DSA/ISE 5113 Advanced Analytics and Metaheuristics
Homework #4

Instructor: Charles Nicholson

Due: See course website for due date

Requirement details

1. Homeworks should be submitted in a clean, clear, concise electronic format. You must show your logic, work, and/or code where appropriate in a PDF file – the PDF is your primary submission. Code files (e.g., AMPL model and/or data files) are also part of your required submission (turn in as separate files).
2. For mathematical programming problems, ensure you clearly define the following elements: (i) **necessary assumptions**, (ii) **decision variables**, (iii) **objective and objective function**, and (iv) **constraints**. Points may be deducted if proper definitions/documentation is missing.
3. Submit one PDF file and all code files. The code files may be zipped, however, do not zip the PDF file. If you have a multi-part problem that requires different code, you may want to provide separate model files for each part.

Question 1: Titan Enterprises Case Study: Part II (100 points)

Mr. Jonathon Lee, President of Titan Enterprises, was quite pleased with the recent analysis of their investment opportunities. He was excited about the mathematical programming model developed (see Figure 1) and wanted to know if it could be extended to address a new concern brought forward by the Investment Review Committee. The model was originally created to optimize investment return over a 4 year investment time horizon based on an example case provided by Mr. Shirazi. The decision variables $x_A, x_B, \ldots x_E$ denote the dollars to be invested in projects A thru E and the variables x_{b1}, x_{b2}, and x_{b3} denote the annual investment allocations to the bank for years 1, 2, and 3, respectively.

$$
\begin{aligned}
\text{maximize } & x_B + 1.4x_E + 1.75x_D + 1.06x_{b3} \\
\text{s.t.} & \\
& x_A + x_C + x_D + x_{b1} = 1000000 \\
& 0.3x_A + 1.1x_C + 1.06x_{b1} = x_B + x_{b2} \\
& x_A + 0.3x_B + 1.06x_{b2} = x_E + x_{b3}
\end{aligned}
$$

Figure 1: Mathematical program for Shirazi's investment scenario

The members of the committee are Mr. Will Zhang, the Treasurer; Ms. Brandi Phillips, the Controller; and Mr. Amin Shirazi, Operations director. Mr. Zhang had some concerns regarding implicit assumptions in the modeling approach, namely, that the model does not account for risk in any of the investments.

Ms. Phillips suggested the use of simple risk scores to be assigned to each project. For example, a risk score of 0.10 for project A would imply that 10% of the invested monies in A are at risk. She then suggested that the model could be tuned to minimize risk instead of maximizing the return.

Mr. Lee noted that a minimum risk option would likely invest all money in the bank and not consider other Project options. He further noted that such a strategy is not an appealing option since the stockholders would not like such a risk averse strategy.

Based on these suggestions, Mr. Shirazi created a set of risk values associated with dollars invested in each of the projects. He assumed that there is no risk associated with investments in the bank. The risk coefficients per dollar invested are provided in Table 1. He further suggested the use of multiple objective optimization to create a set of Pareto optimal investment strategies. The other committee members agreed to consider the approach, but they noted that they would likely need help evaluating the model outcomes.

Table 1: Project risk coefficients

Project	**Risk Value**
A	0.10
B	0.12
C	0.05
D	0.20
E	0.05

(a) Formulate and solve Titan's investment problem as an LP to accomplish the following:
   i. Maximize return only (should be equivalent to the original correct solution from HW #2)
   ii. Minimize risk only (do not consider return, only risk)
   iii. Solve the problem using a scalarized objective function to combine the objectives where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are the weights associated with returns and risks, respectively, and $\lambda_1 + \lambda_2 = 1$. Use the values, $\lambda_1 = 0, 0.25, 0.5, 0.75, 1$.
   iv. Use the ϵ-constraint method to solve the investment problem for 20 values of ϵ. The ϵ values that you choose should represent the entire spectrum of risk and return. That is, one solution in the set should correspond to the maximum returns achieved in (i) and one solution should correspond to the minimum risk achieved in (ii).

(b) Using the outcomes from either the scalarized or ϵ-constraint method, graph the Pareto optimal results on objective space. Clearly label the axes, denote the feasible region and infeasible regions, and provide a brief, but clear, interpretation of the results to help the committee understand the graph, the tradeoffs involved, and how to make decisions regarding investments.

(c) Using the ϵ-constraint results, analyze how the investment portfolio changes across the spectrum of risk-tolerance solutions. Include a clear interpretation to help the decision-makers understand the set of Pareto optimal solutions (visualizations will help!)