An ALE Finite Element Method for 2D Navier-Stokes Equation with Species Transport Equation

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Outline

Introduction

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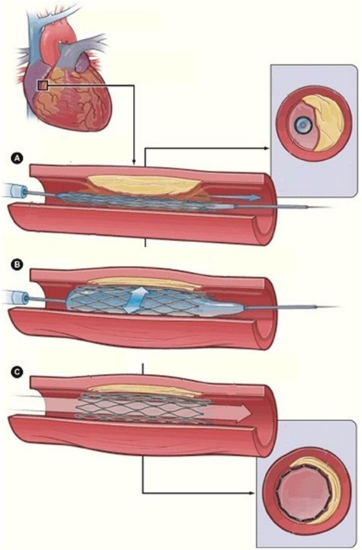
Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Goals:

- To develop a ALE-FE code for 2D Navier-Stokes Equation with Species Transport Equation

- To create new drug-eluting stent design patent



[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

1. Introduction
2. Mathematical Model
3. Results
4. Conclusion

Arbitrary Lagrangian-Eulerian (ALE)

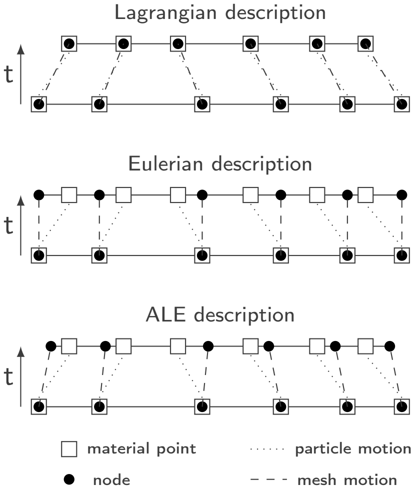
The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment



[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian–Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

Governing Equations

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Assumptions [3]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}-\hat{\mathbf{v}})\cdot\nabla\mathbf{v} = -\nabla p+\frac{1}{Re}\nabla^2\mathbf{v}$$

$$\nabla\cdot\mathbf{v} = 0$$

$$\frac{\partial e}{\partial t}+(\mathbf{v}-\hat{\mathbf{v}})\cdot\nabla e = \frac{1}{ReSc}\nabla^2 e$$

- If the mesh velocity field $\hat{\mathbf{v}} = \mathbf{v}$ (*Lagrangian*) or $\hat{\mathbf{v}} = 0$ (*Eulerian*)

[3] Panton, R. (2013). Incompressible Flow John Wiley & Sons, Ltd

Semi-Lagrangian Method

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The implicit semi-Lagrangian time discretization provides [4]:

$$\frac{\mathbf{v}_i^{n+1} - \mathbf{v}_d^n}{\Delta t} = -\nabla p^{n+1} + \frac{1}{Re}\nabla^2 \mathbf{v}^{n+1}$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\frac{e_i^{n+1} - e_d^n}{\Delta t} = \frac{1}{ReSc}\nabla^2 e^{n+1}$$

Advantages:

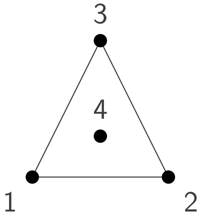
- Symmetric linear systems
- Unconditionnal stability

Disadvantages:

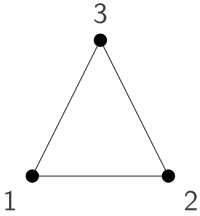
- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed

[4] Pironneau, O. On the transport-diffusion algorithm and its applications to the Navier-Stokes equations. Numer. Math. 38, 309–332 (1982). *https://doi.org/10.1007/BF01396435*

Galerkin FE Method



Mini Element



Linear Element

$$\begin{bmatrix} \frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re} & -\mathbf{G} \\ \mathbf{D} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_i^{n+1} \\ p \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{M}}{\Delta t}\mathbf{v}_d^n \\ 0 \end{bmatrix} + bc$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc}\right] e_i^{n+1} = \frac{\mathbf{M}}{\Delta t} e_d^n + bc_e$$

Mesh Velocity

The computational mesh velocity $\hat{\mathbf{v}}$ is defined as a linear combination of other velocities, such as:

$$\hat{\mathbf{v}} = \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2$$

where, $\mathbf{v}_1$ is the Lagrangian velocity, $\mathbf{v}_2$ is the Laplacian Smoothing velocity, β_1 and β_2 are the parameters control the Lagrangian and Laplacian motion respectively.

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Results

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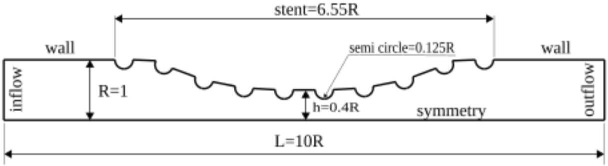

Non-dimensional domain for the analysis of concentration diffusion in coronary artery with drug-eluting stent, where the atherosclerosis was modeled by a sinusoidal equation with 40% of channel obstruction.

Parameters of simulation [5]:

$$R = 0.0015m$$
$$\mu = 0.0035Pa.s$$
$$\rho = 1060kg/m^3$$
$$u = 12cm/s$$

$$\beta_1 = 0$$
$$\beta_2 = 1$$
$$Re = 54.5$$
$$Sc = 1.0$$

numNodes = 1549
numVerts = 577
numElements = 972
interações = 253
dt = 0.025

[5] Bozsak, F.; J-M., C.; Barakat, A. Modeling the transport of drugs eluted from stents: physical phenomena driving drug distribution in the arterial wall. Biomech Model Mechanobiol 2014, 13, 327–347. *doi:10.1007/s10237-013-0546-4*

Conclusion

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1. The ALE description allows to perform simulations in the fluid-structure and moving boundary problems
2. The semi-Lagragian Method showed to be very useful due to the unconditionnal stability and symmetric linear system
3. The Navier-Stokes allows a smooth implementation for 3D cases

Further developments:

1. Sparse matrix implementation
2. 3D Navier-Stokes implementation
3. Blood flow model as a non-Newtonian fluid
4. Blood flow model as a multiphase problem

Thank you!

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