

# A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

Student Researcher: Leandro Marques

Advisors: Gustavo Anjos and Jose Pontes

State University of Rio de Janeiro

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# Outline

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion aa

# Introduction

## Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

## Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting design patent



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# Arbitrary Lagrangian-Eulerian (ALE)

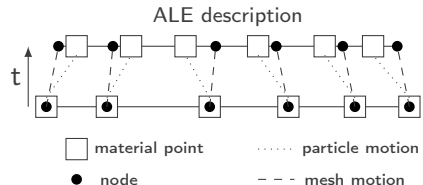
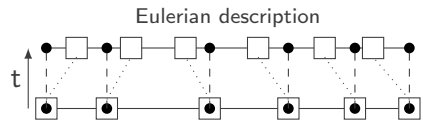
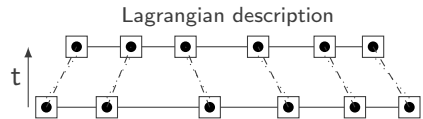
The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment



[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian-Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

# Governing Equations

Assumptions [3]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{\partial \omega}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

$$\frac{\partial c}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla c = \frac{1}{ReSc} \nabla^2 c$$

The material velocity field  $\mathbf{v} = (v_x, v_y)$  is calculated by:  $v_x = \partial \psi / \partial y$  and  $v_y = -\partial \psi / \partial x$

The mesh velocity field  $\hat{\mathbf{v}} \neq \mathbf{v}$  (*Lagrangian*) or 0 (*Eulerian*)

# Semi-Lagrangian Method

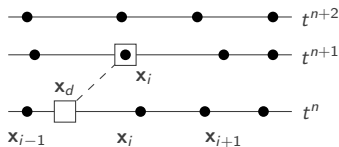
The convective term was replaced by material derivative of  $\omega$  and  $c$  in the direction of characteristic trajectory

$$\frac{D\omega}{Dt} = \frac{1}{Re} \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc} \nabla^2 c$$

The departure node is calculated by  $x_d^n = x_i^{n+1} - (\mathbf{v} - \hat{\mathbf{v}}) \Delta t$ . Then, a searching procedure is required to find  $x_d^n$  using barycentric coordinates



# Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [4]:

Advantages:

- Symmetric linear systems
- Unconditionnal stability

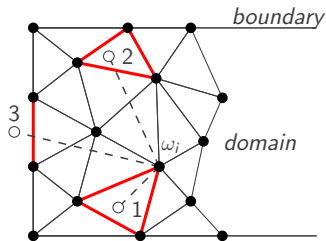
Disadvantages:

- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed

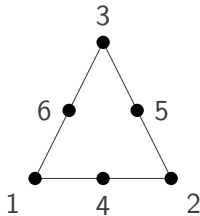
$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re} \nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

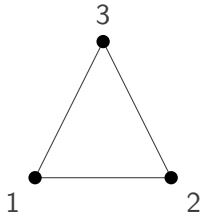
$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc} \nabla^2 c^{n+1}$$







Quad Element



Linear Element

$$\left[ \frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re} \right] \omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t} \omega_d^n$$

$$\mathbf{K}\psi = \mathbf{M}\omega$$

$$\left[ \frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc} \right] c_i^{n+1} = \frac{\mathbf{M}}{\Delta t} c_d^n$$

The material velocity field is calculated by:  $\mathbf{M}v_x = \mathbf{G}_y\psi$  and  $\mathbf{M}v_y = -\mathbf{G}_x\psi$

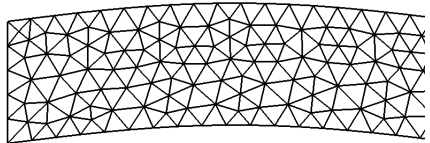
# Laplacian Smoothing

To avoid the fast degradation of the computational elements due to ALE description, it was used the Laplacian Smoothing Method [5]

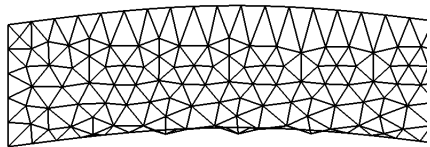
The new node position  $\hat{\mathbf{x}}_i$  can be approximated by:

$$\hat{\mathbf{x}}_i = \sum_{i \in N_1} e_{ij}^{-1} (\mathbf{x}_j - \mathbf{x}_i)$$

where,  $e_{ij}^{-1}$  is the distance between the node and each neighbor in 1-ring  $N_1$



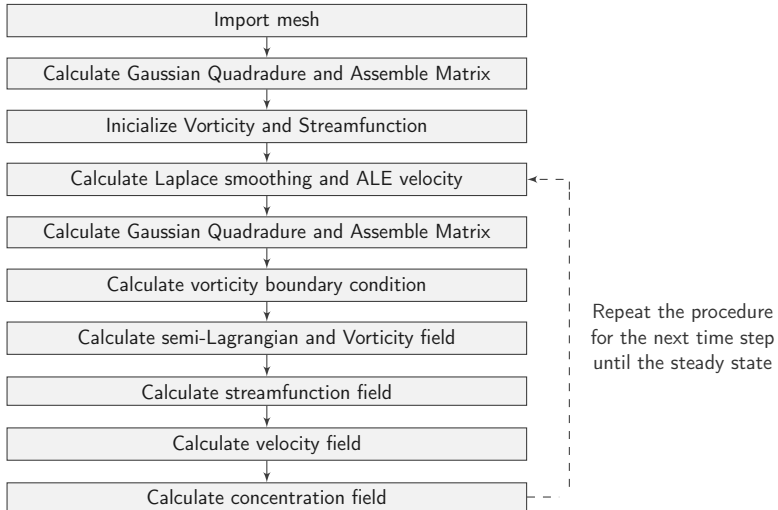
with Laplacian Smoothing



no Laplacian Smoothing

[5] Desbrun, M. Meyer, P. Schröder, A. Barr, Implicit fairing of irregular meshes using diffusion and curvature flow, in: Proceedings of Siggraph, 1999, pp. 317–324

# Solution Algorithm



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# Validation - Poiseuille Flow

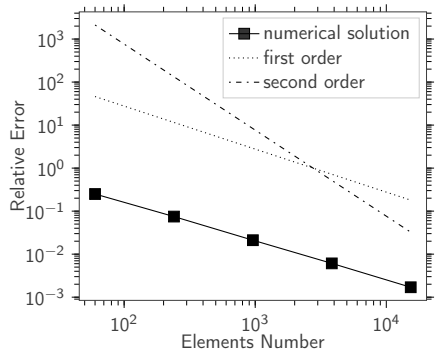
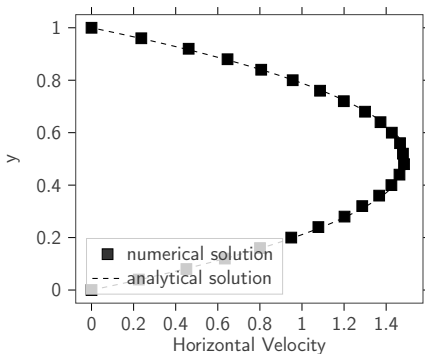
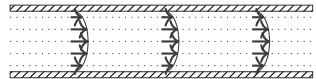
Boundaries Conditions:

Inflow condition:  $u = 1$ ,  $v = 0$  e  $\psi = y$

Outflow condition:  $\psi = y$

Top plate:  $u = 0$ ,  $v = 0$ ,  $\psi = 1$

Bottom plate:  $u = 0$ ,  $v = 0$ ,  $\psi = 0$

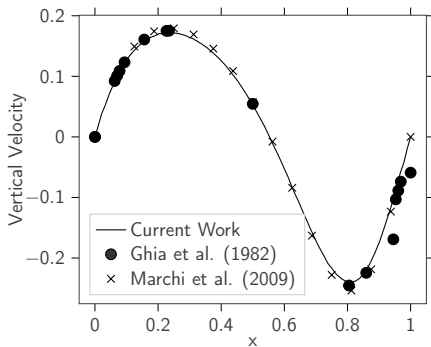
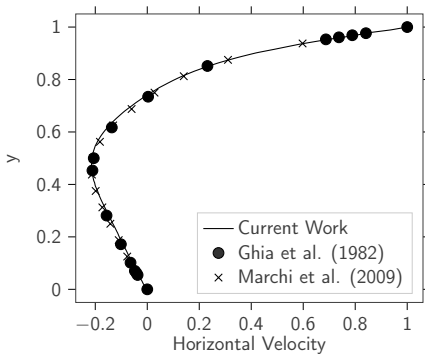
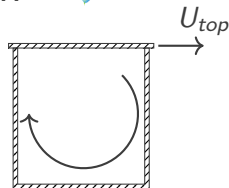


# Validation - Lid Driven Cavity Flow

Boundaries Conditions:

Bottom and side plates:  $u = 0$ ,  $v = 0$  e  $\psi = 0$

Top plate:  $u = 1$ ,  $v = 0$  e  $\psi = 0$



Coming Soon

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1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production
2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation
3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

# Thank you!

marquesleandro67@gmail.com  
gustavo.rabello@mecanica.coppe.ufrj.br  
jose.pontes@uerj.br

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