2. Basic Algebra

EXERCISE - 2.1

1) Classify each element of $\{\sqrt{7}, \frac{-1}{4}, 0, 3.14, 4, \frac{22}{7}\}$ as a member of N, Q, R-Q or Z

Soln:

1) $\sqrt{7} \in R-Q$ 2) $\frac{-1}{4} \in Q$

3) $0 \in Q, Z$ 4) $3.14 \in Q$

5) $4 \in N, Q, Z$ 6) $\frac{22}{7} \in Q$

2) Prove that $\sqrt{3}$ is an irrational number.

Soln: Suppose that $\sqrt{3}$ is a rational number

Let $\sqrt{3} = \frac{m}{n}$, where m, n are positive integers with no common factors greater than 1.

$(\sqrt{3})^2 = \frac{m^2}{n^2} \Rightarrow 3 = \frac{m^2}{n^2} \Rightarrow m^2 = 3n^2$ — ①

m^2 is divisible by 3

m is also divisible by 3

$\therefore m = 3k$, ① $\Rightarrow (3k)^2 = 3n^2$

$3k^2 = n^2 \Rightarrow n^2 = 3k^2$

n^2 is divisible by 3

n is also divisible by 3

$\therefore n = 3l$, m and n are having a Common factor 3.

$\therefore$ we arrived at a contradiction.

$\therefore \sqrt{3}$ is an irrational number.

3) Are there two distinct irrational numbers such that their difference is a rational number? Justify.

Soln: Let $x+y\sqrt{3}$ and $z+y\sqrt{3}$ be two distinct irrational numbers. where x, y and z are rational number. Their difference

$= (x+y\sqrt{3}) - (z+y\sqrt{3})$

$= x + y\sqrt{3} - z - y\sqrt{3} = x - z \in Q$

yes, Their difference is a rational number.

For Example,

Let $4+\sqrt{3}$ and $2+\sqrt{3}$ be two distinct irrational numbers. where 1, 2 and 4 are rational number.

Their difference $= (4+\sqrt{3}) - (2+\sqrt{3})$

$= 4 + \sqrt{3} - 2 - \sqrt{3} = 4 - 2 = 2 \in Q$

yes, Their difference is a rational number.

4) Find two irrational numbers such that their sum is a rational number. Can you find two irrational numbers whose product is a rational number.

Soln:

Let $x+y\sqrt{3}$ and $x-y\sqrt{3}$ be two irrational numbers. where x and y are rational number.

Their sum $= x + y\sqrt{3} + x - y\sqrt{3} = 2x$

which is a rational number.

Their Product $= (x+y\sqrt{3})(x-y\sqrt{3})$

$= x^2 - 3y^2$ which is a rational number.

yes, The two irrational numbers their sum and product are rational number.

For Example,

Let $2+\sqrt{3}$ and $2-\sqrt{3}$ be two irrational numbers. where 1 and 2 are rational number.

Their sum $= 2 + \sqrt{3} + 2 - \sqrt{3} = 2+2 = 4$ which is a rational number.

Their Product $= (2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$ which is a rational number.

yes, The two irrational numbers their sum and product are rational number.

5) Find a positive number smaller than $\frac{1}{2^{1000}}$. Justify.

Soln:

$2^{1000} < 2^{1001}$

$\frac{1}{2^{1000}} > \frac{1}{2^{1001}}$

$\therefore \frac{1}{2^{1001}}$ is a positive number smaller than $\frac{1}{2^{1000}}$

Theorem 2.1 $\sqrt{2}$ is not a rational number.

Soln: Suppose that $\sqrt{2}$ is a rational number.

Let $\sqrt{2} = \frac{m}{n}$, where m and n are positive integers with no common factors greater than 1.

$(\sqrt{2})^2 = \frac{m^2}{n^2} \Rightarrow 2 = \frac{m^2}{n^2} \Rightarrow m^2 = 2n^2$ — ①

m^2 is even and m is also even.

$\therefore m = 2k$, ① $\Rightarrow (2k)^2 = 2n^2$

$2k^2 = n^2 \Rightarrow n^2 = 2k^2$

n^2 is even and n is also even.

$\therefore n = 2l$, m and n are even numbers having a Common factor 2.

$\therefore$ we arrived at a contradiction.

$\therefore \sqrt{2}$ is an irrational number.

2.3 Absolute value

EXERCISE 2.2

1) Solve for x:

(i) $|3-x| < 7$

Soln: Given: $|3-x| < 7$

$-7 < 3-x < 7$

$-7-3 < -x < 7-3$

$-10 < -x < 4$

$10 > x > -4 \Rightarrow -4 < x < 10$

(ii) $|4x-5| \geq -2$

Soln: For all the values of x, $|4x-5|$ is positive.

$\therefore$ The given equation Satisfy for $x \in R$ (or) $x \in (-\infty, \infty)$

(iii) $|3 - \frac{3}{4}x| \leq \frac{1}{4}$

Soln: Given: $|3 - \frac{3}{4}x| \leq \frac{1}{4}$

$-\frac{1}{4} \leq 3 - \frac{3}{4}x \leq \frac{1}{4}$

$-\frac{1}{4} - 3 \leq -\frac{3}{4}x \leq \frac{1}{4} - 3$

$\frac{-13}{4} \leq -\frac{3}{4}x \leq \frac{-11}{4}$

Multiple by 4, $-13 \le -3x \le -11$

$13 \ge 3x \ge 11 \Rightarrow 11 \le 3x \le 13$

$\frac{11}{3} \le \frac{3x}{3} \le \frac{13}{3} \Rightarrow \frac{11}{3} \le x \le \frac{13}{3}$

(iv) $|x|-10<-3$

Soln: Given: $|x|-10<-3$

$|x|<10-3 \Rightarrow |x|<7 \Rightarrow -7<x<7$

2) Solve $\frac{1}{|2x-1|}<6$ and express the Solution using the interval notation

Soln: Given: $\frac{1}{|2x-1|}<6$

$\Rightarrow 1<6|2x-1| \Rightarrow 6|2x-1|>1$

$6(2x-1)>1,\ 6(2x-1)<-1$

$|2x-6>1,\ |2x-6<-1$

$|2x>7,\ |2x<5$

$x>\frac{7}{12},\ x<\frac{5}{12}$

$\therefore x \in (-\infty, \frac{5}{12}) \cup (\frac{7}{12}, \infty)$

$\therefore$ The Solution is $(-\infty, \frac{5}{12}) \cup (\frac{7}{12}, \infty)$

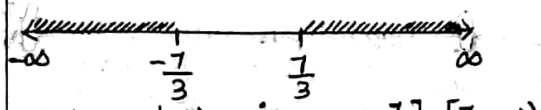
3) Solve $-3|x|+5 \le -2$ and graph the Solution set in a number line

Soln: Given: $-3|x|+5 \le -2$

$-3|x| \le -7 \Rightarrow 3|x| \ge 7 \Rightarrow |x| \ge \frac{7}{3}$

$x \ge \frac{7}{3},\ x \le \frac{-7}{3}$

$\therefore x \in (-\infty, \frac{-7}{3}] \cup [\frac{7}{3}, \infty)$



$\therefore$ The Solution is $(-\infty, \frac{-7}{3}] \cup [\frac{7}{3}, \infty)$

4) Solve $2|x+1|-6 \le 7$ and graph the Solution set in a number line.

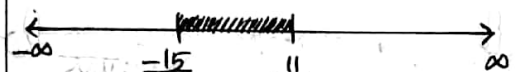
Soln: Given: $2|x+1|-6 \le 7$

$2|x+1| \le 13 \Rightarrow |x+1| \le \frac{13}{2}$

$\frac{-13}{2} \le x+1 \le \frac{13}{2}$

$\frac{-13}{2} - 1 \le x \le \frac{13}{2} - 1$

$\frac{-15}{2} \le x \le \frac{11}{2},\ x \in [\frac{-15}{2}, \frac{11}{2}]$



$\therefore$ The Solution is $\frac{-15}{2} \le x \le \frac{11}{2}$

5) Solve $\frac{1}{5}|10x-2|<1$

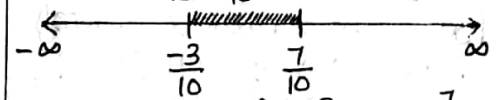
Soln: Given: $\frac{1}{5}|10x-2|<1$

$|10x-2|<5 \Rightarrow -5<10x-2<5$

$-5+2<10x<5+2$

$-3<10x<7 \Rightarrow \frac{-3}{10}<x<\frac{7}{10}$

$\therefore x \in (\frac{-3}{10}, \frac{7}{10})$



$\therefore$ The Solution is $\frac{-3}{10}<x<\frac{7}{10}$

6) Solve $|5x-12|<-2$

Soln:

Given inequation does not satisfy any x value in R

$\therefore x$ has no Solution.

EX 2.1 Solve $|2x-17|=3$ for x

Soln: Given: $|2x-17|=3$

$2x-17=\pm 3$

$2x-17=3,\ 2x-17=-3$

$2x=20,\ 2x=14$

$x=10,\ x=7$

$\therefore x=10$ (or) $x=7$

EX 2.2 Solve $3|x-2|+7=19$ for x

Soln: Given: $3|x-2|+7=19$

$3|x-2|=12 \Rightarrow |x-2|=4 \Rightarrow x-2=\pm 4$

$x-2=4,\ x-2=-4$

$x=6,\ x=-2$

$\therefore$ The Solutions are $x=-2$ and $x=6$

EX 2.3 Solve $|2x-3|=|x-5|$

Soln: Given: $|2x-3|=|x-5|$

$2x-3=\pm(x-5)$

$2x-3=x-5,\ 2x-3=-(x-5)$

$2x-x=3-5,\ 2x-3=-x+5$

$\boxed{x=-2},\ 2x+x=3+5$

$3x=8$

$\boxed{x=\frac{8}{3}}$

$\therefore$ The Solutions are $x=-2$ and $x=\frac{8}{3}$

EX 2.4 Solve $|x-9|<2$ for x

Soln: Given: $|x-9|<2$

$-2<x-9<2 \Rightarrow -2+9<x-9+9<2+9$

$\Rightarrow 7<x<11$

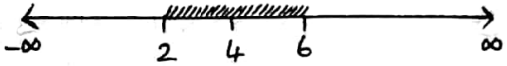
EX 2.5 Solve $\left|\frac{2}{x-4}\right|>1,\ x \neq 4$

Soln: Given: $\left|\frac{2}{x-4}\right|>1 \Rightarrow \frac{2}{|x-4|}>1$

$\Rightarrow 2>|x-4| \Rightarrow |x-4|<2$

$-2<x-4<2 \Rightarrow -2+4<x-4+4<2+4$

$\Rightarrow 2<x<6$ and $x \neq 4$



$\therefore x \in (2,4) \cup (4,6)$

$\therefore$ The Solution set is $(2,4) \cup (4,6)$

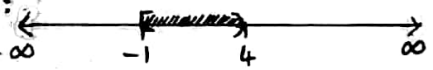
2.4 linear Inequalities

EXERCISE 2.3

1) Represent the following inequalities in the interval notation:

(i) $x \ge -1$ and $x<4$

Soln:



$\therefore x \in [-1,4)$

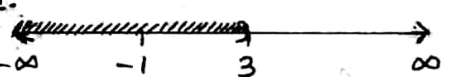
(ii) $x \le 5$ and $x \ge -3$

Soln:

$\therefore x \in [-3,5]$

(iii) $x<-1$ or $x<3$

Soln:



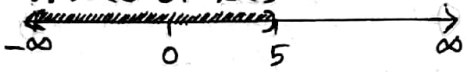
$\therefore x \in (-\infty, 3)$

(iv) $-2x>0$ or $3x-4<11$

Soln: $-2x>0 \Rightarrow 2x<0 \Rightarrow x<0$

Also, $3x<15 \Rightarrow x<5$

$\therefore x<0$ or $x<5$



$\therefore x \in (-\infty, 5)$

2) Solve $23x<100$

(i) x is a natural number

(ii) x is an integer

Soln: Given: $23x<100$

$x<\frac{100}{23} \Rightarrow x<4.347$

(i) $x=1,2,3,4$

(ii) $x=\cdots\cdots -3,-2,-1,0,1,2,3,4$

3) Solve $-2x \geq 9$ when

(i) x is a real number

(ii) x is an integer

(iii) x is a natural number

Soln: Given: $-2x \geq 9$

$2x \leq -9 \Rightarrow x \leq \frac{-9}{2} \Rightarrow x \leq -4.5$

(i) $x \in (-\infty, \frac{-9}{2}]$

(ii) $x=\cdots\cdots -7,-6,-5$

(iii) x has no solution

4) Solve (i) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

(ii) $\frac{5-x}{3} < \frac{x}{2} - 4$

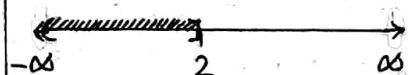
Soln: (i) Given: $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

$9(x-2) \leq 25(2-x)$

$9x-18 \leq 50-25x$

$9x+25x \leq 50+18 \Rightarrow 34x \leq 68$

$\Rightarrow x \leq 2$



$\therefore x \in (-\infty, 2]$

(ii) Given: $\frac{5-x}{3} < \frac{x}{2} - 4$

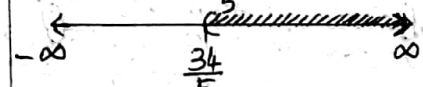
$\frac{5-x}{3} < \frac{x-8}{2}$

$2(5-x) < 3(x-8)$

$10-2x < 3x-24$

$10+24 < 3x+2x$

$34 < 5x \Rightarrow \frac{34}{5} < x \Rightarrow x > \frac{34}{5}$



$\therefore x \in (\frac{34}{5}, \infty)$

5) To secure A grade one must obtain an average of 90 marks or more in 5 subjects each of maximum 100 marks. If one scored 84, 87, 95, 91 in first four subjects. What is the minimum mark one scored in the fifth subject to get A grade in the course?

Soln: Let x be the fifth subject

From the given information,

$\frac{84+87+95+91+x}{5} \geq 90$

$357+x \geq 450 \Rightarrow x \geq 450-357$

$\Rightarrow x \geq 93$

$\therefore$ Minimum marks to be score in Fifth subject for getting A grade is 93

6) A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it. So that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?

Soln: Let x be the required number of litres.

Total amount of mixture $= (600+x)$ litres.

30% of x litres + 12% of 600 litres > 15% of $(600+x)$ litres

$\frac{30}{100}x + \frac{12}{100}(600) > \frac{15}{100}(600+x)$

$\frac{30x+7200}{100} > \frac{9000+15x}{100}$

$30x-15x > 9000-7200$

$15x > 1800 \mid 120 \Rightarrow x > 120$ —①

Also,

30% of x litres + 12% of 600 litres < 18% of $(600+x)$ litres

$\frac{30}{100}x + \frac{12}{100}(600) < \frac{18}{100}(600+x)$

$\frac{30x+7200}{100} < \frac{10800+18x}{100}$

$30x-18x < 10800-7200$

$12x < 3600 \mid 300 \Rightarrow x < 300$ —②

From ① and ② we get,

$120 < x < 300$

$\therefore$ The required solution is between 120 litres and 300 litres

7) Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40.

Soln: Let $(x, x+2)$ be the pair of consecutive odd natural numbers.

Given: $x>10$, $x+2>10$ —①

and $x+x+2<40$

$2x<38 \mid 19 \Rightarrow x<19$ —②

from ① and ② we get,

$10<x<19$

The possible values of x are 11, 13, 15, 17 and the possible value of $x+2$ are 13, 15, 17, 19

$\therefore$ The required pairs of consecutive odd natural numbers are (11, 13), (13, 15), (15, 17) and (17, 19)

8) A model rocket is launched From the ground. The height h reached by the rocket after t seconds from lift off is given by $h(t) = -5t^2+100t$, $0 \leq t \leq 20$. At what time the rocket is 495 feet above the ground?

Soln: Given: $h(t) = -5t^2+100t$, $0 \leq t \leq 20$ and $h(t) = 495$

$-5t^2+100t = 495$

$5t^2-100t+495=0$

$\div 5$, $t^2-20t+99=0$

$99 = -11 \times -9$

$(t-11)(t-9)=0$

$t-11=0$, $t-9=0$

$t=11$, $t=9$

$\therefore$ At $t=9$ seconds and $t=11$ seconds, the rocket is 495 feet above the ground.

9) A plumber can be paid acc-ording to the following sche-mes. In the first scheme he will be paid rupees 500 plus rupees 70 per hour and in the second scheme he will paid rupees 120 per hour. If he works x hours, then for what value of x does the first scheme give better wages?

Soln: Let x be the number of working hours.

I Scheme : $500+70x$

II Scheme : $120x$

From the given information,

$500+70x > 120x$

$500 > 120x-70x$

$10\cancel{500} > \cancel{50}x \Rightarrow x<10$

For less than 10 working hours, first scheme gives better wages.

10) A and B are working on simil-ar jobs but their annual sa-laries differ by more than Rs 6000. If B earns rupees 27000 per month, then what are the possibilities of A's Salary per month?

Soln: Monthly Salary of A = ₹ x

Monthly Salary of B = ₹ 6000

From the given information,

$|A-B| > 6000$

$|x-27000| > 6000$

$x-27000 > 6000$	$x-27000 < -6000$
$x > 27000+6000$	$x < 27000-6000$
$x > 33000$ —①	$x < 21000$ —②

From ① and ②,

The possibilities of A's Salary are less than ₹ 21,000 or greater than ₹ 33,000

EX 2.6 our monthly electricity bill contains a basic charge. which does not change with number of units used and a charge that depe-nds only on how many units we use. Let us say Electricity Board charges Rs 110 as basic charge and charges Rs 4 for each unit we use. If a person wants to keep his electricity bill below Rs 250 then what should be his electricity charge?

Soln: Let x denote the number of units used. For $x \geq 0$ —①

From the given information,

$110+4x < 250 \Rightarrow 4x < 250-110$

$\cancel{4}x < \cancel{140}35 \Rightarrow x < 35$ —②

From ① and ② we get,

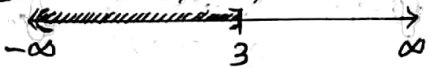
$0 \leq x < 35$

The person should keep his usage below 35 units in order to keep his bill below Rs 250

EX 2.7 Solve $3x-5 \leq x+1$ For x

Soln: Given : $3x-5 \leq x+1$

$3x-x \leq 6 \Rightarrow \cancel{2}x \leq \cancel{4}3 \Rightarrow x \leq 3$



$\therefore x \in (-\infty, 3]$

EX 2.8 Solve the following system of linear inequalities $3x-9 \geq 0$, $4x-10 \leq 6$

Soln: Given : $3x-9 \geq 0$ and $4x-10 \leq 6$

$\cancel{3}x \geq \cancel{9}^{3} \Rightarrow x \geq 3$ —①

Also, $\cancel{4}x \leq \cancel{16}4 \Rightarrow x \leq 4$ —②

From ① and ② we get,

$3 \leq x \leq 4$, $\therefore x \in [3,4]$

EX 2.9 A girl A is reading a book having 446 pages and she has already finished reading 271 pages. She wa-nts to finish reading this book within a week. What is the minim-um number of pages she should read per day to complete reading the book within a week?

Soln: Let x denote the number of pages.

From the given information,

$271+7x \geq 446 \Rightarrow 7x \geq 446-271$

$\Rightarrow \cancel{7}x \geq \cancel{175}25 \Rightarrow x \geq 25$

$\therefore$ She should read atleast 25 pages per day.

2.5 Quadratic Functions

EXERCISE 2.4

1) Construct a quadratic equation with roots 7 and −3

Soln: Let $\alpha = 7$ and $\beta = -3$

$S.R = \alpha+\beta = 7-3 = 4$

$P.R = \alpha\beta = (7)(-3) = -21$

The required quadratic equation is $x^2-(S.R)x+P.R=0$

$x^2-4x-21=0$

2) A quadratic Polynomial has one of its zeros $1+\sqrt{5}$ and it satisfies $P(1)=2$. Find the quadratic polynomial.

Soln: Let $\alpha = 1+\sqrt{5}$ and $\beta = 1-\sqrt{5}$

$S.R = \alpha+\beta = 1+\cancel{\sqrt{5}}+1-\cancel{\sqrt{5}} = 2$

$P.R = \alpha\beta = (1+\sqrt{5})(1-\sqrt{5}) = 1-5 = -4$

$\therefore x^2-(S.R)x+P.R=0$

$x^2-2x-4=0$

The required quadratic Polynomial is $P(x) = k(x^2-2x-4)$ —①

when $x=1$, $P(1) = k(1-2-4)$

Given : $P(1)=2$, $2 = -5k \Rightarrow -2 = 5k$

$\Rightarrow k = \frac{-2}{5}$, ① $\Rightarrow P(x) = \frac{-2}{5}(x^2-2x-4)$

3) If α and β are the roots of the quadratic equation $x^2+\sqrt{2}x+3=0$ form a quadratic Polynomial with zeros $\frac{1}{\alpha}, \frac{1}{\beta}$

Soln: Given : $x^2+\sqrt{2}x+3=0$

$\alpha+\beta = \frac{-b}{a} = \frac{-\sqrt{2}}{1} = -\sqrt{2}$

$\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

$S.R = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-\sqrt{2}}{3}$

$P.R = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = \frac{1}{\alpha\beta} = \frac{1}{3}$

The required Polynomial equation is

$x^2-(S.R)x+P.R=0$

$x^2-(\frac{\sqrt{2}}{3})x+\frac{1}{3}=0 \Rightarrow x^2+\frac{\sqrt{2}}{3}x+\frac{1}{3}=0$

4) If one root of $k(x-1)^2=5x-7$ is double the other root Show that $k=2$ or -25

Soln: Given: $k(x-1)^2=5x-7$

$k(x^2-2x+1)-5x+7=0$

$kx^2-2kx+k-5x+7=0$

$kx^2+(-2k-5)x+k+7=0$

From the given information, one root is α and other root is 2α.

$S.R=\alpha+2\alpha=\frac{-b}{a}=\frac{-(-2k-5)}{k}$

$3\alpha=\frac{2k+5}{k} \Rightarrow \alpha=\frac{2k+5}{3k}$ —①

$P.R=(\alpha)(2\alpha)=\frac{c}{a} \Rightarrow 2\alpha^2=\frac{k+7}{k}$ —②

Substitute equation ① in ②

$2\left(\frac{2k+5}{3k}\right)^2=\frac{k+7}{k}$

$2\left(\frac{(2k+5)^2}{9k^2}\right)=\frac{k+7}{k}$

$2(4k^2+20k+25)=9k(k+7)$

$8k^2+40k+50=9k^2+63k$

$9k^2-8k^2+63k-40k-50=0$

$k^2+23k-50=0$

$(k-2)(k+25)=0$

$k-2=0, k+25=0$

$k=2, k=-25$

$\therefore k=2$ or -25

$-50 = -2 \times 25$

5) If the difference of the roots of the equation $2x^2-(a+1)x+a-1=0$ is equal to their product then Prove that $a=2$

Soln: Let α and β be the roots.

Given: $2x^2-(a+1)x+a-1=0$

$\alpha+\beta=\frac{-b}{a}=\frac{-(-(a+1))}{2}=\frac{a+1}{2}$

$\alpha\beta=\frac{c}{a}=\frac{a-1}{2}$

Given: $\alpha-\beta=\alpha\beta$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$(\alpha\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$\left(\frac{a-1}{2}\right)^2=\left(\frac{a+1}{2}\right)^2-4\left(\frac{a-1}{2}\right)$

$\frac{(a-1)^2}{4}=\frac{(a+1)^2}{4}-2(a-1)$

Multiple by 4, $(a-1)^2=(a+1)^2-8(a-1)$

$a^2-2a+1=a^2+2a+1-8a+8$

$-2a=-6a+8$

$6a-2a=8 \Rightarrow 4a=8 \Rightarrow a=2$

6) Find the condition that one of the roots of ax^2+bx+c may be (i) negative of the other (ii) thrice the other (iii) reciprocal of the other.

Soln: Let α and β be the roots of the given equation ax^2+bx+c

$\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$

(i) The roots are α and $\beta=-\alpha$

$S.R=\alpha+\beta=\frac{-b}{a} \Rightarrow \alpha-\alpha=\frac{-b}{a}$

$0=\frac{-b}{a} \Rightarrow -b=0 \Rightarrow b=0$

∴ The required condition is $b=0$

(ii) The roots are α and $\beta=3\alpha$

$S.R=\alpha+\beta=\frac{-b}{a} \Rightarrow \alpha+3\alpha=\frac{-b}{a}$

$4\alpha=\frac{-b}{a} \Rightarrow \alpha=\frac{-b}{4a}$ —①

$P.R=\alpha\beta=\frac{c}{a} \Rightarrow (\alpha)(3\alpha)=\frac{c}{a}$

$3\alpha^2=\frac{c}{a} \Rightarrow \alpha^2=\frac{c}{3a}$ —②

Substitute equation ① in ②

$\left(\frac{-b}{4a}\right)^2=\frac{c}{3a} \Rightarrow \frac{b^2}{16a^2}=\frac{c}{3a} \Rightarrow 3b^2=16ac$

∴ The required condition is $3b^2=16ac$

(iii) The roots are α and $\beta=\frac{1}{\alpha}$

$P.R=\alpha\beta=\frac{c}{a} \Rightarrow \alpha\left(\frac{1}{\alpha}\right)=\frac{c}{a} \Rightarrow 1=\frac{c}{a}$

$\Rightarrow c=a$

∴ The required condition is $c=a$

7) If the equations $x^2-ax+b=0$ and $x^2-ex+f=0$ have one root in common and if the second equation has equal roots then prove that $ae=2(b+f)$

Soln: Let α and β be the roots of $x^2-ax+b=0$

$S.R=\alpha+\beta=\frac{-b}{a}=\frac{-(-a)}{1}=a$

$\therefore \boxed{\alpha+\beta=a}$

$P.R=\alpha\beta=\frac{c}{a}=\frac{b}{1}=b$

$\therefore \boxed{\alpha\beta=b}$

Also, Let α and $\beta=\alpha$ be the roots of $x^2-ex+f=0$

$S.R=\alpha+\beta=\frac{-b}{a}=\frac{-(-e)}{1}=e$

$\alpha+\alpha=e \Rightarrow \boxed{2\alpha=e}$

$P.R=\alpha\beta=\frac{c}{a}=\frac{f}{1}=f$

$\alpha\alpha=f \Rightarrow \boxed{\alpha^2=f}$

$ae=(\alpha+\beta)2\alpha=2(\alpha^2+\alpha\beta)$

$ae=2(f+b) \Rightarrow ae=2(b+f)$

8) Discuss the nature of roots of (i) $-x^2+3x+1=0$ (ii) $4x^2-x-2=0$ (iii) $9x^2+5x=0$

Soln: (i) Given: $-x^2+3x+1=0$

$a=-1, b=3, c=1$

$\Delta=b^2-4ac=(3)^2-4(-1)(1)$

$\Delta=9+4=13>0$

∴ The roots are real and distinct.

(ii) Given: $4x^2-x-2=0$

$a=4, b=-1, c=-2$

$\Delta=b^2-4ac=(-1)^2-4(4)(-2)$

$\Delta=1+32=33>0$

∴ The roots are real and distinct.

(iii) Given: $9x^2+5x=0$

$a=9, b=5, c=0$

$\Delta=b^2-4ac=(5)^2-4(9)(0)$

$\Delta=25-0=25>0$

∴ The roots are real and distinct

9) Without sketching the graphs, Find whether the graphs of the following functions will interseet the x-axis and if So in how many Points (i) $y=x^2+x+2$ (ii) $y=x^2-3x-7$ (iii) $y=x^2+6x+9$

Soln: (i) Given: $y=x^2+x+2$

Put $y=0$, $x^2+x+2=0$

$a=1, b=1, c=2$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{(1)^2-4(1)(2)}}{2(1)}$

$x=\frac{-1\pm\sqrt{1-8}}{2}=\frac{-1\pm\sqrt{-7}}{2}=\frac{-1\pm i\sqrt{7}}{2}$

∴ The quadratic equation has no real roots.

∴ The graph does not meet x-axis.

(ii) Given: $y=x^2-3x-7$

Put $y=0$, $x^2-3x-7=0$

$a=1, b=-3, c=-7$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-7)}}{2(1)}$

$x=\frac{3\pm\sqrt{9+28}}{2}=\frac{3\pm\sqrt{37}}{2}$

$x=\frac{3+\sqrt{37}}{2}$ and $x=\frac{3-\sqrt{37}}{2}$

The quadratic equation has real and distinct.

∴ The graph intersects x-axis at two points.

(iii) Given: $y=x^2+6x+9$

Put $y=0$, $x^2+6x+9=0$

$(x+3)^2=0\Rightarrow(x+3)(x+3)=0$

$x+3=0, x+3=0\Rightarrow x=-3, x=-3$

∴ The quadratic equation has real and equal

∴ The graph touches x-axis at one point.

10) write $f(x)=x^2+5x+4$ in Completed Square form

Soln: Given: $f(x)=x^2+5x+4$

$f(x)=(x+\frac{5}{2})^2-(\frac{5}{2})^2+4$

$=(x+\frac{5}{2})^2-\frac{25}{4}+4$

$=(x+\frac{5}{2})^2-\frac{9}{4}$

$f(x)=(x+\frac{5}{2})^2-(\frac{3}{2})^2$

EX 2.10 If a and b are the roots of the equation $x^2-px+q=0$. find the value of $\frac{1}{a}+\frac{1}{b}$

Soln: Let a and b be the roots of the equation $x^2-px+q=0$

$S.R=a+b=\frac{-b}{a}=\frac{-(-p)}{1}=p$

$P.R=ab=\frac{c}{a}=\frac{q}{1}=q$

$\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}=\frac{p}{q}$

EX 2.11 Find the complete set of values of a for which the quadratic $x^2-ax+a+2=0$ has equal roots.

Soln: Given: Roots are equal

$\therefore\Delta=b^2-4ac=0$

Given: $x^2-ax+a+2=0$

$a=1, b=-a, c=a+2$

$\Delta=(-a)^2-4(1)(a+2)=0$

$a^2-4a-8=0$

$a=1, b=-4, c=-8$

$a=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-8)}}{2(1)}$

$=\frac{4\pm\sqrt{16+32}}{2}=\frac{4\pm\sqrt{48}}{2}=\frac{4\pm\sqrt{4\times12}}{2}$

$a=\frac{4\pm2\sqrt{12}}{2}=\frac{2(2\pm\sqrt{12})}{2}=2\pm\sqrt{12}$

$\therefore a=2+\sqrt{12}, 2-\sqrt{12}$

EX 2.12 Find the number of Solutions of $x^2+|x-1|=1$

Soln: case (i) For $x\geq1$

Given: $x^2+|x-1|=1$

$x^2+x-1-1=0$ (−2, 2, −1)

$x^2+x-2=0$

$(x+2)(x-1)=0\Rightarrow x+2=0, x-1=0$

$\Rightarrow x=-2, x=1$

∴ $x=-2$ is not possible.

∴ $x=1$

case (ii) For $x<1$

Given: $x^2+|x-1|=1$

$x^2-(x-1)=1\Rightarrow x^2-x+1=1$

$\Rightarrow x^2-x=1-1\Rightarrow x^2-x=0\Rightarrow x(x-1)=0$

$\Rightarrow x=0, x-1=0\Rightarrow x=0, x=1$

∴ $x=1$ is not possible

∴ $x=0$

∴ The Solution Set is $\{0,1\}$

∴ The number of solutions = 2

2.5.2 Quadratic Inequalities

EXERCISE 2.5

1) Solve $2x^2+x-15\leq0$

Soln: Given: $2x^2+x-15\leq0$ (−30, 6, −5)

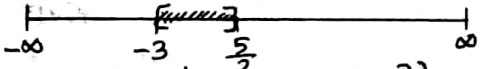
$2x^2+6x-5x-15\leq0$

$2x(x+3)-5(x+3)\leq0$

$(x+3)(2x-5)\leq0$

$\Rightarrow x=-3, x=\frac{5}{2}$

∴ The critical points are $x=-3$ and $x=\frac{5}{2}$



The intervals are $(-\infty,-3)$, $(-3,\frac{5}{2})$ and $(\frac{5}{2},\infty)$

Interval	Sign of $2x^2+x-15$
$(-\infty,-3)$	+
$(-3,\frac{5}{2})$	−
$(\frac{5}{2},\infty)$	+

∴ $x\in[-3,\frac{5}{2}]$ is satisfies $2x^2+x-15\leq0$

∴ The Solution Set is $[-3,\frac{5}{2}]$

2) Solve $-x^2+3x-2\geq0$

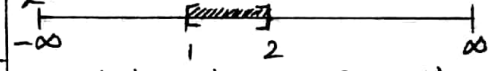
Soln: Given: $-x^2+3x-2\geq0$ (2, −2, −1)

$x^2-3x+2\leq0$

$(x-1)(x-2)\leq0$

$\Rightarrow x=1, x=2$

∴ The critical points are $x=1$ and $x=2$



The intervals are $(-\infty,1)$, $(1,2)$ and $(2,\infty)$

Interval	sign of x^2-3x+2
$(-\infty,1)$	+
$(1,2)$	−
$(2,\infty)$	+

$\therefore x\in[1,2]$ is satisfies $x^2-3x+2\le 0$

$\therefore$ The Solution set is $[1,2]$

EX 2.13 Solve $3x^2+5x-2\le 0$

Soln: Given: $3x^2+5x-2\le 0$

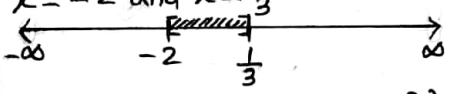
$3x^2+6x-x-2\le 0$ (−6 : 6, −1)

$3x(x+2)-1(x+2)\le 0$

$(x+2)(3x-1)\le 0$

$\Rightarrow x=-2, x=\frac{1}{3}$

$\therefore$ The critical points are $x=-2$ and $x=\frac{1}{3}$



The intervals are $(-\infty,-2)$, $(-2,\frac{1}{3})$ and $(\frac{1}{3},\infty)$

Interval	sign of $3x^2+5x-2$
$(-\infty,-2)$	+
$(-2,\frac{1}{3})$	−
$(\frac{1}{3},\infty)$	+

$\therefore x\in[-2,\frac{1}{3}]$ is satisfies $3x^2+5x-2\le 0$

$\therefore$ The Solution set is $[-2,\frac{1}{3}]$

EX 2.14 Solve $\sqrt{x+14}<x+2$

Soln: Given: $\sqrt{x+14}<x+2$

Now, $x+14\ge 0$ and $x+2\ge 0$

$x\ge -14$ and $x\ge -2$

$\sqrt{x+14}<x+2$

Squaring on both Sides,

$x+14<(x+2)^2 \Rightarrow x+14<x^2+4x+4$

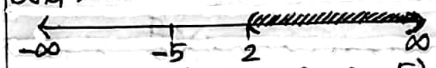
$\Rightarrow 0<x^2+4x-x+4-14 \Rightarrow 0<x^2+3x-10$

$\therefore x^2+3x-10>0$ (−10 : 5, −2)

$(x+5)(x-2)>0$

$\Rightarrow x=-5, x=2$

$\therefore$ The critical points are $x=-5$ and $x=2$



The intervals are $(-\infty,-5)$, $(-5,2)$ and $(2,\infty)$

Interval	sign of $x^2+3x-10$
$(-\infty,-5)$	+
$(-5,2)$	−
$(2,\infty)$	+

since $x\ge -2$, $\therefore x\in(2,\infty)$ is satisfies $x^2+3x-10>0$

$\therefore$ The Solution set is $(2,\infty)$

EX 2.15 Solve the equation $\sqrt{6-4x-x^2}=x+4$

Soln: Given: $\sqrt{6-4x-x^2}=x+4$, $x+4\ge 0$, $x\ge -4$

Squaring on both sides,

$6-4x-x^2=(x+4)^2$

$6-4x-x^2=x^2+8x+16$

$\Rightarrow x^2+x^2+8x+4x+16-6=0$

$\Rightarrow 2x^2+12x+10=0$ (5 : 5, 1)

$\div 2,\ x^2+6x+5=0$

$(x+5)(x+1)=0$

$x+5=0, x+1=0$

$x=-5, x=-1$

$\therefore x=-5$ is not possible.

since $x\ge -4$, The Solution of the given equation is $x=-1$

2.6 Polynomial Functions

EXERCISE 2.6

1) Find the zeros of the polynomial function $f(x)=4x^2-25$

Soln: Given: $f(x)=4x^2-25$

Put $f(x)=0$, $4x^2-25=0$

$\Rightarrow 4x^2=25 \Rightarrow x^2=\frac{25}{4} \Rightarrow x=\pm\frac{5}{2}$

$\therefore$ zeros of the polynomial are $x=\frac{-5}{2}$ and $x=\frac{5}{2}$

2) If $x=-2$ is one root of $x^3-x^2-17x=22$ then Find the other roots of equation

Soln: Given: $x^3-x^2-17x=22$

$x^3-x^2-17x-22=0$

and Given: $x=-2$ is one root.

−2	1	−1	−17	−22
		0	−2	6	22
	1	−3	−11	0

$\therefore (x+2)(x^2-3x-11)$ is a factor.

$\therefore (x+2)(x^2-3x-11)=0$

$x+2=0, x^2-3x-11=0$

$x=-2$, $a=1, b=-3, c=-11$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-11)}}{2(1)}=\frac{3\pm\sqrt{9+44}}{2}$

$x=\frac{3\pm\sqrt{53}}{2}$

$\therefore$ The other two roots are $\frac{3+\sqrt{53}}{2}$ and $\frac{3-\sqrt{53}}{2}$

3) Find the real roots of $x^4=16$

Soln: Given: $x^4=16$

$x^4-16=0 \Rightarrow (x^2)^2-(4)^2=0$

$(x^2-4)(x^2+4)=0$

$x^2-4=0, x^2+4=0$

$x^2=4, x^2=-4$

$x=\pm 2, x=\sqrt{-4} \Rightarrow x=\pm 2i$ which is no real

$\therefore x=2$ and $x=-2$

$\therefore$ The real roots of the given equation are 2 and −2

4) Solve $(2x+1)^2-(3x+2)^2=0$

Soln: Given: $(2x+1)^2-(3x+2)^2=0$

$((2x+1)+(3x+2))((2x+1)-(3x+2))=0$

$(5x+3)(2x+1-3x-2)=0$

$(5x+3)(-x-1)=0$

$5x+3=0, -x-1=0$

$x=\frac{-3}{5}, x=-1$

$\therefore$ The required solutions are $x=\frac{-3}{5}$ or $x=-1$

Method of Undetermined coefficients

EXERCISE 2.7

1) Factorize: x^4+1 (Hint: Try Completing the Square)

Soln: $x^4+1=(x^2)^2+1$

$=(x^2)^2+2x^2+1-2x^2$

$=(x^2+1)^2-(\sqrt{2}x)^2$

$=(x^2+1+\sqrt{2}x)(x^2+1-\sqrt{2}x)$

$=(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)$

2) If x^2+x+1 is a factor of the polynomial $3x^3+8x^2+8x+a$ then find the value of a

Soln:

$$\begin{array}{r|l} & 3x+5 \\ \hline x^2+x+1 & 3x^3+8x^2+8x+a \\ & 3x^3+3x^2+3x \\ \hline & 5x^2+5x+a \\ & 5x^2+5x+5 \\ \hline & a-5 \end{array}$$

$\therefore a-5=0 \Rightarrow a=5$

Ex 2.16 Find a quadratic polynomial $f(x)$ such that $f(0)=1, f(-2)=0$ and $f(1)=0$

Soln: $x=-2, x=1$ are zeros of $f(x)$

$\therefore f(x)=d(x+2)(x-1)$ — ①

Given: $f(0)=1$

when $x=0$, $f(0)=d(2)(-1)$

$\Rightarrow -2d=1 \Rightarrow 2d=-1 \Rightarrow d=\frac{-1}{2}$

① $\Rightarrow f(x)=\frac{-1}{2}(x+2)(x-1)$

$f(x)=\frac{-1}{2}(x^2+x-2)$

$f(x)=\frac{-1}{2}x^2-\frac{1}{2}x+1$

[OR]

Let $f(x)=ax^2+bx+c$

when $x=0$, $f(0)=c \Rightarrow \boxed{c=1}$

when $x=-2$, $f(-2)=a(-2)^2+b(-2)+c$

$4a-2b+1=0 \Rightarrow 4a-2b=-1$ — ①

when $x=1$, $f(1)=a+b+c$

$a+b+1=0 \Rightarrow a+b=-1$ — ②

① $\Rightarrow 4a-2b=-1$

② $\times 2 \Rightarrow 2a+2b=-2$

$6a=-3$

$\boxed{a=-\frac{1}{2}}$

② $\Rightarrow -\frac{1}{2}+b=-1 \Rightarrow b=\frac{1}{2}-1 \Rightarrow \boxed{b=-\frac{1}{2}}$

$\therefore c=1$ and $a=b=-\frac{1}{2}$

$\therefore f(x)=ax^2+bx+c$

$f(x)=-\frac{1}{2}x^2-\frac{1}{2}x+1$

Ex 2.17 Construct a cubic polynomial function with rational coefficients having zeros at $x=\frac{2}{5}, 1+\sqrt{3}$ such that $f(0)=-8$

Soln: Given: $\frac{2}{5}$ and $1+\sqrt{3}$ are zeros of $f(x)$

$\therefore 1-\sqrt{3}$ is also a zero of $f(x)$

$f(x)=a(x-\frac{2}{5})(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))$

$=a(x-\frac{2}{5})((x-1)-\sqrt{3})((x-1)+\sqrt{3})$

$=a(x-\frac{2}{5})((x-1)^2-(\sqrt{3})^2)$

$=a(x-\frac{2}{5})(x^2-2x+1-3)$

$f(x)=a(x-\frac{2}{5})(x^2-2x-2)$ — ①

Given: $f(0)=-8$

when $x=0$, $f(0)=a(-\frac{2}{5})(-2)$

$-8=\frac{4a}{5} \Rightarrow a=-10$

① $\Rightarrow f(x)=-10(x-\frac{2}{5})(x^2-2x-2)$

$=(-10x+4)(x^2-2x-2)$

$=-10x^3+20x^2+20x+4x^2-8x-8$

$f(x)=-10x^3+24x^2+12x-8$

Ex 2.18 Prove that $ap+q=0$ if $f(x)=x^3-3px+2q$ is divisible by $g(x)=x^2+2ax+a^2$

Soln:

$$\begin{array}{r|l} & x-2a \\ \hline x^2+2ax+a^2 & x^3+0x^2-3px+2q \\ & x^3+2ax^2+a^2x \\ \hline & -2ax^2-(3p+a^2)x+2q \\ & -2ax^2-4a^2x-2a^3 \\ \hline & (3a^2-3p)x+(2q+2a^3) \end{array}$$

$\therefore (3a^2-3p)x+(2q+2a^3)=0$

Equating coefficient of x and constant term,

$3a^2-3p=0$, $2q+2a^3=0$

$\div 3$, $a^2-p=0$, $\div 2$, $q+a^3=0$

$p=a^2$, $q+a(a^2)=0 \Rightarrow q+ap=0$

$\therefore ap+q=0$

[OR]

$\frac{f(x)}{g(x)}=x+b \Rightarrow f(x)=(x+b)g(x)$

$x^3-3px+2q=(x+b)(x^2+2ax+a^2)$

$=x^3+2ax^2+a^2x+bx^2+2abx+a^2b$

$x^3-3px+2q=x^3+(2a+b)x^2+(a^2+2ab)x+a^2b$

Equating like coefficients on both sides,

$2a+b=0 \Rightarrow b=-2a$

$a^2+2ab=-3p \Rightarrow a^2+2a(-2a)=-3p$

$a^2-4a^2=-3p \Rightarrow +3a^2=+3p \Rightarrow a^2=p$

Also, $a^2b=2q \Rightarrow a^2(-2a)=2q$

$p(-2a)=2q \Rightarrow -ap=q$

$\Rightarrow ap+q=0$

Ex 2.19 Use the method of undetermined coefficients to find the sum of $1+2+3+\cdots+(n-1)+n, n\in N$

Soln:

Let $S(n)=n+(n-1)+(n-2)+\cdots+2+1$

$=n+(n-1)+(n-2)+\cdots+(n-(n-2))+(n-(n-1))$

$=n\left[1+\frac{n-1}{n}+\frac{n-2}{n}+\cdots+\frac{n-(n-2)}{n}+\frac{n-(n-1)}{n}\right]$

$\le n[1+1+1+\cdots+1]=n(n)=n^2$

since $\frac{n-1}{n}<1, \frac{n-2}{n}<1, \ldots, \frac{n-(n-1)}{n}<1$

$S(n)\le n^2$ — ①

Let $S(n)=a+bn+cn^2$, $a,b,c\in R$

$S(n+1)-S(n)=n+1$

$a+b(n+1)+c(n+1)^2-a-bn-cn^2=n+1$

$a+bn+b+c(n^2+2n+1)-a-bn-cn^2=n+1$

$b+cn^2+2cn+c-cn^2=n+1$

$2cn+b+c=n+1$

Equating coefficient of n and Constant term,

$2c=1 \Rightarrow c=\frac{1}{2}$

$b+c=1 \Rightarrow b+\frac{1}{2}=1 \Rightarrow b=1-\frac{1}{2} \Rightarrow b=\frac{1}{2}$

① $\Rightarrow S(n)\le n^2$ $\quad \therefore b=c=\frac{1}{2}$

when $n=1$, $S(1)\le 1 \Rightarrow S(1)=1$

Also, $S(n)=a+bn+cn^2$

when $n=1$, $S(1)=a+b+c$

$a+\frac{1}{2}+\frac{1}{2}=1 \Rightarrow a+1=1 \Rightarrow a=0$

$\therefore S(n)=0+\frac{1}{2}n+\frac{1}{2}n^2 \Rightarrow S(n)=\frac{n^2+n}{2}$

$\therefore S(n)=\frac{n(n+1)}{2}, n\in N$

Ex 2.20 Find the roots of the Polynomial equation $(x-1)^3(x+1)^2(x+5)=0$ and state their multiplicity.

Soln:

Let $f(x)=(x-1)^3(x+1)^2(x+5)=0$

$(x-1)^3=0$	$(x+1)^2=0$	$x+5=0$
$x=1,1,1$	$x=-1,-1$	$x=-5$
$\therefore x=1$ is a root Multiplicity=3	$\therefore x=-1$ is a root Multiplicity=2	$\therefore x=-5$ is a root Multiplicity=1

Ex 2.21 Solve $x=\sqrt{x+20}$, for $x\in R$

Soln: $x+20\geq 0 \Rightarrow x\geq -20$

Also $x\geq 0$

Given: $x=\sqrt{x+20}$

Squaring on both sides,

$x^2=x+20 \Rightarrow x^2-x-20=0$

$\Rightarrow (x-5)(x+4)=0$ (−20: −5, 4)

$x-5=0,\ x+4=0$

$x=5,\ x=-4$ is not possible

$\therefore x=5$ is only possible Solution

$\therefore$ The required Solution is $x=5$

Ex 2.22 The equations $x^2-6x+a=0$ and $x^2-bx+6=0$ have one root in Common. The other root of the first and the second equations are integers in the ratio 4:3. Find the Common root.

Soln: Let α be the Common root.

Let $\alpha, 4\beta$ be the roots of $x^2-6x+a=0$

Let $\alpha, 3\beta$ be the roots of $x^2-bx+6=0$

$\therefore 4\alpha\beta=a$ and $3\alpha\beta=6 \Rightarrow \alpha\beta=2$ — ①

$4(2)=a \Rightarrow a=8$ (8: −4, −2)

$\therefore x^2-6x+8=0$

$(x-4)(x-2)=0 \Rightarrow x-4=0, x-2=0$

$\Rightarrow x=4, x=2$

when $\alpha=4$, ① $\Rightarrow 4\beta=2 \Rightarrow \beta=\frac{1}{2}\notin Z$

when $\alpha=2$, ① $\Rightarrow 2\beta=2 \Rightarrow \beta=1\in Z$

$\therefore$ The Common root is 2.

Ex 2.23 Find the values of p for which the difference between the roots of the equation $x^2+px+8=0$ is 2

Soln: Let α and β be the roots of the equation $x^2+px+8=0$

$\alpha+\beta=-p$ and $\alpha\beta=8$

Given: $|\alpha-\beta|=2$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$(2)^2=(-p)^2-4(8) \Rightarrow 4=p^2-32$

$\Rightarrow p^2=36 \Rightarrow p=\pm 6$

2.7 Rational Functions

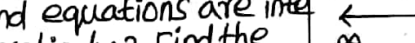
2.7.1 Rational Inequalities

EXERCISE-2.8

1) Find all values of x for which $\frac{x^3(x-1)}{(x-2)}>0$

Soln: Given: $\frac{x^3(x-1)}{(x-2)}$, $x\neq 2$

The critical points are $x=0,1,2$



The intervals are $(-\infty,0), (0,1), (1,2)$ and $(2,\infty)$

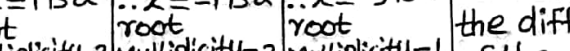
Interval	Sign of $\frac{x^3(x-1)}{(x-2)}$
$(-\infty,0)$	−
$(0,1)$	+
$(1,2)$	−
$(2,\infty)$	+

$\therefore$ The Solution set is $(0,1)\cup(2,\infty)$

2) Find all values of x that Satisfies the inequality $\frac{2x-3}{(x-2)(x-4)}<0$

Soln: Given: $\frac{2x-3}{(x-2)(x-4)}<0$, $x\neq 2$, $x\neq 4$

The critical points are $x=\frac{3}{2},2,4$



The intervals are $(-\infty,\frac{3}{2}), (\frac{3}{2},2), (2,4)$ and $(4,\infty)$

Interval	Sign of $\frac{2x-3}{(x-2)(x-4)}$
$(-\infty,\frac{3}{2})$	−
$(\frac{3}{2},2)$	+
$(2,4)$	−
$(4,\infty)$	+

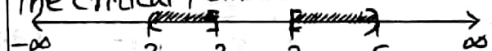
$\therefore$ The Solution set is $(-\infty,\frac{3}{2})\cup(2,4)$

3) Solve $\frac{x^2-4}{x^2-2x-15}\leq 0$

Soln: Given: $\frac{x^2-4}{x^2-2x-15}\leq 0$

$\frac{(x-2)(x+2)}{(x+3)(x-5)}\leq 0$, $x\neq -3, x\neq 5$

The critical points are $x=-2,2,-3,5$



The intervals are $(-\infty,-3), (-3,-2), (-2,2), (2,5)$ and $(5,\infty)$

Interval	Sign of $\frac{x^2-4}{x^2-2x-15}$
$(-\infty,-3)$	+
$(-3,-2)$	−
$(-2,2)$	+
$(2,5)$	−
$(5,\infty)$	+

$\therefore$ The Solution set is $(-3,-2]\cup[2,5)$

Ex 2.24 Solve $\frac{x+1}{x+3}<3$

Soln: Given: $\frac{x+1}{x+3}<3$

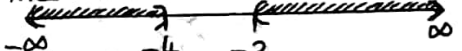
$\frac{x+1}{x+3}-3<0$

$\frac{x+1-3(x+3)}{x+3}<0 \Rightarrow \frac{x+1-3x-9}{x+3}<0$

$\frac{-2x-8}{x+3}<0 \Rightarrow \frac{2x+8}{x+3}>0$

$\frac{2(x+4)}{x+3}>0 \Rightarrow \frac{x+4}{x+3}>0$, $x\neq -3$

The critical points are $x=-3,-4$



The intervals are $(-\infty,-4), (-4,-3)$ and $(-3,\infty)$

Interval	Sign of $\frac{x+4}{x+3}$
$(-\infty,-4)$	+
$(-4,-3)$	−
$(-3,\infty)$	+
$x=-4$	0

$\therefore$ The Solution set is $(-\infty,-4]\cup(-3,\infty)$

2.7.2 Partial Fractions

EXERCISE 2-9

Resolve the following rational expressions into Partial fractions

1) $\frac{1}{x^2-a^2}$

Soln:

$\frac{1}{x^2-a^2}=\frac{1}{(x-a)(x+a)}=\frac{A}{x-a}+\frac{B}{x+a}$ — ①

$\frac{1}{(x-a)(x+a)}=\frac{A(x+a)+B(x-a)}{(x-a)(x+a)}$

$1=A(x+a)+B(x-a)$

when $x=-a$, $1=B(-2a)$
$\Rightarrow -1=B(2a) \Rightarrow B=\frac{-1}{2a}$
when $x=a$, $1=A(2a) \Rightarrow A=\frac{1}{2a}$
$① \Rightarrow \frac{1}{x^2-a^2}=\frac{\frac{1}{2a}}{x-a}+\frac{\left(\frac{-1}{2a}\right)}{x+a}$
$\frac{1}{x^2-a^2}=\frac{1}{2a(x-a)}-\frac{1}{2a(x+a)}$

2) $\frac{3x+1}{(x-2)(x+1)}$
Soln:
$\frac{3x+1}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$ —①
$\frac{3x+1}{(x-2)(x+1)}=\frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$
$3x+1=A(x+1)+B(x-2)$
when $x=-1$, $+2=+3B \Rightarrow B=\frac{2}{3}$
when $x=2$, $7=3A \Rightarrow A=\frac{7}{3}$
$① \Rightarrow \frac{3x+1}{(x-2)(x+1)}=\frac{\frac{7}{3}}{x-2}+\frac{\frac{2}{3}}{x+1}$
$\frac{3x+1}{(x-2)(x+1)}=\frac{7}{3(x-2)}+\frac{2}{3(x+1)}$

3) $\frac{x}{(x^2+1)(x-1)(x+2)}$
Soln:
$\frac{x}{(x^2+1)(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{Cx+D}{x^2+1}$ —①
$\frac{x}{(x^2+1)(x-1)(x+2)}=\frac{A(x+2)(x^2+1)+B(x-1)(x^2+1)+(Cx+D)(x-1)(x+2)}{(x^2+1)(x-1)(x+2)}$
$x=A(x+2)(x^2+1)+B(x-1)(x^2+1)+(Cx+D)(x-1)(x+2)$
when $x=1$, $1=A(3)(2) \Rightarrow 1=6A \Rightarrow A=\frac{1}{6}$
when $x=-2$, $+2=B(-3)(5) \Rightarrow 2=15B \Rightarrow B=\frac{2}{15}$
when $x=0$, $0=2A-B-2D$
$2D=2\left(\frac{1}{6}\right)-\frac{2}{15}$, $\div 2$, $D=\frac{1}{6}-\frac{1}{15}$
$D=\frac{5-2}{30} \Rightarrow D=\frac{3}{30} \Rightarrow D=\frac{1}{10}$

Equating coeff of x^3 on both sides,
$A+B+C=0 \Rightarrow \frac{1}{6}+\frac{2}{15}+C=0$
$\frac{5+4}{30}+C=0 \Rightarrow \frac{9}{30}+C=0 \Rightarrow C=\frac{-3}{10}$
$\therefore A=\frac{1}{6}$, $B=\frac{2}{15}$, $C=\frac{-3}{10}$ and $D=\frac{1}{10}$
$① \Rightarrow \frac{x}{(x^2+1)(x-1)(x+2)}=\frac{\frac{1}{6}}{x-1}+\frac{\frac{2}{15}}{x+2}+\frac{\frac{-3}{10}x+\frac{1}{10}}{x^2+1}$
$\frac{x}{(x^2+1)(x-1)(x+2)}=\frac{1}{6(x-1)}+\frac{2}{15(x+2)}+\frac{-3x+1}{10(x^2+1)}$

4) $\frac{x}{(x-1)^3}$
Soln: $\frac{x}{(x-1)^3}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$ —①
$\frac{x}{(x-1)^3}=\frac{A(x-1)^2+B(x-1)+C}{(x-1)^3}$
$x=A(x-1)^2+B(x-1)+C$
when $x=1$, $1=C \Rightarrow C=1$
when $x=0$, $0=A-B+C \Rightarrow A-B+1=0$
$\Rightarrow A-B=-1$
Equating coeff of x^2 on both sides,
$0=A \Rightarrow A=0$
$\therefore 0-B=-1 \Rightarrow +B=+1 \Rightarrow B=1$
$\therefore A=0$, $B=1$, $C=1$
$① \Rightarrow \frac{x}{(x-1)^3}=\frac{0}{x-1}+\frac{1}{(x-1)^2}+\frac{1}{(x-1)^3}$
$\frac{x}{(x-1)^3}=\frac{1}{(x-1)^2}+\frac{1}{(x-1)^3}$

5) $\frac{1}{x^4-1}$
Soln: $\frac{1}{x^4-1}=\frac{1}{(x^2-1)(x^2+1)}$ —①

Let $x^2=t$
$① \Rightarrow \frac{1}{x^4-1}=\frac{1}{(t-1)(t+1)}=\frac{A}{t-1}+\frac{B}{t+1}$ —②
$\frac{1}{(t-1)(t+1)}=\frac{A(t+1)+B(t-1)}{(t-1)(t+1)}$
$1=A(t+1)+B(t-1)$
when $t=1$, $1=2A \Rightarrow A=\frac{1}{2}$
when $t=-1$, $1=-2B \Rightarrow B=-\frac{1}{2}$
$② \Rightarrow \frac{1}{x^4-1}=\frac{\frac{1}{2}}{x^2-1}+\frac{-\frac{1}{2}}{x^2+1}$
$\frac{1}{x^4-1}=\frac{1}{2(x^2-1)}-\frac{1}{2(x^2+1)}$

6) $\frac{(x-1)^2}{x^3+x}$
Soln: $\frac{(x-1)^2}{x^3+x}=\frac{(x-1)^2}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$ —①
$\frac{(x-1)^2}{x^3+x}=\frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)}$
$(x-1)^2=A(x^2+1)+(Bx+C)x$
when $x=0$, $A=1$
when $x=1$, $2A+B+C=0$
$2(1)+B+C=0 \Rightarrow B+C=-2$ —②
Equating coeff of x^2 on both sides,
$A+B=1 \Rightarrow 1+B=1 \Rightarrow B=1-1 \Rightarrow B=0$
$② \Rightarrow C=-2$
$\therefore A=1$, $B=0$, $C=-2$
$① \Rightarrow \frac{(x-1)^2}{x^3+x}=\frac{1}{x}-\frac{2}{x^2+1}$

7) $\frac{x^2+x+1}{x^2-5x+6}$
Soln:

$$\begin{array}{r|l} & 1 \\ \hline x^2-5x+6 & x^2+x+1 \\ & \mp x^2 \pm 5x \pm 6 \\ \hline & 6x-5 \end{array}$$

$\frac{x^2+x+1}{x^2-5x+6}=1+\frac{6x-5}{x^2-5x+6}$ —①
$\frac{6x-5}{x^2-5x+6}=\frac{6x-5}{(x-3)(x-2)}=\frac{A}{x-3}+\frac{B}{x-2}$ —②
$\frac{6x-5}{x^2-5x+6}=\frac{A(x-2)+B(x-3)}{(x-3)(x-2)}$
$6x-5=A(x-2)+B(x-3)$
when $x=3$, $A=13$
when $x=2$, $-B=7 \Rightarrow B=-7$
$\therefore A=13$, $B=-7$
$② \Rightarrow \frac{6x-5}{x^2-5x+6}=\frac{13}{x-3}-\frac{7}{x-2}$
$① \Rightarrow \frac{x^2+x+1}{x^2-5x+6}=1+\frac{13}{x-3}-\frac{7}{x-2}$

8) $\frac{x^3+2x+1}{x^2+5x+6}$
Soln:

$$\begin{array}{r|l} & x-5 \\ \hline x^2+5x+6 & x^3+0x^2+2x+1 \\ & \mp x^3 \pm 5x^2 \pm 6x \\ \hline & -5x^2-4x+1 \\ & \mp 5x^2 \mp 25x \mp 30 \\ \hline & 21x+31 \end{array}$$

$\frac{x^3+2x+1}{x^2+5x+6}=x-5+\frac{21x+31}{x^2+5x+6}$
$\frac{x^3+2x+1}{x^2+5x+6}=x-5+\frac{21x+31}{(x+2)(x+3)}$ —①
$\frac{21x+31}{(x+2)(x+3)}=\frac{A}{x+2}+\frac{B}{x+3}$ —②
$\frac{21x+31}{(x+2)(x+3)}=\frac{A(x+3)+B(x+2)}{(x+2)(x+3)}$
$21x+31=A(x+3)+B(x+2)$

when $x=-2$, $A=-11$

when $x=-3$, $+B=+32 \Rightarrow B=32$

②$\Rightarrow \frac{21x+31}{(x+2)(x+3)} = \frac{-11}{x+2} + \frac{32}{x+3}$

①$\Rightarrow \frac{x^3+2x+1}{x^2+5x+6} = (x-5) - \frac{11}{x+2} + \frac{32}{x+3}$

9) $\frac{x+12}{(x+1)^2(x-2)}$

Soln:

$\frac{x+12}{(x+1)^2(x-2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2}$ —①

$\frac{x+12}{(x+1)^2(x-2)} = \frac{A(x+1)(x-2)+B(x-2)+C(x+1)^2}{(x+1)^2(x-2)}$

$x+12 = A(x+1)(x-2)+B(x-2)+C(x+1)^2$

when $x=2$, $9C=14 \Rightarrow C=\frac{14}{9}$

when $x=-1$, $-3B=11 \Rightarrow B=-\frac{11}{3}$

Equating coeff of x^2 on both sides,

$A+C=0 \Rightarrow A+\frac{14}{9}=0 \Rightarrow A=-\frac{14}{9}$

$\therefore A=-\frac{14}{9}$, $B=-\frac{11}{3}$ and $C=\frac{14}{9}$

①$\Rightarrow \frac{x+12}{(x+1)^2(x-2)} = \frac{-\frac{14}{9}}{x+1} + \frac{-\frac{11}{3}}{(x+1)^2} + \frac{\frac{14}{9}}{x-2}$

$\frac{x+12}{(x+1)^2(x-2)} = -\frac{14}{9(x+1)} - \frac{11}{3(x+1)^2} + \frac{14}{9(x-2)}$

10) $\frac{6x^2-x+1}{x^3+x^2+x+1}$

Soln: $\frac{6x^2-x+1}{x^3+x^2+x+1} = \frac{6x^2-x+1}{x^2(x+1)+1(x+1)}$

$\frac{6x^2-x+1}{x^3+x^2+x+1} = \frac{6x^2-x+1}{(x+1)(x^2+1)}$

$\frac{6x^2-x+1}{x^3+x^2+x+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$ —①

$\frac{6x^2-x+1}{x^3+x^2+x+1} = \frac{A(x^2+1)+(Bx+C)(x+1)}{x^3+x^2+x+1}$

$6x^2-x+1 = A(x^2+1)+(Bx+C)(x+1)$

when $x=-1$, $2A=8 \Rightarrow A=4$

when $x=0$, $A+C=1 \Rightarrow 4+C=1 \Rightarrow C=-3$

Equating coeff of x^2 on both sides,

$A+B=6 \Rightarrow 4+B=6 \Rightarrow B=2$

①$\Rightarrow \frac{6x^2-x+1}{x^3+x^2+x+1} = \frac{4}{x+1} + \frac{2x-3}{x^2+1}$

11) $\frac{2x^2+5x-11}{x^2+2x-3}$

Soln:

$$\begin{array}{r|l} & 2 \\ \hline x^2+2x-3 & 2x^2+5x-11 \\ & 2x^2 \pm 4x \mp 6 \\ \hline & x-5 \end{array}$$

$\frac{2x^2+5x-11}{x^2+2x-3} = 2 + \frac{x-5}{x^2+2x-3}$ —①

$\frac{x-5}{x^2+2x-3} = \frac{x-5}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$ —②

$\frac{x-5}{x^2+2x-3} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$

$x-5 = A(x-1)+B(x+3)$

when $x=-3$, $-4A=-8 \Rightarrow A=2$

when $x=1$, $4B=-4 \Rightarrow B=-1$

②$\Rightarrow \frac{x-5}{x^2+2x-3} = \frac{2}{x+3} - \frac{1}{x-1}$

①$\Rightarrow \frac{2x^2+5x-11}{x^2+2x-3} = 2 + \frac{2}{x+3} - \frac{1}{x-1}$

12) $\frac{7+x}{(1+x)(1+x^2)}$

Soln: $\frac{7+x}{(1+x)(1+x^2)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$ —①

$\frac{7+x}{(x+1)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x+1)}{(x+1)(x^2+1)}$

$7+x = A(x^2+1)+(Bx+C)(x+1)$

when $x=-1$, $2A=6 \Rightarrow A=3$

when $x=0$, $A+C=7 \Rightarrow 3+C=7 \Rightarrow C=4$

Equating coeff of x^2 on both sides,

$A+B=0 \Rightarrow 3+B=0 \Rightarrow B=-3$

$\therefore A=3$, $B=-3$ and $C=4$

①$\Rightarrow \frac{7+x}{(1+x)(1+x^2)} = \frac{3}{x+1} + \frac{-3x+4}{x^2+1}$

$\frac{7+x}{(1+x)(1+x^2)} = \frac{3}{x+1} + \frac{4-3x}{x^2+1}$

EX 2.25 Resolve into Partial Fractions

$\frac{x}{(x+3)(x-4)}$

Soln: $\frac{x}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$ —①

$\frac{x}{(x+3)(x-4)} = \frac{A(x-4)+B(x+3)}{(x+3)(x-4)}$

$x = A(x-4)+B(x+3)$

when $x=-3$, $+7A=+3 \Rightarrow A=\frac{3}{7}$

when $x=4$, $7B=4 \Rightarrow B=\frac{4}{7}$

①$\Rightarrow \frac{x}{(x+3)(x-4)} = \frac{\frac{3}{7}}{x+3} + \frac{\frac{4}{7}}{x-4}$

$\frac{x}{(x+3)(x-4)} = \frac{3}{7(x+3)} + \frac{4}{7(x-4)}$

EX 2.26 Resolve into Partial Fractions

$\frac{2x}{(x^2+1)(x-1)}$

Soln: $\frac{2x}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ —①

$\frac{2x}{(x^2+1)(x-1)} = \frac{A(x^2+1)+(Bx+C)(x-1)}{(x^2+1)(x-1)}$

$2x = A(x^2+1)+(Bx+C)(x-1)$

when $x=1$, $2A=2 \Rightarrow A=1$

when $x=0$, $A-C=0 \Rightarrow 1-C=0 \Rightarrow C=1$

Equating coeff of x^2 on both sides,

$A+B=0 \Rightarrow 1+B=0 \Rightarrow B=-1$

$\therefore A=1$, $B=-1$ and $C=1$

①$\Rightarrow \frac{2x}{(x^2+1)(x-1)} = \frac{1}{x-1} + \frac{-x+1}{x^2+1}$

$\frac{2x}{(x^2+1)(x-1)} = \frac{1}{x-1} + \frac{1-x}{x^2+1}$

EX 2.27 Resolve into partial fractions

$\frac{x+1}{x^2(x-1)}$

Soln: $\frac{x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$ —①

$\frac{x+1}{x^2(x-1)} = \frac{Ax(x-1)+B(x-1)+Cx^2}{x^2(x-1)}$

$x+1 = Ax(x-1)+B(x-1)+Cx^2$

when $x=0$, $-B=1 \Rightarrow B=-1$

when $x=1$, $C=2$

Equating coeff of x^2 on both sides,

$A+C=0 \Rightarrow A+2=0 \Rightarrow A=-2$

$\therefore A=-2$, $B=-1$ and $C=2$

①$\Rightarrow \frac{x+1}{x^2(x-1)} = -\frac{2}{x} - \frac{1}{x^2} + \frac{2}{x-1}$

2.7.3 Graphical Representation of linear inequalities

EXERCISE 2.10

Determine the region in the plane determined by the inequalities:

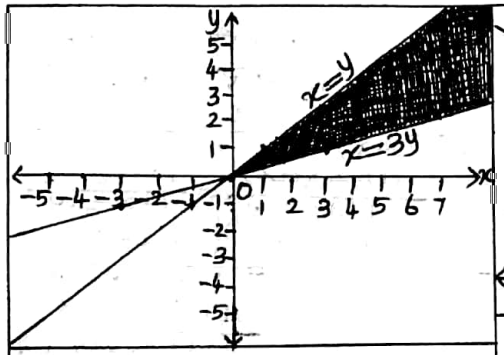
1) $x \le 3y$, $x \ge y$

Soln: Let $x=3y \Rightarrow y=\frac{x}{3}$

x	-3	0	3
y	-1	0	1

Let $x=y$

x	-1	0	1
y	-1	0	1



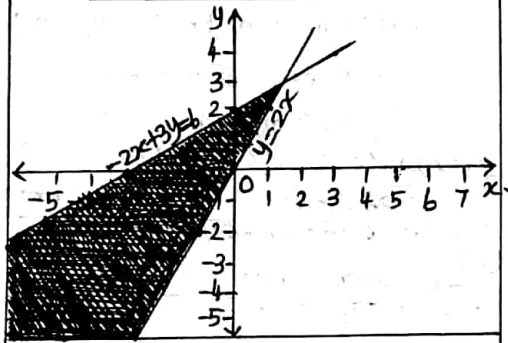
2) $y \geq 2x, -2x+3y \leq 6$

Soln: Let $y=2x$

x	-1	0	1
y	-2	0	2

Let $-2x+3y=6$

x	-3	0	3
y	0	2	4



3) $3x+5y \geq 45, x \geq 0, y \geq 0$

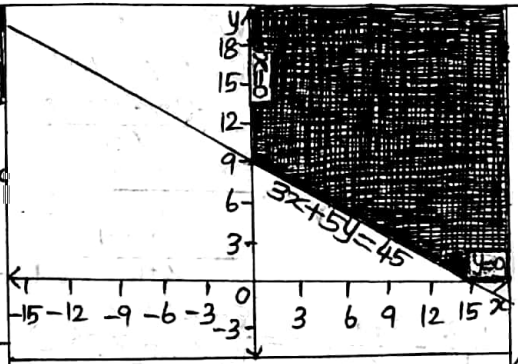
Soln:

Let $3x+5y=45$

x	0	15
y	9	0

$x=0$ represents y-axis and

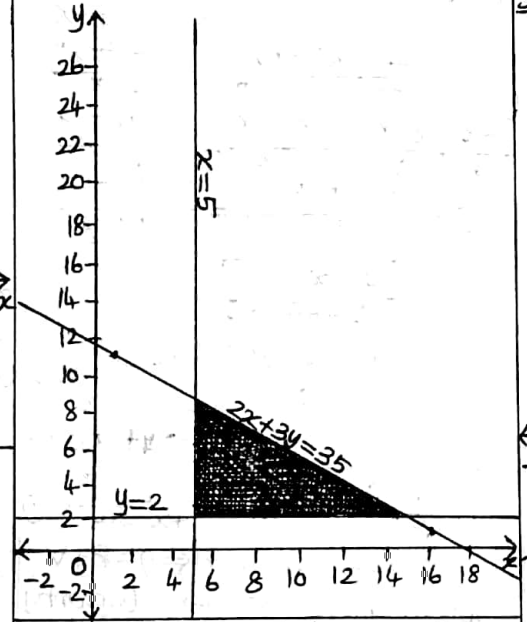
$y=0$ represents x-axis



4) $2x+3y \leq 35, y \geq 2, x \geq 5$

Soln: Let $2x+3y=35$

x	1	16
y	11	1



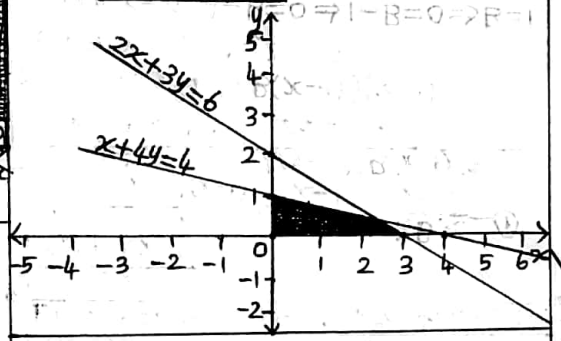
5) $2x+3y \leq 6, x+4y \leq 4, x \geq 0, y \geq 0$

Soln: Let $2x+3y=6$

x	0	3
y	2	0

Let $x+4y=4$

x	0	4
y	1	0



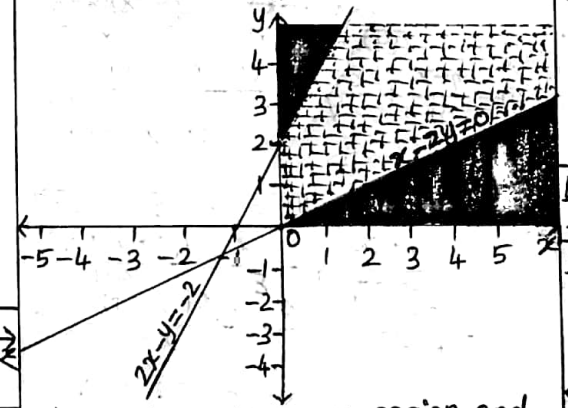
6) $x-2y \geq 0, 2x-y \leq -2, x \geq 0, y \geq 0$

Soln: Let $x-2y=0$

x	0	2
y	0	1

Let $2x-y=-2$

x	0	-1
y	2	0



There is no common region and has no Solution.

7) $2x+y \geq 8, x+2y \geq 8, x+y \leq 6$

Soln:

Let $2x+y=8$

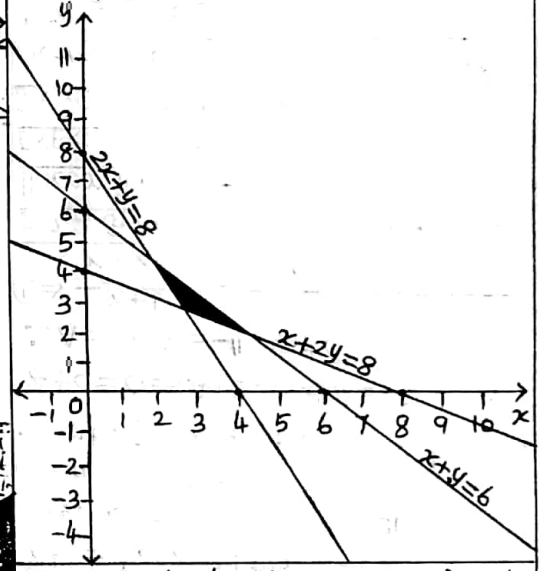
x	0	4
y	8	0

Let $x+2y=8$

x	0	8
y	4	0

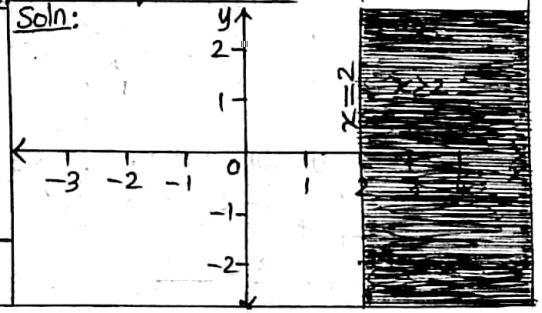
Let $x+y=6$

x	0	6
y	6	0



Ex 2.28 Shade the region given by the inequality $x \geq 2$

Soln:

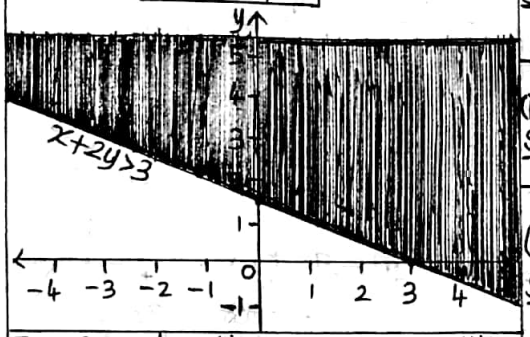


Ex 2.29 Shade the region given by the linear inequality $x+2y>3$

Soln:

Let $x+2y=3$

x	0	3
y	1.5	0



Ex 2.30 Solve the linear inequalities and exhibit the Solution set graphically $x+y\geq 3, 2x-y\leq 5, -x+2y\leq 3$

Soln:

Let $x+y=3$

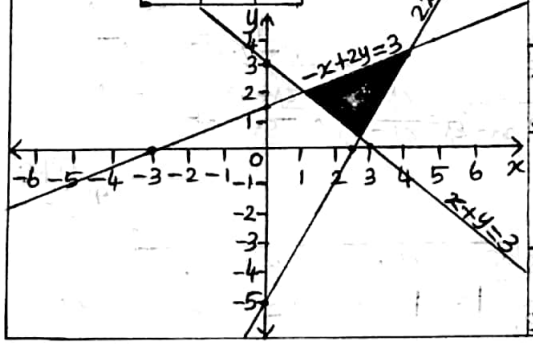
x	0	3
y	3	0

Let $2x-y=5$

x	0	2.5
y	-5	0

Let $-x+2y=3$

x	0	-3
y	1.5	0



2.8 Exponents and Radicals

EXERCISE 2.11

1) Simplify:

(i) $(125)^{\frac{2}{3}}$

Soln: $(125)^{\frac{2}{3}}=(5^3)^{\frac{2}{3}}=(5)^2=25$

(ii) $16^{\frac{-3}{4}}$

Soln: $(16)^{\frac{-3}{4}}=(2^4)^{\frac{-3}{4}}=(2)^{-3}=\frac{1}{2^3}=\frac{1}{8}$

(iii) $(-1000)^{\frac{-2}{3}}$

Soln: $(-1000)^{\frac{-2}{3}}=(-10^3)^{\frac{-2}{3}}=(-10)^{-2}=\frac{1}{(-10)^2}=\frac{1}{100}$

(iv) $(3^{-6})^{\frac{1}{2}}$

Soln: $(3^{-6})^{\frac{1}{2}}=(3^{-2})=\frac{1}{3^2}=\frac{1}{9}$

(v) $\frac{27^{\frac{-2}{3}}}{27^{\frac{-1}{3}}}$

Soln: $\frac{27^{\frac{-2}{3}}}{27^{\frac{-1}{3}}}=\frac{1}{27^{\frac{2}{3}}\cdot 27^{\frac{-1}{3}}}=\frac{1}{27^{\frac{1}{3}}}$

$=\frac{1}{(3^3)^{\frac{1}{3}}}=\frac{1}{3}$

2) Evaluate: $\left(\left((256)^{\frac{-1}{2}}\right)^{\frac{-1}{4}}\right)^3$

Soln:

$\left(\left((256)^{\frac{-1}{2}}\right)^{\frac{-1}{4}}\right)^3=\left((256)^{\frac{1}{8}}\right)^3=\left((2^8)^{\frac{1}{8}}\right)^3$

$=(2)^3=8$

3) If $(x^{\frac{1}{2}}+x^{\frac{-1}{2}})^2=\frac{9}{2}$, then find the value of $(x^{\frac{1}{2}}-x^{\frac{-1}{2}})$ for $x>1$

Soln: Given: $(x^{\frac{1}{2}}+x^{\frac{-1}{2}})^2=\frac{9}{2}$

$(x^{\frac{1}{2}})^2+(x^{\frac{-1}{2}})^2+2x^{\frac{1}{2}}x^{\frac{-1}{2}}=\frac{9}{2}$

$x+\frac{1}{x}+2=\frac{9}{2}\Rightarrow x+\frac{1}{x}=\frac{9}{2}-2\Rightarrow x+\frac{1}{x}=\frac{5}{2}$

consider,

$(x^{\frac{1}{2}}-x^{\frac{-1}{2}})^2=(x^{\frac{1}{2}})^2+(x^{\frac{-1}{2}})^2-2x^{\frac{1}{2}}x^{\frac{-1}{2}}$

$=x+\frac{1}{x}-2=\frac{5}{2}-2=\frac{1}{2}$

$(x^{\frac{1}{2}}-x^{\frac{-1}{2}})^2=\frac{1}{2}\Rightarrow x^{\frac{1}{2}}-x^{\frac{-1}{2}}=\pm\frac{1}{\sqrt{2}}$

Since $x>1$, $x^{\frac{1}{2}}-x^{\frac{-1}{2}}=\frac{1}{\sqrt{2}}$

4) Simplify and hence find the value of n: $\frac{3^{2n}9^2 3^{-n}}{3^{3n}}=27$

Soln: Given: $\frac{3^{2n}9^2 3^{-n}}{3^{3n}}=27$

$3^{2n}(3^2)^2 3^{-n}3^{-3n}=3^3\Rightarrow 3^{2n-n-3n+4}=3^3$

$3^{4-2n}=3^3\Rightarrow 4-2n=3\Rightarrow 2n=4-3\Rightarrow 2n=1$

$\therefore n=\frac{1}{2}$

5) Find the radius of the spherical tank whose Volume is $32\pi/3$ units.

Soln: Volume $=\frac{4}{3}\pi r^3=\frac{32\pi}{3}\Rightarrow r^3=8$

$\Rightarrow r^3=2^3\Rightarrow r=2$

$\therefore$ The radius of the spherical tank $=2$ units

6) Simplify by rationalising the denominator $\frac{7+\sqrt{6}}{3-\sqrt{2}}$

Soln:

$\frac{7+\sqrt{6}}{3-\sqrt{2}}=\frac{7+\sqrt{6}}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}}$

$=\frac{21+7\sqrt{2}+3\sqrt{6}+\sqrt{12}}{9-2}$

$=\frac{21+7\sqrt{2}+3\sqrt{6}+2\sqrt{3}}{7}$

7) Simplify:

$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$

Soln:

$=\left[\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}\right]-\left[\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}\right]+\left[\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}\right]$

$-\left[\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right]+\left[\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}\right]$

$=\left[\frac{3+\sqrt{8}}{9-8}\right]-\left[\frac{\sqrt{8}+\sqrt{7}}{8-7}\right]+\left[\frac{\sqrt{7}+\sqrt{6}}{7-6}\right]-\left[\frac{\sqrt{6}+\sqrt{5}}{6-5}\right]+\left[\frac{\sqrt{5}+2}{5-4}\right]$

$=\left[\frac{3+\sqrt{8}}{1}\right]-\left[\frac{\sqrt{8}+\sqrt{7}}{1}\right]+\left[\frac{\sqrt{7}+\sqrt{6}}{1}\right]-\left[\frac{\sqrt{6}+\sqrt{5}}{1}\right]+\left[\frac{\sqrt{5}+2}{1}\right]$

$=3+\cancel{\sqrt{8}}-\cancel{\sqrt{8}}-\cancel{\sqrt{7}}+\cancel{\sqrt{7}}+\cancel{\sqrt{6}}-\cancel{\sqrt{6}}-\cancel{\sqrt{5}}+\cancel{\sqrt{5}}+2$

$=3+2=5$

8) If $x=\sqrt{2}+\sqrt{3}$ Find $\frac{x^2+1}{x^2-2}$

Soln:

$\frac{x^2+1}{x^2-2}=\frac{(\sqrt{2}+\sqrt{3})^2+1}{(\sqrt{2}+\sqrt{3})^2-2}=\frac{2+3+2\sqrt{6}+1}{2+3+2\sqrt{6}-2}$

$=\frac{6+2\sqrt{6}}{3+2\sqrt{6}}\times\frac{3-2\sqrt{6}}{3-2\sqrt{6}}=\frac{18-12\sqrt{6}+6\sqrt{6}-24}{9-24}$

$=\frac{-6-6\sqrt{6}}{-15}=\frac{+3(2+2\sqrt{6})}{+15\,5}=\frac{2+2\sqrt{6}}{5}$

Ex 2.31 (i) Simplify: $(x^{\frac{1}{2}}y^{-3})^{\frac{1}{2}}$, where $x,y\geq 0$

(ii) Simplify: $\sqrt{x^2-10x+25}$

Soln: (i) since $x,y\geq 0$

$(x^{\frac{1}{2}}y^{-3})^{\frac{1}{2}}=\left(\frac{x^{\frac{1}{2}}}{y^3}\right)^{\frac{1}{2}}=\frac{x^{\frac{1}{4}}}{y^{\frac{3}{2}}}$

(ii) $\sqrt{x^2-10x+25}=\sqrt{(x-5)^2}=|x-5|$

EX 2.32 Rationalize the denominator of $\frac{\sqrt{5}}{\sqrt{6}+\sqrt{2}}$

Soln:

$$\frac{\sqrt{5}}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{5}}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}=\frac{\sqrt{30}-\sqrt{10}}{6-2}$$

$$\frac{\sqrt{5}}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{30}-\sqrt{10}}{4}$$

EX 2.33 Find the Square root of $7-4\sqrt{3}$

Soln: Let $\sqrt{7-4\sqrt{3}}=a+b\sqrt{3}$

Squaring on both Sides,

$7-4\sqrt{3}=(a+b\sqrt{3})^2$

$7-4\sqrt{3}=a^2+3b^2+2ab\sqrt{3}$

$\therefore a^2+3b^2=7$ and $2ab=-4$

$ab=-2$

$a=\frac{-2}{b}$ —①

$a^2+3b^2=7$

$(\frac{-2}{b})^2+3b^2=7\Rightarrow\frac{4}{b^2}+3b^2=7$

$\Rightarrow\frac{4+3b^4}{b^2}=7\Rightarrow 4+3b^4=7b^2$

$\Rightarrow 3b^4-7b^2+4=0$

$a=3, b=-7, c=4$

$b^2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-7)\pm\sqrt{(-7)^2-4(3)(4)}}{2(3)}$

$=\frac{7\pm\sqrt{49-48}}{6}=\frac{7\pm\sqrt{1}}{6}=\frac{7\pm1}{6}$

$b^2=\frac{7\pm1}{6}\Rightarrow b^2=\frac{7-1}{6}$ and $b^2=\frac{7+1}{6}$

$b^2=\frac{6}{6}$ and $b^2=\frac{8}{6}\Rightarrow b^2=1$ and $b^2=\frac{4}{3}$

$b=\pm1$ and $b=\pm\frac{2}{\sqrt{3}}$

Since b is rational, $\therefore b=\pm1$

when $b=1$, ①$\Rightarrow a=\frac{-2}{1}=-2$

when $b=-1$, ①$\Rightarrow a=\frac{+2}{+1}=2$

Since $\sqrt{7-4\sqrt{3}}>0$, $a=2$ and $b=-1$

$\sqrt{7-4\sqrt{3}}=2-\sqrt{3}$

2.9 Logarithm

EXERCISE 2.12

1) Let $b>0$ and $b\neq1$ Express $y=b^x$ in logarithmic form. Also state the domain and range of the logarithmic functions.

Soln: Given: $y=b^x$

$\log_b y=\log_b b^x=x\log_b b=x(1)=x$

$\log_b y=x$

$\therefore$ Domain: $x\in(0,\infty)$

Range: $y\in(-\infty,\infty)$

2) Compute $\log_9 27-\log_{27}9$

Soln: $\log_9 27-\log_{27}9=\log_9 3^3-\log_{27}3^2$

$=3\log_9 3-2\log_{27}3=3\frac{1}{\log_3 9}-2\frac{1}{\log_3 27}$

$=\frac{3}{\log_3 3^2}-\frac{2}{\log_3 3^3}=\frac{3}{2\log_3 3}-\frac{2}{3\log_3 3}$

$=\frac{3}{2(1)}-\frac{2}{3(1)}=\frac{3}{2}-\frac{2}{3}=\frac{9-4}{6}=\frac{5}{6}$

3) Solve $\log_8 x+\log_4 x+\log_2 x=11$

Soln: $\log_8 x+\log_4 x+\log_2 x=11$

$\frac{1}{\log_x 8}+\frac{1}{\log_x 4}+\frac{1}{\log_x 2}=11$

$\frac{1}{\log_x 2^3}+\frac{1}{\log_x 2^2}+\frac{1}{\log_x 2}=11$

$\frac{1}{3\log_x 2}+\frac{1}{2\log_x 2}+\frac{1}{\log_x 2}=11$

$\frac{1}{\log_x 2}\left[\frac{1}{3}+\frac{1}{2}+1\right]=11$

$\frac{1}{\log_x 2}\left[\frac{2+3+6}{6}\right]=11\Rightarrow\frac{1}{\log_x 2}\left[\frac{11}{6}\right]=11$

$\Rightarrow\log_2 x=6\Rightarrow x=2^6\Rightarrow x=64$

4) Solve $\log_4 2^{8x}=2^{\log_2 8}$

Soln: Using property, $a^{\log_a x}=x$

Given: $\log_4 2^{8x}=2^{\log_2 8}$

$8x\log_4 2=8^1\Rightarrow x=\frac{1}{\log_4 2}\Rightarrow x=\log_2 4$

$\Rightarrow x=\log_2 2^2\Rightarrow x=2\log_2 2\Rightarrow x=2(1)\Rightarrow x=2$

5) If $a^2+b^2=7ab$ Show that $\log\frac{a+b}{3}=\frac{1}{2}(\log a+\log b)$

Soln: Given: $a^2+b^2=7ab$

$a^2+b^2+2ab=7ab+2ab$

$(a+b)^2=9ab\Rightarrow a+b=3\sqrt{ab}$

$\Rightarrow\frac{a+b}{3}=(ab)^{\frac{1}{2}}\Rightarrow\log(\frac{a+b}{3})=\log(ab)^{\frac{1}{2}}$

$\log(\frac{a+b}{3})=\frac{1}{2}\log(ab)$

$\log(\frac{a+b}{3})=\frac{1}{2}(\log a+\log b)$

6) Prove $\log\frac{a^2}{bc}+\log\frac{b^2}{ca}+\log\frac{c^2}{ab}=0$

Soln: $\log\frac{a^2}{bc}+\log\frac{b^2}{ca}+\log\frac{c^2}{ab}=\log\left[\frac{a^2}{bc}\cdot\frac{b^2}{ca}\cdot\frac{c^2}{ab}\right]$

$=\log\left[\frac{a^2b^2c^2}{a^2b^2c^2}\right]$

$=\log1=0$

$\log\frac{a^2}{bc}+\log\frac{b^2}{ca}+\log\frac{c^2}{ab}=0$

7) Prove that $\log2+16\log\frac{16}{15}+12\log\frac{25}{24}+7\log\frac{81}{80}=1$

Soln:

$\log2+16\log\frac{16}{15}+12\log\frac{25}{24}+7\log\frac{81}{80}$

$=\log2+\log(\frac{16}{15})^{16}+\log(\frac{25}{24})^{12}+\log(\frac{81}{80})^7$

$=\log2+\log(\frac{2^4}{3\times5})^{16}+\log(\frac{5^2}{3\times2^3})^{12}+\log(\frac{3^4}{5\times2^4})^7$

$=\log2+\log\left[(\frac{2^4}{3\times5})^{16}\cdot(\frac{5^2}{3\times2^3})^{12}\cdot(\frac{3^4}{5\times2^4})^7\right]$

$=\log2+\log\left[\frac{2^{64}\cdot5^{24}\cdot3^{28}}{3^{16}\cdot5^{16}\cdot3^{12}\cdot2^{36}\cdot5^7\cdot2^{28}}\right]$

$=\log2+\log\left[\frac{2^{64}\cdot5^{24}\cdot3^{28}}{2^{64}\cdot5^{23}\cdot3^{28}}\right]=\log2+\log5$

$=\log(2\cdot5)=\log10=1$

$\log2+16\log\frac{16}{15}+12\log\frac{25}{24}+7\log\frac{81}{80}=1$

8) Prove $\log_a a\cdot\log_b b^2\cdot\log_c c^2=\frac{1}{8}$

Soln:

$\log_a a\cdot\log_b b^2\cdot\log_c c^2=\frac{1}{\log_a a^2}\cdot\frac{1}{\log_b b^2}\cdot\frac{1}{\log_c c^2}$

$=\frac{1}{2\log_a a}\cdot\frac{1}{2\log_b b}\cdot\frac{1}{2\log_c c}=\frac{1}{2(1)}\cdot\frac{1}{2(1)}\cdot\frac{1}{2(1)}$

$=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$

$\log_a a\cdot\log_b b^2\cdot\log_c c^2=\frac{1}{8}$

9) Prove $\log a+\log a^2+\log a^3+\cdots+\log a^n=\frac{n(n+1)}{2}\log a$

Soln: $\log a+\log a^2+\log a^3+\cdots+\log a^n$

$=\log a+2\log a+3\log a+\cdots+n\log a$

$=[1+2+3+\cdots+n]\log a$

$=\frac{n(n+1)}{2}\log a$

10) If $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}$ then prove that $xyz=1$

Soln: Let $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}=k$

$\log x=k(y-z)$, $\log y=k(z-x)$, $\log z=k(x-y)$

$\log x+\log y+\log z=k(y-z)+k(z-x)+k(x-y)$

$\log(xyz)=k(y-z+z-x+x-y)$

$\log(xyz)=k(0)=0 \Rightarrow \log(xyz)=\log 1$

$\Rightarrow xyz=1$

11) Solve $\log_2 x-3\log_{\frac{1}{2}} x=6$

Soln: Given: $\log_2 x-3\log_{\frac{1}{2}} x=6$

$\log_2 x-3\frac{1}{\log_x \frac{1}{2}}=6 \Rightarrow \log_2 x-\frac{3}{\log_x 2^{-1}}=6$

$\log_2 x-\frac{3}{-\log_x 2}=6 \Rightarrow \log_2 x+\frac{3}{\log_x 2}=6$

$\log_2 x+3\log_2 x=6 \Rightarrow 4\log_2 x=6$

$\log_2 x=\frac{3}{2} \Rightarrow x=2^{\frac{3}{2}} \Rightarrow x=2\sqrt{2}$

12) Solve $\log_{5-x}(x^2-6x+65)=2$

Soln: Given: $\log_{5-x}(x^2-6x+65)=2$

$x^2-6x+65=(5-x)^2$

$x^2-6x+65=25+x^2-10x$

$10x-6x=25-65 \Rightarrow 4x=-40$

$\therefore x=-10$

EX 2.34 Find the logarithm of 1728 to the base $2\sqrt{3}$

Soln: $\log_{2\sqrt{3}} 1728=\log_{(2\sqrt{3})}(2^6 3^3)=\log_{2\sqrt{3}}(2^6(\sqrt{3})^6)$

$=\log_{2\sqrt{3}}(2\sqrt{3})^6=6\log_{2\sqrt{3}}(2\sqrt{3})=6(1)=6 \Rightarrow \log_{2\sqrt{3}} 1728=6$

EX 2.35 If the logarithm of 324 to base a is 4 then find a

Soln: $\log_a 324=4 \Rightarrow a^4=324$

$a^4=3^4 2^2 \Rightarrow a^4=3^4(\sqrt{2})^4 \Rightarrow a^4=(3\sqrt{2})^4$

$\Rightarrow a=3\sqrt{2}$

EX 2.36 Prove $\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}=\log 2$

Soln:

$\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}$

$=\log\frac{75}{16}+\log\frac{32}{243}-\log\left(\frac{5}{9}\right)^2$

$=\log\left(\frac{75}{16}\times\frac{32}{243}\right)-\log\frac{25}{81}=\log\frac{50}{81}-\log\frac{25}{81}$

$=\log\left(\frac{\frac{50}{81}}{\frac{25}{81}}\right)=\log\left(\frac{50}{25}\right)=\log 2$

$\therefore \log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}=\log 2$

EX 2.37 If $\log_2 x+\log_4 x+\log_{16} x=\frac{7}{2}$ Find the value of x.

Soln: Given: $\log_2 x+\log_4 x+\log_{16} x=\frac{7}{2}$

$\frac{1}{\log_x 2}+\frac{1}{\log_x 4}+\frac{1}{\log_x 16}=\frac{7}{2}$

$\frac{1}{\log_x 2}+\frac{1}{\log_x 2^2}+\frac{1}{\log_x 2^4}=\frac{7}{2}$

$\frac{1}{\log_x 2}+\frac{1}{2\log_x 2}+\frac{1}{4\log_x 2}=\frac{7}{2}$

$\log_2 x+\frac{1}{2}\log_2 x+\frac{1}{4}\log_2 x=\frac{7}{2}$

$\log_2 x\left(1+\frac{1}{2}+\frac{1}{4}\right)=\frac{7}{2}$

$\log_2 x\left(\frac{4+2+1}{4}\right)=\frac{7}{2} \Rightarrow \log_2 x\left(\frac{7}{4}\right)=\frac{7}{2}$

$\log_2 x=\frac{14}{7} \Rightarrow \log_2 x=2 \Rightarrow x=2^2 \Rightarrow x=4$

EX 2.38 Solve $x^{\log_3 x}=9$

Soln: Given: $x^{\log_3 x}=9$

$\log_3 x^{\log_3 x}=\log_3 9 \Rightarrow \log_3 x\cdot\log_3 x=\log_3 3^2$

$(\log_3 x)^2=2\log_3 3 \Rightarrow (\log_3 x)^2=2(1) \Rightarrow (\log_3 x)^2=2$

$\log_3 x=\pm\sqrt{2} \Rightarrow \log_3 x=\sqrt{2}$ and $\log_3 x=-\sqrt{2}$

$x=3^{\sqrt{2}}$ and $x=3^{-\sqrt{2}}$

EX 2.39 Compute $\log_3 5\log_{25} 27$

Soln: Given: $\log_3 5\log_{25} 27=\log_3 5\cdot\log_{25} 3^3$

$=\log_3 5\cdot 3\log_{25} 3=3\log_{25} 3\cdot\log_3 5=3\log_{25} 5$

$=3\frac{1}{\log_5 25}=\frac{3}{\log_5 5^2}=\frac{3}{2\log_5 5}=\frac{3}{2(1)}=\frac{3}{2}$

[OR]

$\log_3 5\log_{25} 27=\frac{\log_a 5}{\log_a 3}\cdot\frac{\log_a 27}{\log_a 25}$

$=\frac{\log_a 5}{\log_a 3}\cdot\frac{\log_a 3^3}{\log_a 5^2}=\frac{\log_a 5}{\log_a 3}\cdot\frac{3\log_a 3}{2\log_a 5}=\frac{3}{2}$

EX 2.40 Given that $\log_{10} 2=0.30103$, $\log_{10} 3=0.47712$ (approximately) Find the number of digits in $2^8\cdot 3^{12}$.

Soln: Let $N=2^8\cdot 3^{12}$ has n+1 digits.

$\log_{10} N=\log_{10}(2^8\cdot 3^{12})=\log_{10} 2^8+\log_{10} 3^{12}$

$=8\log_{10} 2+12\log_{10} 3$

$=8(0.30103)+12(0.47712)$

$=2.40824+5.72544$

$\log_{10} N=8.13368$ —①

Let $N=10^n\times b$, $1\le b<10$

$\log_{10} N=\log_{10}(10^n\times b)$

$=\log_{10} 10^n+\log_{10} b$

$=n\log_{10} 10+\log_{10} b=n(1)+\log_{10} b$

$\log_{10} N=n+\log_{10} b$

① $\Rightarrow n+\log_{10} b=8.13368$, $1\le b<10$

$\therefore$ The number of digit is 9.

EXERCISE 2.13

Choose the correct or the most suitable answer.

1) If $|x+2|\le 9$ then x belongs to

Soln: Given: $|x+2|\le 9$

$-9\le x+2\le 9 \Rightarrow -9-2\le x+2-2\le 9-2$

$\Rightarrow -11\le x\le 7$, $\therefore x\in[-11,7]$

Ans: (2) $[-11,7]$

2) Given that x, y and b are real numbers $x<y$, $b>0$ then

Soln: Given: $x<y$, $b>0 \Rightarrow xb<yb$

Ans: (1) $xb<yb$

3) If $\frac{|x-2|}{x-2}\ge 0$ then x belongs to

Soln: Given: $\frac{|x-2|}{x-2}\ge 0$, but $x-2>0$

$\Rightarrow x>2 \Rightarrow x\in(2,\infty)$

Ans: (2) $(2,\infty)$

4) The Solution of $5x-1<24$ and $5x+1>-24$ is

Soln: Given: $5x-1<24$ and $5x+1>-24$

$\Rightarrow 5x<25$ and $5x>-25$

$\Rightarrow x<5$ and $x>-5 \Rightarrow -5<x<5$

$\therefore x\in(-5,5)$, Ans: (3) $(-5,5)$

5) The Solution set of the following inequality $|x-1|\ge|x-3|$ is

Soln: Given: $|x-1| \geq |x-3|$

$(x-1) \geq (x-3)$ and $(x-1) \geq -(x-3)$

$-1 \geq -3$ and $x-1 \geq -x+3 \Rightarrow 2x \geq 4$

$\Rightarrow x \geq 2$

$\therefore x \in [2, \infty)$, Ans: (2) $[2, \infty)$

6) The value of $\log_{\frac{1}{\sqrt{2}}} 512$ is

Soln: $\log_{\frac{1}{\sqrt{2}}} 512 = \log_{\frac{1}{\sqrt{2}}} 2^9 = \log_{\frac{1}{\sqrt{2}}} ((\sqrt{2})^2)^9$

$= \log_{\frac{1}{\sqrt{2}}} (\sqrt{2})^{18} = 18 \log_{\frac{1}{\sqrt{2}}} \sqrt{2}$

$= 18(1) = 18$

Ans: (2) 18

7) The value of $\log_{3^{\frac{1}{4}}} 81$ is

Soln: $\log_{3^{\frac{1}{4}}} 81 = \log_{(3^4)^{-1}} 81 = -\log_{3^4} 81$

$= -\log_{3^4} 3^4 = -4 \log_{3^4} 3$

$= -4(1) = -4$

Ans: (3) -4

8) If $\log_{\sqrt{x}} 0.25 = 4$ then the value of x

Soln: Given: $\log_{\sqrt{x}} 0.25 = 4$

$(\sqrt{x})^4 = 0.25 \Rightarrow x^2 = 0.25$

$\Rightarrow x = 0.5$, Ans: (1) 0.5

9) The value of $\log_a b \log_b c \log_c a$ is

Soln:

$\log_a b \log_b c \log_c a = \frac{\log b}{\log a} \cdot \frac{\log c}{\log b} \cdot \frac{\log a}{\log c}$

$= 1$

Ans: (2) 1

10) If 3 is the logarithm of 343 then the base is

Soln: $\log_x 343 = 3 \Rightarrow x^3 = 343 \Rightarrow x^3 = 7^3$

$\Rightarrow x = 7$, Ans: (2) 7

11) Find a so that the sum and Product of the roots of the equation $2x^2 + (a-3)x + 3a - 5 = 0$ are equal to

Soln: S.R $= \frac{-(a-3)}{2}$, P.R $= \frac{3a-5}{2}$

Given: S.R = P.R $\Rightarrow \frac{-(a-3)}{2} = \frac{3a-5}{2}$

$\Rightarrow -a + 3 = 3a - 5 \Rightarrow 4a = 8 \Rightarrow a = 2$

Ans: (2) 2

12) If a and b are the roots of the equation $x^2 - kx + 16 = 0$ and satisfy $a^2 + b^2 = 32$, then the value of k is

Soln: Given: $x^2 - kx + 16 = 0$ and $a^2 + b^2 = 32$

S.R $= a + b = \frac{-(-k)}{1} = k \Rightarrow a + b = k$

P.R $= ab = \frac{16}{1} = 16 \Rightarrow ab = 16$

$(a+b)^2 = a^2 + b^2 + 2ab \Rightarrow k^2 = 32 + 2(16)$

$\Rightarrow k^2 = 32 + 32 \Rightarrow k^2 = 64 \Rightarrow k = \pm 8$

$\Rightarrow k = -8, 8$, Ans: (3) $-8, 8$

13) The number of Solutions of $x^2 + |x-1| = 1$ is

Soln: Given: $x^2 + |x-1| = 1$

$x^2 + (x-1) = 1$ and $x^2 + (-(x-1)) = 1$,

$x^2 + x - 2 = 0$ and $x^2 - x + 1 = 1 \Rightarrow x^2 - x = 0$

$(x+2)(x-1) = 0$ and $x(x-1) = 0$

$x + 2 = 0, x - 1 = 0$ and $x = 0, x - 1 = 0$

$x = -2, x = 1$ and $x = 0, x = 1$

$\therefore x = -2, 0, 1$, $x = -2$ is not satisfy the given equation.

$\therefore$ The roots are $x = 0$ and $x = 1$

$\therefore$ The number of Solution is 2

Ans: (3) 2

14) The equation whose roots are numerically equal but opposite in Sign to the roots of $3x^2 - 5x - 7 = 0$ is

Soln: Let α, β are the roots

S.R $= \alpha + \beta = \frac{-(-5)}{3} \Rightarrow \alpha + \beta = \frac{5}{3}$

P.R $= \alpha\beta = -\frac{7}{3}$, Now take the roots are $-\alpha, -\beta$

S.R $= (-\alpha) + (-\beta) = -(\alpha + \beta) = -\frac{5}{3}$

P.R $= (-\alpha)(-\beta) = \alpha\beta = -\frac{7}{3}$

$\therefore x^2 - (S.R)x + P.R = 0 \Rightarrow x^2 - (\frac{-5}{3})x - \frac{7}{3} = 0$

$\times$ by 3, $3x^2 + 5x - 7 = 0$

Ans: (2) $3x^2 + 5x - 7 = 0$

15) If 8 and 2 are the roots of $x^2 + ax + c = 0$ and 3, 3 are the roots of $x^2 + dx + b = 0$ then the roots of the equation $x^2 + ax + b = 0$ are

Soln: Given: $x^2 + ax + c = 0$

S.R $= \frac{-a}{1} = 8 + 2 \Rightarrow -a = 10 \Rightarrow a = -10$

and Given: $x^2 + dx + b = 0$

P.R $= \frac{b}{1} = (3)(3) \Rightarrow b = 9$

Also, Given: $x^2 + ax + b = 0 \Rightarrow x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0 \Rightarrow x - 9 = 0, x - 1 = 0 \Rightarrow x = 9, x = 1$

Ans: (3) 9, 1

16) If a and b are the real roots of the equation $x^2 - kx + c = 0$ then the distance between the Points (a, 0) and (b, 0) is

Soln: Given: $x^2 - kx + c = 0$

S.R $= a + b = \frac{-(-k)}{1} \Rightarrow a + b = k$

P.R $= ab = \frac{c}{1} \Rightarrow ab = c$

distance $= \sqrt{(b-a)^2 + (0-0)^2} = \sqrt{(-(a-b))^2} = \sqrt{(a-b)^2}$

$= \sqrt{(a+b)^2 - 4ab} = \sqrt{k^2 - 4c}$, Ans: (1) $\sqrt{k^2 - 4c}$

17) If $\frac{kx}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$ then the value of K is

Soln: $\frac{kx}{(x+2)(x-1)} = \frac{2(x-1) + (x+2)}{(x+2)(x-1)} \Rightarrow kx = 2x - 2 + x + 2$

$kx = 3x \Rightarrow k = 3$, Ans: (3) 3

18) If $\frac{1-2x}{3+2x-x^2} = \frac{A}{3-x} + \frac{B}{x+1}$ then the value of A+B is

Soln: $\frac{1-2x}{3+2x-x^2} = \frac{A(x+1) + B(3-x)}{(3-x)(x+1)}$

$1 - 2x = A(x+1) + B(3-x)$

when $x = 3$, $4A = -5 \Rightarrow A = -\frac{5}{4}$

when $x = -1$, $4B = 3 \Rightarrow B = \frac{3}{4}$

$A + B = -\frac{5}{4} + \frac{3}{4} = -\frac{2}{4} \Rightarrow A + B = -\frac{1}{2}$

Ans: (1) $-\frac{1}{2}$

19) The number of roots of $(x+3)^4 + (x+5)^4 = 16$ is

Soln: Degree of $(x+3)^4$ is 4 and Degree of $(x+5)^4$ is 4

$\therefore$ The number of roots is 4

Ans: (1) 4

20) The value of $\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81$ is

Soln: $\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81$

$= \frac{\log 11}{\log 3} \cdot \frac{\log 13}{\log 11} \cdot \frac{\log 15}{\log 13} \cdot \frac{\log 27}{\log 15} \cdot \frac{\log 81}{\log 27} = \frac{\log 81}{\log 3}$

$= \log_3 81 = \log_3 3^4 = 4 \log_3 3 = 4(1) = 4$

Ans: (4) 4

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