Marshall Grimmett - Exam 3

a) $$\log L = \sum_{i=1}^{n} \left[Y_i \log\left(\frac{\pi_i}{1-\pi_i}\right) + \log(1-\pi_i) \right]$$

$$1-\pi_i = 1 - \frac{e^{\beta_0}}{1+e^{\beta_0}} = \frac{1+\cancel{e^{\beta_0}} - \cancel{e^{\beta_0}}}{1+e^{\beta_0}}$$

$$= \frac{1}{1+e^{\beta_0}}$$

$$\frac{\pi_i}{1-\pi_i} = \left(\frac{e^{\beta_0}}{\cancel{1+e^{\beta_0}}}\right)\left(\cancel{1+e^{\beta_0}}\right) = e^{\beta_0}$$

$$\log L = \sum_{i=1}^{n} \left[Y_i \log(e^{\beta_0}) + \log\left(\frac{1}{1+e^{\beta_0}}\right) \right]$$

$$\log L = \sum_{i=1}^{n} \left[Y_i \beta_0 - \log(1+e^{\beta_0}) \right]$$

b) $$\frac{\partial \ln L}{\partial \pi_i} = \sum_{i=1}^{n}\left(\frac{y_i}{\pi_i} - \frac{(1-y_i)}{(1-\pi_i)}\right)$$

$$= \sum_{i=1}^{n}\left(\frac{y_i(1+e^{\beta_0})}{e^{\beta_0}} - (1-y_i)(1+e^{\beta_0})\right)$$

The vector derivative will be

$$\pi = \frac{e^{\beta_0}}{1+e^{\beta_0}}$$