A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

Student Researcher: Leandro Marques
Advisors: Gustavo Anjos and Jose Pontes

State University of Rio de Janeiro
June, 22th 2020

GESAR

Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Introduction

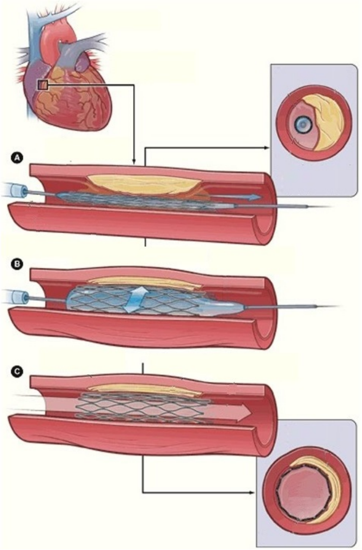
Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach

- To create new drug-eluting design patent

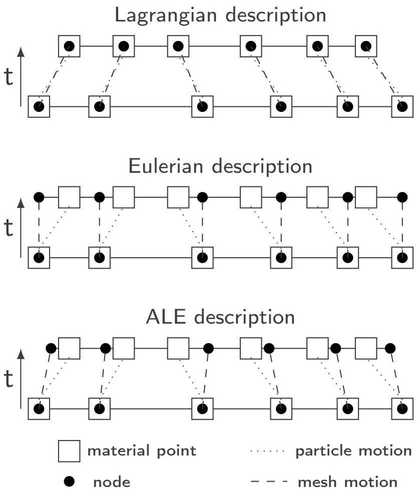


[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Arbitrary Lagrangian-Eulerian (ALE)

The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:



Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment

[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian–Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

Governing Equations

Assumptions [4]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{D\omega}{Dt} = \frac{1}{Re}\nabla^2\omega$$

$$\nabla^2\psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc}\nabla^2 c$$

where, $D(\cdot)/Dt$ is substantive derivative and the material velocity field is calculated by: $v_x = \partial\psi/\partial y$ and $v_y = -\partial\psi/\partial x$

[4] Batchelor, G. (1967). An Introduction to Fluid Dynamics (Cambridge Mathematical Library). Cambridge: Cambridge University Press. *doi:10.1017/CBO9780511800955*

Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [3]:

$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re}\nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

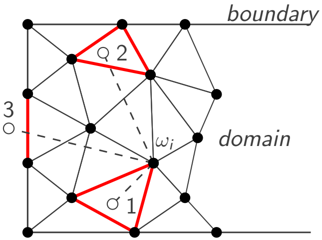
$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc}\nabla^2 c^{n+1}$$

Advantages:

- Symmetric linear systems
- Unconditionnal stability

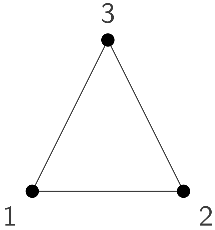
Disadvantages:

- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed



[3] Pironneau, O. On the transport-diffusion algorithm and its applications to the Navier-Stokes equations. Numer. Math. 38, 309–332 (1982). *https://doi.org/10.1007/BF01396435*

Galerkin FE Method



$$N_i = L_i$$

$$i = 1, 2, 3$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re}\right] \omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t} \omega_d^n$$

$$\mathbf{K}\psi = \mathbf{M}\omega$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc}\right] c_i^{n+1} = \frac{\mathbf{M}}{\Delta t} c_d^n$$

The material velocity field is calculated by: $\mathbf{M}v_x = \mathbf{G_y}\psi$ and $\mathbf{M}v_y = -\mathbf{G_x}\psi$

Adaptive Mesh Refinement

Mesh smoothing description and comparative figures

Solution Algorithm

Import mesh
↓
Calculate Gaussian Quadradure and Assemble Matrix
↓
Inicialize Vorticity and Streamfunction
↓
Calculate Laplace smoothing and ALE velocity
↓
Calculate Gaussian Quadradure and Assemble Matrix
↓
Calculate vorticity boundary condition
↓
Calculate semi-Lagrangian and Vorticity field
↓
Calculate streamfunction field
↓
Calculate velocity field
↓
Calculate concentration field

Repeat the procedure
for the next time step
until the steady state

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Validation - Poiseuille Flow

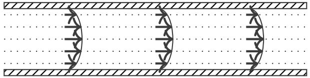
Boundaries Conditions:

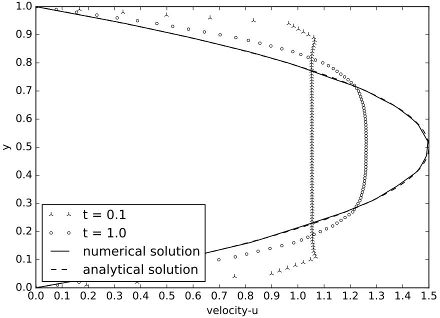
Inflow condition: $u = 1,\ v = 0$ e $\psi = y$

Outflow condition: $\psi = y$

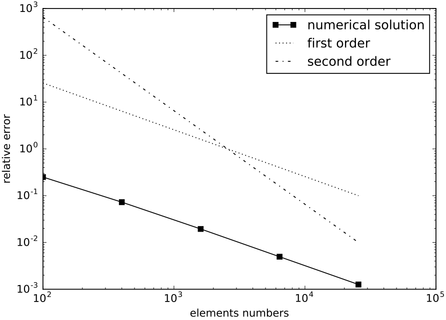
Top plate: $u = 0,\ v = 0,\ \psi = 1$

Bottom plate: $u = 0,\ v = 0,\ \psi = 0$





(a)



(b)

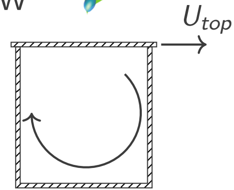
(a) comparison of Poiseuille Flow velocity profile and
(b) log scale graph of convergence order.

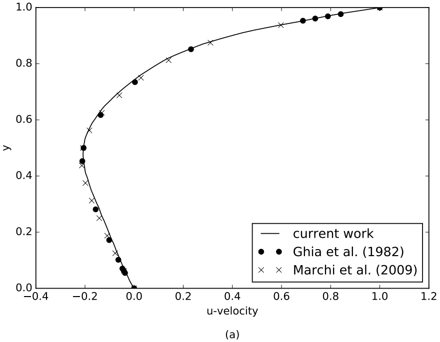
Validation - Lid Driven Cavity Flow

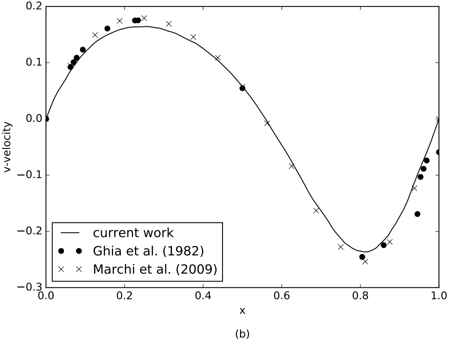
Boundaries Conditions:

Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

Top plate: $u = 1$, $v = 0$ e $\psi = 0$







Centerline velocity profile in a lid-driven cavity for $Re = 100$: (a) u-velocity and (b) v-velocity.

Validation - Backward-facing Step

Coming Soon

Validation - Pulsation Flow

Coming Soon

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Results

Coming Soon

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Conclusion

1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production

2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation

3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

Thank you!

marquesleandro67@gmail.com
gustavo.rabello@mecanica.coppe.ufrj.br
jose.pontes@uerj.br

The authors thank the FAPERJ (Research Support Foundation of the State of Rio de Janeiro) for its financial support