The following must be completed by the student and submitted for marking by Wednesday, 12 April 2023 by midnight.

1. Optimizing the Ackley function

For $x \in \mathbb{R}^d$, the Ackley function is

$$f(x) = -20 \exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_i)\right) + 20 + \mathrm{e}.$$

Let the search domain be the hypercube $\mathcal{D} = [-32, 32]^d$. Compare the performance of the three basic search methods—SRS, LRS, ELRS—on this function in dimensions $d = 2, 4, 6$. Don't forget to tune the parameters in LRS and ELRS to try to improve their performance.

2. Modelling the Nile River

In the R `datasets` package, there is the `Nile` dataset, which tracks the annual flow of the river Nile over 100 years. Use the Robbins-Monro algorithm to model the flow using a linear combination of radial basis functions. That is, for $x \in [1, 100]$, let

$$f_{\theta,\sigma}(x) = \sum_{i=1}^{k} \theta_i \exp\left(-\sigma_i \left| x - \frac{100i}{k+1} \right|\right).$$

The parameters to optimize over are $\theta_1, \ldots, \theta_k$ and $\sigma_1, \ldots, \sigma_k$. Try optimizing with two different loss functions: the squared error and the absolute error,

$$L_2(\theta, \sigma) = \frac{1}{2n}\sum_{j=1}^{100} \left(y_j - f_{\theta,\sigma}(x_j)\right)^2 \quad \text{and} \quad L_1(\theta, \sigma) = \frac{1}{n}\sum_{j=1}^{100} |y_j - f_{\theta,\sigma}(x_j)|,$$

where $x_j = j$ is the jth year and y_j is the jth flow measurement. You can pick the number of basis functions k as you like, but note that there will be $2k$ parameters to optimize over.

Note that if you don't want to use R, the Nile dataset is just a vector of the following numbers: 1120, 1160, 963, 1210, 1160, 1160, 813, 1230, 1370, 1140, 995, 935, 1110, 994, 1020, 960, 1180, 799, 958, 1140, 1100, 1210, 1150, 1250, 1260, 1220, 1030, 1100, 774, 840, 874, 694, 940, 833, 701, 916, 692, 1020, 1050, 969, 831, 726, 456, 824, 702, 1120, 1100, 832, 764, 821, 768, 845, 864, 862, 698, 845, 744, 796, 1040, 759, 781, 865, 845, 944, 984, 897, 822, 1010, 771, 676, 649, 846, 812, 742, 801, 1040, 860, 874, 848, 890, 744, 749, 838, 1050, 918, 986, 797, 923, 975, 815, 1020, 906, 901, 1170, 912, 746, 919, 718, 714, 740