A ALE Finite Element Method for Vorticity-Streamfunction Formulation with Species Transport Equation

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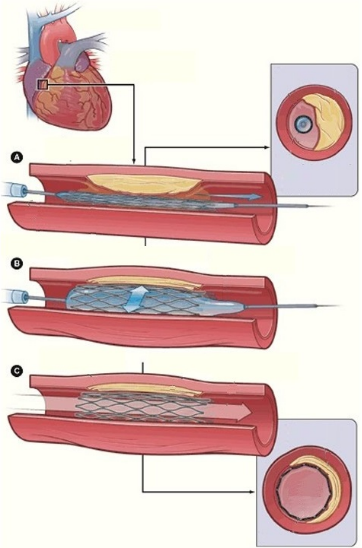
Outline

Introduction

Motivation:

- Ischaemic heart disease and stroke have remained the leading death causes globally in the last 15 years [1]



Aims:

- To develop a Finite Element code for stream-vorticity formulation with species transport equation using the Arbitrary Lagrangian-Eulerian (ALE) approach
- To create new drug-eluting stent design patent

[1] World Health Organization, 2018, "The top 10 causes of death". *www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death* [Online accessed 06-04-20 08:27pm]

Arbitrary Lagrangian-Eulerian (ALE)

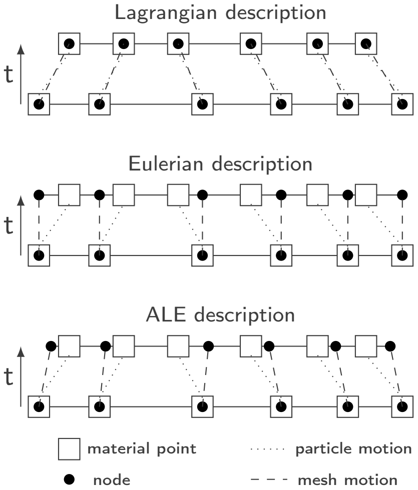
The Arbitrary Lagrangian-Eulerian combines the classical motion descriptions, while it provides [2]:

Advantages:

- Simulations in fluid-structure and moving boundary problems

Disadvantages:

- The computational mesh requires an extensive topological treatment



[2] Donea, J., Huerta, A., Ponthot, J.-P. and Rodríguez-Ferran, A. (2004). Arbitrary Lagrangian–Eulerian Methods. In Encyclopedia of Computational Mechanics *doi:10.1002/0470091355.ecm009*

Governing Equations

Assumptions [3]:

1. Continuum hypothesis
2. Homogeneous and Isotropic
3. Incompressible
4. Newtonian
5. Constant Mass Difusivity
6. Single-phase Flow
7. Two-dimensional flow

$$\frac{\partial \omega}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega$$

$$\nabla^2 \psi = -\omega$$

$$\frac{\partial c}{\partial t} + (\mathbf{v} - \hat{\mathbf{v}}) \cdot \nabla c = \frac{1}{ReSc} \nabla^2 c$$

The material velocity field $\mathbf{v} = (v_x, v_y)$ is calculated by: $v_x = \partial \psi / \partial y$ and $v_y = -\partial \psi / \partial x$

If the mesh velocity field $\hat{\mathbf{v}} = \mathbf{v}$ (*Lagrangian*) or $\hat{\mathbf{v}} = 0$ (*Eulerian*)

[3] Batchelor, G. (1967). An Introduction to Fluid Dynamics (Cambridge Mathematical Library). Cambridge: Cambridge University Press. *doi:10.1017/CBO9780511800955*

Semi-Lagrangian Method

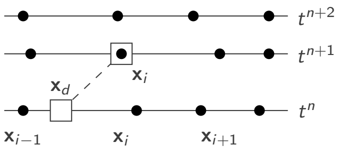
The convective term was replaced by material derivative of ω and c in the direction of characteristic trajectory

$$\frac{D\omega}{Dt} = \frac{1}{Re}\nabla^2\omega$$

$$\nabla^2\psi = -\omega$$

$$\frac{Dc}{Dt} = \frac{1}{ReSc}\nabla^2 c$$

The departure node is calculated by $x_d^n = x_i^{n+1} - (\mathbf{v} - \hat{\mathbf{v}})\,\Delta t$. Then, a searching procedure is required to find x_d^n using barycentric coordinates



Semi-Lagrangian Method

The implicit semi-Lagrangian time discretization provides [4]:

$$\frac{\omega_i^{n+1} - \omega_d^n}{\Delta t} = \frac{1}{Re}\nabla^2\omega^{n+1}$$

$$\nabla^2\psi = -\omega$$

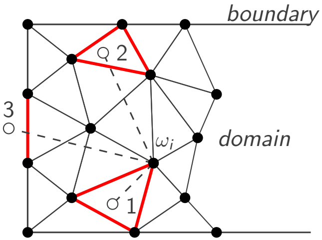
$$\frac{c_i^{n+1} - c_d^n}{\Delta t} = \frac{1}{ReSc}\nabla^2 c^{n+1}$$

Advantages:

- Symmetric linear systems
- Unconditionnal stability

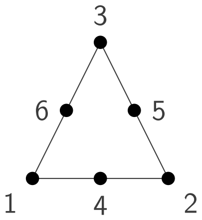
Disadvantages:

- Numerical Diffusion
- Searching procedure may lead to excessive computational cost if it is not well designed

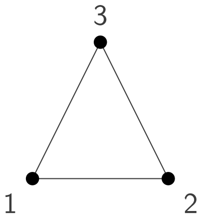


[4] Pironneau, O. On the transport-diffusion algorithm and its applications to the Navier-Stokes equations. Numer. Math. 38, 309–332 (1982). *https://doi.org/10.1007/BF01396435*

Galerkin FE Method



Quad Element



Linear Element

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{Re}\right]\omega_i^{n+1} = \frac{\mathbf{M}}{\Delta t}\omega_d^n$$

$$\mathbf{K}\psi = \mathbf{M}\omega$$

$$\left[\frac{\mathbf{M}}{\Delta t} + \frac{\mathbf{K}}{ReSc}\right]c_i^{n+1} = \frac{\mathbf{M}}{\Delta t}c_d^n$$

The material velocity field is calculated by: $\mathbf{M}v_x = \mathbf{G_y}\psi$ and $\mathbf{M}v_y = -\mathbf{G_x}\psi$

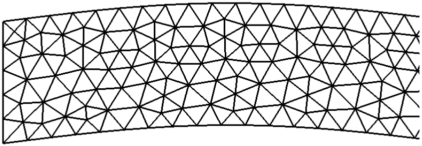
Laplacian Smoothing

To avoid the fast degradation of the computational elements due to ALE description, it was used the Laplacian Smoothing Method [5]

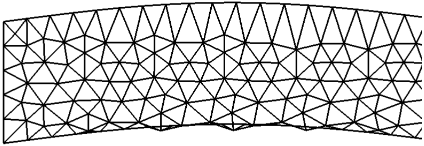
The new node position $\hat{\mathbf{x}}_\mathbf{i}$ can be approximated by:

$$\hat{\mathbf{x}}_\mathbf{i} = \sum_{i \in N_1} e_{ij}^{-1} (\mathbf{x}_j - \mathbf{x}_i)$$

where, e_{ij}^{-1} is the distance between the node and each neighbor in 1-ring N_1



with Laplacian Smoothing



no Laplacian Smoothing

[5] Desbrun, M. Meyer, P. Schröder, A. Barr, Implicit fairing of irregular meshes using diffusion and curvature flow, in: Proceedingsof Siggraph, 1999, pp. 317–324

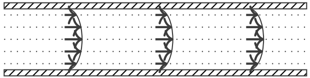
Solution Algorithm

GESAR

Import mesh
↓
Calculate Gaussian Quadrature and Assemble Matrix
↓
Inicialize Vorticity and Streamfunction
↓
Calculate Laplace smoothing and ALE velocity
↓
Calculate Gaussian Quadrature and Assemble Matrix
↓
Calculate vorticity boundary condition
↓
Calculate semi-Lagrangian and Vorticity field
↓
Calculate streamfunction field
↓
Calculate velocity field
↓
Calculate concentration field

Repeat the procedure
for the next time step
until the steady state

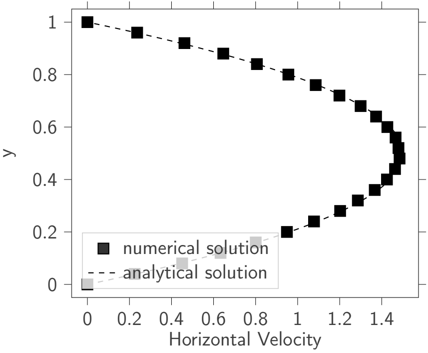
Validation - Poiseuille Flow

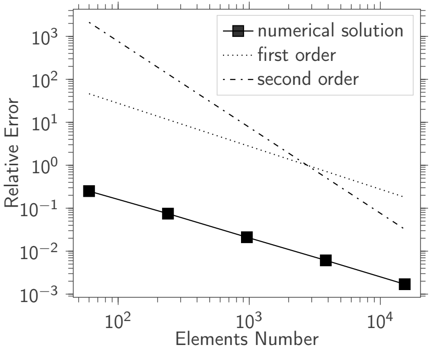


Boundaries Conditions:

Inflow condition: $u = u_{analytical}$, $v = 0$
Top plate: $u = 0$, $v = 0$, $\psi = 1$
Bottom plate: $u = 0$, $v = 0$, $\psi = 0$

Nodes: 1757
Elements: 3263
Relative Error: 0.67%





Validation - Lid Driven Cavity Flow

Boundaries Conditions:

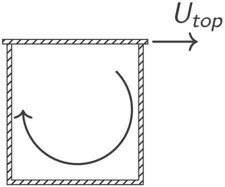
Bottom and side plates: $u = 0$, $v = 0$ e $\psi = 0$

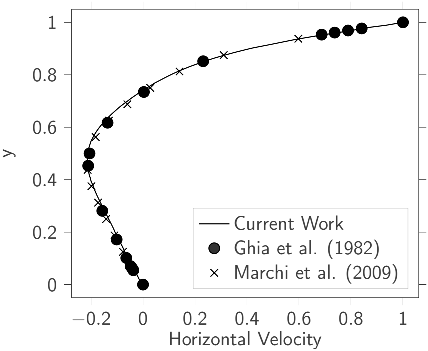
Top plate: $u = 1$, $v = 0$ e $\psi = 0$

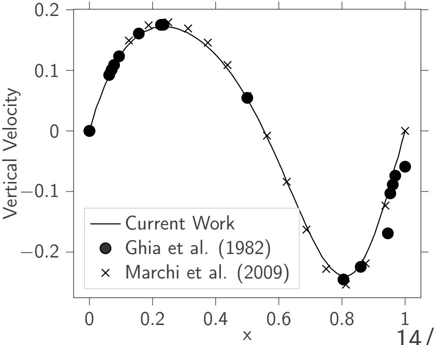
Nodes: 3798

Elements: 7382

Re: 100







Coming Soon

- Backward-Facing Step Validation
- Pulsation Flow Validation
- Drug-Eluting Stent Cases

Work Plan

	2018				2019												2020							
ACTIVITIES	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8
SUBJECTS	■	■	■	■	■	■	■	■	■	■	■	■												
LITERATURE REVIEW													■	■	■	■								
CODE IMPLEMENTATION															■	■	■	■	■	■	■			
CODE VALIDATION																				■	■	■		
RESULTS SIMULATION																							■	
PRESENTATION																								■

SUBJECTS:

- CONTINUUM MECHANICS I
- ADVANCED CALCULUS
- FLUID MECHANICS
- COMPUTATIONAL FLUID MECHANICS I
- COMPUTATIONAL FLUID MECHANICS II
- FINITE ELEMENT METHOD
- COMPUTATIONAL METHODS
- MULTIPHASE FLOWS

LITERATURE REVIEW:

- ARBITRARY LAGRANGIAN-EULERIAN (ALE)
- SEMI-LAGRANGIAN METHOD
- LAPLACIAN SMOOTHING

Thank you!

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