

A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Introduction

Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting design patent

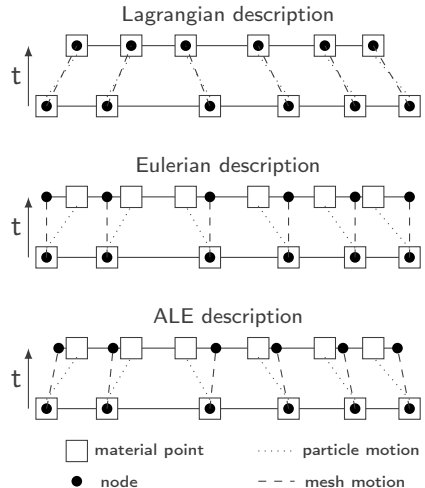


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Arbitrary Lagrangian-Eulerian (ALE)

In ALE description, the referential frame moves with an arbitrary velocity that does not necessarily represent the Lagrangian or Eulerian description.

The convection velocity is calculated by the relative velocity between the material and the mesh velocity respectively [2]



Assumptions:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{D\omega}{Dt} = \frac{1}{Re} \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc} \nabla^2 c$$

where, $D(\cdot)/Dt$ is substantive derivative and the material velocity field is calculated by: $v_x = \partial\psi/\partial y$ and $v_y = -\partial\psi/\partial x$

Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [3]:

Advantages:

- Symmetric linear systems
- Unconditionnal stability

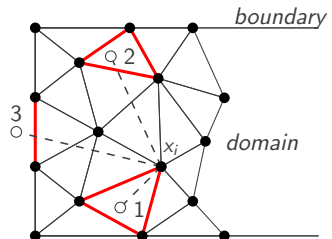
Disadvantages:

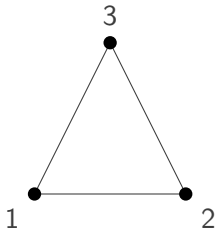
- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed

$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re} \nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc} \nabla^2 c^{n+1}$$





$$N_i = L_i$$
$$i = 1, 2, 3$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re} \right] \omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t} \omega_d^n$$

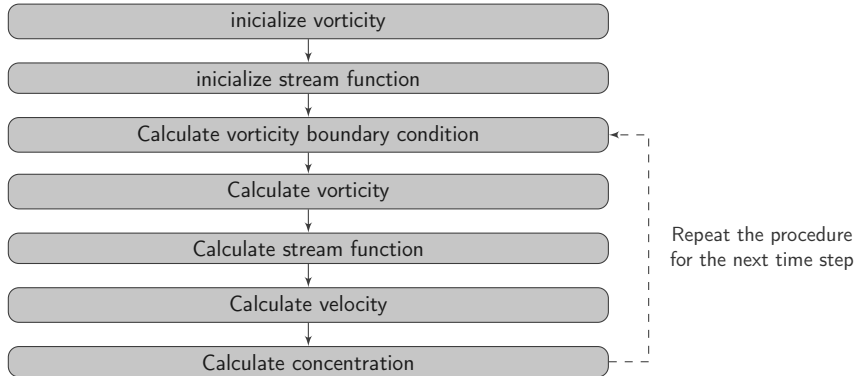
$$\mathbf{K}\psi = \mathbf{M}\omega$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc} \right] c_i^{n+1} = \frac{\mathbf{M}}{\Delta t} c_d^n$$

The material velocity field is calculated by: $\mathbf{M}v_x = \mathbf{G}_y\psi$ and $\mathbf{M}v_y = -\mathbf{G}_x\psi$

Laplace Smoothing description, comparative figures and ALE velocity formula

Solution Algorithm



Streamfunction-Vorticity formulation with species transport equation solution algorithm

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Validation - Poiseuille Flow

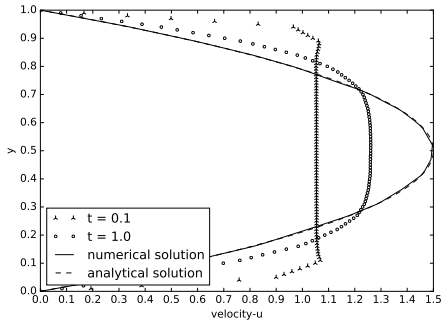
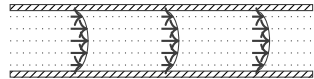
Boundaries Conditions:

Inflow condition: $u = 1, v = 0$ e $\psi = y$

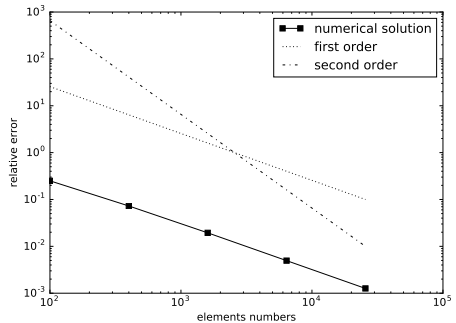
Outflow condition: $\psi = y$

Top plate: $u = 0, v = 0, \psi = 1$

Bottom plate: $u = 0, v = 0, \psi = 0$



(a)



(b)

(a) comparison of Poiseuille Flow velocity profile and

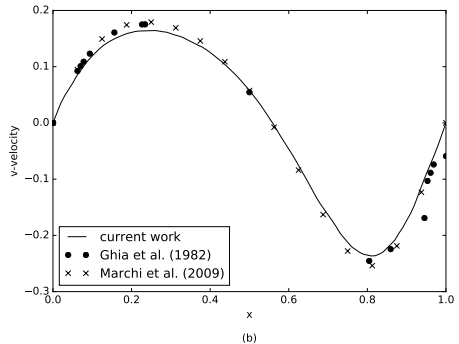
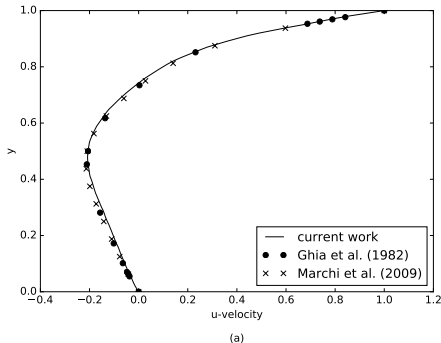
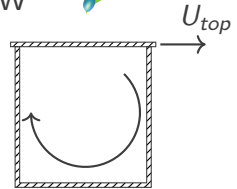
(b) log scale graph of convergence order.

Validation - Lid Driven Cavity Flow

Boundaries Conditions:

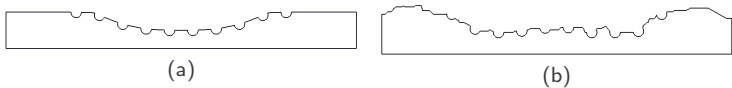
Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

Top plate: $u = 1$, $v = 0$ e $\psi = 0$



Centerline velocity profile in a lid-driven cavity for $Re = 100$:
(a) u-velocity and (b) v-velocity.

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Non-dimensional symmetric geometry for blood flow in coronary artery with drug-eluting stent placed by Wang et al. (2017): (a) Curved Channel with Stent (b) Real Channel with Stent.

Boundaries Conditions:

Inflow condition: $u = 1$, $v = 0$ e $\psi = y$;

Outflow condition: $\psi = y$;

Top plate: $u = 0$, $v = 0$, $\psi = 1$;

Symmetry condition: $v = 0$, $\psi = 0$;

Drug-eluting stent: $u = 0$, $v = 0$, $\psi = 1$ e $c = 1$

$$R = 0.0015m$$

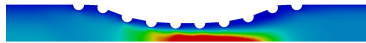
$$\mu = 0.0035Pa.s$$

$$\rho = 1060kg/m^3$$

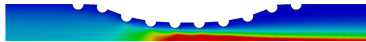
$$u = 12cm/s$$

$$Re = 54.5$$

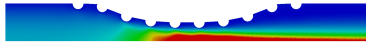
Results - Velocity Field



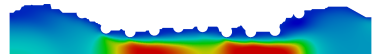
$t = 1.0$



$t = 5.0$



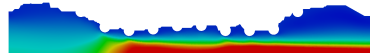
$t = 10.0$



$t = 1.0$

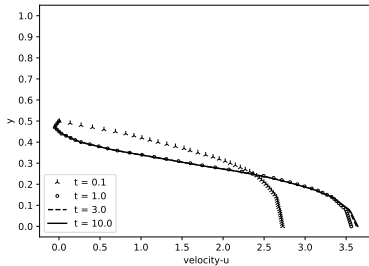


$t = 5.0$

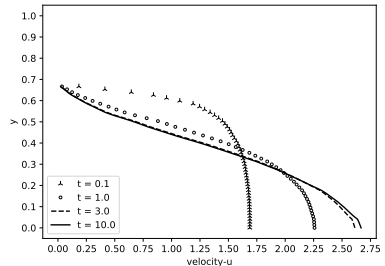


$t = 10.0$

Evolution in time and space of velocity field:
Curved Channel (left column) and Real Channel (right column)



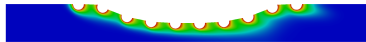
(a)



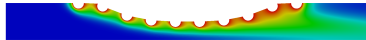
(b)

Evolution of velocity profile in centerline ($x = 0.5L$):
(a) Curved Channel and (b) Real Channel

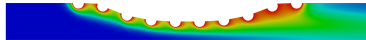
Results - Concentration Field



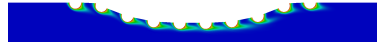
$t = 1.0$



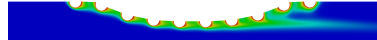
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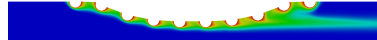
$t = 10.0$



$t = 1.0$

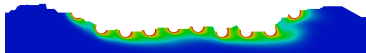


$t = 5.0$

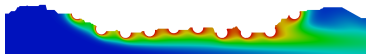


$t = 10.0$

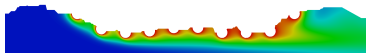
Evolution in time and space of concentration field in Curved Channel:
 $Sc = 1$ (left column) and $Sc = 10$ (right column)



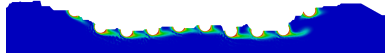
$t = 1.0$



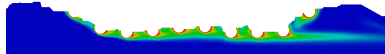
$t = 5.0$



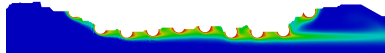
$t = 10.0$



$t = 1.0$



$t = 5.0$



$t = 10.0$

Evolution in time and space of concentration field in Real Channel:
 $Sc = 1$ (left column) and $Sc = 10$ (right column)

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1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production
2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation
3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

Thank you!

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