(Computationally) Simple sigmoid with specific slopes at specific points

This is a refinement of this question: (Computationally) Simple sigmoid.

Ideally, I would like to have relatively simple sigmoid function with the following properties:

1. derivatives at -1 and 1 are zero
2. derivative is not zero between -1 and 1
3. derivative at 0 is 1, maybe can be controlled with a parameter
4. I can control the change in derivative near -1 and 1 within (-1,1), so that the derivative doesn't depart from zero too quickly or too slowly.
5. The curve is "symmetric" in the sense that flipping the curve on [0,1] both vertically and horizontally produces the curve on [0,1].

Approximations to items 1 through 5 are OK. This is for a computer simulation in which values c in [-1,1] cause an increase or decrease in other values e in [-1,1]. I want e to change quickly when it's pushed around in the middle range near 0, but to be pushed up slowly when it's near 1, and similarly when pushed down near -1. I just need a function that works well enough; it doesn't have to perfect.

I've had trouble figuring out how to tune the sigmoids I'm aware of in order to get properties that are close to 1 through 5.

Thanks!

(functions)

asked Apr 20 '13 at 4:33
Mars
140 6

1 One possible solution is $\tanh(\frac{-kx}{x^2-1})$ for positive k. – Belov Apr 20 '13 at 4:44

Beautiful. Thanks @Belov. The slope approaches 0 a little too far from -1 and 1 for my taste, but I can make the approach softer by multiplying x^2 by a constant, even if that means it's not really flat at -1 and 1. You should put this into an answer and get credit. – Mars Apr 20 '13 at 5:19

Did and made some explanation how to get some more if you need. – Belov Apr 20 '13 at 5:31

Will your sigmoid be dealing with arguments outside $[-1, 1]$? – J . M. Apr 20 '13 at 6:28

No, just that interval. – Mars Apr 20 '13 at 16:16

1 Answer

So a possible solution is

$$\tanh \frac{kx}{(1-x^2)^{1/n}}$$

For $k > 0, n \geq 1$ being a constants for adjustment of the function as you like.
In general to explain how you can construct more sigmoid functions. Let's assume we have a sigmoid function $f(x) : \mathbb{R} \to [-1; 1]$ and we want to make it to find $g(x) : [-1; 1] \to [-1; 1]$ what we can do is use a mapping $h(x) : [-1; 1] \to [-\infty; \infty]$ and define $g(x) = f(h(x))$. Such mapping is $h(x) = \frac{kx}{(1-x^2)^{1/n}}$. Using this you can build and more functions for what you need using different well known sigmoids. And you should play a bit with k and n to find a good solution.
And don't forget that for any sigmoid $f(x) : \mathbb{R} \to [0; 1]$ you can transform them to $[-1; 1]$ by taking $2 * f(x) - 1$ and vice versa.

edited Apr 20 '13 at 6:03

answered Apr 20 '13 at 5:31

Belov

445 2 10

Thanks very much for the additional parameter `n` , the additional explanation! – Mars Apr 21 '13 at 4:30

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