Block information in attractor neural networks: A topology-dependent analysis

Mario Gonzalez, *SI2-Lab UDLA,* John Doe, *Fellow, OSA,* and Jane Doe, *Life Fellow, IEEE*

Abstract

The abstract goes here.

Index Terms

IEEE, IEEEtran, journal, LaTeX, paper, template.

I. Introduction

Artificial neural networks as models for associative memory usually use broad connectivity. However from a neurobiological and an implementation perspective, it is logical to minimize the length of inter-nodes connections and consider networks whose connectivity is predominant local. Biologically plausible associative memories must have sparse connectivity, reflecting the situation in the cortex and hippocampus[1], [2]. Small World (SW) networks[3] provide a practical approach to this issue featuring *dense internal* connections and sparse *inter-modular* connections.

SW networks are surprisingly common and have become popular in many areas of science. These networks exhibit characteristics such as a relatively small path length, due to the presence of long range shortcuts, and a high clustering coefficient. As a result small world networks have some areas (*sub-networks*) with highly connected nodes, with a few shortcuts between the different areas. This mimics a lot of networks found in biology including the brain, in which the majority of connections appear to occur between nearby regions and the pathways between different areas are to some degree diffuse[4].

A model of sparsely connected Hopfield-type neural network[5] on the SW topology is presented. As described by Watts and Strogatz[3] the network is a regular lattice where each node is connected to its $K/2$ nearest neighbors on either side. A fraction ω of these connections are then re-wired to other randomly selected nodes. Self connections and repeated connections are not allowed. The result is a network that interpolates between a regular lattice and a completely random graph.

Table I depicts the times between clusters found for the extreme points.

TABLE I
Mean time and distance between clusters in the city of Quito.

from cluster	to cluster	mean time (min)	mean distance (km)	mean speed (km/h)
A	E	35.55	22.61	38.16
A	G	32.32	18.54	34.41
B	F	26.66	23.69	53.32
B	G	44.35	26.26	35.53
C	G	40.40	24.04	35.70
D	F	31.54	18.74	35.65
D	G	39.06	19.66	30.20
E	G	32.62	16.38	30.12

II. Methodology

A. The dynamics and the topology

A neuron i can be in one of two states, firing/quiescent, described by binary variables $\sigma_i \in \{\pm 1\}$. The state of a neuron σ_i is updated in time t according to the sigmoidal activation function:

$$\sigma_i^t = \mathrm{sign}\left(\sum_i h_i^{t-1} - \theta\right), \quad h_i^t \equiv \sum_j J_{ij}\sigma_j^t, \tag{1}$$

where h_i^t is the postsynaptic field arriving at neuron σ_i. Here sign is defined as: $\mathrm{sign(z)} = 1$ if $\mathrm{z} \geq 0$, and $\mathrm{sign(z)} = -1$ if $\mathrm{z} < 0$. The variable θ is the firing threshold which is considered to be zero. A synchronous update is used.

M. Shell was with the Department of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA, 30332 USA e-mail: (see http://www.michaelshell.org/contact.html).
J. Doe and J. Doe are with Anonymous University.
Manuscript received April 19, 2005; revised August 26, 2015.

The synaptic couplings between neurons $i,\ j$ are $J_{ij} \equiv C_{ij}W_{ij}$, where $\mathbf{C} = \{C_{ij}\}$ is the topology matrix and $\mathbf{W} = \{W_{ij}\}$ are the synaptic weights. The topology matrix, with $C_{ij} \in \{0, 1\}$ splits in local and random links. The local links connect each neuron to its K_L nearest neighbors, in a closed ring. The random links connect each neuron to K_R others uniformly distributed along the network[6]. Hence, the network degree is $K = K_L + K_R$. The network topology is then characterized by two parameters: the *connectivity* ratio, and the *randomness* ratio, defined respectively by:

$$\gamma = K/N,\ \omega = K_R/K, \tag{2}$$

where ω plays the role of a rewiring probability in the *small-world* model[3]. The connectivities and the weights are considered to be asymmetrical, $C_{ij} \neq C_{ji}$, and $W_{ij} \neq W_{ji}$.

B. Biased network and block-activity

The weights W_{ij}, of the connections between neurons i and j, are composed of two terms[7]:

$$W_{ij} = cW^r_{ij} + (1 - c)\overline{W}, \tag{3}$$

where W^r_{ij} are generated randomly to be either $+1$ or -1 with equal probability, representing either an excitatory or an inhibitory synapse, respectively. This term is multiplied by a parameter $c \in (0, 1)$, and the bias term $\overline{W}$ is multiplied by $(1 - c)$. $\overline{W} = 1$ is used for all synapses. According to the strength of c, the network is induced into a ferromagnetic state[5] (driven by the bias interaction), competing with a disordered state (driven by the random term).

It is studied the evolution of the network when initialized in b blocks (l_+ and l_-). The blocks are defined as the groups of neighbor neurons initialized as $\sigma_i = +1, i \in l_+$, and $\sigma_i = -1, i \in l_-$. A mesoscopic variable $a_l(t)$ is used to describe the neural activity of block l, with size $L = N/b$, as the fraction of neurons firing at time t,

$$a^t_l = \frac{1}{L}\sum_{i\in l}\sigma^t_i. \tag{4}$$

The macroscopic parameters are:

$$a = \langle a_l\rangle_b,\ v = \langle a^2_l\rangle_b - a^2. \tag{5}$$

where a is the usual global activity[5], [8], and $d = \sqrt{v}$ stands for the block activity. The time index t is dropped for simplicity. For the case of uniform blocks, it can also be defined the global activity of the network as $a = (a_+ + a_-)/2$ and the block activity as $d = (a_+ - a_-)/2$.

The network can be in the following representative phases: global activity (G, with $a \neq 0,\ d = 0$), block activity (B, with $a = 0,\ d \neq 0$) and zero activity (Z, with $a = 0,\ d = 0$).

III. Results

A. Bias interaction

The dynamics in Eq.(1) was simulated for a network with $N = 10^5$, and connectivity $\gamma = 10^{-3}$. The network started in $b = 10$ contiguous blocks of positive/negative activity with $a^{t=0}_l \sim \pm 0.2$. For a fixed value of $c = 0.8$, the evolution in time of the block activities for different values of the randomness ratio is depicted in Fig.1. In each panel the activities are smooth averaged over windows of $N_w = 2 \times 10^2$.

In Fig.1, left panels, the probability of random connections is $\omega = 0.1$. After $t = 10$ time steps, the blocks have been almost completed, and reach a stationary state in which the block structured is maintained, where $a_+ \sim 1$ and $a_- \sim -1$. In the right panels of Fig.1 with $\omega = 0.3$, the block structure is reached after $t \sim 10$ time steps. At $t = 60$ the active blocks were approximately filled with $a_+ \sim 1$, while the inactive blocks were destructed $a_- \sim 0$. In the next steps, the inactive blocks become attracted by the active ones, and the global phase is achieved, where an active ordering is accomplished.

In Fig.2, both global and block activity order parameters are plotted against time evolution for the same values of the variables in the Fig.1. Fig.2-left, shows that for a value of $\omega = 0.1$, a stable block activity is reached, $a \sim 0,\ d \sim 0.93$. It is seen that up to 10^6 time steps, the blocks don't change into global ordering. The behavior of the activity during the network evolution corresponds to the left panels in Fig.1. In Fig.2-right, for a larger value of the randomness parameter $\omega = 0.3$, it is seen that after an initial retrieval of the full block ordering, the network almost suddenly (in a logarithmic time scale) crashes into a switch between B and G phases. An active order is restored, $a \sim 1,\ d \sim 0$. This behavior corresponds to the right panels in Fig.1.

Fig.3 depicts the evolution of a network with $N = 10^4$, starting in $b = 40$ blocks chosen in a symmetric successive structure of large and small blocks, $a^{t=0}_l \sim \pm 0.3$, $N_w = 50$. The values of $\omega = 0.1, c = 0.4$ are fixed to achieve the block phase. The connectivity ratio γ is varied and its effect over the block structure is studied.

The block structure is sensitive to both, the number of blocks and the network dilution γ. The block structure is stable when the network degree K is small, (γ decreases), as shown in the left panels of Fig.3. Increasing K, some of the blocks may loose their stability and they are caught by blocks of larger size. Thus, a stable structure with a few blocks emerges, as seen

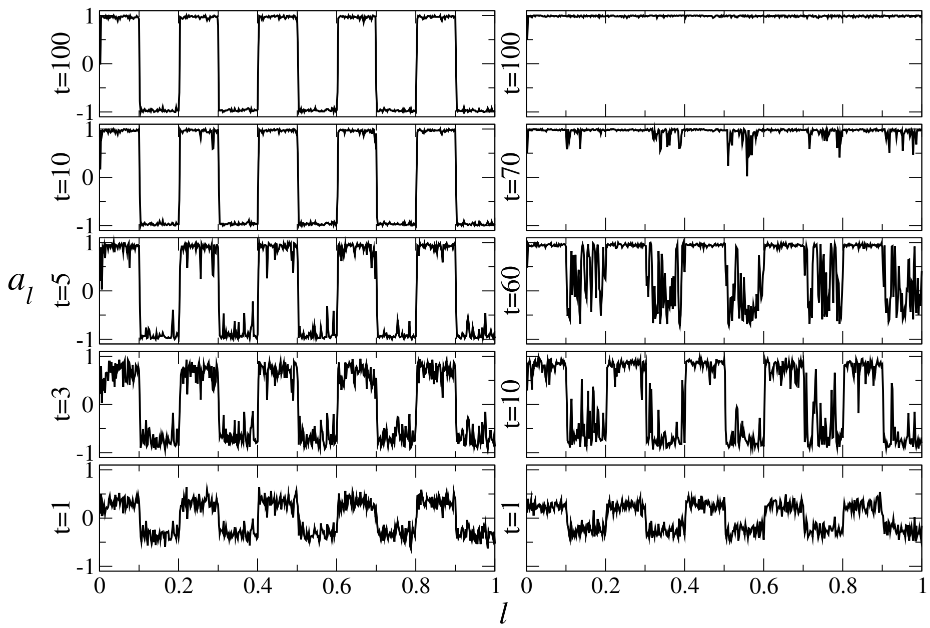

Fig. 1. Left panels: Evolution of the network into a block activity ordering, $\omega = 0.1$ and $c = 0.8$. Right panels: Evolution of the network into a global activity ordering, $\omega = 0.3$ and $c = 0.8$. Network with N= 10^5 and $\gamma = 10^{-3}$. Initial condition: $a_l^{t=0} \sim \pm 0.2$

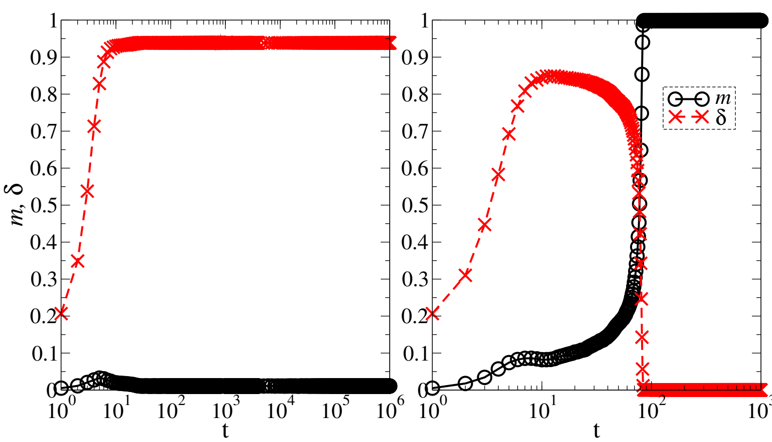

Fig. 2. Network evolution in time. Left: Block activity evolution, $\omega = 0.1, c = 0.8$. Right: Global activity evolution, $\omega = 0.3,\ c = 0.8$. Network with N= 10^5 and $\gamma = 10^{-3}$.

in the middle panels. Further increasing of the connectivity, will lead the network to the global state (right panels). Fig.3 also shows that, the block phase activity is robust for different spatial configurations of the initial block structure, and for a large amount of noise. This phase is also robust to an initial biased structure of blocks, $l_+ > l_-$ or viceversa.

IV. Conclusion

A new type of solution, for an attractor neural network, was studied here: the block activity/retrieval phase (B). When a bias in the synaptic weights is added to random weights, the network becomes ordered in a global activity phase (G), which resembles the ferromagnetic state in a spin system. This phase may coexists with a spin-glass phase, which is microscopically ordered, but without any spatial structure. The B phase, however, is spatially structured: within each (mesoscopic) block, the neurons are ordered. If the connections between each block are less relevant than inside the blocks, as it is the case of small-world networks with few long-range shortcuts, the B phase is stable. If there are enough random long-range connections, the G phase attracts almost all space of configurations: even an initial condition close to a block structure leads to a final state where all neurons are ordered.

It also has been shown that, this Physics model approach is extensible to memory networks for learning of patterns. The overlap between a learning pattern and the neural state is the usual global parameter which measures the network retrieval ability. When the connections are preferentially local, however, there are spatial correlations between neurons which allow for states retrieving blocks of a pattern. It was observed that these block states are stable, with a large basin of attraction.

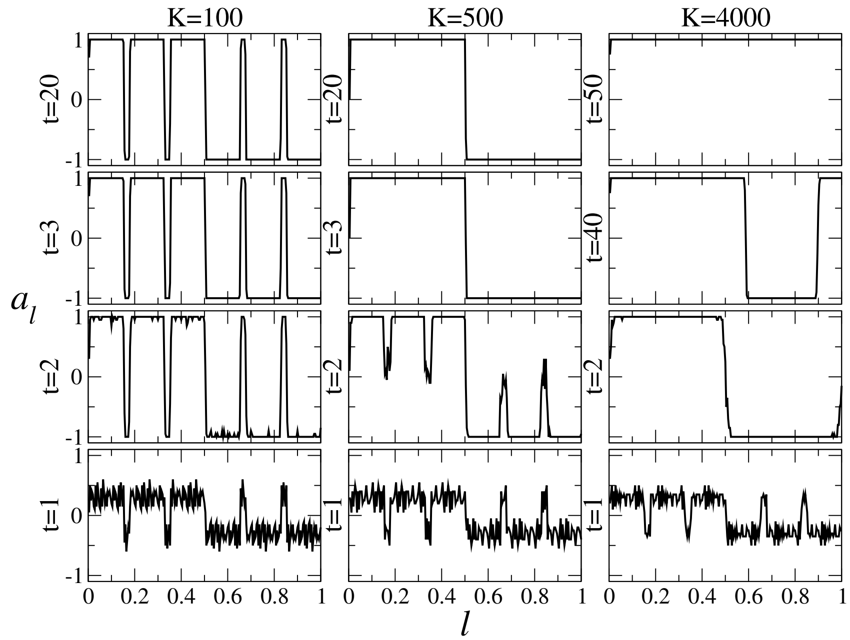

Fig. 3. Evolution of the network into block or global phase according to the network dilution γ. The network starts in $b = 40$ non contiguous blocks of positive/negative activity ($a_l^{t=0} \sim \pm 0.3$). Network with $N = 10^4$, $\omega = 0.1, c = 0.4$

Appendix A
Proof of the First Mean Field Equation

Appendix one text goes here.

Acknowledgment

The authors would like to thank...

References

[1] E. Rolls and A. Treves, *Neural Networks and Brain Function*. England: Oxford University Press, 1998.
[2] N. Levy, D. Horn, and E. Ruppin, "Associative memory in a multimodular network," *Neural Comput.*, vol. 11, no. 7, pp. 1717–1737, 1999.
[3] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, pp. 440–422, 1998.
[4] A. Damasio, *Descartes' Error: Emotion, Reason, and the Human Brain*. New York: Grosset/Putnam, 1994.
[5] D. J. Amit, *Modeling brain function: the world of attractor neural networks*. New York, NY, USA: Cambridge University Press, 1989.
[6] J. P. Hatchett, N. S. Skantzos, and T. Nikoletopoulos, "Dynamic rewiring in small world networks," *Phys. Rev. E*, vol. 72, p. 066105, 2005.
[7] M. Gonzalez, D. Dominguez, and F. B. Rodriguez, "Block activity in metric neural networks," *WASET Proceeding*, vol. 27, pp. 56–59, 2008.
[8] D. R. Paula, A. D. Araujo, J. J. S. Andrade, H. J. Herrmann, and J. A. C. Gallas, "Periodic neural activity induced by network complexity," *Phys. Rev. E*, vol. 74, no. 1, p. 017102, 2006.

Mario Salvador González-Rodríguez was born in León, Nicaragua, in 1981. He received the B.Sc. degree in industrial engineering from the Universidad Nacional de Ingeniería, Managua, Nicaragua, in 2004, and the M.Sc. and Ph.D. degrees in computer science from the Autonomous University of Madrid (UAM), in 2008 and 2012, respectively. He is specialized in the area of artificial intelligence, complex systems, and information processing using neural networks. He was a Visiting Professor with the Universidad de Los Andes, Bogota, Colombia, in 2017. He has been an Associate Professor with the Universidad de las Américas, Quito, Ecuador, since 2016. His research interests include the modeling of attractor networks for pattern retrieval, data analytics, artificial intelligence, and smart transportation systems.

John Doe Biography text here.

Jane Doe Biography text here.