Notes on sigmoid functions

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In response to my question, Belov at math.stackexchange.com[1] suggested at that the following function

$$\tanh \frac{kx}{(1-x^2)^{1/n}}$$

with $k > 0$, $n \geq 1$ would be such that:

- derivatives at -1 and 1 are zero
- derivative is not zero between -1 and 1
- derivative at 0 is 1, maybe can be controlled with a parameter
- I can control the change in derivative near -1 and 1 within $(-1, 1)$, so that the derivative doesn't depart from zero too quickly or too slowly. The curve is "symmetric" in the sense that flipping the curve on $[0, 1]$ both vertically and horizontally produces the curve on $[0, 1]$.

My file GeneralSigmoid.gcx can be used to experiment with this function in OS X Grapher.

Notes on the parameters:

- Increasing k (e.g. from 2) makes the curve steeper in the center.
- Giving k a value between 0 and 1 can make the curve very flat in the center.
- The higher n is, the more gently the curve slopes toward -1 and 1. (This effect depends on the value of k.)
- Small values of n (e.g. less than 1) can create "shelves" along which the curve appears completely flat near -1 and 1. (This violates my original requirements.)

For a nice paradigmatic sigmoid, use e.g. $k = 2$ or $k = 3$ with $n = 2$. For a function that is mostly flat and then rises quickly to shelves near -1 and 1, try e.g. $k = .01$, $n = .35$. The latter might be used to create a "success" function that normally drifts randomly, but creates a valued or rejected cultvar when it reaches an extreme value.

[1] http://math.stackexchange.com/questions/367078/computationally-simple-sigmoid-with-specific-slopes-at-specific-points