A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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GESAR

Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

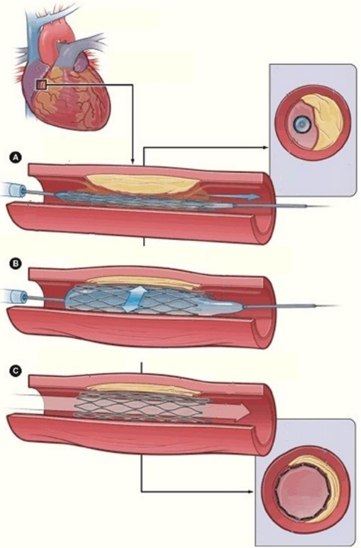
Introduction

Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting design patent



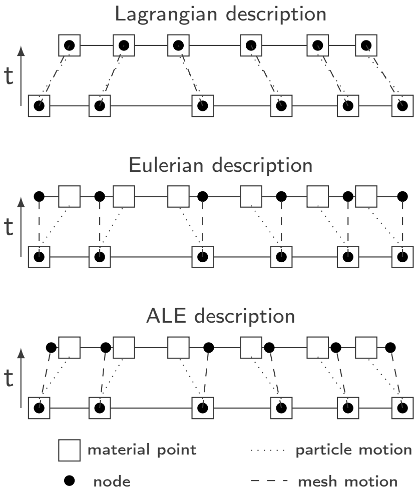
[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

Arbitrary Lagrangian-Eulerian (ALE)

REWRITE

In ALE description, the referential frame moves with an arbitrary velocity that does not necessary represents the Lagrangian or Eulerian description.

The convection velocity is calculated by the relative velocity between the material and the mesh velocity respectively [2]



Governing Equations

Assumptions:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{D\omega}{Dt} = \frac{1}{Re}\nabla^2\omega$$

$$\nabla^2\psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc}\nabla^2 c$$

where, $D(\cdot)/Dt$ is substantive derivative and the material velocity field is calculated by: $v_x = \partial\psi/\partial y$ and $v_y = -\partial\psi/\partial x$

Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [3]:

$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re}\nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

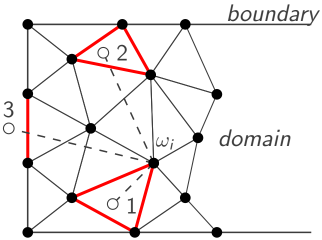
$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc}\nabla^2 c^{n+1}$$

Advantages:

- Symmetric linear systems
- Unconditionnal stability

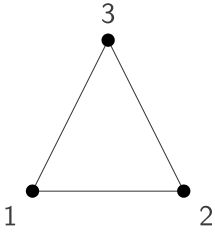
Disadvantages:

- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed



[3] Pironneau, O. On the transport-diffusion algorithm and its applications to the Navier-Stokes equations. Numer. Math. 38, 309–332 (1982). *https://doi.org/10.1007/BF01396435*

Galerkin FE Method



$$N_i = L_i$$

$$i = 1, 2, 3$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re}\right]\omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t}\omega_d^n$$

$$\mathbf{K}\psi = \mathbf{M}\omega$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc}\right]c_i^{n+1} = \frac{\mathbf{M}}{\Delta t}c_d^n$$

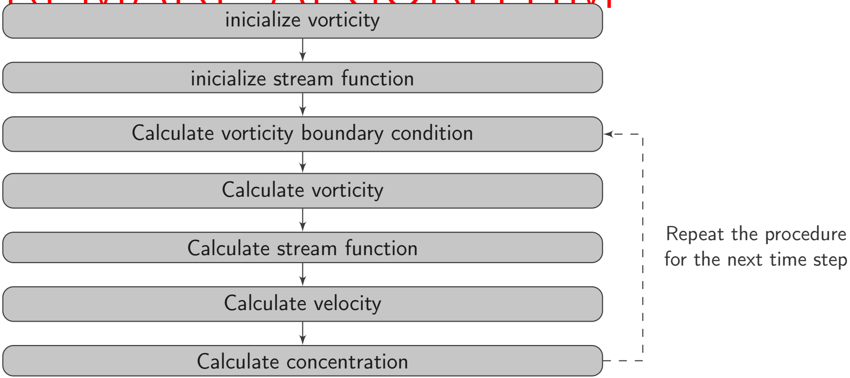
The material velocity field is calculated by: $\mathbf{M}v_x = \mathbf{G_y}\psi$ and $\mathbf{M}v_y = -\mathbf{G_x}\psi$

Adaptive Mesh Refinement

Mesh smoothing description and comparative figures

Solution Algorithm

REMAKE ALGORITHM



Streamfunction-Vorticity formulation with species transport equation solution algorithm

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Validation - Poiseuille Flow

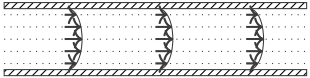
Boundaries Conditions:

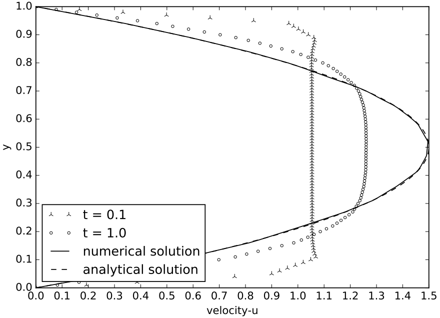
Inflow condition: $u = 1$, $v = 0$ e $\psi = y$

Outflow condition: $\psi = y$

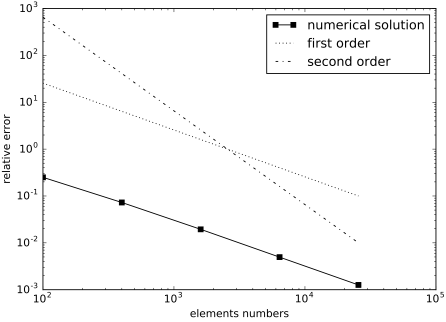
Top plate: $u = 0$, $v = 0$, $\psi = 1$

Bottom plate: $u = 0$, $v = 0$, $\psi = 0$





(a)



(b)

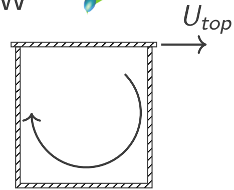
(a) comparison of Poiseuille Flow velocity profile and (b) log scale graph of convergence order.

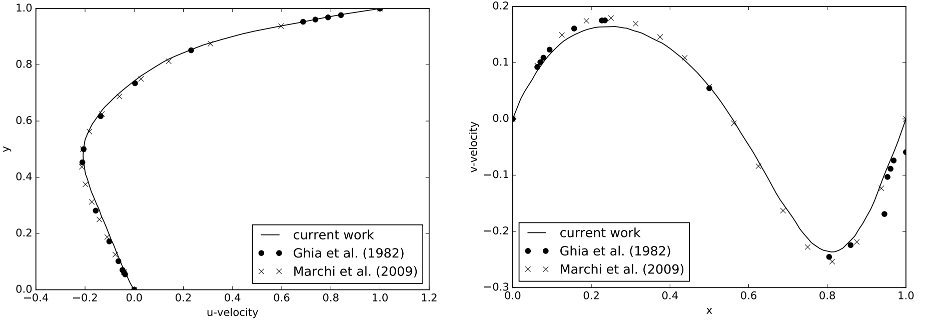
Validation - Lid Driven Cavity Flow

Boundaries Conditions:

Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

Top plate: $u = 1$, $v = 0$ e $\psi = 0$





(a) (b)

Centerline velocity profile in a lid-driven cavity for $Re = 100$:
(a) u-velocity and (b) v-velocity.

Validation - Backward-facing Step

Coming Soon

Validation - Pulsation Flow

Coming Soon

Results

Coming Soon

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Conclusion

1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production
2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation
3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

Thank you!

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