ISLR | Chapter 10 Exercises

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Conceptual

1

NEED TO COME BACK TOO

- **A**. 10.12, illustrated below, is showing the the *within-cluster variation* is equal to twice the squared distance between each data point in cluster k (C_k) and that cluster's centroid, summed across all data points.

$$\frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{P}(x_{i,j}-x_{i',j})^2 = 2\sum_{i\in C_k}\sum_{j=1}^{P}(x_{i,j}-\bar{x}_{k,j})^2 \tag{1}$$

$$\frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{P}(x_{i,j}-x_{i',j})(x_{i,j}-x_{i',j}) = 2\sum_{i\in C_k}\sum_{j=1}^{P}(x_{i,j}-\bar{x}_{k,j})(x_{i,j}-\bar{x}_{k,j}) \tag{2}$$

$$\frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{P}(x_{i,j}^2-2x_{i,j}x_{i',j}+x_{i',j}^2) = 2\sum_{i\in C_k}\sum_{j=1}^{P}(x_{i,j}^2-2\bar{x}_{k,j}x_{i,j}+\bar{x}_{k,j}^2) \tag{3}$$

$$\frac{|C_k|}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{P}(x_{i,j}^2-2x_{i,j}x_{i',j}+x_{i',j}^2) = 2|C_k|\sum_{i\in C_k}\sum_{j=1}^{P}(x_{i,j}^2-2\bar{x}_{k,j}x_{i,j}+\bar{x}_{k,j}^2) \tag{4}$$

$$\sum_{i,i'\in C_k}\sum_{j=1}^{P}(x_{i,j}^2-2x_{i,j}x_{i',j}+x_{i',j}^2) = 2|C_k|\sum_{i\in C_k}\sum_{j=1}^{P}(x_{i,j}^2-2\bar{x}_{k,j}x_{i,j}+\bar{x}_{k,j}^2) \tag{5}$$

$$\sum_{i\in C_k}\sum_{j=1}^{P}x_{i,j}^2 - 2\sum_{i,i'\in C_k}\sum_{j=1}^{P}x_{i,j}x_{i',j} + \sum_{i'\in C_k}\sum_{j=1}^{P}x_{i',j}^2 = 2|C_k|\sum_{i\in C_k}\sum_{j=1}^{P}x_{i,j}^2 - 2|C_k|\sum_{i\in C_k}\sum_{j=1}^{P}2\bar{x}_{k,j}x_{i,j} + 2|C_k|\sum_{i\in C_k}\sum_{j=1}^{P}\bar{x}_{k,j}^2 \tag{6}$$

$$\sum_{i\in C_k}\sum_{j=1}^{P}x_{i,j}^2 - 2\sum_{i,i'\in C_k}\sum_{j=1}^{P}x_{i,j}x_{i',j} + \sum_{i'\in C_k}\sum_{j=1}^{P}x_{i',j}^2 = 2|C_k|\sum_{i\in C_k}\sum_{j=1}^{P}x_{i,j}^2 - 4|C_k|\sum_{i\in C_k}\sum_{j=1}^{P}\bar{x}_{k,j}x_{i,j} + 2|C_k|\sum_{i\in C_k}\sum_{j=1}^{P}\bar{x}_{k,j}^2 \tag{7}$$

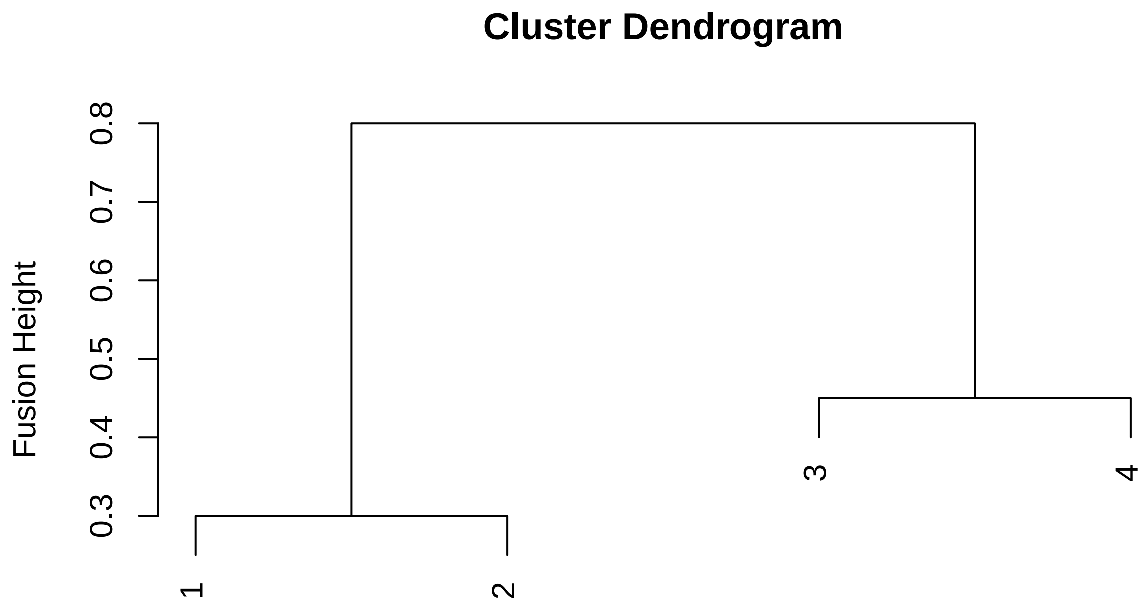
$$\tag{8}$$

2

- **A**.

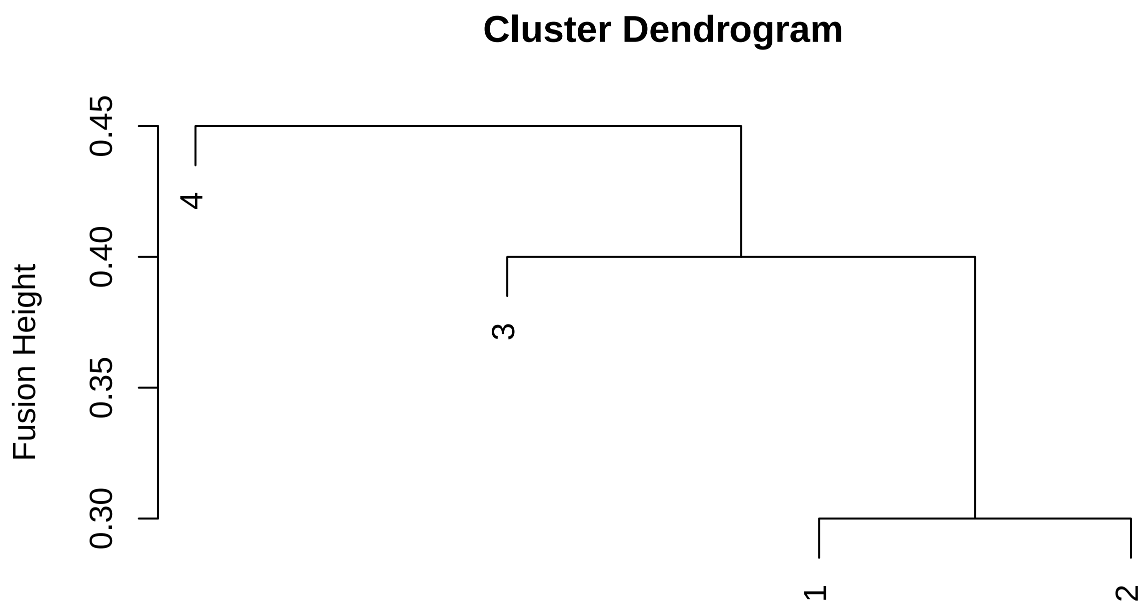
```
x <- matrix(c(0, 0.3, 0.4, 0.7,
              0.3, 0, 0.5, 0.8,
              0.4, 0.5, 0, 0.45,
              0.7, 0.8, 0.45, 0), ncol = 4, nrow = 4)
```

```r
plot(hclust(as.dist(x), method = 'complete'),
     xlab = "",
     sub = "",
     ylab = "Fusion Height")
```



- **B.**

```r
plot(hclust(as.dist(x), method = 'single'),
     xlab = "",
     sub = "",
     ylab = "Fusion Height")
```

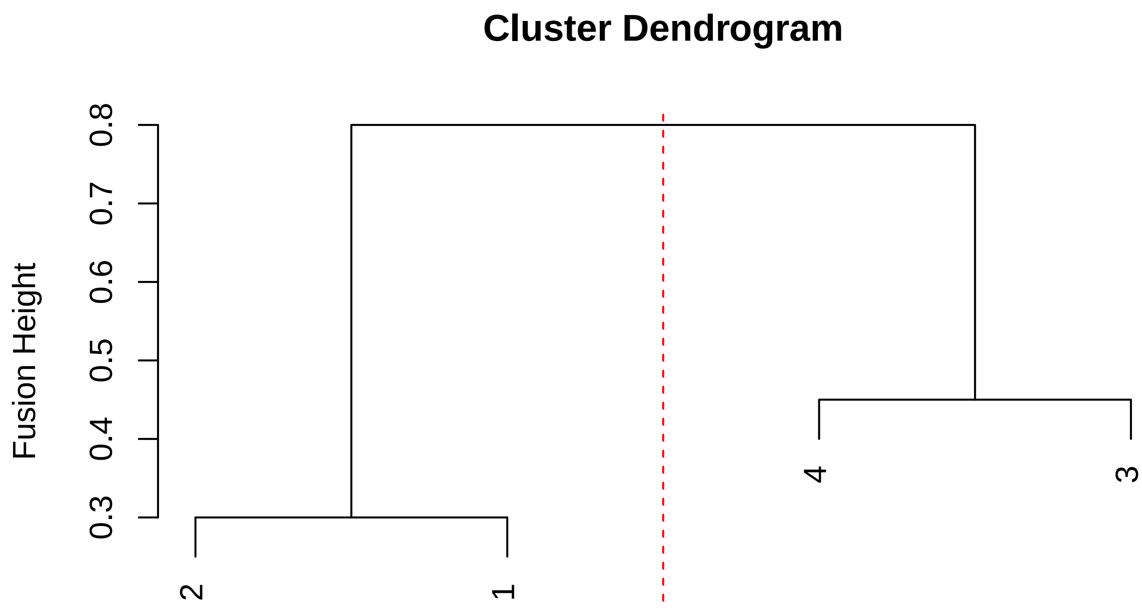


- **C**. Observations 1 & 2 will be in cluster A and observations 3 & 4 will be in cluster B (assuming one cuts the dendrogram at a height greater than 0.45).

- **D**. Although the answer to this question depends on where one cuts the dendrogram, the most likely clusters would contain observations 1 & 2 in cluster A and observations 3 & 4 in cluster B. This would results from a cut at a height greater than 0.3 and less than 0.4, which is the largest vertical distance on the dendrogram. If one were to make a cut between 0.4 and 0.45, then cluster A would contain observations 1, 2 & 3, while cluster B would consist of only observation 4. However, with the distance being greater between cluster's for the first grouping, that would be the more probable grouping.

- **E**. As shown below, one can simply switch the labels of the observations within each cluster to change the dendrogram without changing the meaning of the dendrogram. In addition, one could take the mirror image of the plot displayed along the dotted red line, producing a "new" dendrogram that has the same meaning.

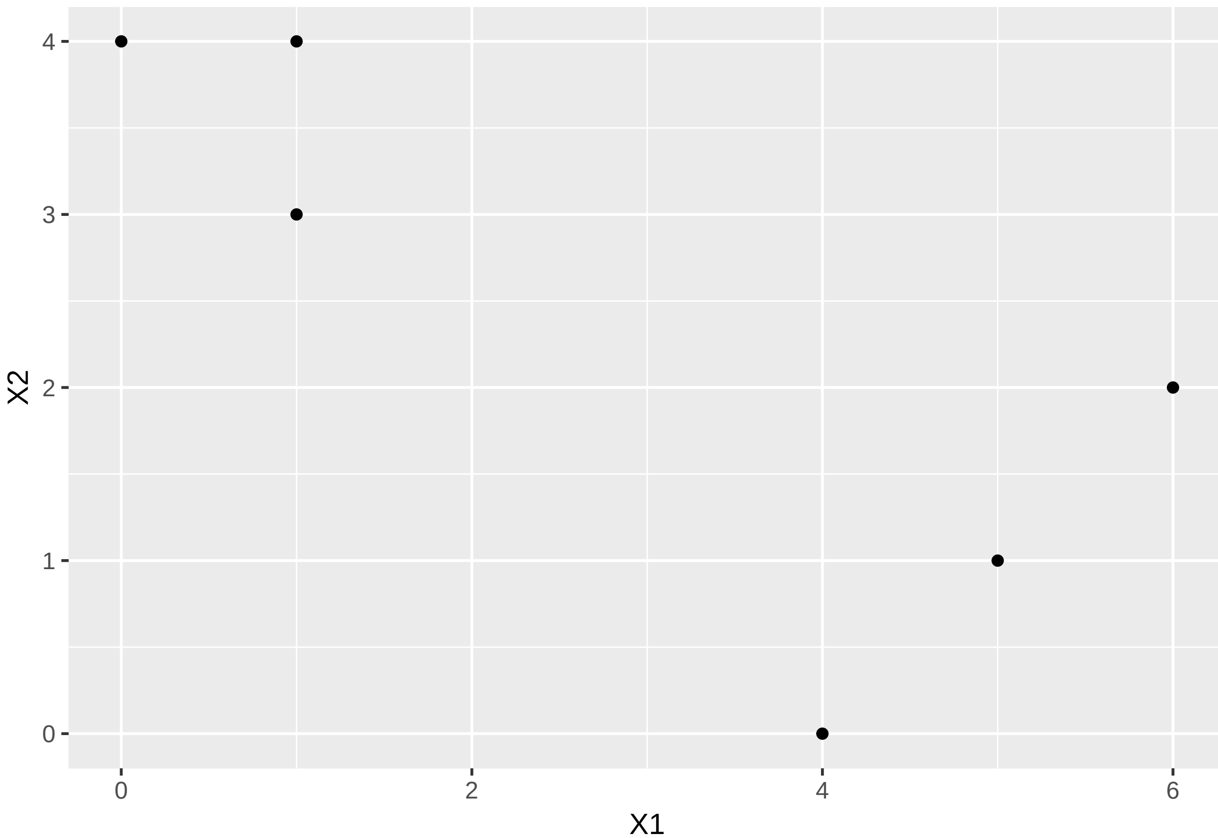
```
plot(hclust(as.dist(x), method = 'complete'),
     xlab = "",
     sub = "",
     ylab = "Fusion Height", labels = c(2,1,4,3))
abline(v = 2.5, col = 'red', lty = 2)
```



3

- **A**.

```
suppressPackageStartupMessages(library(ggplot2))
df <- data.frame(x1 = c(1,1,0,5,6,4),
                 x2 = c(4,3,4,1,2,0))
qplot(df$x1, df$x2, xlab = 'X1', ylab = 'X2')
```



- **B**.

```
suppressPackageStartupMessages(library(knitr))
df$group <- sample(c('A','B'), 6, replace = TRUE)
knitr::kable(df, caption = 'Sample Data with Group Assignments')
```

Table 1: Sample Data with Group Assignments

x1	x2	group
1	4	A
1	3	B
0	4	A
5	1	B
6	2	A
4	0	A

- **C**.

```
group.a <- subset(df, group == 'A')
group.b <- subset(df, group == 'B')
```

```r
# centroid calculation
calculate.centroid <- function(df, variables=c('x1','x2')) {

    coordinates <- NULL

    for (variable in variables) {
        column.mean <- mean(df[, variable])
        coordinates <- c(column.mean, coordinates)
    }

    return(coordinates)
}
group.a.centroid <- calculate.centroid(group.a)
group.b.centroid <- calculate.centroid(group.b)
print(paste("Group A centroid at coordinates",
            round(group.a.centroid[1], 2), 'and', round(group.a.centroid[2], 2)))
```

```
## [1] "Group A centroid at coordinates 2.5 and 2.75"
```

```r
print(paste("Group B centroid at coordinates",
            round(group.b.centroid[1], 2), 'and', round(group.b.centroid[2], 2)))
```

```
## [1] "Group B centroid at coordinates 2 and 3"
```

- **D**.

```r
centroid.matrix <- matrix(c(group.a.centroid, group.b.centroid),
                          nrow = 2,
                          ncol = 2)
rownames(centroid.matrix) <- c('A','B')

reassign.cluster <- function(df,
                             col.idx=c(1,2),
                             cluster.column='group',
                             cluster.labels=c('A','B'),
                             centroids=centroid.matrix) {

    df.matrix <- as.matrix(df[, col.idx])

    updated.labels <- NULL
    for (i in 1:dim(df.matrix)[1]) {

        sqr.manhattan.dist <- (df.matrix[i, ] - centroids)^2
        euclidean.dist <- sqrt(colSums(sqr.manhattan.dist))
        closest.centroid <- which.min(euclidean.dist)
        updated.labels <- c(updated.labels, cluster.labels[closest.centroid])

    }

    return(data.frame(df.matrix, group = updated.labels))

}
```

```r
updated.df <- reassign.cluster(df)
knitr::kable(updated.df, caption = "Sample Data After One Reassignment Iteration")
```

Table 2: Sample Data After One Reassignment Iteration

x1	x2	group
1	4	B
1	3	B
0	4	B
5	1	A
6	2	A
4	0	A

- **E**.

Resources

Dendrograms in R