

# A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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# Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

# Introduction

## Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

## Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting design patent



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# Arbitrary Lagrangian-Eulerian (ALE)

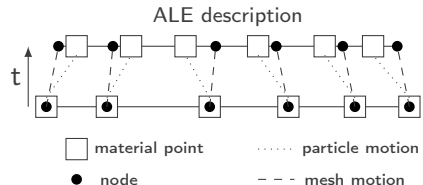
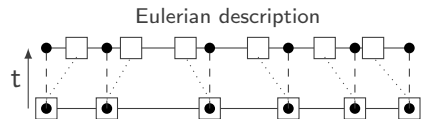
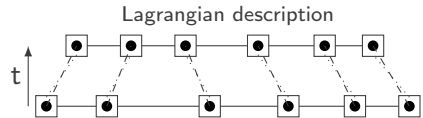
The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment



[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian-Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

# Governing Equations

Assumptions [4]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{D\omega}{Dt} = \frac{1}{Re} \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc} \nabla^2 c$$

where,  $D(\cdot)/Dt$  is substantive derivative and the material velocity field is calculated by:  $v_x = \partial\psi/\partial y$  and  $v_y = -\partial\psi/\partial x$

# Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [3]:

Advantages:

- Symmetric linear systems
- Unconditionnal stability

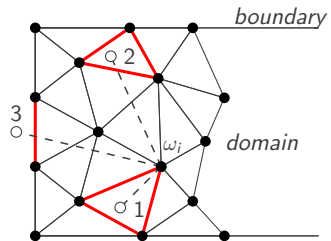
Disadvantages:

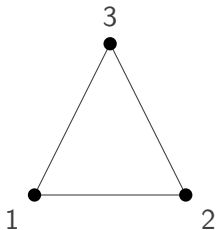
- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed

$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re} \nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc} \nabla^2 c^{n+1}$$





$$N_i = L_i$$
$$i = 1, 2, 3$$

$$\left[ \frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re} \right] \omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t} \omega_d^n$$

$$\mathbf{K}\psi = \mathbf{M}\omega$$

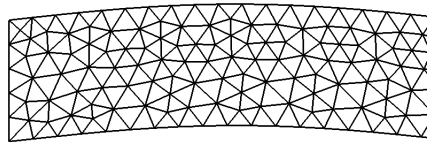
$$\left[ \frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc} \right] c_i^{n+1} = \frac{\mathbf{M}}{\Delta t} c_d^n$$

The material velocity field is calculated by:  $\mathbf{M}v_x = \mathbf{G}_y\psi$  and  $\mathbf{M}v_y = -\mathbf{G}_x\psi$



## Motivation:

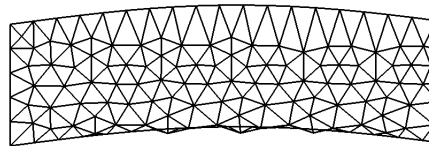
- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]



with ALE

## Aims:

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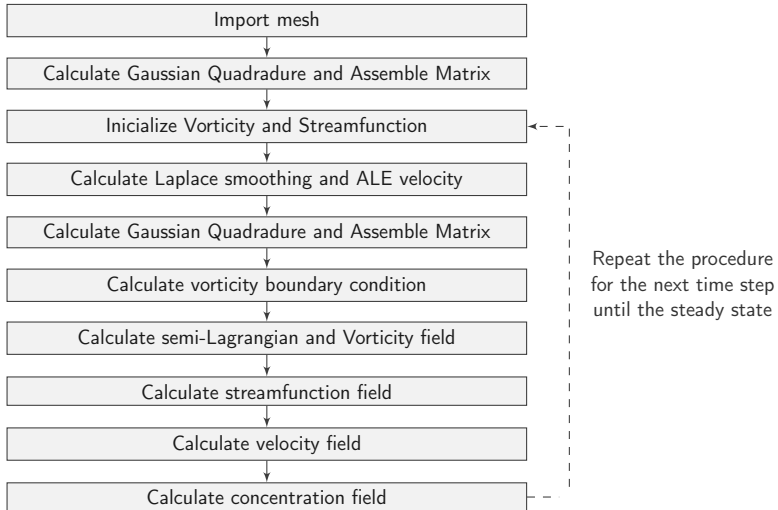


without ALE

[1] World Health Organization, 2018, "The top 10 causes of death". [www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death](http://www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death) [Online accessed 06-04-20 08:27pm]

- To create new drug-eluting design patent

# Solution Algorithm



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# Validation - Poiseuille Flow

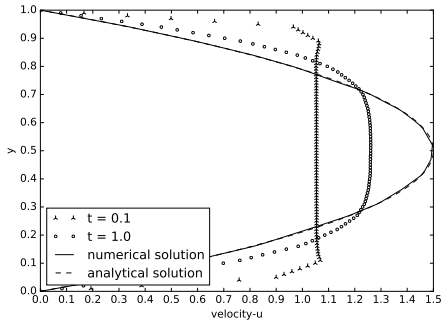
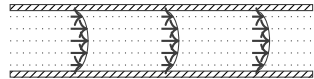
Boundaries Conditions:

Inflow condition:  $u = 1, v = 0$  e  $\psi = y$

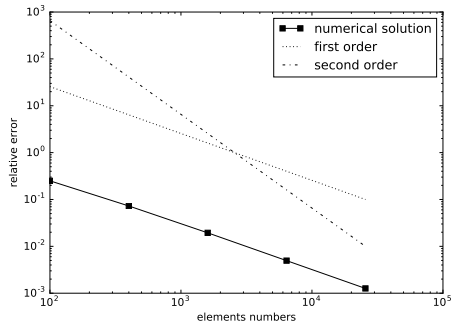
Outflow condition:  $\psi = y$

Top plate:  $u = 0, v = 0, \psi = 1$

Bottom plate:  $u = 0, v = 0, \psi = 0$



(a)



(b)

(a) comparison of Poiseuille Flow velocity profile and

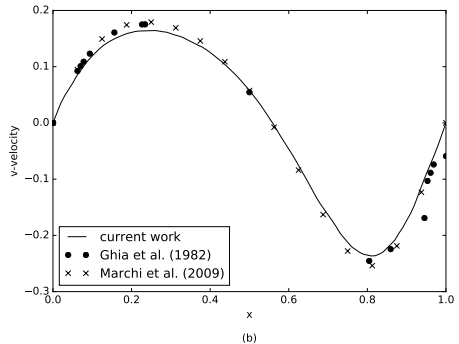
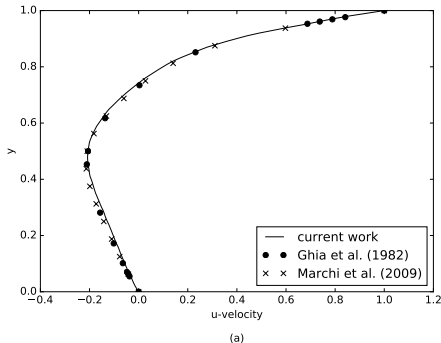
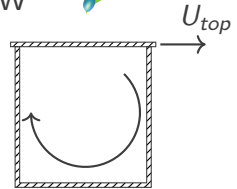
(b) log scale graph of convergence order.

# Validation - Lid Driven Cavity Flow

Boundaries Conditions:

Bottom and side plates:  $u = 0$ ,  $v = 0$  e  $\psi = 0$

Top plate:  $u = 1$ ,  $v = 0$  e  $\psi = 0$



Centerline velocity profile in a lid-driven cavity for  $Re = 100$ :  
(a) u-velocity and (b) v-velocity.

Coming Soon

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1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production
2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation
3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

# Thank you!

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