An ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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GESAR

Outline

Introduction

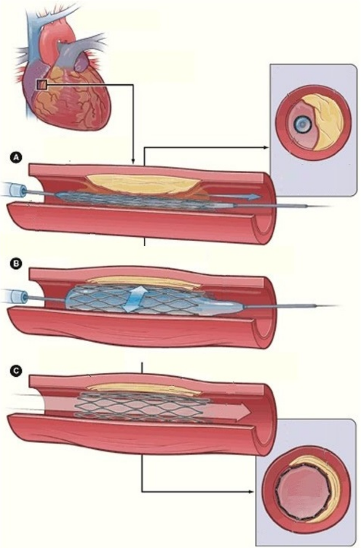
Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach

- To create new drug-eluting stent design patent



[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

Arbitrary Lagrangian-Eulerian (ALE)

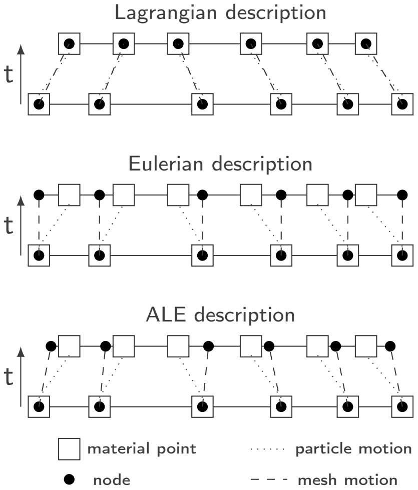
The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment



[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian–Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

Governing Equations

Assumptions [3]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{\partial \omega}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

$$\frac{\partial c}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla c = \frac{1}{ReSc} \nabla^2 c$$

- If the mesh velocity field $\hat{\mathbf{v}} = \mathbf{v}$ (*Lagrangian*) or $\hat{\mathbf{v}} = 0$ (*Eulerian*)
- The material velocity field $\mathbf{v} = (v_x, v_y)$ is calculated by: $v_x = \partial\psi/\partial y$ and $v_y = -\partial\psi/\partial x$

[3] Batchelor, G. (1967). An Introduction to Fluid Dynamics (Cambridge Mathematical Library). Cambridge: Cambridge University Press. *doi:10.1017/CBO9780511800955*

Semi-Lagrangian Method

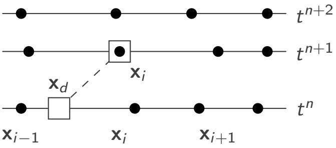
The convective term was replaced by material derivative in the direction of characteristic trajectory, that is: $\frac{D(\cdot)}{Dt} = \frac{\partial(\cdot)}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla(\cdot)$

$$\frac{D\omega}{Dt} = \frac{1}{Re}\nabla^2\omega$$

$$\nabla^2\psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc}\nabla^2 c$$

The departure node is calculated by $x_d^n = x_i^{n+1} - (\mathbf{v} - \hat{\mathbf{v}})\,\Delta t$. Then, a searching procedure is required to find x_d^n using barycentric coordinates



Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [4]:

$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re} \nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

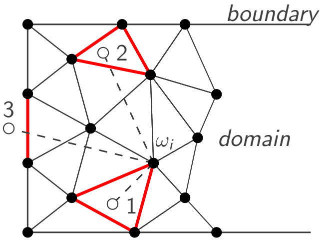
$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc} \nabla^2 c^{n+1}$$

Advantages:

- Symmetric linear systems
- Unconditionnal stability

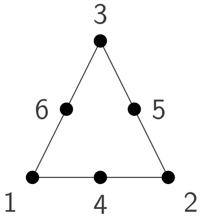
Disadvantages:

- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed

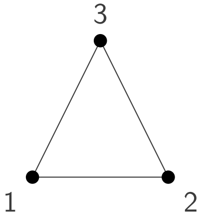


[4] Pironneau, O. On the transport-diffusion algorithm and its applications to the Navier-Stokes equations. Numer. Math. 38, 309–332 (1982). *https://doi.org/10.1007/BF01396435*

Galerkin FE Method



Quad Element



Linear Element

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re}\right]\omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t}\omega_d^n$$

$$\mathbf{K}\psi = \mathbf{M}\omega$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc}\right]c_i^{n+1} = \frac{\mathbf{M}}{\Delta t}c_d^n$$

- The material velocity field is calculated by:
  $\mathbf{M}v_x = \mathbf{G_y}\psi$
  $\mathbf{M}v_y = -\mathbf{G_x}\psi$

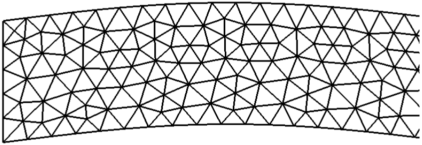
Laplacian Smoothing

To avoid the fast degradation of the computational elements due to ALE description, it was used the Laplacian Smoothing Method [5]

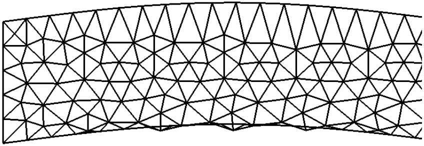
The new node position $\hat{\mathbf{x}}_{\mathbf{i}}$ can be approximated by:

$$\hat{\mathbf{x}}_{\mathbf{i}} = \sum_{i \in N_1} e_{ij}^{-1} (\mathbf{x}_j - \mathbf{x}_i)$$

where, e_{ij}^{-1} is the distance between the node and each neighbor in 1-ring N_1



with Laplacian Smoothing



no Laplacian Smoothing

[5] Desbrun, M. Meyer, P. Schröder, A. Barr, Implicit fairing of irregular meshes using diffusion and curvature flow, in: Proceedingsof Siggraph, 1999, pp. 317–324

Solution Algorithm

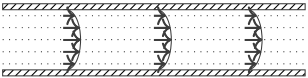
1. Import mesh
2. Calculate Gaussian Quadradure and Assemble Matrix
3. Inicialize Vorticity and Streamfunction
4. Calculate Laplacian smoothing and ALE velocity
5. Move Nodes
6. Calculate Gaussian Quadradure and Assemble Matrix
7. Calculate vorticity boundary condition
8. Calculate semi-Lagrangian and Vorticity field
9. Calculate streamfunction field
10. Calculate velocity field
11. Calculate concentration field

Repeat the procedure
for the next time step
until the steady state

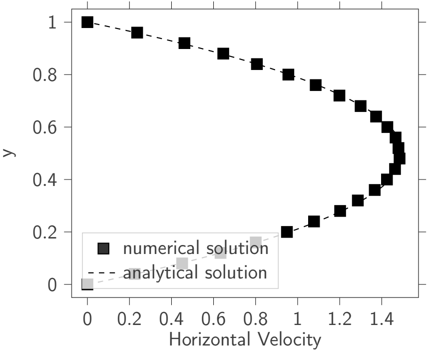
Validation - Poiseuille Flow

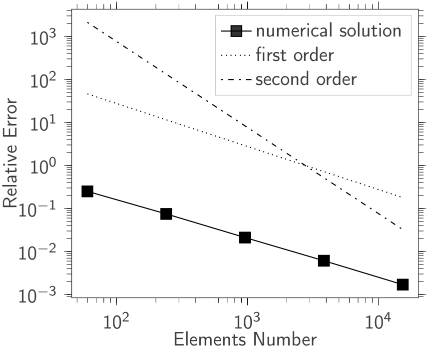
Boundaries Conditions:

Inflow condition: $u = u_{analytical}$, $v = 0$
Top plate: $u = 0$, $v = 0$, $\psi = 1$
Bottom plate: $u = 0$, $v = 0$, $\psi = 0$



Nodes: 1757
Elements: 3263
Relative Error: 1.2%





Validation - Lid Driven Cavity Flow

Boundaries Conditions:

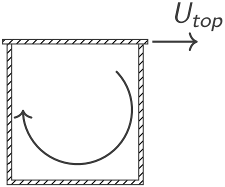
Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

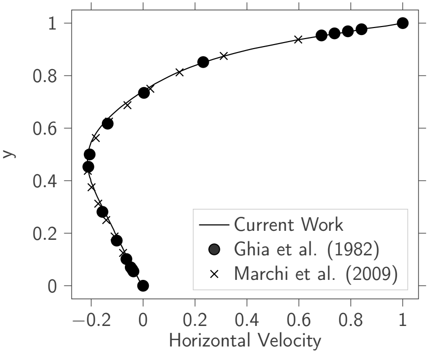
Top plate: $u = 1$, $v = 0$ e $\psi = 0$

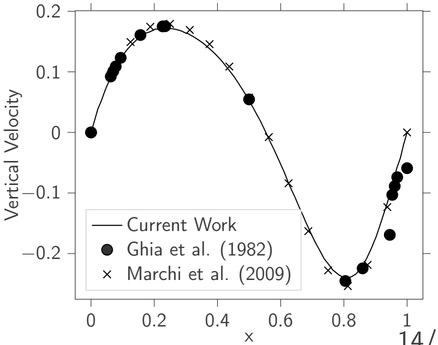
Nodes: 3798

Elements: 7382

Re: 100







Coming Soon

- Backward-Facing Step Validation
- Pulsation Flow Validation
- Drug-Eluting Stent Cases

Work Plan

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	2018				2019												2020							
ACTIVITIES	**9**	**10**	**11**	**12**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
SUBJECTS	■	■	■	■	■	■	■	■	■	■	■	■												
LITERATURE REVIEW													■	■	■	■								
CODE IMPLEMENTATION															■	■	■	■	■	■	■			
CODE VALIDATION																				■	■	■		
RESULTS SIMULATION																							■	
PRESENTATION																								■

SUBJECTS:

- CONTINUUM MECHANICS I
- ADVANCED CALCULUS
- FLUID MECHANICS
- COMPUTATIONAL FLUID MECHANICS I
- COMPUTATIONAL FLUID MECHANICS II
- FINITE ELEMENT METHOD
- COMPUTATIONAL METHODS
- MULTIPHASE FLOWS

LITERATURE REVIEW:

- ARBITRARY LAGRANGIAN-EULERIAN (ALE)
- SEMI-LAGRANGIAN METHOD
- LAPLACIAN SMOOTHING

Final Presentation

Expected Date for Final Presentation

August 2020

	S	M	T	W	T	F	S
31	26	27	28	29	30	31	1
32	2	3	4	5	6	7	8
33	9	10	11	12	13	14	15
34	16	17	18	19	20	21	22
35	23	24	25	26	27	28	29
36	30	31	1	2	3	4	5

Thank you!

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