A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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GESAR

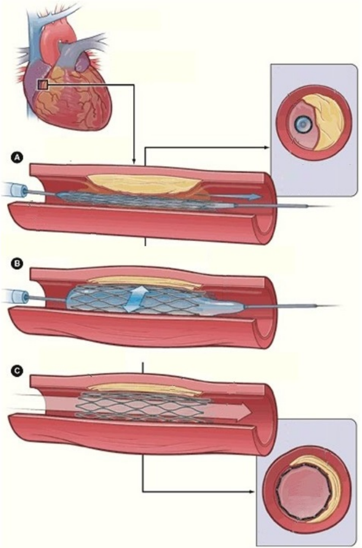
Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Introduction

Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]



Aims:

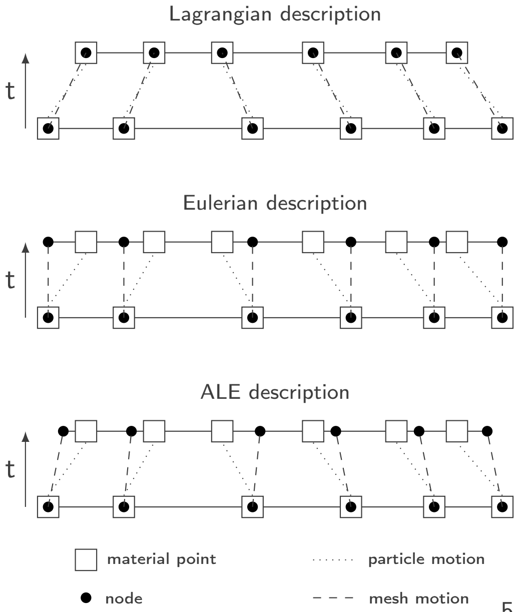
- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting design patent

[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

Arbitrary Lagrangian-Eulerian (ALE)

In ALE description, the referential frame moves with an arbitrary velocity that does not necessary represents the Lagrangian or Eulerian description.

The convection velocity $\mathbf{c}$ is the relative velocity between the material and the mesh velocity, that is, $\mathbf{c} = \mathbf{v} - \hat{\mathbf{v}}$



Governing Equations

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{\partial \omega}{\partial t} + \mathbf{c} \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega \qquad \nabla^2 \psi = -\omega$$

$$\frac{\partial \kappa}{\partial t} + \mathbf{c} \cdot \nabla \kappa = \frac{1}{ReSc} \nabla^2 \kappa$$

The material velocity field is calculated by: $v_x = \partial \psi / \partial y$ and $v_y = -\partial \psi / \partial x$

The semi-Lagragian Method

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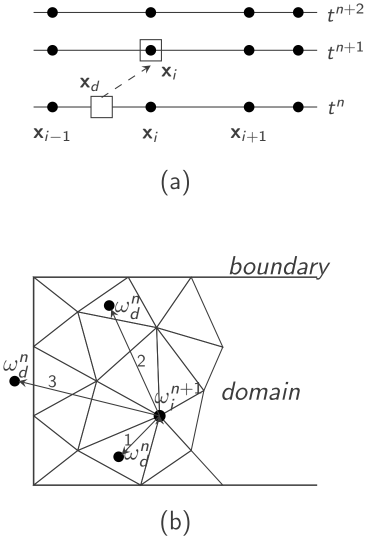
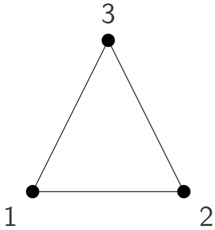


Finite Element Method



$$N_i = L_i$$

$$i = 1, 2, 3$$

$$\frac{\mathbf{M}}{\Delta t}\dot{\omega} = -\mathbf{v} \cdot \mathbf{G}\omega^n - \frac{1}{Re}\mathbf{K}\omega^n - \frac{\Delta t}{2}\mathbf{K_s}\omega^n \qquad \mathbf{K}\psi = \mathbf{M}\omega$$

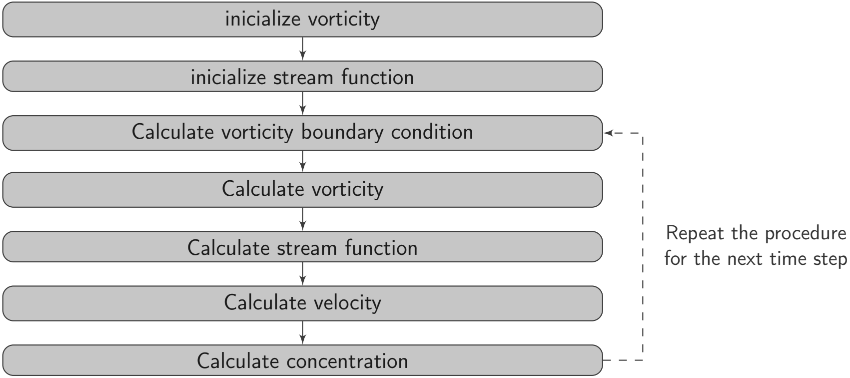
$$\frac{\mathbf{M}}{\Delta t}\dot{c} = -\mathbf{v} \cdot \mathbf{G}c^n - \frac{1}{ReSc}\mathbf{K}c^n - \frac{\Delta t}{2}\mathbf{K_s}c^n \qquad \mathbf{M}\mathbf{v} = \mathbf{G}\psi$$

Where $\mathbf{K_s}$ is stability matrix to decrease spurious oscillations

Adaptive Mesh Refinement

Laplace Smoothing description, comparative figures and ALE velocity formula

Solution Algorithm



Streamfunction-Vorticity formulation with species transport equation solution algorithm

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Validation - Poiseuille Flow

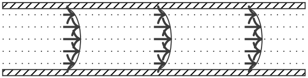
Boundaries Conditions:

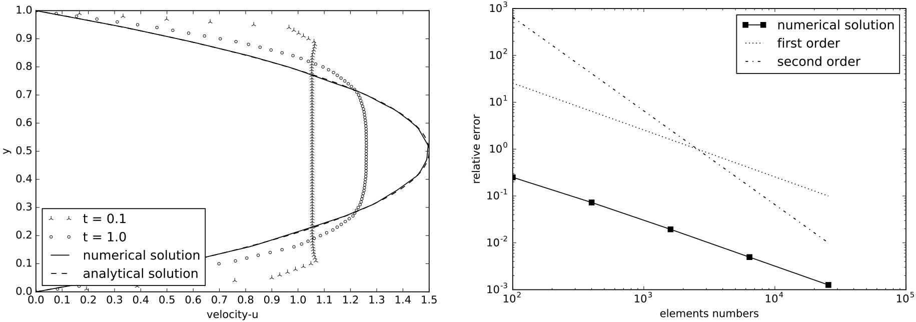
Inflow condition: $u = 1,\ v = 0$ e $\psi = y$

Outflow condition: $\psi = y$

Top plate: $u = 0,\ v = 0,\ \psi = 1$

Bottom plate: $u = 0,\ v = 0,\ \psi = 0$





(a) (b)

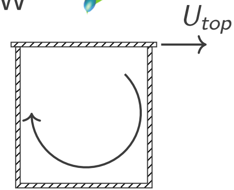
(a) comparison of Poiseuille Flow velocity profile and
(b) log scale graph of convergence order.

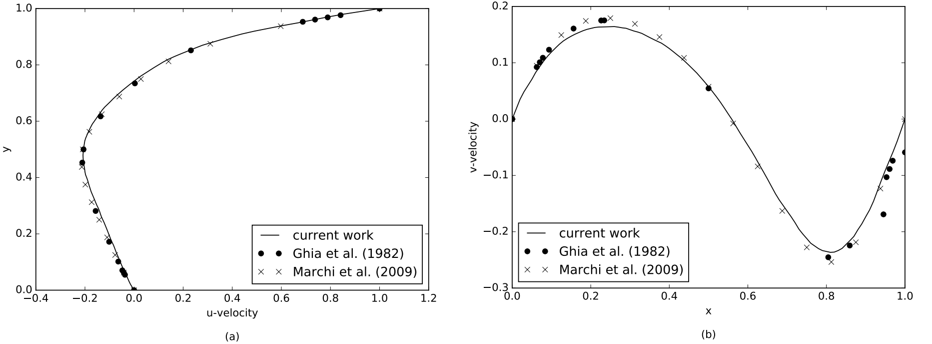
Validation - Lid Driven Cavity Flow

Boundaries Conditions:

Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

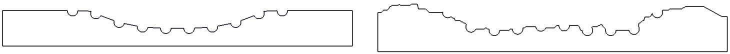
Top plate: $u = 1$, $v = 0$ e $\psi = 0$





Centerline velocity profile in a lid-driven cavity for $Re = 100$: (a) u-velocity and (b) v-velocity.

Results



(a) (b)

Non-dimensional symmetric geometry for blood flow in coronary artery with drug-eluting stent placed by Wang et al. (2017): (a) Curved Channel with Stent (b) Real Channel with Stent.

Boundaries Conditions:

Inflow condition: $u = 1$, $v = 0$ e $\psi = y$;

Outflow condition: $\psi = y$;

Top plate: $u = 0$, $v = 0$, $\psi = 1$;

Symmetry condition: $v = 0$, $\psi = 0$;

Drug-eluting stent: $u = 0$, $v = 0$, $\psi = 1$ e $c = 1$

$R = 0.0015m$

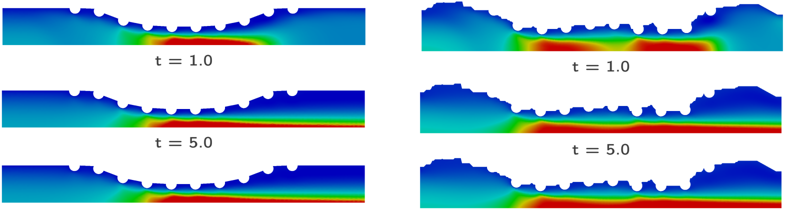
$\mu = 0.0035Pa.s$

$\rho = 1060kg/m^3$

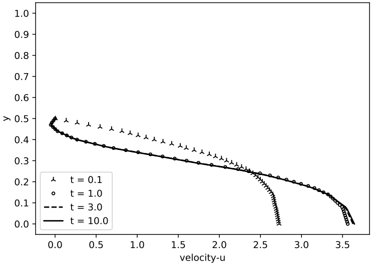
$u = 12cm/s$

$Re = 54.5$

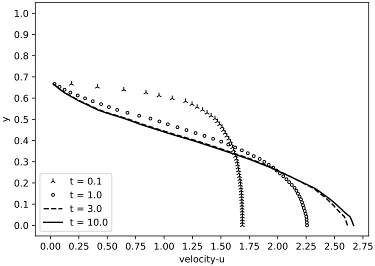
Results - Velocity Field



Evolution in time and space of velocity field:
Curved Channel (left column) and Real Channel (right column)



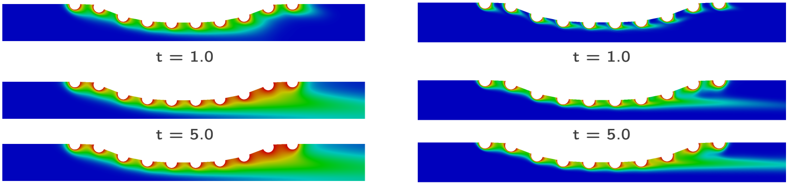
(a)



(b)

Evolution of velocity profile in centerline ($x = 0.5L$):
(a) Curved Channel and (b) Real Channel

Results - Concentration Field

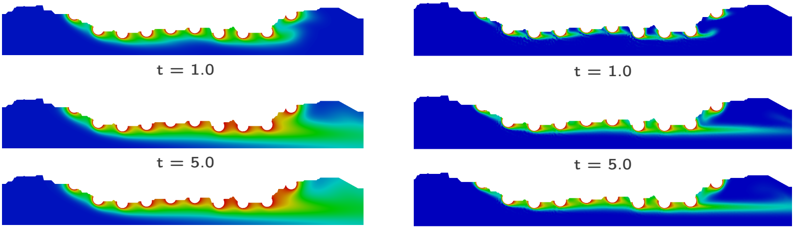


t = 1.0 t = 1.0

t = 5.0 t = 5.0

t = 10.0 t = 10.0

Evolution in time and space of concentration field in Curved Channel:
$Sc = 1$ (left column) and $Sc = 10$ (right column)



t = 1.0 t = 1.0

t = 5.0 t = 5.0

t = 10.0 t = 10.0

Evolution in time and space of concentration field in Real Channel:
$Sc = 1$ (left column) and $Sc = 10$ (right column)

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Conclusion

1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production
2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation
3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

Thank you!

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