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BLOOD FLOW NUMERICAL SIMULATION USING ALE-FE METHOD FOR 2D NAVIER-STOKES EQUATION

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Abstract. *The present work aims at developing a computational framework to simulate the drug diffusion in the bloodstream in a coronary artery with drug-eluting stent implanted. The blood was modeled as a single-phase, incompressible and Newtonian fluid and the Navier-Stokes equation was approximated according to the Finite Element Method (FEM) in an Arbitrary Lagrangian-Eulerian (ALE) description. The dynamics of drug-eluting concentration in bloodstream was investigated using the drug-eluting stent in microchannels with atherosclerosis. The results reveal the possibility of other simulations using complex geometries.*

Keywords: *Navier-Stokes; Finite Element Method; Semi-Lagrangian; Drug-Eluting Stent; Arbitrary Lagrangian-Eulerian.*

1. INTRODUCTION

According to the World Health Organization (WHO), cardiovascular diseases have remained the leading death causes globally in the last 15 years. It is estimated that 15.2 million people died from CVD in 2016, representing 26.7% of all deaths in the world. In Brazil, about 38% of deaths due to CVD is in the procutive age range (18 to 65 years) and the estimated costs of CVD were R$ 37.1 billion in 2015, that is, 0.7% GDP (Siqueira *et al.*, 2017). About 60% of CVD deaths occurred due to coronary artery disease (CAD). The main cause of CAD is the atherosclerosis which consists of the accumulation of fat plaques inside the artery wall causing a decrease in lumen diameter. The Atherosclerosis can be prevented with a change in harmful habits such as: cigarette smoking, physical inactivity/low fitness and poor dietary habits (Spring *et al.*, 2013). For a corrective approach, however, two treatments can be performed: the *Coronary Artery Bypass Graft* (CABG) or a procedure minimally invasive called *Percutaneous Transluminal Coronary Angioplasty* (PTCA) (Gruntzig *et al.*, 1979), where a wire tube (*stent*) is placed.

In 2001, Hwang *et al.* presented a numerical simulation in a coronary artery, where the stent strut was coated with the drug. Such procedure proved to be a promising option for the treatment of atherosclerosis and reestonosis. This new type of stent would be known as *drug-eluting stent*. In the following years, Zunino *et al.* (2009) presented a complete overview of mathematical models and finite element numerical simulation applied to the modeling of drug eluting stens and of their interaction with the coronary arteries, take into account the stent expansion, fluid dynamics around the stent and drug release. The numerical simulation showing recirculation zones downstream has important consequences on the drug release process. The authors concluded that the drug released into the lumem does not significantly contribute to the permanent drug deposition into the arterial wall and only an small fraction of the total amount drug stored into the stent was effectively delivered to the artery. In 2018, Lucena *et al.* presented the simulation of the transport of the drug *sirolimus* on the wall of an artery modeled as a porous and anisotropic medium. The authors estimated that about 47% of the drug is diffused in the lumen and it is lost in the bloodstream. The spatial distribution of the drug, however, is greatly influenced by blood flow and the properties of the artery wall. Thus, such results are susceptible to the patient health conditions. In the following years, Marques and R. Anjos (2021) presented the a numerical study on the drug diffusion in the bloodstream in a coronary artery with drug-eluting stent implanted, using the streamfunction-vorticity formulation. The atherosclerosis has a direct influence on the drug-eluting concentration field, modifying the dynamics of the drug diffusion in the bloodstream and the horizontal velocity reaches three times the blood velocity in coronary artery without atherosclerosis. Therefore, we propose the development of a numerical code using an ALE-FE method for 2D Navier-Stokes Equation with the drug diffusion in bloodstream.

The dynamics of concentration diffusion in coronary arteries requires a robust numerical method to compute the solution of the differential equations in a relevant model, especially due to the thin concentration boundary layer in the vicinity of the implanted stents that must be accurately capture. The equations that govern the dynamics of blood flow in a coronary artery were developed according to continuum media assumption. Thus, the universal conservation laws such as mass, momentum and species transport were used in an Arbitrary Lagrangian-Eulerian context. The blood was modeled as single-phase, incompressible, Newtonian and a diffusion coefficient were taken under consideration to evaluate different drug transport alongside the blood flow. Thus, the Navier–Stokes equation is presented with drug-eluting transport equation using the Finite Element method. The domain was discretized on an unstructured mesh using the GMSH open source software, where the triangular element with linear interpolation was used for pressure field and with cubic interpolation was used for velocity and concentration fields. Therefore, the Babuska-Brezzi was satisfied. The

equations were discretized in spatial domain using the Galerkin formulation and in temporal domain the semi-Lagrangian scheme was used to discretized the material derivative using first order backward difference scheme. The computational code was developed in Python Language using the Object-Oriented Paradigm (OPP) and the linear system of equations that comes from implementing the FEM is solved through direct solver available in the public library for scientific tools SciPy, using the sparse matrices.

The dynamics of concentration diffusion in a coronary artery with drug-eluting stent was investigated in a complex geometry, using a two-dimensional approach. That is, the coronary artery was approximated as a plannar domain. Due to symmetry of problem, only half domain was simulated. The case is called the *Curved Channel with Stent*, where this geometry is a model of the real coronary artery and an atherosclerosis was considered using a sinusoidal equation to model 40% of channel obstruction. The concentration diffusion was investigated using a drug concentration diffusity, whose *Schmidt numbers* Sc was: $Sc = 1$, which are equivalent to drug-lumen mass diffusion rate of $D = 3{\times}10^6$ m$^2/s$ respectively.

2. MATHEMATICAL MODEL

The dynamics of concentration diffusion in coronary arteries require a robust numerical method to compute the solution of the differential equations in a relevant model. The equations that govern the dynamics of blood flow in a coronary artery were developed according to continuum media assumption. Thus, the universal conservation laws such as mass, momentum and species transport were used in an Arbritary Lagragian-Eulerian context. The blood was modeled as single-phase, incompressible, Newtonian and a drug diffusion coefficient was taken under consideration to evaluate the drug transport alongside the blood flow. The Navier-Stokes equation is shown with species transport equation in a 2-dimensional approach:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\frac{D\mathbf{v}}{Dt} = -\nabla p + \frac{1}{Re}\nabla^2 \mathbf{v} \tag{2}$$

$$\frac{De}{Dt} = \frac{1}{ReSc}\nabla^2 e \tag{3}$$

where, **v** is the material velocity field, *p* is the pressure field, *e* is the concentration field, *D/Dt* is the material derivative, $Re = \rho u \mathrm{R}/\mu$ is the Reynolds number, where ρ is the blood density (kg/m^3), u is the blood velocity in the coronary artery (m/s), R is the lumen radius of the coronary artery (m) and μ is the blood viscosity (N.s/m^2), $Sc = \mu/\rho D$ is the Schmidt number, where D is the mass diffusion rate of the drug (m^2/s) and *x* and *y* are the spatial variables.

The boundaries conditions used were:

- *inflow condition*: this condition was specified in the left line of the geometries in this work, where an mass inflow is desired. For such a condition, $u = u_o$ and $v = v_o$, where $u_o = 1.5[1 - (y/R)^2]$ and $y_o = 0$
- *wall condition*: this condition is specified at wall boundary, that is, the top line of the geometries in this work, where the no-slip condition is set. All the velocity components are specified with null values.
- *outflow condition*: this condition represents a state where is close to a fully developed profile, that is, the right line of the geometries in this work. No prescribed value is specified for the velocity field and the pressure field is set the null value.
- *free-slip condition (symmetry)*: this condition is specified at the symmetric axis, that is, the bottom line of the geometries in this work. The vertical velocity component is null and the derivative of the horizontal component is also null value.
- *strut condition (semi-circles)*: this condition is used on the semi-circles of drug-eluting stent. The vertical and horizontal velocity components are specified with null value. The concentration field is specified as $e = e_o$, where $e_o = 1$.

To solve these governing equations, a numerical method is necessary. In this work, the Finite Element Method was used to approximate the differential equations. The domain was discretized on an unstructured mesh using the GMSH open source software, where the triangular element with linear interpolation was used for pressure field and with cubic interpolation was used for velocity and concentration fields. Therefore, the Babuska-Brezzi was satisfied. The equations

were discretized in spatial domain using the Galerkin formulation. Thus, we present the semi-discrete matricial form of the Navier–Stokes using the Galerkin method as follows:

$$D\mathbf{v} = 0 \tag{4}$$

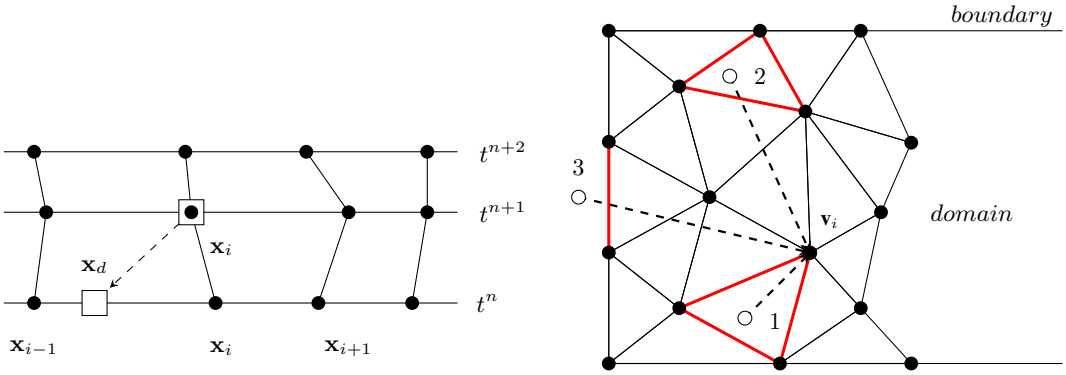
$$M\frac{D\mathbf{v}}{Dt} + \frac{1}{Re}K\mathbf{v} - Gp = 0 \tag{5}$$

$$M\frac{De}{Dt} + \frac{1}{ReSc}Ke = 0 \tag{6}$$

where, M is mass matrix, D is divergent matrix, G is gradient matrix and K is stiffness matrix. In this moment, the velicity $\mathbf{v}$, the pressure p and the drug concentration field e field are discretized in the spatial domain but continuos in the temporal domain. Then, for time discretization, the semi-Lagrangian Method was used. This method was proposed by Sawyer in 1963 for atmospheric flow numerical simulation using vorticity-advection equation, allowing to use large time steps without numerical instability. However, because of a limited computer capability, the use of such methodology to model several fluid flow problems, with high order differences and fine mesh, came later in the 1980s through the work of Robert (1981) and Pironneau (1982), where the semi-lagrangian scheme would be able to run models faster than the Eulerian scheme, besides being unconditionally stable and only symmetric linear systems to solve. Therefore the semi-lagrangian method discretized and computed the material derivative along the trajectory

$$\frac{D\mathbf{v}}{Dt} \approx \frac{\mathbf{v}_i^{n+1} - \mathbf{v}_d^n}{\Delta t} \tag{7}$$

where, $D\mathbf{v}/Dt$ is material derivative of $\mathbf{v}$ and the right-hand side equation is material derivative discretized using first order backward difference scheme. The variable t is time, $\mathbf{v}_i^{n+1}$ is the velocity field calculated in current time step at the current node position and $\mathbf{v}_d^n$ is the velocity field calculated in previous time step at the departure node position. The departure node is found by solving equation $\mathbf{x}_d^n = \mathbf{x}_i^{n+1} - \mathbf{v}\Delta t$, using the initial condition $\mathbf{x}_i^{n+1} = \mathbf{x}(t^{n+1})$ as shown in Figure 1a . A algorithm must be used to find the element that the departure node be, then the velocity field in departure node ($\mathbf{v}_d^n$) is calculed by barycenter coordinates interpolation between nodes of element found. As shown in Figure 1b , three situations may occur depending on the trajectory: the first and the second situations are similar, differentiating only the trajectory length. In the first situation, the departure node is inside near element from current node, while the second situation the departure node is inside far element from current node. The third situation, the departure node is outside domain then the vorticity field in departure node receives the boundary condition value of nearest node to departure node.



(a) (b)

Figure 1. In (a), an one-dimensional space scheme where the departure node x_d is found by integrating the mesh backward time. In (b), a two-dimensional space scheme where three situations may occur in searching procedure.

The triangular element with linear interpolation was used for the pressure field and a modified cubic element (2D MINI) was used for the velocity field, in order to respect the Babuska-Brezzi restriction. These elements consists in three nodes in the vertices and one node in the baricentric for MINI element case. The elementary matrices of this element are calculated using the Gaussian Quadrature, whose parameters can be found in the literature (Cowper, 2021). This element is represented by the Figure 2, where a linear combination of the area coordinates L_1, L_2 and L_3 generates the cubic shape functions N_1 to N_4. The velocity $\mathbf{v}$ and concentration e fields are evaluated and computed at all nodes, whereas the pressure field p is computed at vertices nodes;

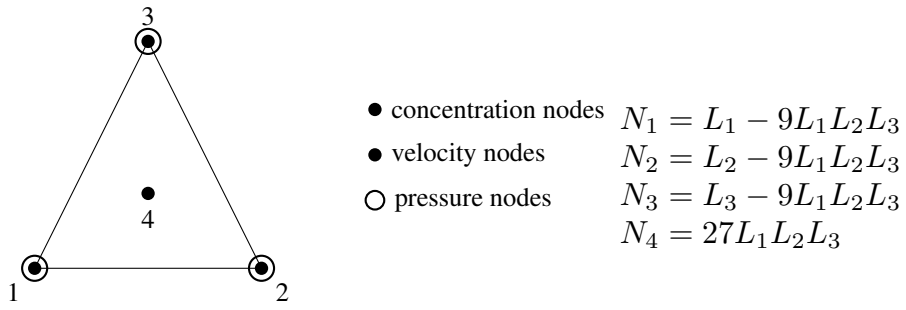


Figure 2. The linear triangle finite element for pressure field and the MINI element for velocity field. The L_1, L_2 and L_3 are the linear combination of the triangle area coordinates and N_1, N_2, N_3 and N_4 are the resulting shape functions.

Thus, we present the final version of the discrete matrix form of the dimensionless governing equations used in this work:

$$\begin{bmatrix} \frac{M}{\Delta t} + \frac{K}{Re} & -G \\ D & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_i^{n+1} \\ p \end{bmatrix} = \begin{bmatrix} \frac{M}{\Delta t}\mathbf{v}_d^n \\ 0 \end{bmatrix} + bc$$

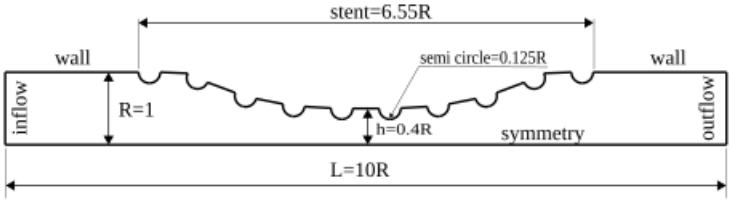
$$\left[\frac{M}{\Delta t} + \frac{K}{ReSc}\right] e_i^{n+1} = \frac{M}{\Delta t} e_d^n + bc_e$$

The direct solution of these equations results in the computation of the velocity **v**, pressure p and the drug concentration e at all mesh nodes.

3. RESULTS AND DISCUSSION

The dynamics of drug concentration diffusion in a coronary artery with drug-eluting stent was investigated in a complex geometry using a two-dimensional approach. That is, the coronary artery was approximated as a plannar domain. Due to symmetry of problem, only half domain was simulated. The blood was modeled as single-phase, incompressible and Newtonian fluid, the diffusion coefficient was considered as constant. The lumen radius of the coronary artery used was $R = 0.0015m$, the viscosity used was $\mu = 0.0035Pa.s$ and the density used was $\rho = 1060kg/m^3$ as suggested Bozsak *et al.* (2014). According to Kessler *et al.* (1998), the blood velocity in the coronary artery is $u = 12cm/s$. Thus, the Reynolds number used will be $Re = 54.5$.

The lumen radius was used as characteristic length; therefore, all geometry parameters will be presented as a lumen radius function. Thus, the width between the symmetric axis (bottom line) and the wall (top line) was equal to R = 1 (nondimensional value). The length of channel, that is, the difference between the inflow (left line) and the outflow (right line) was equal to L = 10 R. The dynamics of concentration diffusion in a coronary artery with drug-eluting stent was carried out in a complex geometry. The complex geometry is a simplified model of the real coronary artery, where it does not require a image processing; therefore, the mathematical equation use to describe the curve is sufficient. The width and length of this geometry is L=10R, whereas the drug-eluting stent of length equal to 6.55 R was modeled in this geometry by 10 uniform spaced semi-circles with radius equal to 0.125 R. In addition, an atherosclerosis was considered for this case, where a sinusoidal equation was used to model 40% of channel obstruction. This complex geometry is called the *Curved Channel with Stent* and it is represented by Figure 3.



(a) Curved Channel

Figure 3. Nondimensional domains for the dynamics analysis of concentration diffusion in coronary artery with drug-eluting stent. The lumen dimensionless width used was R = 1 and the lumen length was L = 10 R. The drug-eluting stent of length equal to 6.55 R was modeled by 10 uniform spaced semi-circles with radius equal to 0.125 R. An atherosclerosis was considered, where a sinusoidal equation was used to model 40% of channel obstruction.

Figure 4 presents the spatial and temporal distribution of the blood horizontal (left column) and vertical (right column) velocities for several time steps. The red color refers the maximum value and the blue color refers the minimum value. It is possible observed that the maximum horizontal velocity is localized in the maximum obstrution of channel, due to the atherosclerosis. Whereas for the vertical velocity component, the drug-eluting stent implantation makes the blood is initially directed downwards and changes the direction close to the end of channel. The maximum dimensionless horizontal velocity for the Curved Channel with Stent is $u = 3.23$. Conveting to dimensional values, the maximum horizontal velocity assumes $u = 38.8cm/s$, that is, more than 3 times the blood velocity in coronary artery without atherosclerosis and drug-eluting stent implanted. This increase can influence the dynamics of blood flow and its biological processes; therefore, a more detailed analysis should be performed. In addition, it is also possible to observe an inversion of the horizontal velocity field direction at the top of the figure. This inversion occurs in the region that is located between the stents strut semi-circles and a possible coagulation should be checked.

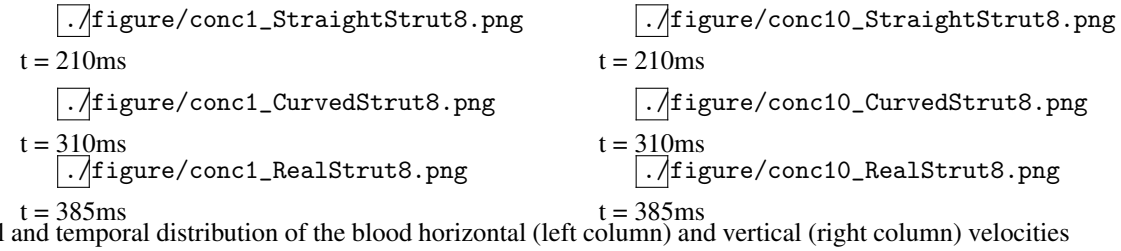


Figure 4. The spatial and temporal distribution of the blood horizontal (left column) and vertical (right column) velocities for several time steps. The red color refers the maximum value and the blue color refers the minimum value. The maximum dimensionless horizontal velocity for the Curved Channel with Stent is more than 3 times the blood velocity in coronary artery without atherosclerosis and drug-eluting stent implanted.

Figure 5 presents the spatial and temporal distribution of the blood horizontal (left column) and vertical (right column) velocities for the several time steps. the red color represents 100% and the blue color represents 0% of the diffused concentration in the bloodstream. Is possible to observe that the concentration diffusion acquires the steady state about 210 milliseconds. In addition, a large portion of the diffused drug in bloodstream is quickly spread and the concentration field is more dispersed at the end of the channel due to the direction of the blood flow. This diffusion affects the density and viscosity of the blood and consequently the Reynolds number is changed. Therefore, the velocity field would also br affected. However, this influence is not considered in this work.

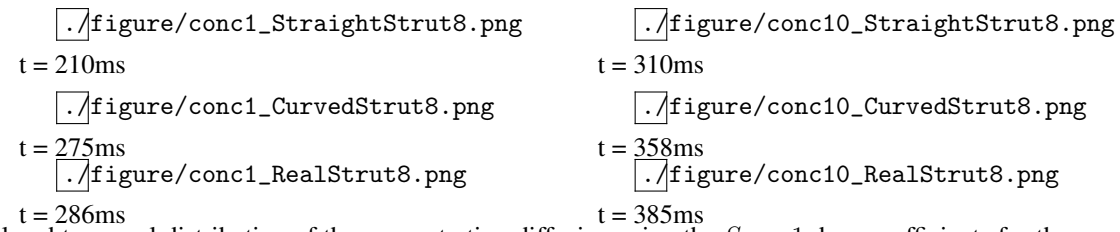


Figure 5. The spatial and temporal distribution of the concentration diffusion using the $Sc = 1$ drug coefficients for the Curved channel with the drug-eluting stent. Is possible to observe that the concentration diffusion acquires the steady state about 210 milliseconds. A large portion of the diffused drug in bloodstream is quickly spread and the concentration field is more dispersed at the end of the channel due to the direction of the blood flow.

4. PRELIMINARY CONCLUSION

In this work, the finite element simulation using the Navier-Stokes equations with high order elements to compute biological flows found in coronary artery diseases was performed. The dynamics of concentration diffusion in coronary arteries requires a robust numerical method to compute the solution of the differential equations in a relevant model specially due to the presence of the thin drug concentration boundary layer that must be accurately captured.

The domain was discretized on an unstructured mesh using the GMSH open source software, where the triangular element with linear interpolation was used for pressure field and with cubic interpolation was used for velocity and concentration fields. Therefore, the Babuska-Brezzi restriction was satisfied. The equations were discretized in spatial domain using the Galerkin formulation and for time discretization, the semi-Lagrangian Method was used.

According to the results presented, the numerical code developed proved to be able to perform simulations in complex geometries such as those found in problems involving cardiovascular diseases.

5. ACKNOWLEDGEMENTS

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