

A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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June, 22th 2020

Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Introduction

Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting design patent



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Arbitrary Lagrangian-Eulerian (ALE)

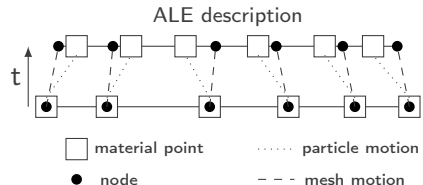
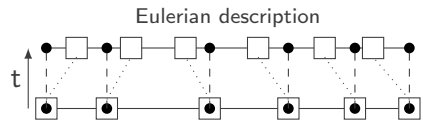
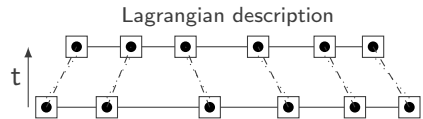
The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment



[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian-Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

Governing Equations

Assumptions [3]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{\partial \omega}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

$$\frac{\partial c}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla c = \frac{1}{ReSc} \nabla^2 c$$

The material velocity field $\mathbf{v} = (v_x, v_y)$ is calculated by: $v_x = \partial \psi / \partial y$ and $v_y = -\partial \psi / \partial x$

The mesh velocity field $\hat{\mathbf{v}} \neq \mathbf{v}$ (*Lagrangian*) or 0 (*Eulerian*)

Semi-Lagrangian Method

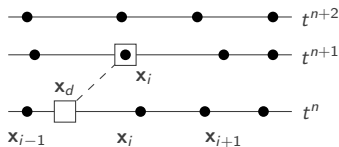
The convective term was replaced by material derivative of ω and c in the direction of characteristic trajectory

$$\frac{D\omega}{Dt} = \frac{1}{Re} \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc} \nabla^2 c$$

The departure node is calculated by $x_d^n = x_i^{n+1} - (\mathbf{v} - \hat{\mathbf{v}}) \Delta t$. Then, a searching procedure is required to find x_d^n using barycentric coordinates



Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [4]:

Advantages:

- Symmetric linear systems
- Unconditionnal stability

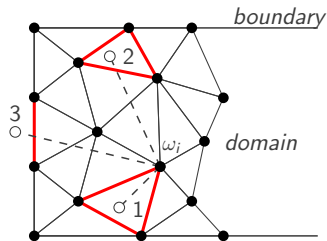
Disadvantages:

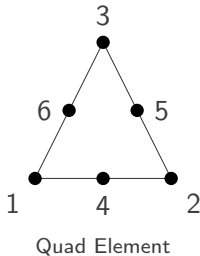
- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed

$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re} \nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

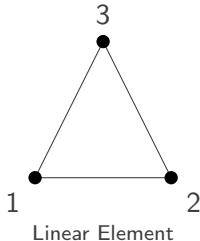
$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc} \nabla^2 c^{n+1}$$





$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re} \right] \omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t} \omega_d^n$$

$$\mathbf{K}\psi = \mathbf{M}\omega$$



$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc} \right] c_i^{n+1} = \frac{\mathbf{M}}{\Delta t} c_d^n$$

The material velocity field is calculated by: $\mathbf{M}v_x = \mathbf{G}_y\psi$ and $\mathbf{M}v_y = -\mathbf{G}_x\psi$

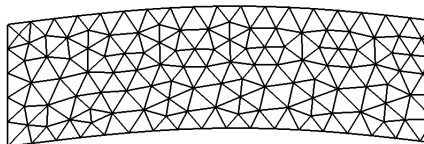
Laplacian Smoothing

To avoid the fast degradation of the computational elements due to ALE description, it was used the Laplacian Smoothing Method [5]

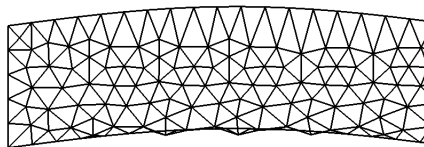
The new node position $\hat{\mathbf{x}}_i$ can be approximated by:

$$\hat{\mathbf{x}}_i = \sum_{i \in N_1} e_{ij}^{-1} (\mathbf{x}_j - \mathbf{x}_i)$$

where, e_{ij}^{-1} is the distance between the node and each neighbor in 1-ring N_1



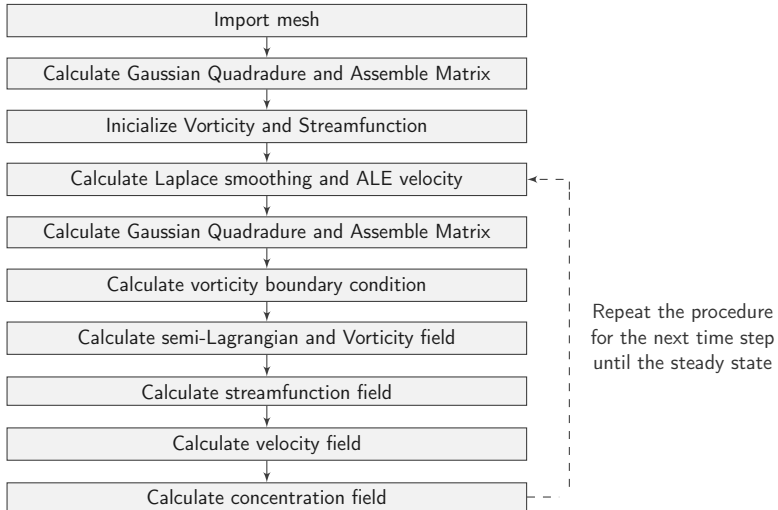
with Laplacian Smoothing



no Laplacian Smoothing

[5] Desbrun, M. Meyer, P. Schröder, A. Barr, Implicit fairing of irregular meshes using diffusion and curvature flow, in: Proceedings of Siggraph, 1999, pp. 317–324

Solution Algorithm



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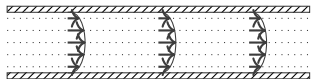
Validation - Poiseuille Flow

Boundaries Conditions:

Inflow condition: $u = u_{analytical}$, $v = 0$

Top plate: $u = 0$, $v = 0$, $\psi = 1$

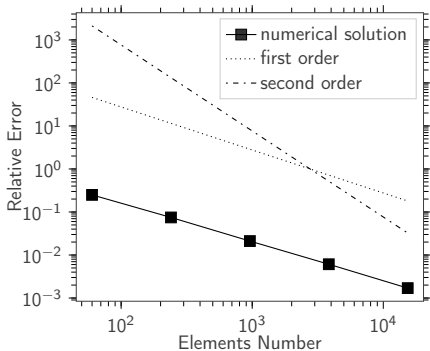
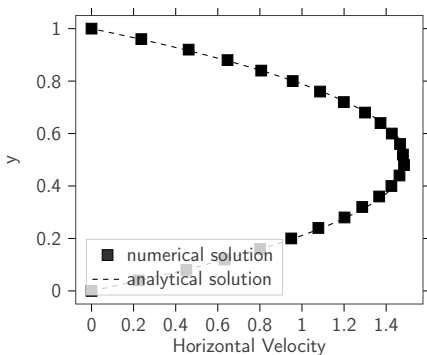
Bottom plate: $u = 0$, $v = 0$, $\psi = 0$



Nodes: 1757

Elements: 3263

Relative Error: 0.67%



Validation - Lid Driven Cavity Flow

Boundaries Conditions:

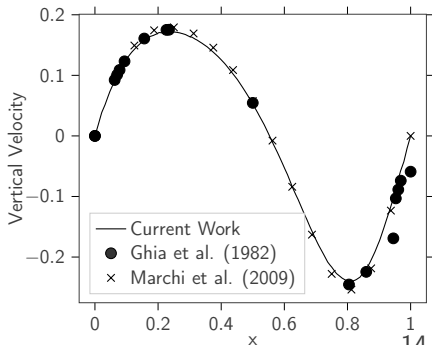
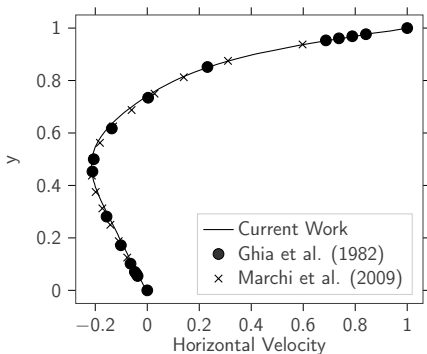
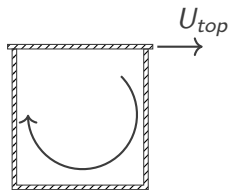
Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

Top plate: $u = 1$, $v = 0$ e $\psi = 0$

Nodes: 3798

Elements: 7382

Re: 100



Coming Soon

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1. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation
2. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)
3. The ALE description allows moving boundary problems to be simulated

Thank you!

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The authors thank the FAPERJ (Research Support Foundation of the State of Rio de Janeiro) for its financial support

