

ISLR | Chapter 7 Exercises

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Conceptual

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- **A.** The cubic piecewise polynomial:

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x - \xi)_+^3 \quad \text{where} \quad (x - \xi)_+^3 = \begin{cases} 0, & x \leq \xi \\ (x - \xi)^3, & \text{otherwise} \end{cases}$$

...can be broken up and rewritten to be:

$$f(x) = \begin{cases} f(x)_1 = a_1 + b_1 x + c_1 x^2 + d_1 x^3, & x \leq \xi \\ f(x)_2 = a_2 + b_2 x + c_2 x^2 + d_2 x^3, & \text{otherwise} \end{cases}$$

In $f(x)_1$, since $(x - \xi)_+^3 = 0$ (because $x \leq \xi$), the fifth term (of $f(x)$) zeroes out and the coefficients can be expressed as $a_1 = \beta_0$, $b_1 = \beta_1$, $c_1 = \beta_2$ and $d_1 = \beta_3$.

- **B.** Expanding the fifth term in $f(x)$ allows for the various powers of x to be grouped together and then recondensed. a_2 , b_2 , c_2 and d_2 are expressed in terms of the coefficients below.

$$f(x)_2 = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x - \xi)^3 \quad (1)$$

$$= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x - \xi)(x - \xi)(x - \xi) \quad (2)$$

$$= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^2 - 2x\xi + \xi^2)(x - \xi) \quad (3)$$

$$= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^3 - x^2\xi - 2x^2\xi + 2x\xi^2 + \xi^2 x - \xi^3) \quad (4)$$

$$= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^3 - 3x^2\xi + 3x\xi^2 - \xi^3) \quad (5)$$

$$= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^3 - \beta_4 3x^2\xi + \beta_4 3x\xi^2 - \beta_4 \xi^3 \quad (6)$$

$$= (\beta_0 - \beta_4 \xi^3) + (\beta_1 x + \beta_4 3x\xi^2) + (\beta_2 x^2 - \beta_4 3x^2\xi) + (\beta_3 x^3 + \beta_4 x^3) \quad (7)$$

$$= (\beta_0 - \beta_4 \xi^3) + (\beta_1 + 3\beta_4 \xi^2)x + (\beta_2 - 3\beta_4 \xi)x^2 + (\beta_3 + \beta_4)x^3 \quad (8)$$

$$f(x)_2 = a_2 + b_2 x + c_2 x^2 + d_2 x^3 \quad \text{where} \quad \begin{cases} a_2 = \beta_0 - \beta_4 \xi^3 \\ b_2 = \beta_1 + 3\beta_4 \xi^2 \\ c_2 = \beta_2 - 3\beta_4 \xi \\ d_2 = \beta_3 + \beta_4 \end{cases} \quad (9)$$