

Marshall Grinnett - Exam 3

a) $\log L = \sum_{i=1}^n \left[y_i \log\left(\frac{\pi_i}{1-\pi_i}\right) + \log(1-\pi_i) \right]$

$$\frac{1-\pi_i}{1-\pi_i} = 1 - \frac{e^{\beta_0}}{1+e^{\beta_0}} = \frac{1+e^{\beta_0} - e^{\beta_0}}{1+e^{\beta_0}} = \frac{1}{1+e^{\beta_0}}$$

$$\frac{\pi_i}{1-\pi_i} = \left(\frac{e^{\beta_0}}{1+e^{\beta_0}} \right) (1+e^{\beta_0}) = e^{\beta_0}$$

$$\log L = \sum_{i=1}^n \left[y_i \log(e^{\beta_0}) + \log\left(\frac{1}{1+e^{\beta_0}}\right) \right]$$

$$\log L = \sum_{i=1}^n \left[y_i \beta_0 - \log(1+e^{\beta_0}) \right]$$

b) ~~$$\frac{\partial \log L}{\partial \pi_i} = \sum_{i=1}^n \left(\frac{y_i - \pi_i}{\pi_i (1-\pi_i)} \right) = \sum_{i=1}^n \left(\frac{y_i - \frac{e^{\beta_0}}{1+e^{\beta_0}}}{\frac{e^{\beta_0}}{1+e^{\beta_0}} \left(\frac{1}{1+e^{\beta_0}} \right)} \right)$$~~

~~$$= \sum_{i=1}^n \left(\frac{y_i (1+e^{\beta_0}) - e^{\beta_0}}{(1+e^{\beta_0}) e^{\beta_0}} \right) = \sum_{i=1}^n \left(\frac{y_i (1+e^{\beta_0}) - e^{\beta_0}}{e^{\beta_0} (1+e^{\beta_0})} \right)$$~~

$$\begin{aligned}
 b) \quad \frac{2 \ln L}{2 \pi_i} &= \sum_{i=1}^n \left(\frac{y_i}{\pi_i} - \frac{(1-y_i)}{(1-\pi_i)} \right) \\
 &= \sum_{i=1}^n \left(\frac{y_i(1+e^{\beta_0})}{e^{\beta_0}} - (1-y_i)(1+e^{\beta_0}) \right)
 \end{aligned}$$

The vector derivative will be

$$\pi = \frac{e^{\beta_0}}{1+e^{\beta_0}}$$