| Section A | Concepts and Skills | 150 marks |
|---|---|---|

Answer **all six** questions from this section.

**Question 1** **(25 marks)**

Eimear earns a gross wage of €40 000 per annum with Company A.

**(a)** Eimear pays income tax at a rate of 20% on income up to the standard rate cut-off point of €35 300. She pays tax at a rate of 40% on the remainder.
She has annual tax credits of €1650.
Find how much income tax she pays per annum.

€35,300 × 0·2 = €7,060

€40,000 − €35,300 = €4,700 remaing amount

€4,700 × 0·4 = €1,880

remaing amount taxed at higher rate

€7,060 + €1,880 = €8,940 total tax

income tax = €8,940 − €1,650 tax credits
= €7,290

**(b)** Eimear pays her health insurance which costs her €1500 net. Find her annual income after paying income tax and health insurance (i.e. her net annual income).

gross income − income tax − health insurance

€40,000 − €7,290 − €1,500

= €31,210 net annual income

**(c)** Eimear is planning to change jobs. She is offered a job by Company B with a gross wage of €38 000 and a bonus of €1500 (tax free to Eimear) to be paid by the company, which she would use to pay her health insurance. Her tax rates and credits would remain the same. Find by how much Eimear's net annual income (after paying income tax and health insurance) will increase if she accepts the job with Company B.

€35,300 × 0·2 = €7,060

€38,000 − €35,300 = €2,700

€2,700 × 0·4 = €1,080

total tax = €7,060 + €1,080

= €8,140

tax credits

income tax = €8,140 − €1650

= €6,490

gross wage − income tax + bonus − health insurance

€38,000 − €6,490 + €1,500 − €1,500

net income = €31,510

increase = €31,510 − €31,210 = €300

## Question 2

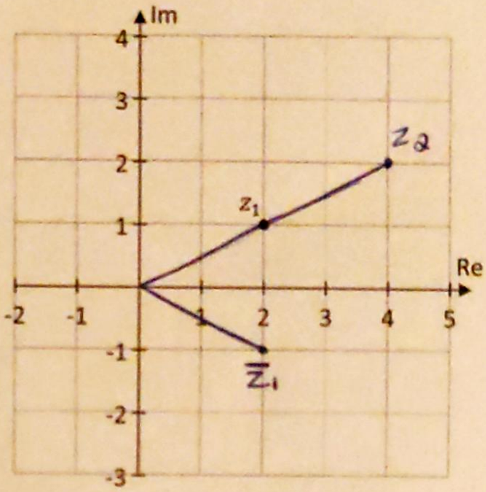
**(25 marks)**

The complex number $z_1 = 2 + i$, where $i^2 = -1$, is shown on the Argand Diagram below.

**(a)** **(i)** $z_2 = 2z_1$.

Find the value of $z_2$, and **plot and label** it on the Argand Diagram.

$z_2 = 2(2+i)$
$= 4 + 2i$

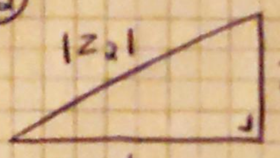



**(ii)** $\bar{z}_1$ is the complex conjugate of $z_1$.
Write down the value of $\bar{z}_1$, and **plot and label** it on the Argand Diagram.

$\bar{z}_1 =$ $\boxed{2 - i}$

**(iii)** Investigate if $|z_2| = |z_1 + \bar{z}_1|$.

$z_2$




$|z_2|^2 = 4^2 + 2^2$
$|z_2|^2 = 16 + 4$
$|z_2|^2 = 20$
$|z_2| = \sqrt{20}$
$= 2\sqrt{5}$

$z_1 + \bar{z}_1 = (2+i) + (2-i)$
$= 4 + 0i = 4$
$\sqrt{(4)^2 + (0)^2} = |z_1 + \bar{z}_1|$
$\sqrt{16} = |z_1 + \bar{z}_1|$
$4 = |z_1 + \bar{z}_1|$

$2\sqrt{5} \neq 4 \quad \therefore |z_2| \neq |z_1 + \bar{z}_1|$
they do not equal

**(b)** Show that $z_1 = 2 + i$ is a solution of the equation $z^2 - 4z + 5 = 0$.

$a = 1$

$b = -4$

$c = 5$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

note $\sqrt{-1} = i$

Substitute:

~~$-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}$~~

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$\frac{4 \pm \sqrt{16 - 20}}{2}$$

$$\frac{4 \pm \sqrt{-4}}{2}$$

$$\frac{4 \pm \sqrt{4}\sqrt{-1}}{2}$$

$$\frac{4 \pm 2i}{2}$$

$$2 \pm i$$

$\boxed{z = 2 + i}$ $z = 2 - i$

**Question 3** **(25 marks)**

The function $f$ is defined as $f(x) = -x^3 + 4x^2 + x - 2$, where $x \in \mathbb{R}$.

**(a)** **(i)** Complete the table below for the values of $f$ in the domain $-1 \leq x \leq 4$ and hence draw the graph of the function $f(x)$ in the domain $-1 \leq x \leq 4, x \in \mathbb{R}$.

| $x$ | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $f(x)$ | 2 | -2 | 2 | 8 | 10 | 2 |

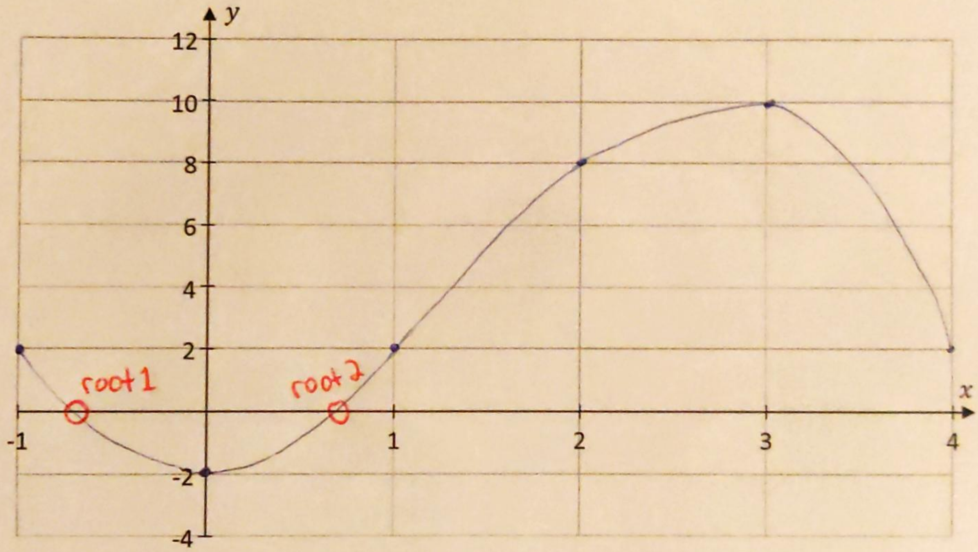
$x = -1 \quad f(-1) = -(-1)^3 + 4(-1)^2 + (-1) - 2 = 1 + 4 - 1 - 2 = 2$

$x = 0 \quad f(0) = -(0)^3 + 4(0)^2 + (0) - 2 = -2$

$x = 1 \quad f(1) = -(1)^3 + 4(1)^2 + (1) - 2 = -1 + 4 + 1 - 2 = 2$

$x = 2 \quad f(2) = -(2)^3 + 4(2)^2 + (2) - 2 = -8 + 16 + 2 - 2 = 8$

$x = 4 \quad f(4) = -(4)^3 + 4(4)^2 + (4) - 2 = -64 + 64 + 4 - 2$
$= 2$




**(ii)** Use your graph to estimate the two roots of $f(x)$ which are in the domain $-1 \leq x \leq 4$.

~~rot~~ root 1 $\approx -0.75$

root 2 $\approx 0.75$

approximately

**(b)** Find the value of $x$ for which $f''(x) = 0$, where $f''(x)$ is the second derivative of $f(x)$.

$f(x) = -x^3 + 4x^2 + x - 2$

$f'(x) = -3x^2 + 8x + 1$

$f''(x) = -6x + 8$

$f''(x) = 0 \qquad -6x + 8 = 0$

$\cancel{-6x + 8} \qquad -6x = -8$

$6x = 8$

$x = \frac{8}{6}$

$x = \frac{4}{3}$

**Question 4** **(25 marks)**

**(a)** Solve for $x$:

$$\frac{3x+1}{5}+\frac{x-2}{2}=\frac{47}{10}$$

$$\frac{3x+1}{5}+\frac{x-2}{2}=\frac{47}{10}$$

x10 everything

$$\frac{\overset{2}{\cancel{10}}(3x+1)}{\cancel{5}}+\frac{\overset{5}{\cancel{10}}(x-2)}{\cancel{2}}=\frac{(47)\cancel{10}}{\cancel{10}}$$

$$2(3x+1)+5(x-2)=47$$

$$6x+2+5x-10=47$$

$$11x-8=47$$

$$11x=55$$

$$\underline{\underline{x=5}}$$

**(b)** Solve the simultaneous equations:

$$x - 5y = -13 \quad \text{Linear}$$
$$x^2 + y^2 = 13. \quad \text{Quadratic}$$

$x - 5y = -13$

① $x = 5y - 13$

Sub ① into Quadratic

$(5y-13)^2 + y^2 = 13$

$(5y-13)(5y-13) + y^2 = 13$

$25y^2 - 65y - 65y + 169 + y^2 = 13$

$26y^2 - 130y + 169 = 13$ $\div 13$ make it easier to work with

$2y^2 - 10y + 13 = 1$

$2y^2 - 10y + 12 = 0$ $a \times c = 24$ Solve!

$2y^2 - 6y - 4y + 12 = 0$ $-6 \times -4 = 24$

$2y(y-3) - 4(y-3) = 0$ $(-6)+(-4) = -10$ correct

$(2y-4)(y-3) = 0$

$2y - 4 = 0$ | $y - 3 = 0$

$2y = 4$ | $y = 3$ → Sub y values into ① to find x values.

$y = 2$

$y = 3$
$x = 5y - 13$
$x = 5(3) - 13$
$x = 15 - 13$
$x = 2$

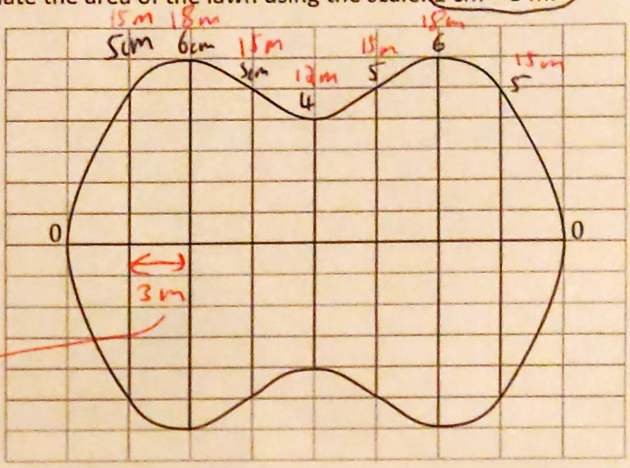
$y = 2$
$x = 5y - 13$
$x = 5(2) - 13$
$x = 10 - 13$
$x = -3$

$$\boxed{\begin{array}{ll} x=2 & y=3 \\ x=-3 & y=2 \end{array}}$$

**Question 5** **(25 marks)**

Harry draws a scale diagram of the portion of his garden that is covered in lawn.
His diagram is shown below.
Each box on the grid is 1 cm × 0·5 cm.
Each cm on Harry's diagram represents 3 m.
In order to estimate the area of the lawn Harry divides the diagram into eight sections.

**(a)** Use the trapezoidal rule to estimate the area of the lawn using the scale: 1 cm = 3 m.




$3m \rightarrow \frac{x}{2}\left[0+0+2(15+18+15+12+15+18+15)\right]$

$A = 324 \text{ m}^2$

**(b)** Nuala can walk at a speed of 1·6 metres per second.
Write this speed in kilometres per hour.

1·6 m/s

1·6 m/s X 60 seconds X 60 minutes = 5,760m/h

1,000m in 1km

$\therefore$ 5,760m/h = 5·76 km/h

**Question 6** **(25 marks)**

**(a)** Solve the following inequality for $x \in \mathbb{R}$ **and** show your solution on the numberline below:

$$2(3 - x) < 8.$$

$2(3-x) < 8$

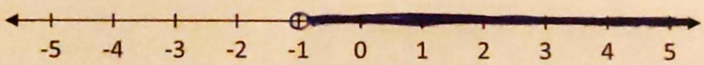
$6 - 2x < 8$

$6 - 8 < 2x$

$-2 < 2x$

$-1 < x$

x greater than $-1$



**(b)** Solve for $x$:

$$2^{2x-1} = 64.$$

note: $\log_2 y = x$ , $2^x = y$

$\cancel{x} = 2x-1$   $y = 64$

$\log_2 64 = 2x - 1$

$6 = 2x - 1$

$7 = 2x$

$\frac{7}{2} = x$

$3.5 = x$

Check: $2^{2(\frac{7}{2})-1} = 64$

$2^{7-1} = 64$ , $2^6 = 64$ , $64 = 64$.

| Section B | Contexts and Applications | 150 marks |
|---|---|---|

Answer **all three** questions from this section.

**Question 7** **(55 marks)**

A camogie goalkeeper, on a level pitch, hit a ball straight up into the air.
The path that the ball travelled can be modelled by the function:

$$f(t) = -4t^2 + 16t + 1, \quad t \in \mathbb{R},$$

where $t$ is the time, in seconds, from when the ball is hit and $f(t)$ was the height of the ball, in metres, above the pitch. The ball landed on the ground without being hit again.

**(a)** At what height was the ball when it was hit by the goalkeeper?

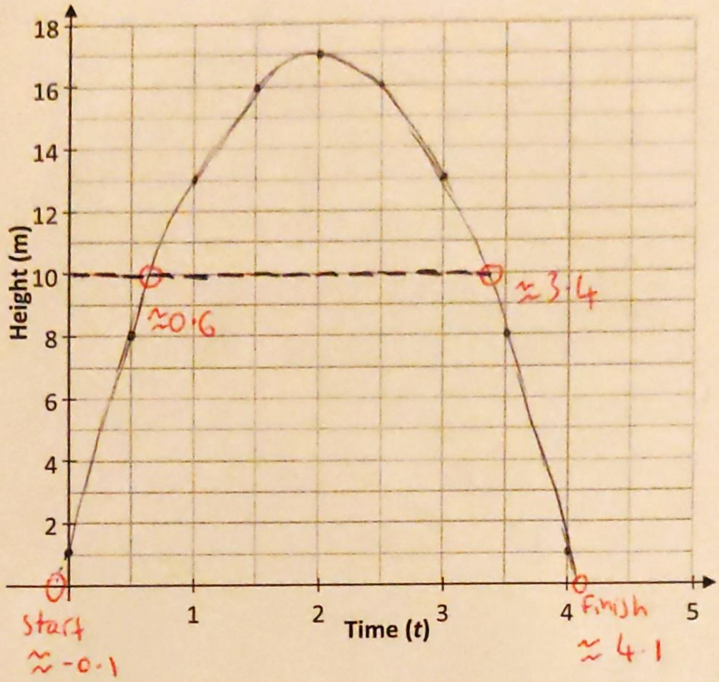
$t = 0 \quad f(0) = -4(0)^2 + 16(0) + 1 = 1\text{m}$

**(b)** **(i)** Complete the table below to show the height of the ball at various intervals during the first 4 seconds of its flight.

| Time ($t$) | 0 | 0·5 | 1 | 1·5 | 2 | 2·5 | 3 | 3·5 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| Height (m) | 1 | 8 | 13 | 16 | 17 | 16 | 13 | 8 | 1 |

example: $f(0{\cdot}5) = -4(0{\cdot}5)^2 + 16(0{\cdot}5) + 1$
$= -4(0{\cdot}25) + 8 + 1$
$= -1 + 8 + 1$
$f(0{\cdot}5) = 8$

**(ii)** On the grid below draw a graph to show the height of the ball while it was in the air.




**(c)** Use your graph to estimate:
(Show your work on the graph above)

**(i)** the length of time the ball was in the air from the time it was hit until it landed on the ground

~~-0.1~~ 0 → 4.1 = ~~4.2~~ 4.1 Seconds.

**(ii)** the length of time the ball was 10 m, or more, above the ground.

0.6 → 3.4 = 2.8 seconds

*This question continues on the next page.*

**(d)** **(i)** Find $f'(t)$, the derivative of $f(t) = -4t^2 + 16t + 1$.

$f'(t) = -8t + 16$

**(ii)** Use your answer from **part (d)(i)** to find the speed of the ball when it had been in the air for 4 seconds. Give your answer in metres per second.

$f'(t) = -8t + 16$

$f'(4) = -8(4) + 16$

$= -32 + 16$

$= -16$

$\underline{16 \text{ m/s} \downarrow \text{ down.}}$

**(iii)** Use your answer from **part (d)(i)** to find the value of $t$ for which the ball was descending and travelling at a speed of 8 metres per second.

$f'(t) = -8t + 16$

$-8 = -8t + 16$

$-24 = -8(t)$

$\underline{3 = t}$

**Question 8** **(55 marks)**

**(a)** The power ($P$) of an engine is measured in horsepower using the formula:

$$P = \frac{R \times T}{5252}$$

where $R$ is the engine speed measured in revolutions per minute (RPM) and $T$ is the torque measured in appropriate units.

**(i)** Find the power of an engine that generates 480 units of torque at 2500 RPM. Give your answer correct to the nearest whole number.

$$P = \frac{2500 \times 480}{5252}$$

$$= 228 \cdot 48$$

$$= \underline{\underline{228}}$$

**(ii)** Rearrange the formula to write $R$ in terms of $P$ and $T$.

$$P = \frac{R \times T}{5252}$$

$$(5252)P = R \times T$$

$$\frac{(5252)P}{T} = R$$

*This question continues on the next page.*

**(b)** A company was set up in January 2016 to repair engines. In the first month of its existence the company made a loss of €4000. This loss reduced by €250 a month for each month that the company traded.

**(i)** Complete the table below to show the company's loss/profit for each of the first six months of trading.

| Month | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Profit (€) | −4000 | −3750 | -3500 | −3250 | -3000 | -2750 |

**(ii)** Show that the profit the company makes in month $n$ is given by the formula $T_n = 250n - 4250$.

$T_1 = 250(1) - 4250 = 250 - 4250 = 4000$

$T_2 = 250(2) - 4250 = 3750$

$T_3 = 250(3) - 4250 = 3500$

.. ...

add 250 each month, formula makes sense!

**(iii)** What profit does the company make in January 2018 (i.e. month 25)?

$T_{25} = 250(25) - 4250$

$6250 - 4250$

$= €2,000$ profit

**(iv)** Find the month in which the company breaks even (i.e. €0 profit).

$T_n = 250n - 4250$

$0 = 250n - 4250$

$4250 = 250n$

$17 = n$

17 months

**(v)** Find $S_n$, the general term for the **total** profit of the company after $n$ months.

$S_n = \frac{n}{2}[2(-4{,}000) + (n-1)250]$

$= \frac{n}{2}[-8000 + 250n - 250]$

$= \frac{n}{2}[-8250 + 250n]$

**(vi)** Hence, or otherwise, find the total profit of the company at the end of January 2019 (i.e. month 37).

$S_n = \frac{n}{2}[-8{,}250 + 250n]$

$= \frac{37}{2}[-8{,}250 + 250(37)]$

$= 18{,}500$

**Question 9** **(40 marks)**

Avril has a website. For a certain period of time, the total number of registered users of the website, $U(m)$, can be estimated by using the formula:

$$U(m) = 3000(1{\cdot}8)^m$$

where $m$ is the number of months from when the website was launched.

**(a)** Use the formula to estimate the number of registered users 8 months after the website was launched.

$3000(1{\cdot}8)^8$

$= 330598$

**(b)** There are 31 493 registered users at the end of a particular month.
Estimate how many users there will be one month later.

multiplied by 1·8 each month

$31,493 \times 1{\cdot}8 = 56687{\cdot}4$

**(c)** Users can register free of charge on the website.
Through advertising, Avril earns €0·0012 **each month**, for every user who is registered.

**(i)** How much would she earn in a month when there were 600 000 registered users?

$600,000 \times 0{\cdot}0012 = €720$

**(ii)** Avril earns €1285·37 for a particular month.
How many registered users did the website have in that month.

n = number of users

$(0{\cdot}0012)n = 1285{\cdot}37$

$n = \dfrac{1285{\cdot}37}{0{\cdot}0012}$

$n = 1071141{\cdot}67$

**(iii)** A web design company charges Avril €80 per month to maintain and host the site. How much will Avril make or lose in month 4 and in month 12 from the website.

| Month 4: | Month 12: |
| --- | --- |
| users = 3000(1·8)$^4$ = 31492·8 | users = 3000(1·8) = 3470494 |
| profit = 31492·8 × 0·0012 | 3470494 × 0·0012 |
| = €37·79 | = €4164.59 |
| €80 − €37·79 = €42·21 | €4164·59 − €80 = €4084·59 |
| loss | profit |

**(d)** Another web design company in the UK offers to host and maintain her website for £55 sterling per month. Avril uses the exchange rate on a particular day to find how much this would cost in euro. She finds that it would cost €62·70.
Find the exchange rate for **1 euro** on that day. Give your answer correct to 4 decimal places.

£55 = €62·70

$\frac{£55}{62·7}$ = €1

0·8772 = €1

€1 = £ 0·8772