STATS 415 - Homework 7 - Regression Splines

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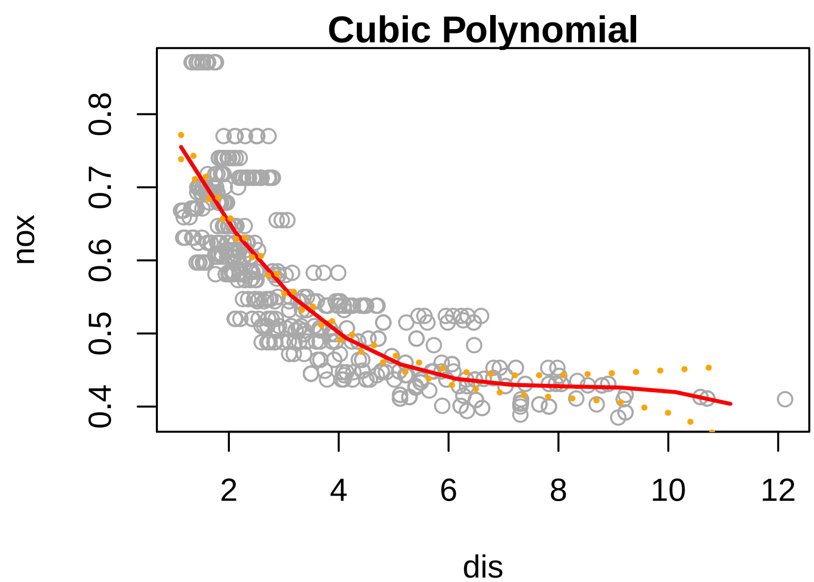
1. This question uses the variables `dis` (the weighted mean of distances to five Boston employment centers) and `nox` (nitrogen oxides concentration in parts per 10 million) from the `Boston` data. We will treat `dis` as the predictor and `nox` as the response.

(a) Use the `poly()` function to fit a cubic polynomial regression to predict `nox` using `dis`. Report and comment on the regression output, and plot the resulting data and polynomial fits.

```
cubic_fit <- lm(nox ~ poly(dis, 3), data = Boston)
coef(summary(cubic_fit))
```

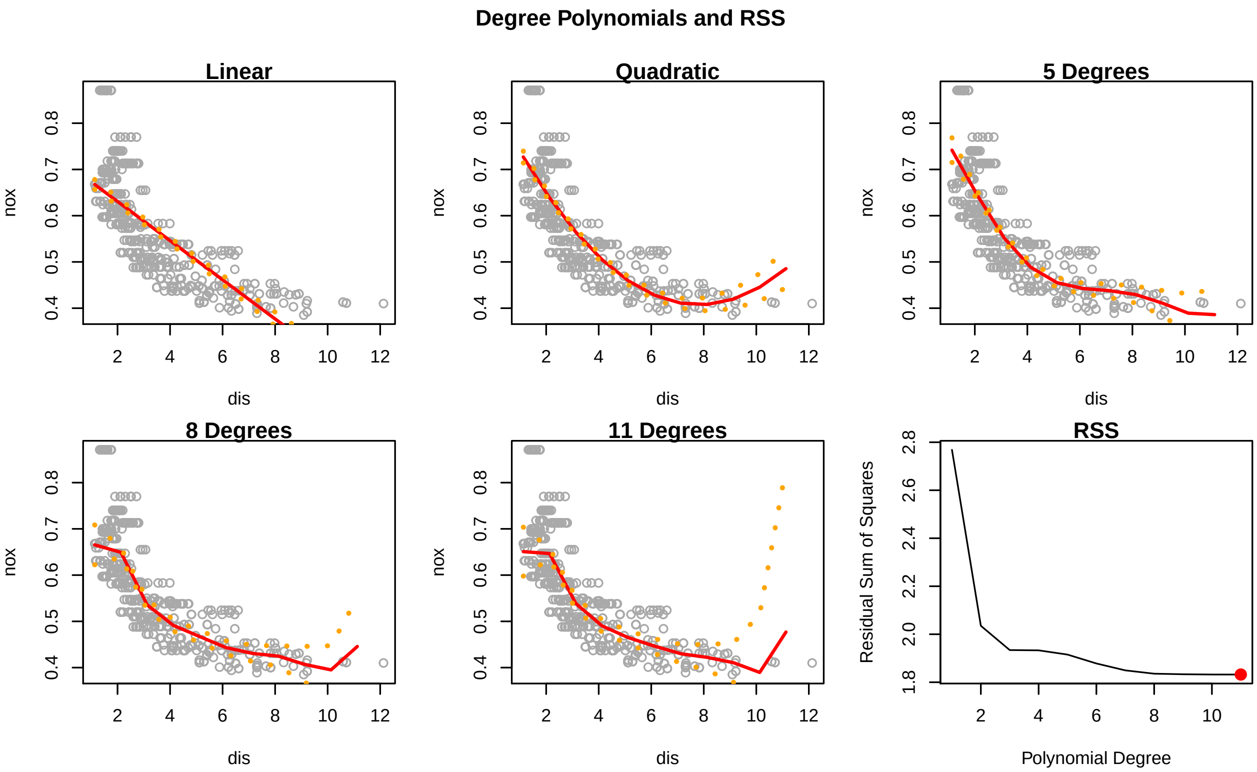
```
##                  Estimate Std. Error    t value      Pr(>|t|)
## (Intercept)     0.5546951 0.00275939 201.020894  0.000000e+00
## poly(dis, 3)1 -2.0030959 0.06207094 -32.271071 1.597201e-124
## poly(dis, 3)2  0.8563300 0.06207094  13.795987  6.133104e-37
## poly(dis, 3)3 -0.3180490 0.06207094  -5.123959  4.274950e-07
```

```
dislims <- range(dis)
dis_grid <- seq(from = dislims[1], to = dislims[2])
cubic_pred <- predict(cubic_fit, newdata = list(dis = dis_grid), se = TRUE)
se_bands <- cbind(cubic_pred$fit + 2*cubic_pred$se.fit,
                  cubic_pred$fit - 2*cubic_pred$se.fit)
par(mar = c(4.5,4.5,1,1), oma = c(0,0,2,0))
plot(dis, nox, xlim = dislims, col = "darkgrey", xlab = "dis", ylab = "nox")
title("Cubic Polynomial", outer = FALSE) # title that spans both plots
lines(dis_grid, cubic_pred$fit, lwd = 2, col = "red")
matlines(dis_grid, se_bands, lwd = 3, col = "orange", lty = 3)
```



The summary of the cubic fit above shows that all of the polynomial coefficients are significant in predicting `nox` from `dis`. The plot shows a smooth curve that fits the data well with confidence intervals that are small until the upper limit of the data.

(b) Plot the polynomial fits for a range of different polynomial degrees (say, from 1 to 10); report and comment on the associated residual sum of squares.

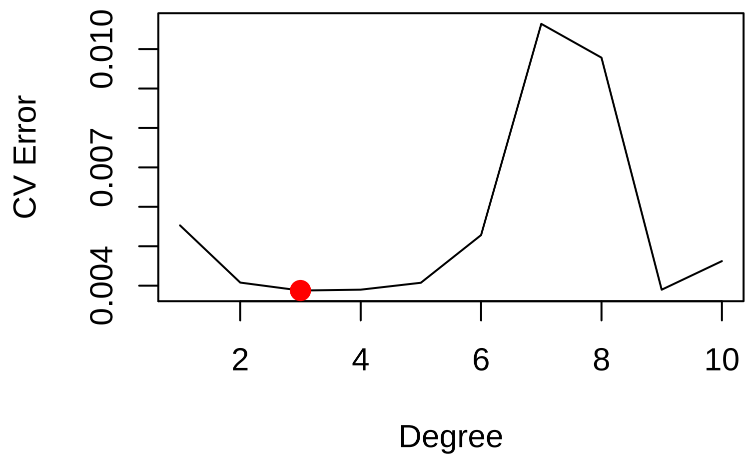


The RSS decreases as the degree of polynomial increases and thus, the highest polynomial has the lowest RSS. However, it is clear from the plots that the small and high `dis` *values are more and more uncertain (higher confidence intervals on the extremes) with increasing polynomials.*

(c) Perform cross-validation or another approach to select the optimal degree for the polynomial, and explain your results.

Using a 10-fold cross validation

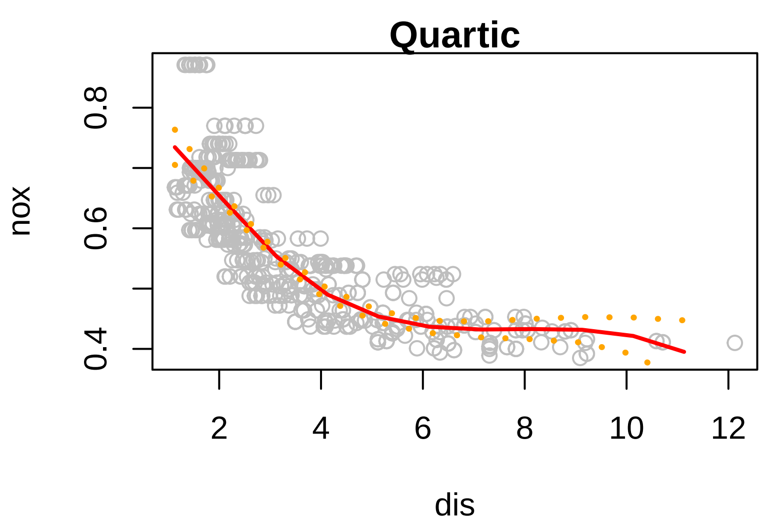
```
prediction_error <- rep(0, 10)
for (i in 1:10){ # Run all the polynomial models and store them
  # Use the glm function for poly models instead of lm so we can use cv.glm
   poly_fit <- glm(nox ~ poly(dis, i), data = Boston)
   prediction_error[i] <- cv.glm(Boston, poly_fit, K = 10)$delta[1]}
par(mfrow = c(1,1),  mar = c(4.5,4.5,1,1), oma = c(0,0,2,0)) # plot it!
plot(1:10, prediction_error, xlab = "Degree", ylab = "CV Error", type = "l")
d.min <- which.min(prediction_error)
points(which.min(prediction_error), prediction_error[which.min(prediction_error)],
       col = "red", cex = 2, pch = 20)
```



According to a 10-fold cross validation, the CV error reduces from degrees 1-3 and then increases afterwards until 9 degrees, when it decreases and starts to go up again. The 3rd polynomial is the best model for predicting `nox` from `dis` based on 10-fold CV.

(d) Use the `bs()` function to fit a regression spline to predict `nox` using `dis`. Report and comment on the output for the fit using four degrees of freedom. How did you choose the knots? Plot the resulting fit.

```
spline_fit <- lm(nox ~ bs(dis, df = 4), data = Wage)
spline_pred <- predict(spline_fit, newdata = list(dis = dis_grid), se = TRUE)
par(mar = c(4.5,4.5,1,1), oma = c(0,0,2,0))
plot(dis, nox, col = "gray");title("Quartic", outer = FALSE)  # Plot the output
lines(dis_grid, spline_pred$fit, lwd = 2, col = "red")
lines(dis_grid, spline_pred$fit + 2* spline_pred$se, lwd = 3, col = "orange", lty = 3)
lines(dis_grid, spline_pred$fit - 2* spline_pred$se,lwd = 3, col = "orange", lty = 3)
```




```
attr(bs(dis, df = 4), "knots")
```

```
##     50%
## 3.20745
```

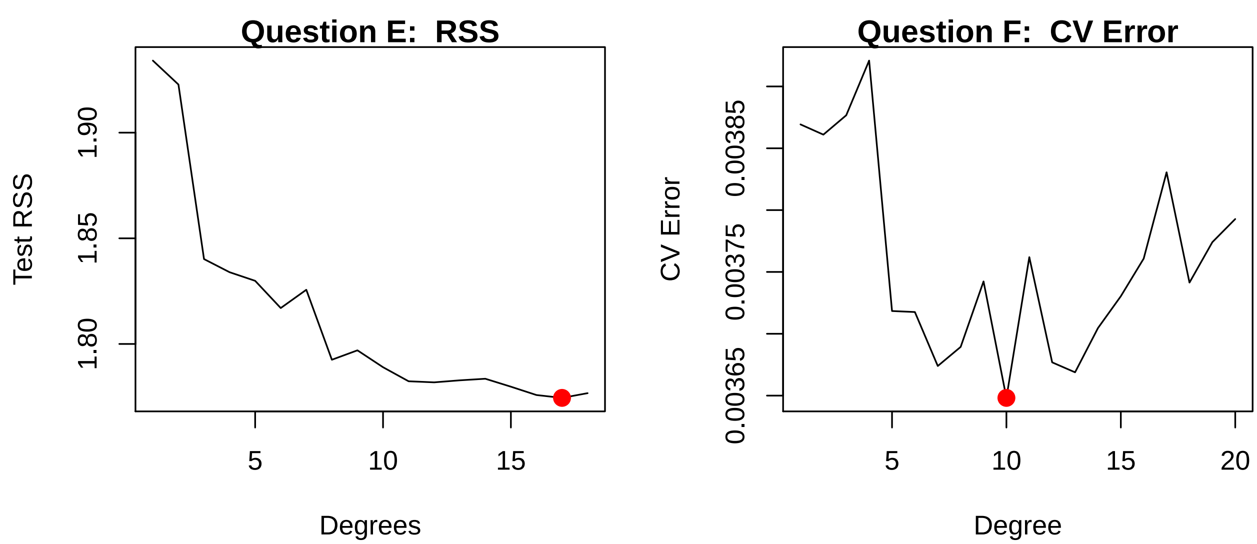
*Above **R** chooses a knot at a **dis** of 3.2 which corresponds to the 50th percentile of **dis**.*

(e) Now fit a regression spline for a range of degrees of freedom, and plot the resulting fits; report and comment on the resulting RSS. Describe the results obtained.

Let's fit polynomial regression splines with degrees of freedom between 3 and 20.

```
# Code for *Question E*
RSS_reg_splines <- rep(NA, 18)
for (i in 3:20) {
    reg_spline_fit <- lm(nox ~ bs(dis, df = i), data = Boston)
    RSS_reg_splines[i] <- sum(reg_spline_fit$residuals^2)}
par(mfrow = c(1,2), mar = c(4.5,4.5,1,1), oma = c(0,0,4,0))
RSS <- RSS_reg_splines[-c(1, 2)]
plot(1:18, RSS, xlab = "Degrees", ylab = "Test RSS", type = "l")
d.min <- which.min(RSS); title("Question E:  RSS", outer = FALSE)
points(which.min(RSS), RSS[which.min(RSS)], col = "red", cex = 2, pch = 20)

# Code for *Question F*
prediction_error <- rep(0, 20); set.seed(232)
for (i in 1:20){ # Run all the polynomial models and store them
  # Use the glm function for poly models instead of lm so we can use cv.glm
   reg_spline_fit <- glm(nox ~ bs(dis, df = i), data = Boston)
   prediction_error[i] <- cv.glm(Boston, reg_spline_fit, K = 10)$delta[1]}
plot(1:20, prediction_error, xlab = "Degree", ylab = "CV Error", type = "l")
d.min <- which.min(prediction_error); title("Question F:  CV Error", outer = FALSE)
points(which.min(prediction_error), prediction_error[which.min(prediction_error)],
       col = "red", cex = 2, pch = 20)
```



From the above plot on the left, The RSS decreases monotonically with a minimum RSS at 17 degrees of freedom.

(f) Perform cross-validation or another approach in order to select the best degrees of freedom for a regression spline on this data. Describe your results.

From the above plot on the right, the CV error is very unstable as the degrees of freedom increases. However, the minimum CV error is 10 degrees of freedom.