ACI650 - Modelos y Simulación

Generating Discrete Random Variables

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UNIVERSIDAD DE LAS AMÉRICAS
Laureate International Universities®

April 18, 2016

Learning Objectives

- Generate discrete probability distributions.
- Hands-on workshop.
- Simulation examples.

Simulating a Bernoulli random variable

- Let $U \sim Unif(0,1)$, we can transform U so that the result is a Bernoulli random variable.

$$x = \begin{cases} 1 & U < p \\ 0 & U \geq p \end{cases}$$

- Pseudocode:

```
function Bernoulli(p)
    if rand() < p
        return 1
    else
        return 0
    endif
```

Simulating a Binomial random variable

- Suppose we want to generate samples from a binomial random variable ($X \sim Bin(n, p)$) with parameters n and p. This r.v. has the same distribution as the sum of n. Bernoulli r.v.s.
- Therefore we can use the following pseudocode:

```
function Binomial(n, p)
    x = 0
    for k = 1 to n
        x = x + Bernoulli(p)
    endfor
    return x
```

Geometric and Negative Binomial

- Since geometric/negative binomial random variables are just the time until the first/rth success of a sequence of Bernoulli trials it is also easy to generate such r.v.s.
- Simulating geometric r.v. $X \sim Geom(p)$:

```
function Geom(p)
    x = 0
    while Bernoulli(p) == 0
        x = x + 1
    endwhile
    return x
```

- Simulating negative binomial r.v. $X \sim NBin(r, p)$:

```
function NBin(r, p)
    x = 0
    for k = 1 to r
        x = x + Geom(p)
    endfor
    return x
```

Generating a Poisson r.v. with parameter λ

- Recall that if $X \sim Poisson(\lambda)$ then

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, ...$$

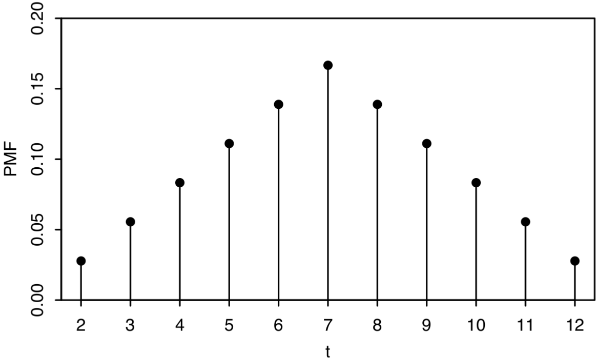
- Pseudocode:

```
function Poisson(lambda)
    u = rand()
    cp = exp(-lambda)
    k = 0
    factorial_k = 1
    while cp < u
        k = k + 1
        factorial_k = factorial_k * k
        cp = cp + exp(-lambda) * lambda^k / factorial_k
    endwhile
    return k
```

- See Generating random numbers with a Poisson distribution for an explanation.

Exercises - Simulate I

- Simulate three coin tossing.
- Simulate the sum of rolling two dices.



- Implement the pseudocodes in this presentation.

Exercises - Simulate II

- Simulate the process of extracting balls from a urn without replacement (Hypergeometric distribution).
- Reproduce
  - Last session slide page 35.
  - Last session slide page 41.
  - Last session slide page 48.