ISLR | Chapter 7 Exercises

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Conceptual

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- **A**. The cubic piecewise polynomial:

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x-\xi)^3_+ \quad where \quad (x-\xi)^3_+ = \begin{cases} 0, & x \le \xi \\ (x-\xi)^3, & otherwise \end{cases}$$

  …can be broken up and rewritten to be:

$$f(x) = \begin{cases} f(x)_1 = a_1 + b_1 x + c_1 x^2 + d_1 x^3, & x \le \xi \\ f(x)_2 = a_2 + b_2 x + c_2 x^2 + d_2 x^3, & otherwise \end{cases}$$

  In $f(x)_1$, since $(x-\xi)^3_+ = 0$ (because $x \le \xi$), the fifth term (of $f(x)$) zeroes out and the coefficients can be expresses as $a_1 = \beta_0,\ b_1 = \beta_1,\ c_1 = \beta_2$ and $d_1 = \beta_3$.

- **B**. Expanding the fifth term in $f(x)$ allows for the various powers of x to be grouped together and then recondensed. $a_2,\ b_2,\ c_2$ and d_2 are expressed in terms of the cofficients below.

$$\begin{aligned}
f(x)_2 &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x-\xi)^3 && (1)\\
&= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x-\xi)(x-\xi)(x-\xi) && (2)\\
&= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^2 - 2x\xi + \xi^2)(x-\xi) && (3)\\
&= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^3 - x^2\xi - 2x^2\xi + 2x\xi^2 + \xi^2 x - \xi^3) && (4)\\
&= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^3 - 3x^2\xi + 3x\xi^2 - \xi^3) && (5)\\
&= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^3 - \beta_4 3x^2\xi + \beta_4 3x\xi^2 - \beta_4 \xi^3 && (6)\\
&= (\beta_0 - \beta_4\xi^3) + (\beta_1 x + \beta_4 3x\xi^2) + (\beta_2 x^2 - \beta_4 3x^2\xi) + (\beta_3 x^3 + \beta_4 x^3) && (7)\\
&= (\beta_0 - \beta_4\xi^3) + (\beta_1 + 3\beta_4\xi^2)x + (\beta_2 - 3\beta_4\xi)x^2 + (\beta_3 + \beta_4)x^3 && (8)\\
f(x)_2 &= a_2 + b_2 x + c_2 x^2 + d_2 x^3 \quad where \quad \begin{cases} a_2 = \beta_0 - \beta_4\xi^3 \\ b_2 = \beta_1 + 3\beta_4\xi^2 \\ c_2 = \beta_2 - 3\beta_4\xi \\ d_2 = \beta_3 + \beta_4 \end{cases} && (9)
\end{aligned}$$

- **C**. Showing that $f(x)$ is continuous at ξ is illustrated by showing that $f(\xi)_1 = f(\xi)_2$.

$$
\begin{aligned}
f(\xi)_1 &= a_1 + b_1(\xi) + c_1(\xi)^2 + d_1(\xi)^3 && (10)\\
&= \beta_0 + \beta_1(\xi) + \beta_2(\xi)^2 + \beta_3(\xi)^3 && (11)\\
& && (12)\\
f(\xi)_2 &= a_2 + b_2(\xi) + c_2(\xi)^2 + d_2(\xi)^3 && (13)\\
&= (\beta_0 - \beta_4\xi^3) + (\beta_1 + 3\beta_4\xi^2)(\xi) + (\beta_2 - 3\beta_4\xi)(\xi)^2 + (\beta_3 + \beta_4)(\xi)^3 && (14)\\
&= (\beta_0 - \beta_4\xi^3) + (\beta_1\xi + 3\beta_4\xi^3) + (\beta_2\xi^2 - 3\beta_4\xi^3) + (\beta_3\xi^3 + \beta_4\xi^3) && (15)\\
&= \beta_0 - \beta_4\xi^3 + \beta_1\xi + 3\beta_4\xi^3 + \beta_2\xi^2 - 3\beta_4\xi^3 + \beta_3\xi^3 + \beta_4\xi^3 && (16)\\
&= \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3 + 3\beta_4\xi^3 - 3\beta_4\xi^3 + \beta_4\xi^3 - \beta_4\xi^3 && (17)\\
&= \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3 + (3\beta_4\xi^3 - 3\beta_4\xi^3) + (\beta_4\xi^3 - \beta_4\xi^3) && (18)\\
f(\xi)_2 &= \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3 && (19)
\end{aligned}
$$

$$f(\xi)_2 = \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3 = f(\xi)_1$$