A Visual, Intuitive Guide to Imaginary Numbers

by Kalid Azad, http://betterexplained.com/articles/a-visual-intuitive-guide-to-imaginary-numbers/

Fun Fact	Negative Numbers (-x)	Complex Numbers (a +bi)
Invented to answer	"What is 3 – 4?"	"What is sqrt(-1)?"
Strange because…	*How can you have less than nothing?*	*How can you take the square root of less than nothing?*
Intuitive meaning	"Opposite"	"Rotation"
Considered absurd until	1700s	Today ☺
Multiplication cycle [& general pattern]	`1, -1, 1, -1…` `X, -X, X, -X…`	`1, i, -1, -i…` `X, Y, -X, -Y…`
Use in coordinates	Go backwards from origin	Rotate around origin
Measure size with	Absolute value $\sqrt{(-x)^2}$	Pythagorean Theorem $\sqrt{a^2+b^2}$

Really Understanding Negative Numbers

Negative numbers aren't easy. Imagine you're a European mathematician in the 1700s. You have 3 and 4, and know you can write 4 – 3 = 1. Simple.

But what about 3-4? What, exactly, does that mean? How can you take 4 cows from 3? *How could you have less than nothing?*

Negatives were considered **absurd**, something that "darkened the very whole doctrines of the equations" (Francis Maseres, 1759). Yet today, it'd be **absurd** to think negatives aren't logical or useful. Try asking your teacher whether negatives corrupt the very foundations of math.

What happened? We invented a *theoretical number that had useful properties*. Negatives aren't something we can touch or hold, but they describe certain relationships well (like debt). **It was a useful fiction**.

Rather than saying "I owe you 30" and reading words to see if I'm up or down, I can write "-30" and know it means I'm in the hole. If I earn money and pay my debts (-30 + 100 = 70), I can record the transaction easily. I have +70 afterwards, which means I'm in the clear.

The positive and negative signs **automatically keep track of the direction** — you don't need a sentence to describe the impact of each transaction. Math became easier, more elegant. It didn't matter if negatives were "tangible" — they had useful properties, and we used them until they became everyday items. Today you'd call someone obscene names if they didn't "get" negatives.

But let's not be smug about the struggle: negative numbers were a huge mental shift. Even Euler, the

genius who discovered e and much more, didn't understand negatives as we do today. They were considered "meaningless" results (he later made up for this in style).

It's a testament to our mental potential that today's children are *expected* to understand ideas that once confounded ancient mathematicians.

Enter Imaginary Numbers

Imaginary numbers have a similar story. We can solve equations like this all day long:

$$x^2 = 9$$

The answers are 3 and -3. But suppose some wiseguy puts in a teensy, tiny minus sign:

$$x^2 = -9$$

Uh oh. This question makes most people cringe the first time they see it. *You want the square root of a number less than zero? That's absurd!* (Historically, there were real questions to answer, but I like to imagine a wiseguy.)

It seems crazy, just like negatives, zero, and irrationals (non-repeating numbers) must have seemed crazy at first. There's no "real" meaning to this question, right?

Wrong. So-called "imaginary numbers" are as normal as every other number (or just as fake): they're a tool to describe the world. In the same spirit of assuming -1, .3, and 0 "exist", let's assume some number *i* exists where:

$$i^2 = -1$$

That is, you multiply *i* by itself to get -1. What happens now?

Well, first we get a headache. But playing the "Let's pretend *i* exists" game actually makes math easier and more elegant. **New relationships emerge** that we can describe with ease.

You may not believe in *i*, just like those fuddy old mathematicians didn't believe in -1. New, brain-twisting concepts are **hard** and they don't make sense immediately, even for Euler. But as the negatives showed us, strange concepts can still be useful.

I dislike the term "imaginary number" — it was considered an insult, a slur, designed to hurt*i*'s feelings. The number i is just as normal as other numbers, but the name "imaginary" stuck so we'll use it.