# A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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GESAR

# Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

# Introduction

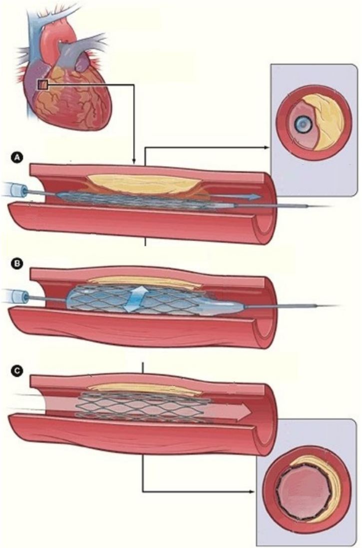
Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach

- To create new drug-eluting design patent




[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

# Arbitrary Lagrangian-Eulerian (ALE)

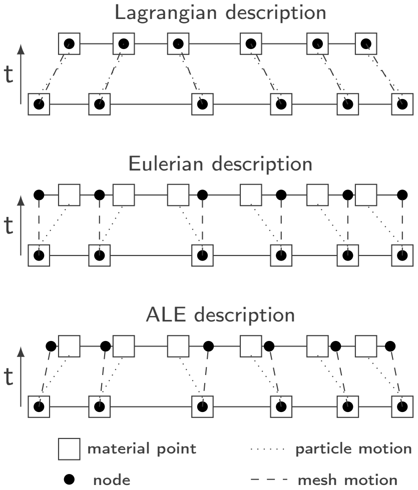
The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment





[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian–Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

# Governing Equations

Assumptions [3]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{D\omega}{Dt} = \frac{1}{Re}\nabla^2\omega$$

$$\nabla^2\psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc}\nabla^2 c$$

where, $D(\cdot)/Dt$ is substantive derivative and the material velocity field is calculated by: $v_x = \partial\psi/\partial y$ and $v_y = -\partial\psi/\partial x$

[3] Batchelor, G. (1967). An Introduction to Fluid Dynamics (Cambridge Mathematical Library). Cambridge: Cambridge University Press. *doi:10.1017/CBO9780511800955*

# Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [4]:

$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re}\nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

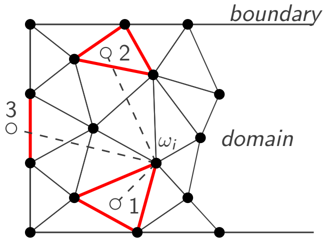
$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc}\nabla^2 c^{n+1}$$

Advantages:

- Symmetric linear systems
- Unconditionnal stability

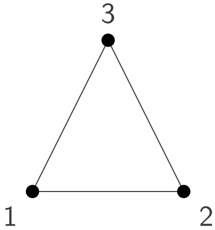
Disadvantages:

- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed




[4] Pironneau, O. On the transport-diffusion algorithm and its applications to the Navier-Stokes equations. Numer. Math. 38, 309–332 (1982). *https://doi.org/10.1007/BF01396435*

# Galerkin FE Method




$$N_i = L_i$$

$$i = 1, 2, 3$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re}\right]\omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t}\omega_d^n$$

$$\mathbf{K}\psi = \mathbf{M}\omega$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc}\right]c_i^{n+1} = \frac{\mathbf{M}}{\Delta t}c_d^n$$

The material velocity field is calculated by: $\mathbf{M}v_x = \mathbf{G_y}\psi$ and $\mathbf{M}v_y = -\mathbf{G_x}\psi$

# Mesh Smoothing

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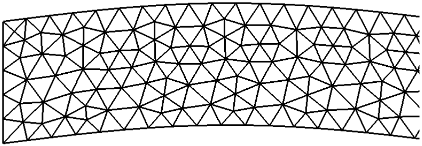

with Mesh Smoothing

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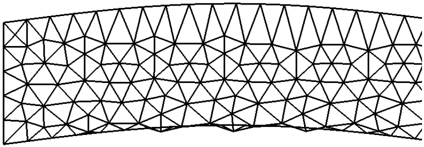


no Mesh Smoothing

[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

- To create new drug-eluting design patent

# Solution Algorithm

Import mesh
↓
Calculate Gaussian Quadrature and Assemble Matrix
↓
Inicialize Vorticity and Streamfunction
↓
Calculate Laplace smoothing and ALE velocity
↓
Calculate Gaussian Quadrature and Assemble Matrix
↓
Calculate vorticity boundary condition
↓
Calculate semi-Lagrangian and Vorticity field
↓
Calculate streamfunction field
↓
Calculate velocity field
↓
Calculate concentration field

Repeat the procedure
for the next time step
until the steady state

# Validation - Poiseuille Flow

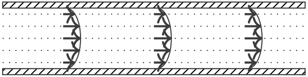
Boundaries Conditions:

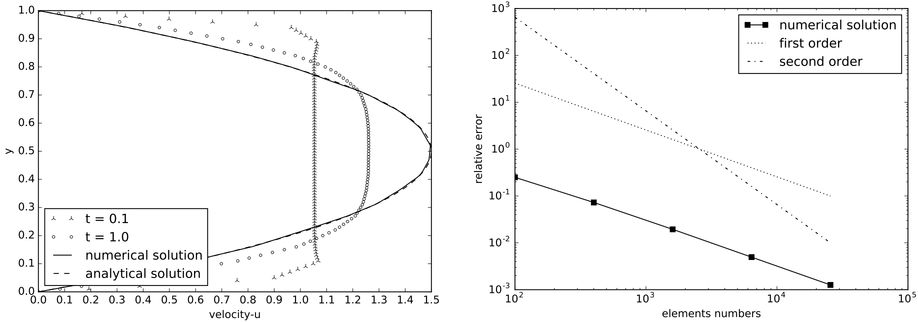
Inflow condition: $u = 1$, $v = 0$ e $\psi = y$

Outflow condition: $\psi = y$

Top plate: $u = 0$, $v = 0$, $\psi = 1$

Bottom plate: $u = 0$, $v = 0$, $\psi = 0$






(a) (b)

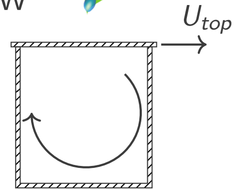
(a) comparison of Poiseuille Flow velocity profile and
(b) log scale graph of convergence order.

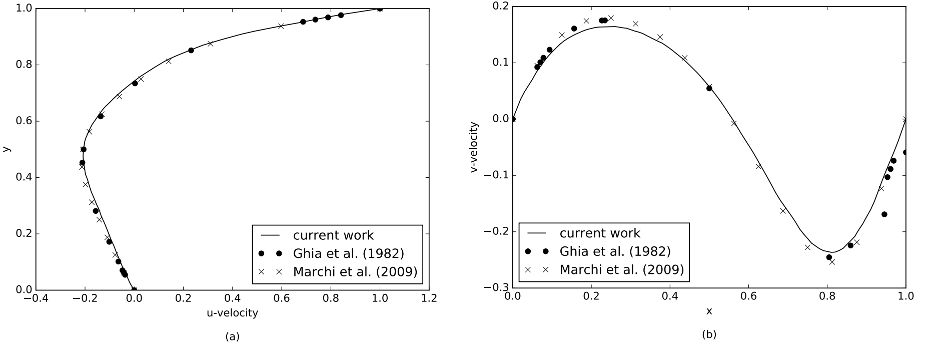
# Validation - Lid Driven Cavity Flow

Boundaries Conditions:

Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

Top plate: $u = 1$, $v = 0$ e $\psi = 0$




Centerline velocity profile in a lid-driven cavity for $Re = 100$:
(a) u-velocity and (b) v-velocity.

# Validation - Backward-facing Step

Coming Soon

# Validation - Pulsation Flow

Coming Soon

# Results

Coming Soon

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# Conclusion

1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production
2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation
3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

# Thank you!

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