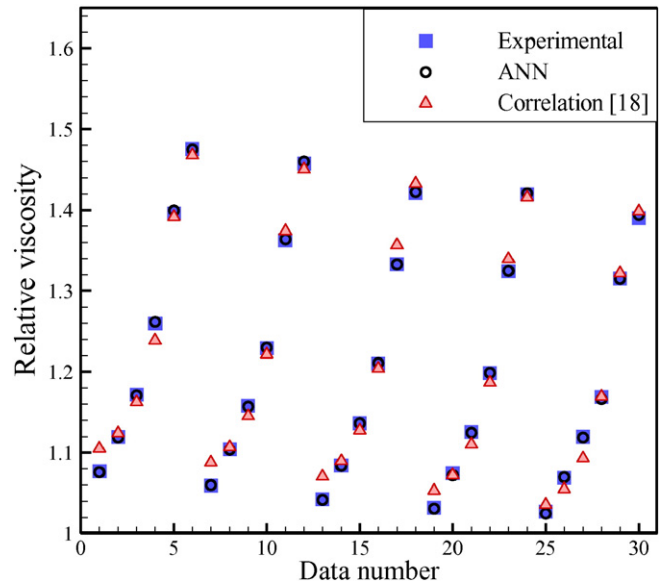


Fig. 8. Comparison of ANN outputs with experimental data and empirical correlation outputs.



model, designed in this study, is more accurate compared to the correlation proposed by previous work [18].

#### 4. Conclusion

In the present work, at the first, an accurate and efficient artificial neural network was designed to predict the relative viscosity of MWCNTs/water nanofluid. Solid volume fraction and temperature were used as input variables and relative viscosity was employed as output variable. The results gained from the optimal artificial neural network showed a maximum deviation margin of 0.28%. Then, the ANN outputs were compared with results obtained from the previous empirical correlation and experimental data. It was found from this comparison that the optimal artificial neural network model is more accurate compared to the previous empirical correlation.

#### Acknowledgement

The first and third authors are grateful to Najafabad Branch, Islamic Azad University, Najafabad, Iran for support in obtaining the experimental data. The fourth and fifth authors gratefully acknowledge the High Impact Research Grant “UM.C/HIR/MOHE/ENG/23” and Faculty of Engineering, University of Malaya, Malaysia for support in conducting this research work.

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