A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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Outline

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Introduction

Motivation:

- ▶ Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- ▶ To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach

- ▶ To create new drug-eluting design patent

[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

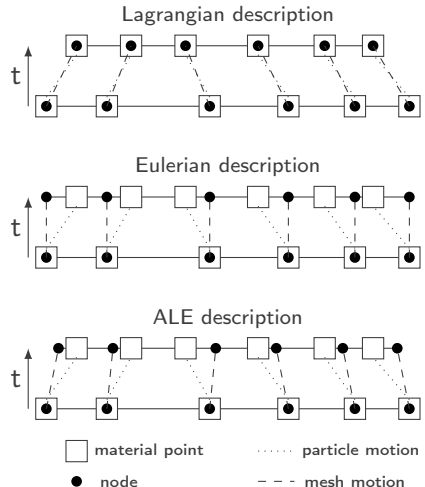# Arbitrary Lagrangian-Eulerian (ALE) 🌿 **GESAR**

The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

▶ Simulations in fluid-structure and moving boundary problems

Disadvantages:

▶ The computational mesh requires an extensive topological treatment

Lagrangian description

Eulerian description

ALE description

□ material point   ⋯⋯ particle motion
● node   – – – mesh motion

[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian–Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

Governing Equations

Assumptions [3]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{D\omega}{Dt} = \frac{1}{Re}\nabla^2\omega$$

$$\nabla^2\psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc}\nabla^2 c$$

where, $D(\cdot)/Dt$ is substantive derivative and the material velocity field is calculated by: $v_x = \partial\psi/\partial y$ and $v_y = -\partial\psi/\partial x$

[3] Batchelor, G. (1967). An Introduction to Fluid Dynamics (Cambridge Mathematical Library). Cambridge: Cambridge University Press. *doi:10.1017/CBO9780511800955*

Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [4]:

Advantages:

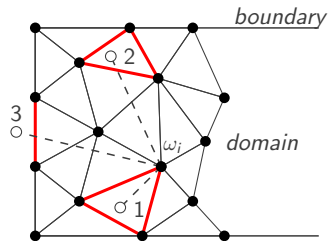▶ Symmetric linear systems

▶ Unconditionnal stability

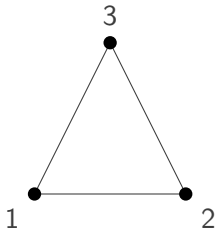$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re}\nabla^2 \omega^{n+1}$$

$$\nabla^2 \psi = -\omega$$

$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc}\nabla^2 c^{n+1}$$

Disadvantages:

▶ Numerical Diffusion

▶ Searching procedure may lead to excessive computational cost if it is not well designed

[4] Pironneau, O. On the transport-diffusion algorithm and its applications to the Navier-Stokes equations. Numer. Math. 38, 309–332 (1982). *https://doi.org/10.1007/BF01396435*

Galerkin FE Method

$N_i = L_i$

$i = 1, 2, 3$

$$\left[\frac{\mathsf{M}}{\Delta t} + \frac{\mathsf{K}}{Re}\right] \omega_i^{n+1} = \frac{\mathsf{M}}{\Delta t}\omega_d^n$$

$$\mathsf{K}\psi = \mathsf{M}\omega$$

$$\left[\frac{\mathsf{M}}{\Delta t} + \frac{\mathsf{K}}{ReSc}\right] c_i^{n+1} = \frac{\mathsf{M}}{\Delta t}c_d^n$$

The material velocity field is calculated by: $\mathsf{M}v_x = \mathsf{G_y}\psi$ and $\mathsf{M}v_y = -\mathsf{G_x}\psi$
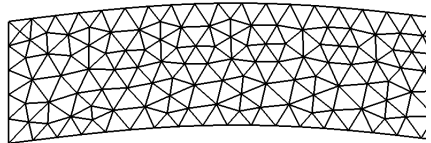
Laplacian Smoothing

To avoid the fast degradation of the computational elements due to ALE description, it was used the Laplacian Smoothing Method [5]
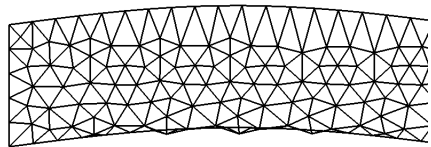

with Laplacian Smoothing

The new node position $\hat{x}_i$ can be approximated by:

$$\hat{x}_i = \sum_{i \in N_1} e_{ij}^{-1}(x_j - x_i)$$

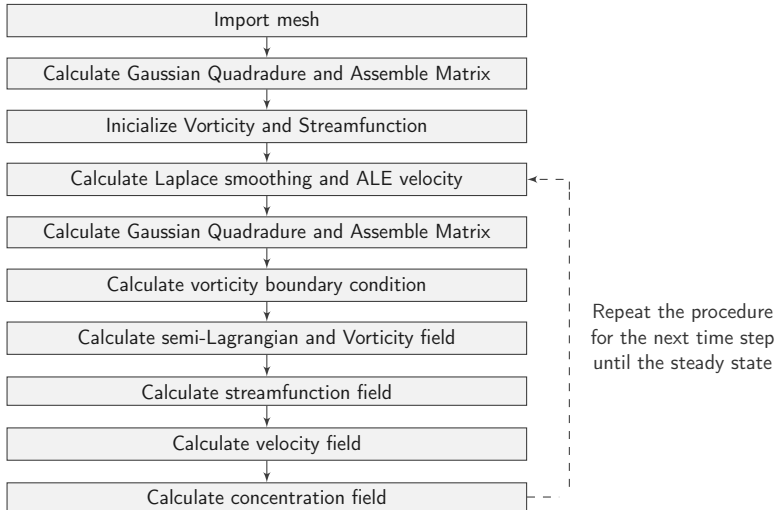where, e_{ij}^{-1} is the distance between the node and each neighbor in 1-ring N_1

no Laplacian Smoothing

[5] Desbrun, M. Meyer, P. Schröder, A. Barr, Implicit fairing of irregular meshes using diffusion and curvature flow, in: Proceedingsof Siggraph, 1999, pp. 317–324

Solution Algorithm

Import mesh
Calculate Gaussian Quadradure and Assemble Matrix
Inicialize Vorticity and Streamfunction
Calculate Laplace smoothing and ALE velocity
Calculate Gaussian Quadradure and Assemble Matrix
Calculate vorticity boundary condition
Calculate semi-Lagrangian and Vorticity field
Calculate streamfunction field
Calculate velocity field
Calculate concentration field

Repeat the procedure
for the next time step
until the steady state

Validation - Poiseuille Flow

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Boundaries Conditions:

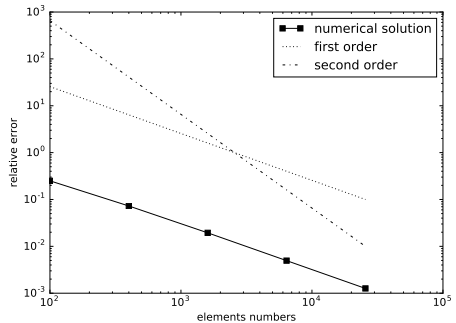Inflow condition: $u = 1$, $v = 0$ e $\psi = y$

Outflow condition: $\psi = y$

Top plate: $u = 0$, $v = 0$, $\psi = 1$

Bottom plate: $u = 0$, $v = 0$, $\psi = 0$
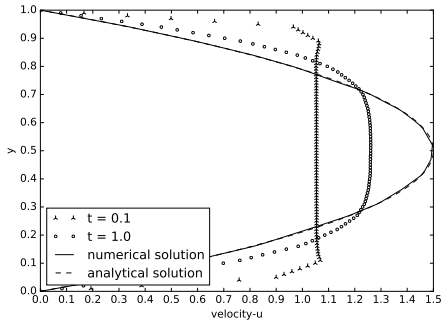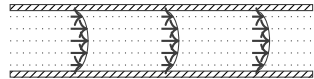
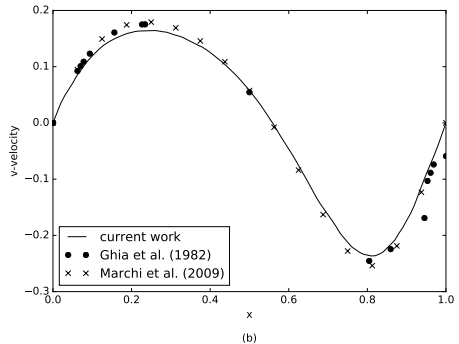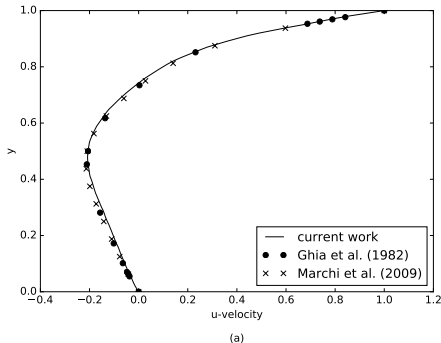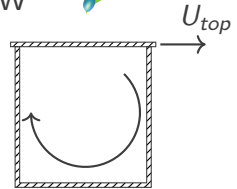(a) comparison of Poiseuille Flow velocity profile and
(b) log scale graph of convergence order.

Validation - Lid Driven Cavity Flow

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U_{top}

Boundaries Conditions:

Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

Top plate: $u = 1$, $v = 0$ e $\psi = 0$

Centerline velocity profile in a lid-driven cavity for $Re = 100$: (a) u-velocity and (b) v-velocity.

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Coming Soon

Coming Soon

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Coming Soon

Conclusion

1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production

2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation

3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

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Thank you!

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