A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

Student Researcher: Leandro Marques
Advisors: Jose Pontes and Gustavo Anjos

State University of Rio de Janeiro
June, 17th 2020

GESAR

Outline

GESAR

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

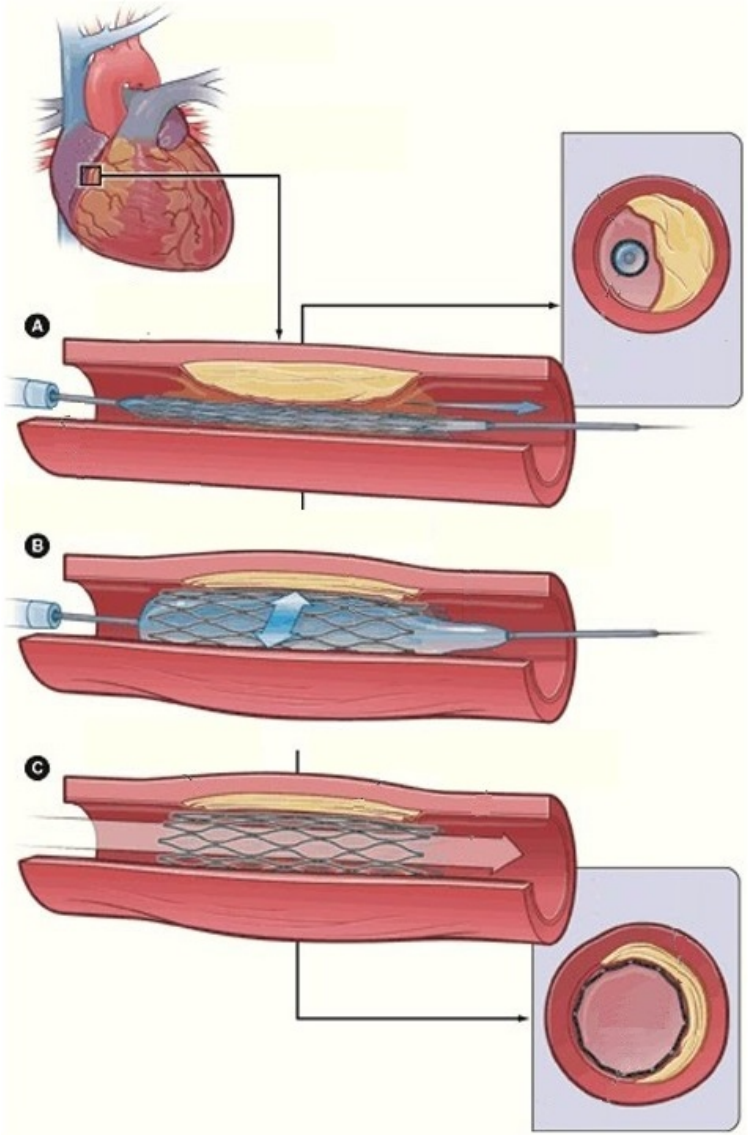
Introduction

Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]

Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting design patent



[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

GESAR

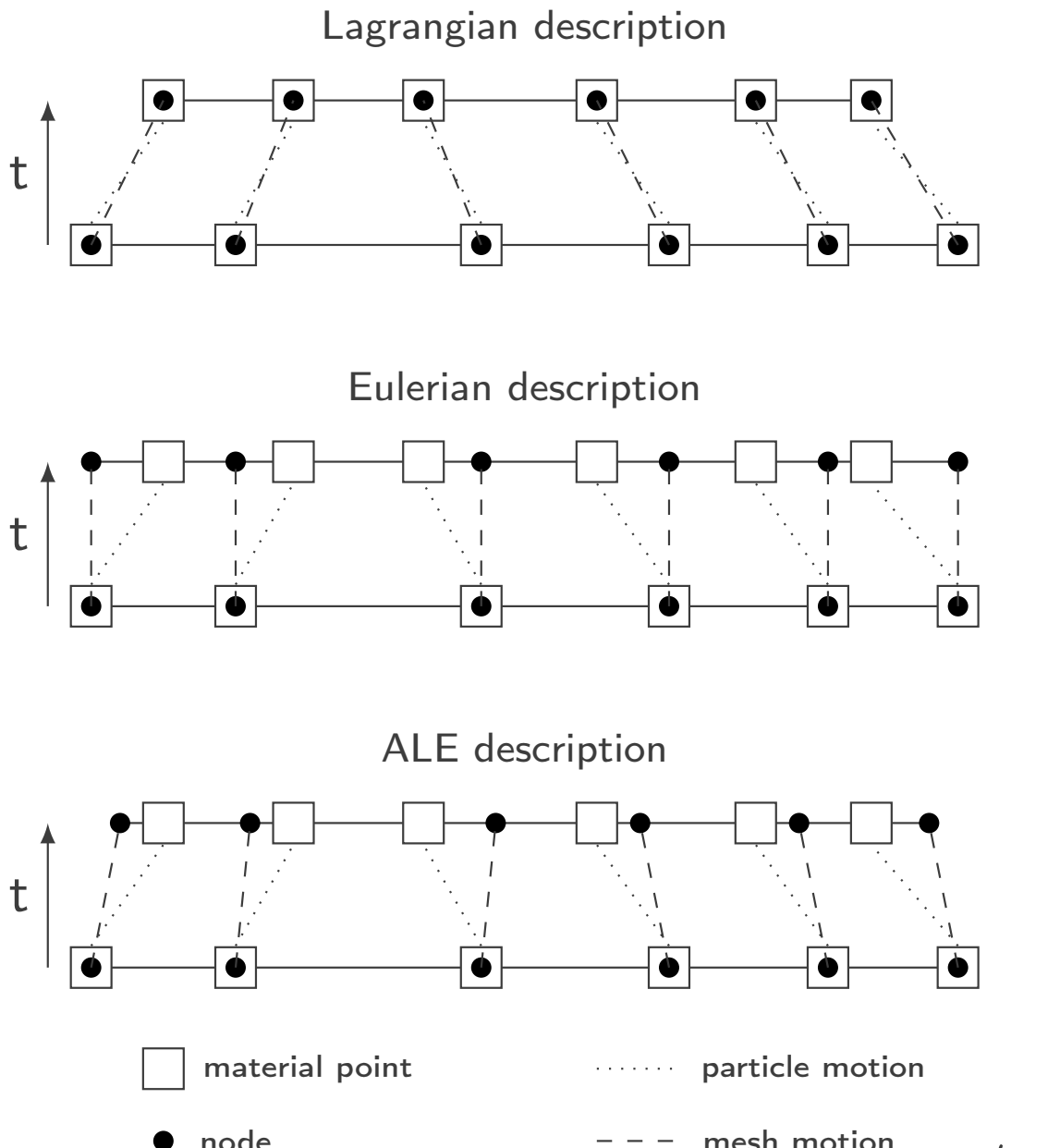
1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Arbitrary Lagrangian-Eulerian (ALE)

In ALE description, the referential frame moves with an arbitrary velocity that does not necessary represents the Lagrangian or Eulerian description.

The convection velocity $\mathbf{c}$ is the relative velocity between the material and the mesh velocity respectively, that is:

$$\mathbf{c} = \mathbf{v} - \hat{\mathbf{v}}$$



Governing Equations

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{\partial \omega}{\partial t} + \mathbf{c} \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega \qquad \nabla^2 \psi = -\omega$$

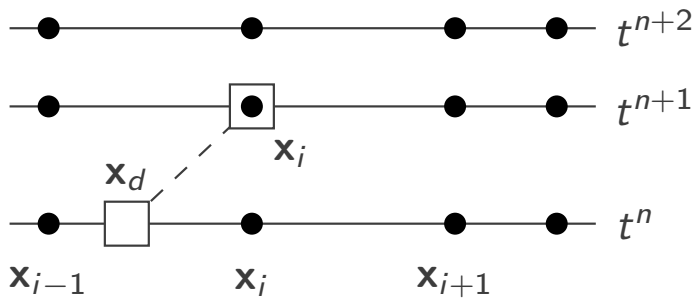
$$\frac{\partial \kappa}{\partial t} + \mathbf{c} \cdot \nabla \kappa = \frac{1}{ReSc} \nabla^2 \kappa$$

The material velocity field is calculated by: $v_x = \partial\psi/\partial y$ and $v_y = -\partial\psi/\partial x$

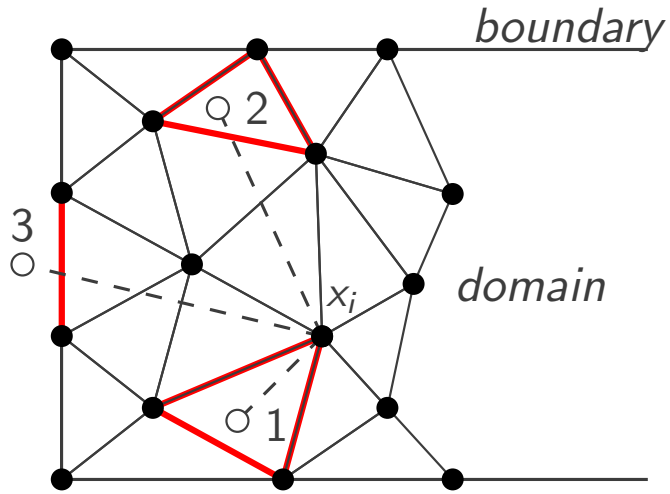
The semi-Lagragian Method

In ALE description, the referential frame moves with an arbitrary velocity that does not necessary represents the Lagrangian or Eulerian description.

The convection velocity $\mathbf{c}$ is the relative velocity between the material and the mesh velocity, that is, $\mathbf{c} = \mathbf{v} - \hat{\mathbf{v}}$

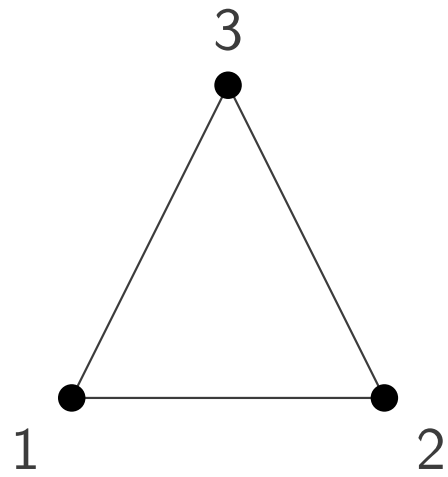


(a)



(b)

Finite Element Method



$$N_i = L_i$$

$$i = 1, 2, 3$$

$$\frac{\mathbf{M}}{\Delta t}\dot{\omega} = -\mathbf{v}\cdot\mathbf{G}\omega^n - \frac{1}{Re}\mathbf{K}\omega^n - \frac{\Delta t}{2}\mathbf{K_s}\omega^n \qquad \mathbf{K}\psi = \mathbf{M}\omega$$

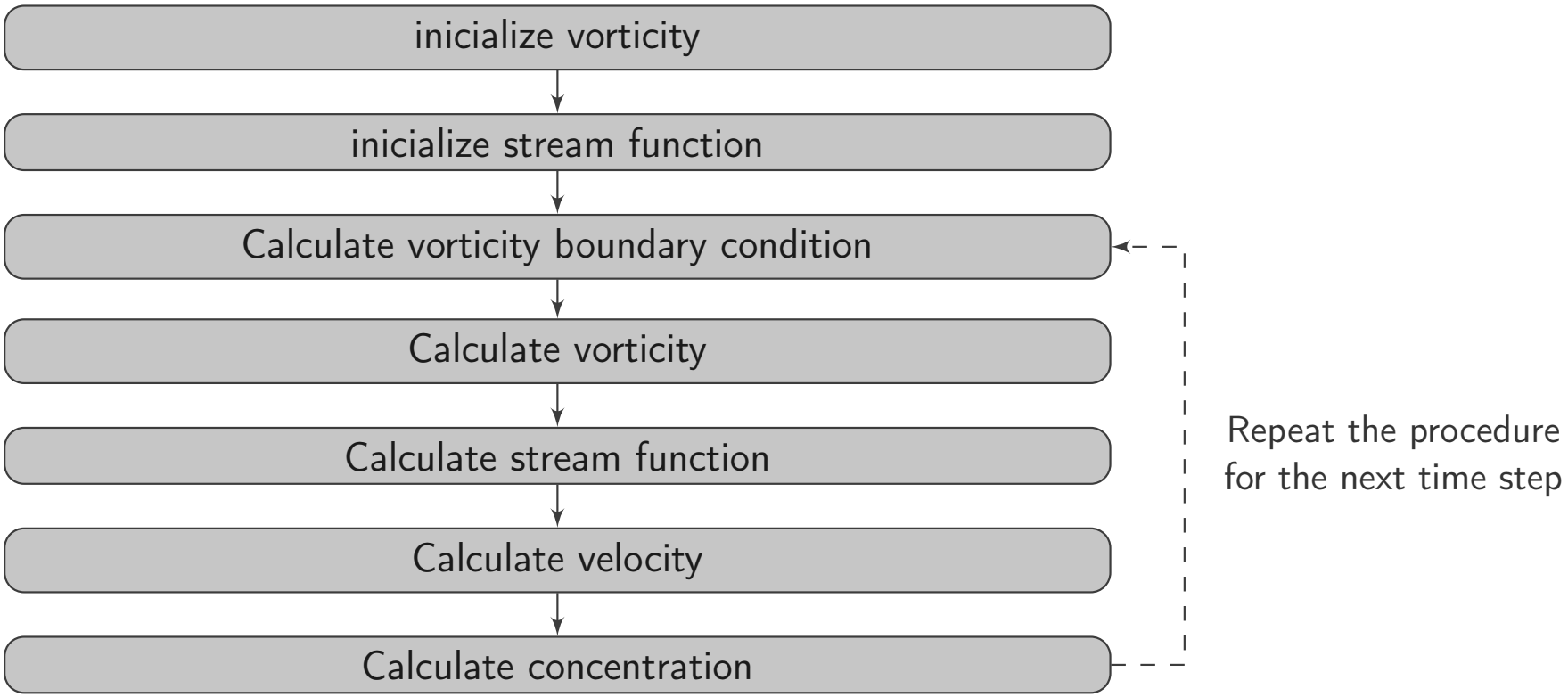
$$\frac{\mathbf{M}}{\Delta t}\dot{c} = -\mathbf{v}\cdot\mathbf{G}c^n - \frac{1}{ReSc}\mathbf{K}c^n - \frac{\Delta t}{2}\mathbf{K_s}c^n \qquad \mathbf{M}\mathbf{v} = \mathbf{G}\psi$$

Where $\mathbf{K_s}$ is stability matrix to decrease spurious oscillations

Adaptive Mesh Refinement

Laplace Smoothing description, comparative figures and ALE velocity formula

Solution Algorithm



Streamfunction-Vorticity formulation with species transport equation solution algorithm

GESAR

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Validation - Poiseuille Flow

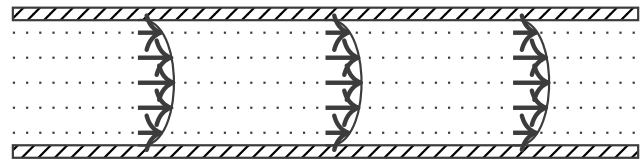
Boundaries Conditions:

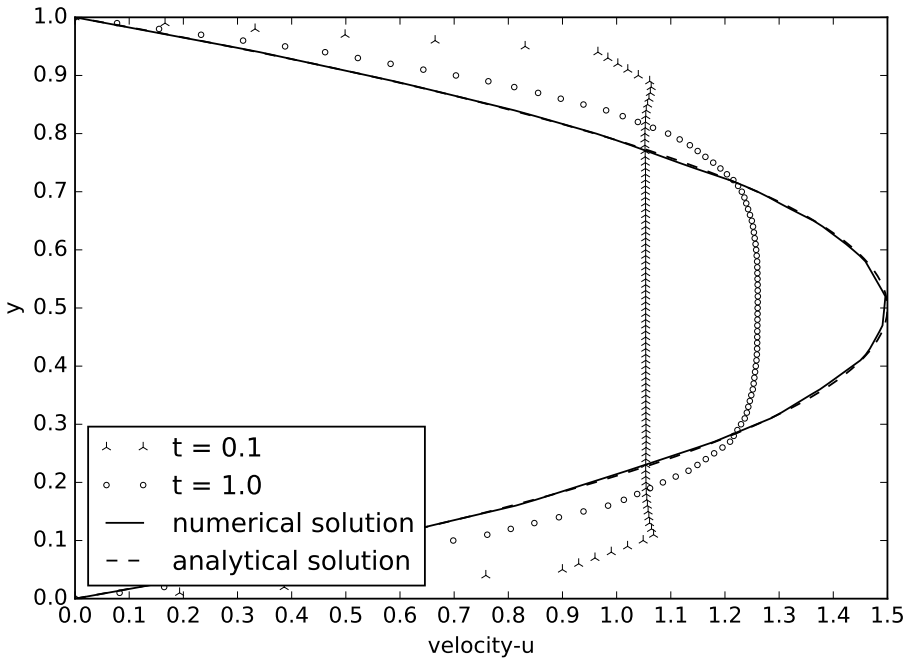
Inflow condition: $u = 1$, $v = 0$ e $\psi = y$

Outflow condition: $\psi = y$

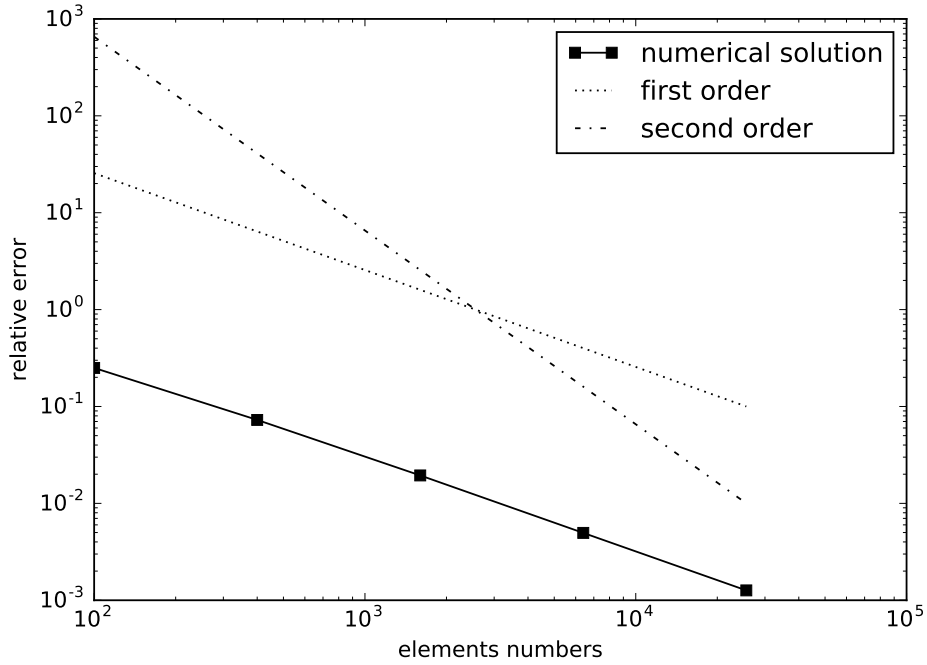
Top plate: $u = 0$, $v = 0$, $\psi = 1$

Bottom plate: $u = 0$, $v = 0$, $\psi = 0$





(a)



(b)

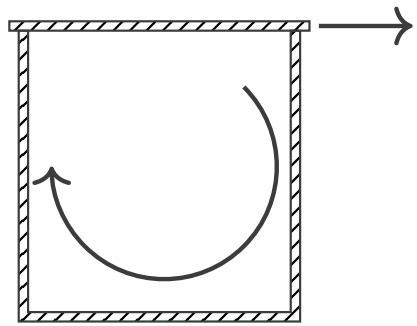
(a) comparison of Poiseuille Flow velocity profile and
(b) log scale graph of convergence order.

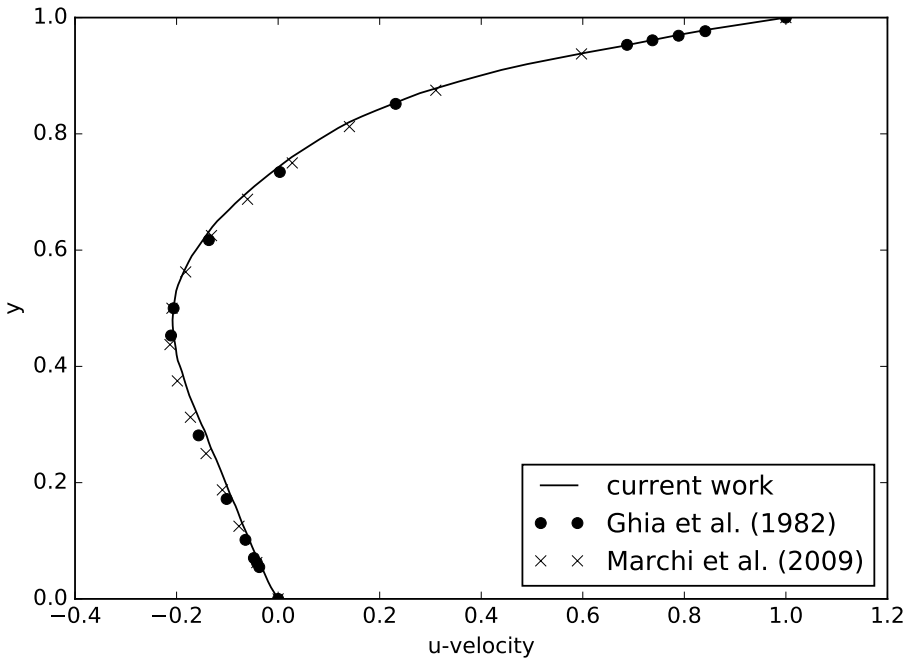
Validation - Lid Driven Cavity Flow

Boundaries Conditions:

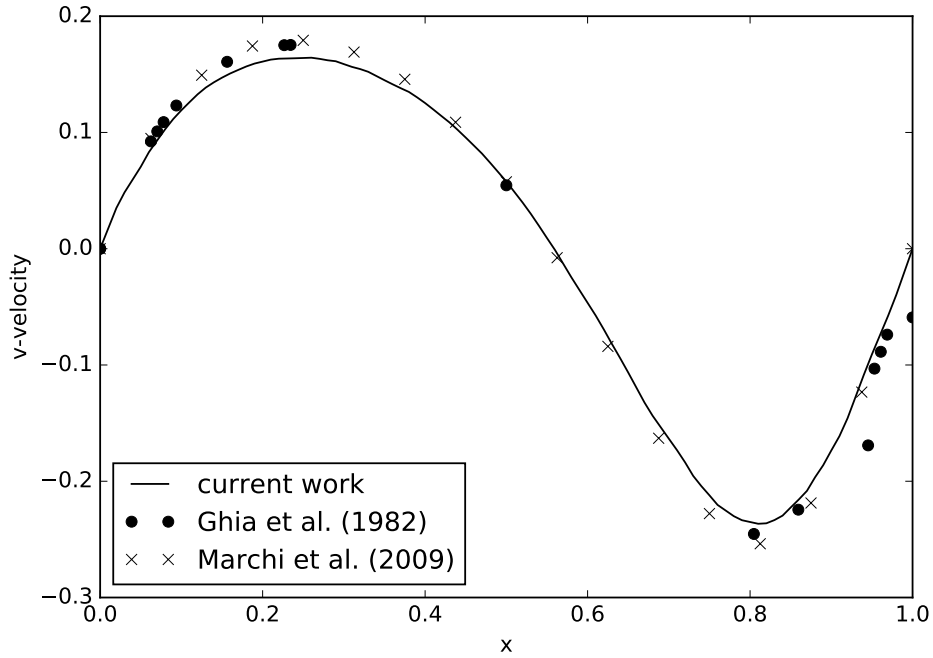
Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

Top plate: $u = 1$, $v = 0$ e $\psi = 0$





(a)

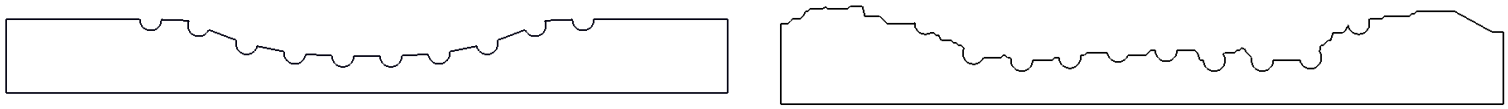


(b)

Centerline velocity profile in a lid-driven cavity for $Re = 100$:
(a) u-velocity and (b) v-velocity.

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Results



(a) (b)

Non-dimensional symmetric geometry for blood flow in coronary artery with drug-eluting stent placed by Wang et al. (2017): (a) Curved Channel with Stent (b) Real Channel with Stent.

Boundaries Conditions:

Inflow condition: $u = 1$, $v = 0$ e $\psi = y$;

Outflow condition: $\psi = y$;

Top plate: $u = 0$, $v = 0$, $\psi = 1$;

Symmetry condition: $v = 0$, $\psi = 0$;

Drug-eluting stent: $u = 0$, $v = 0$, $\psi = 1$ e $c = 1$

$R = 0.0015m$

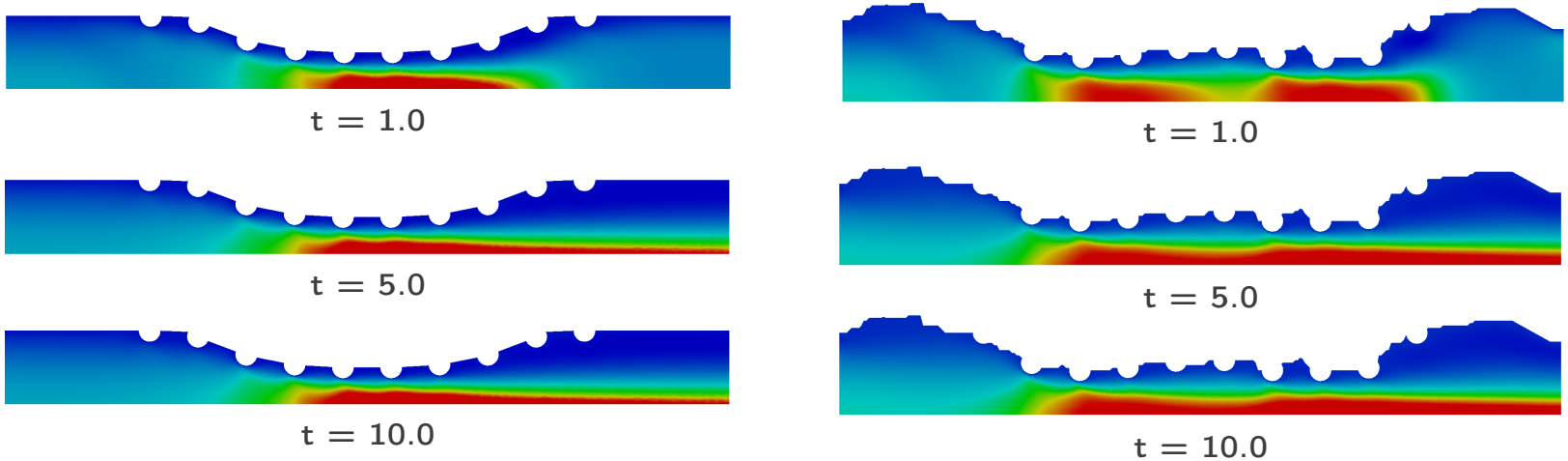
$\mu = 0.0035Pa.s$

$\rho = 1060kg/m^3$

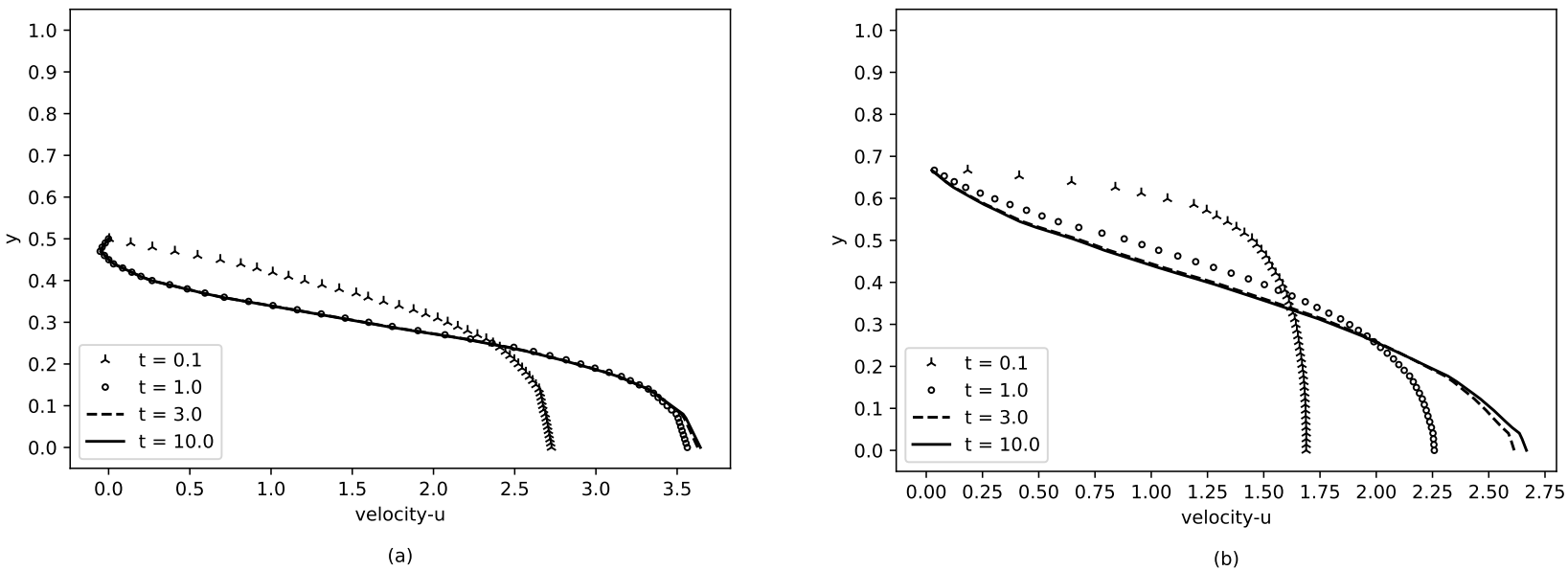
$u = 12cm/s$

$Re = 54.5$

Results - Velocity Field

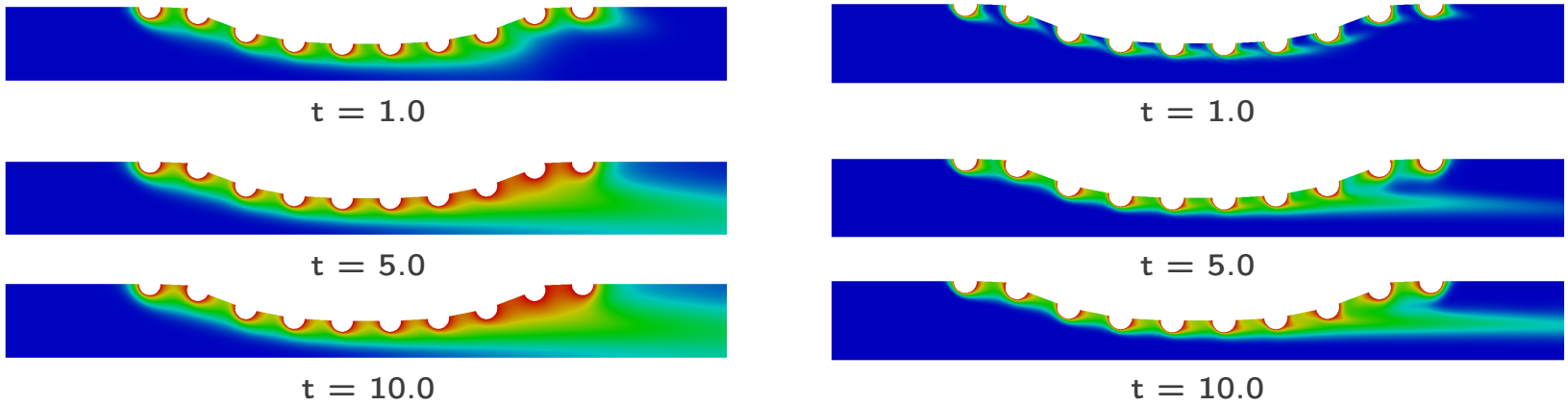


Evolution in time and space of velocity field:
Curved Channel (left column) and Real Channel (right column)

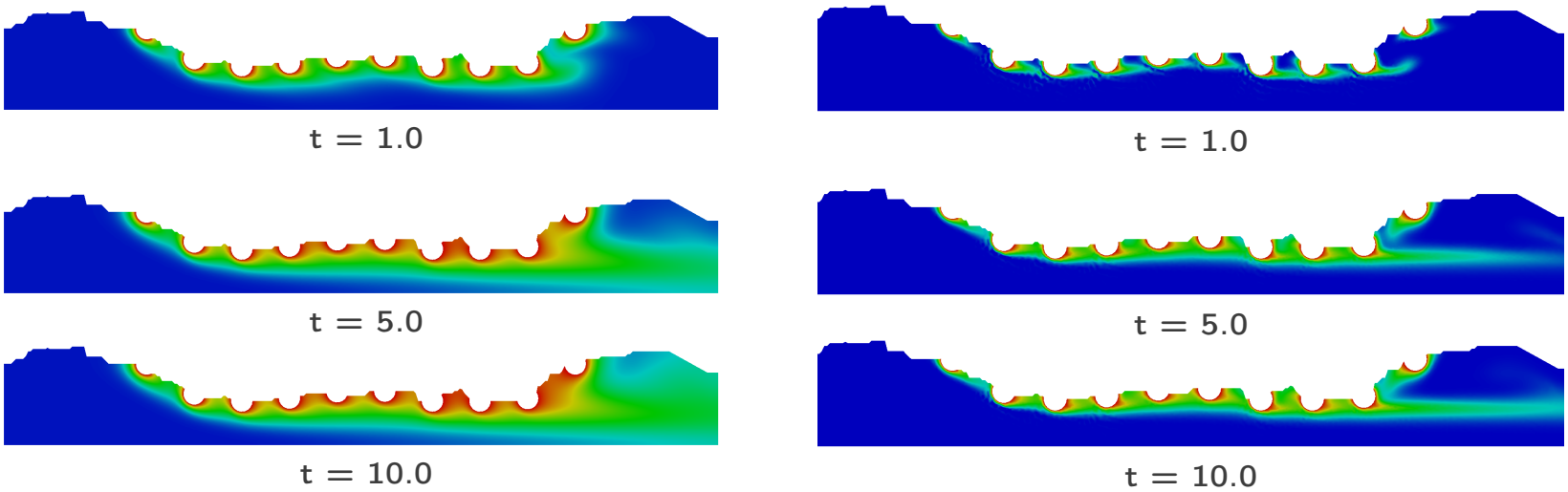


Evolution of velocity profile in centerline ($x = 0.5L$):
(a) Curved Channel and (b) Real Channel

Results - Concentration Field



Evolution in time and space of concentration field in Curved Channel:
$Sc = 1$ (left column) and $Sc = 10$ (right column)



Evolution in time and space of concentration field in Real Channel:
$Sc = 1$ (left column) and $Sc = 10$ (right column)

GESAR

1. Introduction
2. Mathematical Model
3. Validation
4. Results
5. Conclusion

Conclusion

1. Was observed that the species transport in blood flow is directly influenced by drug used in stent production

2. The streamfunction-vorticity formulation showed an useful approach for to calculate the velocity and concentration fields since the variables are scalars allowing a smooth implementation

3. Due to generalized construction of the code, the simulator is able to describe drug-eluting stent problem in coronary artery as well as flows of Newtonian fluids with scalar transport (concentration or temperature)

Thank you!

marquesleandro67@gmail.com
jose.pontes@uerj.br
gustavo.rabello@mecanica.coppe.ufrj.br

The authors thank the FAPERJ (Research Support Foundation of the State of Rio de Janeiro) for its financial support