An ALE Finite Element Method for 2D Navier-Stokes Equation with Species Transport Equation

Student Researcher: Leandro Marques
Advisors: Gustavo Anjos

Federal University of Rio de Janeiro
April, 22th 2021

COPPE
UFRJ

Outline

1. Introduction
2. Mathematical Model
3. Results
4. Conclusion

Introduction

COPPE UFRJ

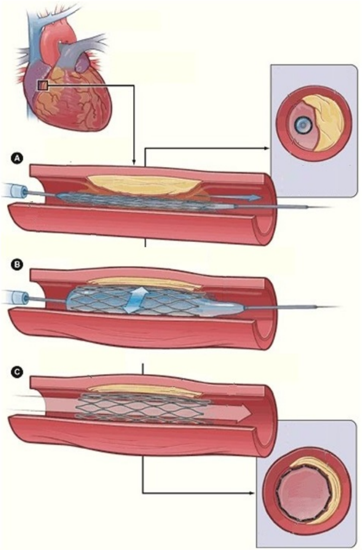
Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Goals:

- To develop a ALE-FE code for 2D Navier-Stokes Equation with Species Transport Equation

- To create new drug-eluting stent design patent



[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

Arbitrary Lagrangian-Eulerian (ALE)

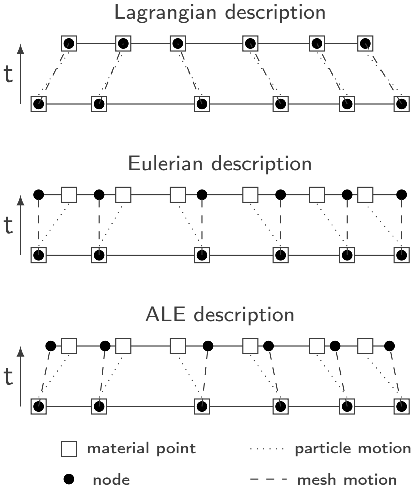
The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment



[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian–Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

Governing Equations

COPPE UFRJ

Assumptions [3]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla \mathbf{v} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{v}$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\frac{\partial e}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla e = \frac{1}{ReSc} \nabla^2 e$$

- If the mesh velocity field $\hat{\mathbf{v}} = \mathbf{v}$ (*Lagrangian*) or $\hat{\mathbf{v}} = 0$ (*Eulerian*)

[3] Panton, R. (2013). Incompressible Flow John Wiley & Sons, Ltd

Semi-Lagrangian Method

COPPE UFRJ

The implicit semi-Lagrangian time discretization provides [4]:

Advantages:

- Symmetric linear systems
- Unconditionnal stability

Disadvantages:

- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed

$$\frac{\mathbf{v}_i^{n+1} - \mathbf{v}_d^n}{\Delta t} = -\nabla p^{n+1} + \frac{1}{Re}\nabla^2 \mathbf{v}^{n+1}$$

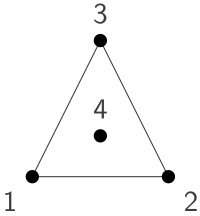
$$\nabla \cdot \mathbf{v} = 0$$

$$\frac{e_i^{n+1} - e_d^n}{\Delta t} = \frac{1}{ReSc}\nabla^2 e^{n+1}$$

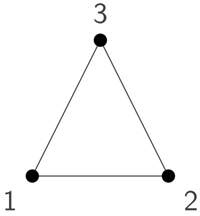
[4] Pironneau, O. On the transport-diffusion algorithm and its applications to the Navier-Stokes equations. Numer. Math. 38, 309–332 (1982). *https://doi.org/10.1007/BF01396435*

Galerkin FE Method

COPPE
UFRJ



Mini Element



Linear Element

$$\begin{bmatrix} \frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re} & -\mathbf{G} \\ \mathbf{D} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_i^{n+1} \\ p \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{M}}{\Delta t}\mathbf{v}_d^n \\ 0 \end{bmatrix} + bc$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc} \right] e_i^{n+1} = \frac{\mathbf{M}}{\Delta t} e_d^n + bc_e$$

Mesh Velocity

COPPE UFRJ

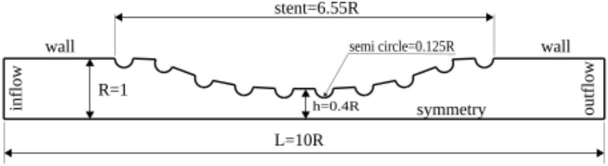
The computational mesh velocity $\hat{\mathbf{v}}$ is defined as a linear combination of other velocities, such as:

$$\hat{\mathbf{v}} = \beta_1 \mathbf{v}_1 + \beta_2 \mathbf{v}_2$$

where, $\mathbf{v}_1$ is the Lagrangian velocity, $\mathbf{v}_2$ is the Laplacian Smoothing velocity, β_1 and β_2 are the parameters control the Lagrangian and Laplacian motion respectively.

Results

COPPE UFRJ



Non-dimensional domain for the analysis of concentration diffusion in coronary artery with drug-eluting stent, where the atherosclerosis was modeled by a sinusoidal equation with 40% of channel obstruction.

Parameters of simulation [5]:

$R = 0.0015m$	$\beta_1 = 0$	numNodes $= 0$
$\mu = 0.0035Pa.s$	$\beta_2 = 1$	numVerts $= 1$
$\rho = 1060kg/m^3$	$Re = 54.5$	numElements $= 54.5$
$u = 12cm/s$	$Sc = 1.0$	dt $= 0$ nt $= 0$

[5] Bozsak, F.; J-M., C.; Barakat, A. Modeling the transport of drugs eluted from stents: physical phenomena driving drug distribution in the arterial wall. Biomech Model Mechanobiol 2014, 13, 327–347. *doi:10.1007/s10237-013-0546-4*

Conclusion

COPPE UFRJ

1. The ALE description allows to perform simulations in the fluid-structure and moving boundary problems, in addition it is possible to assign several mesh velocity values in specific regions of the problem in order to improve the accuracy of the numerical results

2. The searching procedure of the semi-Lagrangian Method used in this work was well designed since it corresponds to only 2% of average computational cost of the numerical simulation. Therefore, the methos showed to be very useful due to the unconditionnal stability and symmetric linear system

3. As expected, the chemical species transport in blood flow is directly influenced by Schmidt number, where the diffusion is increased as the Schmidt number decreases. It is possible that the density and viscosity of the blood are affected by chemical species diffusion. However, this influence is not considered in this work

Further developments:

COPPE UFRJ

1. Increase assembly performance
2. The use of primitive variables in the 3D Navier-Stokes equation
3. Blood flow model as a multiphase problem
4. Blood flow model as a non-Newtonian fluid

Thank you!

marquesleandro67@gmail.com