

## BACKTRACKING

- It is one of the problem solving / algorithm design strategy.
  - Backtracking is an algorithm technique to solve a problem by an incremental way..
  - It was a recursive approach to solve the problem.
  - We can say that backtracking is used to find all the possible combinations to solve an optimization problem.
- In backtracking, the problem can be categorized into 3 categories
- 1) decision problem:- Here we find whether there is any feasible solution.
  - 2) Optimization problem:- Here we find whether there exists any best solution.
  - 3) Enumeration problem:- Here we find all possible feasible solutions.

It incrementally builds candidates to the solutions, and abandons a candidate ("backtracks") as soon as it determines that the candidate cannot possibly be completed to a valid solution.

## Applications of backtracking

- 1) N-Queens problems.
- 2) Graph colouring.
- 3) 0/1 Knapsack problem
- 4) Hamiltonian cycles.
- 5) Sum of subsets.

## Working of backtracking with an example

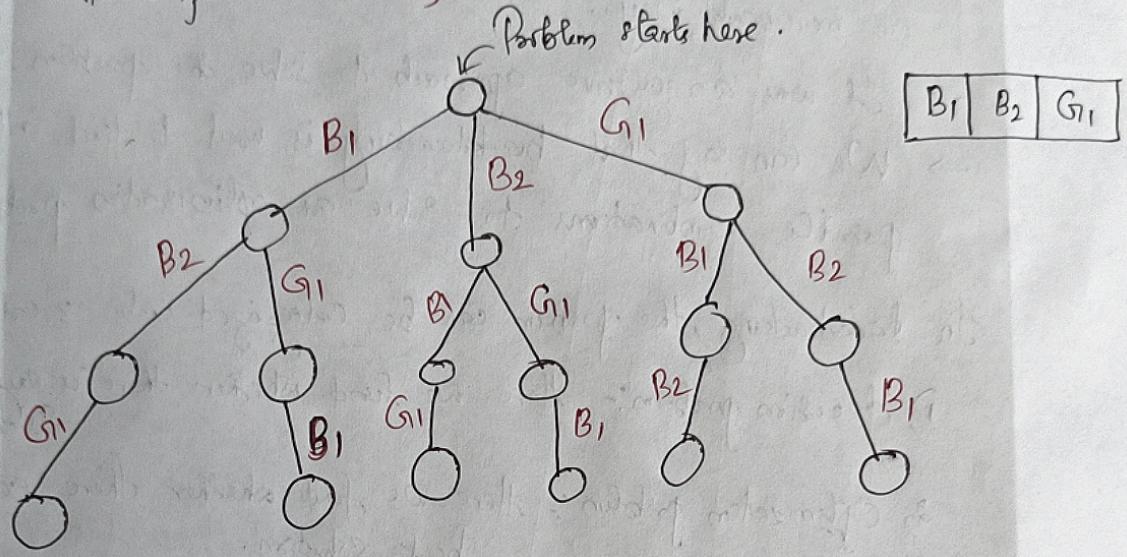
3 students → 2 boys.  
& → 1 girl  
3 chairs are there.

\* We have to arrange them in these chairs.

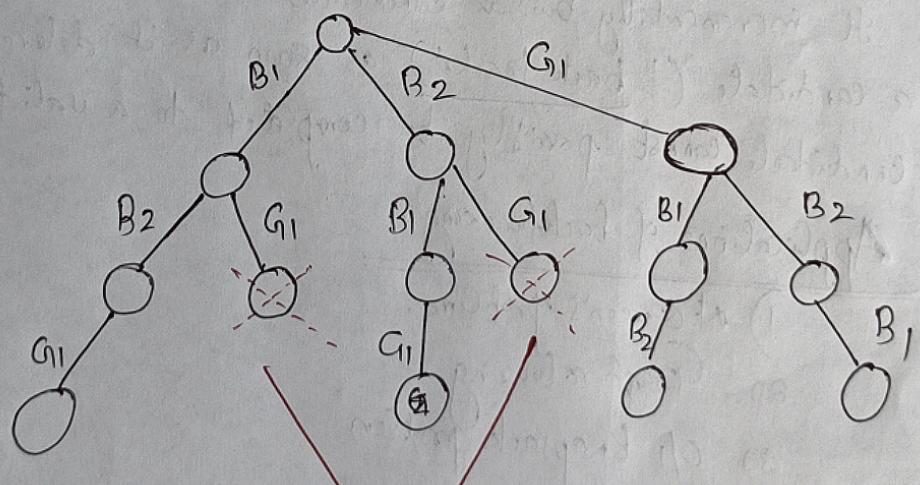
Prob :- How many ways we can arrange them

$$\text{Soln} :- 3! \text{ ways} = 3 \times 2 \times 1 = \underline{\underline{6 \text{ ways}}}$$

We can represent the solution in the form of State Space tree.  
 { A state space tree is the tree of the construction of the solution from partial solution, starting with the root with ~~root with~~ with no component solution }



Next, we are applying a constraint that the girl should not be in between the boys.



Bounding function.  $\rightarrow$  is needed to kill some live nodes without actually expanding them.

## N-Queens Problem

N-Queens problem is to place n-queens in such a manner or an  $N \times N$  chessboard that no queens attack each other by being in the same row, column or diagonal. It can be seen that for  $n=1$ , the problem has trivial solution and solution exists for  $n=2$  and  $n=3$ .

So first we will consider the 4 queens problem.

Given a  $4 \times 4$  chessboard and number the rows and columns of the chessboard 1 through 4.

	1	2	3	4
1	•			
2				
3				
4				

$4 \times 4$  chessboard.

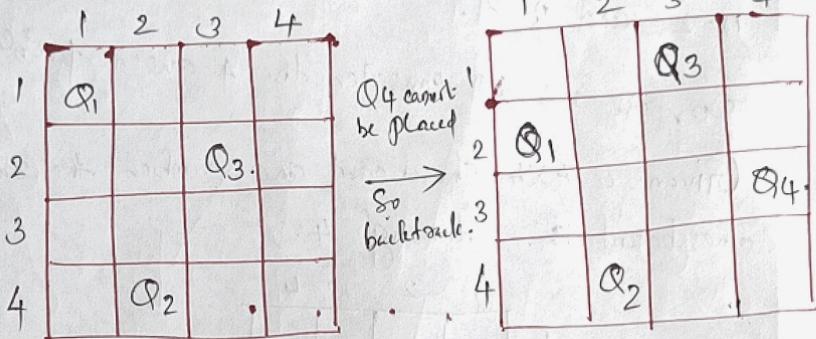
Since we have to place 4 queens such as  $Q_1, Q_2, Q_3$  and  $Q_4$  on the chessboard, such that no two queens attack each other. In such a condition each queen must be placed on a different row. i.e., we put queen "1" on row 1.

Now, we place queen  $Q_1$  in the very first acceptable position (1,1). (Whenever we place a ~~red~~ queen, increment the column and find a position for the next queen). Next we put queen  $Q_2$  so that both these queens do not attack each other.

	1	2	3	4
1	Q <sub>1</sub>			
2				
3		Q <sub>2</sub>		
4				

We find that if we place  $Q_2$  in column 2 & row 3, ~~dead end~~ is encountered. The first acceptable position is for  $Q_2$ . Then increment the column and when we try to place  $Q_3$ , then the dead end is encountered. So we

backtrack one step and replace the last placed queen  $Q_2$ . (by incrementing row). Then we obtain the position for  $Q_2$  in the 4<sup>th</sup> row and now we can place  $Q_3$  in 3<sup>rd</sup> column and row. These steps are continued until all the queens are placed in the safe position.



### Algorithm

- 1) Start in the leftmost column
- 2) If all queen are placed  
return true.
- 3) Try all row in the current column  
Do following for every tried row.
  - a) If the queen can be placed safely in this row  
then mark this [row, column] as part of the solution &  
recursively check if placing queen here leads to a solution.
  - b) If placing the queen in [row, column] leads to a solution  
then return true.
  - c) If placing queen doesn't lead to a solution then unmark  
this [row, column] (Backtrack) and go to step (a) to try  
other rows.
- 4) If all rows have been tried and nothing worked,  
return false to trigger backtracking.