How many utterances does each listener receive?

Let there be N popco persons all in the same fully interconnected group. (For the moment, ignore pundits.) Assume that each person has max-talk-to = m.

Then on any given tick, there are N speakers with up to m utterances each, i.e. up to mN utterances in total, distributed between N listeners. The probability that any given listener l will receive a particular utterance is $p = \frac{1}{N}$.

How many utterances does each listener receive?

The probability that a given listener l will receive k utterances during a tick follows a binomial distribution:

$$\mathsf{P}(X = k) = \binom{mN}{k} \left(\frac{1}{N}\right)^k \left(1 - \frac{1}{N}\right)^{mN-k} .$$

The expected number of utterances received by a given listener is

$$\mathsf{E}(X) = \sum_{k=0}^{mN} k \binom{mN}{k} \left(\frac{1}{N}\right)^k \left(1 - \frac{1}{N}\right)^{mN-k} = mNp = mN\frac{1}{N} = m ,$$

and the variance is

$$\sigma^2(X) = mNp(1-p) = mN\frac{1}{N}\left(1 - \frac{1}{N}\right) = m\left(\frac{N-1}{N}\right) .$$

The standard deviation is therefore

$$\sigma(X) = \sqrt{m}\sqrt{\frac{N-1}{N}} .$$

For a slightly large N, the second term is close to 1. For example, the NetLogo Bali model has $N = 172$ subaks, so if max-talk-to $= m = 4$, and all subaks generate that the max number ofutterances, the mean number of utterances per listener will be 4, and the standard deviation will be slightly less than 2.

(All correct?)

What fraction of utterances are selected by success bias?

Similarly, since the number of utterances produced by any one speaker is m, the expectation and variance of the number of utterances received by a given listener l from any one speaker s, are

$$\mathsf{E}(X_s) = mp = \frac{m}{N}$$

$$\sigma^2(X_s) = m\frac{1}{N}\left(1 - \frac{1}{N}\right) = \frac{m(N-1)}{N^2}$$

and the standard deviation is

$$\sigma(X_s) = \frac{\sqrt{m(N-1)}}{N}.$$

If we assume that a listener's success-bias function always selects a single speaker's utterances on any given tick (which would generally obtain if success varies continuously and is rarely equal to its maximum value), then we might we treat the ratio between the above two means as an estimate of the average fraction of most-sucessful utterance among all utterances received by a listener:

$$\frac{\frac{m}{N}}{m} = \frac{m^2}{N}$$

For example, for $N = 100$ persons, with $m = 5$ utterances per person, roughly 1/4 of the utterances received, on average, by a listener, would be the most successful for that listener on that tick.

(Yeah, but that's not the real way to calculate this. A ratio of expectations isn't an expectation, in general.)

So let's try again.

The probability that l gets j utterances from speaker s, where $j \leq m$, is:

$$\mathsf{P}(X_s = j) = \binom{m}{j} \left(\frac{1}{N}\right)^j \left(1 - \frac{1}{N}\right)^{m-j} .$$

(j must also be $\leq k$.)

The probability that a given listener l will receive $k - j$ utterances from any person but s during a tick is:

$$\mathsf{P}(X = (k-j)) = \binom{m(N-1)}{k-j} \left(\frac{1}{N-1}\right)^{k-j} \left(1 - \frac{1}{N-1}\right)^{m(N-1)-(k-j)} .$$

The probability that l receives k utterances, and j of them are from s, is the probability that l receives j utterances from s and $k-j$ utterances from all of the other speakers is

$$\mathsf{P}(X_s = j)\,\mathsf{P}(X = (k-j))$$

since each person generates utterances independently of every other. (Is this correct?)

$$= \binom{m}{j}\binom{m(N-1)}{k-j} \left(\frac{1}{N}\right)^j \left(\frac{N-1}{N}\right)^{m-j} \left(\frac{1}{N-1}\right)^{k-j} \left(\frac{N-2}{N-1}\right)^{m(N-1)-(k-j)} .$$

Yow.