

Ejercicio 5.

$$H(z) = \frac{z}{z - 0,8}$$

$$X[n] = 20 \cdot \cos\left(\frac{\pi \cdot n}{2} + 30^\circ\right)$$

$$X[n] = 20 \cdot \sin\left(\frac{\pi n}{2} + 60^\circ\right) + 120^\circ$$

$$X[n] = 20 \cdot \cos\left(\frac{\pi n}{2} - 330^\circ\right)$$

$$X[n] = 20 \cos\left(\frac{\pi}{2} n - \frac{330 \cdot \pi}{360}\right)$$

$$X[n] = 20 \cdot \cos\left(\frac{\pi}{2} \left[n - \frac{11}{3}\right]\right)$$

$$X(z) = 20 \cdot z^{-\frac{11}{3}} \cdot \frac{z^2 - z \cdot \cos\left(\frac{\pi}{2}\right)}{z^2 - 2z \cos(\pi/2) + 1}$$

$$X(z) = 20 \cdot z^{-\frac{11}{3}} \cdot \frac{z^2}{z^2 + 1}$$

$$Y(z) = X(z) \cdot H(z) = 20 \cdot z^{-\frac{11}{3}} \cdot \frac{z^2}{z^2 + 1} \cdot \frac{z}{z - 0,8}$$

$$Y(z) = \frac{A \cdot z + B}{z^2 + 1} + \frac{C}{z - 0,8} = \frac{(z - 0,8)(A \cdot z + B) + C \cdot (z^2 - 1)}{(z^2 + 1)(z - 0,8)}$$

$$(z - 0,8)(A \cdot z + B) + C \cdot (z^2 - 1) = A \cdot z^2 + B \cdot z - 0,8A \cdot z - 0,8B + z^2 \cdot C - C$$

$$20z^2 = A \cdot z^2 + C \cdot z^2$$

$$0 \cdot z = B \cdot z - 0,8A \cdot z \rightarrow A = B / 0,8$$

$$0 = -0,8B - C \rightarrow C = -0,8B$$

$$20 = \frac{B}{0,8} - 0,8B \rightarrow 16 = B - 0,64B$$

$$16 = 0,36B \rightarrow B = \frac{400}{9} ; A = \frac{500}{9} ; C = -\frac{320}{9}$$

$$Y(z) = z^{-\frac{11}{3}} \cdot z \cdot \left(\frac{\frac{500}{9} z + \frac{400}{9}}{z^2 + 1} + \frac{-\frac{320}{9}}{z - 0,8} \right)$$

$$Y(z) = z^{-\frac{\pi}{3}} \left(\frac{\frac{500}{9} z + \frac{400}{9}}{z^2 + 1} - \frac{\frac{320}{9}}{z - 0,8} \right)$$

$$Y[n] = \frac{500}{9} \cdot \sin \left(\frac{\pi}{2} \left[n - \frac{\pi}{3} \right] \right) - \frac{320}{9} \cdot (0,8)^{n - \frac{\pi}{3}}$$
