

A ALE-FE Method for Axisymmetric Vorticity-Streamfunction Formulation with Species Transport Equation

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Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

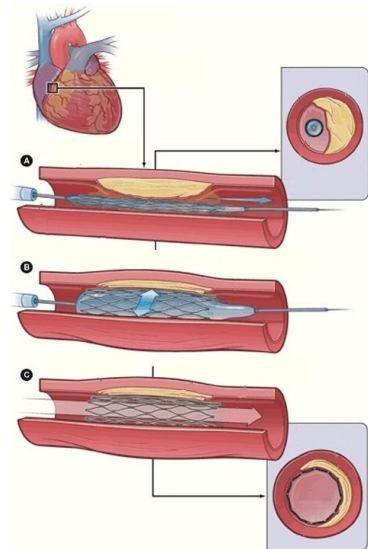
Introduction

Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- To develop a Finite Element code for axisymmetric vorticity-streamfunction formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting design patent



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Assumptions

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Axisymmetric flow

Vorticity Transport:

$$\frac{D\omega_{\theta}}{Dt} = \frac{\omega_{\theta}v_r}{r} + \frac{1}{Re} \left[\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \omega_{\theta}}{\partial r} + \frac{\partial^2 \omega_{\theta}}{\partial z^2} - \frac{\omega_{\theta}}{r^2} \right]$$

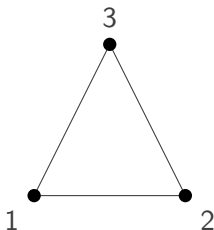
Streamfunction:

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} - \frac{2}{r} \frac{\partial \psi}{\partial r} = -r\omega_{\theta}$$

Species Transport:

$$\frac{Dc}{Dt} = \frac{1}{ReSc} \nabla^2 c$$

where D/Dt is the material derivator operator.



$$N_i = L_i$$

$$i = 1, 2, 3$$

$$\frac{\mathbf{M}}{\Delta t} \dot{\omega} = -\mathbf{v} \cdot \mathbf{G} \omega^n - \frac{1}{Re} \mathbf{K} \omega^n - \frac{\Delta t}{2} \mathbf{K}_s \omega^n \quad \mathbf{K} \psi = \mathbf{M} \omega$$

$$\frac{\mathbf{M}}{\Delta t} \dot{c} = -\mathbf{v} \cdot \mathbf{G} c^n - \frac{1}{ReSc} \mathbf{K} c^n - \frac{\Delta t}{2} \mathbf{K}_s c^n \quad \mathbf{M} \mathbf{v} = \mathbf{G} \psi$$

Where \mathbf{K}_s is stability matrix to decrease spurious oscillations

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5. Conclusion

Validation - Poiseuille Flow

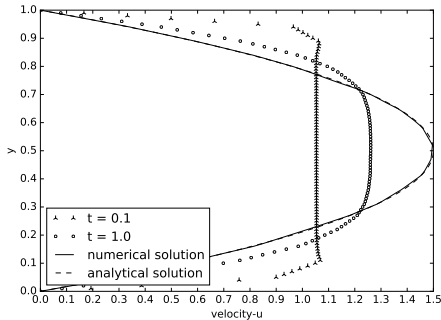
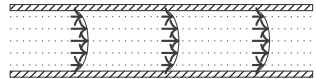
Boundaries Conditions:

Inflow condition: $u = 1$, $v = 0$ e $\psi = y$

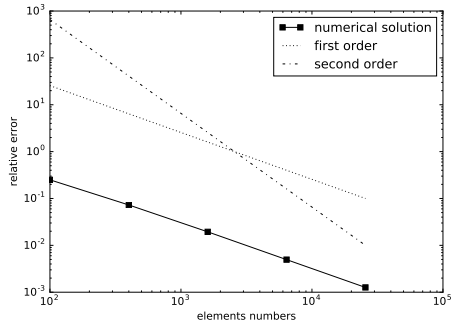
Outflow condition: $\psi = y$

Top plate: $u = 0$, $v = 0$, $\psi = 1$

Bottom plate: $u = 0$, $v = 0$, $\psi = 0$



(a)



(b)

(a) comparison of Poiseuille Flow velocity profile and

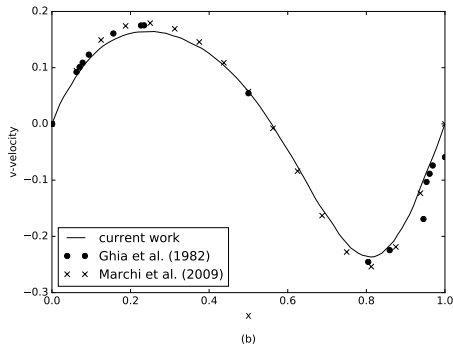
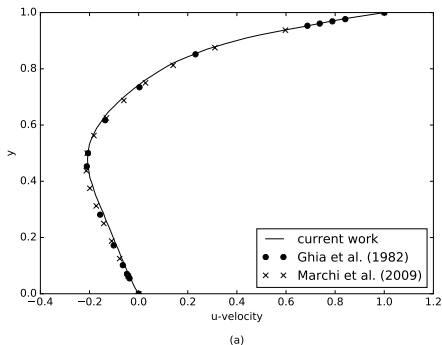
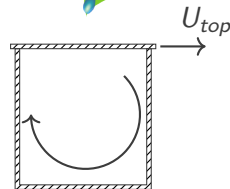
(b) log scale graph of convergence order.

Validation - Lid Driven Cavity Flow

Boundaries Conditions:

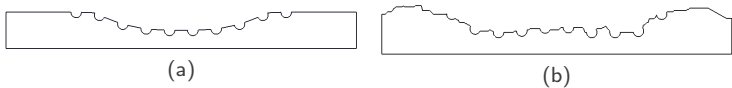
Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

Top plate: $u = 1$, $v = 0$ e $\psi = 0$



Centerline velocity profile in a lid-driven cavity for $Re = 100$:
(a) u-velocity and (b) v-velocity.

1. Introduction
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Non-dimensional symmetric geometry for blood flow in coronary artery with drug-eluting stent placed by Wang et al. (2017): (a) Curved Channel with Stent (b) Real Channel with Stent.

Boundaries Conditions:

Inflow condition: $u = 1$, $v = 0$ e $\psi = y$;

Outflow condition: $\psi = y$;

Top plate: $u = 0$, $v = 0$, $\psi = 1$;

Symmetry condition: $v = 0$, $\psi = 0$;

Drug-eluting stent: $u = 0$, $v = 0$, $\psi = 1$ e $c = 1$

$$R = 0.0015m$$

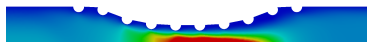
$$\mu = 0.0035Pa.s$$

$$\rho = 1060kg/m^3$$

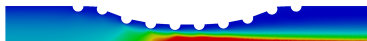
$$u = 12cm/s$$

$$Re = 54.5$$

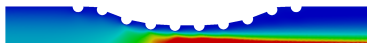
Results - Velocity Field



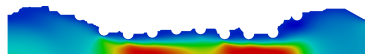
$t = 1.0$



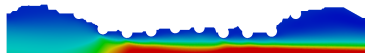
$t = 5.0$



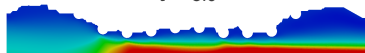
$t = 10.0$



$t = 1.0$

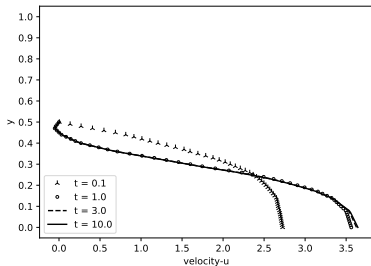


$t = 5.0$

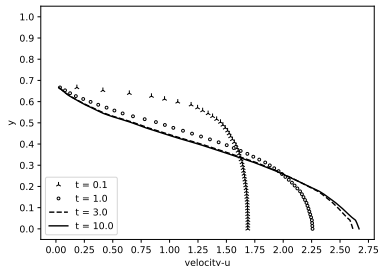


$t = 10.0$

Evolution in time and space of velocity field:
Curved Channel (left column) and Real Channel (right column)



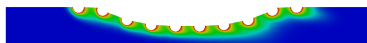
(a)



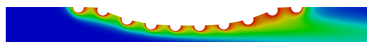
(b)

Evolution of velocity profile in centerline ($x = 0.5L$):
(a) Curved Channel and (b) Real Channel

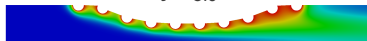
Results - Concentration Field



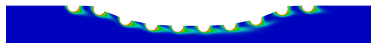
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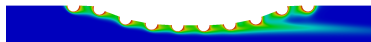
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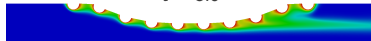
$t = 10.0$



$t = 1.0$

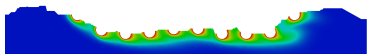


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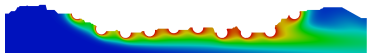


$t = 10.0$

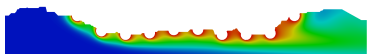
Evolution in time and space of concentration field in Curved Channel:
 $Sc = 1$ (left column) and $Sc = 10$ (right column)



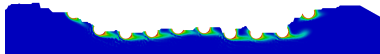
$t = 1.0$



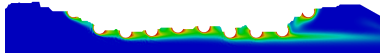
$t = 5.0$



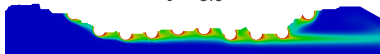
$t = 10.0$



$t = 1.0$



$t = 5.0$



$t = 10.0$

Evolution in time and space of concentration field in Real Channel:
 $Sc = 1$ (left column) and $Sc = 10$ (right column)

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1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production
2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation
3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

Thank you!

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