

AA 609 : Computational Methods for Astronomy and Space Sciences

Lab Practical 3

Due date : February 07, 2022

1. Flux Limiters : Shock Capturing Schemes

We have seen in the earlier problem of advecting square pulse that higher-order schemes gives rise to oscillations at the sharp gradient. The following technique will help resolve the issue and also ensure flux conservation.

- Explain what is Total Variation Diminishing Scheme? Define Slope Limiters and Flux Limiters. Is there any difference between the two limiters?
- The general expression of the flux to be estimated at the left interface (i-1/2) for a flux conserving TVD scheme is given by

$$f_{i-1/2}^{n+1/2} = \frac{1}{2}u_{i-1/2} [(1 + \theta_{i-1/2})q_{i-1}^n + (1 - \theta_{i-1/2})q_i^n] + \frac{1}{2}|u_{i-1/2}| \left(1 - \left|\frac{u_{i-1/2}\Delta t}{\Delta x}\right|\right) \Phi(r_{i-1/2}^n)(q_i^n - q_{i-1}^n)$$

- Adopt the following Flux limiter functions and see how the advection solution of square pulse is affected. Use the same grid definition and advection velocity $\lambda = 3.0$.

Name	$\Phi(r)$
Donner-Cell	0
Lax-Wendroff	1.0
Beam-Warming	r
Fromm	$\frac{1}{2} (1 + r)$
Minmod	minmod(1,r)
Superbee	max(0, min(1,2r), min(2,r))
MC	max(0, min((1 + r)/2, 2, 2r))
van Leer	$(r + r)/(1 + r)$

2. 1D Isothermal Shock Tube

In this example, we will develop an algorithm for solving a set of 1D Isothermal Hydrodynamic equation. To do that adopt the following steps :

- Write the 1D Isothermal Hydrodynamic Equations in its Conservative form using Primitive variables ρ , v and P . Assume an Isothermal Sound Speed c_0 .
- Adopt the appropriate CFL time condition assuming the signal speed as c_0 and ensure Δt is chosen appropriately with a CFL factor 0.5.
- Solve the continuity Equation numerically taking into account any choice of above Flux Limiter.
- Solve the momentum conservation equation in two steps: a) Solve the transport step by advecting momentum ρv numerically using same Flux limiter as adopted for continuity equation. b) Solve the Source step using the second update of velocity.
- Test the above algorithm for a simple top-hat profile with 100 physical grid cells (+ boundaries), $\Delta x = 1$, $c_0 = 1$. Adopt closed boundaries (reflective) with following initial conditions

$$\begin{aligned}\rho(x, t = 0) &= 2\Theta(50 - x) + 1 \\ v(x, t = 0) &= 0\end{aligned}$$

where, $\Theta(x)$ is a top-hat function with value 1 for $x > 0$ and 0 otherwise. Integrate upto time $t = 30$.